UNIVERSITÄT DORTMUND

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A Library of Multiobjective Functions with Corresponding Graphs

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A Library of Multiobjective Functions with Corresponding Graphs

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1 Introduction

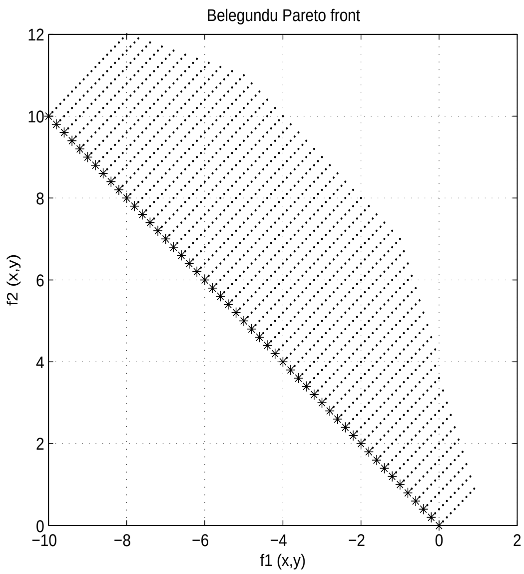
Testing multiobjective evolutionary algorithms with benchmark functions improves the knowledge about algorithms and allows a standardized comparison. In the following, an alphabetically ordered and updated list of multiobjective test functions is given (based on [7]). The graphs of the Pareto fronts and Pareto sets are displayed. Additionally, a short characterization of the Pareto front and Pareto set is given. Especially for benchmarking new algorithms, it can be very helpful to have an intuitive knowledge about the properties of the test functions. Often Pareto sets are neglected although it is generally advisable to have a look at the solution sets.

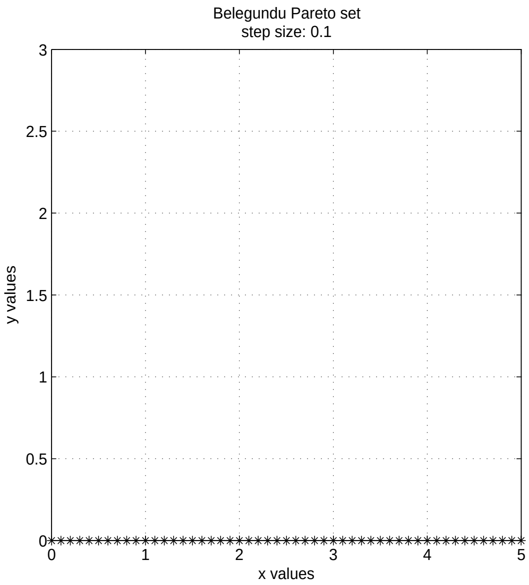
The graphs are generated by simple grid scanning of the decision space. In contrast to random scanning (see [8]), the equidistant grid scanning generates a neighborhood relation that increases the ability to interpret the solution structures graphically. The transform of all scanned points of the decision space is plotted in the objective space using small dots. The Pareto fronts are filtered by an effective scanner which uses a fast selection scheme that is based on Borwein's Theorem (see [11]). The Pareto front is indicated by stars in the graphs in the decision space. The corresponding Pareto sets are plotted in the decision space also using stars. All functions and the scanner are written in standard C++. The plots were generated with Matlab.

The functions are titled as commonly found in the literature. Short descriptions of the type of the Pareto front PF_{true} and the Pareto set P_{true} are given in the corresponding columns of the tables. In general the functions have been cited as published in the original papers. In special cases parameter ranges have been adapted for a better understanding of the idea behind the functions. A list of references is given at the end of this paper.

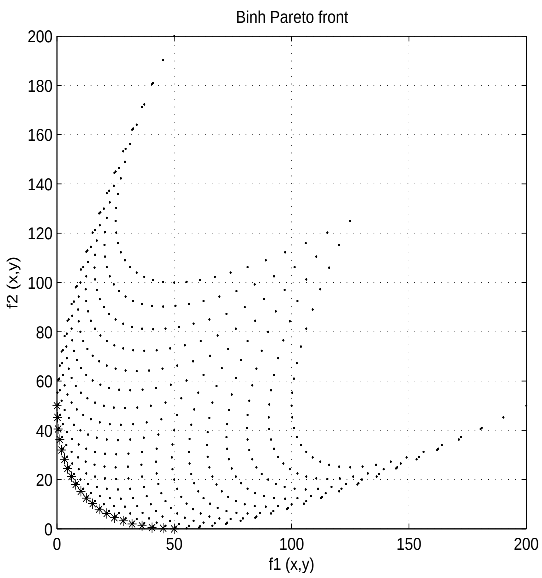
2 Functions and Graphs

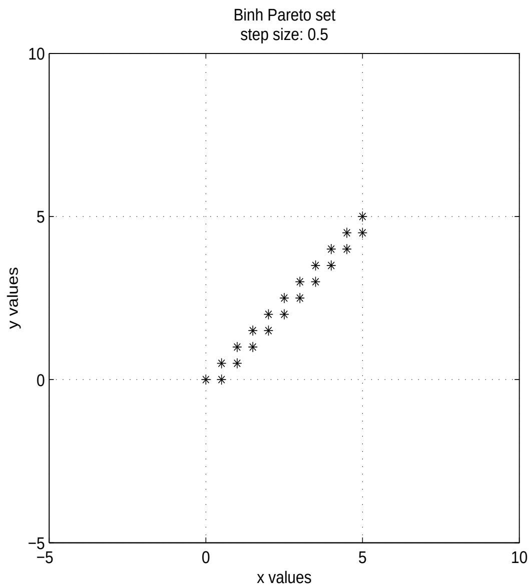
name of problem / authors	**definition and restrictions**	**characteristics**
Belegundu [1] A. D. Belegundu, D. V. Murthy, R. R. Salagame, E. W. Constanst.	$F = (f_1(x,y), f_2(x,y))$, with $$\begin{aligned} f_1(x,y) &= -2x + y, \\ f_2(x,y) &= 2x + y \end{aligned}$$ restrictions: $0 \leq x \leq 5$, $0 \leq y \leq 3$, $0 \geq -x + y - 1$, $0 \geq x + y - 7$	PF_{true} linear and conntected; P_{true} linear and connected



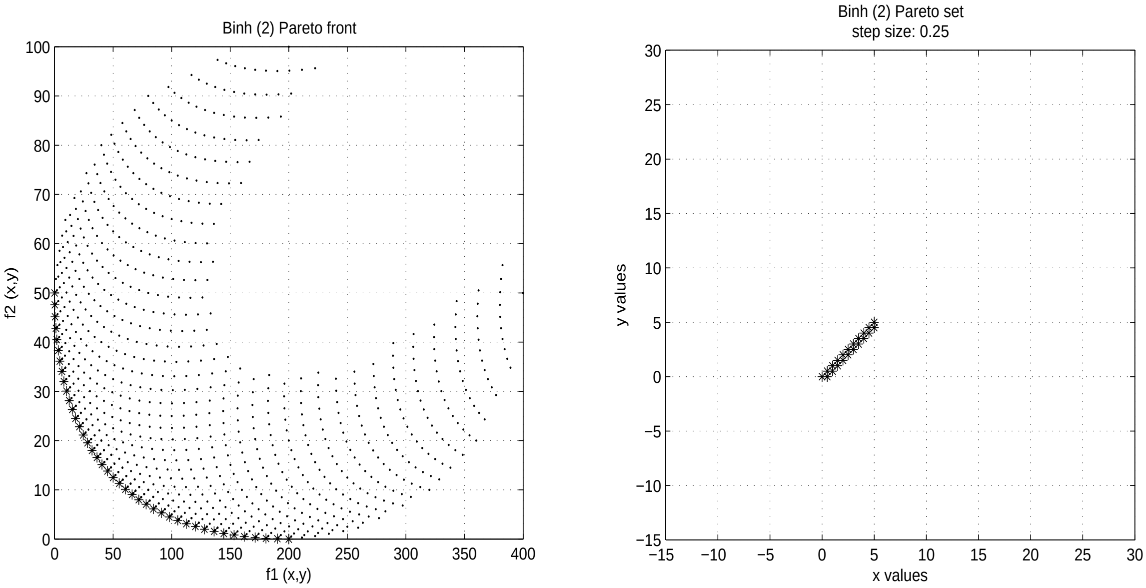


name of problem / authors	**definition and restrictions**	**characteristics**
Binh (1) [4] T.T. Binh, U. Korn.	$F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= x^2 + y^2, \\ f_2(x,y) &= (x-5)^2 + (y-5)^2 \end{aligned}$$ restrictions: $-5 \leq x, y \leq 10$	PF_{true} connected and convex; P_{true} connected and linear

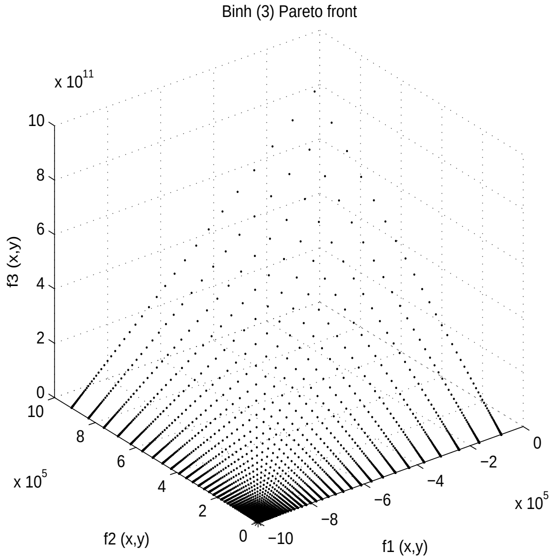


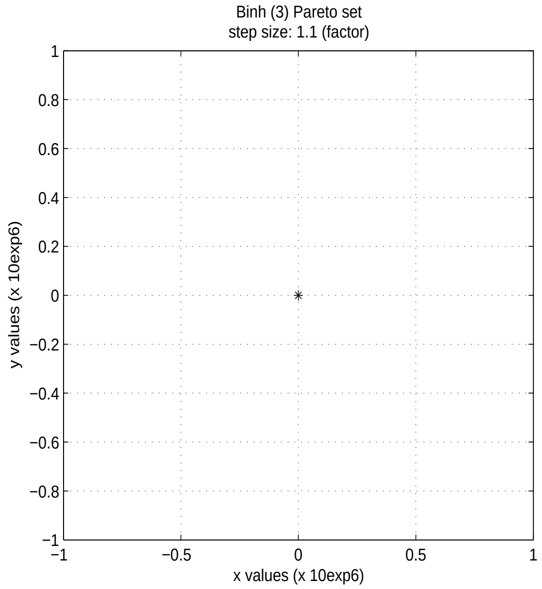


name of problem / authors	definition and restrictions	characteristics
Binh (2) [5] T.T. Binh, U. Korn.	$F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = 4x^2 + 4y^2,$$ $$f_2(x,y) = (x-5)^2 + (y-5)^2$$ restrictions: $0 \leq x \leq 5$, $0 \leq y \leq 3$, $0 \geq (x-5)^2 + y^2 - 25$, $0 \geq -(x-8)^2 - (y+3)^2 + 7.7$	PF_{true} connected and convex; P_{true} connected and linear

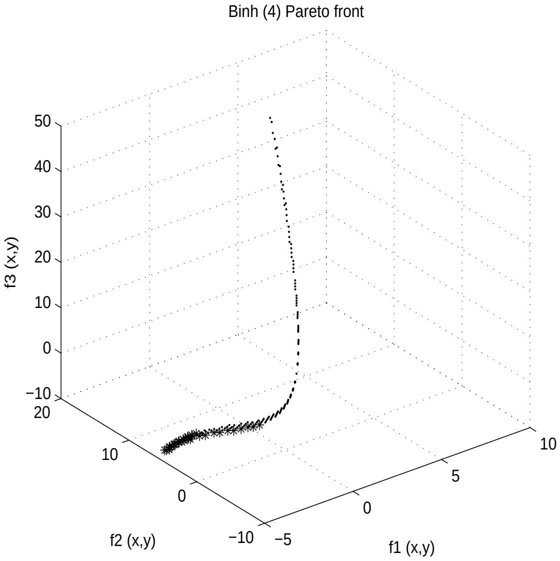


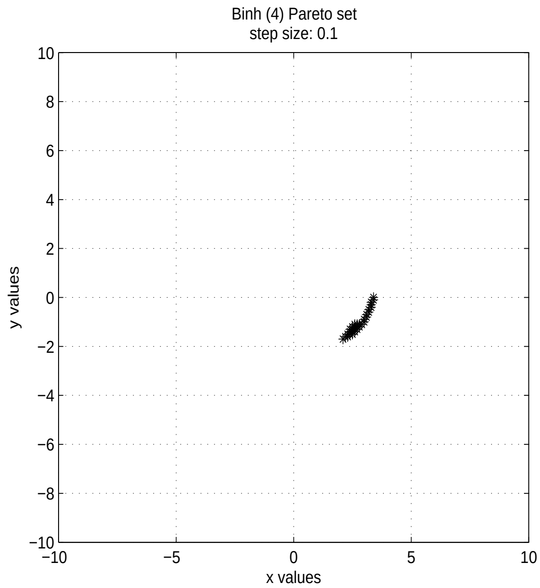
name of problem / authors	definition and restrictions	characteristics
Binh (3) [6, 7, 3] T.T. Binh, U. Korn.	$F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $$\begin{aligned} f_1(x,y) &= x - 10^6, \\ f_2(x,y) &= y - 2 * 10^{-6}, \\ f_3(x,y) &= xy - 2 \end{aligned}$$ restrictions: $10^{-6} \leq x, y \leq 10^6$	PF_{true} is weak Pareto optimal (point selected); P_{true} point solution selected



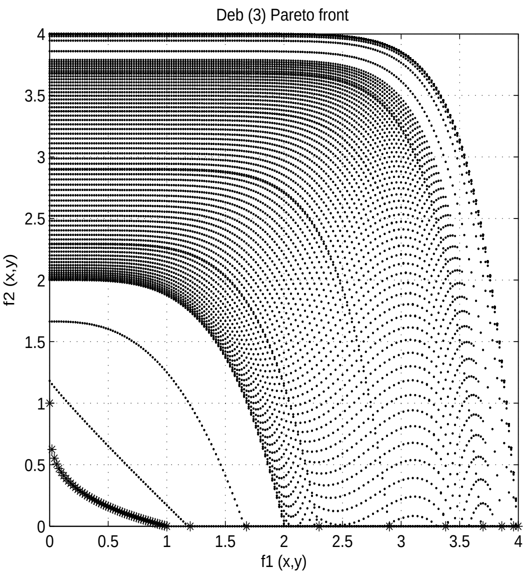


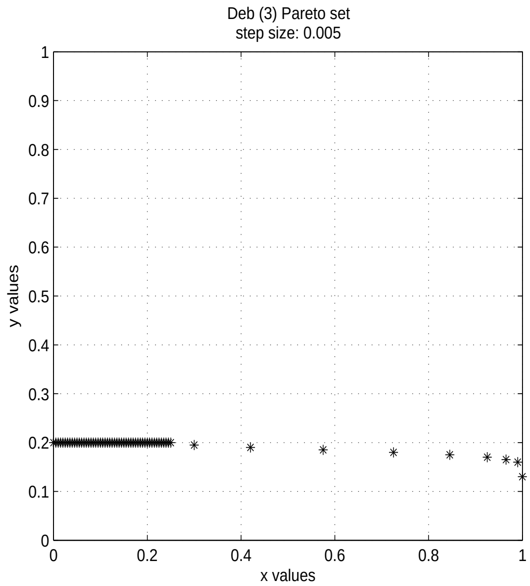
name of problem / authors	definition and restrictions	characteristics
Binh (4) [2] T.T. Binh, U. Korn.	$F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = x+y,$$ $$f_2(x,y) = 1 - exp(-4x)sin^4(5\pi x)$$ restrictions: $-10 \le x, y \le 10$	PF_{true} is a curve; P_{true} is a curve



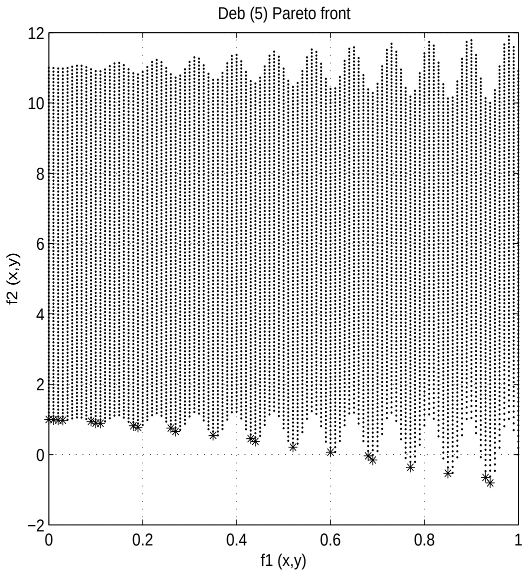


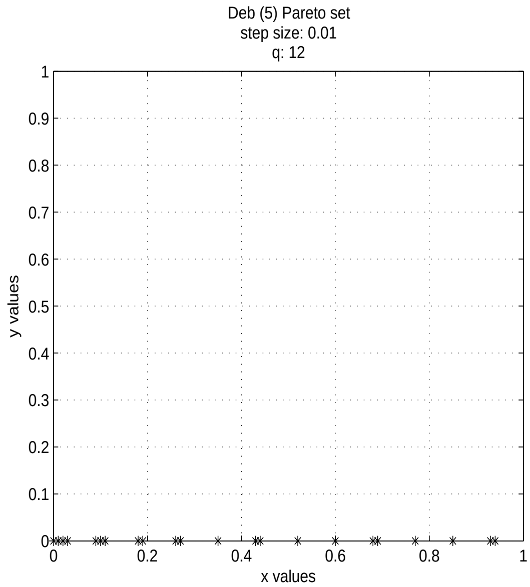
name of problem / authors	**definition and restrictions**	**characteristics**
Deb (3) [8] K. Deb	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $f_1(\mathbf{x}) = f(x),$ $f_2(\mathbf{x}) = g(y) \cdot h(f, g)$ and $f(x) = 4 \cdot x,$ $g(y) = \begin{cases} 4 - 3 \cdot \exp(-(\frac{y-0.2}{0.02})^2) & 0 \leq y \leq 0.4 \\ 4 - 2 \cdot \exp(-(\frac{y-0.7}{0.2})^2) & 0.4 \leq y \leq 1 \end{cases}$ $h(f,g) = \begin{cases} 1 - (\frac{f}{\beta \cdot g})^{\alpha} & \text{if } f \leq \beta \cdot g \\ 0 & \text{else} \end{cases}$ with $\alpha = 0.25 + 3.75 \cdot (g(y) - 1)$ and $\beta = 1$ restrictions: $0 \leq x, y \leq 1$	PF_{true} connected line; problem difficult due to change of expected PF-shape from concave to convex; multi-front problem; P_{true} connected curve



