Confidence Intervals for Combined Univariate Economic Forecasts

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Abstract

Research in combining of economic forecasts made by several institutes on the same economic variable has focused on estimation using mainly regression based methods, hoping that the combined forecast will be improved by incorporating the expert opinions of the institutes. We provide confidence intervals on the combined forecast using analysis of variance techniques. A scoring of the individual institutes is proposed by taking into account the historical performance of the institutes in forecasting the quantity in question. It is remarkable that no information is needed about the individual precision or the variance of the forecasts.

Key words: Combining information; Expert opinion; Heteroscedastic variances.

1 Introduction

Consider the situation where one has several forecasts on the same quantity, e. g. several economic research institutes forecast an important economic variable. This situation is the rule in many econometric applications. Interest is now in combining the individual forecasts to improve the accuracy of the forecast. Since by way of combining several forecasts one takes into account several expert opinions, one expects a better performance of the combined forecast. The most popular models and methods for the combination are regression based. However, it is rarely observed that the simple average of the individual forecasts is beaten by more sophisticated methods, see Clemen and Winkler (1986). As Chatfield (2001), section 4.3, points out, it is quite difficult to estimate the variances of the individual predictions which are needed as the weights in a weighted combination of the forecasts, and as a consequence, confidence and prediction intervals are hard to derive, see Chatfield (2001), section 4.3. Thus using a simple average of the predictions is an easy and common way to avoid these difficulties in practice.

Different methods of combining forecasts in the context of GNP forecasts from four major econometric models are applied by Clemen and Winkler

(1986). Klapper (1999) discusses rank–based techniques for combining forecasts. A review on combining predictions is provided by Clemen (1989) and Granger (1989). However, despite its practical importance, there is little known about confidence intervals on the combined forecast in this crucial area.

In this paper, we cope with the thorny task of combining the information from different sources and give several confidence intervals on the combined forecast which are of approximate nature. The confidence intervals are derived using analysis of variance as a main tool. It is notable that for constructing the confidence intervals, we do not have to expect that each forecast is accompanied by its precision or variance. For illustration, the confidence intervals are applied to German economic data. Some simulation results on the properties of the confidence intervals are also provided.

2 A statistical model for combining forecasts

We consider the following model

$$y_i \sim N(\mu, \alpha_i), \tag{2.1}$$

where y_i denotes the forecast of the i–th institute and the forecasts are independent, and μ represents the true unknown quantity, $i = 1, \ldots, K$, $K > 2$. We assume that each individual forecast y_i follows a normal distribution. But this assumption is not restrictive, at least asymptotically, because, in general, the forecasting techniques provide normally distributed quantities since the estimators are of least–squares or maximum likelihood type. We want to emphasize that we do not assume that the institutes use the same technique or method to forecast the variable in question, and we also do not assume that the precisions or the variances of the forecasts are known. Such an information may not be available in practice, simply because it is rarely reported. The variances α_i of the forecasts are assumed to be heteroscedastic to reflect the different ability and quality of the institutes in providing, more or less, precise forecasts.

Since there is no information available on the precision or variance of each individual forecast, the situation we consider is rather non–standard, and thus, makes the already difficult task of combining forecasts more challenging. Confidence intervals and tests on the variance components are discussed in a related heteroscedastic ANOVA model by Hartung and Argaç (2002 a, b).

We estimate the mean μ by a weighted average of the individual forecasts y_i,

$$y = \sum_{i=1}^{K} b_i^2 \cdot y_i, \tag{2.2}$$

where b_i^2 denotes the weight which is given to the i–th institute by some scoring process. We assume that the weights are normed

$$\sum_{i=1}^{K} b_i^2 = 1, \tag{2.3}$$

and we also assume that $b_i^2 < 1/2$, that is we exclude the possibility that one particular institute dominates the rest. However, the assumption that the weights b_i^2 are less than 0.5 does *not* mean that we assume away the possibility that one institute predicts better than the rest; if there is one institute that always forecasts much better than all others, then one need not combine the predictions. But there is some practical evidence that the combination of forecasts gives better prediction results than the individual forecasts, see Chatfield (2001), section 4.3. If we choose the weights as $b_i^2 = 1/K$, then we obtain the simple average of the predictions as a special case.

