The Yield of Ten-Year T-Bonds: Stumbling Towards a ‘Good’ Forecast

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Abstract

Due to their status as “the” benchmark yield for the world’s largest government bond market and its importance for US monetary policy, the interest in a “good” forecast of the constant maturity yield of the 10-year U.S. Treasury bond (“T-bond yields”) is immense. This paper assesses three univariate time series models for forecasting the yield of T-bonds: It shows that a simple SETAR model proves to be superior to the random walk and an ARMA model. However, dividing the sample of bond yields, dating from 1962 to 2005, into a training sample and a test sample reveals the forecast to be biased. A new bias-corrected version is developed and forecasts for March 2005 to February 2006 are presented. In addition to point estimates forecast limits are also given.

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1. Introduction

Due to their status as "the" benchmark yield for the world's largest government bond market and its importance for US monetary policy[1] (see Fleming 2000), the interest in a "good" forecast of the constant maturity yield of the 10-year U.S. Treasury bond ("T-bond yields") is immense. Hence, it does not surprise that a large body of literature is devoted to forecasting T-bond yields.[2] If a random walk process describes bond yields accurately, then much of the efforts devoted to forecasting stock returns and bond yields are of questionable value.[3] However, the literature (see Aburachis/Kish 1999) shows that bonds yields do not follow a pure random walk. Theoretical and empirical evidence exists – at least for the long term – for the dependence of interest rates on fundamental factors (see Warnock/Warnock (2005), Hoffmann/MacDonald (2006)).

The goal of the present study is to provide additional technical insight in the statistical behaviour of 10-year T-bonds. It describes step-by-step the approach towards a "good" technical forecast, and also discusses some intermediary simple models with less forecasting accuracy. The simplicity might be appealing in situations where the effort to establish and maintain the forecast is considered disproportional to the increase in power. Three univariate time series models for forecasting the yield of 10-year Treasury bonds are assessed. The paper is structured as follows: Section 2 gives a short survey of the statistical characteristics of T-bond yields. Section 3 discusses different methods and models to forecast these characteristics. It is shown that a simple SETAR (Self-Existing Threshold Autoregressive – SETAR) model proves to be superior to the random walk and an ARMA model.[4]

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[1] See e.g. Greenspan (2005), Poole (2002), and Kliesen (2005). Diebold et al. (2005) deliver a survey about modelling bond yields; Fleming/Remolona (1997) do the same for drivers of the US Treasury bond market. Wu (2005) shows the importance of U.S. Treasury Bonds for institutional investors and central banks. The existing empirical literature approaches the problem of bond yield determination via (1) exploring fundamental factors, (2) high-frequency data, (3) international transmission of shocks with respect to bond markets, and (4) combinations of bond modelling strategies from a finance and macro perspective. See Clostermann/Seitz (2005, 2) and the corresponding references. A large part of the literature covers forecasting the yield curve and not specifically the 10-year yield of U.S. Treasury bonds. See e.g. Bernadell et al. (2005).

[2] See e.g. Ilmanen (1997). Cambell (1995) gives an overview of the U.S. yield curve.

[3] How difficult it is to forecast systematically more accurately than a random walk long-term interest rates shows the quarterly evaluation of the corresponding forecasts of German financial institutions by the German think tank ZEW. See http://www.zew.de/de/publikationen/bankprognosen/bewertungprognose.php.

[4] SETAR models belong to the staple of standard financial econometrics textbooks. See e.g. Brooks (2002).

However, dividing the sample of bond yields, dating from 1962 to 2005, into a training sample and a test sample reveals the forecast to be biased. A new bias-corrected version is developed and forecasts for March 2005 to February 2006 are presented. In addition to point estimates forecast limits are also given. Section 4 gives some critical comments on the practical use of model-based forecasts of T-bond yields.

2. A First Glance at T-Bonds Yields

We analyze 10-year T-bond constant maturity yields between February 1962 and February 2005. The 517 observations are monthly average yields (see figure 1).

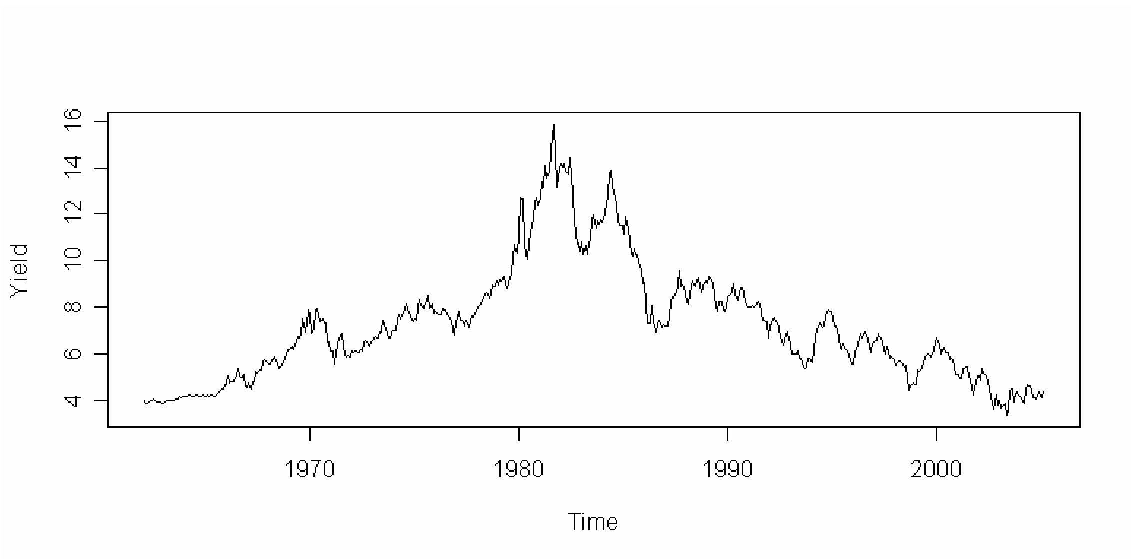

Figure 1: Average monthly T-bond yield, Feb. 1962 through Feb. 2005 , Source: Bloomberg.