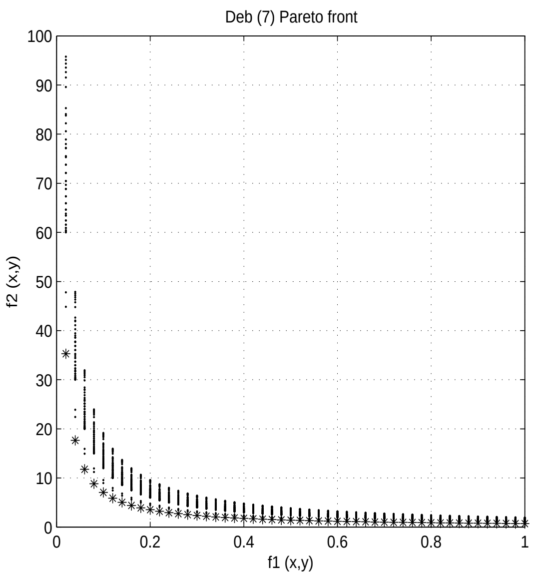


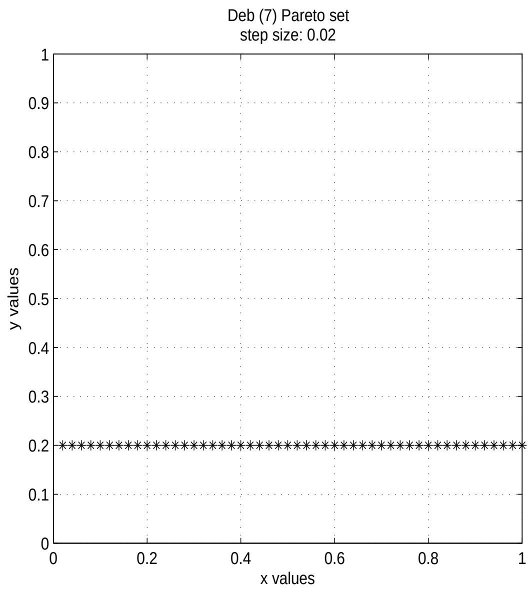
name of problem / authors	definition and restrictions	characteristics
Deb (5) [8] K. Deb	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= f(x), \\ f_2(\mathbf{x}) &= g(y) \cdot h(f,g) \\ \text{and} \\ f(x) &= x \\ g(y) &= 1 + 10 \cdot y \\ h(f,g) &= 1 - \left(\frac{f}{g}\right)^{\alpha} - \left(\frac{f}{g}\right) \cdot \sin(2\pi \cdot q \cdot f) \end{aligned}$$ with: q defined the number of lags in the interval [0,1] $\alpha = 2$ is a typical choice restrictions: $0 \le x, y \le 1$	PF_{true} not connected; P_{true} not connected



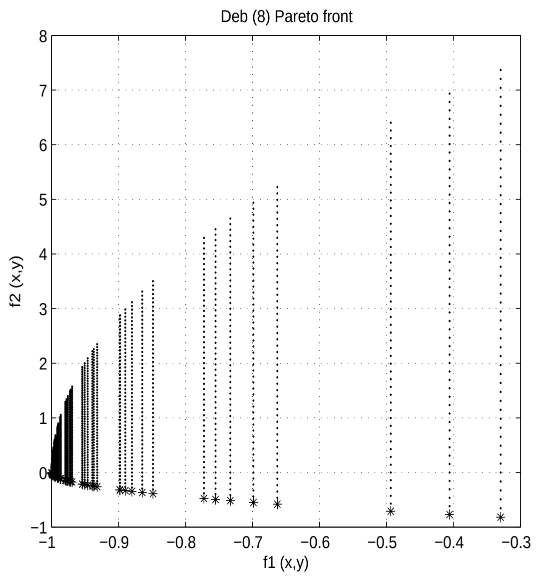


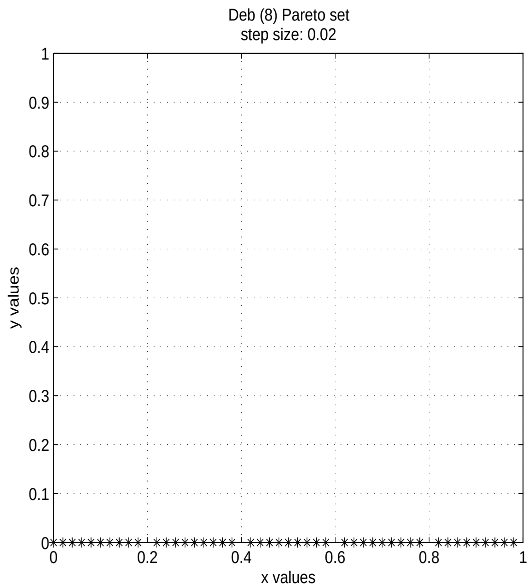
name of problem / authors	definition and restrictions	characteristics
Deb (7) [8] K. Deb	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= f(x), \\ f_2(\mathbf{x}) &= g(y) \cdot h(f, g) \\ \text{and} & \\ f(x) &= x \\ g(y) &= 2 - \exp\left(-\left(\frac{y-0.2}{0.04}\right)^2\right) \\ &\quad -0.8 \cdot \exp\left(-\left(\frac{y-0.6}{0.4}\right)^2\right) \\ h(f,g) &= \frac{1}{f} \end{aligned}$$ Restrictions: $0 < x \leq 1, 0 \leq y \leq 1$	PF_{true} "ghost" tradeoff surfaces, multi-front problem; P_{true} connected line



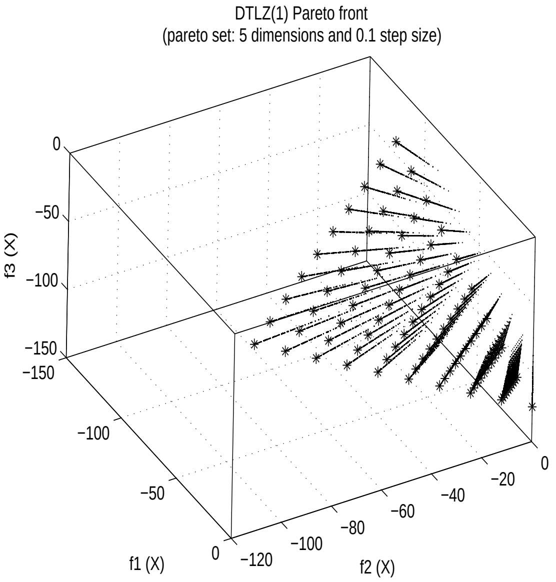


name of problem / authors	**definition and restrictions**	**characteristics**
Deb (8) [8] K. Deb	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= -f(x), \\ f_2(\mathbf{x}) &= -g(y) \cdot h(f,g) \\ \text{and} & \\ f(x) &= 1 - \exp(-4 \cdot x) \cdot \sin^4(5\pi \cdot x) \\ g(y) &= 1 - 10 \cdot y \\ h(f,g) &= \sqrt{1-f} \end{aligned}$$ restrictions: $0 \leq x, y \leq 1$	PF_{true} not connected convex curve; P_{true} line

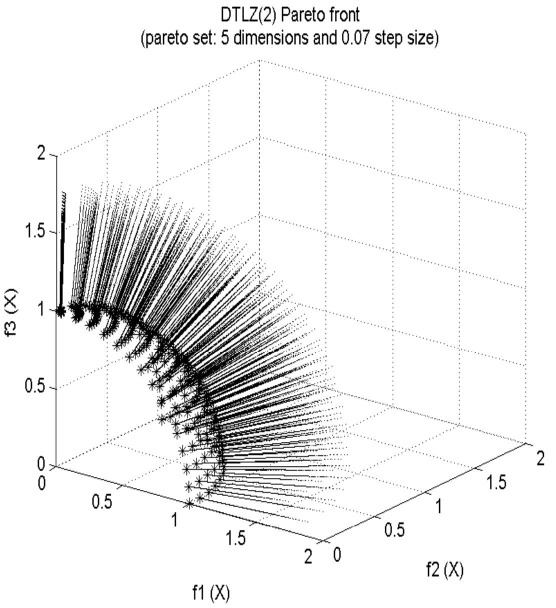




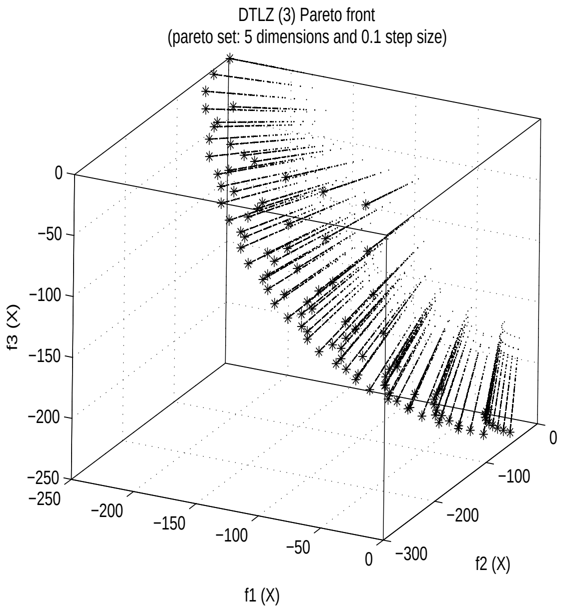
name of problem / authors	definition and restrictions	characteristics
DTLZ (1) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimiere $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, wo $f_1(\mathbf{x}) = \frac{1}{2}x_1x_2\cdots x_{M-1}(1+g(\mathbf{x}_M))$, $f_2(\mathbf{x}) = \frac{1}{2}x_1x_2\cdots(1-x_{M-1})(1+g(\mathbf{x}_M))$, $\vdots$ $f_{M-1}(\mathbf{x}) = \frac{1}{2}x_1(1-x_2)(1+g(\mathbf{x}_M))$, $f_M(\mathbf{x}) = \frac{1}{2}(1-x_1)(1+g(\mathbf{x}_M))$ $g(\mathbf{x}_M) = 100(\lvert\mathbf{x}_M\rvert + \sum_{x_i\in\mathbf{x}_M}((x_i-0.5)^2 - cos(20\pi(x_i-0.5))))$ restrictions: $0 \leq x_i \leq 1$ for $i = 1, 2, ..., n$. $\forall \quad g \geq 0$: $g(\mathbf{x}_M)$ assumes $\lvert\mathbf{x}_M\rvert = k$ variables. Total number of variables is $n = M + k - 1$.	after scheme from [10] PF_{true} is a (triangular) hyperplane with $\sum_{m-1}^{M} f_m^* = 0.5$; P_{true} high dimensional



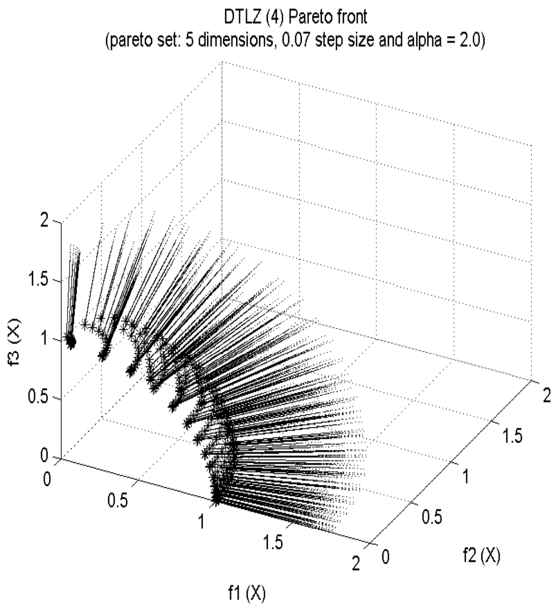
name of problem / authors	definition and restrictions	characteristics
DTLZ (2) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $f_1(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(x_1\pi/2)\cdots\cos(x_{M-2}\pi/2)\cos(x_{M-1}\pi/2),$ $f_2(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(x_1\pi/2)\cdots\cos(x_{M-2}\pi/2)\sin(x_{M-1}\pi/2),$ $f_3(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(x_1\pi/2)\cdots\sin(x_{M-2}\pi/2),$ $\vdots$ $f_M(\mathbf{x}) = (1 + g(\mathbf{x}_M))\sin(x_1\pi/2),$ mit $g(\mathbf{x}_M) = \sum_{x_i \in \mathbf{x}_M} (x_i - 0.5)^2$ restrictions: $0 \leq x_i \leq 1$ for all $i = 1, 2, ..., n$.	generated following [10] PF_{true} spherical surface; P_{true} high dimensional



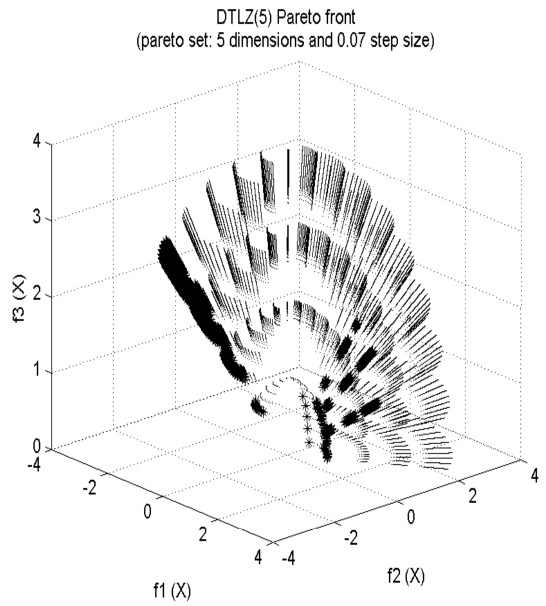
name of problem / authors	**definition and restrictions**	**characteristics**
DTLZ (3) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $f_1(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \cos(x_1\pi/2) \cdots \cos(x_{M-2}\pi/2) \cos(x_{M-1}\pi/2)$, $f_2(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \cos(x_1\pi/2) \cdots \cos(x_{M-2}\pi/2) \sin(x_{M-1}\pi/2)$, $f_3(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \cos(x_1\pi/2) \cdots \sin(x_{M-2}\pi/2)$, $\vdots$ $f_M(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \sin(x_1\pi/2)$, mit $g(\mathbf{x}_M) = 100\Big(\lvert\mathbf{x}_M\rvert + \sum_{x_i \in \mathbf{x}_M} ((x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)))\Big)$ restrictions: $0 \leq x_i \leq 1$ for all $i = 1, 2, ..., n$.	generated following [10] PF_{true} spherical surface; P_{true} high dimensional



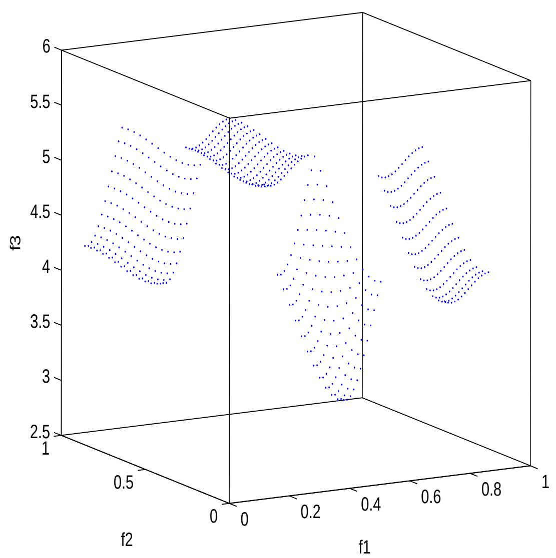
name of problem / authors	definition and restrictions	characteristics
DTLZ (4) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $$f_1(\mathbf{x}) = (1+g(\mathbf{x}))\cos(x_1^\alpha \pi/2)\cdots\cos(x_{M-2}^\alpha \pi/2)\cos(x_{M-1}^\alpha \pi/2),$$ $$f_2(\mathbf{x}) = (1+g(\mathbf{x}))\cos(x_1^\alpha \pi/2)\cdots\cos(x_{M-2}^\alpha \pi/2)\sin(x_{M-1}^\alpha \pi/2),$$ $$f_3(\mathbf{x}) = (1+g(\mathbf{x}))\cos(x_1^\alpha \pi/2)\cdots\sin(x_{M-2}^\alpha \pi/2)$$ $$\vdots$$ $$f_M(\mathbf{x}) = (1+g(\mathbf{x}))\sin(x_1^\alpha \pi/2),$$ with $g(\mathbf{x}) = \sum_{x_i \in \mathbf{x}} (x_i - 0.5)^2$ and e.g. $\alpha = 1.0$ restrictions: $0 \le x_i \le 1$ for $i = 1, 2, ..., n$.	generated following [10] PF_{true} spherical surface; P_{true} high dimensional



name of problem / authors	definition and restrictions	characteristics
DTLZ (5) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $f_1(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \cos(\theta_1 \pi/2) \cdots \cos(\theta_{M-2}\pi/2) \cos(\theta_{M-1}\pi/2)$, $f_2(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \cos(\theta_1 \pi/2) \cdots \cos(\theta_{M-2}\pi/2) \sin(\theta_{M-1}\pi/2)$, $f_3(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \cos(\theta_1 \pi/2) \cdots \sin(\theta_{M-2}\pi/2)$, $\vdots$ $f_M(\mathbf{x}) = (1 + g(\mathbf{x}_M)) \sin(\theta_1 \pi/2)$, with $\theta_i = \frac{\pi}{4(1+g(r))}(1 + 2g(r)x_i)$, $r = 1 + 2g(x_{i-1})$ for $i = 2, 3, ...(M-1)$, $\theta_1 = x_1 \frac{\pi}{2}$ $g(\mathbf{x}_M) = \sum_{x_i \in \mathbf{x}_M} x_i^{0.1}$ restrictions: $0 \leq x_i \leq 1$ for all $i = 1, 2, ..., n$, $n = M + k - 1$ with $k = \|x_M\|$.	generated following [10] PF_{true} circular curve; P_{true} high dimensional

