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Article

Decoherence: An Introduction

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Abstract: In this paper I review the fundamentals of decoherence theory. Decoherence is viewed as a straightforward application of the general kinematical concept of a quantum wave function. Classical notions (such as "particle") as well as secondary quantum concepts (such as "observable", superselection rule etc.) can be derived. Special emphasis is put on a precise and consistent interpretation of quantum states and processes.

Keywords: interpretations of quantum mechanics, classical physics, decoherence, quantum measurement, irreversibility, quantum-classical relation

1 Introduction

During the last two decades, interest in decoherence studies has been steadily rising. Not only the new field of "quantum information" and "quantum computing" has lead to increased activity. In contrast to the decades before, attention to interpretational issues (d'Espagnat 1995) woke up again. Early work in decoherence (then called "continuous measurement") had its origin in a quest for understanding the quantum measurement process and the relationship between quantum and classical mechanics. In the following I will present an overview, concentrating on the most important conceptual issues.

What is decoherence? The essential mechanism defining decoherence is the (irreversible) creation of entanglement between a quantum system and its (quantum) environment. If an initially factorizing state evolves into an entangled one, the properties of this state – and therefore the "behavior" of the subsystems – differ very much from that of a factorizing state. In particular, the always present entanglement of macroscopic systems with their natural environment allows the (partial) derivation of classical physics from quantum physics (Joos et al. 2003).

Since a product state of two interacting systems is a very special state, the unitary evolution according to the Schrödinger equation will generally lead to entanglement,

$$|\varphi\rangle|\Phi\rangle \quad \stackrel{t}{\longrightarrow} \quad \sum_{n,m} c_{nm}|\varphi_n\rangle|\Phi_m\rangle$$
$$= \sum_n \sqrt{p_n(t)}|\tilde{\varphi}_n(t)\rangle|\tilde{\Phi}_n(t)\rangle\,. \tag{1}$$

The rhs of Eq. (1) can no longer be written as a single product in the general case. This can also be described by using the Schmidt representation, shown in the second line, where the presence of more than one component is equivalent to the existence of quantum correlations.
If many degrees of freedom are involved in this process, this entanglement will become practically irreversible, except for very special situations. Decoherence is thus a quite normal and, moreover, ubiquitous, quantum mechanical process. Historically, the important observation was the fact that this de-separation of quantum states happens extremely fast for macroscopic objects (Zeh 1970). The natural environment cannot simply be ignored or treated as a classical background in this case.
Eq. (1) shows that there is an intimate connection to the theory of irreversible processes. However, decoherence must not be identified or confused with dissipation: decoherence precedes dissipation by acting on a much faster timescale, while requiring initial conditions which are essentially the same as those responsible for the thermodynamic arrow of time (Zeh 2007).
With respect to observations at one of the two systems, the consequences of this entanglement are numerous. What can be observed at the subsystem alone is conveniently described by its density matrix ρ, calculated by "tracing out" all other degrees of freedom from the global state $|\Psi\rangle$ in Eq. (1),

$$\rho = Tr_\Phi\,|\Psi\rangle\,\langle\Psi| = \sum p_n\,|\tilde{\varphi}_n\rangle\,\langle\tilde{\varphi}_n|\;, \tag{2}$$

where the Schmidt states $|\tilde{\varphi}_n\rangle$ define the density matrices. First of all, a subsystem will no longer obey a Schrödinger equation, the local dynamics is in general very complicated, but can often be approximated by some sort of master equation. The most important effect is the disappearance of phase relations (i.e., interference) between certain subspaces of the Hilbert space of the system. Hence the resulting superselection rules can be understood as emerging from a dynamical, approximate and time-directed process. If the coupling to the environment is very strong, the internal dynamics of the system may become slowed down or even frozen. This is now usually called the quantum Zeno effect, which apparently does not occur in our macroscopic world.
The details of the dynamics depend on the kind of coupling between the system we consider and its environment. In many cases – especially in the macroscopic domain – this coupling leads to an evolution similar to a measurement process,

because the state of the environment evolves in one-to-one correspondence to the state of the considered system. Therefore it is appropriate to recall the essential elements of the quantum theory of measurement.

2 Measurement-like Processes

A fully quantum-mechanical description of measurement was outlined by von Neumann already in 1932 (von Neumann 1932). No classical concepts are required (or permitted) in such a treatment. Only under this provision, the measurement process can be analyzed consistently, despite many claims to the contrary.
Consider a set of system states $|n\rangle$ which our apparatus is built to discriminate.

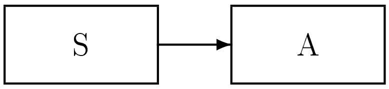

Figure 1: Original form of the von Neumann measurement model. "Information" about the state of the measured system S is transferred to the measuring apparatus A through an appropriate interaction.

To each state $|n\rangle$ of the measured system there exists a corresponding pointer state $|\Phi_n\rangle$ (more precisely, for each "quantum number" n there exists a large set of macrostates $|\Phi_n^{(\alpha)}\rangle$, α describing microscopic degrees of freedom). If the measurement is repeatable or "ideal" the dynamics of the measurement interaction must look like

$$|n\rangle|\Phi_0\rangle \quad \stackrel{t}{\longrightarrow} \quad |n\rangle|\Phi_n(t)\rangle\,. \tag{3}$$

From linearity it follows immediately what happens for a general initial state of the measured system, namely

$$\left(\sum_n c_n|n\rangle\right)|\Phi_0\rangle \quad \stackrel{t}{\longrightarrow} \quad \sum_n c_n|n\rangle|\Phi_n(t)\rangle\,. \tag{4}$$

The rhs of Eq. (4) does not describe a unique measurement result, but a superposition. Through unitary evolution, a correlated (and still pure) state results, which contains all possible results as components. Of course such a superposition must not be interpreted as an ensemble. The transition from this superposition to a single component – which is what we observe – constitutes the quantum measurement problem. As long as there is no collapse we have to deal with the whole superposition – and it is well known that a superposition has very different properties compared to any of its components. Simply put: quantum correlations are *not* statistical correlations.
Quantum correlations are also often misinterpreted as (quantum) "noise". This is wrong, however: Noise would mean that the considered system is in a certain

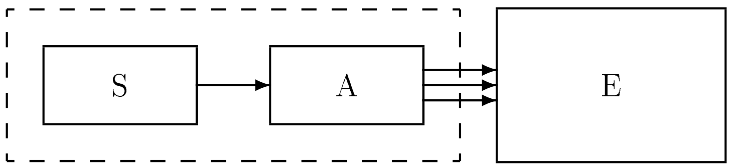

Figure 2: Realistic extension of the von Neumann measurement model. "Information" about the state of the measured system S is transferred to the measuring apparatus A and then very rapidly sent to the environment E. The back-reaction on the (local) system S+A originates entirely from quantum nonlocality.

state, which may be unknown and/or evolve in a complicated way. Such an interpretation is untenable and contradicts all experiments which show the nonlocal features of quantum-correlated (entangled) states.
Von Neumann's treatment, as described so far, is unrealistic since it does not take into account the essential openness of macroscopic objects. This deficiency can easily be remedied by extending the above scheme.

