# Balanced Growth and Empirical Proxies of the Consumption-Wealth Ratio

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**Abstract**

Empirical proxies of the aggregate consumption-wealth ratio in terms of a cointegrating relationship between consumption ($c$), asset wealth ($a$) and labour income ($y$), commonly referred to as *cay*-residuals, play an important role in recent empirical research in macroeconomics and finance. This paper shows that the balanced-growth assumption made in deriving *cay* implies a second cointegrating relationship between the three variables; the three great ratios $c - a$, $c - y$ and $a - y$ should all be individually stationary In U.S. data I find evidence for this second cointegrating relationship once I control for deterministic trends and a structural break. The fact that *cay* is a linear combination of two stationary great ratios has a number of important implications. First, without additional identifying restrictions, the residual from a cointegrating regression can no longer be interpreted as an approximation of the aggregate consumption-wealth ratio. I discuss an identifying assumption that may still allow to do so. Secondly, predictive regressions of asset prices on a combination of two stationary great ratios, must do at least as well as regressions on *cay* alone. Still, *cay* proves remarkably robust as an indicator of aggregate asset price cycles. The findings here also inform a recent debate about the role of look-ahead bias in *cay*: in order to identify transitory components in asset prices, households do not need to identify the parameters of the *cay*-relation at all.

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# 1 Introduction

In two very influential recent papers, Lettau and Ludvigson (2001, 2004) have used an empirical characterization of the consumption-wealth ratio in terms of a cointegrating relationship between consumption, asset wealth and labour income known as the *cay*-residual. One key assumption made in deriving this cointegrating relationship from the budget constraint of the average household is that the economy follows a balanced growth path in which the capital and labour shares are constant in the long-run. I show that this implies that the consumption-asset ($ca$), the consumption income ($cy$)and the income asset ($ya$) ratio are all individually stationary. Hence, there should be two linearly independent long-run (cointegrating) relationships between the three variables – consumption, income and asset wealth should share a single stochastic trend.

I re-examine the U.S. data set used by Lettau and Ludvigson (2004) in search of evidence for this second cointegrating relationship dimensions: first, I recognize that cointegration tests could have very low power in samples of the size considered here. Once I override the results of these tests, I find indeed quite consistent evidence that supports the notion that there are two stationary linear combinations between the three variables. Secondly, I allow for possibility of deterministic trend terms in *cay* as well as in the three great ratios. Third, I re-examine evidence – suggested in some recent papers – about a structural break in the joint dynamics of consumption, income and asset wealth at around the mid-point of the sample, i.e. around 1978. This structural break affects the trend growth rates of consumption and labour income and also introduces a break in the long-run relationships . Once this structural break is explicitly modelled, extant tests indicate the two cointegrating relationships predicted by the theoretical framework.

One the one hand, these results constitute an important empirical corroboration of the theoretical approach underlying the two Lettau-Ludvigson studies – if the second cointegrating relationship can indeed be found in the data, then consumption, asset wealth and income follow a single stochastic trend, as is implied by the balanced-growth assumption. First, this is a necessary condition for the construction of an empirical proxy of the consumption wealth ratio. Secondly, it also substantially facilitates the interpretation of the joint long-run dynamics of the three variables in the light of standard macroeconomic theory.

One the other hand, the potential presence of a second cointegrating relationship also complicates the interpretation of estimated *cay* residuals along at least two dimensions: if the cointegrating space is two-dimensional, then the *cay*-proxy of the consumption-wealth ratio will not be econometrically identified. Without further identifying restrictions, the researcher will therefore not be able to estimate the share of physical assets and human capital in total wealth. I argue that such an identifying restriction is to assume that economic agents chose a minimum variance portfolio of physical assets and human capital. I construct such a portfolio and show that under these conditions – at least in the U.S. data set used here – *cay* proxies the implied consumption-wealth ratio remarkably well.

Secondly, the balanced-growth assumption could potentially also have implications for the predictability of stock returns and for the size of transitory components in asset prices. Since *cay* is just a particular linear combination in a two-dimensional cointegrating space, any combination of two of the three great ratios *ca*, *cy* and *ya* must do at least as well as *cay* in predicting stock returns. Interestingly enough, however, my results suggests that alternative linear combinations of *cy* and *ya* do not significantly outperform *cay* in predicting stock markets. Hence, even though the second cointegrating relationship would seem to draw into question the validity of *cay* as a proxy of the consumption-wealth ratio and as predictor of asset prices, the results here suggest that *cay* – though generally estimated from an econometric setup that is misspecified under the maintained null – proves remarkably robust in both respects.

Finally, the considerations here also have immediate implications for the recent debate about look-ahead-bias in the *cay*-residual. If *cay* is just a linear combination of two observable stationary great ratios, say *cy* and *ay*, then in order to identify transitory components in asset prices, a forecaster does not have to estimate the parameters of the *cay*-relation first. She can use *cy* and *ay* directly and does therefore not need long spans of data to identify the parameters of the *cay*-relationship.

# 2 The issue

## 2.1 The framework

The framework used by L&L builds on Campbell and Mankiw (1989) and assumes that the aggregate consumption-wealth ratio is a stationary variable. Starting from the aggregate budget constraint

$$W_{t+1} = (1 + r_{t+1})(W_t - C_t) \tag{1}$$

where $W_t$ is aggregate wealth, $r_t$ its net return and $C_t$ consumption, C&M use this assumption to log-linearize the intertemporal budget constraint around the long-run mean of $c - w = \ln(C_t/W_t)$, so that

$$c_t - w_t = \mathbf{E}_t \sum_{j=1}^{\infty} \rho^j \left(r_{t+j} - \Delta c_{t+j}\right) \tag{2}$$

where $\rho$ is a constant smaller than one. According to (2), the consumption wealth ratio predicts returns to wealth or future declines in consumption. The assumption made in deriving (2) as well as the fact that consumption growth and returns can usually be characterized as non-integrated ($I(0)$) variables suggests that $c$ and $w$ should cointegrate. Aggregate wealth is, however, not directly observable, since it is composed of both physical (asset) wealth as well as human capital:

$$W_t = A_t + H_t \tag{3}$$

where $A_t$ is asset wealth and $H_t$ is human capital. To proxy for $W_t$ in terms of observable variables, L&L as well as virtually the entire literature inspired by their papers assume that the shares of physical (asset) wealth and the share of human wealth in total wealth are if not constant so to the least stationary, so that (3) can be log-linearized to obtain

$$w_t = \gamma a_t + (1 - \gamma) h_t$$

where $\gamma = E(\exp(a_t - w_t))$ is the long-run mean of the share of asset wealth in total wealth. In a second step, L&L then assume that the stochastic trend in human capital can be captured by its dividend – labour income. Denoting log-labour income with $y_t$ and assuming that $z_t = y_t - h_t$ is $I(0)$, we get

$$w_t = \gamma a_t + (1 - \gamma) y_t + (1 - \gamma) z_t$$

Plugging into (2), one obtains

$$c - \gamma a_t - (1 - \gamma) y_t = \mathbf{E}_t \sum_{j=1}^{\infty} \rho^j \left[r_{t+j} - \Delta c_{t+j}\right] + (1 - \gamma)\, z_t. \tag{4}$$

Since the RHS of this equation just differs from the RHS of (2) only by the $I(0)$ process $z_t$, $c - \gamma a_t - (1 - \gamma) y_t$ should be $I(0)$ as well. Hence, the logs of consumption, asset wealth and labour income should cointegrate with cointegrating vector $[1, -\gamma, -(1 - \gamma)]'$. This is the *cay*-relationship that is the focus of L&L. To the extent that $z_t$ – the transitory part of labour income – is small, *cay* should therefore capture the variation in the consumption wealth ratio $c - w$. Lettau and Ludvigson show that this is indeed the case: in U.S. data, labour income is hardly predictable. Since $c$ is also behaves almost like a random walk, *cay* mainly predicts changes in asset wealth.

## 2.2 A second cointegrating relationship

One key assumption made in obtaining the prediction that the *cay*-residual is a cointegrating relationship and that it proxies for the aggregate consumption wealth ratio is that the shares of asset wealth and human capital in total wealth , $\gamma$ and $(1-\gamma)$, are constant in the long run. This is tantamount to saying that the log of $A/W$, given by $a_t - w_t$ must be an $I(0)$ process. The same must be true for $h - w$ and – since $z_t = y_t - h_t$ is assumed stationary as well – also for $y - w$. Hence, all three variables $c$, $a$ and $y$ will cointegrate pairwise with $w_t$ :

$$c_t - w_t \sim I(0) \tag{5a}$$
$$a_t - w_t \sim I(0) \tag{5b}$$
$$y_t - w_t \sim I(0) \tag{5c}$$

The first is just a restatement of (2) above. The second and third follow from the fact that the portfolio shares of human and physical capital in total wealth are assumed constant in the long-run. Clearly, any linear combination of $c - w$, $a - w$ and $y - w$ must therefore also be stationary. Since $w_t$ is unobservable, it its sufficient to concentrate on those three linear combinations of (5) that eliminate $w_t$:

$$c - a \sim I(0) \tag{6a}$$
$$c - y \sim I(0) \tag{6b}$$
$$y - a \sim I(0) \tag{6c}$$

Only two of these linear combinations are, however, linearly independent. The first and the second of these great ratios are the (log) consumption-asset and the consumption-income ratios to which, in analogy to L&L, we refer as *ca* and *cy* respectively. The third one is the income-asset ratio (*ya*). Clearly, any linear combination of *ca*, *cy*, *ya* will also constitute a valid representation of one of the two cointegrating relationships, so that in particular *cay* can be written as

$$cay = \gamma ca + (1-\gamma)cy = cy - \gamma ay = ca - (1-\gamma)ya$$

Hence, the assumptions made in obtaining the representation (4) of *cay* actually imply the presence of two cointegrating relationships between consumption, asset wealth and income. This point has so far been overlooked in the literature. It poses a big challenge to the validity of the entire framework presented in the previous subsection, since neither L&L nor any of the studies applying this framework to other countries and data sets have actually detected this second cointegrating relationship in the data. Therefore, the second cointegrating relationship raises two questions: first, why has the second cointegrating relationship been so elusive in the data? And secondly, how does its presence affect the interpretation of *cay* as a proxy of the consumption-wealth ratio and as predictor of asset price changes?