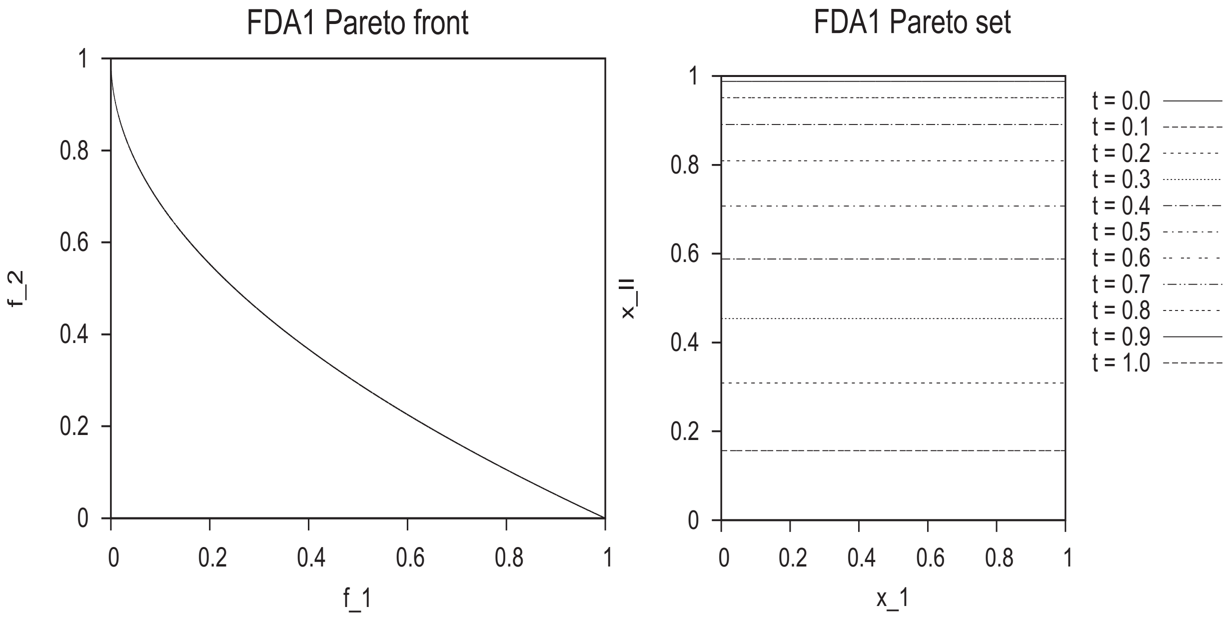


name of problem / authors	**definition and restrictions**	**characteristics**
DTLZ (6) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $\begin{aligned} f_1(x_1) &= x_1, \\ &\vdots \\ f_{M-1}(x_{M-1}) &= x_{M-1}, \\ f_M(\mathbf{x}) &= (1+g(\mathbf{x}_M))h(f_1, f_2, ..., f_{M-1}, g), \\ \text{mit} & \\ g(\mathbf{x}_M) &= 1 + \frac{9}{\lvert\mathbf{x_M}\rvert} \sum_{x_i \in \mathbf{x}_M} x_i, \\ h &= M - \sum_{i=1}^{M-1} \left[\frac{f_i}{1+g}(1+\sin(3\pi f_i)) \right] \end{aligned}$ restrictions: $0 \leq x_i \leq 1$ for $i = 1, 2, ..., n$. $g(\mathbf{x}_M)$ assumes $\lvert\mathbf{x}_M\rvert = k$ variables and the total number of variables is $n = M + k - 1$.	generated following [10] PF_{true} disconnected regions; P_{true} high dimensional

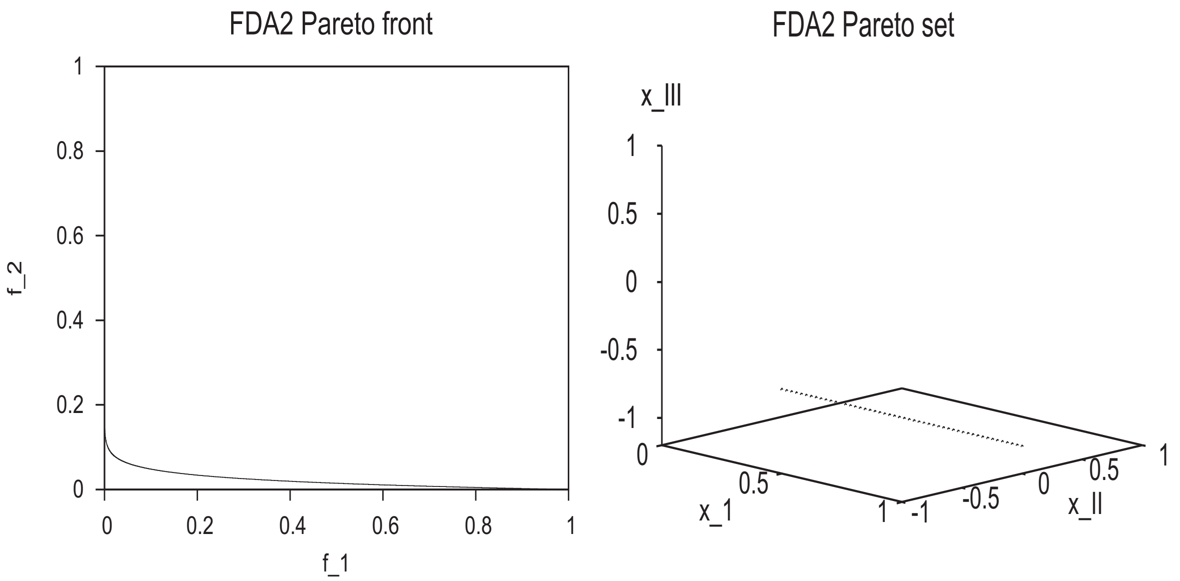


name of problem / authors	definition and restrictions	characteristics
DTLZ (7) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $$\begin{aligned} f_j(\mathbf{x}) &= \frac{1}{\lfloor \frac{n}{M} \rfloor} \sum_{i=\lfloor (j-1)\frac{n}{M} \rfloor}^{\lfloor j\frac{n}{M} \rfloor} x_i,\ j = 1, 2, ..., M \\ g_j(\mathbf{x}) &= f_M(\mathbf{x}) + 4f_j(\mathbf{x}) - 1 \geq 0, \\ & \quad j = 1, ..., (M-1) \text{ mit} \\ g(\mathbf{x}_M) &= 2f_M(\mathbf{x}) \\ & \quad + \min{}_{i,j=1,\ i\neq j}^{M-1} [f_i(\mathbf{x}) + f_j(\mathbf{x})] - 1 \geq 0 \end{aligned}$$ restrictions: $0 \leq x_i \leq 1$ for $i = 1, 2, ..., n$.	generated following [10] PF_{true} straight line and hyperplane
DTLZ (8) [10] K. Deb, L. Thiele, M. Laumanns, E. Zitzler.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$, where $$\begin{aligned} f_j(\mathbf{x}) &= \sum_{i=\lfloor (j-1)\frac{n}{M} \rfloor}^{\lfloor j\frac{n}{M} \rfloor} x_i^{0.1},\ j = 1, 2, ..., M \\ g_j(\mathbf{x}) &= f_M^2(\mathbf{x}) + f_j^2(\mathbf{x}) - 1 \geq 0 \\ & \quad \text{for } j = 1, 2, ..., (M-1) \end{aligned}$$ restrictions: $0 \leq x_i \leq 1$ for $i = 1, 2, ..., n$ with e. g. $n = 10M$.	generated following [10] PF_{true} curve with $f_1 = f_2 = \dots f_{M-1}$

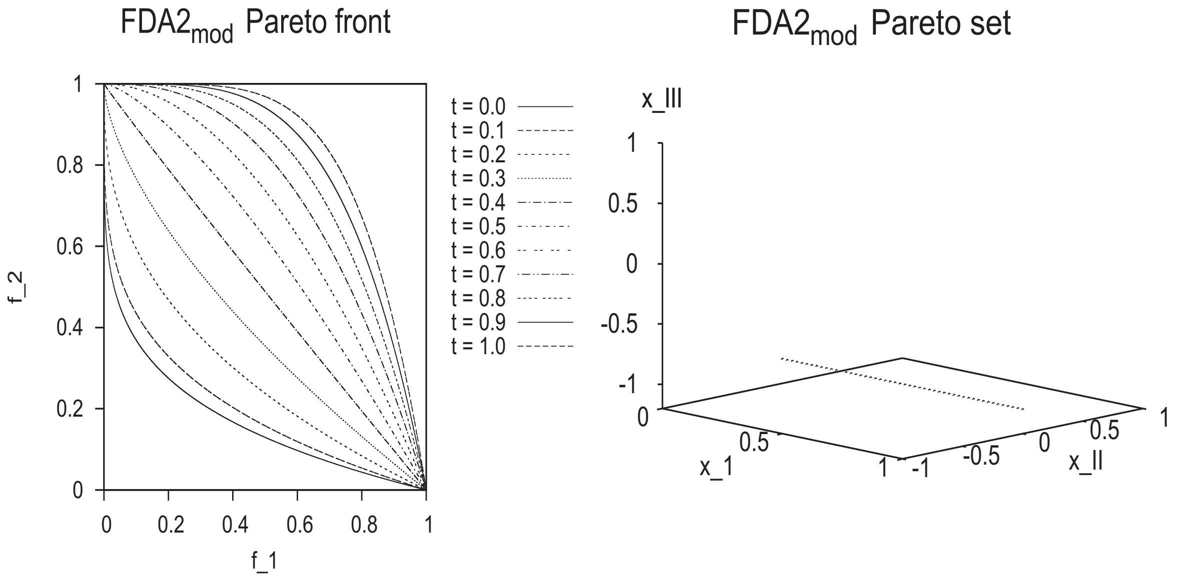
name of problem / authors	definition and restrictions	characteristics
FDA1: [12] M. Farina and K. Deb and P. Amato	$f_1(\mathbf{x_I}) = x_1, g(\mathbf{x_{II}}) = 1 + \sum_{x_i \in \mathbf{x_{II}}} (x_i - G(t))^2$, $h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}},\ G(t) = sin(0.5\pi t)$, $f_2 = g * h(f_1, g)$ $t = \frac{1}{n_t} \lfloor \frac{\tau}{\tau_T} \rfloor$, τ is a generation counter and τ_T the number of generations, for which t is constant. $\mathbf{x_I} = (x_1)^T, x_1 \in [0, 1], \mathbf{x_{II}} = (x_2, \ldots, x_n)^T$, $x_2, \ldots, x_n \in [-1, 1]$	dynamic fitness function; Typ I: PF_{true} is static and convex Pareto front ($f_2 = 1 - \sqrt{f_1}$); P_{true} oszillating parallel lines



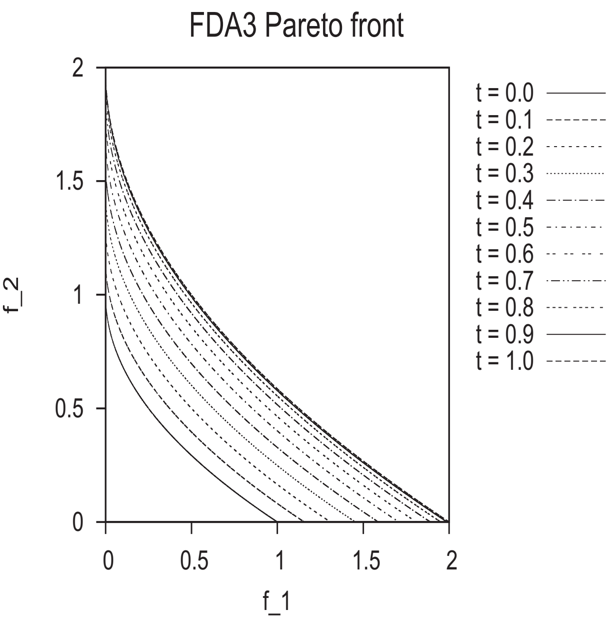
name of problem / authors	definition and restrictions	characteristics
FDA2: [12] M. Farina and K. Deb and P. Amato	$f_1(\mathbf{x_I}) = x_1 \quad g(\mathbf{x_{II}}) = 1 + \sum_{x_i \in \mathbf{x_{II}}} x_i^2$ $h(\mathbf{x_{III}}, f_1, g) = 1 - (\frac{f_1}{g})^{e(t,\mathbf{x_{III}})}$ $e(t, \mathbf{x_{III}}) = \left(H(t) + \sum_{x_i \in \mathbf{x_{III}}} (x_i - H(t))^2\right)^{-1}$ $H(t) = 0.75 + 0.7sin(0.5\pi t), \quad t = \frac{1}{n_t}\lfloor\frac{\tau}{\tau_T}\rfloor,$ $f_2 = g * h(f_1, g)$ τ is a generation counter and τ_T the number of generations, for which t is constant. $\mathbf{x_I} = (x_1)^T, x_1 \in [0,1], \mathbf{x_{II}} \in [-1,1]^{r_2},$ $\mathbf{x_{III}} \in [-1,1]^{r_3}, 1 + r_2 + r_3 = n$ e. g. $r_2 = r_3 = 15$ (dimensions) $\mathbf{x_I}, \mathbf{x_{II}}, \mathbf{x_{III}}$ in disjoint spaces	dynamic fitness function; Typ III: PF_{true} are convex curves; P_{true} connected static line, $\forall x_i \in \mathbf{x_{III}}$: $x_i = -1$

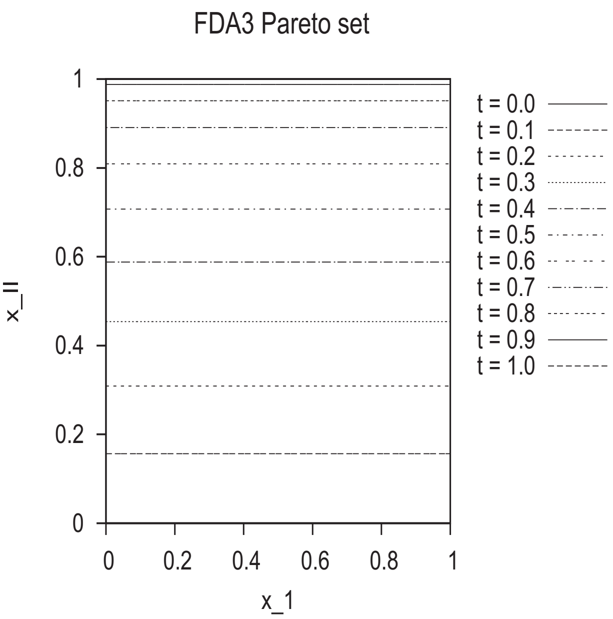


name of problem / authors	**definition and restrictions**	**characteristics**
$FDA2_{modi}$**:** J. Mehnen, T. Wagner	$f_1(\mathbf{x_I}) = x_1$ $g(\mathbf{x_{II}}) = 1 + \sum_{x_i \in \mathbf{x_{II}}} x_i^2 + \sum_{x_i \in \mathbf{x_{III}}} (x_i + 1)^2$ $h(f_1, g) = 1 - (\frac{f_1}{g})^{H(t)}$ $H(t) = 0.2 + 4.8t^2, \quad t = \frac{1}{n_t} \lfloor \frac{\tau}{\tau_T} \rfloor,$ $f_2 = g * h(f_1, g)$ τ is a generation counter and τ_T the number of generations, for which t is constant. $\mathbf{x_I} = (x_1)^T, x_1 \in [0,1], \mathbf{x_{II}} \in [-1,1]^{r_2}$, $\mathbf{x_{III}} \in [-1,1]^{r_3}, 1 + r_2 + r_3 = n$ e. g. $r_2 = r_3 = 15$ (dimensions) $\mathbf{x_I}, \mathbf{x_{II}}, \mathbf{x_{III}}$ in disjoint spaces	dynamic fitness function; Typ II: PF_{true} is changing from a convex to a nonconvex curve; P_{true} connected static line, $\forall x_i \in \mathbf{x_{III}}$: $x_i = -1$

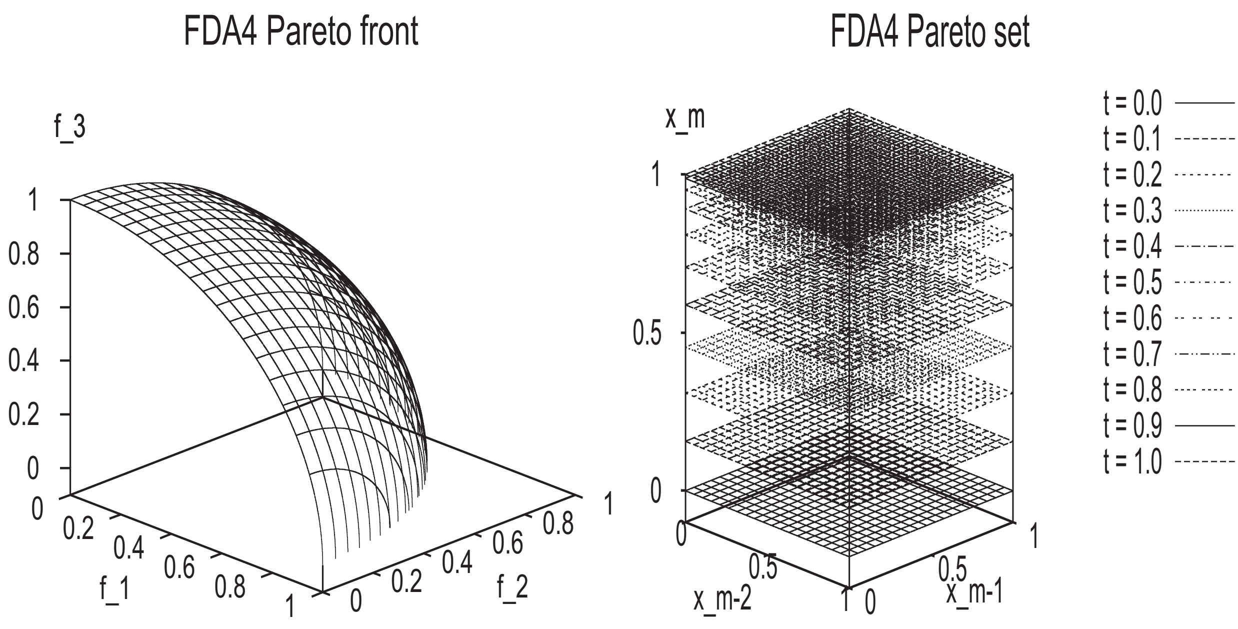


name of problem / authors	definition and restrictions	characteristics
FDA3: [12] M. Farina and K. Deb and P. Amato	$f_1(\mathbf{x_I}) = \sum_{x_i \in \mathbf{x_I}} x_i^{F(t)}$, $g(\mathbf{x_{II}}) = 1 + G(t) + \sum_{x_i \in \mathbf{x_{II}}} (x_i - G(t))^2$, $h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}}$ $G(t) = sin(0.5\pi t)$, $F(t) = 10^{2sin(0.5\pi t)}$, $t = \frac{1}{n_t} \lfloor \frac{\tau}{\tau_T} \rfloor$, $f_2 = g * h(f_1, g)$ τ is a generationen counter and τ_T the number of generations, for which t is constant. $\mathbf{x_I} = (x_1)^T, x_1 \in [0,1]^{r_1}, \mathbf{x_{II}} \in [-1,1]^{r_2}$, $r_1 + r_2 = n$ e. g. $r_1 = 5, r_2 = 25$ (dimensions) $\mathbf{x_I}, \mathbf{x_{II}}$ in disjoint spaces	dynamic fitness function; Typ II, PF_{true} is a convex line moving up and down; P_{true} moving lines