3 Classical Properties through Decoherence

If one takes into account the observation that the apparatus A, since macroscopic, is coupled to its environment E, which also acts like a measurement device, the phase relations are (extremely fast) further dislocalized into the total system – finally into the entire universe, according to

$$\left(\sum_n c_n|n\rangle|\Phi_n\rangle\right)|E_0\rangle \quad \stackrel{t}{\longrightarrow} \quad \sum_n c_n|n\rangle|\Phi_n\rangle|E_n\rangle \tag{5}$$

(see Fig. 2 as an illustration). The behavior of the local part system+apparatus is then described by the density matrix

$$\rho_{SA} \approx \sum_n |c_n|^2 |n\rangle\langle n| \otimes |\Phi_n\rangle\langle\Phi_n| \qquad \text{if} \qquad \langle E_n|E_m\rangle \approx \delta_{nm}\,, \tag{6}$$

which is identical to that of an ensemble of measurement results $|n\rangle|\Phi_n\rangle$.
Of course, this does not solve the measurement problem! This density matrix describes only an "improper" ensemble, i.e., with respect to all possible observations at S+A it *appears* that a certain measurement result has been achieved. A acts dynamically on E, but the back-action arises entirely from quantum nonlocality (as long as the measurement is "ideal", that is, Eq. 4 is a good approximation). Nevertheless, the system S+A acquires classical behavior, since interference terms are absent with respect to local observations if the above process is irreversible (Zurek 1981 and Joos/Zeh 1985).
Needless to say, the interference terms still exist globally in the total (pure) state, although they are unobservable at either system alone – a situation which may be characterized by the statement (Joos/Zeh 1985, p. 224),

"The interference terms still exist, but they are not *there*."

4 Decoherence versus Genuine Measurements

Both decoherence and measurements in the usual sense lead to destruction of interference. In the case of genuine measurements, this is trivial, however. Since the result of a measurement is assumed definite (thereby accompanied by a collapse of the wave function), disappearance of interference is a trivial consequence because all components (required for interference) except one have ceased to exist. On the other hand, decoherence does not require a collapse of the wave function. No definite outcome is assumed. In contrast to a proper measurement, the environment usually acts in an uncontrollable way (so there is no "pointer"). Even if the environment is described by a thermal distribution, interference can be destroyed locally.
One should also avoid the common misunderstanding that decoherence has something to do with "noise". Quantum entanglement is quite distinct from anything which can be called noise (in the sense of an uncontrollable disturbance).

5 Observables as Derived Concepts

In most treatments of quantum mechanics the notion of an observable plays a central role. Do observables represent a fundamental concept or can they be derived? If a measurement is described as a certain kind of interaction, then observables should not be required as an essential ingredient of quantum theory. In a sense this was also done by von Neumann, but not used later very much because of restrictions enforced by the Copenhagen school (e.g., the demand to describe a measurement device in classical terms instead of seeking for a consistent treatment in terms of wave functions).
Two elements are necessary to derive an observable that discriminates certain (orthogonal) system states $|n\rangle$. First, one needs an appropriate interaction which is diagonal in the eigenstates of the measured "observable" and is able to "move the pointer", so that we have as above

$$|n\rangle|\Phi_0\rangle \quad \xrightarrow{H_{int}} \quad |n\rangle|\Phi_n\rangle \, . \tag{7}$$

This can be achieved by Hamiltonians of the form

$$H_{int} = \sum_n |n\rangle\langle n| \otimes \hat{A}_n \tag{8}$$

with appropriate $\hat{A}_n$ leading to orthogonal pointer states (note that Eq. 7 defines only the eigenbasis of an observable, the eigenvalues represent merely scale factors and are therefore of minor importance). The second condition that must be fulfilled is dynamical stability of pointer states against decoherence, that is, the pointer states must only be passively recognized by the environment according to

$$|\Phi_n\rangle|E_0\rangle \quad \stackrel{decoherence}{\longrightarrow} \quad |\Phi_n\rangle|E_n\rangle\,. \tag{9}$$

Both conditions must be fulfilled. For example, a measurement device which acts according to Eq. (7) would be totally useless, if it were not stable against decoherence: Consider a Schrödinger cat state as pointer state! Such a superposition of "macroscopically different" states would rapidly decohere since its components (in this case a dead and a living cat) are immediately correlated with vastly different environmental states. Hence, the *same* basis states $|\Phi_n\rangle$ must be distinguished as dynamically relevant in Eq. (7) as well as in Eq. (9).
These mechanisms explain *dynamically* why certain observables may "not exist" operationally. For a general discussion of the relation between quantum states and observables see Sect. 2.2 of Joos et al. (2003). Arguments along these lines also lead to the conclusion that one should not attribute a fundamental status to the Heisenberg picture – contrary to widespread belief – despite its *phenomenological* equivalence with the Schrödinger picture.

6 Localization

One of the most important – and by now standard – examples of decoherence is the localization of macroscopic objects. Why do macroscopic objects always appear localized in (ordinary) space? Through decoherence, phase relations between macroscopically different positions are destroyed (dislocalized) *very* rapidly because of the strong influence of scattering processes.

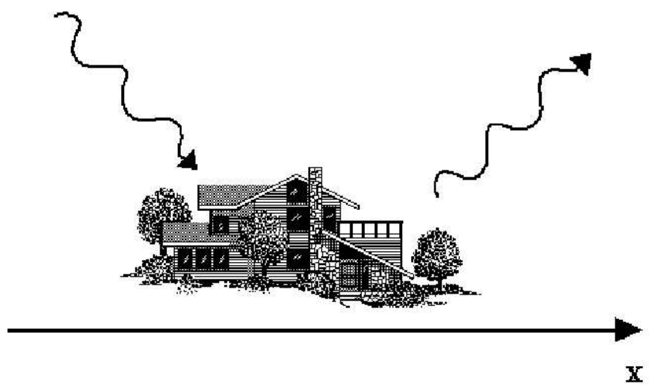

Figure 3: Macroscopic objects are under constant scrutiny by their natural environment. Scattering of photons and molecules leads to rapid quantum entanglement.

A simplified description may proceed as follows. Let $|x\rangle$ be the position eigenstate of a macroscopic object, and $|\chi\rangle$ the state of an incoming particle. Following the von Neumann scheme (Eq. 2), the scattering of such particles off an object located at position x can be written as

$$|x\rangle|\chi\rangle \stackrel{t}{\longrightarrow} |x\rangle|\chi_x\rangle = |x\rangle S_x|\chi\rangle\,, \tag{10}$$

where the scattered state may conveniently be calculated by means of an appropriate S-matrix. For the more general initial state of a wave packet we have then

$$\int d^3x\ \varphi(x)|x\rangle|\chi\rangle \xrightarrow{t} \int d^3x\ \varphi(x)|x\rangle S_x|\chi\rangle\,. \tag{11}$$

Therefore, the reduced density matrix describing our object changes into

$$\rho(x,x') = \varphi(x)\varphi^*(x')\left\langle \chi|S^\dagger_{x'}S_x|\chi\right\rangle\,. \tag{12}$$

Of course, a single scattering process will usually not resolve a small distance, so in most cases the matrix element on the right-hand side of Eq. (12) will be close to unity. If the contributions of many scattering processes are added up, however, an exponential damping of spatial coherence results:

$$\rho(x,x',t) = \rho(x,x',0)\exp\left\{-\Lambda t(x-x')^2\right\}\,. \tag{13}$$

The strength of this effect is described by a single parameter Λ that may be called "localization rate". It is given by

$$\Lambda = \frac{k^2 N v \sigma_{eff}}{V}\,. \tag{14}$$

Here, k is the wave number of the incoming particles, Nv/V the flux, and σ_{eff} is of the order of the total cross section (for details see Joos/Zeh 1985 or Sect. 3.2.1 and Appendix 1 of Joos et al. 2003, updated calculations can be found in Hornberger/Sipe 2003 and Adler 2006). Some values of Λ are given in the following table.