What does figure 1 show? Yields develop smoothly until 1966; afterwards, oscillations get more and more pronounced and a positive trend becomes visible as well. Between 1979 and 1986 yields reach their highest volatility and reach their peak in the early 1980s. After 1987 the development is similar to that between 1967 and 1979, with a negative trend, however.

Today, there is a consensus that this evolution of yields mirrors the level and volatility of inflation expectations due to successes and failures of monetary policy to anchor inflation expectations: Various misconceptions of monetary policy makers about the macro economy and the monetary transmission mechanism allowed inflation to get out of control in the 1960s and 1970s with the result of increasing bond yields; and the fight against inflation and the pursuit of price stability led to a reversal of the positive trend in bond yields since the early 1980s.[5]

The behaviour of yields can be described further with the spectral density, i.e. on basis of the periodogram (see figure 2). Here, seasonal effects are not visible – the exponential increases towards low frequencies, i.e. long swings indicate a trend component.

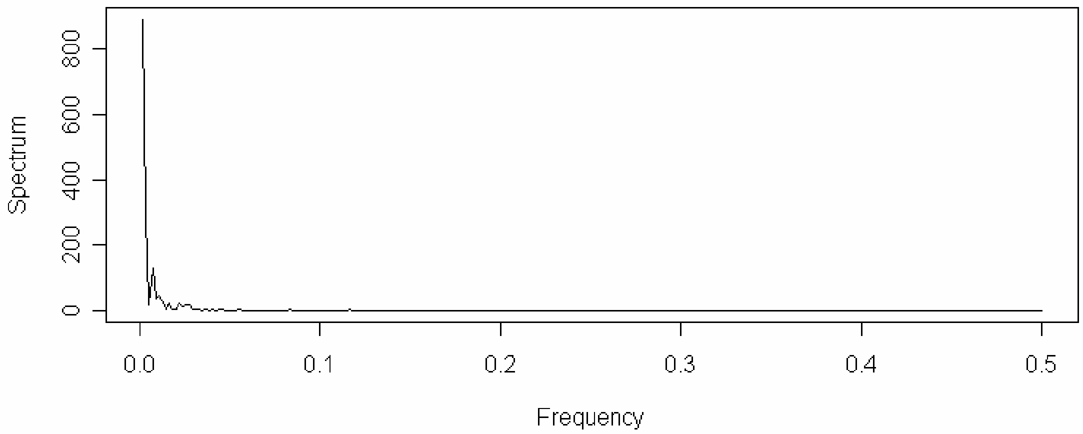

Figure 2: Periodogram of T-bond yields.

Trends are usually eliminated by considering returns, rather than the original yields. Figure 3 displays the daily log-returns $\log(yield_t/yield_{t-1}) = \Delta\log(yield)$ and their corresponding periodogram. We find seasonal components for 4.7, 5.6, 6, 8.6 and 36 months, e.g. the inverse of the 36-month period is 0.028 and is the location of the first peek in the lower picture of figure 3. An economic reason is beyond our experience; hence we refrain from filtering these seasonal effects with band-pass-filters. Additionally, doing so would reduce the amount of information contained in the data. Alias-effects could not be ruled out either.

A second possibility to eliminate trends is differencing. For the first difference we find almost identical spectral characteristics as for the log-returns. We plan to find a sparse model to enable efficient forecasting. For this purpose, an important intermediary step is a stationary time series which does not change its statistical behaviour over time. We consider only weak stationarity, i.e. the mean function must be constant and the covariance function depends on the lag – the distance between two series values. Unfortunately, many financial time series are not stationary. A persistent positive trend for yields is economically not obvious, in contrast e.g. to stock prices, and inspection of

[5] See Meltzer (2005), Nelson (2004), Goodfriend (1993), Bernanke (2006), Wheelock (2005), Guidolin (2005), Bordo/Dewald (2001).

figure 1 exhibits no such persistent trend. However, the volatility does not appear to be constant and apart from this exploratory assessment we will perform a test.