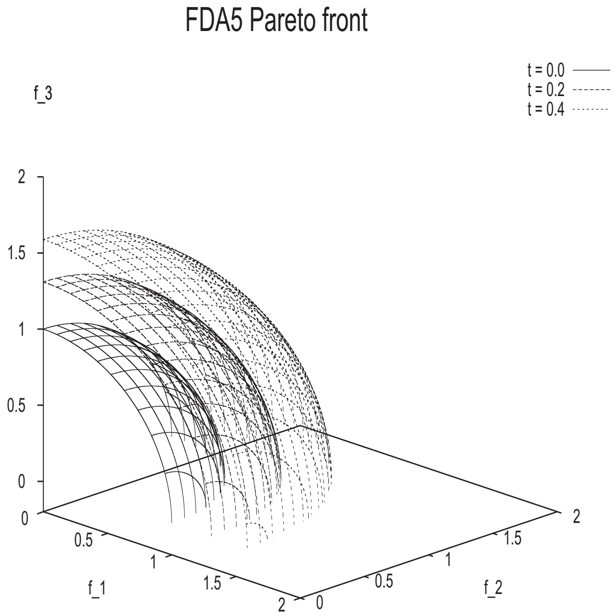


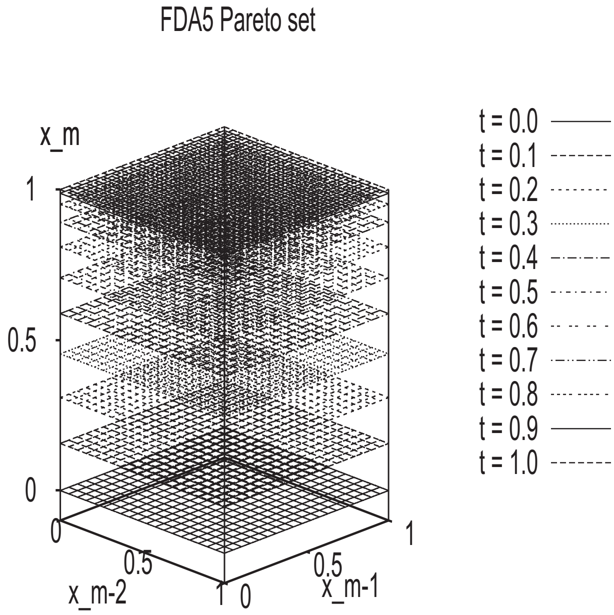


name of problem / authors	definition and restrictions	characteristics
FDA4: [12] M. Farina and K. Deb and P. Amato	$min_{\mathbf{x}}\ f_1(\mathbf{x}) = (1+g(\mathbf{x_{II}}))\prod_{i=1}^{m-1} cos(x_i\pi/2)$ $min_{\mathbf{x}}\ f_k(\mathbf{x}) = (1+g(\mathbf{x_{II}}))\left(\prod_{i=1}^{m-k} cos(x_i\pi/2)\right) sin(x_{m-k+1}\ \pi/2),\quad k=2,\ldots,m-1$ $min_{\mathbf{x}}\ f_m(\mathbf{x}) = (1+g(\mathbf{x_{II}}))sin(x_1\pi/2)$ $g(\mathbf{x_{II}}) = 1+\sum_{x_i\in\mathbf{x_{II}}}(x_i+G(t))^2$ $G(t) = \lvert sin(0.5\pi t)\rvert, \mathbf{x_{II}} = (x_m,\ldots,x_n)$, $x_i \in [0,1], i=1,\ldots,n$ $t = \frac{1}{n_t}\lfloor\frac{\tau}{\tau_T}\rfloor$, τ is a generation counter and τ_T is the number of generations, for which t is constant.	dynamic fitness function; Typ I: PF_{true} static concave spherical Pareto front; P_{true} moving planes

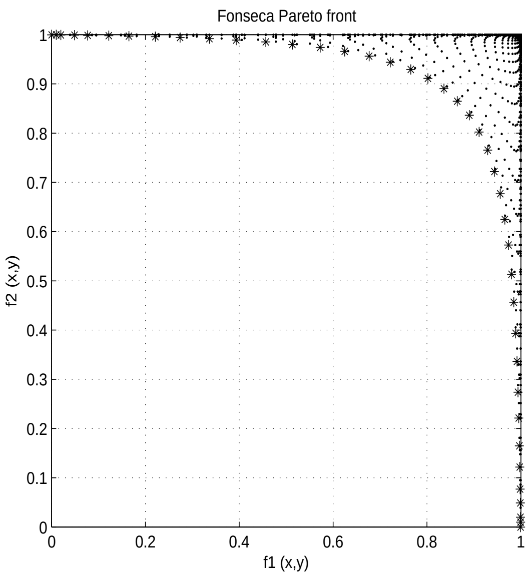


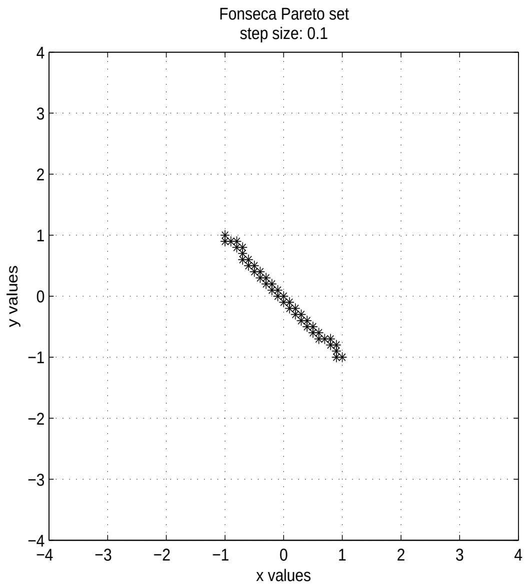
name of problem / authors	definition and restrictions	characteristics
FDA5: [12] M. Farina and K. Deb and P. Amato	$min_{\mathbf{x}}\ f_1(\mathbf{x}) = (1 + g(\mathbf{x_{II}})) \prod_{i=1}^{m-1} cos(y_i\pi/2)$ $min_{\mathbf{x}}\ f_k(\mathbf{x}) = (1 + g(\mathbf{x_{II}})) \left(\prod_{i=1}^{m-k} cos(y_i\pi/2)\right)$ $sin(y_{m-k+1}\ \pi/2),\quad k = 2, \ldots, m-1$ $min_{\mathbf{x}}\ f_m(\mathbf{x}) = (1 + g(\mathbf{x_{II}}))sin(y_1\pi/2)$ $g(\mathbf{x_{II}}) = 1 + G(t) + \sum_{x_i \in \mathbf{x_{II}}} (x_i - 0.5)^2$ $y_i = x_i^{F(t)}, G(t) = \lvert sin(0.5\pi t)\rvert,$ $F(t) = 1 + 100sin^4(0.5\pi t),$ $\mathbf{x_{II}} = (x_m, \ldots, x_n), x_i \in [0, 1], i = 1, \ldots, n,$ $t = \frac{1}{n_t}\lfloor\frac{\tau}{\tau_T}\rfloor$ τ is a generation counter and τ_T the maximum number of generations	dynamic fitness function; Typ II: PF_{true} concave spheres with changing diameter; P_{true} moving planes



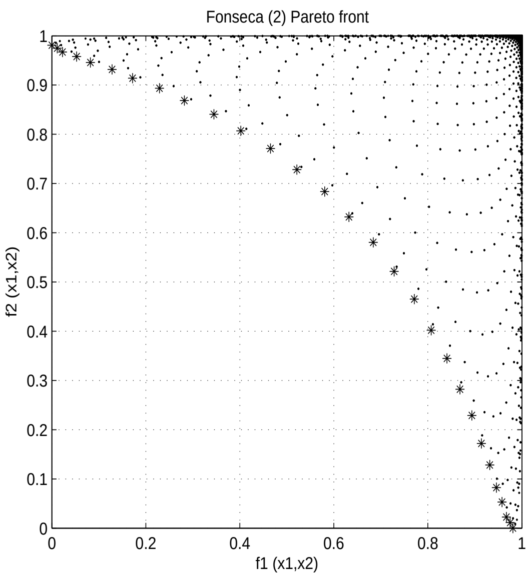


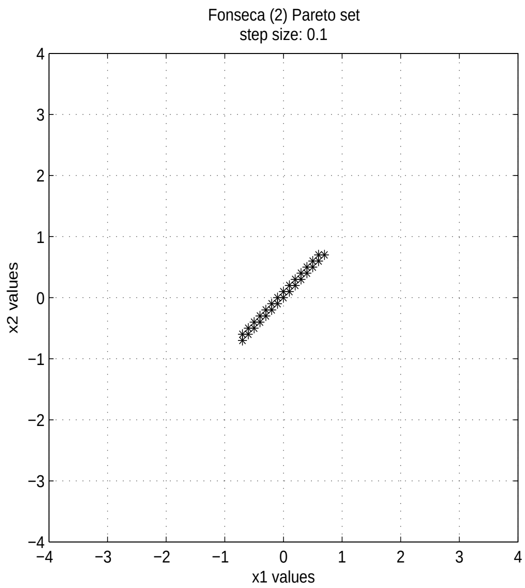
name of problem / authors	definition and restrictions	characteristics
Fonseca [13] C.M. Fonseca, P.J. Fleming.	Minimize $F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= 1 - \exp\left(-(x-1)^2 - (y+1)^2\right), \\ f_2(x,y) &= 1 - \exp\left(-(x+1)^2 - (y-1)^2\right) \end{aligned}$$ restrictions: none	PF_{true} concave and connected; P_{true} connected curve



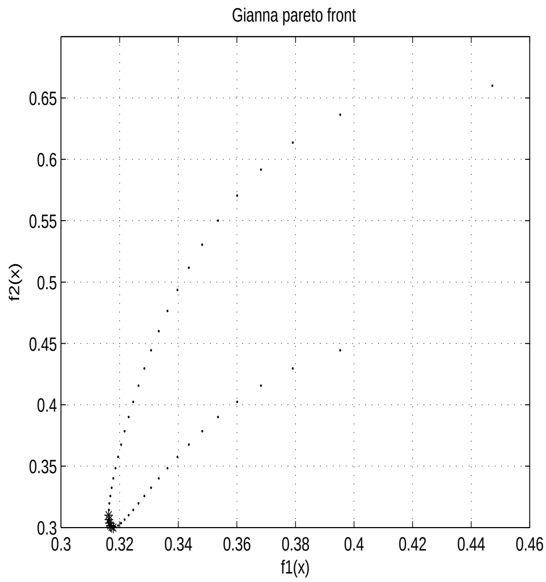


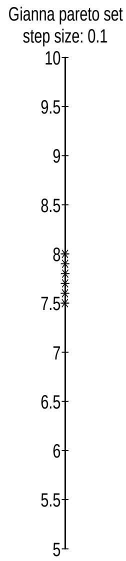
name of problem / authors	**definition and restrictions**	**characteristics**
Fonseca (2) [14] C.M. Fonseca, P.J. Fleming.	Minimize $F = (f_1(\overrightarrow{x}), f_2(\overrightarrow{x}))$, where $f_1(\overrightarrow{x}) = 1 - \exp\left(-\sum_{i=1}^{n}(x_i - \frac{1}{\sqrt{n}})^2\right)$, $f_2(\overrightarrow{x}) = 1 - \exp\left(-\sum_{i=1}^{n}(x_i + \frac{1}{\sqrt{n}})^2\right)$ restrictions: $-4 \leq x_i \leq 4$	PF_{true} concave and connected curve; P_{true} connected curve analytic solution possible



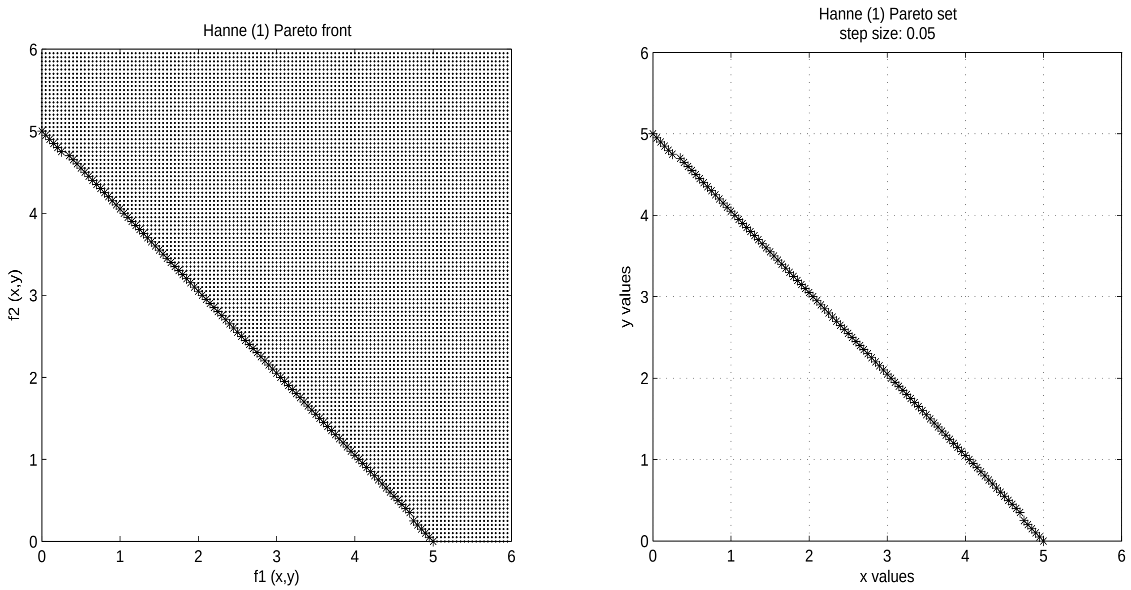


name of problem / authors	definition and restrictions	characteristics
Gianna [15] A.P. Giotis K.C. Giannakoglou	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $f_1(\mathbf{x}) = \frac{1}{\sqrt{10-x}+\sqrt{x-5}}$ $f_2(\mathbf{x}) = 0.04\,(x-8)^2 + 0.3$ restrictions: $5 \leq x \leq 10$	PF_{true} connected convex line; P_{true} connected line

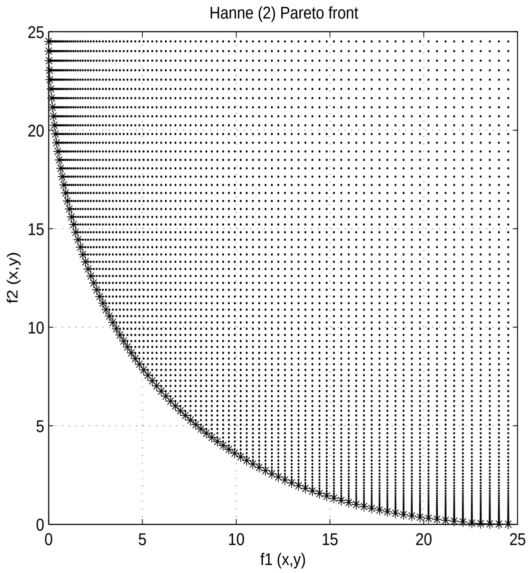


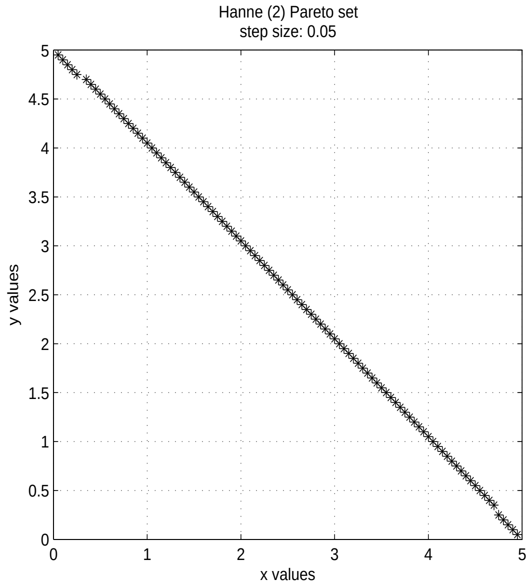


name of problem / authors	definition and restrictions	characteristics
Hanne (1) [8] Th. Hanne	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= x, \\ f_2(\mathbf{x}) &= y \end{aligned}$$ restrictions: $x + y \geq 5$ and $x, y \geq 0$	PF_{true} connected line; P_{true} connected line