Localization rate Λ in $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ for three sizes of "dust particles" and various types of scattering processes (from Joos/Zeh 1985, p. 234). This quantity measures how fast interference between different positions disappears as a function of distance in the course of time.

	$a = 10^{-3}$cm dust particle	$a = 10^{-5}$cm dust particle	$a = 10^{-6}$cm large molecule
Cosmic background radiation	10^{6}	10^{-6}	10^{-12}
300 K photons	10^{19}	10^{12}	10^{6}
Sunlight (on earth)	10^{21}	10^{17}	10^{13}
Air molecules	10^{36}	10^{32}	10^{30}
Laboratory vacuum (10^3 particles/cm^3)	10^{23}	10^{19}	10^{17}

Most of the numbers in the table are quite large, showing the extremely strong coupling of macroscopic objects, such as dust particles, to their natural environment. Even in intergalactic space, the 3K background radiation cannot simply be neglected.

The upshot is obviously:

Macroscopic objects are not even approximately isolated.

A consistent unitary description must therefore include the environment and finally the whole universe.[1]
If we combine this damping of coherence with the "free" Schrödinger dynamics we arrive at an equation of motion for the density matrix that to a good approximation simply adds these two contributions,

$$i\frac{\partial \rho}{\partial t} = [H_{internal}, \rho] + i\frac{\partial \rho}{\partial t}\bigg|_{scatt.} . \tag{15}$$

In the position representation this equation reads in one space dimension

$$i\frac{\partial \rho(x,x',t)}{\partial t} = \frac{1}{2m}\left(\frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2}\right)\rho - i\Lambda(x-x')^2\rho\,. \tag{16}$$

For macroscopic objects decoherence acts so fasts that its onset cannot be observed. This situation changes in laboratory situations where now even *quantitative* checks of the required models can be performed.
Eq. (13) describes the effect of scattering for small distances $k|x-x'| \ll 1$, leading to a typical decoherence timescale $t_{\rm dec} \approx \frac{1}{\Lambda|x-x'|^2}$. In the opposite limit $k|x-x'| \gg 1$, where a single scattering event destroys coherence, the decoherence timescale is just given by the scattering rate, that is $t_{\rm dec} \approx \frac{V}{Nv\sigma_{\rm tot}} \approx \frac{k^2}{\Lambda}$.
For example, consider a tiny dust particle of the size of a virus (10^{-5}cm). Under normal conditions, scattering of air molecules leads to a decoherence timescale of the order of 10^{-13} s. Since this value scales with the particle density, in a laboratory situation radiation effects may become dominant. In this example, the 300K thermal background (Rayleigh scattering) yields a decoherence time of about 1 s for a distance of 10^{-6}cm, 10^{-4} s for 10^{-4} cm, and 10^{-5} s for $|x-x'| > 10^{-2}$ cm, respectively.
Indeed, experiments with C_{60} and C_{70} fullerene molecules (Hackermüller et al. 2004) showed the expected interference patterns, in line with the above estimates.

[1] One of the first stressing the importance of the dynamical coupling of macro-objects to their environment was Dieter Zeh, who wrote in his 1970 Found. Phys. paper (Zeh 1970), "Since the interactions between macroscopic systems are effective even at astronomical distances, the only 'closed system' is the universe as a whole. [...] It is of course very questionable to describe the universe by a wavefunction that obeys a Schrödinger equation. Otherwise, however, there is no inconsistency in measurement, as there is no theory."

This is now more or less commonplace, but this was not the case some 30 years ago, when he sent an earlier version of this paper to the journal Il Nuovo Cimento. I quote from the referee's reply: "The paper is completely senseless. It is clear that the author has not fully understood the problem and the previous contributions in this field" (H. D. Zeh, private communication).

Twenty years later, Gell-Mann/Hartle (1990), p. 455 wrote, "Quantum theory is best and most fundamentally understood in the framework of quantum cosmology." Clearly the situation has improved, although there still is no consensus about the meaning of quantum theory.

The most important decoherence mechanism for C_{60} is emission of (thermal) radiation from the internally hot molecule. These and other examples demonstrate the strong dependence of decoherence effects on the actual situation.
So far this treatment represents *pure* decoherence, following directly the von Neumann scheme. If recoil is added as a next step, we arrive at models including friction, that is, quantum Brownian motion. There are several models for the quantum analog of Brownian motion, some of which are even older than the first decoherence studies. Early treatments did not, however, draw a distinction between decoherence and friction (decoherence alone does *not* imply friction.). As an example, consider the equation of motion derived by Caldeira/Leggett (1983),

$$i\frac{\partial\rho}{\partial t} = [H,\rho] + \frac{\gamma}{2}[x,\{p,\rho\}] - im\gamma k_B T[x,[x,\rho]]\,, \tag{17}$$

which reads for a "free" particle

$$\begin{aligned} i\frac{\partial\rho(x,x',t)}{\partial t} &= \left[\frac{1}{2m}\left(\frac{\partial^2}{\partial x'^2}-\frac{\partial^2}{\partial x^2}\right) - i\Lambda(x-x')^2\right. \\ &\quad \left. +i\gamma(x-x')\left(\frac{\partial}{\partial x'}-\frac{\partial}{\partial x}\right)\right]\rho(x,x',t)\,, \end{aligned} \tag{18}$$

where γ is the damping constant, and here $\Lambda = m\gamma k_B T$.
If one compares the effectiveness of the two terms representing decoherence and relaxation for a distance δx, one finds that their ratio is given by

$$\frac{\text{decoherence rate}}{\text{relaxation rate}} = mk_B T(\delta x)^2 \propto \left(\frac{\delta x}{\lambda_{th}}\right)^2\,, \tag{19}$$

where λ_{th} denotes the thermal de Broglie wavelength of the considered object. This ratio has for a typical macroscopic situation ($m = 1$g, $T = 300$K, $\delta x = 1$cm) the enormous value of about 10^{40}! This shows that in these cases decoherence is *far more important* than dissipation.

7 Molecular Structure

Not only the center-of-mass position of dust particles becomes "classical" via decoherence. The spatial structure of molecules represents another most important example. Consider a simple model of a chiral molecule.
If the Hamiltonian commutes with parity P, $[H,P] = 0$, then non-degenerate energy eigenstates always are also parity eigenstates. On the other hand, right- and left-handed versions of the molecule both have a rather well-defined spatial structure and exchange their roles under parity transformation. In particular, the ground state would then – for symmetry reasons – be a superposition of both chiral states. These chiral configurations are usually separated by a tunneling barrier, which is so high that under normal circumstances tunneling is very improbable, as was already shown by Hund (1927). But this alone does not explain why chiral (and, indeed, most) molecules are never found in energy eigenstates!