# 3 Another look at the data

## 3.1 Stationarity of the great ratios?

The data set used here is the one used in Lettau's and Ludvigson's (2004) paper and ranges from 1952Q4 to 2003Q1.

Figure (1) presents the graphs of the three potentially stationary relations, $ca = c - a$, $cy = c - y$ and $ya = y - a$. I formally examine the stationarity of $ca$, $cy$ and $ya$ in table 1. Based on Johansen's (1991) test the null of no cointegration cannot be rejected in any of the three pairs of variables. Once one cointegrating relationship is imposed, however, the cointegrating vector estimated using Johansen's procedure come relatively close to their theoretical value $\begin{bmatrix} 1 & -1 \end{bmatrix}'$ and for the system consisting of $c$ and $y$ and the pair $y$,$a$ we cannot reject the hypothesis that the cointegrating vector is actually $\begin{bmatrix} 1 & -1 \end{bmatrix}$.

Ocular inspection of figure (1) suggests that at least two of the three great ratios are individually trending; $cy$ seems to trend more strongly in the first half of the sample period, whereas the downward drift in $ca$ is more evenly spread. While it is beyond the scope of this paper to identify the economic forces that may induce this gradual drift in the three great ratios, it is important to note that a trend in either of the great ratios is not necessarily at odds with the intertemporal budget constraint on which the log-linearization of the consumption-wealth ratio is based. As recently emphasized by Hahn and Lee (2006), the intertemporal budget constraint (1) does have to hold for each individual household, but if households are heterogeneous and if the structure of household heterogeneity drifts slowly over time, then this may induce a trend in the aggregate consumption-wealth ratio. Indeed, Hahn and Lee (2006) find that a deterministic trend cannot actually be excluded from the trivariate cointegrating relationship between $c$, $a$ and $y$ and they conclude that the coefficients of the *cay*-relationship as estimated by Lettau and Ludvigson are likely to be biased by the omission of this linear trend.

Clearly, if the *cay*-relationship is trending, then it is certainly not implausible that the great ratios of which *cay* is a linear combination are individually trending. Table 2, panel I, reports regressions of the three great ratios on a deterministic trend and a constant. In all three cases this trend is found to be highly significant. But even the inclusion of deterministic trend does not generally allow me to reject the non-stationarity of the great ratios: as is apparent from the last lines of table (2), standard unit-root tests on the ensuing regression residual do not generally reject the null and if so, they are just marginally significant. Based on Johansen's test for cointegration, I cannot reject non-stationarity in a single one of the three cases.

What I wish to argue here is that the trend in the three great ratios may have been subject to at least one major break over the sample period and that this break may have contributed to our failure to detect the second cointegrating relationship in the data. In fact, the recent literature on the consumption-wealth ratio reports considerable evidence for a structural break in the *cay* relationship. Hahn and Lee (2001, 2006) show that the coefficients of the *cay*-relation are unstable between the first and the second half of their sample period. Brennan and Xi (2005) report that the forecasting power of *cay* for stock markets is considerably weaker in the second half of the sample period. The ocular evidence from figure (1) supports the view that the first half and the second half of the sample period are different: in particular the drift in *cy* seems much steeper during the first half. Formal stability tests for the bivariate VECMs also support the notion that there is a structural break in the second half of the 1970s. I therefore allow the trend in the three great ratios to break in 1978Q1. Panel II of Table (2) reports regressions of *cy*, *ca* and *ya* on a deterministic trend and a trend break variable that takes the values $t - t_0$ for $t < t_0$ and $t = 0$ for $t > t_0$ where $t_0$ is the date of the break which – following Hahn and Lee – I locate in 1977Q4. As is apparent, both trend terms are found to be significant in all three great ratios. Now, I clearly reject the unit root when I run ADF-tests on the regression residuals and the Johansen tests are significant at least at the 90 percent level.[1] This suggests that all three great ratios can indeed be characterized as trend stationary, once a break in the deterministic trend is explicitly modelled. Figure (2, a-c) conveys an optical impression of the great ratios, once with only a deterministic trend , once with both the trend cum break removed. As is apparent, the inclusion of a trend break makes a major difference, in particular for *cy*: while *cy* would only cross its mean twice during the sample period if only a linear trend is removed, it looks much more clearly mean reverting if the trend break is explicitly modelled. A similar, though somewhat less pronounced pattern emerges for the other

[1] Strictly speaking the critical values for the Johansen test are not valid if a trend break is included in the cointegrating space. In the trivariate system studied below, I also report tests based on simulated critical vaues, with very similar conclusions.

two great ratios. This is only additional informal evidence but it seems to speak very strongly; a merely trend stationary process would seem unlikely to display the pattern observed here.

Allowing for the presence of deterministic trends in the great ratios and in particular, for a break in these trends seems to go a long way in explaining why extant tests have failed to detect the second cointegrating relationship that is implied by the balanced growth assumption on which the entire L&L approach is based. As discussed here and elsewhere, such trends are not *a priori* incompatible with the intertemporal budget constraint. It should therefore now appear natural to impose the second cointegrating relationship in a trivariate characterization of the dynamics of the three variables in a vector error correction framework. I explore the implications of this approach next.

## 3.2 A trivariate VECM specification

The deterministic terms in the great ratios imply a VECM-specification of the form

$$\boldsymbol{\Gamma}(\mathbf{L})\boldsymbol{\Delta}\mathbf{x}_t = \boldsymbol{\alpha} \begin{bmatrix} \boldsymbol{\beta}' & \boldsymbol{\delta}_{trend} & \boldsymbol{\delta}_{break} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ t \\ \min(0, t - t_0) \end{bmatrix} + \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 stepdum + \boldsymbol{\varepsilon}_t \quad (7)$$

where $\mathbf{x}_t = \begin{bmatrix} c_t & a_t & y_t \end{bmatrix}$, $\boldsymbol{\Gamma}(\mathbf{L})$ is a $3 \times 3$ matrix polynomial in the lag operator, $\boldsymbol{\beta}$ is the $3 \times 2$-matrix of cointegrating vectors and $\boldsymbol{\alpha}$ the $3 \times 2$ matrix of error correction adjustment loadings. The vectors $\delta_{trend}$ and $\delta_{break}$ give the coefficients on the two deterministic trend terms. The trend shift in the great ratios modelled by the deterministic terms must be caused by a shift in the trend growth rate of at least one of the three endogenous variables. Therefore, an additional step dummy has to be included in the short-run dynamics which is loaded with the vector of coefficients $\boldsymbol{\mu}_1$. Finally, $\boldsymbol{\mu}_0$ is a vector of intercept terms.

It is well known that the inclusion of trend breaks and similar deterministic terms invalidates the standard critical values used in cointegration testing. I therefore simulate critical values for the particular configuration of deterministic terms considered here using the program Disco available from Bent Nielsen's web page. Table (3), panel I, provides the cointegration tests. As is apparent, these strongly signal the presence of two cointegrating relationships, as implied by the theoretical framework. Once again, the importance of the trend break is highlighted by these tests. The table also reports the

Johansen tests for the case when only a deterministic but non-breaking trend is included. In this case, very much as in the Lettau and Ludvigson-paper, I can only detect a single cointegrating relation. This suggests that the presence of structural breaks could indeed be responsible for the fact that extant tests fail to signal the presence of a second cointegrating relationship in the *cay* framework.

Panel II of Table (3) also gives the estimated cointegrating vectors. These, indeed, come close to the ones that we have estimated from the bivariate relationships. One is virtually the consumption income ratio, the other one can be interpreted as proxying the portfolio share, $a - y$. Though the point estimates of the coefficients on $y$ seem to deviate somewhat from their theoretical value unity, I can accept the hypothesis that $\beta_y = 1$ at high probability levels. The table therefore also gives the cointegrating vectors estimated under this restriction. In this case, the coefficients on the two trend terms turn out to be virtually identical to the coefficients obtained from the regression of $cy$ and $-ya$ respectively on the deterministic trends.[2] This further supports the notion that it is the great ratios that form the stationary directions in the VECM, even though their stochastic stationarity may have been affected by gradual shifts and exogenous breaks.

As memorandum items for my discussion below, the second column of the same table also gives the results obtained from the model that only includes the non-breaking. In accordance with the above tests results and by way of comparison with earlier studies, this model is estimated with only one cointegrating relation. If the trend term is left unrestricted, the cointegrating vector aligns very well with the results reported in Hahn and Lee, I estimate $[1, -0.16, -0.58]$ and I find the deterministic trend coefficient highly significant. The lower part of the column also reports the cointegrating vector obtained when the trend coefficient is restricted to zero; this yields $\boldsymbol{\beta}' = \begin{bmatrix} 1 & -0.26 & -0.63 \end{bmatrix}$.