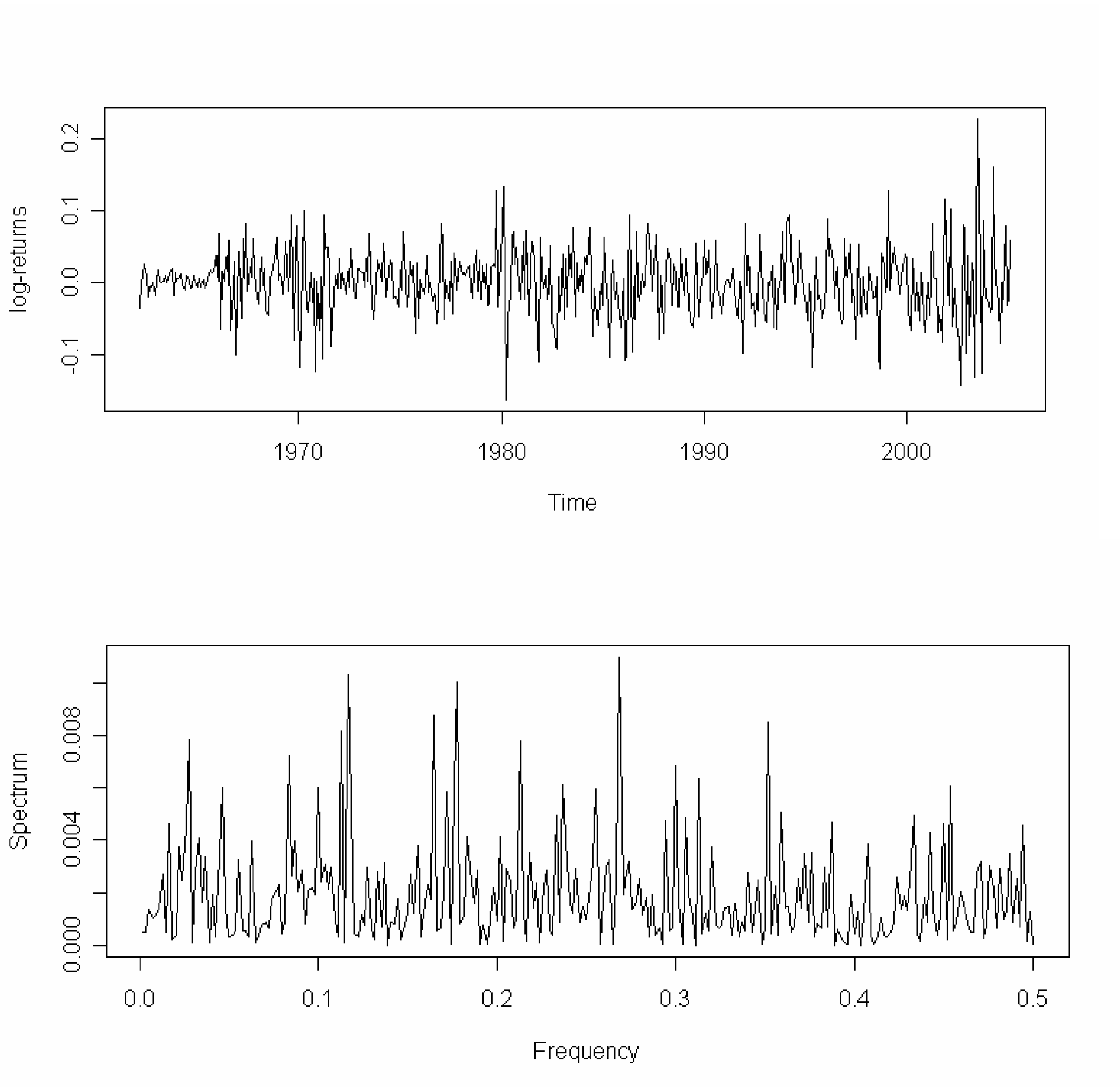

Figure 3: Log-rates of return for T-bond yields and periodogram.

The industry standard to test for stationarity is the Dickey-Fuller test (Dickey and Fuller (1979), Hamilton (1994)). In brief, one restricts to testing the mean stationarity and focuses on a potential stochastic trend. In an AR-process stationarity, e.g a missing trend, corresponds to the existence of a unit root. This can be is tested for with a simple t-test using an auxiliary regression. In this context, T-bond yields are fitted to an AR-process and we use the coefficient of determination R^2 for assessing the fit of the model. By R^2 one measures in economcetrics the extent to which the variation of the dependent variable is explained by the model (see e.g. Theil (1971)). In time series analysis the use R^2 has been critizised (Franses (1998)) and modified model selection criteria have been proposed (Harvey (1989)). However, due to its attractive interpretation be restrict ourselves here to R^2. For an AR(2)-process R^2 is for our series 98.1%. An AR(2) process is a reasonable model and the Dickey-Fuller test is applicable, its p-value of 0.43 does no allow rejecting the hypothesis of a unit-root. We cannot prove that yields are stationary for this model.

A glance at the yields in figure 1 shows that mean and variance are time-dependent. We try to find transformations of the yield series that result in a stationary series. Here, difference filters $Y_t = \Delta^d X_t = (1-B)^d X_t$ of order *d* are applicable as well as seasonal filters $Y_t = \Delta_s^D X_t = (1-B^s)^D X_t$. *B* denotes the back shift operator. Combining the filters results in $Y_t = \Delta_s^D \Delta^d X_t = (1-B^s)^D (1-B)^d X_t$ and $Y_t = \Delta^d \Delta_s^D X_t = (1-B)^d (1-B^s)^D X_t$, respectively . Prior to finding optimal *d* and *D*, we focus on variance stabilization which is infeasible after filtering. As a typical method we use a Box-Cox transformation.

The selection of the parameters *d* and *D* follows the minimization of the series variance.[6] Tables A1 and A2 in the appendix show the results for *d*=1,2,3 and *D*=1,2,3. Interestingly, the simple first difference, i.e. *d*=1 and *D*=0, is optimal. Figure 4 shows the differenced time series of Box-Cox transformed yields and reveals a stationary behaviour (after an initial regime of relatively low volatility). Stationary models are more plausible for those series, although application of the Dickey-Fuller test is not possible due to the small R^2 of the fitting AR(1)-process.

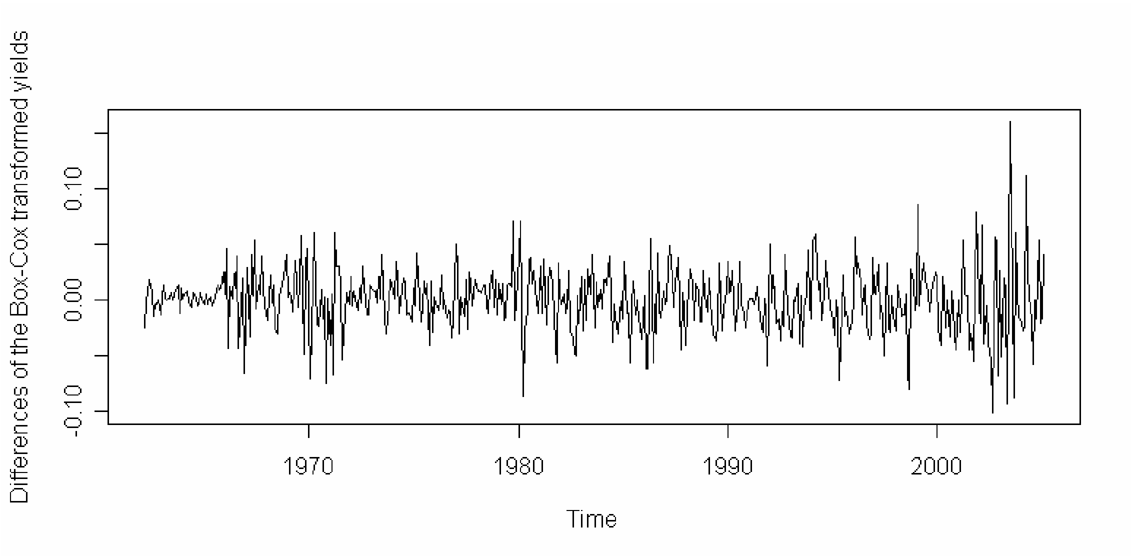

Figure 4: Differences of the Box-Cox transformed yields.

