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Evolutionary Optimization of Dynamic Multiobjective Functions

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Evolutionary Optimization of Dynamic Multiobjective Functions

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Abstract. Many real-world problems show both multiobjective as well as dynamic characteristics. In order to use multiobjective evolutionary optimization algorithms (MOEA) efficiently, a systematic analysis of the behavior of these algorithms in dynamic environments is necessary. Dynamic fitness functions can be classified into problems with moving Pareto fronts and Pareto sets having varying speed, shape, and structure. The influence of the dimensions of the objective and decision space is considered. The analysis will focus on standard benchmark functions and newly designed test functions. Convergence and solution distribution features of modern MOEA, namely NSGA-II, SPEA 2 and MSOPS using different variation operators (SBX and Differential Evolution), will be characterized using Pareto front metrics. A new path integral metric is introduced. Especially the ability of the algorithms to use historically evolved population properties will be discussed.

1 Introduction

An effective optimization of dynamic multiobjective functions using population based optimization algorithms demands for a systematic case study. New generic functions are needed to test the distribution and approximation properties of multiobjective optimization algorithms when dynamically varying Pareto front and Pareto set structures or changing restrictions for low as well as high dimensions of the decision and objective spaces are used. The application of new multiobjective quality measures (metrics) as well as improved genetic operators is the consequence of this research. The following empirical analyses give a brief insight into this field. The application of a standard programming environment (PISA) allows the adaptation of the results to real-world applications.

2 Dynamic Test Functions

Dynamic multiobjective function optimization is rarely discussed in current literature. Jin and Sendhoff [1] introduce an open scheme for generating dynamic test functions from static functions. Farina, Deb and Amato discuss a generic dynamic scheme that follows the concept of Deb [2],[3]:

$$D \subset \mathbb{R}_+^s, E \subset \mathbb{R}_+^{n-s-1}, D \cap E = \emptyset$$
$$f_1 : D \rightarrow \mathbb{R}, f_2 : D \cup E \rightarrow \mathbb{R}$$
$$g : E \rightarrow \mathbb{R}, h : D \cup E \subset \mathbb{R}^n \rightarrow \mathbb{R}$$

$$\begin{aligned} &\min\ f_1(\mathbf{x}) = f_1(x_1, \ldots, x_s) \\ &\min\ f_2(\mathbf{x}) = g(x_{s+1}, \ldots, x_n) \cdot h(f_1(x_1, \ldots, x_s), g(x_{s+1}, \ldots, x_n)) \end{aligned} \quad (1)$$

The shape of the Pareto front depends on the values of h. A property of the function set is that for fixed f_1 the values of h are weakly and monotonically increasing in g and for fixed g the values of h are monotonically decreasing in f_1. The first property implies that the Pareto front corresponds with the global minimum of g. The second property is a precondition for a conflicting situation between f_1 and f_2, i.e. for the generation of a multiobjective problem.

Farina et al. use this concept for the generation of bi-objective dynamic test functions. Their approach follows the ZDT-functions, which are concrete static realizations of the concept of Deb. They classify the type of dynamics of multi-objective problems into four classes:

Type I: Static Pareto front, dynamic Pareto set
Type II: Dynamic Pareto front, dynamic Pareto set
Type III: Dynamic Pareto front, static Pareto set
Type IV: Static Pareto front, static Pareto set. The fitness topology may change.

The dynamics can further be classified into sub-classes that correspond to the dynamics in shape or structure (e.g. connectivity) of the Pareto fronts and Pareto sets. None, static or dynamic definitions of the feasible set D and the restrictions of the fitness functions introduce additional subclasses.

In this paper the functions of Farina, Deb and Amato – called FDA1, FDA2 to FDA5 – are used as test cases with known properties. They are representatives for type I, type II and type III behavior. For an explicit definition of the FDA functions please refer to [2] and [3]. Additionally, new functions such as DSW and DTF will be introduced next in order to analyse a generalized set of test cases, functions of different dimensions with moving Pareto sets, separating Pareto sets, as well as separating Pareto fronts or dynamic restrictions.

