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Article

Einstein's Objections against Quantum Mechanics

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Abstract: After the discovery of quantum mechanics by Heisenberg and Schrödinger in 1925, Einstein raised again and again objections to this theory. Obviously, he had the impression that (a) quantum mechanics does not adequately grasp reality, that it is (b) based on probabilistic laws of nature and that it is (c) for this reason incomplete. Einstein must have obtained this impression from many presentations of quantum mechanics in the first decade after its discovery. – However, technical refutations of Einstein's objections were not possible when these arguments were put forward, since the necessary formal tools were not yet available at this time. Instead, the advocates of quantum mechanics tried to disprove Einstein merely by intuitive and less rigorous arguments. – In the light of current physics we find that the objections (a) and (b) are irrelevant since, in accordance with Einstein's intentions, quantum mechanics does refer to reality and is not based on probabilistic laws. Only the incompleteness argument is incorrect. However, for technical reasons a convincing refutation of this objection only became possible thirty years after its formulation and ten years after Einstein's death.

Keywords: Quantum mechanics, Einstein, Reality Criterion, Bell's inequalities

1 Introduction: Einstein's Contributions to Quantum Physics

Without any doubt, Einstein made crucial contributions to quantum theory. In 1905, the year when also his paper on special relativity appeared, Einstein published an article with the title *Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt* (Einstein 1905). In order to give a better explanation for light electrical phenomena, Einstein in this publication developed the idea that the light hitherto described by Maxwell's theory consists of discrete energy quanta, the light quanta. In a further paper *Zur Theorie der*

Lichterzeugung und Lichtabsorption that appeared in 1906 (Einstein 1906), these considerations were continued and related to Planck's radiation formula. In the beginning, however, the significance of the light quantum hypothesis was just noted by few physicists. This presumably contributed to the fact that Einstein was only in 1921 awarded the Nobel prize for this discovery.
In the subsequent years, Einstein predominantly dealt with the theory of relativity and apparently he did not follow up the light quantum problem. Only when in 1924 the young Indian physicist S. N. Bose sent a manuscript titled *Planck's Law and the Light Hypothesis* to him, with the request to review and forward it to a journal, Einstein's interest woke up again. He was obviously very impressed by Bose's article, translated it, and submitted it for publication to the *Zeitschrift für Physik* (Bose 1924). Immediately afterwards, in the course of two weeks, Einstein wrote an own manuscript titled *Quantentheorie des einatomigen Gases* and sent it to the *Preußische Akademie der Wissenschaften* (Einstein 1924). A second article with the same title followed shortly afterwards. Bose's and Einstein's articles developed the statistics of an ideal quantum gas, which today we call Bose-Einstein statistics (for many historical details see Stachel 2002, in particular pp. 427–444 and 519–538). In spite of his own significant contributions to the theory of light quanta, Einstein's attitude toward quantum theory drastically changed when in 1925 Heisenberg, Schrödinger, Bohr, and others developed the quantum mechanics proper. From the very beginning Einstein stood in critical opposition to this new theory. During the subsequent years up to his death in 1955, again and again he put forward important objections against quantum mechanics. – Here, these objections will be investigated in more detail.

2 Does Quantum Mechanics Grasp Reality? – The Principle of Reality

Einstein believed that quantum mechanics does not grasp reality adequately, if it does at all. With this assessment, Einstein either became a victim of the supposed positivism of quantum mechanics[1] or of one of the many misunderstandable remarks of Bohr's.[2] In his review of Schilpp's volume on Einstein (Schilpp 1949), C. F. von Weizsäcker remarked already in 1950 (von Weizsäcker 1960, p. 208):

> "Einstein's tragical error seems to be that he thinks this *(i.e., to remove the concept of reality from physics, P.M.)* would happen in quantum mechanics."

We will see that not only Einstein was caught by this error.
In the first years after the formulation of quantum mechanics, the discussions between Bohr and Einstein made it clear that according to quantum mechan-

[1] For example, in Heisenberg (1927), p. 197, the remark following remark is found: "Die Physik soll nur den Zusammenhang der Wahrnehmungen formal beschreiben."

[2] See e.g. the Como lecture, Bohr (1928), p. 46: "Nach der Quantentheorie kommt eben wegen der nicht zu vernachlässigenden Wechselwirkung mit dem Messmittel bei jeder Beobachtung ein ganz neues unkontrollierbares Element hinzu."

ics, in general no direct experimental access to a physical quantity (observable) is possible and that in any measurement it is unavoidable to disturb the object system.[3] In order to exclude all objections of this type, some years later Einstein formulated a criterion of reality, the so-called principle of reality (Einstein/Podolsky/Rosen 1935, 777, emphasis deleted):

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

This formulation of this criterion alone caused a storm of protest among the discoverers of quantum physics: Pauli, Heisenberg and especially Niels Bohr. But let us first see what exactly Einstein's criterion expresses in the context of quantum mechanics.