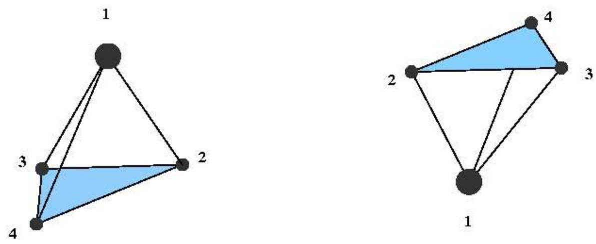

Figure 4: Most molecules show a well-defined spatial structure and are therefore never observed in energy eigenstates.

In a simplified model with low-lying nearly-degenerate eigenstates $|1\rangle$ and $|2\rangle$, the right- and left-handed configurations may be given by

$$
\begin{aligned}
|L\rangle &= \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \\
|R\rangle &= \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \,.
\end{aligned} \tag{20}
$$

Because the environment recognizes the spatial structure via scattering processes, only chiral states are stable against decoherence,

$$
|R, L\rangle|\Phi_0\rangle \xrightarrow{t} |R, L\rangle|\Phi_{R,L}\rangle \,. \tag{21}
$$

The dynamical instability of energy (i.e., parity) eigenstates of molecules represents a typical example of "spontaneous symmetry breaking" induced by decoherence. Additionally, transitions between spatially oriented states are suppressed by the quantum Zeno effect (see Sect. 3.3.1 of Joos et al. 2003 or Joos 1984).

8 Classical and Quantum Chaos

The relation between quantum and classical physics is particularly critical for the case of chaotic systems. The popular limiting procedure $\hbar \to 0$ fails, and there have been heated debates whether quantum chaos does exist at all. It was soon found that "noise" leads to a behavior resembling much more that of the classical situation. Obviously decoherence also here plays an important role for establishing the quantum-classical connection.

The strong dependence on initial conditions in nonlinear systems leads to the well-known exponential divergence in phase space, where an initially small compact volume is deformed locally in the way that it will shrink exponentially in one direction and expand exponentially in others, formally described by Lyapunov exponents. Since phase space volume is preserved, expansion is always accompanied by contraction (the sum of Lyapunov exponents is zero for Hamiltonian systems).

In order to get an idea how instability in phase space translates into the quantum regime, consider a localized wave packet with an initial width $\Delta p(0)$ in the momentum distribution. If this momentum interval is squeezed exponentially with a (positive) Lyapunov exponent λ_L,

$$\Delta p(t) = \Delta p(0) \exp(-\lambda_L t) \tag{22}$$

then the uncertainty relations require an exponential divergence in position,

$$\Delta x(t) \geq \frac{\hbar}{\Delta p(0)} \exp(+\lambda_L t)\,. \tag{23}$$

The classical description certainly breaks down as soon as Δx becomes so large that the nonlinearity of the potential becomes relevant. This scale may be estimated as

$$\chi \approx \sqrt{\frac{\partial_x V}{\partial_x^3 V}}\,. \tag{24}$$

The coherence length will reach the scale χ at the so-called Ehrenfest time

$$t_E = \frac{1}{\lambda_L} \log \frac{\Delta p(0)\chi}{\hbar}\,. \tag{25}$$

At this time a nonclassical situation will emerge: The wave packet is broad (in a sense like a Schrödinger cat state) and the notion of a trajectory becomes untenable. Even at the level of expectation values, classical behavior will be lost, since the Ehrenfest theorem ceases to be valid.
It is obvious that taking the naive limit $\hbar \to 0$ (leading to $t_E \to \infty$) is incorrect. Classicality does not follow from such as simple formal operation.
To put these heuristic arguments on a more solid basis one may consider the evolution of the system by means of the Wigner function. Its equation of motion reads

$$\begin{aligned} \dot{W}(x,p,t) &= \{H, W\}_{MB} \\ &= -i \sin(i\hbar\{H,W\}_{PB})/\hbar \\ &= \{H,W\}_{PB} + \sum_{n\geq 1} \frac{\hbar^{2n}(-1)^n}{2^{2n}(2n+1)!} \partial_x^{2n+1} V(x) \partial_p^{2n+1} W(x,p)\,. \end{aligned} \tag{26}$$

The so-called Moyal bracket $\{H,W\}_{MB}$ is equivalent to the von Neumann commutator $[H,\rho]$. The Poisson bracket term $\{H,W\}_{PB}$ is identical to the classical Liouville expression. Hence – ignoring problems of interpretation for the moment – one may reason that corrections to the classical motion are small, if the Wigner function is smooth enough, so that correction terms containing higher derivatives do not contribute significantly. But as argued before, this is impossible for times larger than the Ehrenfest time. At least at this time, quantum "corrections" can no longer be ignored – not even at the level of expectation values. An overestimate for the Ehrenfest time is

$$t \approx \frac{1}{\lambda_L} \log \frac{\text{action}}{\hbar}\,, \tag{27}$$

Figure 5: Hyperion shows, like any other macroscopic object, a well-defined orientation (it tumbles chaotically). According to the Schrödinger equation its state of rotation would be smeared out over all orientations, like a Schrödinger cat state, after circa 20 years.

with "action" being some typical value characterizing the macroscopic system. These estimates are quite insensitive to the precise numerical values of the action (or $\hbar$, respectively), since the action enters only logarithmically (by contrast, the Ehrenfest time for non-chaotic systems scales according to some power law). After this time motion should become completely nonclassical.
A particularly striking example is the chaotic rotational motion of Hyperion (a moon of Saturn). An overestimate of the relevant action may be given by the product of Hyperion's orbital kinetic energy and its orbital period. This yields an estimate of $t_E \approx 20$ years. Thus one would expect to find Hyperion in an extremely nonclassical state of rotation.
Taking into account the macroscopic nature of objects such as Hyperion, decoherence can now be included as described before. The equation of motion then reads

$$\begin{aligned} \frac{dW(x,p)}{dt} = \quad & \{H,W\}_{Poisson} && \text{Liouville} \\ & + \left(-\tfrac{\hbar^2}{24}\,\partial_x^3 V\,\partial_p^3 W + ...\right) && \textit{large}\text{ “quantum corrections”} \\ & + \Lambda\,\tfrac{\partial^2 W}{\partial p^2} && \text{decoherence}\ . \end{aligned} \tag{28}$$

The effect of squeezing in phase space in counteracted by decoherence, so that the non-classical spreading of the wave-packet does not occur.

9 Decoherence of Fields

In Quantum Electrodynamics we find two (related) situations,

- "Measurement" of charges by fields;

- "Measurement" of fields by charges.