In the following the original FDA2 function (type III) was redesigned to get a Pareto front that is clearly changing in $t \in [0, 1]$ from a convex shape to a concave shape. In the experiments $FDA2_{mod}$ was used.

$FDA2_{mod}$**:** Test function with dynamic PF_{true} and static P_{true}

$$FDA2_{mod}: \begin{cases} f_1(\mathbf{x_I}) & = & x_1 \\ f_2(g,h) & = & g \cdot h(f,g) \\ g(\mathbf{x_{II}}) & = & 1 + \sum_{x_i \in x_{II}} x_i^2 + \sum_{x_i \in x_{III}} (x_i+1)^2 \\ h(f_1,g) & = & 1 - \left(\frac{f_1}{g}\right)^{H(t(\tau))} \\ H(t) & = & 0.2 + 4.8\, t(\tau)^2 \\ & & \quad\quad \tau \quad \text{current generation} \\ t(\tau) & = & \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_t} \right\rfloor, \ \tau_t^{-1} \ \text{substep frequenz} \\ & & \quad\quad n_t \quad \text{total number of generations} \\ & & x_I = (x_1)^T, x_1 \in [0,1] \\ & & x_{II} = (x_2, \ldots, x_{r_1})^T \in [-1,1]^{r_1-1} \\ & & x_{III} = (x_{r_1+1}, \ldots, x_n)^T \in [-1,1]^{n-r_1-1} \end{cases} \tag{2}$$

τ is the current generation of the evolutionary process and n_t is the total number of generations. Therefore, time t is always in the interval $[0,1]$. In order to realize a step wise increase of t, a subdivision of the interval in τ_t steps is useful. In order to simplify the notation, in the paper $t := t(\tau)$ is used.

The new functions $DSW1$ to $DSW3$ are motivated by the multiobjective and static function of Schaffer [4]. In order to extend the dimension of the decision space of the Schaffer function from 1 to n, a hyper parabolic term can be added. The basically parabolic character of the functions (also known as sphere model) is used as a typical test case for the analysis in continuous optimization. The analytical solution of the static and convex PF_{true} of the Schaffer function yields $f_2^* = (\sqrt{f_1^*} - 2)^2$ for $f_1^* \in [0,4]$, $P_{true} = [0,2]$.
The general scheme (DSW) used for functions with moving Pareto sets is:

$$DSW: \begin{cases} f_1(\mathbf{x}) = (a_{11}x_1 + a_{12}|x_1| - b_1 \cdot G(t))^2 + \sum_{i=2}^{n} x_i^2 \\ f_2(\mathbf{x}) = (a_{21}x_1 + a_{22}|x_1| - b_2 \cdot G(t) - 2))^2 + \sum_{i=2}^{n} x_i^2 \end{cases} \tag{3}$$

where $G(t): \mathbb{R} \to \mathbb{R}$ is a continuous function with monotonously increasing values. In the following the definition $G(t) := t(\tau) \cdot s$ with $t(\tau) = \frac{1}{n_t}\left\lfloor \frac{\tau}{\tau_T} \right\rfloor$ is used. In the analyses the DSW functions have been defined as follows:
DSW1: $\mathbf{x} \in [-50,50]^n, a_{11} = 1, a_{12} = 0, a_{21} = 1, a_{21} = 0, b_1 = b_2 = 1$
DSW2: $\mathbf{x} \in [-50,50]^n, a_{11} = 0, a_{12} = 1, a_{21} = 0, a_{21} = 1, b_1 = b_2 = 1$
DSW3: $\mathbf{x} \in [-50,50]^n, a_{11} = 1, a_{12} = 0, a_{21} = 1, a_{21} = 0, b_1 = 0, b_2 = 1$
Analysis with an extended decision space $n > 1$ of the corresponding functions $DSW1$ to $DSW3$ are addressed in the text, respectively.