Consider now the following quadratic form

$$u_{ib}^2 = b_i^2 \cdot \left(y_i - \sum_{j=1}^{K} b_j^2 \cdot y_j \right)^2 , \tag{2.4}$$

which can be interpreted as a weighted quadratic deviation of each individual forecast from the weighted average of the individual forecasts of each institute. Let $\beta_i = b_i^2 \cdot \alpha_i$, and one can derive explicitly the important moments of the quadratic form u_{ib}^2 (Hartung, Böckenhoff and Knapp 2003),

$$E(u_{ib}^2) = (1 - 2 \cdot b_i^2) \cdot \beta_i + b_i^2 \cdot \sum_{j=1}^{K} b_j^2 \cdot \beta_j =: e_i(b, \beta), \tag{2.5}$$

$$var(u_{ib}^2) = 2 \cdot e_i(b, \beta)^2, \tag{2.6}$$

$$Cov(u_{ib}^2, u_{jb}^2) = 2 \cdot b_i^2 \cdot b_j^2 \cdot \left(\sum_{k=1}^{K} b_k^2 \cdot \beta_k - \beta_i - \beta_j \right)^2 =: 2 \cdot e_{ij}(b, \beta)^2. \tag{2.7}$$

We need to estimate the variance of the combined forecast and of each individual forecast, and for this purpose we use quadratic functions of the individual forecasts as estimators. The variance of the combined forecast is estimated with an unbiased positive (PSD–MINQUE) variance estimator which is given by, see also Hartung, Böckenhoff and Knapp (2003),

$$\widehat{var}(y)_{psd} = \frac{1}{1+\sum_{k=1}^{K}\frac{b_k^4}{1-2b_k^2}} \cdot \sum_{i=1}^{K} \frac{b_i^2}{1-2b_i^2} \cdot u_{ib}^2 \tag{2.8}$$

$$= \sum_{i=1}^{K} d_i \cdot u_{ib}^2. \tag{2.9}$$

The estimator of the variance of each forecast is derived as a positive minimum biased variance estimator (PSD–MINQMBE)

$$\widehat{\alpha}_i = \frac{\sum_{j=1}^{K} u_{jb}^2 + \sum_{j=1}^{K} d_j \cdot u_{jb}^2}{\sum_{j=1}^{K} b_j^2 \cdot \widetilde{\alpha}_j} \cdot \widetilde{\alpha}_i, \tag{2.10}$$

where

$$\widetilde{\alpha}_i = \frac{1}{b_i^2} \cdot \frac{(1-b_i^2)^2}{(1-b_i^2)^4 + b_i^4 \sum_{j\neq i} b_j^4} \cdot u_{ib}^2. \tag{2.11}$$

Note here also that both estimators are given explicitly, see Hartung, Böckenhoff and Knapp (2003) for a detailed derivation of the estimators. Under suitable conditions these estimators are consistent, exist always and are unique. It is remarkable and surprising that the variance of the combined forecast can be unbiasedly estimated *without* any information about the variances of the individual forecasts! Note that for the results obtained above, we have not made use of the normality assumption. The only assumption needed up to now is the existence of the first and second moments of the predictions. Essentially, the normality assumption is only needed for deriving the confidence intervals.

We will construct the confidence interval using a pivotal quantity, and for this purpose we have to determine the distribution of the variance estimators. We will approximate the distribution of the variance estimators by suitable χ^2–distributions using moment matching, see Satterthwaite (1946) and Patnaik (1949). This leads to

$$\nu \cdot \sum_{i=1}^{K} d_i \cdot u_{ib}^2 / E(\sum_{i=1}^{K} d_i \cdot u_{ib}^2) \overset{appr}{\sim} \chi_\nu^2, \tag{2.12}$$

where

$$\nu = 2 \, \frac{[E(\sum_{i=1}^{K} d_i \cdot u_{ib}^2)]^2}{var(\sum_{i=1}^{K} d_i \cdot u_{ib}^2)}. \tag{2.13}$$