In both cases, the entanglement between charge and field states leads to decoherence.
In quantum optics experiments it is possible to prepare and study superpositions of different classical field states, quantum-mechanically represented by coherent states $|\alpha\rangle$, for example "Schrödinger cat" states of the form

$$|\Psi\rangle = N(|\alpha\rangle + |-\alpha\rangle)\,, \tag{29}$$

which can be realized as field states in a cavity. In these experiments (see e.g. Brune et al. 1996) decoherence can be turned on gradually by coupling the cavity to a reservoir. Typical decoherence times are in the range of about 100 μs.
For *true* cats the decoherence time is much shorter (in particular, it is *very much* shorter than the lifetime of a cat!). This leads to the appearance of *quantum jumps*, although all underlying processes are smooth in principle since they are governed by the Schrödinger equation.
In experimental situations of this kind we find a gradual transition from a superposition of different decay times (seen in "collapse and revival" experiments) to a local mixture of decay times (leading to "quantum jumps") according to the following scheme.

theory	experiment
superposition of different decay times	collapse and revivals
⇓	⇓
ensemble of different decay times	quantum jumps

A similar scheme can be put up for decoherence in space:

theory	experiment
superposition of different positions	double-slit experiments
⇓	⇓
ensemble of different positions	particles (Wilson chamber)

10 Spacetime and Quantum Gravity

In quantum theories of the gravitational field, no classical spacetime exists at the most fundamental level. Since it is generally assumed that the gravitational field has to be quantized for consistency reasons, the question again arises how the corresponding classical properties can be understood.

Genuine quantum effects of gravity are expected to occur for scales of the order of the Planck length $\sqrt{G\hbar/c^3}$. It is therefore often argued that the spacetime structure at larger scales is automatically classical. However, this Planck scale argument is as insufficient as the large mass argument in the evolution of free wave packets. As long as the superposition principle is valid (and even superstring theory leaves this untouched), superpositions of different metrics should occur at any scale.

The central problem can already be demonstrated in a simple Newtonian model (Joos 1986). Consider a cube of length L containing a homogeneous gravitational field with a quantum state ψ such that at some initial time $t = 0$

$$|\psi\rangle = c_1|g\rangle + c_2|g'\rangle\,, \tag{30}$$

where g and g' correspond to two different field strengths. A particle with mass m in a state $|\chi\rangle$, which moves through this volume, "measures" the value of g, since its trajectory depends on the acceleration g:

$$|\psi\rangle|\chi^{(0)}\rangle \to c_1|g\rangle|\chi_g(t)\rangle + c_2|g'\rangle|\chi_{g'}(t)\rangle\,. \tag{31}$$

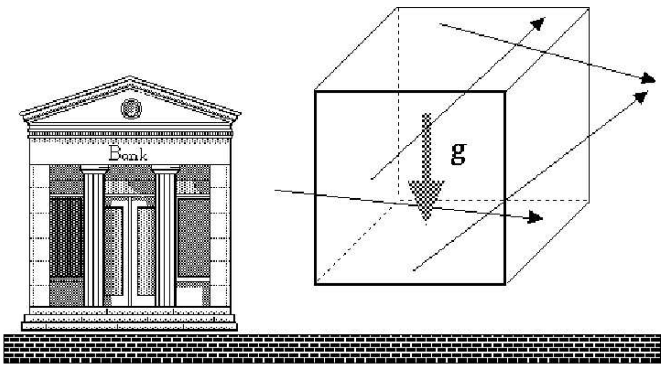

Figure 6: The trajectories of air molecules depend on the value of the gravitational acceleration g. Therefore matter always "measures" the gravitational field, rendering it classical.

This correlation destroys the coherence between g and g', and the reduced density matrix can be estimated to assume the following form after many such interactions are taken into account:

$$\rho(g, g', t) = \rho(g, g', 0)\exp\left(-\Gamma t(g - g')^2\right)\,, \tag{32}$$

where

$$\Gamma = nL^4\left(\frac{\pi m}{2k_BT}\right)^{3/2}$$

for a gas with particle density n and temperature T. For example, air under ordinary conditions, $L = 1$ cm, and $t = 1$ s yields a remaining coherence width of $\Delta g/g \approx 10^{-6}$ (Joos 1986).

Thus, matter does not only tell space to curve but also to behave classically. This is also true in full quantum gravity.
In a fully quantized theory of gravity (Kiefer 2007), for example in the canonical approach described by the Wheeler-deWitt equation,

$$H|\Psi(\Phi,^{(3)}\mathcal{G})\rangle = 0\,, \tag{33}$$

where Φ describes matter and $^{(3)}\mathcal{G}$ is the three-metric, everything is contained in the "wave function of the universe" Ψ. Here we encounter new problems: There is neither an external time parameter, nor is there an external observer. Apart from these interpretational problems, important decoherence effects have been studied. For example, in the expansion of a quantized Friedmann universe, the scale factor becomes classical through decoherence (except near the big bang/big crunch).

11 True, False, and Fake Decoherence

The term "decoherence" is often identified with "disappearance of interference effects", formally described by damping of non-diagonal terms in a density matrix in a certain basis. This characterization hides important conceptual aspects. It is a good idea to clearly discriminate the physical mechanisms leading to the disappearance of coherence. These may be classified into three categories, from which only the first one deserves to be called "true" decoherence.
Pure states are coherent almost by definition. Parts, i. e. certain components, can act in a way which is observably different from the behavior of their superposition ("interference"). Let the parts be given by the Hilbert space vectors $|1\rangle$ and $|2\rangle$. The general superposition

$$|\Psi\rangle = a\,|1\rangle + b\,|2\rangle = e^{i\alpha}(\cos\frac{\Theta}{2}\,|1\rangle + e^{i\Phi}\sin\frac{\Theta}{2}\,|2\rangle) \tag{34}$$

shows coherence between its components, conveniently expressed by the non-diagonal part ρ_{12} contained in the density matrix. Decoherence in this basis means that the off-diagonal part ρ_{12} is reduced or completely eliminated,

$$\frac{1}{2} \geq |\rho_{12}| = |ab^*| \longrightarrow 0\,. \tag{35}$$

True decoherence. The fundamental decoherence mechanism is "pure" entanglement with the environment – *without any dynamical change* of the component states,

$$\begin{aligned} |1\rangle\,|\Phi\rangle &\longrightarrow |1\rangle\,|\Phi_1\rangle \\ |2\rangle\,|\Phi\rangle &\longrightarrow |2\rangle\,|\Phi_2\rangle\ , \end{aligned} \tag{36}$$

hence

$$(a\,|1\rangle + b\,|2\rangle)\,|\Phi\rangle \longrightarrow a\,|1\rangle\,|\Phi_1\rangle + b\,|2\rangle\,|\Phi_2\rangle\ . \tag{37}$$

Locally, coherence is lost,

$$\rho_{12} \longrightarrow \rho_{12} \langle \Phi_2 | \Phi_1 \rangle \longrightarrow 0\,. \tag{38}$$

The components *still exist*, but can no longer interfere, since the required phase relations are delocalized. This process has no analog in classical physics. The damping of ρ_{12} often follows a Lindblad master equation with *hermitean* generators.

False decoherence. Coherence is trivially lost if one of the required components disappears. An important situation of this kind is represented by relaxation processes, for example, a decay of state $|2\rangle$ into state $|1\rangle$,

$$\begin{aligned} |1\rangle\,|\Phi\rangle &\longrightarrow |1\rangle\,|\Phi'\rangle \\ |2\rangle\,|\Phi\rangle &\longrightarrow |1\rangle\,|\Phi''\rangle\ . \end{aligned} \tag{39}$$

This is often called "amplitude damping". Clearly, interference must disappear together with the decay of component $|2\rangle$. Therefore the timescales T_1 for "longitudinal" and T_2 for "transversal" decay in the commonly used Bloch equations,

$$\begin{aligned} \dot\rho_{22} &= -\frac{1}{T_1}\rho_{22} \\ \dot\rho_{12} &= -i\omega\rho_{12} - \frac{1}{T_2}\rho_{12} \end{aligned} \tag{40}$$

are connected by the well-known relation $T_2 = 2\,T_1$.