[6] See Schlittgen/Streitberg (1997) for an explanation of the modelled series variance relative to the original variance as a measure of stationarity.

Table 1: Stationary assessment of yields, transformed yields and differences.

Time series	Stationarity	Evaluation
$Yield$	Stationarity not accepted	Dickey-Fuller-Test
$\log(yield)$	Stationarity not accepted	Dickey-Fuller-Test
$\frac{1-yield^{-0.25}}{0.25}$	Stationarity not accepted	Dickey-Fuller-Test
$\Delta yield$	Stationarity acceptable	Exploratory
$\Delta \log(yield)$	Stationarity acceptable	Exploratory
$\Delta \frac{1-yield^{-0.25}}{0.25}$	Stationarity acceptable	Exploratory

The second series transformation to achieve homoskedasticity – apart from the Box-Cox transformation – is the log-transform. Here again, stationarity can not be proven (Dickey-Fuller p-value=0.44 for AR(1) with R^2=98.2%). Differencing results in stationarity as far as pure description is taken into account. Table 1 summarizes the stationarity assessment for the transformations.

3. Forecasting US-Treasury Yields: Methods, Models, and Results

In this section we develop and assess forecasts based on six formulations. We fit several univariate time series models to T-bond yields and compare them in terms of their prognostic power. First, we divide the data set into a training sample and a test sample. The test sample is over the period of June 1992 to February 2005; it contains 172 observations. Our goal is to deliver forecasts for the one-to-twelve-month period – this is the "standard" forecasting horizon, h, of financial institutions[7]. The goodness-of-fit for the forecasts is evaluated using R^2. The autoregression (x_t, x_{t+h}) – to which R^2 refers to – evolves over t in the test sample (until t+h exceeds February 2005). Additionally, we estimate the bias. All models are applied to all transformations introduced in the preceding paragraph.

Motivated by no-arbitrage arguments, our baseline model for the yields is a random walk. In terms of the differenced series this is equal to a white noise process. We find a value for R^2 of around 94% for the one-month horizon and of 51% for the one-year horizon in the case of interest rates, log-rates and Box-Cox-transformed rates. The first differences of the three transformations deliver values for R^2 of around 0.2%. Unbiased

[7] See e.g. the published forecasts of the large German asset manager DekaBank: http://www.dekabank.de/db/de/economics/publikationen/index_pub.jsp?CATEGORY_ID=2.

estimation is only achieved on the one-month horizon and only for the log-rates and Box-Cox-transformed rates.

Our second model is an autoregressive moving average (ARMA(p,q)) process. A first device to assess the appropriateness of the model stems from the (partial) autocorrelation function. Here, figure 5 shows the estimates for T-bond yields. The results for the log-rates and the Box-Cox-transformed ratres are similar. The picture reveals an AR(p)-structure for p=1 or 2. The MA-component seems to be negligible as the partial autocorrelations (apart from the first) lie in the 95% confidence interval for no correlation.

Assessing the goodness-of-fit of this second model via R^2, however, is rather disappointing. We find values for R^2 of around 94% for the one-month horizon and 51% for the one-year horizon in the case of interest rates, log-rates and Box-Cox-transformed rates. These are almost identical to those of the random walk model. On the 8-12 month horizons, the forecast is better, but only by around 0.2%. Again, only the one-month forecast for the log-rates and the Box-Cox-transformation is unbiased. The differences do not show any ARMA-characteristics. Figure 6 shows that white noise might be a sufficient model and we refrain from further fitting. On average the ARMA(p,q) model does not show to be superior to the random walk.

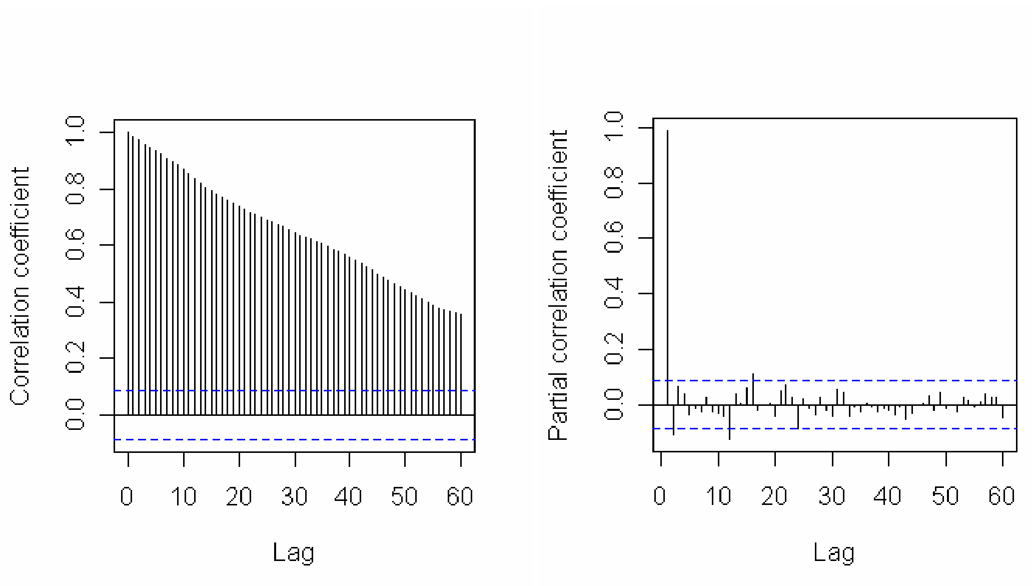

Figure 5: Autocorrelation function (left), partial autocorrelation function of T-bond yields, and 95% limits for lacking correlation.