$DSW1$ is a test function with a static PF_{true} and a shifting P_{true} having a one-dimensional decision space. In the case of $n = 1$ the problem of the $DSW1$ function is to keep the P_{true} values in the interval $[G(t), G(t)+2]$.

The $DSW2$ problem has the two separated Pareto sets. These sets depart diametrically with speed $G(t)$. PF_{true} does not change in time and is identical to

$DSW1$. With $n = 1$ the Pareto set is $P_{true} = [-G(t) - 2, G(t)] \cup [G(t), G(t) + 2]$. For periodical $G(t)$ the Pareto sets will join and depart periodically.

In problem $DSW3$ the right border of the interval of the Pareto set is moving while the other interval border is static. This is realized by setting $b_1 = 0$. The corresponding convex Pareto front increases in size and curvature and moves slightly upward with increasing t.

The new generic scheme DTF is a generalization of the FDA functions and allows a variable scaling of the complexity of the dynamic properties. The number ψ of separated Pareto front sections, the number ω of multi-fronts, the curvature α of the Pareto front and the optimal argument value $\gamma(t)$ for all $\mathbf{x_{II}}$ can be adjusted to describe additional characteristics of dynamical problems. The moving Pareto sets of the DTF functions are identical to the FDA1 Pareto sets.

$$DTF : \begin{cases} f_1(x_I) &= x_1^{\beta(t)} \\ f_2(g,h) &= g \cdot h(f,g) \\ g(x_{II}) &= 1 + \sum_{x_i \in x_{II}} ((x_i - \gamma(t))^2 \\ & \quad -cos(\omega(t(\tau)) \cdot \pi \cdot (x_i - \gamma(t))) + 1) \\ h(f_1,g) &= 2 - \left(\frac{f_1}{g}\right)^{\alpha(t)} - \left(\frac{f_1}{g}\right) \cdot |\sin(\psi(t)\pi f_1)|^{\alpha(t)} \\ & \quad x_I = (x_1)^T, x_1 \in [0,1] \\ & \quad x_{II} = (x_2, \ldots, x_n)^T \in [-1,1]^{n-1} \end{cases} \tag{4}$$

A typical definition are e.g. $n = 20$, $\alpha(t) = 0.2 + 4.8t^2$, $\beta(t) = 10^{2sin(0.5\pi t)}$, $\gamma(t) = sin(0.5\pi t)$, $\psi(t) = t \cdot s$, $s \in \mathbb{R}_+$, and $\omega(t) \propto \psi(t)$.

In practical applications restrictions introduce severe difficulties into optimization problems. The introduction of dynamic restrictions is also motivated by the idea to analyze the empirical behavior of Pareto-dominance based algorithms such as the NSGA II and SPEA 2, which explicitly utilize distribution properties of Pareto front approximations. The following restrictions can be scaled easily in positions, size and number. Here, infeasible areas in the objective space of dimension m are defined by $j = 1, \ldots, k$ m-dimensional spheres with radii r_j and center points $\{c_{1_j}, \ldots, c_{m_j}\}$. The position of the spheres may change over time.

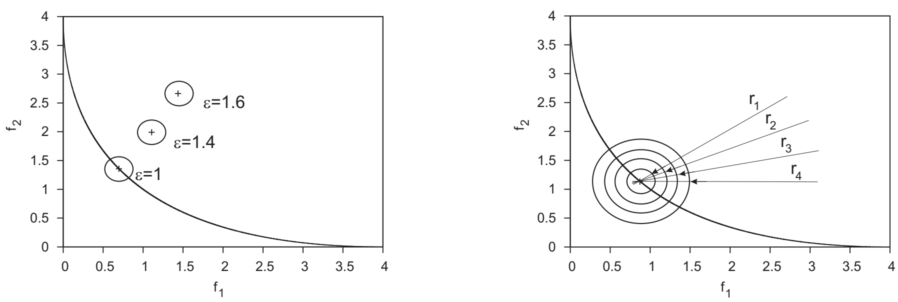

Fig. 1. Dynamic restrictions with varying position (left) or radii (right).