Now, this gives us the following pivot (nominator and denominator are nearly independent)

$$\frac{y - \mu}{\sqrt{\sum_{i=1}^{K} d_i \cdot u_{ib}^2}} \overset{appr}{\sim} t_\nu, \tag{2.14}$$

where

$$\nu = \frac{\left\{\sum_{i=1}^{K} d_i \cdot e_i(b,\beta)\right\}^2}{\sum_{i=1}^{K} d_i^2 \cdot e_i(b,\beta)^2 + \sum_{\substack{i=1 \\ i\neq j}}^{K} \sum_{j=1}^{K} d_i \cdot d_j \cdot e_{ij}(b,\beta)^2}. \tag{2.15}$$

In practice, the unknown quantities have to be replaced by estimators, and here we use $\widehat{\beta}_i = b_i^2 \cdot \widehat{\alpha}_i$.

Now, we are in the position to derive the confidence intervals which are of course of approximate nature only.

$$I_1: \quad y \mp t_{\nu;1-\alpha/2} \cdot \sqrt{\sum_{i=1}^{K} d_i \cdot u_{ib}^2}, \tag{2.16}$$

$$I_2: \quad y \mp u_{1-\alpha/2} \cdot \sqrt{\sum_{i=1}^{K} d_i \cdot u_{ib}^2}, \tag{2.17}$$

$$I_3: \quad y \mp t_{K-1;1-\alpha/2} \cdot \sqrt{\sum_{i=1}^{K} d_i \cdot u_{ib}^2}. \tag{2.18}$$

The first interval is the approximate interval derived using the t–distributed pivot, the second version is obtained if one replaces the quantile of the t–distribution with the quantile of the standard normal distribution for large degrees of freedom. The last version is obtained if one ignores the heteroscedastic variances in calculating the degrees of freedom ν. The approximate degrees of freedom ν might become too small, and hence we used $\max(\nu, 2)$ as the degrees of freedom instead of ν in all the computations in the next sections, since we assume that the number of predictions exceeds two.

Now, we have to specify the weights b_i^2, the crucial part in assessing the institutes' performance. We suggest the following choice of the weights:

$$b_i^2 = \frac{\sum_{j=1}^{L} \left(y_{ij} - y_j^w\right)^{-2}}{\sum_{i=1}^{K} \sum_{j=1}^{L} \left(y_{ij} - y_j^w\right)^{-2}}, \tag{2.19}$$

where y_{ij} denotes the forecast of the i–th institute for some economic variable in the j–th time period or year and y_j^w denotes the true realized value in the j–th time period or year. Hence, each institute is scored taking into account its historical performance in forecasting in the last L time periods or years. We consider quadratic deviations of each forecast from the true value. When choosing the time periods, subject matter knowledge should be taken into consideration.

Note that we have assumed that $b_i^2 < 0.5$. This assumption may be violated in practice, for example if one forecast is close to the true value of the variable in one of the L time periods. Thus, a modification of the weights is needed to guarantee $b_i^2 < 0.5$. For this purpose, choose a strictly positive constant θ with $0 < \theta < 0.5 - 1/K$, and then consider the following procedure to modify the weights which guarantees $b_i^2 < 0.5$:

$$b_i^2 = \begin{cases} b_i^2 & \text{, if } b_i^2 < 0.5 - \theta \\ 0.5 - \theta & \text{, for } i = i_0 \text{ with } b_{i_0}^2 = \max\limits_{j=1,\dots,K} \{b_j^2 | b_j^2 < 0.5 - \theta\} \\ (0.5 + \theta) \cdot b_i^2 / \sum_{\substack{j=1 \\ j \neq i_0}}^{K} b_j^2 & \text{, for } i \in \{1, \dots, K\} \setminus \{i_0\} \end{cases}$$

This procedure gives $b_i^2 < 0.5 - \theta$ for all $i = 1, \dots, K$. If not, then replace θ by $\theta/2$ and start the procedure again. If the index i_0 is not unique, we choose the largest one. Possible choices for θ are $\theta = 1/K^2$ or $\theta = 1/K^3$.