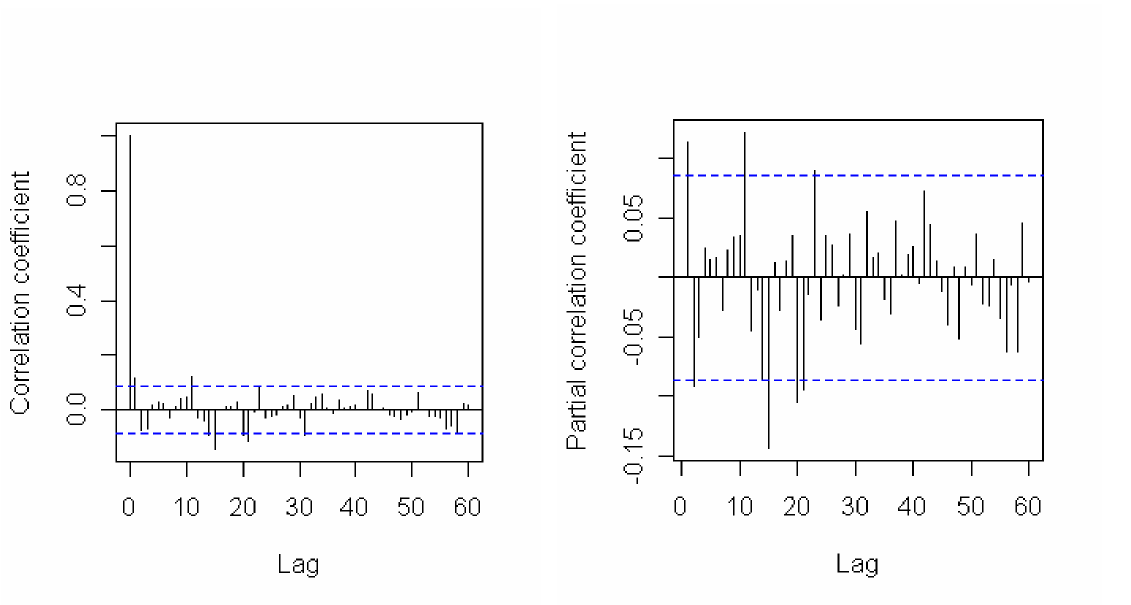

Figure 6: Autocorrelation function (left), partial autocorrelation function of the first difference of T-bond yields, and 95% limits for lacking correlation.

Our aim is an unbiased estimation of T-bond yields for all horizons up to 12 months. However, the first two simple models do not prove to satisfy this goal. Presumably, the non-linearity of economic growth prohibits a linear time series model. Therefore, the question is which methodological model extension may be adequate.

Using exploratory methods for yields (see Weißbach 2006) we find that two groups are present. An economic interpretation for this characteristic of T-bond yields might be structural breaks. We refrain from dating those breaks (as e.g. Zeileis et al. 2005). We rather use threshold autoregressive models (TAR) as proposed by Tong and Lim (1980). Here, we restrict ourselves to the self-existing TAR (SETAR) as described in Priesley (1988).

A times series (X_t) follows a SETAR$[l, k_1, \ldots, k_l]$ model if it is described by

$$X_t = \alpha_0^{(j)} + \sum_{i=1}^{k_j} \alpha_i^{(j)} X_{t-i} + \varepsilon_t \text{ für } X_{t-d} \in R^{(j)}, \; j = 1, \ldots, l,$$

where $R^{(j)}$ are level sets and d is a lag parameter. Due to our explorative finding of two groups we use two level sets. An additional reason for the use of just two sets is the danger of oversmoothing and the burden of computational effort in the use of many sets. The SETAR[$2, k_1, k_2$] model is written as

$$X_t = \begin{cases} \alpha_0^{(1)} + \alpha_1^{(1)} X_{t-1} + \ldots + \alpha_{k_1}^{(1)} X_{t-k_1} + \varepsilon_t & \textit{für} \quad X_{t-d} \le r \\ \alpha_0^{(2)} + \alpha_1^{(2)} X_{t-1} + \ldots + \alpha_{k_2}^{(2)} X_{t-k_2} + \varepsilon_t & \textit{für} \quad X_{t-d} > r \end{cases},$$

where ε_t is the white noise process with $E(\varepsilon_t) = 0$, $Var(\varepsilon_t) = \sigma^2$, and r is the threshold.

The estimation of parameters follows an iterative procedure: First, for initial values of d, r, k_1 and k_2, we achieve estimates for the α's via the method of least squares. Second, for given d and r, minimizing Akaike's information criterion leads to estimates for k_1 and k_2. The last step is repeated for all possible values of r, i.e. over the co-domain of (X_t). Minimization leads to the optimal r. The most outer loop derives the estimate of d.

Evaluating the prognostic power follows the same principle as for the first two models. For the ease of reading, however, some additional complications are not described here. Results for the SETAR model differ now from those of the first two models: The one-month horizon model – and only that – results in an unbiased forecast using the original yields. Astonishingly, the same is true for the forecast for all horizons for the differenced yields. The value of R^2 is 1.4% "better" for the one-year horizon than the random walk as a model for the yields; R^2 for the first difference is more than four times better than for the white noise model. Additionally, the bias for the forecasts on the longer horizons is lower as well. The log- and Box-Cox-transformations are worse for this model. Hence, we do not consider them here anymore.

So far we have mainly compared models. However, our ultimate goals are the out-of-sample forecasts for one month to one year. R^2 suggests – consistently over all time horizons – that the SETAR model using the original yields is superior to all other models. Unfortunately, the forecasts are biased starting from the two-month horizon. As a bias is a very serious draw-back in practice, we need to enhance our model and use the fact that the forecasts for all horizons are unbiased for the differenced series. The bias is displayed in figure 7 for the one-month and one-year horizon.