I now proceed to estimating the dynamic adjustment parameters in the VECM by imposing detrended and break-adjusted great ratios $cy$ and $-ya$ as the stationary directions in the system, so that

$$\begin{bmatrix} \widetilde{ca} \\ \widetilde{ay} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\beta}' & \boldsymbol{\delta}_{trend} & \boldsymbol{\delta}_{break} \end{bmatrix}] \begin{bmatrix} \mathbf{x}_{t-1} \\ t \\ \min(0, t-t_0) \end{bmatrix} = \begin{bmatrix} c - a - 0.00t - 0.002tb_t \\ a - y - 0.002t - 0.005tb_t \end{bmatrix}$$

where the tilde denotes the purely stochastic component of the respective great ratio and $tb_t = \min(0, t - t_0)$ is the trend break variable.

[2]Note also that restricting the trend on the system without strcutural break to zero generates the very cointegrating vector estimated by Lettau and Ludvigson, the very point highlighted by Hahn and Lee (2006).

Table (4) presents the coefficient estimates of the VECM, $\mathbf{\Gamma(L)}$, and in particular of the two vectors of adjustment loadings $\boldsymbol{\alpha}$ in (7) above. A first key feature of the results is that the adjustment coefficients in the asset wealth equation is large and significant on both cointegrating relationships. This is in line with the findings reported in Lettau and Ludvigson who also find a big role for asset wealth in the error correction dynamics of their system. Note also that, quite in line with most economic theories, consumption does not seem to react significantly to past cointegration errors. But unlike in the L&L model, here I also find the coefficient on $cy$ in the income equation to be highly significant.[3] Furthermore, it is also worth noting that the step dummy is highly significant in the labour income equation, reflecting the significance of the structural break.

I now turn to exploring the implications of the second cointegrating relationship for the dynamics of the three variables.

**a) The size of transitory components:** I start by examining how the size and variability of the transitory component of the three variables is affected. first, I identify permanent and transitory shocks in the VECM and conduct variance decompositions. Secondly, I also perform a decomposition of consumption, income and asset wealth into a trend and a cycle component.

I identify transitory shocks following the (equivalent) procedures outlined in Johansen (1995) and Gonzalo and Ng (2001). This identification starts from the insight that the error correction term in the VECM can be annihilated by premultipliyng (7) with the orthogonal complement of the matrix of adjustment loadings $\boldsymbol{\alpha}$, that I denote with $\boldsymbol{\alpha}'_{\perp}$. Hence, permanent shocks must be identified as

$$\boldsymbol{\pi} = \boldsymbol{\alpha}'_{\perp}\boldsymbol{\varepsilon}_t$$

Then, imposing that the vector of transitory shocks $\tau$, be orthogonal to $\pi$, one can obtain the following representation of $\tau$ as

$$\boldsymbol{\tau} = \boldsymbol{\alpha}'\boldsymbol{\Omega}^{-1}\boldsymbol{\varepsilon}_t$$

where $\Omega$ is the variance-covariance matrix of the residuals $\boldsymbol{\varepsilon}_t$.

As shown e.g. in Becker and Hoffmann (2006), this identification is sufficient to conduct variance decompositions. Note that it will not be sufficient to uncover impulse responses to all three shocks. For example, in the model studied here, there is one permanent and two transitory shocks and any nonsingular transformation $\mathbf{S}\boldsymbol{\tau}$ of $\boldsymbol{\tau}$ will also qualify as a vector of transitory

[3] Even though our income concept comprises only labour income, this finding seems reminiscent of Cochrane's observation that the consumption-GNP ratio mainly predicts fluctuations.in aggregate income.

shocks that is orthogonal to $\boldsymbol{\pi}$. Hence, in the present model, impulse responses to the permanent shock are readily obtained – I return to this issue below – but identification of transitory shocks would require an additional identifying assumption.

Table (4) presents the contribution of permanent and transitory shocks to the variability of the three variables. The variance decompositions suggest a considerable role of transitory shocks for the dynamics of income. Conversely, the role of transitory shocks for asset wealth appears much more subdued than it is in Lettau and Ludvigson (2004) who find that the only of the three variables with a sizeable transitory component is asset wealth, while both labour income and consumption are almost random walks. The results reported here seem to line up much more with Hahn and Lee's (2006) who suggest that once deterministic components in the *cay* relationship are explicitly modelled, the predictive power of *cay* for asset prices and asset wealth is substantially reduced.

As noted by Gonzalo and Ng, however, variance decompositions can be very sensitive to small changes in the adjustment coefficients $\boldsymbol{\alpha}$ and also to the short-run dynamics capture by $\boldsymbol{\Gamma}(\mathbf{L})$. As an additional exercise, I therefore set all insignificant coefficients in either $\boldsymbol{\Gamma}(\mathbf{L})$ or $\boldsymbol{\alpha}$ to zero. Panel II reports the variance decompositions for this case: now the transitory component in asset wealth appears more important, mainly at the expense of the transitory component in consumption. However, in stark contrast to Lettau and Ludvigson, transitory shocks continue to drive the dynamics of income at business cycle frequencies. I draw two conclusions from these results. first, modelling the second cointegrating relationship implied by the *cay*-approach generates more sizeable components in labour income. Secondly, the size of the transitory components in asset wealth and consumption appear sensitive to the particular parameter restrictions imposed on the model.

While the contribution of transitory shocks is one way to assess the importance of temporary components in the three variables, it is important to note that permanent shocks may also help explain the cycles in the three variables if adjustment to permanent shocks is not immediate. I therefore further investigate the size of the transitory components by conducting a permanent-transitory (P-T) decomposition of the cointegrated system along the lines of Gonzalo and Granger (1995) who suggest to decompose $\mathbf{x}_t$ as

$$\begin{aligned} \mathbf{x}_t &= \left[\boldsymbol{\beta}_\perp(\boldsymbol{\alpha}_\perp\boldsymbol{\beta}'_\perp)^{-1}\boldsymbol{\alpha}_\perp + \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\right]\mathbf{x}_t \\ &= \mathbf{x}_t^P + \mathbf{x}_t^T \end{aligned} \tag{8}$$

where '$\perp$ ' again denotes the orthogonal complement of a matrix and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the adjustment loadings and cointegrating relations from (7) above.

Two points are worth noting from (8): in systems with more than one cointegrating relationship, the transitory component of the three variables will no longer be a scalar multiple of the cointegrating relationship but rather a particular linear combination of all the cointegrating relations in the system. This also implies that the transitory components of the three variables here are not necessarily perfectly correlated as they would be in Lettau's and Ludvigson's VECM.

Figure (3) plots the transitory components obtained from the above decomposition. As is clearly apparent, the presence of a second cointegrating relationship generates substantial transitory components not only in asset wealth but also in labour income and – to a lesser extent in consumption. Still, the transitory component in asset wealth appears to be the most volatile and most sizeable in terms of its average absolute deviation from the mean.

In fact, the transitory components of asset prices is almost unaffected by the modelling of the second cointegrating relationship and the structural breaks. For comparison, figure (4) plots the the Lettau-Ludvigson transitory component obtained from a model with one cointegrating relationship and no trends in the cointegrating space. It also reproduces the transitory component of asset prices from figure (3). The two transitory components virtually have the same size and their correlation almost reaches 0.9. Note that the L&L transitory component is nothing else than a negative multiple of *cay*. Hence, the interpretation of *cay* as a transitory component of asset wealth is not affected by the second cointegrating relationship nor by the drift terms or breaks! Only the size of transitory components in consumption and in particular in income is!

Another feature worth noting is that the transitory components of consumption and income in figure 3 are almost perfectly correlated. One way to interpret the close comovement between consumption and labour income is as evidence for the presence of credit market constraints or rule-of-thumb consumers along the lines of Campbell and Mankiw (1989). Regressing the unrestricted transitory component in consumption on that of income yields a coefficient of 0.38 and an $R^2$ of virtually unity. In the metric of Mankiw and Campbell this suggests that around 40 percent of labour income accrues to rule-of-thumb or credit-constrained consumers.

The distinction between rule-of-thumb consumers and forward looking consumers may be useful in interpreting the time trend in the great ratios. As suggested by Hahn and Lee, some household heterogeneity may be required to reconcile the trends in the *cay*-relationship with the aggregate budget constraint (1).

**b) A single stochastic trend** The second cointegrating relationship substantially facilitates the interpretation of the joint long-run dynamics of consumption, income and asset wealth: consumption, income and asset wealth must share a single common stochastic trend. It is therefore straightforward to study the response of the three variables to the common permanent shock $\pi_t$ along the lines of King et al. (1991). Figure (4) plots the impulse responses of the three variables. First it is noteworthy that the response of consumption is broadly consistent with macroeconomic theory – after a trend shock, consumption quickly reaches its new long-run level, while income and asset wealth adjust somewhat more sluggishly. Still, the adjustment in consumption is not immediate. This is in line with the existence of a transitory component in consumption as discussed above. Fully forward-looking consumers should adjust their consumption level immediately. It is also worth noting that asset wealth shows some interesting non-monotonic adjustment. Assets seem to overshoot their long-run level, a result that is consistent with the observed short-run volatility of asset prices. As both income and consumption settle onto their long-run levels, however, so does asset wealth. The shape of the asset wealth response is consistent with a substantial temporary component in asset prices that could be triggered by what is ultimately a permanent shock to consumption, income and asset wealth. This feature of the asset response may therefore also help explain why I find a relatively lower role of transitory shocks for asset wealth even though the size of the transitory component in $a$ is unchanged vis-a-vis Lettau and Ludvigson's paper.