3 Data analysis

We applied the confidence intervals to German economic data. We used the data on GDP from seven major economic research institutes from 1984–1996, see Table 1. For the first forecast, that is for 1987, we used the data from 1984–1986 to score the institutes. We give the weights b_i^2 in Table 2, the variance estimators $\widehat{\alpha}_i$ in Table 3, and finally, the combined forecast y, the confidence intervals I_1, I_2 and I_3 at the nominal level of 95% and the true values of the GDP variable for every year in Table 4. From Table 2, it is obvious that the fifth institute performs best through the whole period compared to the other institutes, the second best institute is the third institute. The second institute improves its performance in the period 1993–1996 considerably.

4 Monte Carlo results

Since the intervals we constructed are of approximate nature, we conducted a simulation experiment to check their validity with respect to the actual confidence coefficients and lengths of the proposed confidence intervals. We considered $K = 7$ and $K = 14$ institutes. In the first scenario, the weights b_i^2 are chosen to be equal ($b_i^2 = 1/7$, $i = 1, \ldots, 7$), in a second scenario the weight of the first institute is large compared to the other weights which are chosen to be equal ($b_1^2 = 0.49$, $b_i^2 = 0.51/6$, $i = 2, \ldots, 7$), and finally, the weights are chosen to be almost equal ($b_i^2 = 0.49/3$, $i = 1, 2, 3$ and $b_i^2 = 0.51/4$, $i = 4, \ldots, 7$). The variances of the individual forecasts are homoscedastic and heteroscedastic. We paired small variances with small weights and small variances with large weights, see Table 5. The simulation scheme for $K = 14$ follows the pattern of the simulations for $K = 7$ institutes.

We draw samples from the normal distribution and from a centered χ^2–distribution, i. e. $\chi^2_{\nu_i} - \nu_i$, with a few degrees of freedom to cover also the case of non–normal observations. The degrees of freedom of the centered χ^2–distribution were chosen in such a way that the variances are the same as in the corresponding case with normal data.

The number of repetitions in the simulations is 10000. We provide the empirical confidence level of the intervals at the nominal level of 95% and the corresponding widths of the confidence intervals, see Tables 6–9.

The main result is that, in general, the intervals I_1 and I_3 are conservative for normal data and can become slightly liberal in case of the centered χ^2–distribution; the confidence interval I_2 also attains acceptable levels, but might become too liberal for non–normal data when the weights b_i^2 are chosen to be equal or nearly equal and the variances α_i are homoscedastic. For $K = 14$ institutes, we obtained similar results as for $K = 7$ institutes concerning the empirical confidence coefficients and widths of the confidence intervals, see Tables 8 and 9.

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Table 1: The forecasts of seven economic institutes on GDP (1984–1996).

	Institute						
Year	1	2	3	4	5	6	7
1984	2.00	2.25	2.50	2.00	2.50	1.30	2.00
1985	2.00	2.25	2.00	2.00	3.00	2.10	2.75
1986	3.00	3.00	3.00	3.00	3.00	3.00	3.25
1987	1.50	2.25	3.00	3.00	2.00	2.20	3.00
1988	1.00	1.00	2.50	2.00	1.50	1.60	1.50
1989	2.50	2.25	2.50	2.00	2.50	1.70	2.50
1990	3.50	3.00	3.00	3.00	3.00	2.80	3.20
1991	3.50	3.25	3.50	3.00	3.50	2.70	3.00
1992	1.00	1.50	1.50	2.00	2.50	2.20	1.80
1993	−1.00	−0.50	0.00	0.50	0.00	0.90	0.50
1994	−0.50	1.00	1.00	1.00	0.00	0.50	0.40
1995	2.00	3.00	3.50	3.00	3.00	3.84	2.80
1996	1.00	1.75	1.70	2.50	2.00	2.50	2.40

Table 2: The weights b_i^2 for the GDP data.