The value $\epsilon \geq 0$ introduces a shift of the center points of the spheres. Moving circular obstacles in the objective space is another way to analyze the convergence robustness of the optimizing algorithms. The corresponding inequalities for the restrictions are (see Fig. 1):

$$g_j : r(t)^2 - (\epsilon \cdot c_{1_j} - f_1(\mathbf{x}))^2 - \cdots (\epsilon \cdot c_{m_j} - f_m(\mathbf{x})^2) \leq 0, \quad j = 1, \ldots, k \qquad (5)$$

3 Multiobjective Evolutionary Algorithms

Today, NSGA-II [5] and SPEA 2 [6] surely belong to the most commonly applied multiobjective evolutionary algorithms. The analyzes of these algorithms implies a good comparability and applicability of the following results for many research fields that intend to allow dynamics in optimization problems.
Additionally, the MSOPS (Multiple Single Objective Pareto Sampling) strategy [7] is used to get an impression of the behavior of a stochastic population based algorithm that does not utilize the Pareto-dominance principle. MSOPS works with a weighting and ranking scheme of the fitness function values. In contrast to conventional linear aggregation approaches, the algorithm is able to find solutions also for non convex Pareto fronts. The multiobjective PISA environment [8] served as a common interface to well tested genetic operators such as SBX recombination or polynomial mutation [10]. Differential evolution variation operators DE1 and DE2 [9] were not provided by PISA and added by the authors. In PISA variation operators can be combined with specific NSGA II, SPEA 2 and MSOPS selection operators. The MSOPS was not provided in PISA and was added for a comprehensive analysis.

4 Metrics

The solution of multiobjective optimization problems generally focuses on the approximation of Pareto fronts (a posteriori approach) or the selection of specific points or areas on the Pareto front (a priori approach). The quality of a Pareto front approximation is measured by functions. In the MOEA terminology these measures are typically called metrics[3]. Different metrics characterize the distance as well the distribution properties of a MOEA Pareto front approximation PF_{approx}.

The generational distance metric was introduced by van Veldhuizen and Lamont [11]. This measure calculates the average distance of PF_{approx} to the real Pareto front PF_{true}. Given a set of discrete N solutions of a MOEA in generation j, the generational distance metric G_j is defined as follows:

$$G_j := \frac{\sqrt{\sum_{i=1}^{N} d_i^2}}{N} \qquad (6)$$

[3] The strict mathematical definition of a real metric is not always guaranteed by MOEA-metrics.

The d_i are the minimum euclidean distances of one discrete solution to the Pareto front measured in the objective space.

In the following a new distribution metric – the PL-metric – is introduced. This approach uses an analytical description for calculating the distances of solutions on the Pareto front via path integrals. A precondition for the calculations is an analytic closed description of PF_{true}. This is e.g. true for the DSW1 and DSW2 (where $f_2^* = (\sqrt{f_1^*} - 2)))$ as well as for the FDA1-3 functions.

A path between $[a, b]$ can be defined by a continuous parametric function $\gamma : [a, b] \subseteq \mathbb{R} \to \mathbb{R}^m$, $\gamma(t) = (\gamma_1(t), \ldots, \gamma_m(t))^T$. For multiobjective problems with two objective functions with a continuous Pareto front $f_2(f_1)$ the corresponding path on the Pareto front follows $\gamma(t) = (t, f_2(t))$.