In the discussion of the 1930s, many physicists emphasized that the measurement process gives rise to an uncontrollable disturbance of the object, due to which no exact predictions are possible. A measurement process connects a quantum system S to a measuring device M by a interaction U that exists for a short period of time. This interaction must be appropriate for the measurement of the observable A under consideration (which in the simplest case is discrete and non-degenerate). In order to guarantee this, at first an unspecified process of interaction (which corresponds to a scattering process) must obey to some postulates. Here, primarily the *calibration postulate* is important.

> "If the preparation φ of the system S is already an eigenstate of the observable $A = \Sigma A_i P[\varphi_i^A]$, that is, if $\varphi = \varphi_n^A$ for a certain n, then the measurement of A must give rise to the result A with certainty. If it is a repeatable measurement, this means that the system S after the measurement is in the state φ_n^A and objectively has the value A_n of the quantity A."

This requirement may easily expressed in formal terms. If Φ is the preparation state of the pointer of the measuring device and $Z = \Sigma Z_i P[(\Phi_i]$ a discrete, non-degenerate pointer observable, then the application of a unitary interaction operator U_A (which is appropriate for the measurement of A) to the tensor product $\Psi(S + M) = \varphi \otimes \Phi$ of the preparation states has the effect

$$U_A(\varphi \otimes \Phi) = \varphi_n^A \otimes \Phi_n.$$

According to this, the pointer eigenvalue Z_n of an appropriate pointer function $f(Z_i)$ belonging to the state Φ_n provides the wanted value $A_n = f(Z_n)$ of the measurement. Here we do always presuppose that we deal with a repeatable measurement, in which the object only is subject to the influences indispensable for a measurement, but not to other disturbances. For the calibrating measurement considered here this means that the state $\varphi = \varphi_n^A$ of S does not change at all.

[3] Later, Bohr reported about the so-called Bohr-Einstein debate in his contribution to Schilpp (1949).

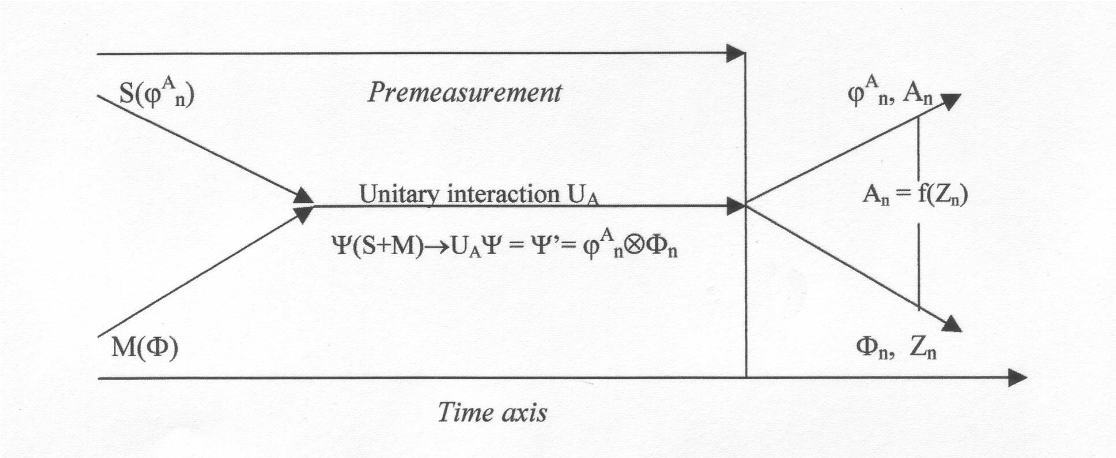

Figure 1: Calibrating Measurement of the Observable $A = \Sigma A_i P[\varphi_i^A]$.

The calibration postulate refers to situations in which the object system is prepared in an eigenstate φ_n^A of the respective observable A. In all other cases, in which the preparation φ of the object is no eigenstate of A, after the interaction the state $\Psi'(S+M) = U_A\Psi'(S+M)$ is a superposition of product states $\varphi_i^A \otimes \Phi_i$ which does not allow to predict the measurement outcome, i.e., the state Φ_n of the pointer and with it the pointer value, with certainty.
Hence, for any repeatable unitary measurement that obeys this calibration postulate the following relation holds. If it is possible to predict for a system S the value A_n of an observable $A = \Sigma A_i P[\varphi_i^A]$ with certainty without changing the system in any way, then the state $\Psi'(S+M)$ of the joint system $S+M$ after the repeatable measurement must be a product state