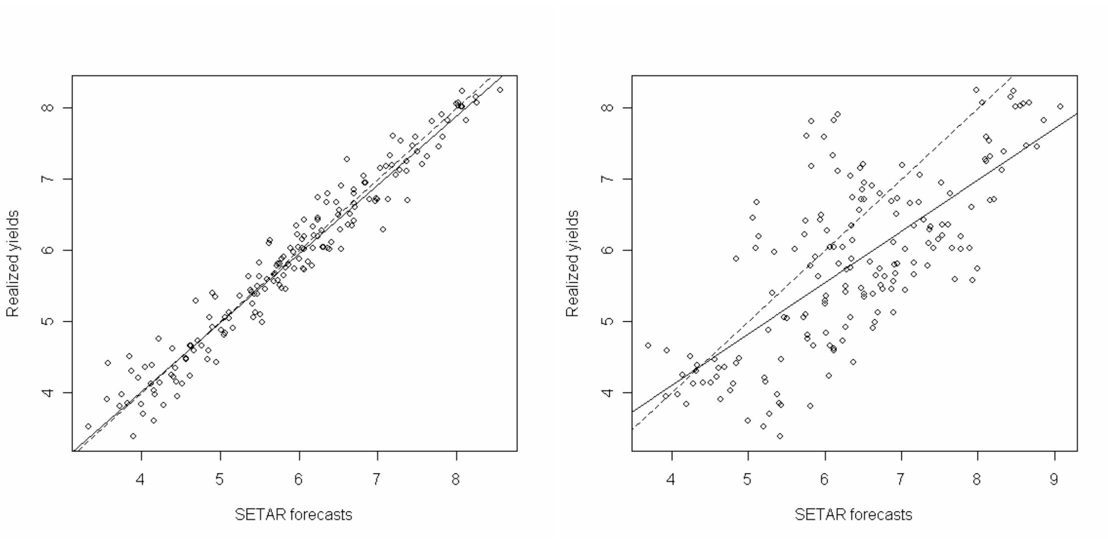

Figure 7: Linear regression of SETAR forecast and realized yields (solid lines) based on the test sample for the one-month horizon (left) and one-year horizon (right). Dotted bisecting lines represent the unbiased estimates.

Figure 7 shows that the bias is approximately linear and we can correct for the bias essentially by turning the regression line onto the bisecting line. As a result we need to estimate the slope and the intercept in the representation $X_t = c + c_1 \hat{X}_{t;h} + \eta$.[8] In practice, this model is proven to be unstable because the linear coefficients c und c_1 change their values over the time.

Another model shows better results. Here we correct the forecasts through summation with the expectation of the residual mean, which we can calculate from the first part of the test sample. The final forecast is of the form $\hat{X}_{t;h}^{bias-corr} = \widehat{E(\varepsilon)} + \hat{X}_{t;h}$.

Along with the point estimate, confidence intervals provide further insight into the model-dependent forecast uncertainty. The Kolmogorov-Smirnov-test for the model fit shows that the residuals for all the horizons are normally distributed. This enables us to derive the constant volatility $\sigma_{t;h}$ for each time horizon. Figure 8 visualizes the procedure. For the second part of the test sample, corrected forecasts based on the training sample, their 95% confidence limits $\hat{X}_{t;h}^{bias-corr} \pm z_{1-\alpha/2}\sigma_{t;h}$, and the observed yields are displayed, where $z_{1-\alpha/2}$ is the $1-\alpha/2$ - quantil of the normal distribution.

[8] For another application of this bias-correction method in financial econometrics see Fan (2005).

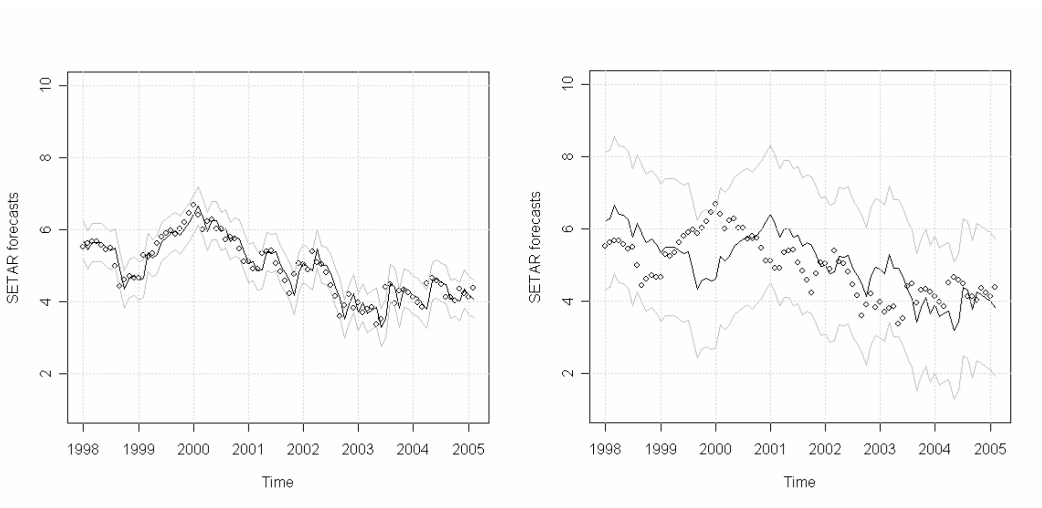

Figure 8: Bias-corrected SETAR forecasts (solid line), 95%-confidence limits (grey line) and observed values (bubbles) for the test sample: One-month horizon (left) and one-year horizon (right).