Given a path $\gamma(t) : \mathbb{R} \to \mathbb{R}^m$ that is continuously differentiable in $[a, b]$ (i.e. $\gamma \in C^1[a, b]$). Then the corresponding path length function s is continuously differentiable in $[a, b]$, i.e. $s(t) = |\dot{\gamma}(t)|$. The length $L(\gamma, a, b)$ of a path between $[a, b]$ on γ is

$$L(\gamma, a, b) := \int_b^a |\dot{\gamma}| dt = \int_b^a \sqrt{\dot{\gamma}_1^2 + \ldots + \dot{\gamma}_m^2} dt \quad (7)$$

with $\dot{\gamma}_i$ is the derivative of γ_i in t and $|\cdot|$ is the euclidean norm.

The PL-metric is defined by the normalized product of the lengths of each subsection $\xi = L(\gamma, f(x_i), f(x_{i+1}))$ between sorted neighboring points on the true Pareto front (see formula 9) adding 1 to guaranties that a new solutions increase the value of the metric. Due to the approximative character of MOEA, a solutions is said to be actually on PF_{true}, if it is in an ϵ-region near PF_{true}, e.g. $\epsilon = 0.01$. It can be shown that the PL-metric has a maximum, if the path of the total length found on the true Pareto Front $L_{PF_{true}}$ is divided into equidistant sections. In this case the value is

$$\lim_{|PF_{true}| \to \infty} \left(1 + \frac{L_{PF_{true}}}{|PF_{true}| - 1}\right)^{|PF_{true}|-1} = e^{L_{PF_{true}}}. \quad (8)$$

With this condition, the PL-metric can be normalized in $[0, 1]$, yielding

$$PL := \frac{\sum_{f(x_i) \in PF_{true}} ln(\xi_{x_i})}{L_{PF_{true}}}. \quad (9)$$

Schott's spacing metric S (see e.g. [11]) was used as a standard method for the estimation of the distribution of PF_{approx}.

5 Experimental Setup

The analysis have been performed on a standard PC using Linux/Unix. The modules MSOPS and differential evolution (DE1 and DE2) were implemented for PISA in C++. The metrics were implemented in MATLAB. Table 5 shows the parameter settings. Each experiment was performed 20 times and the arithmetic average solution was used.

Table 1. Setting for the Selection- and Variation Operators

Selection	Parameter	Setting	Variation	Parameter	Setting
General	α pop. size	100	SBX	p_c crossover rate	0,5
	λ parents	100		η_c dist. index	15
	μ offspring	100	Uniform	p_{swap} prob.	0,5
	κ age	$\kappa = \infty$	Poly. Mut.	p_m mutation rate	$1/\lvert\mathbf{x}\rvert$
	s_t tournament	2		η_m dist. index	20
			DE1	F weight	0,7
MSOPS	T target vectors	50		CR insert length prob.	0,5
			DE2	F weight	0,85
SPEA2	$\overline{N}$ archive size	100		λ_{DE} weight	0,85
				CR insert length prob.	1

6 Results

The analyses focus on the ability of the MOEA to follow the moving or changing PF_{true} or P_{true}. In the first experiments the speed s of the Pareto set is varied. The functions $DSW1$, $DSW2$, $FDA1$, $FDA2_{mod}$, $FDA4$ and DTF functions were tested in combination with NSGA-II, SPEA 2 and MSOPS. A linear increase c of speed s (starting with $s = 0$) is indicated by $s+ = c$.

Analysis with $DSW1$ and $DSW2$:
Figure 2 shows that for the DSW1 problem NSGA-II is able keeping the solutions in the Pareto set for small speed s. From $s > 6$ the distances G_i increase about quadratic because the solution cannot be placed in the Pareto set. The values of the distribution (PL-metric) show stable good values for small s and decrease about linearly with $s > 6$. NSGA-II and SPEA 2 show nearly equally good distribution values over all s. Even for small s the MSOPS distribution values are worse than that of the other two algorithms. This may be caused by the explicit distribution measure which is explicitly used in NSGA-II and SPEA 2. MSOPS focused its points more in the middle of the convex Pareto front. For high s the performance of MSOPS is similar to NSGA-II and SPEA 2. For high s values the variation operators SBX with polynomial mutation perform better on the problems than the DE operators because DE1 and DE2 cause a too small diversity in the solution sets. For small speed ($s = 0.1$) the generational distance values of the SPEA2 with DE1 show the best distance metric values while NSGA-II with DE1 show best distribution values.