# 4 Reinterpreting *cay*

The Lettau-Ludvigson approach suggests two different, though intimately related interpretations of *cay*: first, at a theoretical level, *cay* is an approximation of the unobservable aggregate consumption-wealth ratio $c - w$. Secondly, it is also an empirically successful indicator of transitory fluctuations in financial assets and in particular, in asset prices. It is the coincidence of these two interpretations that accounts for much of the theoretical appeal of the *cay* approach: a variable that is so central in many macroeconomic models – the consumption wealth ratio – uncovers temporary variation in asset prices. I now address in turn, how the presence of a second cointegrating relationship affects both of these interpretations.

## 4.1 *cay* as a proxy of the consumption-wealth ratio

If the cointegrating space is two-dimensional, it would appear that a proxy of the consumption-wealth ratio cannot be consistently estimated from a simple cointegrating regression. However, if only one cointegrating relationship between $c$, $a$ and $y$ is specified in an econometric model, the estimated relation will generally not only be stationary, it will also reflect a linear combination of minimum-variance in the cointegrating space. My argument here is that this minimum-variance property allows us to interpret the *cay*-residual as a factor that mimics the consumption-wealth ratio that is associated with the wealth portfolio with the smallest variance. Hence, if we introduce the additional assumption that the average household holds a portfolio of human capital and assets that minimizes the (short-term) variability of total wealth, then the estimated *cay* will also be a proxy of $c - w$.[4]

I now construct a minimum-variance portfolio from the transitory components obtained from the VECM with two stationary relations. This can be done by backing out $\gamma$ from the minimization problem

$$\min_{\gamma} \left\{ var \left[ w^T \right] \right\} = \min_{\gamma} \left\{ var \left[ \gamma a^T + (1-\gamma) y^T \right] \right\}$$

where $a^T = a - a^p$ and $y^T = y - y^p$ are the cyclical or transitory components of assets and income obtained from the VECM with two cointegrating relations respectively, $w^T$ is the transitory component of total wealth and the superscript $p$ denotes the permanent component. The solution for $\gamma$ is

$$\gamma = \frac{var(y^T) - cov(a^T, y^T)}{var(a^T - y^T)}$$

Based on my estimates of $a^T$ and $y^T$ identified from the Granger-Gonzalo decomposition, I calculate $\gamma = 0.26$. I then obtain $w^T$ and construct a measure of the consumption-wealth ratio as

$$c - w = c^T - w^T = \boldsymbol{\gamma}' \mathbf{x}_t^T$$

[4]A possible rationale for economic agents to care about the short-term variability of their wealth portfolio is the balanced growth property that provides the very motivation for this paper: if consumption, income and physical and financial asssets follow a single stochastic trend, then the composition of the wealth portfolio is irrelevant for the long-run variance of wealth and therefore for the long-run variance of consumption. The choice of the portfolio weights $\gamma$ and $(1-\gamma)$ will then only affect the cyclical variability of wealth and it is only this variability that optimising agents will be able to minimize.

where I have defined $\boldsymbol{\gamma}' = [1, -\gamma,\ -1-\gamma]'$.[5] Figure (5) plots my estimates of $c^T - w^T$ and $w^T$ against the sample estimate of the *cay*-relation .[6] The correlation of$c^T - w^T$ with *cay* is 0.99 , that of $w^T$ with *cay* is 0.83!

These findings strongly suggest that the *cay*-residuals – even though it may have been estimated from what is a misspecified model under the maintained assumptions – is likely to be an excellent approximation of the consumption-wealth ratio– provided the average household holds a portfolio that minimizes the cyclical variability of wealth. Interestingly, this holds true even though such residuals will generally have been estimated without allowance for structural breaks and deterministic trend terms. One potential explanation for this finding is that the minimum-variance property of the *cay*-residual will also seek to minimize the in-sample variability induced by deterministic terms.

Note that my estimate of $\gamma = 0.26$ also seems highly plausible as measure of the portfolio share of assets in total wealth. Lettau and Ludvigson argue that along a balanced growth path, the portfolio weights $\gamma$ and $1-\gamma$ should correspond to the long-run capital and labour shares of the economy. While this may not necessarily be true if capital is, e.g. more risky than human capital, my estimate of $\gamma = 0.26$ is still close to the values for the capital share typically used in the RBC literature. Furthermore, $\gamma = 0.26$ exactly corresponds to the coefficient on $a$ in the *cay*-relation. Indeed, under the maintained assumption that the great ratios are stationary, the cointegrating space is spanned by

$$\boldsymbol{\beta}' = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

and the portfolio vector $\boldsymbol{\gamma}$ is an exact linear combination of $\boldsymbol{\beta}'$ so that $\boldsymbol{\gamma} = \boldsymbol{\beta}\mathbf{R}$ for some non-singular matrix $\mathbf{R}$. I can then write

$$c^T - w^T = \boldsymbol{\gamma}'\boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\mathbf{x}_t = \mathbf{R}'\boldsymbol{\beta}'\boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\mathbf{x}_t = \boldsymbol{\gamma}'\mathbf{x}_t = c_t - \gamma a_t - (1-\gamma) y_t$$

Hence, if the great ratios define the stationary relations, the vector $\boldsymbol{\gamma}$ of portfolio weights must also defines the very linear combination of the *levels* of the process that approximates the consumption-wealth ratio. To the extent

[5] Here, I have used that $c - w$ cointegrates so that $c^P - w^P$ is constant. For convenience, I assume this constant to be zero.

[6] To take account of the deterministic terms, the vector of transitory components is constructed as

$$\mathbf{x}_t^T = \alpha(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1} \left[\ \boldsymbol{\beta}' \quad \boldsymbol{\delta}_{trend} \quad \boldsymbol{\delta}_{break}\ \right]] \begin{bmatrix} \mathbf{x}_{t-1} \\ t \\ \min(0, t-t_0) \end{bmatrix}$$

that the transitory part in consumption is not too volatile, so that minimizing the variability in $w^T$ comes close to also minimizing the variability in $c^T - w^T$, this linear combination can then be estimated by means of a cointegrating regression.

While the findings reported here rehabilitate *cay*-like residuals as empirical proxies of the transitory component in aggregate wealth, this rehabilitation comes at some cost: it has often been claimed that the derivation of the *cay* residual rests on minimal theoretical assumptions because it is based on the log-linearization of the intertemporal budget constraint (1) alone. The results put forward in this paper suggest that things are not that simple. The balanced-growth assumption made in deriving *cay* from (1) implies a second cointegrating relationship which makes it econometrically impossible to identify the consumption-wealth ratio without a further identifying assumption. Such an assumption will almost inevitably be based on economic theory, e.g. on optimizing behaviour by economic agents. In this respect, the *cay*-residual is much more than just the log-linearized version of a budget constraint.

## 4.2 *cay* and asset prices

Under the maintained hypothesis that *cy* and *ay* are individually stationary, *cay* is just a particular linear combination of these two great ratios. This also implies that predictive regressions of equity premia or asset prices on *cy* and *ay* will perform at least as well as a regression on *cay* alone. This should affect the measurement of transitory components in asset prices. The second cointegrating relationship also informs the recent debate about the role of look-ahead bias for the predictive power of *cay*: since the great ratios *ca*, *cy* and *ay* are – in principle – directly observable, their joint predictive power cannot be subject to look-ahead bias; it is not necessary to first estimate the parameters of *cay* from a long sample in order to do at least as well as *cay* in forecasting excess returns.

Under the theoretical assumptions made in section 2, it is trivially true that *cay* is a linear combination of the great ratios. But it is not clear *a priori* to what extent it is true if the great ratios and are subject to deterministic drifts and breaks. Table (6) therefore reports regressions of the *cay* residual on $\widetilde{cy}$ and $\widetilde{ya}$ and the deterministic trend terms $t$ and $\min(t - t_{0,}0)$. I consider two different measures of *cay*. The first is the original *cay* used by Lettau and Ludvigson (2004) which is constructed with the cointegrating vector $\beta = [1, -0.30, -0.60]$ estimated from a dynamic OLS regression. I refer to this residual as $cay_{LL}$.The second one is constructed based on the cointegrating vector $\beta = [1, -0.26, -0.63]$ which is estimated by Johansen's

FIML procedure. This is the cointegrating vector also reported in table (3) above and the associated *cay*-residual is the one that has been used in the paper so far.

In both regressions, the two detrended great ratios are highly significant and also have very similar coefficients. Furthermore, the fit of both regressions is overwhelming with and $R^2$ of 0.97 and 0.98. There is, however, an interesting difference in as far as the deterministic terms are concerned. In the original Lettau-Ludvigson $cay_{LL}$,both the trend and the break term are highly significant. In the *cay*-residual based on $\beta = [1, -0.26, -0.63]$ only the coefficient on the trend break term remains marginally significant but it is much smaller than in the Lettau-Ludvigson *cay* so that this version of the *cay* residual can essentially be written as[7]

$$cay = 0.80\widetilde{cy} + 0.26\widetilde{ya} + 0.28$$

Hahn and Lee (2006) argue that the (unmodelled) deterministic components in the Lettau-Ludvigson *cay* are a main driver behind the predictive power of the residual for asset prices. This point is examined in table (7), where I report regressions of asset returns on the two *cay* measures. My analysis is based on two different measures of asset prices. The first are excess returns on the CRSP index. The second is broad measure of asset returns that I construct from the asset data used in Lettau and Ludvigson.[8] This broad measure has the advantage that it does not only capture fluctuation in stock markets but also fluctuations in other financial and, in particular, in physical assets such as housing.