$$\Psi' = U_A\Psi = \varphi_n^A \otimes \Phi_n.$$

Then, the object system is in state φ_n^A and the pointer M in state Φ_n. From the pointer value Z_n it can be concluded with certainty that after measurement the object system S objectively posses the value $A_n = f(Z_n)$. But, from the explicit form of the state Ψ' of $S+M$ after measurement also follows that the state $\Psi'(S+M)$ of $S+M$ before measurement has the shape

$$\Psi = U_A^{-1}\Psi' = \varphi_n^A \otimes \Phi$$

and that already before the measurement the object system S is in state φ_n^A and has the value A_n. Hence, the property $E(A_n) := P[(\varphi_n^A]$ exists as a real property of S.
Up to different linguistic presentations, however, this relation is exactly Einstein's reality criterion or principle. It is a sufficient criterion for the objective reality of a property $E(A_N)$, which directly follows from the calibration postulate. Therefore, one may say that Einstein's reality criterion does indeed hold in quantum theory. It indicates all the situations in which in spite of all quantum mechanical restrictions make it possible to speak about the objective reality of the properties of a system.

The way in which the quantum physicists felt disturbed by Einstein's reality criterion is hard to understand. As an example, we may cite Niels Bohr's reaction of 1935 (Bohr 1935, p. 65):

> "[...] that the procedure of measurements has an essential influence on the conditions on which the very definition of the physical quantities in question rests. Since these conditions must be considered as an inherent element of any phenomenon to which the term 'physical reality' can be unambiguously applied, the conclusion of the above mentioned authors would not appear to be justified."

Similar remarks of many physicists may be found in the 1930s. Obviously many of them really believed what Einstein was afraid of, namely that the concept of reality does no longer apply in quantum mechanics. An objectively reasonable reaction that might have consisted in an incorporation of this criterion into quantum mechanics is not found anywhere. Indeed, formally such an incorporation became only possible much later. However, these reactions also show that an objectively quite correct observation was rejected without any substantial reasons.

3 Is Quantum Mechanics a Probabilistic Theory? – God Does Not Play Dice!

For Einstein, a particularly annoying feature of quantum mechanics was its supposed probabilistic nature. Since in quantum mechanics it is usually impossible to predict individual events, the (classical) predictions for the outcomes of individual measurements are replaced by statistical propositions. The probabilities or expectation values of measurement results can be calculated in well-known ways according to the laws of quantum mechanics. Since quantum mechanics does in general not determine the individual events but only a large collective of many events, Pauli talks of "statistical causality" in this context, meaning that in quantum mechanics the large collectives still show lawlike behavior (Pauli 1948). Einstein did not so much query the occurrence of probabilities as rather the fact that the quantum mechanical probabilities underly certain laws. In an unpublished draft of a statement to Max Born's contribution to the volume *Albert Einstein: Philosopher–Scientist* edited by Schilpp in 1949 (Schilpp 1949), Einstein wrote:

> "This article is a moving hymn to a beloved friend [...] one who will not believe that God plays dice. In one point, however, Born does me an injustice, namely, when he thinks that I have been untrue to myself in this respect since certain I often availed myself of statistical methods. In truth, I never believed that the foundations of physics could consist of laws of statistical nature." (quoted from Stachel 2002, p. 390).

In a letter of 1942, Einstein made his discontent with the statistical laws of quantum mechanics more precise, indicating that for him it would be much more plausible if the world were completely chaotic and lawless (letter to F. Reiche, 1942; quoted from Stachel 2002, p. 390):

> "I still do not believe that the Lord God plays dice. If he had wanted to do this, then he would have done it thoroughly and not stopped with a plan for gambling [...]. Then we wouldn't have to search for laws at all."

Let us see due to which assertions of quantum mechanics these objections are and whether they actually apply to quantum mechanics. The use of probabilities in quantum mechanics and the loss of an individual causal laws were described by Pauli in terms of "statistical causality", as mentioned. This means the facts essential on quantum mechanics that the outcome of a measurement at an individual object system may be completely undetermined, whereas the behavior of a large number of objects is nevertheless determined by strict statistical law. In a letter to Fierz (26. 09. 1949) Pauli wrote (Pauli 1985b, p. 709):

> "Jenes statistische Verhalten der vielen gleichen Einzelsysteme, die keinerlei Kontakt miteinander haben ("fensterlose Monaden"), ohne doch anderseits kausal determiniert zu sein, ist ja in der Quantenmechanik als letzte, nicht weiter reduzierbare Tatsache aufgefasst."