	Weights						
Year	b_1^2	b_2^2	b_3^2	b_4^2	b_5^2	b_6^2	b_7^2
1987	0.036	0.069	0.332	0.036	0.343	0.033	0.151
1988	0.040	0.067	0.241	0.028	0.466	0.048	0.111
1989	0.040	0.067	0.241	0.028	0.464	0.048	0.111
1990	0.042	0.068	0.240	0.029	0.460	0.048	0.112
1991	0.044	0.069	0.240	0.030	0.458	0.049	0.112
1992	0.082	0.067	0.248	0.029	0.432	0.043	0.099
1993	0.061	0.173	0.300	0.028	0.304	0.034	0.101
1994	0.063	0.173	0.299	0.028	0.303	0.034	0.100
1995	0.063	0.173	0.298	0.029	0.302	0.034	0.100
1996	0.167	0.154	0.264	0.026	0.268	0.030	0.090

Table 3: The variance estimators $\widehat{\alpha}_i$ for the GDP data.

	Variance estimators						
Year	$\widehat{\alpha}_1$	$\widehat{\alpha}_2$	$\widehat{\alpha}_3$	$\widehat{\alpha}_4$	$\widehat{\alpha}_5$	$\widehat{\alpha}_6$	$\widehat{\alpha}_7$
1987	0.894	0.069	0.369	0.192	0.464	0.089	0.245
1988	0.401	0.424	0.779	0.068	0.091	0.009	0.040
1989	0.005	0.034	0.007	0.177	0.013	0.532	0.006
1990	0.212	0.001	0.002	0.001	0.003	0.054	0.031
1991	0.015	0.015	0.023	0.128	0.040	0.433	0.152
1992	0.911	0.205	0.304	0.001	0.635	0.050	0.028
1993	0.905	0.257	0.005	0.288	0.005	0.866	0.335
1994	0.963	0.262	0.351	0.192	0.434	0.001	0.016
1995	1.161	0.011	0.271	0.008	0.015	0.507	0.091
1996	0.738	0.001	0.010	0.466	0.075	0.470	0.395

Table 4: Results of the data analysis of the GDP data.

			Confidence bounds		Confidence
Year	True Value	Forecast	Lower bound	Upper bound	Interval
			1.117	3.934	I_1
1987	1.9	2.525	1.884	3.167	I_2
			1.724	3.326	I_3
			0.758	2.654	I_1
1988	3.7	1.706	1.274	2.138	I_2
			1.167	2.245	I_3
			2.155	2.706	I_1
1989	3.3	2.430	2.305	2.556	I_2
			2.274	2.587	I_3
			2.891	3.177	I_1
1990	4.7	3.034	2.969	3.099	I_2
			2.953	3.115	I_3
			2.891	3.855	I_1
1991	3.7	3.373	3.153	3.593	I_2
			3.099	3.647	I_3
			0.191	3.740	I_1
1992	1.6	1.966	1.157	2.774	I_2
			0.957	2.975	I_3
			−0.409	0.303	I_1
1993	−1.7	−0.053	−0.274	0.169	I_2
			−0.329	0.224	I_3
			−0.732	1.782	I_1
1994	2.4	0.525	−0.047	1.098	I_2
			−0.190	1.240	I_3
			2.345	3.844	I_1
1995	1.9	3.095	2.753	3.436	I_2
			2.668	3.521	I_3
			1.064	2.496	I_1
1996	1.4	1.780	1.454	2.106	I_2
			1.373	2.187	I_3

Table 5: Sample design ($K = 7$).