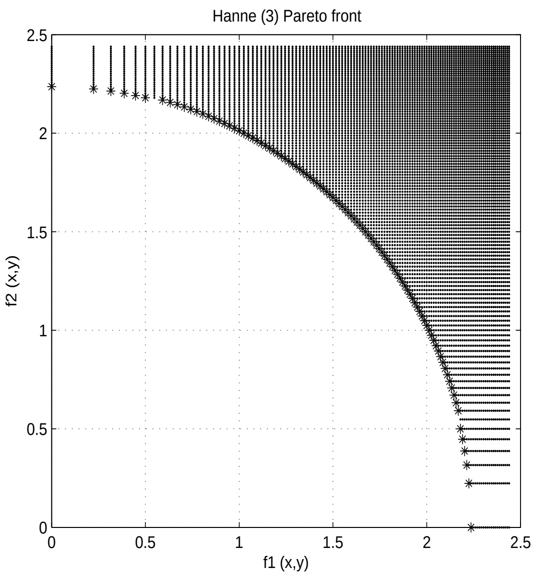


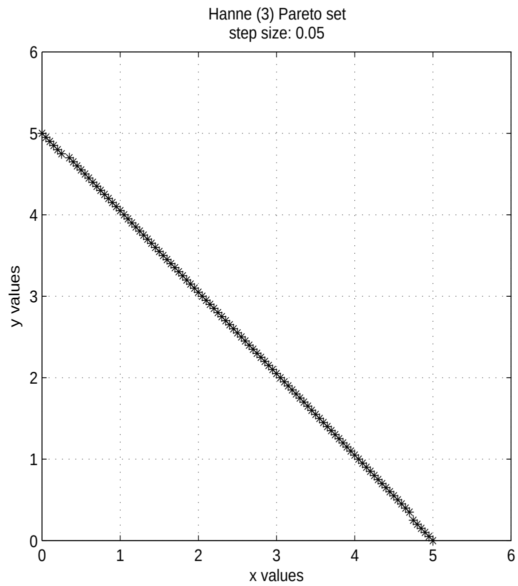
name of problem / authors	definition and restrictions	characteristics
Hanne (2) [8] Th. Hanne	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= x^2, \\ f_2(\mathbf{x}) &= y^2 \end{aligned}$$ restrictions: $x + y \geq 5$ and $x, y \geq 0$	PF_{true} convex connected curve; P_{true} connected line



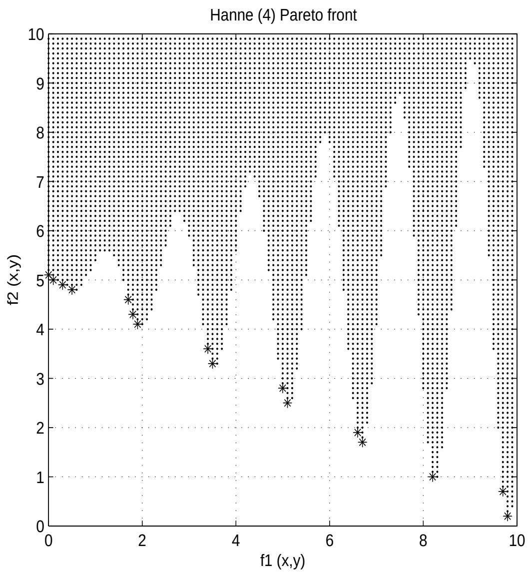


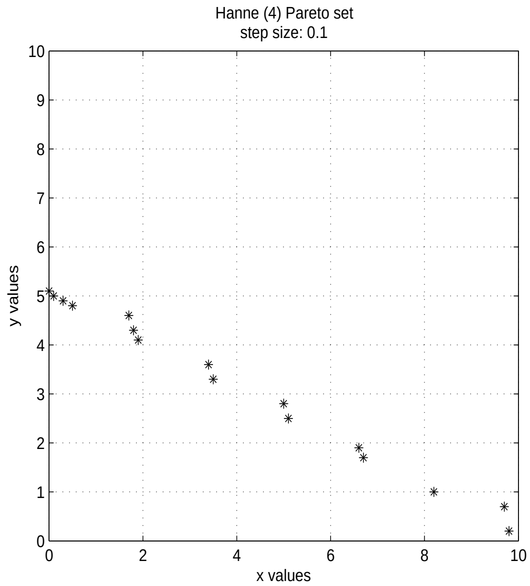
name of problem / authors	definition and restrictions	characteristics
Hanne (3) [8] Th. Hanne	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= \sqrt{x}, \\ f_2(\mathbf{x}) &= \sqrt{y} \end{aligned}$$ restrictions: $x + y \geq 5$ and $x, y \geq 0$	PF_{true} concave connected curve; P_{true} connected line



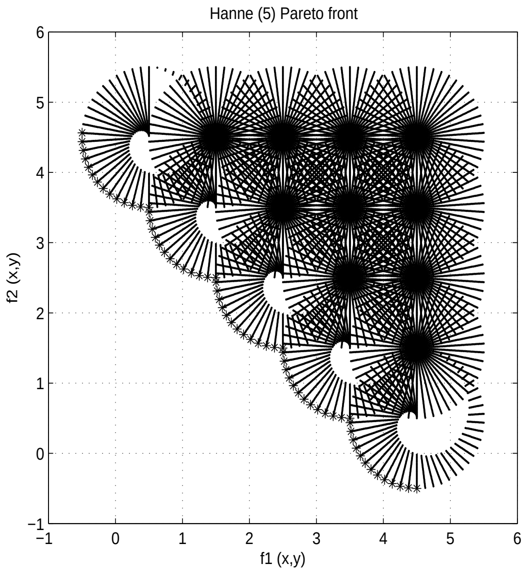


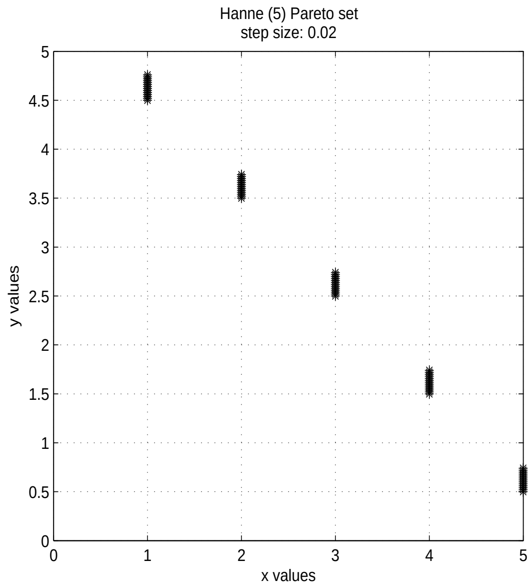
name of problem / authors	definition and restrictions	characteristics
Hanne (4) [8] Th. Hanne	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$f_1(\mathbf{x}) = x,$$ $$f_2(\mathbf{x}) = y$$ restrictions: $y - 5 + 0.5 \cdot x \cdot \sin(4 \cdot x) \geq 0$ and $x, y \geq 0$	PF_{true} not connected curves; P_{true} not connected curves



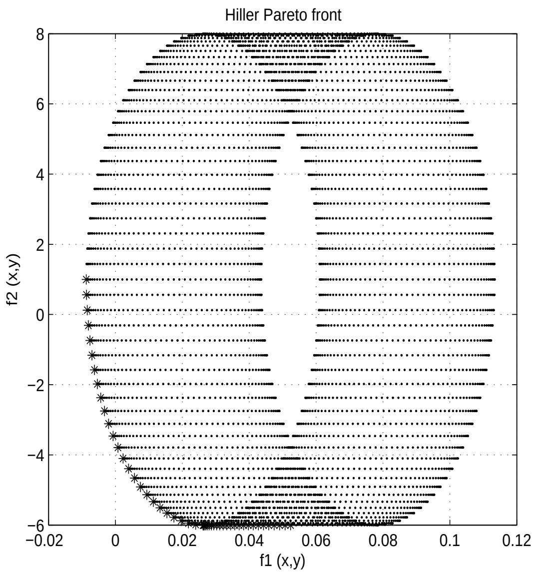


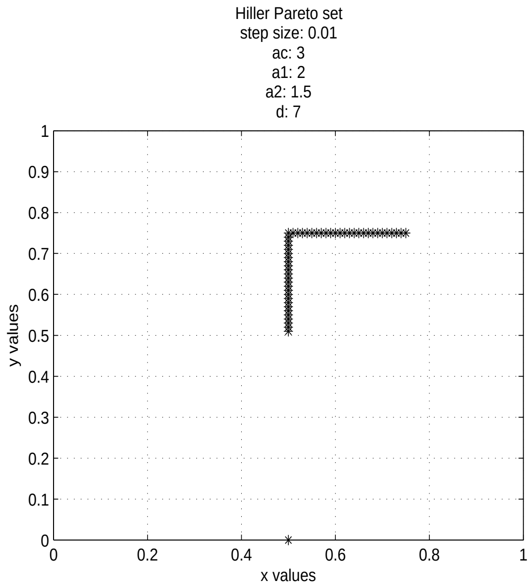
name of problem / authors	definition and restrictions	characteristics
Hanne (5) [8] Th. Hanne	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= \text{int}(x) + 0.5 + (x - \text{int}(x)) \cdot \\ &\quad \sin(2\pi \cdot (y - \text{int}(y))) \\ f_2(\mathbf{x}) &= \text{int}(y) + 0.5 + (x - \text{int}(x)) \cdot \\ &\quad \cos(2\pi \cdot (y - \text{int}(y))) \end{aligned}$$ with: int(x) integer part of x restrictions: $x + y \geq 5$ and $x, y \geq 0$	PF_{true} piecwise circular connected curves; P_{true} not connected lines



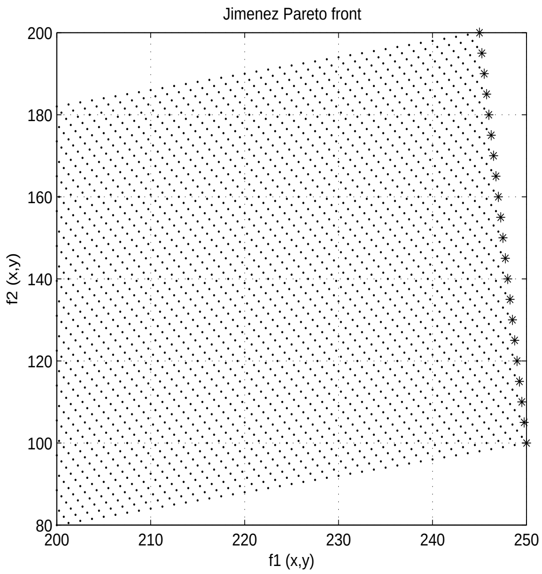


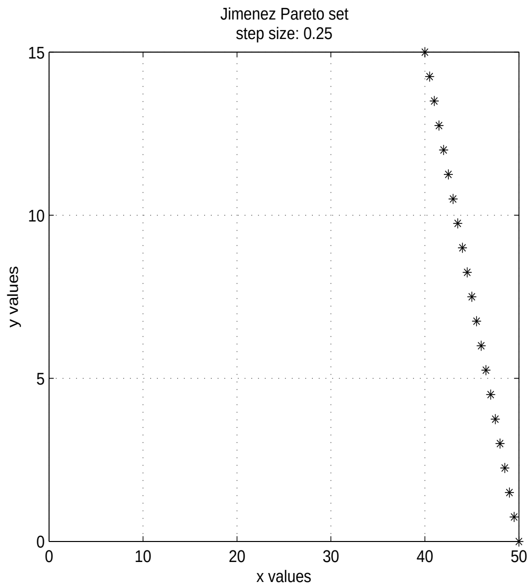
name of problem / authors	definition and restrictions	characteristics
Hiller [16] C. Hillermeier	Minimize: $F = (f_1(x), f_2(x))$, where $$F \quad : \quad \begin{cases} \Re^2 & \rightarrow \quad \Re^2 \\ x & \mapsto \quad \begin{pmatrix} \cos((f_1(x))\cdot(f_2(x)) \\ \sin((f_1(x))\cdot(f_2(x)) \end{pmatrix} \end{cases}$$ $$f_1(x) \quad = \quad \frac{2\pi}{360}[a_c + a_1 \cdot \sin(2\pi x_1) + a_2 \cdot \sin(2\pi x_2)]$$ $$f_2(x) \quad = \quad 1 + d \cdot \cos(2\pi x_1)$$ with: a_c, a_1, a_2 and d constants restriktions: $0 \leq x, y \leq 1$	PF_{true} connected line; P_{true} connected lines



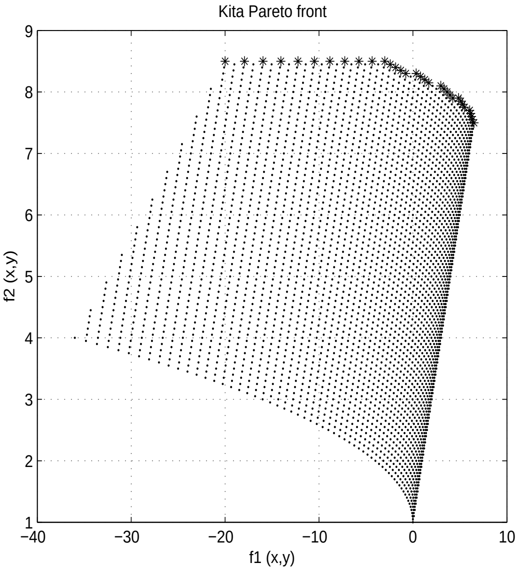


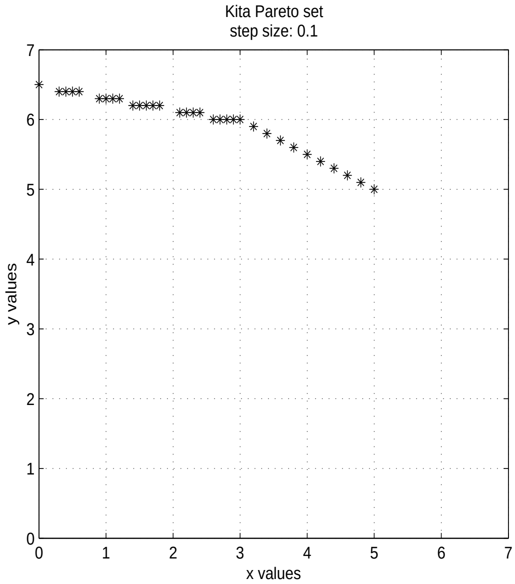
name of problem / authors	definition and restrictions	characteristics
Jimenez [17] F. Jiménez, J. L. Verdegay.	Maximize $F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = 5x + 3y,$$ $$f_2(x,y) = 2x + 8y$$ restrictions: $x, y \geq 0$, $0 \geq x + 4y - 100$, $0 \geq 3x + 2y - 150$, $0 \geq 200 - 5x - 3y$, $0 \geq 75 - 2x - 8y$	PF_{true} connected line; P_{true} connected line



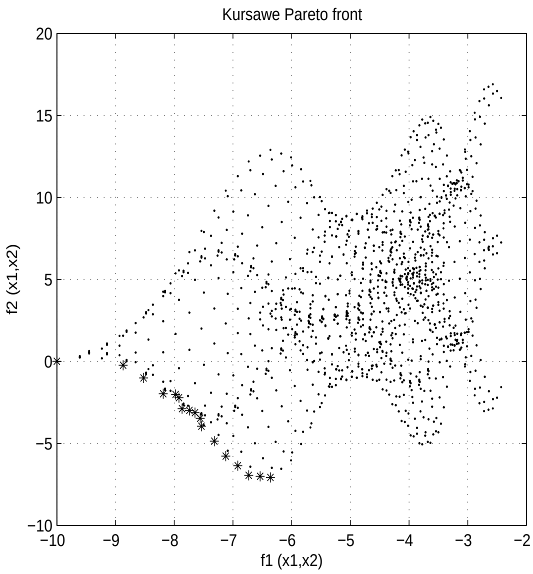


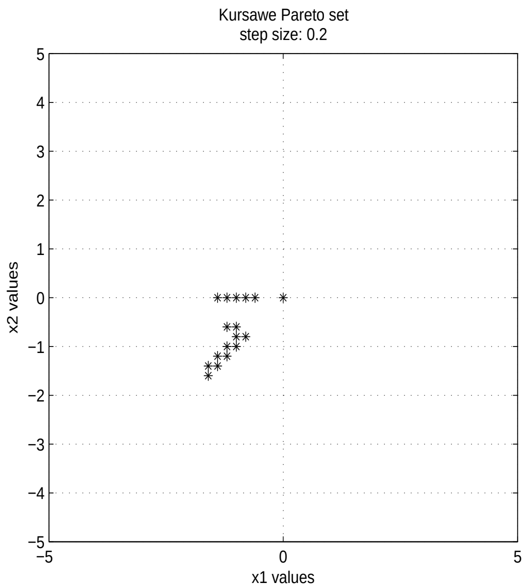
name of problem / authors	definition and restrictions	characteristics
Kita [18] H. Kita, Y. Yabumoto, N. Mori, Y. Nishikawa.	Maximize $F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= -x^2 + y, \\ f_2(x,y) &= \frac{1}{2}x + y + 1 \end{aligned}$$ restrictions: $x, y \geq 0$, $0 \geq \frac{1}{6}x + y - \frac{13}{2}$, $0 \geq \frac{1}{2}x + y - \frac{15}{2}$, $0 \geq 5x + y - 30$	PF_{true} concave line; P_{true} connected curve