Consequently, Pauli does not offer any rational explanation for this "irreducible fact". Instead, he attempts to make this non-explainable phenomenon comprehensible in terms of an analogy to the pre-established harmony of Leibniz' windowless monads. For a large number of measurements of a non-objective property at many identically prepared systems, the outcome of each individual measurement is completely undetermined. And even though the many individual systems do not have any contact with each other, together they satisfy a strict statistical law. It is surely unsatisfactory to understand this situation as an "irreducible fact", as Pauli put it.

Much later, in 1983, J. A. Wheeler was searching for an explanation of this at first look quite strange behavior, introducing the expression "law without law" (Wheeler 1983). Even though the individual systems do not obey any recognizable law, Wheeler suspected that for a collective of many systems only due to the large number of many individual systems emerges a strict statistical law. Wheeler illustrates his suspicion for an interference experiment with photons, without giving any explanation in terms of a theoretical or physical deduction. Instead, he remarks that the relation between the individuals without law and the collective determined by laws is established by an unspecified "grand regulating principle".

> "[...] you will be left as much in the dark as I am, what the grand regulating principle is that produces the laws."

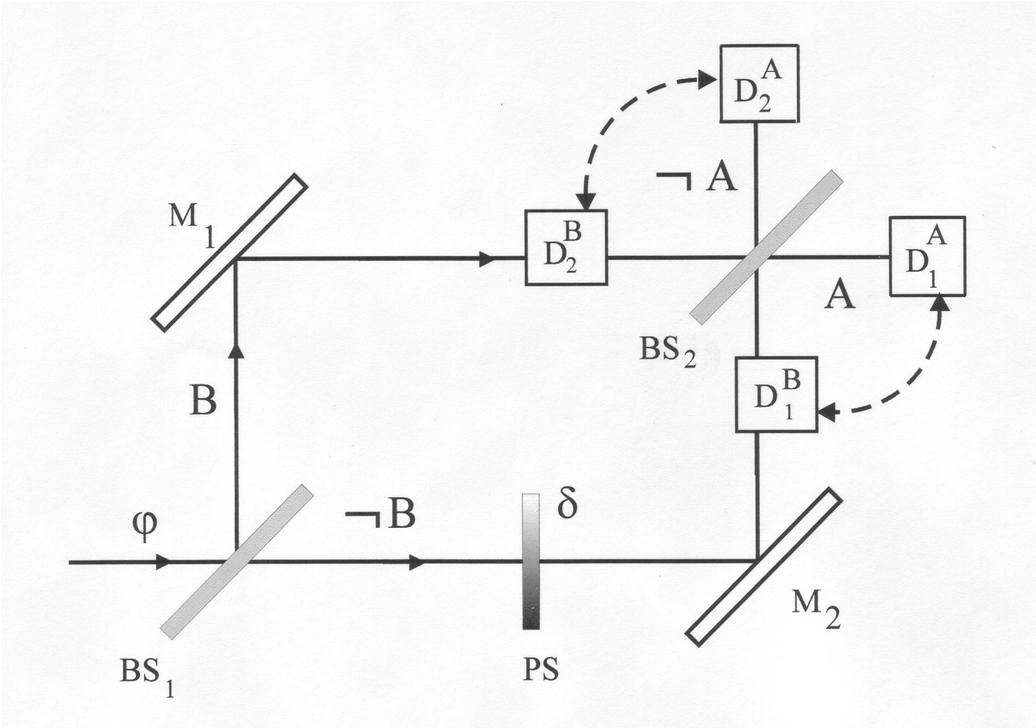

Figure 2: Beam Splitter Experiment in a Mach-Zehnder Interferometer.

However, it is indeed possible to bring light into the dark Wheeler is talking about here and to find a satisfying explanation for a quantum mechanical "law without law". In the following, I will briefly explain this, employing the interference experiment mentioned by Wheeler.

Let us consider a beam-splitter experiment in a Mach-Zehnder interferometer (Fig. 2). A semi-transparent mirror BS_1 splits the state φ of the incoming photon into two orthogonal components $\varphi(B)$ and $\varphi(\neg B)$ which correspond to the two possible paths B and $\neg B$. Then, by means of two totally reflecting mirrors M_1 and M_2 the two partial beams are led to a second semi-transparent mirror BS_2. There, they are recombined with a phase shift δ inserted to the partial beam $\neg B$. Therefore, $\varphi = 1/\sqrt{2}(\varphi(B) + e^{i\delta}\varphi(\neg B))$. By means of placing two detectors D_1 and D_2 in two different positions it is now possible to measure different observables.

If the two detectors are in the positions D_1^B and D_2^B, then one measures which path (B or $\neg B$) the photon has taken. But if the detectors are in the positions D_1^A and D_2^A, then one measures an interference pattern, i.e., intensities that depend on the phase δ.