Based on the broad measure of returns, I find that the Lettau-Ludvigson $cay_{LL}$ seems to outperform *cay* by a wide margin. This result highlights the importance of the Hahn-Lee caveat. But it should be noted that *cay* remains an important predictor of aggregate asset prices with $R^2$ peaking at

[7]Since this measure of the *cay*-residual appears immune against the Hahn-Lee caveat, I continue to use it as my preferreed measure in the remainder of this paper. Whenever a distinction is necessary and may matter for the results, I abbreviate the Lettau-Ludvigson residual with $cay_{LL}$.

[8]he law of motion for asset wealth can be written as $A_{t+1} = (1 + r_{t+1})(A_t + Y_t - C_t)$. Dividing through with $A_t$, taking logarithms and solving backwards it is straightforward to show that $a_{t+1} = \sum_{l=1}^{t+1} r_{t+l} + a_0 + \sum_{l=0}^{t} \log(1 + (Y_l - C_l)/A_l)$. The aggregate asset price measure I construct is $p_t = a_t - \sum_{l=0}^{t} \log(1 + (Y_l - C_l)/A_l)$. Under the null that asset returns are unpredictable, $r_{t+k} = r + v_{t+k}$, where $r$ is a constant and $v_{t+k}$ is *i.i.d.*. Then $\mathbf{E}_t(p_{t+k} - p_t) = kr$, i.e. $p_t$ follows a random walk with drift and should therefore not be predictable from *cay* or other variables.

0.19 at the 2 year horizon. The lower two panels report similar regressions for equity (excess) returns. The predictive power of both *cay* measures is now very similar. Interestingly, the $R^2$on excess returns are generally higher than those obtained on the broad asset return measures , which supports Lettau's and Ludvigson's claim that *cay* is, in particular, a good indicator of the equity risk premium.

Table (8) presents long-horizon regressions of the broad return measure on the observable great ratios *cy* and *ay*. For each forecasting horizon, line *I* reports regressions on the detrended versions of *cy* and *ay*, whereas line *II* gives the regressions on $\widetilde{cy}$ and $\widetilde{ay}$, i.e. taking account of both a linear trend and the break in 1978. While controlling for a linear trend alone would suggest that the joint predictive power of the observable great ratios still by far exceeds that of *cay*(or even $ca_{LL}$), this is not so clearly the case for $\widetilde{cy}$ and $\widetilde{ay}$. Certainly, in the regressions in the second line, the adjusted $R^2$ measure at all forecasting horizons exceeds the $R^2$ from a regression with *cay* (as reported in the previous table). This fact per se should not be surprising, since *cay* is almost an exact linear combination of $\widetilde{cy}$ and $\widetilde{ay}$. It is however, doubtful that $\widetilde{cy}$ and $\widetilde{ya}$ really explain a significantly larger portion of the variation in asset returns than does *cay*: at short horizons, the combination of $\widetilde{cy}$ and $\widetilde{ay}$ generates an adjusted $R^2$ that only exceeds that of the *cay*-regression by a factor of $1.2 - 1.4$. Though this factor increases to 2 at the five year horizon, the regressions are only significant up to an horizon of up to $3 - 4$ years,very much as the *cay*-only based regressions. Indeed, as I show in lines III and IV, none of the two great ratios makes an independent contribution to predicting asset prices. if it is included along with *cay* as a regressor.

Table (9) provides long-horizon regressions of excess returns on the observable great ratios. Based on the linear trend alone (line I) , *cy* and *ay* outperform *cay*by a wide margin, but the results are even more pronounced once the trend break is also controlled for (line II respectively). In this case, $R^2$ reaches 0.6 at the 6-year horizon. As lines III and IV show, both great ratios also make a significant independent contribution to predicting excess returns if they are included along with *cay* as a regressor.

While *cy* and *ay* seem able to uncover predictable dynamics in stock prices and in particular in equity risk premia to a degree that by far exceeds the predictive power of *cay*, this is not generally true for a broader concept of asset prices. For the broader concept, the great ratios together are just as good as *cay*. One possible interpretation for this finding could be based on Cochrane et al.'s (2005) recent argument that return predictability may arise from portfolio adjustment alone in a model with several Lucas trees. Under the balanced growth assumption asset wealth and human capital share a single common trend and their long-run portfolio shares are fixed. So, an asymmetric shock to, say, asset wealth, must either forebode a similar adjustment in human capital (if the shock is permanent), or a re-adjustment to the permanent value of $a$ (if the shock is transitory). Hence, we would expect that relative returns – such as that of equity versus bonds – are even more highly predictable from the interaction of the error-correction terms than are are aggregate returns on a broad measure of assets. The aggregate consumption-wealth ratio predicts aggregate asset price fluctuations and, in particular, risk premia; the observable great ratios also predict the price effects of relative portfolio adjustment, so that they outperform *cay* on this account. Still, *cay* remains remarkably robust as an indicator of transitory components in aggregate asset prices.

**Look-ahead bias in cay?** The results reported here also inform the recent debate about look-ahead bias in *cay*. As long as *cay* is an exact linear combination of stationary and directly observable great ratios, , a household (or a researcher seeking to identify household expectations) does not have to estimate the coefficients of the *cay*-relation in order to identify the transitory component in asset prices. Hence, if the balanced-growth assumption holds, look-ahead bias in *cay* cannot actually be a problem. Certainly, in practice, the great ratios may only be trend stationary, so that there may be some uncertainty about the actual slope of the long-run trend which could make it hard to correctly identify transitory components in asset prices in a given sub-sample. I examine this issue next.

I compare the predictive power of the great ratios in a subsample to that of the *cay* residual estimated from the entire sample. Since the results are qualitatively similar, I confine myself to presenting only the regressions obtained based on stock market excess returns. I start by following the split suggested in Hahn and Lee and Brennan and Xia, i.e. I consider the first half of the sample period, 1952Q4:1977Q4. Table (10) reports the results: panel I the predictive regressions on *cay*; panel II the regressions on *cy* and *ay*. In the latter set of regressions, a sub-sample specific trend is removed from *cy* and *ay*.

The combination of the detrended observable great ratios does not generally do much worse than does the predictive regression based on the 'look-ahead biased' *cay*-residual. The pattern as well as the magnitude of the $R^2$ coefficients is similar across the two sequences of regressions. This suggests that look-ahead bias does not appear to be an issue for this subperiod. To see whether the same holds true if the break in the deterministic trend is included in the subsample, I repeat the same exercise for the sample period

1952Q4:1990Q4. Again, the detrended great ratios do at least as well as *cay*. This conclusion remains unaltered if I also remove the trend break from *cy* and *ay*.

# 5 Discussion and Conclusion

Lettau and Ludvigson have suggested a by now very popular approach to approximating the consumption-wealth ratio as a cointegrating relationship between consumption, asset wealth and labour income commonly called the *cay* residual. One key assumption that underlies the interpretation of *cay* as an approximation of the consumption-wealth ratio is that the shares of human capital and asset wealth in total wealth are constant in the long run. In this note I have demonstrated that this actually implies that the consumption asset (*ca*) , the consumption-labour income (*cy*) and the income-asset (*ya*) ratios should all be individually stationary. Hence, there should be two linearly independent cointegrating relations between the three variables – consumption, income and asset wealth should share a single stochastic trend.

I have explored the reasons why earlier studies have not generally detected this second cointegrating relationship. While cointegration tests could generally have very low power, a structural break in the trend growth rates of consumption and income the particular sample used by Lettau and Ludvigson can provide an explanation for this low power; once this break is explicitly modelled, the second cointegrating relationship predicted by the theoretical framework is picked up by the extant tests.

While these results simplify the interpretation of the joint long-run dynamics of consumption, income and asset wealth in the light of standard economic theory, they strongly affect the interpretation of *cay*-like residuals as approximations of the consumption wealth ratio: in the presence of a second cointegrating relationship, *cay* becomes a particular linear combination in a two-dimensional cointegrating space and we cannot generally hope to obtain an approximation of the consumption -wealth ratio from a simple cointegrating regression alone.

However, I have shown that *cay* remains a good indicator of transitory components in asset prices and that – under the additional assumption that the average household holds a minimum-variance portfolio of physical assets and human wealth – one may still be able to interpret it as a proxy of the consumption-wealth ratio. This assumption however, shows that the interpretation of *cay* as a proxy of the consumption-wealth ratio does not rest on the rather innocuous log-linearization of an intertemporal budget constraint alone.

Finally, the results provided here should be informative with respect to the recent debate about look-ahead bias in *cay*: if *cy* and *ay* are individually stationary, then in order to identify transitory components in asset prices, households and researchers do not need to identify the parameters of the *cay*-relation first – which may only be estimable *ex post* from very long samples of data. Rather, the great ratios carry at least the same amount of information.

# References


[1] Becker, Sascha and Mathias Hoffmann (2006).'Intra- and International Risk-Sharing in the Short Run and the Long Run', previously circulated as CESifo Working Paper Series No. 1111 (2003). Forthcoming in *European Economic Review*.

[2] Brennan, Michael and Yihong Xia (2005) '*tay*'s as good as *cay*', *Finance Research Letters*, vol. 2, 1–14.

[3] Campbell, John Y. and Gregory N. Mankiw (1989). 'Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence,' in *NBER Macroeconomics Annual*: 1989, ed. by O. Blanchard, and S. Fischer, pp. 185-216. MIT Press, Cambridge, MA.