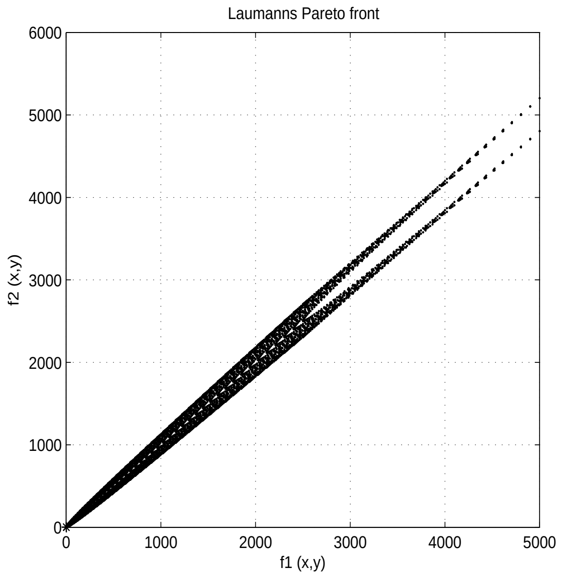


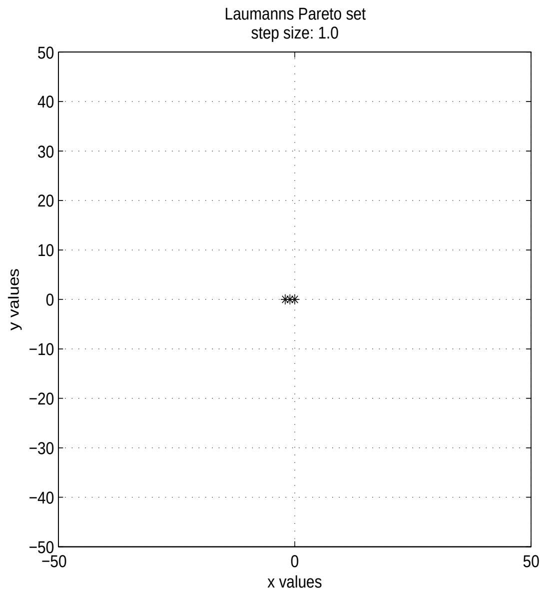
name of problem / authors	definition and restrictions	characteristics
Kursawe (1) [19] F. Kursawe.	$F = (f_1(\overrightarrow{x}), f_2(\overrightarrow{x})$, where $$f_1(\overrightarrow{x}) = \sum_{i=1}^{n-1}(-10e^{(-0.2)} \cdot \sqrt{x_i^2 + x_{i+1}^2}),$$ $$f_2(\overrightarrow{x}) = \sum_{i=1}^{n}(\|x_i\|^{0.8} + 5\sin(x_i)^3)$$ restrictions: none	PF_{true} partially convex or concave curve; P_{true} non connected



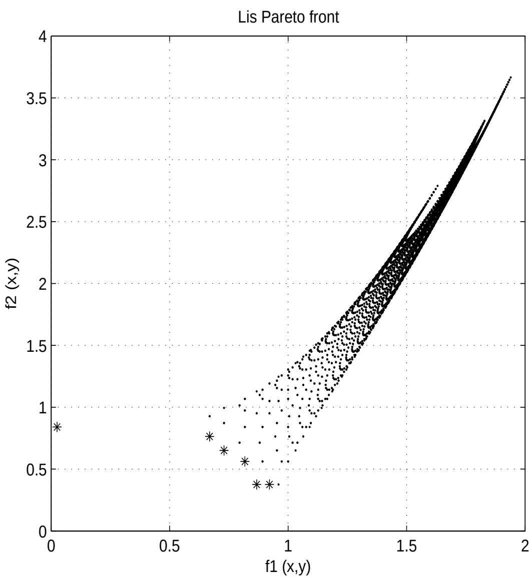


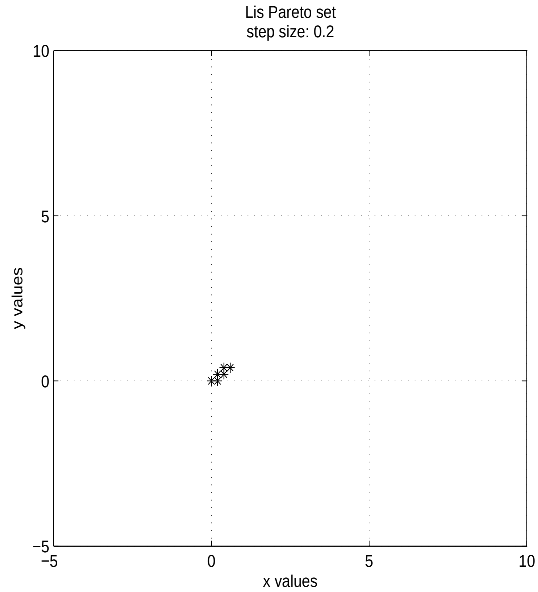
name of problem / authors	**definition and restrictions**	**characteristics**
Laumanns [20] M. Laumanns, G. Rudolph, H.-P. Schwefel.	Minimize: $F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= x^2 + y^2, \\ f_2(x,y) &= (x+2)^2 + y^2 \end{aligned}$$ restrictions: $-50 \le x, y \le 50$	PF_{true} small connected convex curve; P_{true} line



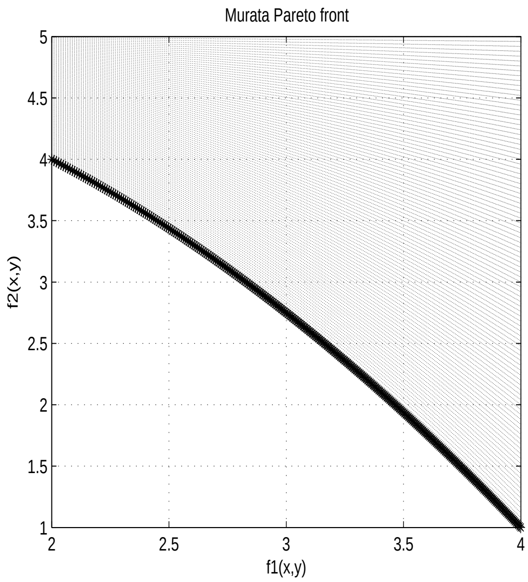


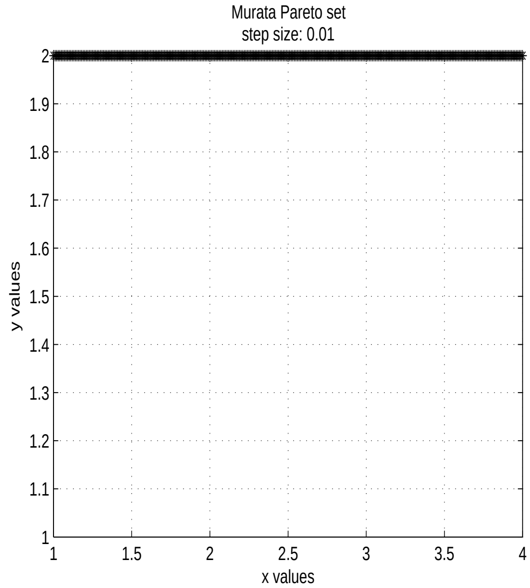
name of problem / authors	definition and restrictions	characteristics
Lis [21] J. Lis, A. E. Eiben.	$F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= \sqrt[8]{x^2+y^2}, \\ f_2(x,y) &= \sqrt[4]{(x-0.5)^2+(y-0.5)^2} \end{aligned}$$ restrictions: $-5 \leq x, y \leq 10$	PF_{true} concave connected line; P_{true} line



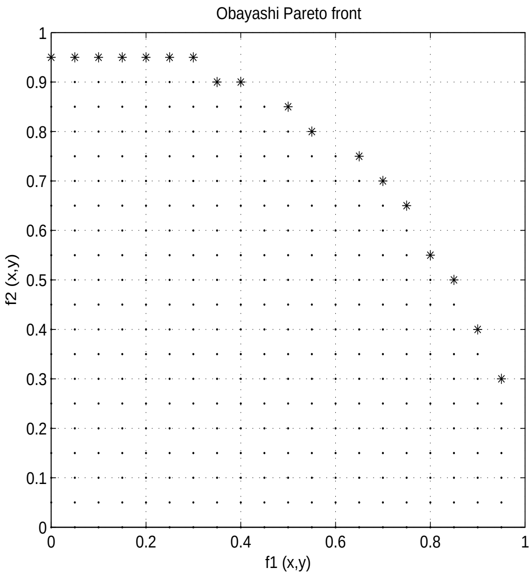


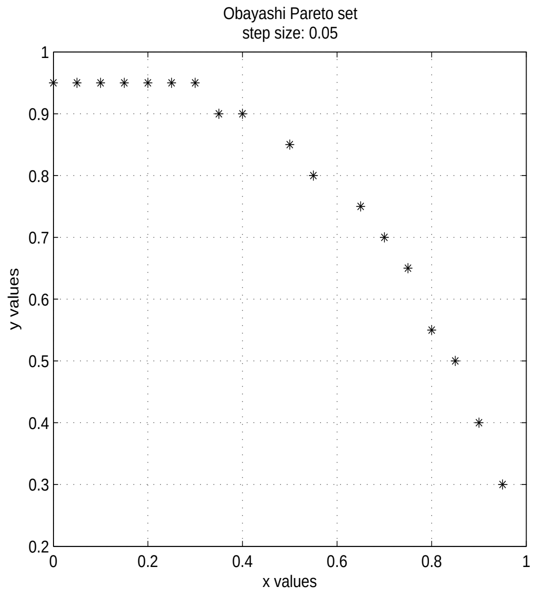
name of problem / authors	definition and restrictions	characteristics
Murata [22] T. Murata, H. Ishibuchi.	Minimize $F = (f_1(x, y), f_2(x, y))$, where $$\begin{aligned} f_1(x, y) &= 2\sqrt{x}, \\ f_2(x, y) &= x(1 - y) + 5 \end{aligned}$$ restrictions: $1 \leq x \leq 4$, $1 \leq y \leq 2$	PF_{true} concave curve; P_{true} connected line



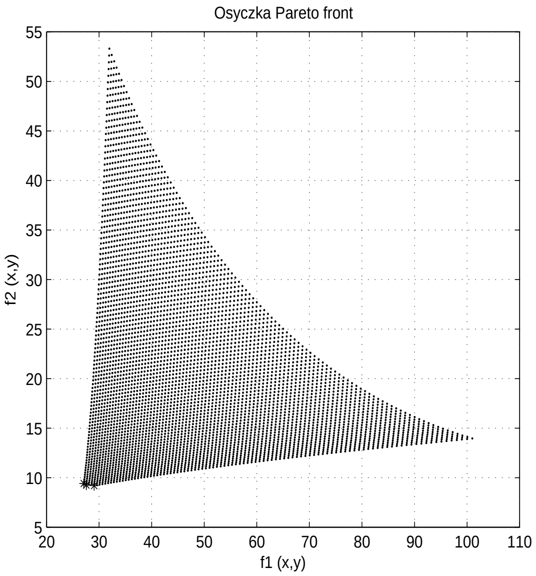


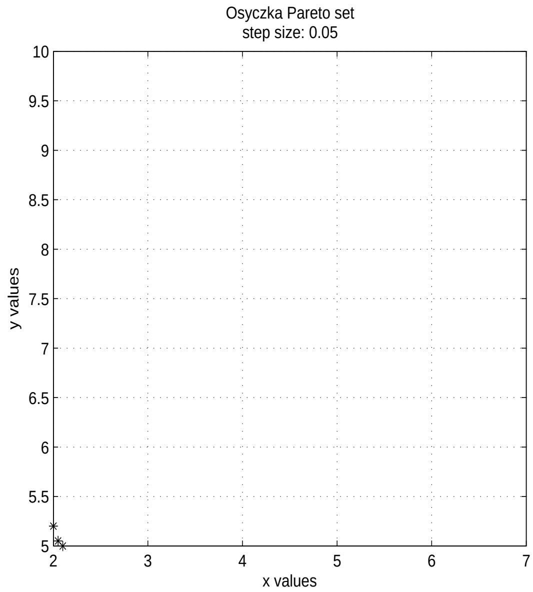
name of problem / authors	definition and restrictions	characteristics
Obayashi [23] S. Obayashi.	Maximize $F = (f_1(x, y), f_2(x, y))$, where $$\begin{aligned} f_1(x, y) &= x, \\ f_2(x, y) &= y \end{aligned}$$ restrictions: $0 \leq x, y \leq 1$, $x^2 + y^2 \leq 1$	PF_{true} convex connected curve; P_{true} connected curve



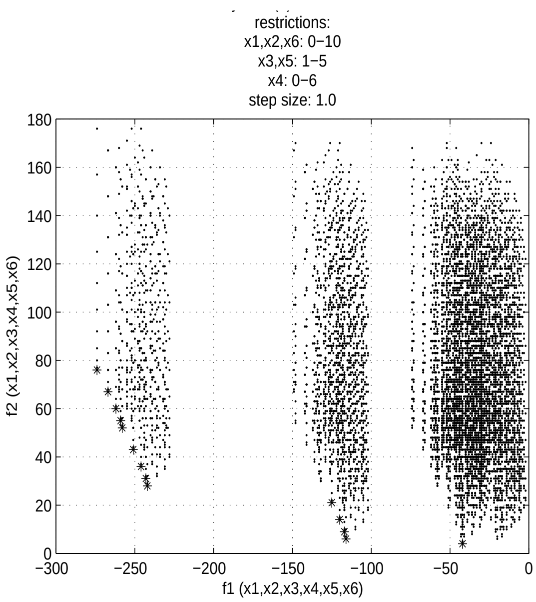


name of problem / authors	definition and restrictions	characteristics
Osyczka [24] A. Osyczka, S. Kundu.	Minimize $F = (f_1(x,y), f_2(x,y))$, where $f_1(x,y) = x + y^2,$ $f_2(x,y) = x^2 + y$ restrictions: $2 \leq x \leq 7$, $5 \leq y \leq 10$, $0 \leq 12 - x - y$, $0 \leq x^2 + 10x - y^2 + 16y - 80$	PF_{true} connected line; P_{true} curve

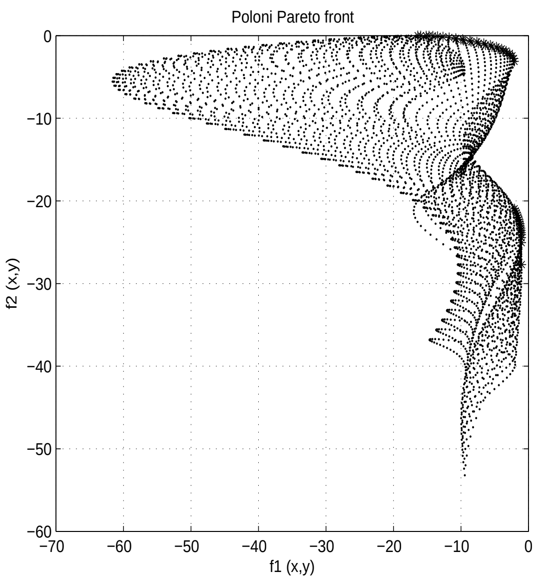


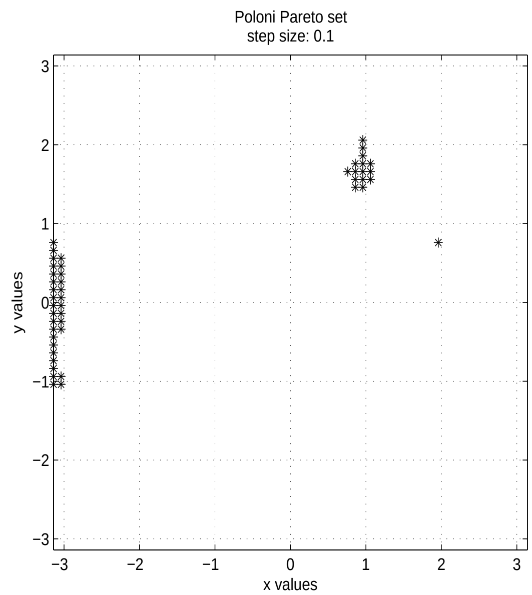


name of problem / authors	definition and restrictions	characteristics
Osyczka (2) [24] A. Osyczka, S. Kundu.	Minimize: $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= -25(x_1-2)^2 + (x_2-2)^2 + (x_3-1)^2 \\ &\quad +(x_4-4)^2 + (x_5-1)^2, \\ f_2(\mathbf{x}) &= x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 \end{aligned}$$ restrictions: $0 \le x_1, x_2, x_6 \le 10$, $1 \le x_3, x_5 \le 5$, $0 \le x_4 \le 6$, $0 \le x_1 + x_2 - 2$, $0 \le 6 - x_1 - x_2$, $0 \le 2 - x_2 + x_1$, $0 \le 2 - x_1 + 3x_2$, $0 \le 4 - (x_3-3)^2 - x_4$, $0 \le (x_5-3)^2 + x_6 - 4$	PF_{true} not connected curves; P_{true} seven dimensional

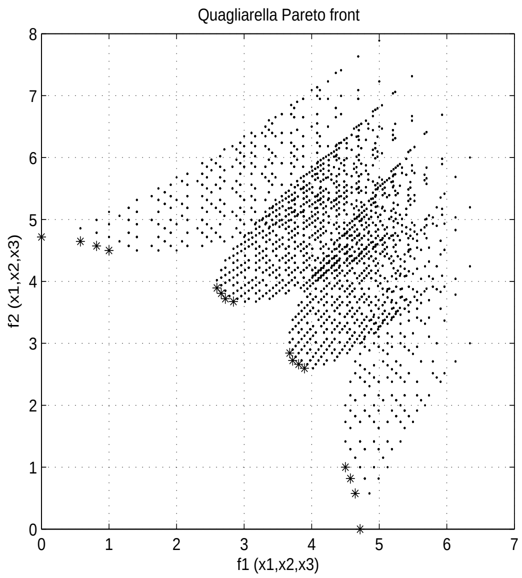


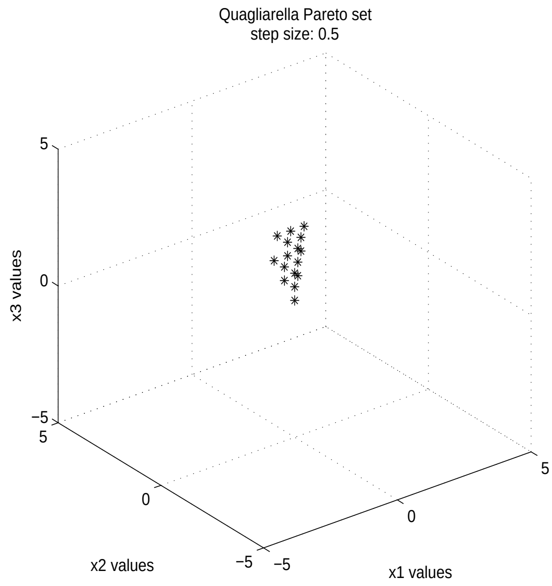
name of problem / authors	definition and restrictions	characteristics
Poloni [25] C. Poloni, G. Mosetti, S. Consetti, V. Pediroda.	Maximize: $F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= -[1 + (A_1 - B_1)^2 + (A_2 - B_2)^2], \\ f_2(x,y) &= -[(x+3)^2 + (y+1)^2] \end{aligned}$$ restrictions: $-\pi \le x, y \le \pi$, $A_1 = 0.5 \sin 1 + 2 \cos 1 + \sin 2 - 1.5 \cos 2$, $A_2 = 1.5 \sin 1 - \cos 1 + 2 \sin 2 - 0.5 \cos 2$, $B_1 = 0.5 \sin x - 2 \cos x + \sin y - 1.5 \cos y$, $B_2 = 1.5 \sin x - \cos x + 2 \sin y - 0.5 \cos y$	PF_{true} not connected curves, generally concave shape; P_{true} not connected