The influence of departing Pareto sets was tested with the DWS2 function. In the case of SBX recombination combined with polynomial mutation all algorithms show a similarly good distribution and approximation values for small s. For larger s the algorithms tend to focus on one part of the Pareto set only. DE2 improves the generational distance values of all algorithms also for large s (see Fig. 3), while DE1 degrade the performances. This may be due to the fact that DE2 utilizes – in contrast to DE1 – the position of the Pareto 'best' individuals. Interactions with these individuals improve the performance of the MOEA with DE2 on DSW2 significantly.

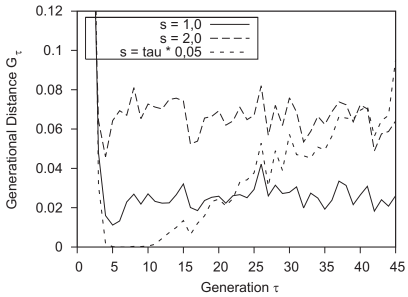

Fig. 2. Values of the generational distance G_τ. NSGA-II (using polynomial mutation and SBX) applied to $DSW1$ for constant and linearly increased shifting speed s of the Pareto set.

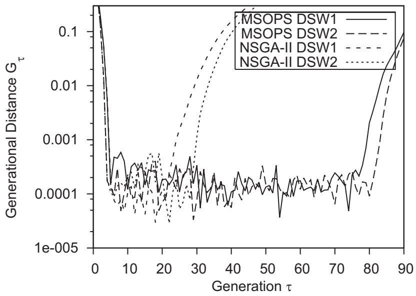

Fig. 3. Values of the generational distance G_i. NSGA-II and MSOPS applied to DSW2 using the DE1 and DE2 operators with speed $s+ = 0.01$ for separating P_{true}.

A stepwise movement of the Pareto set ($s = 10$ and $\tau_t = 5$) together with a restart after each step to reinitializes the population structure was tested on DSW1 to analyze the influence of the generational evolution (population memory effect). The results show that the algorithms using SBX and polynomial mutation are not affected much by reinitialization. A restart in combination with DE2 yields significant improvement in the distribution as well as in the distance values because the structural properties of the DSW1 problem can be exploited. Only for very high s this effect is reduced.

In order to analyze the influence of the decision space dimensions $n > 2$ on the solution of the dynamic problem DSW1 the values $n = 5, 6, \ldots, 20$ and $s = 0.1$ were chosen. Using SBX and polynomial mutation yields good distance values. The distribution quality decreases with increasing n. In contrast to the suggestions in literature to increase the mutation rate per parameter $(1/|\mathbf{x}|)$ for higher dimensions, the dynamic high dimensional DSW1 problem show better convergence speed with decreased rates. The application of the differential evolution operators do not show sufficient results for $n > 5$.
Analysis with $FDA1$, $FDA2_{mod}$ and $FDA4$:
The FDA functions are more complex than DSW. $FDA1$ has a dynamically varying Pareto set. The influence of the memory effect of population based optimizers is tested with the restarting approach. In the experiments with $FDA1$ the speed was set to $s = 0.1$ and the dimensions to $n = 20$. For $\tau_t = 1$ no algorithm succeeds in approximating the Pareto set of $FDA1$ sufficiently. Only a small area in the middle between the borders of the oscillation limits of P_{true} is found. The high variation speed yields an averaging effect. An increase of the saturation phases to $\tau_t = 10$ improves the chance to follow P_{true}. The algorithms tend to loose the optimal solutions after some generations and a delayed averaging effect appears. From about $\tau_t = 50$ the MOEA are able to follow the Pareto set correctly. The NSGA-II is the fastest algorithms to place a solution near the Pareto front (1 percent tolerance) followed by MSOPS and SPEA 2. In $FDA1$