[4] Cochrane, John H., Longstaff, Francis and Pedro Santa Clara (2005). 'Two Trees: Asset Price Dynamics Induced by Market Clearing', manuscript, University of Chicago dated April.

[5] Gonzalo, Jesus and Granger, Clive W., (1995), 'Estimation of Common Long-Memory Components in Cointegrated Systems,' *Journal of Business and Economics Statistics* 13: 27-35.

[6] Gonzalo, Jesus and Serena Ng (2001). 'A Systematic Framework for Analyzing the Dynamic Effects of Permanent and Transitory Shocks ', *Journal of Economic Dynamics and Control*, 25 (10), 1527-1546.

[7] Hahn, Jaehoon and Hangyong Lee (2006). 'Interpreting the predictive power of the consumption-wealth ratio', *Journal of Empirical Finance* 13, 183-202.

[8] Hahn, Jaehoon and Hangyong Lee (2001). "On the Estimation of the Consumption-Wealth Ratio: Cointegrating Parameter Instability and its Implications for Stock Return Forecasting", mimeo, University of Washington Business School.

[9] Johansen, Søren and Bent Nielsen (1993). DisCo, Institute of Mathematical Statistics, University of Copenhagen. Download from *http://www.nuffield.ox.ac.uk/Users/Nielsen/Disco.html*

[10] Johansen, Søren, (1995). *Likelihood-based inference in cointegrated vector autoregressive models*. Oxford University Press: Oxford.

[11] Johansen, Søren, (1991). 'Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models', *Econometrica*, 56, 1551-1580.

[12] King, Robert G., Charles I. Plosser, James H. Stock and Mark W. Watson, (1991),"Stochastic Trends and Economic Fluctuations", American Economic Review,81,pp.819-840.

[13] Lettau, Martin and Sidney Ludvigson (2001). 'Consumption, Aggregate Wealth and Expected Stock Returns', *Journal of Finance*, LVI (3), 815-849.

[14] Lettau, Martin and Sidney Ludvigson (2004). 'Understanding Trend and Cycle in Asset Values: Reevaluating the Wealth Effect on Consumption', *American Economic Review 94, 276-299.*

**Table1: Cointegration results for great ratios**

Panel I: Johansen's tests for cointegration

| | $[c, y]$ | $[c, a]$ | $[y, a]$ |
|---|---|---|---|
| Trace Test | 3.84 | 5.53 | 4.55 |
| p-val | [0.9091] | [0.7512] | [0.8503] |

Panel II: Estimates of CI-vector $\beta = \begin{bmatrix} 1 & \beta_2 \end{bmatrix}'$

| | $[c, y]$ | $[c, a]$ | $[y, a]$ |
|---|---|---|---|
| $\beta_2$ | -0.938 | -0.832 | -0.842 |
| std. deviation | (0.047) | (0.073) | (0.117) |
| p-Value of $\beta_2 = -1$ | [0.18] | [0.031] | [0.2405] |

**Table 2: Trend stationarity of great ratios**

| | Panel I: deterministic trend | | | Panel II: deterministic trend and break | | |
|---|---|---|---|---|---|---|
| | $c-a$ | $c-y$ | $y-a$ | $c-a$ | $c-y$ | $y-a$ |
| $t$ | -0.0009 | -0.0005 | -0.0004 | -0.0025 | 0.0004 | -0.0029 |
| | (-10.8604) | (-14.2166) | (-3.3694) | (-21.4555) | (9.3622) | (21.8682) |
| $\min(t-t_0)$ | —- | —- | —- | 0.0034 | -0.0017 | 0.0051 |
| | | | | (15.8234) | (-25.0342) | (21.4090) |
| *const* | -1.5735 | 0.2515 | -1.8250 | -1.3195 | 0.1203 | -1.4399 |
| | (-168.4468) | (60.8328) | (-144.1113) | (-76.6371) | (21.4005) | (-74.6037) |
| ADF $t-$test | -2.0611$^{**}$ | -1.4260 | -1.6974$^{*}$ | -3.1317$^{***}$ | -3.5337$^{***}$ | -3.2189$^{***}$ |
| Johansen test | 11.97 | 14.21 | 14.32 | 26.81$^{*}$ | 26.81 $^{**}$ | 25.31$^{*}$ |

NOTES: Regressions of the great ratios on deterministic components, t-values in parentheses. The last two lines give unit root tests on the regression residuals (t-stat of an augmented Dicke-Fuller test with two lags) and of Johansen's system cointegration test with a trend restricted to the cointegrating space. 1,2 or 3 Stars denote significance at the 90, 95% and the 99% levels respectively. The corresponding critical values for the ADF test are $-1.67$, $-1.99$ and $-2.65$ respectively. Those for the Johansen test are 23.32, 25.73 and 30.67.

**Table 3: Cointegrating results in the trivariate VECM**

Panel I: Cointegration tests

| | trend cum break | 90% | 95% | CV | trend only | 90% | 95% |
|---|---|---|---|---|---|---|---|
| $r \geq 1$ | 43.83 | 33.17 | 35.95 | | 31.49 | 39.73 | 42.77 |
| $r \geq 2$ | 24.16 | 14.63 | 16.62 | | 11.60 | 23.32 | 25.73 |

Panel II: Estimated cointegrating vectors

| 2 CI relations imposed, cum break | | | | | | 1 CI-relation imposed, no break | | | |
|---|---|---|---|---|---|---|---|---|---|
| Coefficients (unrestricted) | | | | | p-values | Coefficients | | | |
| $\beta_c$ | $\beta_a$ | $\beta_y$ | $\delta_{trend}$ | $\delta_{break}$ | $H_o : \beta_y = -1$ | $\beta_c$ | $\beta_a$ | $\beta_y$ | $\delta_{trend}$ |
| 1 | – | –0.67 | -0.002 | 0.001 | [0.18] | 1 | -0.16 | -0.58 | -0.001 |
| – | 1 | -0.77 | -0.003 | 0.005 | [0.15] | | | | |
| Coefficients (restricted) | | | | | | Coeffs (restricted) | | | |
| 1 | — | -1 | -0.000 | 0.002 | | 1 | -0.26 | -0.63 | — |
| | 1 | -1 | -0.002 | 0.005 | | | | | |

**Table 4: VECM with 2 stationary relations**

| | Equation | | |
|---|---|---|---|
| | $\Delta c_t$ | $\Delta a_t$ | $\Delta y_t$ |
| $\Delta c_{t-1}$ | **0.1770** | 0.1023 | **0.3377** |
| | (2.2373) | (0.2803) | (2.1662) |
| $\Delta a_{t-1}$ | **0.0416** | 0.0863 | **0.0782** |
| | (2.5821) | (1.1613) | (2.4646) |
| $\Delta y_{t-1}$ | 0.0729 | -0.0336 | -0.1055 |
| | (1.7728) | ( -0.1770) | (-1.3021) |
| $\widetilde{cy}_{t-1}$ | 0.0058 | **0.3635** | **0.1074** |
| | (0.2259) | (3.0483) | (2.1083) |
| $-\widetilde{ya}_{t-1}$ | -0.0107 | **0.1014** | -0.0208 |
| | (-1.3992) | (2.8742) | (-1.3769) |
| *step* | 0.0010 | 0.0009 | **0.0026** |
| | (0.5559) | (0.3103) | (2.1358) |
| *const* | **0.0030** | 0.0044 | **0.0026** |
| | (5.6764) | (1.8366) | (2.5340) |
| $\overline{R}^2$ | 0.17 | 0.04 | 0.15 |

**Table 5: Variance decompositions based on 2 stationary relations**

Variance share of transitory component

| Panel I: unrestricted model | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Horizon $k$ in quarters | | | | | | |
| | 1 | 2 | 4 | 8 | 12 | 16 | 24 |
| $c_{t+k} - \mathbf{E}_t(c_{t+k})$ | 0.2479 | 0.2129 | 0.1560 | 0.1059 | 0.0802 | 0.0652 | 0.0499 |
| $a_{t+k} - \mathbf{E}_t(a_{t+k})$ | 0.1341 | 0.1527 | 0.1719 | 0.1912 | 0.1938 | 0.1882 | 0.1713 |
| $y_{t+k} - \mathbf{E}_t(y_{t+k})$ | 0.7099 | 0.5837 | 0.4727 | 0.3603 | 0.2861 | 0.2330 | 0.1671 |

| Panel II: restricted model | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 4 | 8 | 12 | 16 | 24 |
| $c_{t+k} - \mathbf{E}_t(c_{t+k})$ | 0.0000 | 0.0049 | 0.0078 | 0.0252 | 0.0523 | 0.0800 | 0.1248 |
| $a_{t+k} - \mathbf{E}_t(a_{t+k})$ | 0.3389 | 0.3865 | 0.4471 | 0.5134 | 0.5356 | 0.5360 | 0.5154 |
| $y_{t+k} - \mathbf{E}_t(y_{t+k})$ | 0.5071 | 0.4710 | 0.3985 | 0.2774 | 0.2040 | 0.1718 | 0.1666 |

**Table 6: cay** as linear combination of great ratios

| | $cay_{LL}$ | $cay$ |
|---|---|---|
| $\widetilde{cy}_t$ | 0.82 | 0.80 |
| | (88.53) | (78.41) |
| $\widetilde{ay}_t$ | 0.29 | 0.26 |
| | (106.45) | (86.53) |
| $t$ | -0.0001 | -0.0000 |
| | (-29.30) | (-1.08) |
| $\min(t - t_0, 0)$ | 0.0002 | 0.0000 |
| | (19.27) | (4.60) |
| $const$ | 0.908 | 0.28 |
| | (254.21) | (70.78) |
| $\overline{R}^2$ | 0.99 | 0.98 |

NOTES: OLS regression of $cay_{LL}$ and $cay$ on the deterministic terms and on the great ratios. $cay_{LL} = c - 0.3a - 0.6y$ is the Lettau-Ludvigson-cay, whereas $cay = c - 0.26a - 0.63y$ is based on the cointegarting vector estimated by FIML.