In this experiment the object system S is prepared in the state φ. Here, two incommensurable observables exist which are not objective in the preparation state φ, namely the path observable **B** with eigenstates $\varphi(B)$ and $\varphi(\neg B)$ and the interference observable **A** with eigenstates $\varphi(A)$ and $\varphi(\neg A)$. The probabilities to find B in D_2^B or $\neg B$ in D_1^B are:

$$p(\varphi, B) = p(\varphi, \neg B) = 1/2.$$

The probabilities to find A in D_2^A or $\neg A$ in D_1^A are:

$$p(\varphi, A) = \cos^2(\delta/2), \quad p(\varphi, \neg A) = \sin^2(\delta/2).$$

This means that the relative frequency of photons arriving at the detector D_1^A is approximately $\cos^2(\delta/2)$ and the relative frequency of arriving at the detector D_2^A is approximately $\sin^2(\delta/2)$.
This is the usual quantum mechanical description of the above experiment. It makes use of the fact that the calculable probabilities, e.g. $p(\varphi, A)$, are reproduced in the relative frequencies of the measurement results, however, it does not explain it. At first it is only certain that the observables $\mathbf{A}$ and $\mathbf{B}$ are not objective in the state φ and that the measurement result of these quantities is completely undetermined for any individual measurement. If for example a repeatable measurement with an appropriate interaction operator U_A is carried out at the system S prepared in state φ, then the state of the system S after the premeasurement, i.e., after switching off the measurement interaction, is given by the mixed state

$$W_s(\varphi, A) = p(\varphi, A)P[\varphi(A)] + p(\varphi, \neg A)P[\varphi(\neg A)].$$

Now, the orthodox interpretation claims without giving any reason that the coefficients $p(\varphi, A_i)$ are the relative frequencies of the measurement results A_i in a large collective of measurements.
However, this claim does not state a new law of probability but a provable theorem. In order to see this, let us consider a large number N of independent, identically prepared systems S with preparation φ^i to be a compound system S^N in the product state

$$(\varphi)^N = \varphi^1 \otimes \varphi^2 \otimes ... \otimes \varphi^N.$$

A premeasurement of $\mathbf{A}$ transforms the state φ^i of every individual system in a mixed state

$$W^i = p(\varphi^i, A)P[\varphi^i(A)] + p(\varphi^i, \neg A)P[\varphi^i(\neg A)].$$

If $\mathbf{A}$ is measured for all of the N systems S_i, then after the premeasurements the individual systems are in mixed states W^i, and thus the compound system S^N is in the product state $(W)^N = W^1 \otimes ... \otimes W^N$.
Now, in the Hilbert space H^N of the compound system S^N an operator $\mathbf{f}_k^N$ "relative frequency of systems with the property A_k" may be constructed, the eigenvalues of which are the relative frequencies of the values A_k within the compound system.[4]
It is easy to see that the expectation value of the operator $\mathbf{f}_k^N$ in the product state $(W)^N$ is given by (φ, A_k). But $(W)^N$ is no eigenstate of the operator $\mathbf{f}_k^N$ with eigenvalue $p(\varphi, A_k)$; that is,

$$T_k^N := tr\{(W)^N(\mathbf{f}_k^N - p(\varphi, A_k))^2\} \neq 0.$$

[4] For details see Mittelstaedt (1998), pp. 47 and Mittelstaedt/Weingartner (2005), chapter 12.

Hence, the relative frequencies of the measurement results A_k are no objective properties of S^N in state $(W)^N$. However, after a lengthy calculation one finds that

$$T_k^N = p(\varphi, A_k)(1 - p(\varphi, A_k))/N.$$

Therefore, in the limit $N \to \infty$ the state $(W)^N$ becomes an eigenstate of $\mathbf{f}_k^N$ and the compound system S^N possesses the relative frequency $p(\varphi, A_k)$ of A_k as an objective property.

In quantum mechanics the following facts are thus to be recognized. The measurement of a quantity that is undetermined in the preparation state of a system gives rise to a measurement result which is by no law determined. Now, by means of mere formal considerations from this follows that in a sufficiently large ensemble of equal measurements the relative frequencies of the measurement results are already determined by the preparation of the object systems and the measured observable. – Strangely enough, however, this is exactly what Einstein wanted. He did not admit that physics is based on independent statistical laws. Instead, he was more ready to accept complete lawlessness. In quantum mechanics the individual measurements are not subject to a law and the probabilistic laws stem from this only due to statistical considerations and the large number of objects. Therefore, the probabilistic laws of quantum mechanics are not independent, but, on the statistical level, they reflect the fact that the individual systems are not determined by any law. – Unfortunately, nobody was able to explain this situation to Einstein at the time when he expressed his thoughts.