4. Where Does This Leave Us?

Starting point for the discussion is the out-of-sample forecast for the last twelve months for the period March 2005 through February 2006, based on the SETAR model

$$\begin{cases} yield_t = 6.60 + 1.28\,yield_{t-1} - 1.51\,yield_{t-2} + 0.54\,yield_{t-3} + 0.25\,yield_{t-4} + \varepsilon_t,\ yield_t \geq 13.763 \\ yield_t = 0.07 + 1.11\,yield_{t-1} - 0.14\,yield_{t-2} - 0.01\,yield_{t-3} + 0.08\,yield_{t-4} - 0.05\,yield_{t-5} + \varepsilon_t,\ yield_t < 13.763 \end{cases},$$

where ε_t is a white noise process with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = const$. The parameters are estimated according to the procedure described in section 3. Table 2 displays the forecasts together with the bias-corrected modification described in the last paragraph.

Table 2: Out-of-Sample forecasts based on SETAR model.

Month	SETAR-forecast	Bias-corrected SETAR-forecast	95%-forecast interval	Observed values
March 2005	4.455	4.411	[3.739; 4.084]	4.481
April 2005	4.480	4.373	[3.371; 5.374]	4.198
May 2005	4.495	4.323	[3.099; 5.547]	3.981
June 2005	4.530	4.314	[2.918; 5.710]	3.913
July 2005	4.562	4.305	[2.746; 5.864]	4.276
August 2005	4.591	4.282	[2.572; 5.993]	4.014
September 2005	4.618	4.259	[2.408; 6.110]	4.324
October 2005	4.646	4.244	[2.257; 6.230]	4.551
November 2005	4.673	4.232	[2.122; 6.342]	4.484
December 2005	4.701	4.224	[1.994; 6.453]	4.391
January 2006	4.727	4.217	[1.870; 6.563]	4.515
February 2006	4.754	4.210	[1.735; 6.685]	4.551

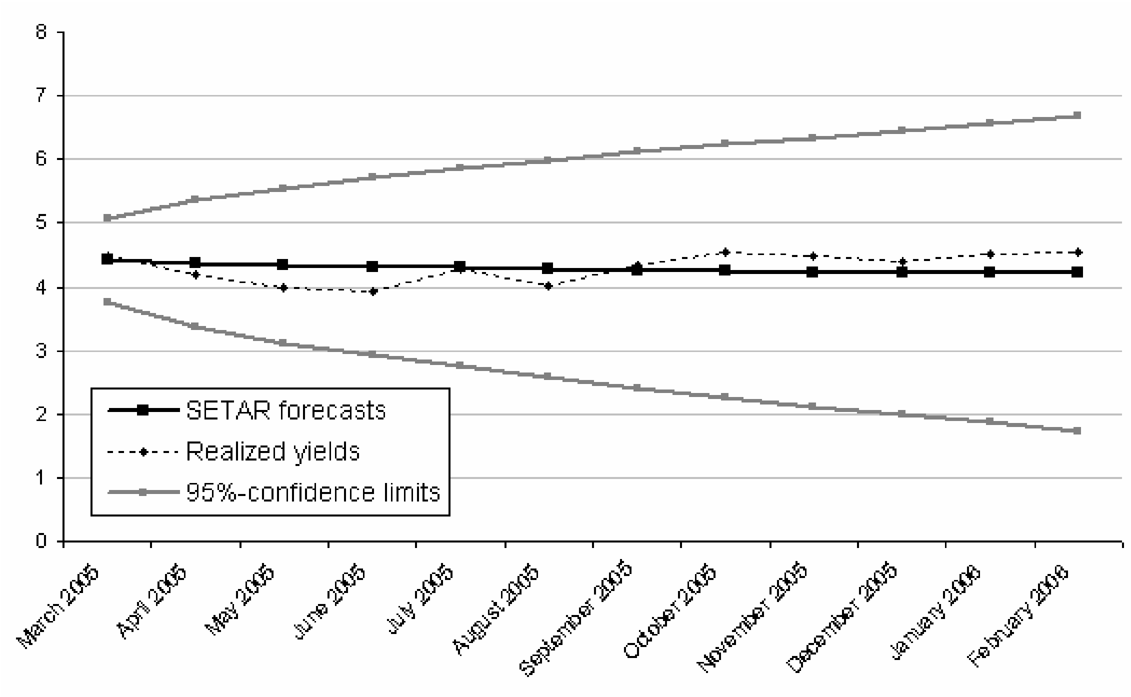

Figure 9: Out-of-Sample forecasts based on SETAR model.

Figure 9 shows the bias-corrected SETAR forecast, the actual observed values, and the upper and lower 95%-forecast intervals for the T-bond yield. The picture clearly shows how difficult it is to give a "good" forecast for bond yields. Although the model performs quite well for the period September 2005 to February 2006, the model could not account for the steeper decrease of interest rates in the period of March 2005 to June 2005. In fact, our model forecasted a persistent, moderate negativ trend in bond yields.

In this context, we want to make two critical points regarding the practical use of our models and results: First, even if the model would deliver an accurate forecast of the path of long-term interest rates, one has to ask what value such a forecast would have for an investor, given that the forecast changes in yields are relatively small and are practically undistinguishable from a random walk. In this context, the interplay between arbitrage and modelling is crucial. Because, if every "right" model gives at least theoretically the possibility to infinite gains, it provokes a countervailing force via the channel of "supply and demand". For this reason, in efficient markets "right" models only have validity for a very short term. This helps to explain the minimal deviations of the models' results from a random walk.

Second, particularly the period from March 2005 to June 2005 shows how cautiously strict-model based forecasts like ours have to be evaluated in practice. During this period the “conundrum” of decreasing bond yields in the wake of interest rate hikes of the Federal Reserve puzzled central banks and investors as well (see Wu 2005). Part of an explanation for this puzzle are changes in the institutional environment (e.g. the purchases of T-bonds by foreign central banks) (see Rudebusch et al. 2006 and Krozner 2006). Institutional features obviously matter when making a forecast. Institutional factors and unforeseen structural breaks make forecasting bond yields not only a science, but also an art in that “good” interest rate forecasting needs besides good econometric modelling a lot of judgement. Models like ours can certainly be only serving as guidelines for forecasting bond yields in practice.