The role of the "environment" can also be played by other states of the *same* system, if the dynamics leads to disappearance of one component from the relevant subspace. For example, such "internal decoherence" may have the form

$$\begin{aligned} |1\rangle &\longrightarrow |1\rangle \\ |2\rangle &\longrightarrow \sum_{n>2} c_n\,|n\rangle\ , \end{aligned} \tag{41}$$

generated by an appropriate Hamiltonian.

Finally, the most direct way to remove a component is represented by a collapse of the wave function, for example during a measurement. If the measurement result is ignored (or unknown), one proceeds with

$$\rho = \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}\ , \tag{42}$$

which has exactly the same form as the density matrix resulting from complete entanglement. But interference terms are missing here for the trivial reason that one or the other component no longer exists.

Fake decoherence. Decoherence often arises from some averaging process. Two typical situations are noteworthy. The ensemble either consists of members undergoing the same unitary evolution but with different initial states or an ensemble of identically prepared states subjected to different Hamiltonians is employed. In

both cases the fundamental dynamics of a *single* system is unitary, hence there is *no decoherence at all* from a microscopic point of view.
If instead of a single state (Eq. 34) an ensemble $\{|\Psi_j\rangle\}$ of such states with different relative phases Φ_j is prepared in some way[2], ensemble averages for measurements are usually calculated from a density matrix (assuming all phases Φ_j equally likely) where

$$\rho = \frac{1}{N}\sum_{j=1}^{N} |\Psi_j\rangle\langle\Psi_j| = \begin{pmatrix} \cos^2\frac{\Theta}{2} & \frac{1}{2}\sin\Theta\ \sum_{j=1}^{N} e^{i\Phi_j} \\ \frac{1}{2}\sin\Theta\ \sum_{j=1}^{N} e^{-i\Phi_j} & \sin^2\frac{\Theta}{2} \end{pmatrix} \approx \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}. \tag{43}$$

This sort of "dephasing" has its root only in the *incomplete* description by this averaged density matrix and leads always to a shorter dephasing time $T_2^* < T_2$. Note also that the correct microscopic description would be given by a tensor product state

$$|\Psi\rangle = \otimes_j |\Psi_j\rangle\ . \tag{44}$$

The above state is observably different as shown in many "echo"-experiments, since for each member of this ensemble the relative phases remain well-defined, even if they may be hard to access experimentally.
As another simple example consider a particle subjected to "random kicks" acting on an individual object. The corresponding *ensemble of unitary evolutions* represents again a non-unitary evolution of the density matrix.
Let a particle be subjected to "random kicks". This model is chosen in analogy to classical Brownian motion, although here it is applied to wave functions. If the original state of the particle is described by a wave packet $\varphi(x)$, a "kick", i.e., an instantaneous shift in momentum by Δp, introduces a phase factor,

$$\varphi(x) \rightarrow \varphi(x)e^{i\Delta p\, x}\,, \tag{45}$$

or for the density matrix,

$$\rho(x,x') \rightarrow \rho(x,x')e^{i\Delta p(x-x')}\,. \tag{46}$$

The average action of kicks distributed according to a probability distribution of momentum transfers $P(q)$ is then given by

$$\rho(x,x') \rightarrow \int dq\, P(q)\rho(x,x')e^{iq(x-x')} =: f(x-x')\rho(x,x')\,. \tag{47}$$

Whatever the shape of $P(q)$, for such "kicks" a damping of *spatial* coherence results, given by the Fourier transform of the momentum transfer distribution.

[2] For example, consider an ensemble of spins rotating with different angular velocities in an NMR experiment, leading to inhomogeneous broadening of a magnetic resonance signal.

For a Gaussian distribution $P(q) = \sqrt{\frac{\lambda}{\pi}} \exp\left(-\lambda q^2\right)$ we find the well-known result

$$\frac{\Delta\rho(x,x')}{\Delta t} \propto \left(1 - \exp\left[-\frac{(x-x')^2}{4\lambda}\right]\right)\rho(x,x')\,. \tag{48}$$

Since the distribution of kicks here does not depend on the state of the system, this treatment does not include any recoil. It corresponds to a Langevin equation with a stochastic force, but without a frictional term, and hence cannot describe approach to equilibrium.

It may thus appear that decoherence can also be obtained from "classical perturbations" (kicks) of the quantum system. This formal equivalence of density matrix equations hides once again the essential conceptual difference between the two types of interactions. For "classical noise" the system follows a *unitary* (even though uncontrollable in practice) dynamics; in each individual case it stays in a pure state (that may remain unknown because of an insufficiently known Hamiltonian). For a series of random successive kicks to the *same* particle, one would still obtain the result (Eq. 45) with Δp simply representing the sum of all kicks,

$$\varphi(x) \to \varphi(x)e^{i(\sum \Delta p)x}\,. \tag{49}$$

In contrast, decoherence leads deterministically to an entangled state that has quite different properties. "Noise" models are in fact only used in situations where this difference cannot be observed

The popular opinion that the measurement process causes an uncontrollable disturbance of the measured system seems to go back to Heisenberg's arguments in support of the uncertainty relations. This has lead to the myth of "quantum randomness", which finds its expression in many often-used terms such as "quantum fluctuations" etc.

12 Superselection rules can be derived

What is a superselection rule? One way to characterize a superselection rule is to say, that certain states $|\Psi_1\rangle$, $|\Psi_2\rangle$ are found in nature, but never general superpositions $|\Psi\rangle = \alpha|\Psi_1\rangle + \beta|\Psi_2\rangle$. This means that all observations can be described by a density matrix of the form $\rho = p_1|\Psi_1\rangle\langle\Psi_1| + p_2|\Psi_2\rangle\langle\Psi_2|$. Clearly such a density matrix is exactly what is obtained through decoherence in appropriate situations.

There are many examples, where it is hard to find certain superpositions in the real world. The most famous example has been given by Schrödinger: A superposition of a dead and an alive cat

$$|\Psi\rangle = |\text{dead cat}\rangle + |\text{alive cat}\rangle \tag{50}$$

is never observed, contrary to what should be possible according to the superposition principle (and, in fact, *must* necessarily occur according to the Schrödinger equation). Another drastic situation is given by a state like

$$|\Psi\rangle = |\text{cat}\rangle + |\text{dog}\rangle\,. \tag{51}$$

Such a superposition looks truly absurd, but only because we never observe states of this kind, so we are not used to it! (The obvious objection that one cannot superpose states of "different systems" seems to be inappropriate. For example, nobody hesitates to superpose states with different numbers of particles, in particular, photons.) A more down-to-earth example is given by the position of large objects, which are never found in states

$$|\Psi\rangle = |\text{here}\rangle + |\text{there}\rangle\,, \tag{52}$$

with "here" and "there" macroscopically distinct. Under realistic circumstances such objects are always well described by a localized density matrix $\rho(x,x') \approx p(x)\delta(x-x')$, as already outlined above.