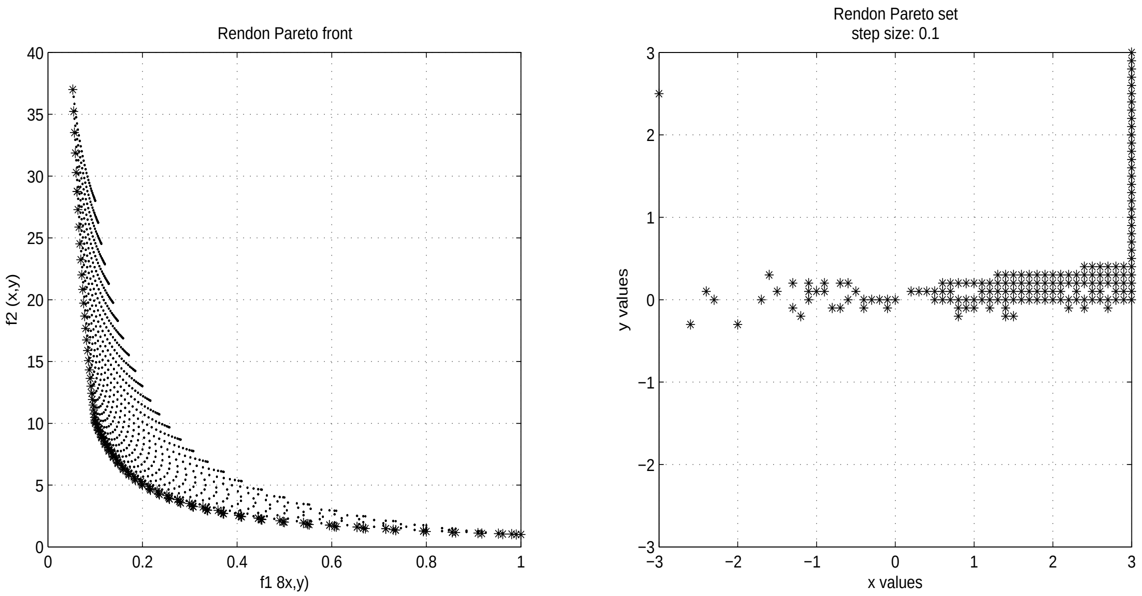


name of problem / authors	**definition and restrictions**	**characteristics**
Quagliarella [26] D. Quagliarella, A. Vicini.	Minimize $F = (f_1(\overrightarrow{x}), f_2(\overrightarrow{x}))$, where $$f_1(\overrightarrow{x}) = \sqrt{\frac{A_1}{n}}, \quad f_2(\overrightarrow{x}) = \sqrt{\frac{A_2}{n}}$$ restrictions: $A_1 = \sum_{i=1}^{n}[(x_i^2) - 10\cos[2\pi(x_i)] + 10]$, $A_2 = \sum_{i=1}^{n}[(x_i - 1.5)^2 - 10\cos[2\pi(x_i - 1.5)] + 10]$ $-5.12 \leq x_i \leq 5.12, n = 16$	PF_{true} not connected curves; P_{true} connected

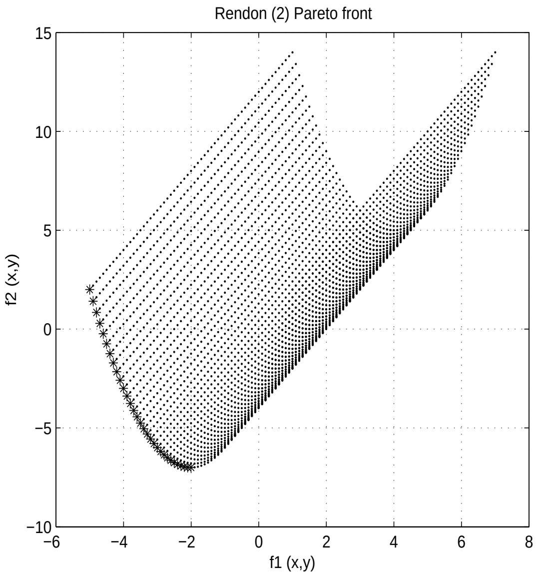


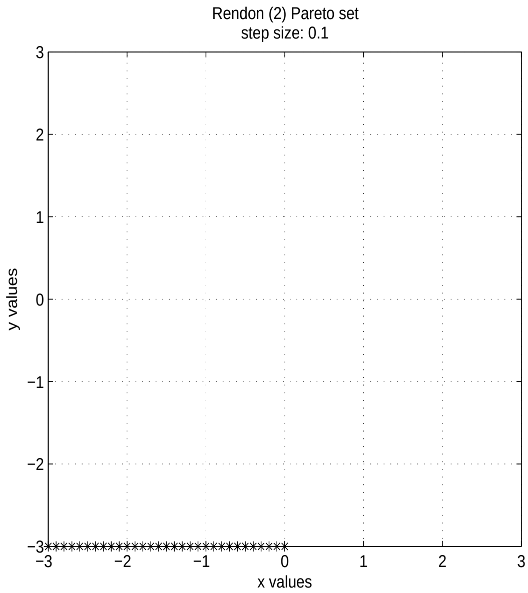


name of problem / authors	definition and restrictions	characteristics
Rendon [32] M. Valenzuela-Rendón, E. Uresti-Charre.	Minimize $F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = \frac{1}{x^2+y^2+1}, \quad f_2(x,y) = x^2+3y^2+1$$ restrictions: $-3 \leq x, y \leq 3$	PF_{true} convex; P_{true} complexly shaped

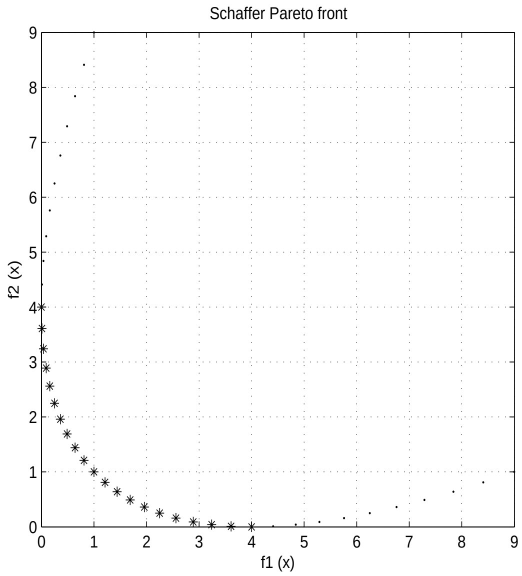


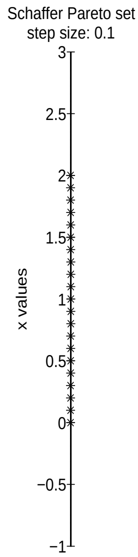
name of problem / authors	definition and restrictions	characteristics
Rendon (2) [32] M. Valenzuela-Rendón, E. Uresti-Charre.	Minimize $F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= x + y + 1, \\ f_2(x,y) &= x^2 + 2y - 1 \end{aligned}$$ restrictions: $-3 \leq x, y \leq 3$	PF_{true} convex; P_{true} connected line



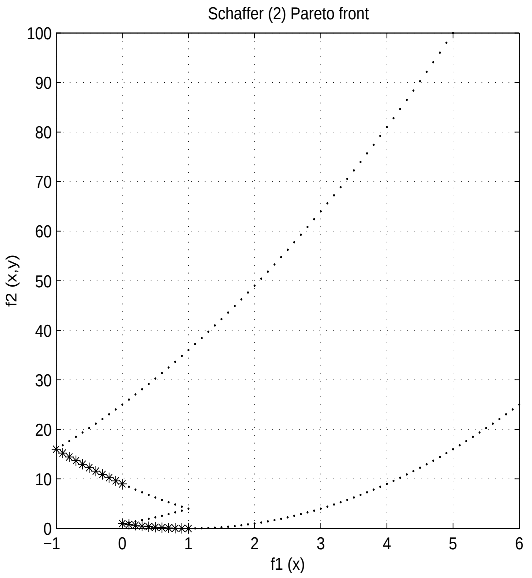


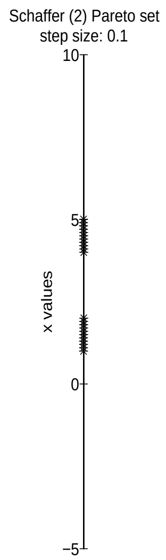
name of problem / authors	definition and restrictions	characteristics
Schaffer [27] J. D. Schaffer.	Minimize $F = (f_1(x), f_2(x))$, where $$\begin{aligned} f_1(x) &= x^2, \\ f_2(x,y) &= (x-2)^2 \end{aligned}$$ restrictions: none	PF_{true} convex, analytic solution known; P_{true} connected interval [0,2]



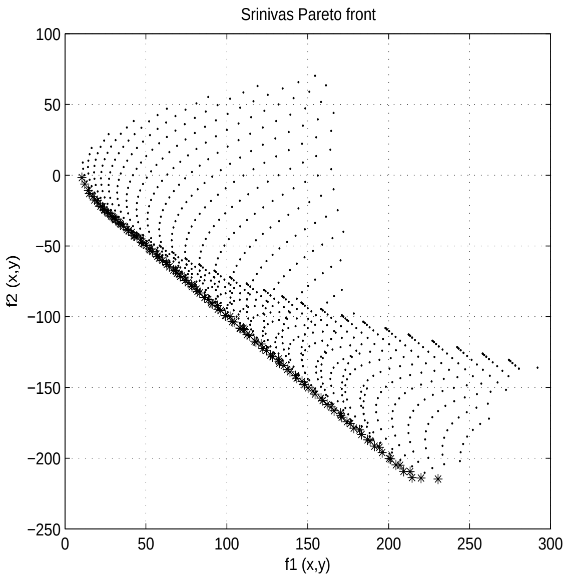


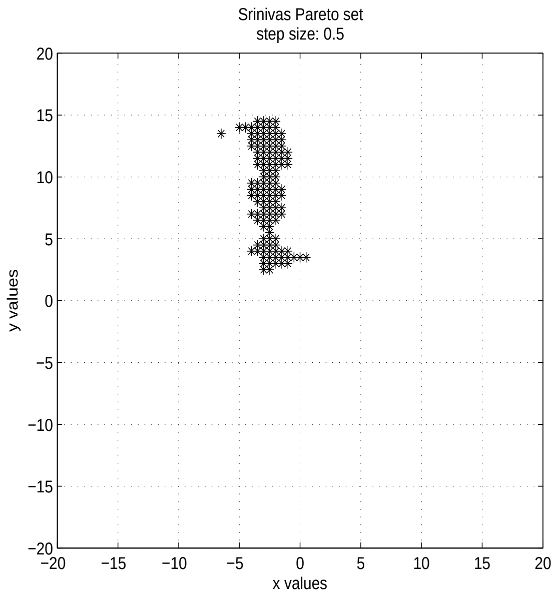
name of problem / authors	definition and restrictions	characteristics
Schaffer (2) [28] J. D. Schaffer.	Minimize $F = (f_1(x), f_2(x))$, where $$\begin{aligned} f_1(x) &= -x, \text{ if } x \leq 1, \\ &= -2 + x, \text{ if } 1 \leq x \leq 3, \\ &= 4 - x, \text{ if } 3 \leq x \leq 4, \\ &= -4 + x, \text{ if } x \geq 4, \\ f_2(x) &= (x-5)^2 \end{aligned}$$ restrictions: $-5 \leq x \leq 10$	PF_{true} not connected; P_{true} not connected



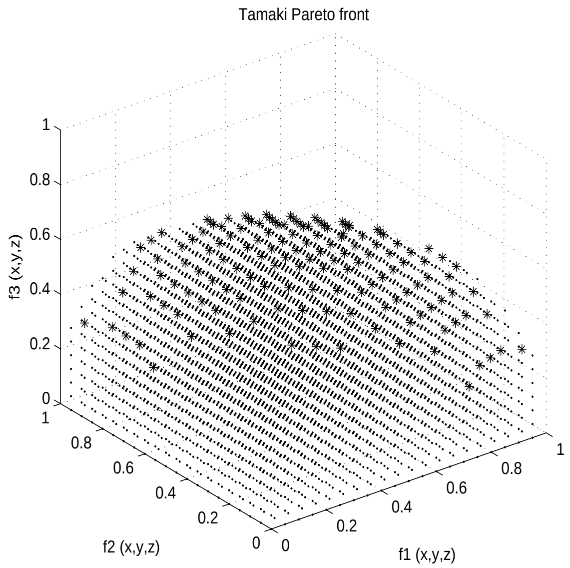


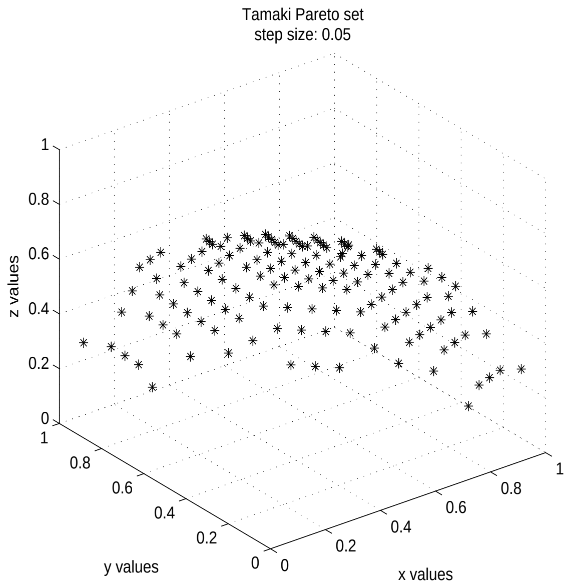
name of problem / authors	definition and restrictions	characteristics
Srinivas [29] N. Srinivas, K. Deb.	Minimize $F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = (x-2)^2 + (y-1)^2 + 2,$$ $$f_2(x,y) = 9x - (y-1)^2$$ restrictions: $-20 \leq x, y \leq 20$, $0 \geq x^2 + y^2 - 225$, $0 \geq x - 3y + 10$	P_{true} complex shape; PF_{true} connected curve



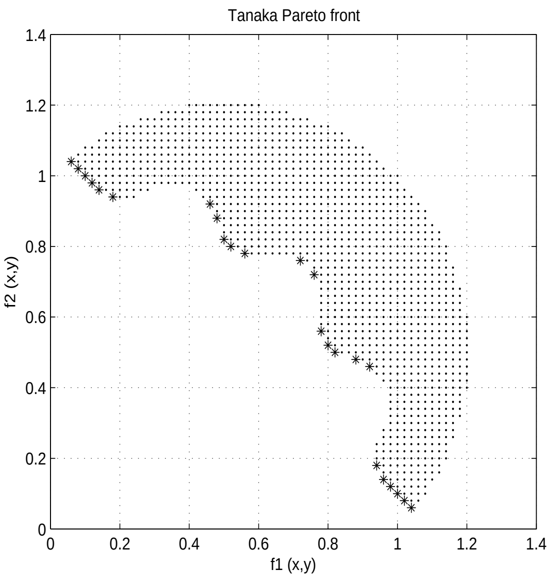


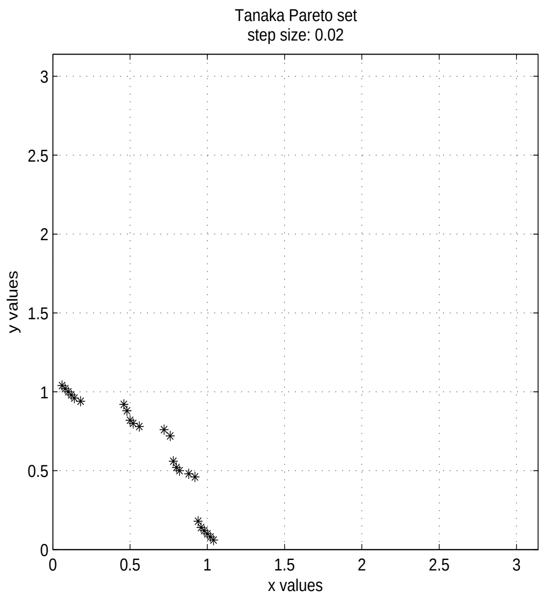
name of problem / authors	definition and restrictions	characteristics
Tamaki [30] H. Tamaki, H. Kita, S. Kobayashi.	Maximize $F = (f_1(x,y,z), f_2(x,y,z), f_3(x,y,z))$, where $$\begin{aligned} f_1(x,y,z) &= x, \\ f_2(x,y,z) &= y, \\ f_3(x,y,z) &= z \end{aligned}$$ restrictions: $0 \leq x, y, z$, $x^2 + y^2 + z^2 \leq 1$	PF_{true} connected spherical surface; P_{true} connected spherical surface