		i						
Design		1	2	3	4	5	6	7
A	b_i^2	0.143	0.143	0.143	0.143	0.143	0.143	0.143
	α_i	*2*	*2*	*2*	*2*	*2*	*2*	*2*
B	b_i^2	0.143	0.143	0.143	0.143	0.143	0.143	0.143
	α_i	*2*	*4*	*6*	*8*	*10*	*12*	*14*
C	b_i^2	0.49	0.085	0.085	0.085	0.085	0.085	0.085
	α_i	*2*	*2*	*2*	*2*	*2*	*2*	*2*
D	b_i^2	0.49	0.085	0.085	0.085	0.085	0.085	0.085
	α_i	*2*	*4*	*6*	*8*	*10*	*12*	*14*
E	b_i^2	0.49	0.085	0.085	0.085	0.085	0.085	0.085
	α_i	*14*	*12*	*10*	*8*	*6*	*4*	*2*
F	b_i^2	0.163	0.163	0.163	0.128	0.128	0.128	0.128
	α_i	*2*	*2*	*2*	*2*	*2*	*2*	*2*
G	b_i^2	0.163	0.163	0.163	0.128	0.128	0.128	0.128
	α_i	*2*	*4*	*6*	*8*	*10*	*12*	*14*
H	b_i^2	0.163	0.163	0.163	0.128	0.128	0.128	0.128
	α_i	*14*	*12*	*10*	*8*	*6*	*4*	*2*

Table 6: Simulated actual confidence coefficients (%) and lengths (*cursive*); normal distribution, $K = 7$.

Design	I_1	I_2	I_3
A	99.7	95.0	98.8
	0.48	*0.29*	*0.36*
B	99.8	95.1	99.3
	1.02	*0.57*	*0.71*
C	99.8	98.6	99.6
	2.28	*1.07*	*1.33*
D	99.8	97.8	99.5
	2.38	*1.09*	*1.36*
E	99.9	99.3	99.7
	6.28	*2.80*	*3.50*
F	99.8	96.0	99.6
	0.53	*0.30*	*0.38*
G	99.9	94.8	98.9
	0.94	*0.54*	*0.67*
H	99.9	95.4	99.3
	1.33	*0.64*	*0.80*

Table 7: Simulated actual confidence coefficients (%)

and lengths (*cursive*); centered χ^2–distribution, $K = 7$.

Design	I_1	I_2	I_3
A	93.2	85.1	92.0
	0.64	*0.27*	*0.33*
B	99.9	90.8	97.9
	1.11	*0.55*	*0.69*
C	99.8	98.9	99.6
	1.86	*0.94*	*1.17*
D	99.9	97.0	98.9
	2.09	*0.94*	*1.17*
E	99.8	98.6	99.4
	5.97	*2.76*	*3.45*
F	96.3	84.9	92.0
	0.68	*0.28*	*0.35*
G	99.4	93.2	98.5
	1.04	*0.53*	*0.66*
H	99.9	92.1	99.3
	1.44	*0.63*	*0.79*

Table 8: Simulated actual confidence coefficients (%)

and lengths (*cursive*); normal distribution, $K = 14$.

Design	I_1	I_2	I_3
A	98.6	95.2	97.3
	0.13	*0.10*	*0.11*
B	99.7	96.4	97.9
	0.28	*0.20*	*0.23*
C	99.9	99.8	99.9
	2.39	*1.07*	*1.18*
D	99.9	99.2	99.4
	2.42	*1.08*	*1.19*
E	99.8	99.5	99.7
	5.96	*2.78*	*3.07*
F	99.3	95.6	97.6
	0.15	*0.11*	*0.12*
G	99.6	96.2	98.0
	0.30	*0.21*	*0.23*
H	99.2	95.1	97.7
	0.32	*0.22*	*0.25*

Table 9: Simulated actual confidence coefficients (%)

and lengths (*cursive*); centered χ^2–distribution, $K = 14$.

Design	I_1	I_2	I_3
A	92.4	89.5	91.2
	0.20	*0.10*	*0.11*
B	97.2	92.4	94.7
	0.31	*0.20*	*0.22*
C	99.8	99.6	99.6
	1.99	*0.93*	*1.03*
D	99.8	99.4	99.4
	1.85	*0.90*	*0.99*
E	99.9	99.7	99.9
	6.28	*2.72*	*3.00*
F	91.8	88.5	90.6
	0.20	*0.10*	*0.11*
G	97.7	92.8	95.0
	0.33	*0.21*	*0.23*
H	98.8	93.5	96.0
	0.36	*0.22*	*0.24*