4 Is Quantum Mechanics Complete? The EPR Argument

The third objection of Einstein's against quantum mechanics which is discussed up to the present day claims that quantum mechanics is incomplete. It was published by Einstein together with Podolsky and Rosen in 1935 and will be called the EPR argument here (Einstein/Podolsky/Rosen 1935). In his debate with Bohr (see Bohr 1949) Einstein had to admit that in general the direct experimental access to a certain observable is impossible. For several thought experiments suggested by Einstein Bohr was able to demonstrate that the uncertainty relation can not be circumvented. – But Einstein pursued also Heisenberg's thesis of an allegedly unavoidable uncontrollable disturbance of the object system.[5] Einstein took both points of view into account when he formulated his principle of reality mentioned above, which is the first step of the argument in the EPR article of 1935. This principle of reality R was:

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

[5] In 1952 Einstein still writes in a letter to Besso: "[...] indeed every measurement signifies an uncontrollable real intervention in the system (Heisenberg)" (quoted from Stachel 2002, p. 390).

The second argument is less explicitly used. It is a principle of locality that derives from the idea of a local, causally determined relativistic field. The principle of locality L may e.g. be expressed as follows:[6]

> "If two systems can not interact with each other, then a measurement of one system can not change the state of the other system."

Based on the principles of reality R and locality L, the EPR article now attempts to show that quantum mechanics is incomplete. The argument makes use of a thought experiment which was later simplified by Bohm and Aharonov (Bohm/Aharonov 1957). In order to understand it, let us consider two spin 1/2 particles S_1 and S_2 (e.g., proton and neutron) prepared in a ${}^1S_0$ state $\Psi(S_1 + S_2)$ (with the total spin 0), however, which do no longer interact. If a measurement of the spin $\sigma_1(\mathbf{n})$ in direction $\mathbf{n}$ (with eigenstates $\varphi^{(1)}_{\mathbf{n}}, \varphi^{(1)}_{\mathbf{-n}}$) of the system S_1 results in the value $s_1 = +1/2$, then in a subsequent measurement of the spin $\sigma_2(\mathbf{n})$ (with eigenstates $\varphi^{(2)}_{\mathbf{n}}, \varphi^{(2)}_{\mathbf{-n}}$) of the system S_2 in the same direction one with certainty obtains the value $s_2 = -1/2$. If the spin measurement at S_2 is carried out in in another direction $\mathbf{n'} \neq \mathbf{n}$, then for $\sigma(\mathbf{n'})$ the measurement outcome $-1/2$ can no longer predicted with certainty, but only with the probability $p(\mathbf{n}, \mathbf{-n'}) = 1/2(1 + \mathbf{n} \cdot \mathbf{n}')$.

In order to show by means of this thought experiment that quantum mechanics is incomplete, we have to take into account that from a logical point of view both principles we presupposed are implications. Accordingly, we write

$$R = R_1 \rightarrow R_2, \quad L = L_1 \rightarrow L_2.$$

When both partial systems have a sufficiently large distance of each other they can no longer interact. Then, the premiss L_1 of L is satisfied and hence also L_2. Therefore, a measurement of $\sigma_1(\mathbf{n})$ at S_1 can not change S_2. Since furthermore the outcome s_1 of a $\sigma_1(\mathbf{n})$-measurement determines the value $s_2 = -s_1$ of the observable $\sigma_2(\mathbf{n})$, the premiss R_1 of R is satisfied. Hence, R_2 holds, too; that is, the value s_2 von $\sigma_2(\mathbf{n})$ is an objective property of the system S_2 with preparation Ψ. Because this argument may be applied to the spin observables $\sigma_1(\mathbf{n})$ and $\sigma_2(\mathbf{n})$ for any direction $\mathbf{n}$, it has to be concluded that the value s_2 of $\sigma_2(\mathbf{n})$ for any direction $\mathbf{n}$ objectively exists in the system S_2 after preparing the state Ψ.

Hence, one the one hand the value s_2 of $\sigma_2(\mathbf{n})$ in S_2 is objectively determined, even if the observer subjectively does not know it. However, on the other hand quantum mechanics does not permit to determine this value but only its probability from the preparation state $\Psi(S_1 + S_2)$. Therefore, quantum mechanics is not complete. The scheme given in Fig. 3 summarizes this result.

When this result was published, it met a storm of protest from the physicists. For example Pauli was obviously very agitated when he wrote to Heisenberg (Pauli 1985a, pp. 402):

[6] Related to a concrete instance, Einstein/Podolsky/Rosen (1935), 779 put it as follows: "since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system".