**Table 7: Long-horizon regressions of asset returns on** *cay* **and** $cay_{LL}$

$$\sum_{l=1}^{k} r_{t+l} - r^f_{t+l} = \mathbf{z}_t' \boldsymbol{\delta}_k + v_{kt}$$

| $\mathbf{z}_t$ | Horizon $k$ in quarters | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 4 | 8 | 12 | 16 | 20 |
| | Panel I: Aggregate asset returns on $cay_{LL}$ | | | | | | |
| $cay_{LL}$ | 0.5106 | 1.0009 | 1.8167 | 3.3771 | 4.3417 | 4.3623 | 4.5481 |
| | 3.7715 | 3.6621 | 3.3145 | 4.1877 | 3.8874 | 2.8581 | 2.7926 |
| $\overline{R}^2$ | 0.1077 | 0.1772 | 0.2353 | 0.3106 | 0.2601 | 0.1549 | 0.1033 |
| | Panel II: Aggregate asset returns on $cay$ | | | | | | |
| $cay$ | 3.7715 | 3.6621 | 3.3145 | 4.1877 | 3.8874 | 2.8581 | 2.7926 |
| | 3.2536 | 2.9835 | 2.4702 | 2.5356 | 2.0190 | 1.4046 | 1.0997 |
| $\overline{R}^2$ | 0.0702 | 0.1135 | 0.1430 | 0.1903 | 0.1338 | 0.0551 | 0.0230 |
| | Panel III: Excess stock market returns on $cay_{LL}$ | | | | | | |
| $cay_{LL}$ | 1.8160 | 3.4859 | 6.2120 | 10.3546 | 12.8615 | 13.8875 | 16.9512 |
| | 3.9625 | 3.9511 | 3.8185 | 5.2619 | 7.3338 | 6.8522 | 5.4022 |
| $\overline{R}^2$ | 0.0855 | 0.1526 | 0.2580 | 0.4052 | 0.4255 | 0.3704 | 0.3529 |
| | Panel IV: Excess stock market returns on $cay$ | | | | | | |
| $cay$ | 2.0324 | 3.8553 | 6.7739 | 11.2227 | 13.3658 | 13.7944 | 15.6080 |
| | 4.6595 | 4.6116 | 4.4898 | 6.0746 | 5.9340 | 5.2692 | 4.9552 |
| $\overline{R}^2$ | 0.0858 | 0.1491 | 0.2475 | 0.3914 | 0.4134 | 0.3666 | 0.3325 |

**Table 8: Predictive regresssions of aggregate asset returns on great ratios**

$$\sum_{l=1}^{k} r_{t+l} - r^{f}_{t+l} = \mathbf{z}_t'\boldsymbol{\delta}_k + v_{kt}$$

| Horizon $k$ in quarters | Regression number | trend only $cy$ | trend only $ay$ | trend and break $cy$ | trend and break $ay$ | $cay$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| $k = 1$ | I | 0.4875 | 0.1273 | | | | 0.1086 |
| | | (4.2695) | (3.1564) | | | | |
| | II | | | 0.5126 | 0.1217 | | 0.0942 |
| | | | | (3.5712) | (3.0610) | | |
| | III | | | | -0.0282 | 0.5278 | 0.0682 |
| | | | | | (-0.6143) | (2.8388 ) | |
| | IV | | | 0.1704 | | 0.3814 | 0.0759 |
| | | | | (1.2655 ) | | (2.5691) | |
| $k = 4$ | I | 1.7445 | 0.4609 | | | | 0.2478 |
| | | (3.5704 ) | (2.6248) | | | | |
| | II | | | 1.5032 | 0.5084 | | 0.2012 |
| | | | | (2.5269 ) | (2.9458) | | |
| | III | | | | 0.0990 | 1.3685 | 0.1439 |
| | | | | | (0.5513) | (1.7611 ) | |
| | IV | | | 0.1085 | | 1.5330 | 0.1392 |
| | | | | (0.1924 ) | | ( 2.2596) | |
| $k = 8$ | I | 3.3120 | 0.8518 | | | | 0.3546 |
| | | (4.0414 ) | ( 2.5665) | | | | |
| | II | | | 2.5186 | 1.0168 | | 0.2842 |
| | | | | (2.3365 ) | ( 3.3375 ) | | |
| | III | | | | 0.3561 | 2.1391 | 0.2124 |
| | | | | | ( 0.8754) | (1.4743) | |
| | IV | | | -0.2671 | | 3.0599 | 0.1876 |
| | | | | (-0.2040 ) | | ( 2.3397) | |
| $k = 16$ | I | 3.7753 | 0.5779 | | | | 0.3409 |
| | | (3.3751) | ( 1.0346) | | | | |
| | II | | | 3.0232 | 1.2342 | | 0.1399 |
| | | | | (1.4402) | (2.5496) | | |
| | III | | | | 0.4993 | 1.7838 | 0.0728 |
| | | | | | (0.5853) | ( 0.6522) | |
| | IV | | | 0.0659 | | 2.6370 | 0.0500 |
| | | | | (0.0241) | | ( 1.1984) | |

**Table 9: Predictive regresssions of excess returns on great ratios**

$$\sum_{l=1}^{k} r_{t+l} - r_{t+l}^{f} = \mathbf{z}_t' \boldsymbol{\delta}_k + \upsilon_{kt}$$

| Horizon $k$ in quarters | Regression number | trend only | | trend and break | | | |
|---|---|---|---|---|---|---|---|
| | | $cy$ | $ay$ | $cy$ | $ay$ | $cay$ | $R^2$ |
| $k=1$ | I | 1.8035 | 0.5559 | | | | 0.0943 |
| | | (5.2850) | (4.5282) | | | | |
| | II | | | 1.7548 | 0.5649 | | 0.0914 |
| | | | | (3.9625) | (4.2529) | | |
| | III | | | | 0.0270 | 1.9730 | 0.0813 |
| | | | | | (0.1737) | (3.5025) | |
| | IV | | | 0.1945 | | 1.9564 | 0.0857 |
| | | | | (0.9377) | | (4.4070) | |
| $k=4$ | I | 6.0814 | 1.9264 | | | | 0.2929 |
| | | (5.1683) | (4.7750) | | | | |
| | II | | | 4.6921 | 2.2045 | | 0.3064 |
| | | | | (3.3196) | (4.6042) | | |
| | III | | | | 0.8163 | 4.9746 | 0.2776 |
| | | | | | (1.8903) | (2.7870) | |
| | IV | | | 0.5197 | | 6.5664 | 0.2517 |
| | | | | (0.8493) | | (4.4351) | |
| $k=8$ | I | 10.1177 | 3.1932 | | | | 0.4690 |
| | | (7.4920) | (6.9389) | | | | |
| | II | | | 6.9072 | 3.8350 | | 0.5214 |
| | | | | (4.1951) | (6.2500) | | |
| | III | | | | 1.8112 | 7.2038 | 0.4826 |
| | | | | | (2.6845) | (3.4218) | |
| | IV | | | 0.9186 | | 10.8365 | 0.4023 |
| | | | | (1.2117) | | (5.6751) | |
| $k=16$ | I | 12.7711 | 3.9858 | | | | 0.4788 |
| | | (7.7646) | (5.8669) | | | | |
| | II | | | 9.1543 | 5.2963 | | 0.5650 |
| | | | | (4.8109) | (10.8392) | | |
| | III | | | | 2.5911 | 9.1572 | 0.5056 |
| | | | | | (3.0523) | (3.5522) | |
| | IV | | | 1.5518 | | (12.4482) | 0.3908 |
| | | | | (1.4603) | | (4.1925) | |

**Table 10: Long horizon regressions of excess returns on *cay* vs *cy* and *ay***

$$\sum_{l=1}^{k} r_{t+l} - r_{t+l}^{f} = \mathbf{z}_t' \boldsymbol{\delta}_k + v_{kt}$$