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5. Appendix

$(1-B^s)^D(1-B)^d$						
$100 \cdot s_y^2 / s_x^2$	$(1-B^2)^0$	$(1-B^2)^1$	$(1-B^2)^2$	$(1-B^3)^0$	$(1-B^3)^1$	$(1-B^3)^2$
$(1-B)^0$	100	4.01	8.73	100	5.68	12.10
$(1-B)^1$	1.91	4.27	13.31	1.91	3.85	11.58
$(1-B)^2$	3.64	8.33	26.35	3.64	6.00	20.81
$100 \cdot s_y^2 / s_x^2$	$(1-B^4)^0$	$(1-B^4)^1$	$(1-B^4)^2$	$(1-B^5)^0$	$(1-B^5)^1$	$(1-B^5)^2$
$(1-B)^0$	100	7.34	15.73	100	9.09	19.18
$(1-B)^1$	1.91	3.73	11.23	1.91	3.97	12.03
$(1-B)^2$	3.64	6.89	20.37	3.64	7.61	22.58
$100 \cdot s_y^2 / s_x^2$	$(1-B^6)^0$	$(1-B^6)^1$	$(1-B^6)^2$	$(1-B^7)^0$	$(1-B^7)^1$	$(1-B^7)^2$
$(1-B)^0$	100	10.72	21.87	100	12.30	24.48
$(1-B)^1$	1.91	3.90	11.71	1.91	3.90	11.52
$(1-B)^2$	3.64	7.15	21.21	3.64	7.52	22.95
$100 \cdot s_y^2 / s_x^2$	$(1-B^8)^0$	$(1-B^8)^1$	$(1-B^8)^2$	$(1-B^9)^0$	$(1-B^9)^1$	$(1-B^9)^2$
$(1-B)^0$	100	13.82	28.09	100	15.47	32.53
$(1-B)^1$	1.91	3.70	11.00	1.91	3.56	10.33
$(1-B)^2$	3.64	7.25	22.01	3.64	6.80	19.54
$100 \cdot s_y^2 / s_x^2$	$(1-B^{10})^0$	$(1-B^{10})^1$	$(1-B^{10})^2$	$(1-B^{11})^0$	$(1-B^{11})^1$	$(1-B^{11})^2$
$(1-B)^0$	100	17.42	38.07	100	19.30	43.03
$(1-B)^1$	1.91	3.93	11.85	1.91	3.62	10.25
$(1-B)^2$	3.64	7.93	24.32	3.64	6.74	19.02
$100 \cdot s_y^2 / s_x^2$	$(1-B^{12})^0$	$(1-B^{12})^1$	$(1-B^{12})^2$			
$(1-B)^0$	100	21.33	49.36			
$(1-B)^1$	1.91	3.78	10.90			
$(1-B)^2$	3.64	7.10	20.11			

Table A1. Ratio of time series variance after and prior to filtering for loss-pass filter of order d and seasonal filter of order D.

$(1-B)^d(1-B^s)^D$						
$100 \cdot s_y^2 / s_x^2$	$(1-B^2)^0$	$(1-B^2)^1$	$(1-B^2)^2$	$(1-B^3)^0$	$(1-B^3)^1$	$(1-B^3)^2$
$(1-B)^0$	100	4.01	8.73	100	5.68	12.10
$(1-B)^1$	1.91	4.27	13.31	1.91	3.85	11.58
$(1-B)^2$	3.64	8.33	26.35	3.64	7.00	20.81
$100 \cdot s_y^2 / s_x^2$	$(1-B^4)^0$	$(1-B^4)^1$	$(1-B^4)^2$	$(1-B^5)^0$	$(1-B^5)^1$	$(1-B^5)^2$
$(1-B)^0$	100	7.34	15.73	100	9.10	19.18
$(1-B)^1$	1.91	3.73	11.23	1.91	3.97	12.03
$(1-B)^2$	3.64	6.89	20.37	3.64	7.61	22.58
$100 \cdot s_y^2 / s_x^2$	$(1-B^6)^0$	$(1-B^6)^1$	$(1-B^6)^2$	$(1-B^7)^0$	$(1-B^7)^1$	$(1-B^7)^2$
$(1-B)^0$	100	10.72	21.87	100	12.30	24.48
$(1-B)^1$	1.913	3.90	11.71	1.91	3.91	11.53
$(1-B)^2$	3.64	7.15	21.22	3.64	7.52	22.95
$100 \cdot s_y^2 / s_x^2$	$(1-B^8)^0$	$(1-B^8)^1$	$(1-B^8)^2$	$(1-B^9)^0$	$(1-B^9)^1$	$(1-B^9)^2$
$(1-B)^0$	100	13.82	28.09	100	15.47	32.53
$(1-B)^1$	1.91	3.71	11.00	1.91	3.56	10.33
$(1-B)^2$	3.64	7.25	22.01	3.64	6.80	19.54
$100 \cdot s_y^2 / s_x^2$	$(1-B^{10})^0$	$(1-B^{10})^1$	$(1-B^{10})^2$	$(1-B^{11})^0$	$(1-B^{11})^1$	$(1-B^{11})^2$
$(1-B)^0$	100	17.42	38.07	100	19.30	43.03
$(1-B)^1$	1.91	3.93	11.85	1.91	3.62	10.25
$(1-B)^2$	3.64	7.93	24.32	3.64	6.74	19.02
$100 \cdot s_y^2 / s_x^2$	$(1-B^{12})^0$	$(1-B^{12})^1$	$(1-B^{12})^2$			
$(1-B)^0$	100	21.33	49.36			
$(1-B)^1$	1.91	3.78	10.90			
$(1-B)^2$	3.64	7.10	20.11			

Table A2. Ratio of time series variance after and prior to filtering for seasonal filter of order D and loss-pass filter of order d.