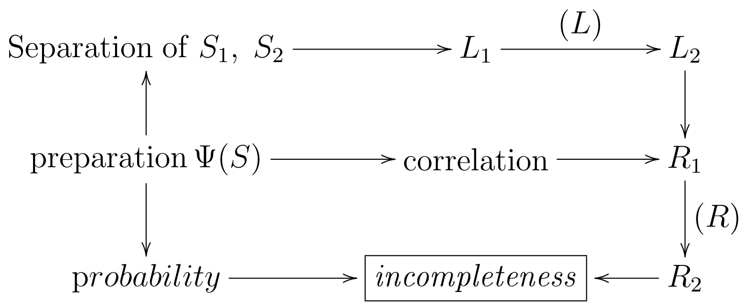

Figure 3: The EPR Argument for Incompleteness.

> "Einstein hat sich wieder einmal zur Quantenmechanik öffentlich geäußert, [...] bekanntlich ist das jedes Mal eine Katastrophe, wenn das geschieht."

And furthermore:

> "Immerhin möchte ich ihm zugestehen, dass ich, wenn mir ein Student in jüngeren Semestern solche Einwände machen würde, diesen für ganz intelligent und hoffnungsvoll halten würde."

And finally:

> "Überhaupt spukt bei älteren Herren wie Laue und Einstein die Idee herum, die Quantenmechanik sei zwar richtig, aber unvollständig. Man könnte sie durch in ihr nicht enthaltene Aussagen ergänzen [...]"

And then Pauli attempts to persuade Heisenberg in this letter to write a "pedagogical reply":

> "Vielleicht könntest Du [...] einmal in autoritativer Weise klarstellen, dass eine solche Ergänzung der Quantenmechanik nicht möglich ist, ohne ihren Inhalt abzuändern."

Even though Heisenberg wrote the draft of such a reply, he never published it, presumably for good reasons.

Countless letters to Einstein and the publications of many authors tried to refute the EPR argument in order to protect quantum mechanics from the stigma of incompleteness. Einstein himself was quite amused that all the writings he received immediately after publication of the EPR article expressed absolute certainty that his arguments were wrong, but all of them gave different reasons (Jammer 1974, p. 187). However, all the critics (and Einstein himself, too) overlooked a little detail in the EPR argument that might have contributed to bringing the controversy to an end on objective grounds.

In order to see this, let us reconsider the last step of the argument that led from R_2 to incompleteness. R_2 states that for every $\mathbf{n}$ the system S_2 has an objective value $s_2 = \{+1/2, -1/2\}$ of $\sigma_2(\mathbf{n})$, with probability $1/2$. Hence, the partial system S_2 is in a mixed state $W_2(S_2) = 1/2P[\varphi^{(2)}_{\mathbf{n}}] + 1/2P[\varphi^{(2)}_{\mathbf{-n}}]$ admitting of an ignorance interpretation, i.e., in a "proper" mixture. This means that the observable $\mathbf{1} \otimes \sigma_2$

related to $S_1 + S_2$ may also be objectified in the compound system $S_1 + S_2$ with preparation Ψ, where $S_1 + S_2$ is in the mixed state

$$W_\Psi(S_1 + S_2) = 1/2P[\varphi^{(1)}_{\mathbf{n}} \otimes \varphi^{(2)}_{\mathbf{\text{-}n}}] + 1/2P[\varphi^{(1)}_{\mathbf{\text{-}n}}] \otimes \varphi^{(2)}_{\mathbf{n}}].$$

Therefore, for a calculation of the expectation values of the compound system the states Ψ and W_Ψ are equivalent. This claim may easily be checked. For the specific test observable

$$B(\mathbf{n'},\mathbf{n''}) := \sigma_1(\mathbf{n'}) \otimes \sigma_2(\mathbf{n''}) = \Sigma_{ik} B_{ik} P_{ik}(\mathbf{n'},\mathbf{n''})$$

with

$$P_{ik}(\mathbf{n'},\mathbf{n''}) := P[\varphi^{(1)}_{i\mathbf{n'}} \otimes \varphi^{(2)}_{k\mathbf{n''}}] \qquad i, k \in \{+, -\},$$

the expectation values with regard to Ψ and W_Ψ must be identical,

$$tr\{P[\Psi]P_{ik}\,(\mathbf{n'},\mathbf{n''})\} = tr\{W_\Psi \cdot P_{ik}\,(\mathbf{n'},\mathbf{n''})\}.$$