| $\mathbf{z}_t$ | Horizon $k$ in quarters | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 4 | 8 | 12 | 16 | 20 |
| **Panel I: Subperiod 1952Q4:1977Q4** | | | | | | | |
| | $\mathbf{z}_t' = [\ cy_t \quad ay_t\ ] - \boldsymbol{\delta}_0' trend$ | | | | | | |
| *cy* | 2.1099 | 4.1301 | 7.4019 | 10.1881 | 9.5124 | 8.2356 | 11.1714 |
| | (3.8032) | (3.9832) | (4.2520) | (4.5219) | (3.8565) | (3.5577) | (3.9783) |
| *ay* | 0.8818 | 1.9058 | 3.7400 | 6.0057 | 5.8389 | 5.3162 | 7.4478 |
| | (4.1311) | (5.0173) | (5.2409) | (5.2987) | (5.8012) | (3.4377) | (6.8727) |
| $\overline{R^2}$ | 0.1186 | 0.2398 | 0.4581 | 0.5911 | 0.4467 | 0.2819 | 0.3835 |
| | $\mathbf{z}_t' = cay_t = c_t - 0.26a_t - 0.63y_t$ | | | | | | |
| *cay* | 2.9321 | 5.8351 | 10.5471 | 14.9677 | 14.5348 | 12.3368 | 16.7322 |
| | (4.2112) | (4.4267) | (4.9232) | (4.7763) | (4.6128) | (5.8309) | (5.0611) |
| $\overline{R^2}$ | 0.1343 | 0.2385 | 0.4033 | 0.4351 | 0.3567 | 0.2348 | 0.2566 |
| **Panel II: Subperiod 1952Q4:1990Q4** | | | | | | | |
| | $\mathbf{z}_t' = [\ cy_t \quad ay_t\ ] - \boldsymbol{\delta}_0' trend$ | | | | | | |
| *cy* | 2.2315 | 4.3771 | 7.8496 | 11.3470 | 11.4053 | 11.0117 | 13.2569 |
| | (4.6781) | (4.6010) | (5.1920) | (6.0362) | (5.7872) | (7.5289) | (6.9488) |
| *ay* | 0.8494 | 1.7187 | 3.0502 | 4.2218 | 3.8877 | 3.2767 | 4.4852 |
| | (4.2950) | (4.9416) | (5.6125) | (5.6714) | (5.5298) | (6.1253) | (5.1137) |
| $\overline{R^2}$ | 0.1041 | 0.1926 | 0.3448 | 0.4437 | 0.3894 | 0.3437 | 0.3483 |
| | $\mathbf{z}_t' = cay_t = c_t - 0.26a_t - 0.63y_t$ | | | | | | |
| *cay* | 2.4587 | 4.8296 | 8.7436 | 13.0296 | 12.9001 | 11.7960 | 13.8095 |
| | (3.7842) | (3.7366) | (4.2139) | (4.9528) | (4.3877) | (5.2991) | (4.8916) |
| $\overline{R^2}$ | 0.0818 | 0.1399 | 0.2488 | 0.3365 | 0.2911 | 0.2284 | 0.2235 |

Notes: *cy* and *ay* have been deterministically detrended using data from the respective sub-sample only. *cay* is based on the whole sample

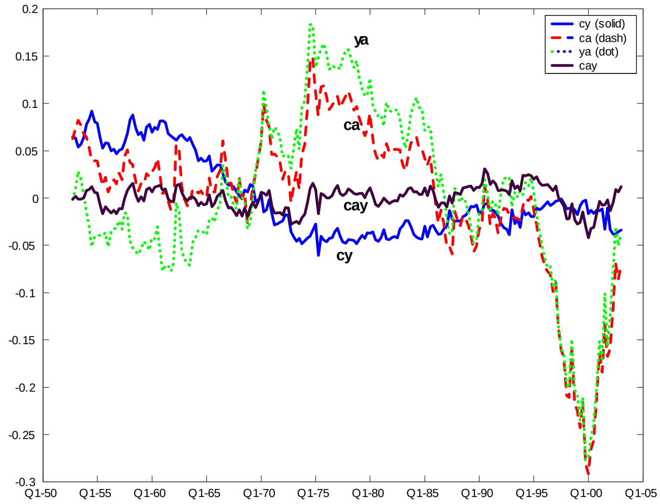


Figure 1: The Great Ratios $cy = c - y$, $ca = c - a$ and $ya = y - a$ along with the $cay$-residual

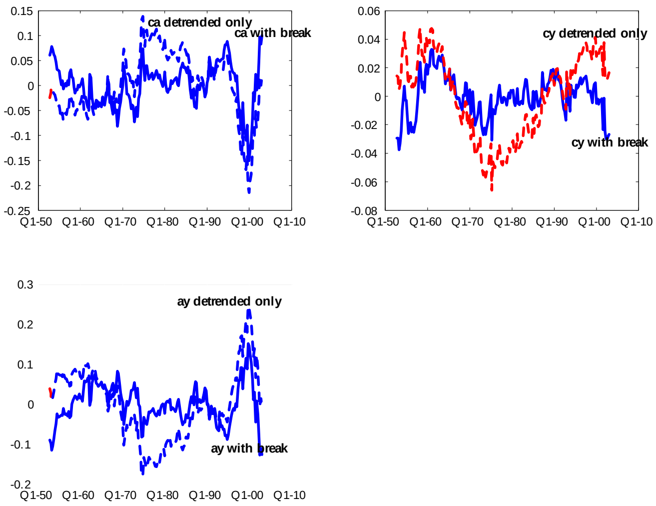


Figure 2: Great Ratios purged of deterministic terms. Trend and break removed (solid /blue line) and trend only removed (dashed/red line).

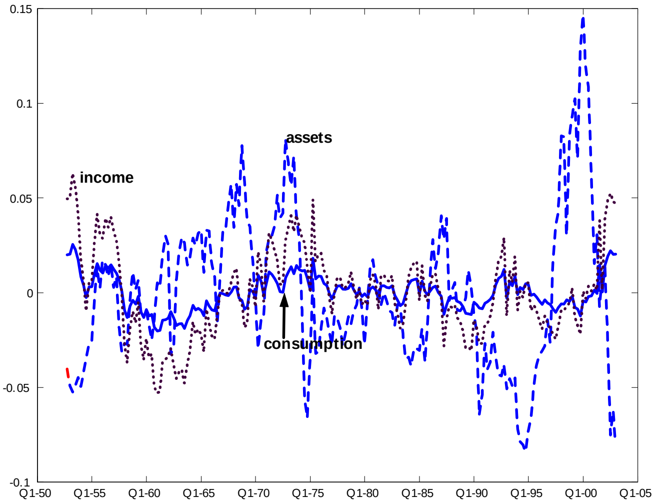


Figure 3: Transitory components from the VECM with 2 stationary relations

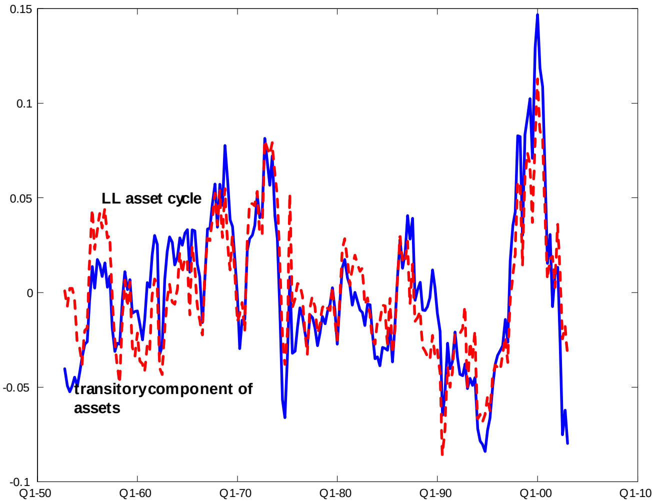


Figure 4: Asset Cycles from Lettau Ludvigson model and from the VECM with 2 stationary relations

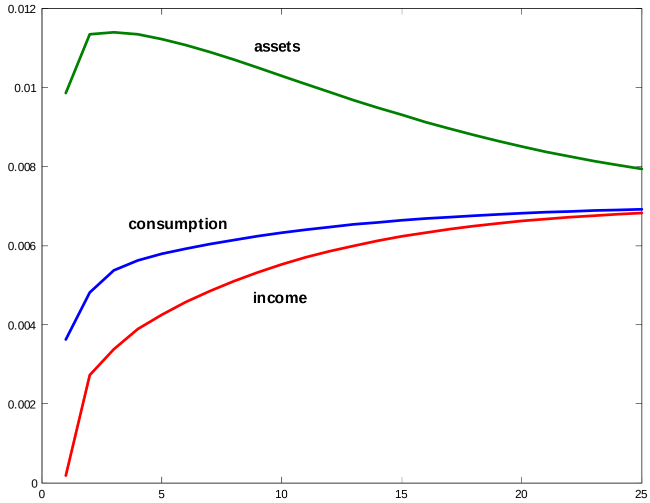


Figure 5: Impulse responses to a permanent shock

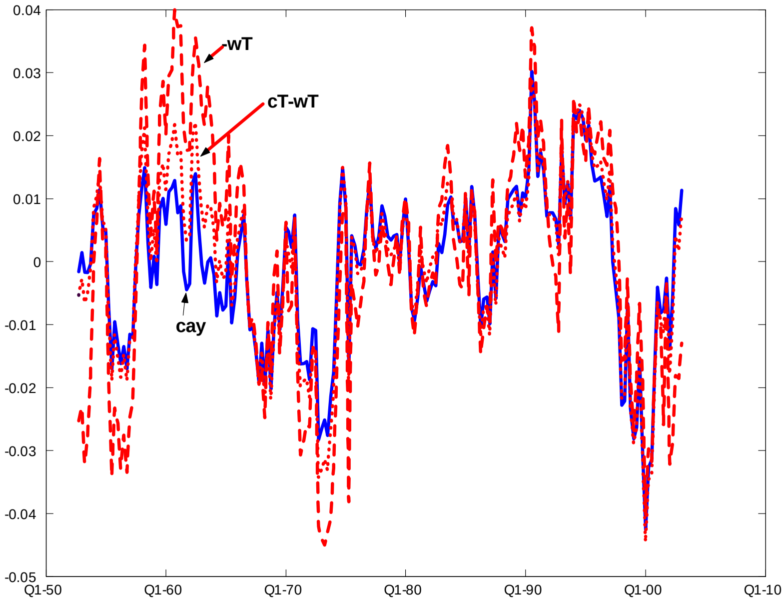


Figure 6: The *cay*-residual, the (negative) minimum-variance wealth portfolio, $-w^T$, and the implied consumption-wealth ratio $c^T - w^T$.