Exact superselection rules, that is, strict absence of interference can only be expected for discrete quantities. One important example is electric charge. Can this be understood via decoherence? We know from Maxwell's theory, that every charge carries with itself an associated electric field, so that a superposition of charges may be written in the form

$$\begin{aligned} \sum_q c_q|\Psi_q^{total}\rangle &= \sum_q c_q|\chi_q^{bare}\rangle|\Psi_q^{field}\rangle \\ &= \sum_q c_q|\chi_q^{local}\rangle|\Psi_q^{farfield}\rangle\,. \end{aligned} \tag{53}$$

Since we can only observe the local dressed charge, it has to be described by the density matrix

$$\rho = \sum_q |c_q|^2 |\chi_q^{local}\rangle\langle\chi_q^{local}|\,. \tag{54}$$

If the far fields are orthogonal (distinguishable), coherence would be absent locally. So the question arises: Is the Coulomb field only part of the kinematics (implemented via the Gauss constraint) or does it represent a quantum dynamical degree of freedom so that we have to consider decoherence via a retarded Coulomb field?

What do experiments tell us? A superposition of the form as in Eq. (52) can be observed for charged particles. On the other hand, the classical (retarded) Coulomb field would contain information about the path of the charged particle, destroying coherence. The situation does not appear very clear-cut. Hence one essential question remains:

What is the quantum *physical role of the Coulomb field?*

A similar situation arises in quantum gravity, where we can expect that superpositions of different masses (energies) are decohered by the spatial curvature.

Another important "exact" superselection rule forbids superposing states with integer and half-integer spin, for example

$$|\Psi\rangle = |\text{ spin } 1\rangle + |\text{ spin } {}^1\!/_2\rangle\,, \tag{55}$$

which would transform under a rotation by 2π into

$$|\Psi_{2\pi}\rangle = |\text{ spin } 1\rangle - |\text{ spin } {}^1\!/_2\rangle\,, \tag{56}$$

clearly a different state because of the different relative phase. If one *demands* that such a rotation should not change anything, such a state must be excluded. This is one standard argument in favor of the "univalence" superselection rule. On the other hand, one *has* observed the sign-change of spin ${}^1\!/_2$ particles under a (relative) rotation by 2π in *certain* experiments. Hence we are left with two options: Either we view the group SO(3) as the proper rotation group also in quantum theory. Then nothing must change if we rotate the system by an angle of 2π. Hence we can derive this superselection rule from symmetry. But this may merely be a classical prejudice. The other choice is to use SU(2) instead of SO(3) as rotation group. Then we are in need of explaining why those strange superpositions never occur. This last choice amounts to keeping the superposition principle as the fundamental principle of quantum theory. In more technical terms we should then avoid using groups with non-unique ("ray"[3]) representations, such as SO(3). In supersymmetric theories, bosons and fermions are treated on an equal footing, so it would be natural to superpose their states (what is apparently never done in particle theory).

In a similar manner one could undermine the well-known argument leading from the Galilean symmetry of nonrelativistic quantum mechanics to the mass superselection rule. In this case we could maintain the superposition principle and replace the Galilei group by a larger group.

13 What is achieved by decoherence?

What insights can be drawn from decoherence studies? It should be emphasized that decoherence derives from a straightforward application of standard quantum theory to realistic situations. It seems to be a historical accident, that the importance of the interaction with the natural environment was overlooked for such a long time. Certainly the still prevailing attitudes enforced by the Copenhagen school played a (negative) role here, for example by outlawing a physical analysis of the measurement process in quantum-mechanical terms (only).

Because of the strong coupling of macroscopic objects, a quantum description of macroscopic objects *requires* the inclusion of the natural environment. A fully unitary quantum theory is only consistent if applied to the whole universe. This does not preclude local phenomenological descriptions. However, their derivation from a universal quantum theory and the interpretation assigned to such descriptions have to be analyzed very carefully.

[3] The widely used argument that physical states are to be represented by rays, not vectors, in Hilbert space because the phase of a state vector cannot be observed, is misleading. Since relative phases are certainly relevant, one should prefer a vector as a *fundamental* physical state concept, rather than a ray. Rays cannot even be superposed without (implicitly) using vectors.

We have seen that typical classical properties, such as localization in space, are *created* by the environment in an irreversible process, and are therefore not inherent attributes of macroscopic objects. The features of the interaction define *what* is classical by selecting a certain basis in Hilbert space. Hence superselection sectors emerge from the dynamics. In all "classical" situations, the relevant decoherence time is extremely short, so that the smooth Schrödinger dynamics leads to apparent discontinuities like "events", "particles" or "quantum jumps".
The conclusion thus can be drawn that a consistent treatment in terms of wave function(al)s allows to understand the appearance of classical states *within* quantum theory. No mystical or inconsistent concepts, such as "uncertainty", "duality", "quantum logic", or "complementarity" are needed any more.
The unusual properties of quantum states lead to some at first sight strange consequences. *Local* classical properties find their explanation in the *nonlocal* features of quantum states. Usually quantum objects are considered as fragile and easy to disturb, whereas macroscopic objects are viewed as the rock-solid building blocks of empirical reality. However, the opposite is true: macroscopic objects are extremely sensitive and immediately decohered.
On the practical side, decoherence also has its disadvantages. It makes testing alternative theories difficult (more on that below), and it represents a major obstacle for people trying to construct a quantum computer. Building a really big one may well turn out to be as difficult as detecting other Everett worlds!

14 What is an observer?

One advantage of the considerations presented so far was the fact that the results are quite independent of any interpretation of quantum theory and therefore they are largely agreeable (except for technicalities). But the task of physics is to draw a conclusive and consistent picture of nature, and for that reason we have to examine critically whatever we have achieved so far. The density matrices used above, for example, are just an aid for the calculation of measurement probabilities. But what is a measurement? It is often pointed out that the final and decisive authority is the perception of an observer. This problem was early recognized as being essential in the discussions about quantum theory. For example, Heisenberg reports in his autobiography "Der Teil und das Ganze" on a conversation with Einstein, in which Einstein urges him:

> "[...] it may be of heuristic value to recall what one really observes. But from a principal point of view it is quite wrong to insist on founding a theory on observed quantities alone. In reality just the opposite is true. Only the theory decides about what can be observed. [...] On the entire long path from a process up to our conscious perception we need to know how Nature is working in order to claim that we have observed anything at all."[4]

[4] "Es mag heuristisch von Wert sein, sich daran zu erinnern, was man wirklich beobachtet. Aber vom prinzipiellen Standpunkt aus ist es ganz falsch, eine Theorie nur auf beobachtbare

This is a most important statement!
Subjective perception has clearly something to do with our brain. Does quantum theory also hold there?