After a short calculation, from this derives

$$\mathbf{n'} \cdot \mathbf{n''} - (\mathbf{n} \cdot \mathbf{n'})(\mathbf{n} \cdot \mathbf{n''}) = 0 \qquad \text{(VO)}$$

as the condition for value objectivation (VO) of σ_1 and σ_2. Except for some very specific triples (**n, n' n''**), this equation is violated in quantum mechanics. Hence, the EPR argument does *not* result in the incompleteness of quantum mechanics, but in a *contradiction*. An elementary calculation shows that Bell's inequalities (BELL), which contradict to quantum mechanics for appropriate triples of values, derive from (OV):

$$\mid \mathbf{n'} \cdot \mathbf{n} - \mathbf{n''} \mid \leq \mathbf{n} \cdot (\mathbf{n} - \mathbf{n''}); \qquad \mid \mathbf{n'} \cdot \mathbf{n} + \mathbf{n''} \mid \leq \mathbf{n} \cdot (\mathbf{n} + \mathbf{n''}) \qquad \text{(BELL)}$$

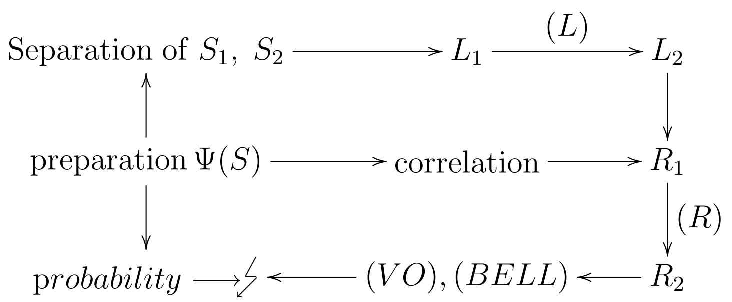

Figure 4: The EPR Argument Gives Rise to a Contradiction.

It is obvious that without knowing Bell's inequalities nobody could refute the EPR argument by good reasons. But Bell's inequalities were only discovered in 1964, that is, almost 30 years after the EPR paper and 10 years after Einstein's dead in 1955. Bell clearly recognized that the incompleteness thesis concerning quantum mechanics can only be refuted by a "NO GO theorem" for additional hidden parameters (Bell (1964), p. 195):

> "[...] additional variables were to restore to the theory causality and locality. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics."

Today, Bell's original proof for this suspicion may be presented in the strongly simplified version given above.
Of course the contradiction revealed by these considerations must be eliminated. Whereas in the years after 1935 primarily the principle of reality was at stake, now it became clear that the principle of locality must be abandoned. Due to this insight, the fundamental non-locality of quantum mechanics entered into the physicists' general consciousness. Furthermore, the non-locality of quantum mechanics was convincingly confirmed since the 1980s up to the present day by a tremendous number of experiments that were increasingly improved.[7]
However, all this was unknown when the EPR argument was published in 1935. Therefore, at that time nobody was able to put forward any convincing counter-arguments against Einstein. Therefore, the objections against EPR were made of lumpy arguments and philosophical conjectures such as the assumption that the principle of reality is untenable in quantum mechanics. Einstein's ironic remark that the authors of all letters to him were convinced that the EPR argument is wrong, by giving all completely different reasons for this, is definitely due to the lack of argument of his opponents.

5 Concluding Remarks

In the years between 1925 and 1955, Einstein again and again put forward arguments against quantum mechanics because he had the impression that this theory does not sufficiently refer to reality; that it is based on probabilistic laws; and that it is therefore incomplete, even though not wrong. Einstein must obtained this impression from the presentations of quantum mechanics in the first decade after its discovery, since they considered a positivistic interpretation of the theory to be unavoidable[8] and the unresolvable connection of object and subject in what Bohr called a "phenomenon" to be essential for any understanding of quantum mechanics.
Hence, Einstein's objections against quantum mechanics are not directed against the theory, as we know and understand it today, but against an interpretational phantom that emerged from several mystifications in the early years of quantum mechanics. Concerning them, Einstein's objections were absolutely justified. However, at the time when Einstein made these objections it was impossible to refute them for good reasons, because the formal tools needed for doing so were not yet available. But these technical deficiencies were not recognized by the advocates of the quantum physics of these days; and this is why many of them

[7] Here, I think of the experiments from Aspect (1982) up to Zeilinger et al. (see Weihs et al. 1998).
[8] See Heisenberg (1969), pp. 91–100.

thought to be able to refute Einstein by lumpy arguments. The refutation campaign against the EPR argument clearly illustrates this observation.
The principle of reality formulated by Einstein is satisfied in quantum mechanics, even though neither Einstein nor his opponents were able to recognize this. His criticism of the probabilistic structure of the theory is irrelevant, too, since just his ideas about chaos and lawfulness in quantum mechanics are realized. But this could also not be known by anybody in Einstein's days. Only the reproach of incompleteness is objectively wrong. However, an adequate refutation became only possible 30 years later.

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