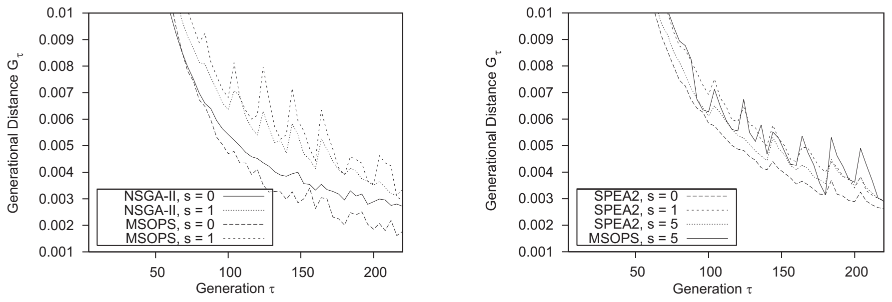

Fig. 4. Values of the generational distance G_τ for NSGA-II, MSOPS and SPEA 2 when applied to DTF for different s.

a memory seems useful for NSGA-II because a restarting strategy reduces its approximation property.

The $FDA2_{mod}$ shows a Pareto front with oscillating curvature. Setting $g \equiv 1$, $\tau_t = 5$ and $n_t = 10$ implies that every solution is in P_{true}. The algorithms only have to keep a good distribution on PF_{true}. Algorithms using SBX and polynomial mutation show good distributions after a short tuning time. SPEA2 shows a significant inertness when the curvature changes from convex to concave. Here, NSGA-II and MSOPS are more tolerant. The differential mutation operators DE1 and DE2 seem to be sensitive to dynamic changes in the curvature and show an inferior behavior.

$FDA4$ has an increased dimension $m > 1$ of the objective space. In the experiments the decision space dimension was reduced to $n = 12$. For $s = 0.1$ and $\tau_t = 50$ all algorithms are able to approximate the Pareto set after each position shift. Here, the MSOPS algorithm finds good solutions on the Pareto front about four times faster than NSGA-II or SPEA 2. This may be explained by the fact that an increased number of dimensions reduces the number of dominated solutions found [7]. If only non-dominated solutions exist, all solutions have the same ranking and only the distance criteria have an effect. This reduces the convergence efficiency of the Pareto dominance based algorithms. The distribution quality of MSOPS is significantly worse than the values measured for NSGA-II and SPEA 2.

Analysis with DTF:

The idea of the DFT experiments is to analyze the effects of a dynamically varying Pareto front structure. The parameter ψ, which characterizes the number of Pareto front sections, is defined to oscillate in $[0, 5]$. The parameters $\tau_t = 20, \gamma = 0, \alpha = 0.5, \beta = 1, \omega = 0, s \in \{0, 1, 5\}$, and $n = 20$ were used. The structure is oscillating between one connected and five separated Pareto front segments. The dominance based algorithms can separate the solutions on the dividing Pareto fronts. The speed of the separation has a significant influence on the solution distribution abilities of the algorithms. A delaying effect can be watched (see Fig. 4). While MSOPS gets best results for static fronts ($s = 0$) its concept seems to be disadvantageous for fast separating Pareto fronts.

7 Summary and Outlook

A comprehensive empirical analysis is performed on standard benchmark functions (FDA) as well as on new functions (DSW and DTF). A new integral PL-metric is introduced. In many cases alternative operators such as differential evolution (DE) and selection methods (MSOPS) show an advantageous behavior on dynamic functions when compared to classic operator combinations. A statistical search for the best parameter settings as well as the analysis of the population structure of the MOEA is matter of future research.

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