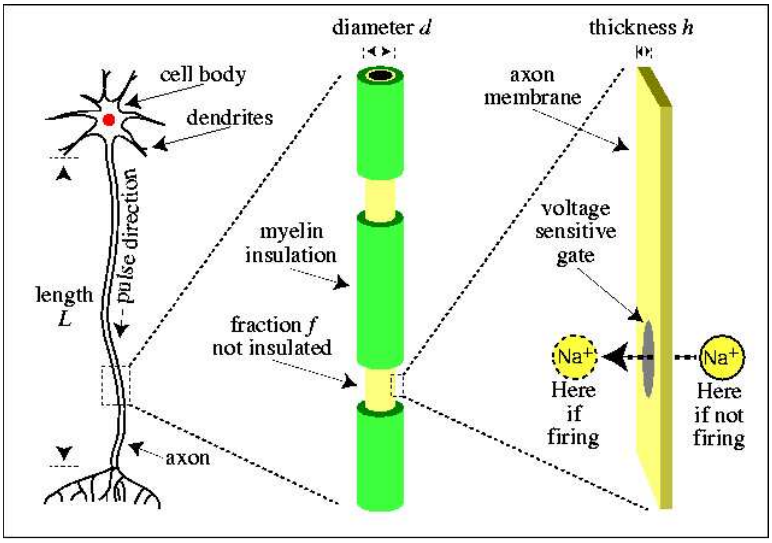

Figure 7: An electrical pulse along an axon represents communication between neurons. The difference between "firing" and "not-firing" is equivalent to the presence of about 10^6 sodium or potassium ions on different sides of a membrane. This difference is rapidly recognized by the environment (illustration from Tegmark 2000, p. 4194).

There are many models for the communication between neurons in the brain (neuronal networks). Practically all of them are using classical pictures in the sense that neurons are always in a classically describable (in particular, local) state, which changes in the course of time following certain laws. However, quantum theory allows many non-classical states, to which - when set in *parallel* with subjective perception - no obvious meaning can be attributed. From a quantum-theoretical point of view the superposition

$$|\Psi = |\text{neuron is firing}\rangle + |\text{neuron is not firing}\rangle \tag{57}$$

is, for example, a totally legal state. But what would it mean?
It turns out again that the surroundings distinguish the two classical alternatives in such a state very fast and therefore destroy coherence. Estimated time scales go down to values as low as $t \approx 10^{-20}s$. This means that we have no chance to make "strange" perceptions, because the above superposition is far too unstable. Obviously, quantum theory can be successfully extended into the brain of the observer.

Größen gründen zu wollen. Denn es ist ja in Wirklichkeit genau umgekehrt. Erst die Theorie entscheidet darüber, was man beobachten kann. [...] Auf dem ganzen langen Weg vom Vorgang bis zur Fixierung in unserem Bewußtsein müssen wir wissen, wie die Natur funktioniert, wenn wir behaupten wollen, daß wir etwas beobachtet haben." (Heisenberg 1969, p. 80).

15 Which interpretations make sense?

One could also ask: what interpretations are left from the many that have been proposed during the decades since the invention of quantum theory? I think, we do not have much of a choice at present, *if* we restrict ourselves to use only wavefunctions as kinematical concepts (that is, we ignore hidden-variable theories, for example).
There seem to be only the two possibilities either (1) to alter the Schrödinger equation to get something like a “real collapse” (Ghirardi/Rimini/Weber 1986; Pearle 2007a and Pearle 2007b), or (2) to keep the theory unchanged and try to establish some variant of the Everett interpretation. Both approaches have their pros and cons, some of them are listed in Table 1. In a collapse theory, the global wave function (which is a consequence of unitary evolution, including the entire chain of interactions up to the observer) is somehow reduced to just a single component,

$$\begin{aligned}\sum_n c_n |\varphi_n\rangle \left|\Phi_n^{(1)}\right\rangle \left|\Phi_n^{(2)}\right\rangle \left|\Phi_n^{(3)}\right\rangle \ldots \left|\Phi_n^{(observer)}\right\rangle \\ \longrightarrow |\varphi_k\rangle \left|\Phi_k^{(1)}\right\rangle \left|\Phi_k^{(2)}\right\rangle \left|\Phi_k^{(3)}\right\rangle \ldots \left|\Phi_k^{(observer)}\right\rangle .\end{aligned} \tag{58}$$

If a collapse occurs before the information enters the consciousness of an observer, one can maintain some kind of psycho-physical parallelism by assuming that what is experienced subjectively is parallel to the physical state $\left|\Phi_k^{(observer)}\right\rangle$ of certain (local) objects, e.g., parts of the brain. The last resort is to view consciousness as *causing* collapse, an interpretation which can more or less be traced back to von Neumann. In any case, the collapse happens with a certain probability (and with respect to a certain basis in Hilbert space) and this element of the theory comprises an *additional* axiom.
How would we want to test such theories? One would look for collapse-like deviations from the unitary Schrödinger dynamics. However, similar *apparent* deviations are also produced by decoherence, in particular in the relevant meso- and macroscopic range. So it is hard to discriminate these *true* changes to the Schrödinger equation from the *apparent* deviations brought about by decoherence (Joos 1987). So far the superposition principle was found valid wherever it could be tested.
In the Everett interpretation one keeps all components in the global wave function

$$\sum_n c_n |\varphi_n\rangle \left|\Phi_n^{(1)}\right\rangle \left|\Phi_n^{(2)}\right\rangle \left|\Phi_n^{(3)}\right\rangle \ldots \left|\Phi_n^{(observer)}\right\rangle . \tag{59}$$

Instead of specifying the collapse one has to define precisely how the wavefunction is to be split up into branches. Decoherence can help here by selecting certain directions in Hilbert space as dynamically stable (and others as extremely fragile – branches with macroscopic objects in nonclassical states immediately decohere), but the location of the observer in the holistic quantum world is always a decisive ingredient. It must be assumed that what is subjectively experienced is parallel to

collapse models	Everett
traditional psycho-physical parallelism: What is perceived is parallel to the observer's physical *state*	new form of psycho-physical parallelism: Subjective perception is parallel to the observer state in a *component* of the universal wave function
probabilities put in by hand	probabilities must also be postulated (existing "derivations" are circular)
problems with relativity	peaceful coexistence with relativity
experimental check: look for collapse-like deviations from the Schrödinger equation $\Downarrow$ hard to test because of decoherence	experimental check: look for macroscopic superpositions $\Downarrow$ hard to test because of decoherence

Table 1: Some pros and cons of collapse models and variants of the Everett interpretation.

certain states (observer states) in a certain *component* of the global wave function. The probabilities (frequencies) we observe in repeated measurements form also an additional axiom[5]. The peaceful coexistence with relativity seems not to pose much problems, since no collapse ever happens and all interactions are local in (high-dimensional) configuration space. But testing Everett means testing the Schrödinger equation in particular with respect to macroscopic superpositions, and this again is made difficult by decoherence.

Decoherence has brought us a much better understanding of the relation between classical and quantum physics. The fundamental interpretational problem of quantum mechanics, however, still remains to be solved.

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[5] There are repeated attempts to derive probabilities in the Everett interpretation. I think these proofs are circular. Consider a sequence of N measurements on copies of a two-state system, all prepared in the initial state $a|1\rangle + b|2\rangle$. The resulting correlated state contains 2^N components, where each pointer state shows one of the 2^N possible sequences of measurement results (e.g. as a computer printout). But these pointer states are *always the same*, independently of the values of a and b! Only if each branch is given a *weight* involving the norms $|a|^2$ and $|b|^2$ one may recover the correct frequencies.

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