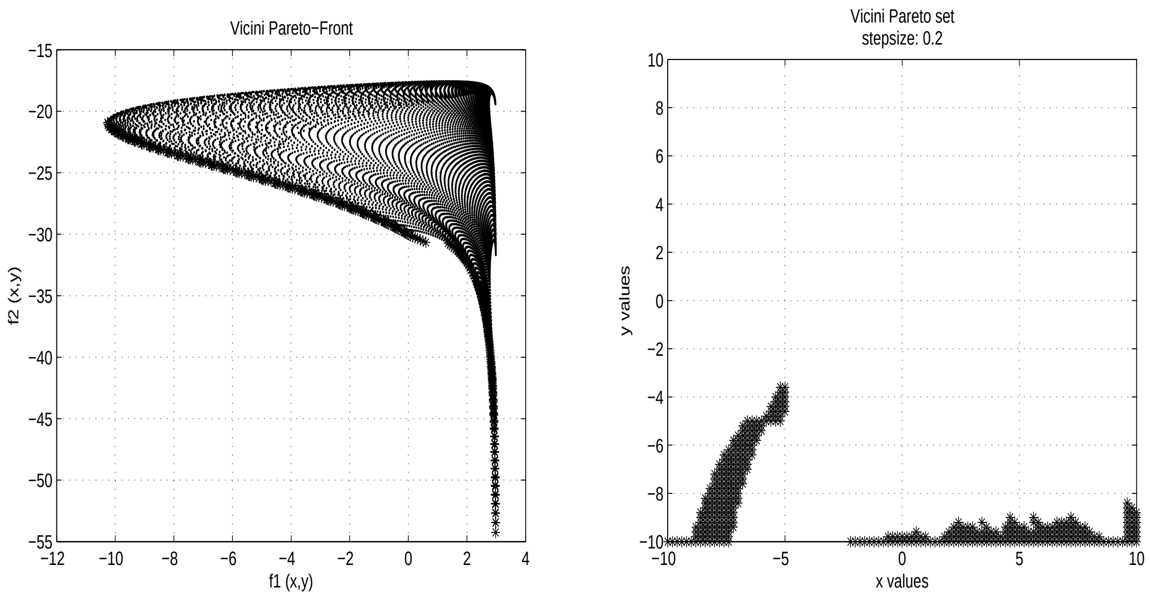


name of problem / authors	definition and restrictions	characteristics
Tanaka [31] M. Tanaka, H. Watanabe, Y. Furukawa, T. Tanino.	Maximize $F = (f_1(x,y), f_2(x,y))$, where $$\begin{aligned} f_1(x,y) &= x, \\ f_2(x,y) &= y \end{aligned}$$ restrictions: $0 \leq x, y \leq \pi$, $0 \geq -(x^2) - (y^2) + 1 + 0.1 * \cos(16 \arctan \frac{x}{y})$, $\frac{1}{2} \geq (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2$	PF_{true} not connected, P_{true} not connected

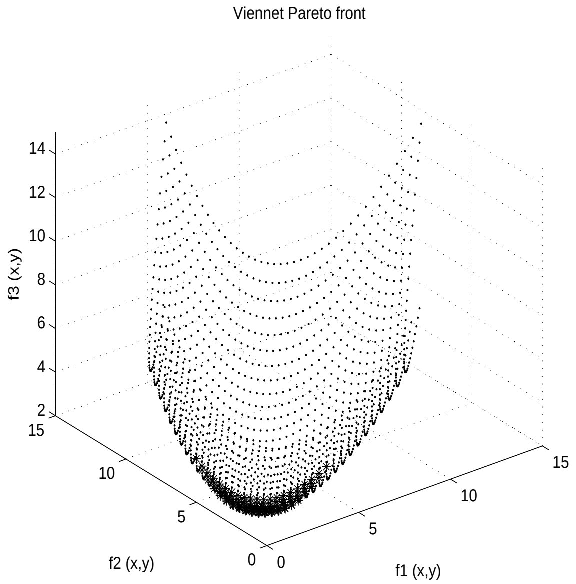


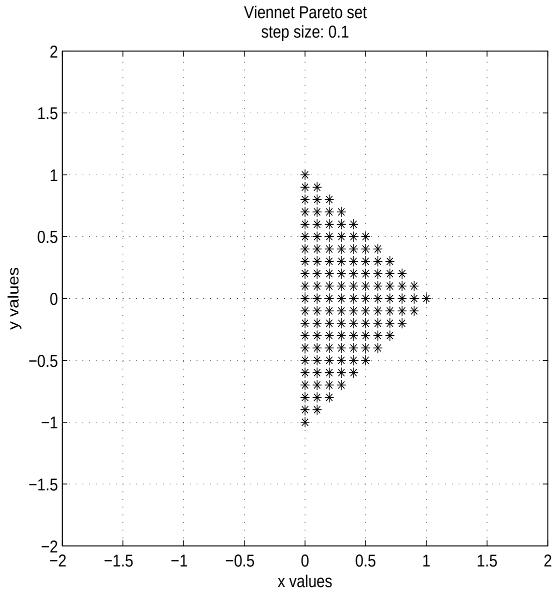


name of problem / authors	definition and restrictions	characteristics
Vicini [33] A. Vicini, D. Quagliarella.	Minimize $F = (f_1(x,y), f_2(x,y))$, where $f_1(x,y) = -\left(\sum_{i=1}^{K} \exp\left(H_i \frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_i^2}\right)\right)+3$ $f_2(x,y) = -\left(\sum_{i=1}^{K} \exp\left(H_i' \frac{(x-x_i')^2+(y-y_i')^2}{2{\sigma_i'}^2}\right)\right)+3$ restrictions: $0 \leq H_i, H_i' \leq 1$, $-10 \leq x, x_i, x_i', y, y_i, y_i' \leq 10$, K=20 $1.5 \leq \sigma_i, \sigma_i' \leq 2.5$ (e.g. $x_i = 0, -0.5, -9.5$, $x_i' = 0, -0.5, -9.5$, $y_i = 0, -0.35, -6.65$, $y_i' = 0, 0.4688, 8.9072$, $H_i = -1.2, -0.1, -3.1$, $H_i' = 1.4, 0.1, 3.3$, $\sigma_i = 5, \sigma_i' = 25$	PF_{true} with these values concave and disconnected; P_{true} connected

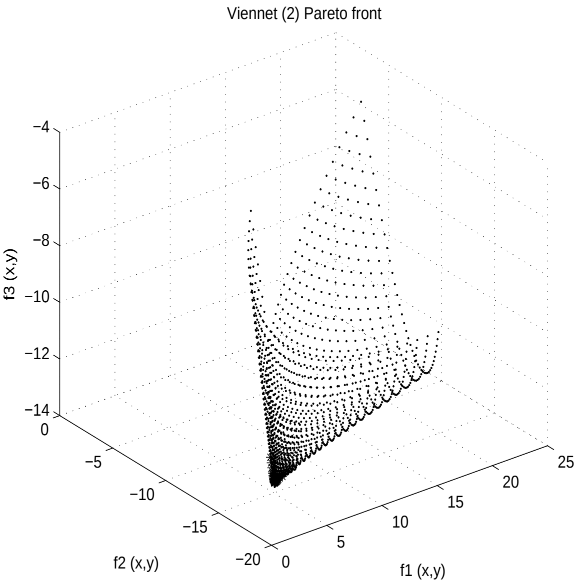


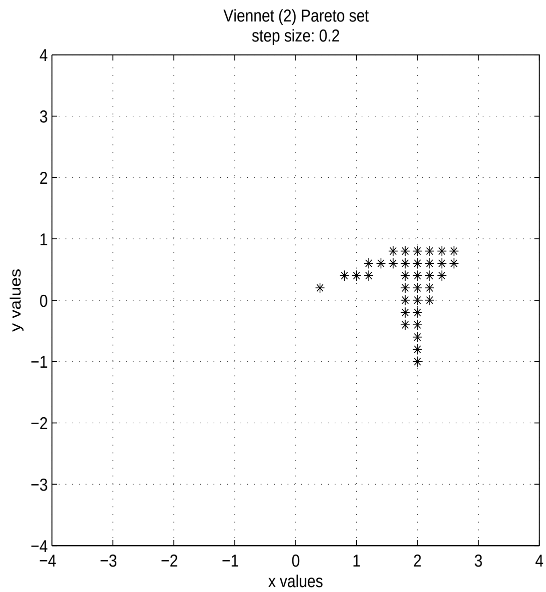
name of problem / authors	definition and restrictions	characteristics
Viennet [34] R. Viennet, C. Fontiex, I. Marc.	Minimize: $F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $f_1(x,y) = x^2 + (y-1)^2,$ $f_2(x,y) = x^2 + (y+1)^2 + 1,$ $f_3(x,y) = (x-1)^2 + y^2 + 2$ restrictions: $-2 \leq x, y \leq 2$	PF_{true} bended surface; P_{true} connected and symmetric



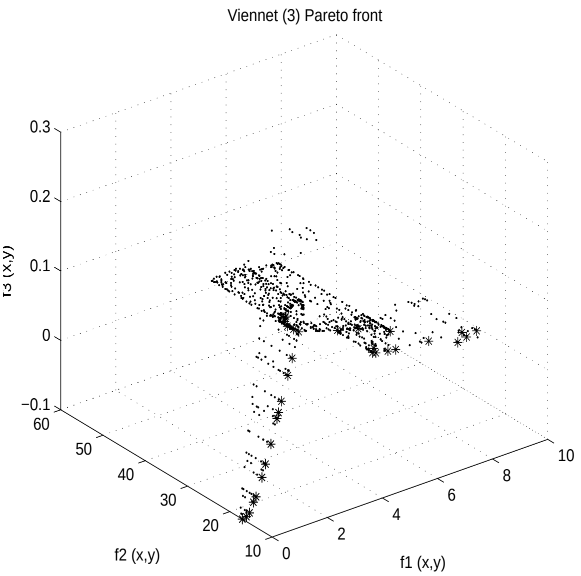


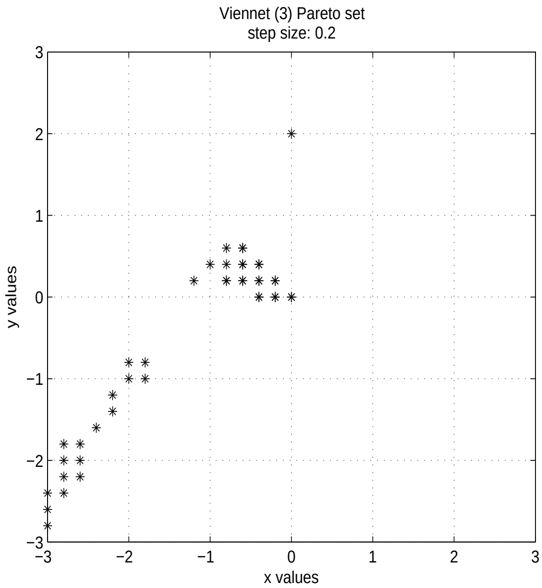
name of problem / authors	definition and restrictions	characteristics
Viennet (2) [34] R. Viennet, C. Fontiex, I. Marc.	Minimize: $F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $f_1(x,y) = \frac{(x-2)^2}{2} + \frac{(y+1)^2}{13} + 3,$ $f_2(x,y) = \frac{(x+y-3)^2}{36} + \frac{(-x+y+2)^2}{8} - 17,$ $f_3(x,y) = \frac{(x+2y-1)^2}{175} + \frac{(2y-x)^2}{17} - 13$ restrictions: $-4 \le x, y \le 4$	PF_{true} bended surface; P_{true} complexly shaped set



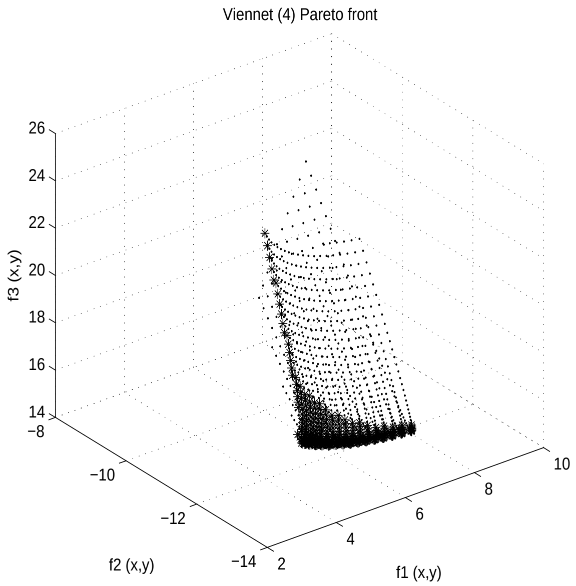


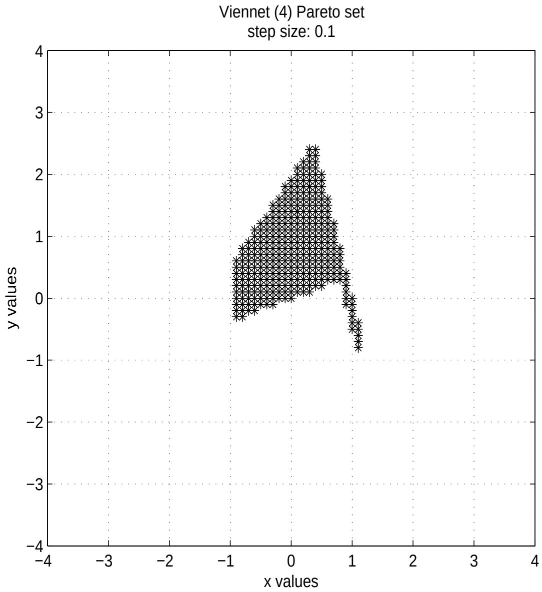
name of problem / authors	definition and restrictions	characteristics
Viennet (3) [34] R. Viennet, C. Fontiex, I. Marc.	Minimize $F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $f_1(x,y) = 0.5 * (x^2+y^2) + \sin(x^2+y^2)$, $f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15$, $f_3(x,y) = \frac{1}{(x^2+y^2+1)} - 1.1e^{(-x^2-y^2)}$ restriktions: $-3 \leq x,y \leq 3$	PF_{true} bended curve P_{true} complexly shaped set



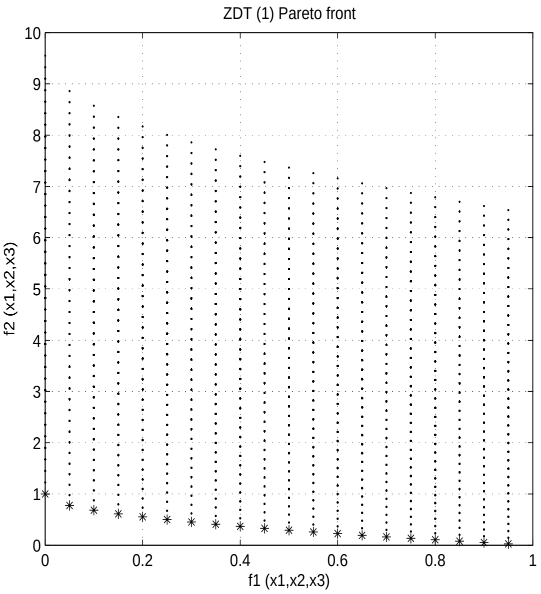


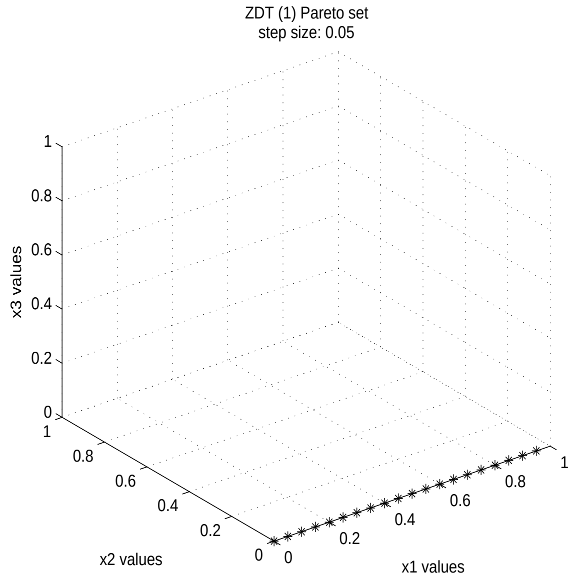
name of problem / authors	definition and restrictions	characteristics
Viennet (4) [34] R. Viennet, C. Fontiex., I. Marc	Minimize $F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $$f_1(x,y) = \frac{(x-2)^2}{2} + \frac{(y-1)^2}{13} + 3,$$ $$f_2(x,y) = \frac{(x+y-3)^2}{175} + \frac{(2y-x)^2}{17} - 13,$$ $$f_3(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15$$ restrictions: $-4 \leq x, y \leq 4$, $y < -4x + 4$, $x > -1$, $y > x - 2$	PF_{true} bended surface; P_{true} connected asymmetric set



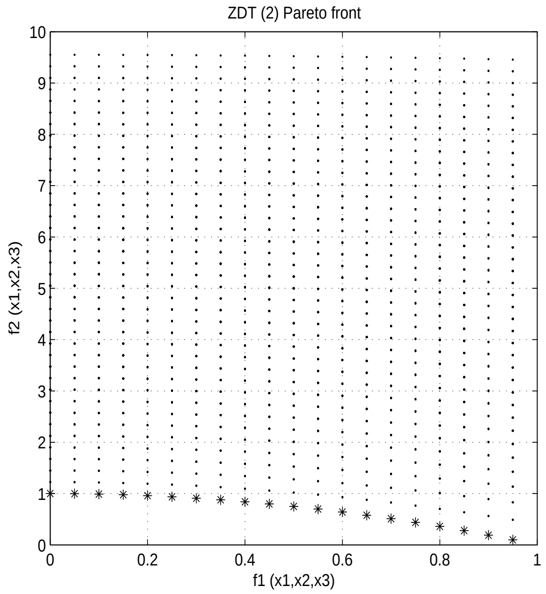


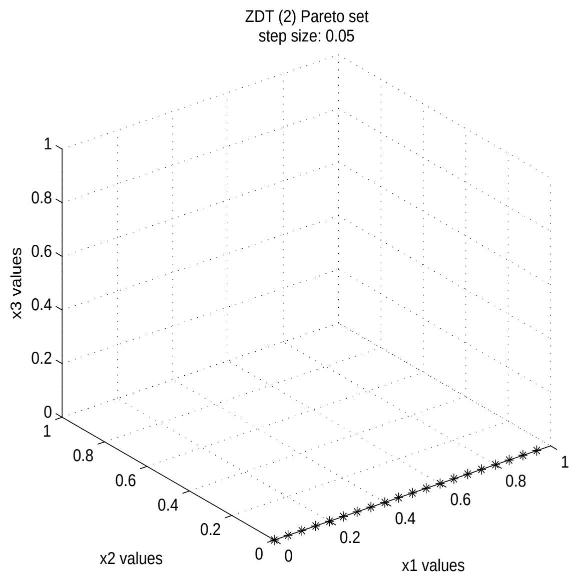
name of problem / authors	**definition and restrictions**	**characteristics**
ZDT (1) [9, 35] E. Zitzler, K. Deb, L. Thiele.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $f_1(\mathbf{x}) = x_1,$ $f_2(\mathbf{x}) = g(\mathbf{x})h(f_1, g),$ $g(\mathbf{x}) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i,$ $h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}}$ restrictions: $0 \leq x_i \leq 1$ and $n = 30$	following the scheme of [35] PF_{true} convex connected curve. $f_2 = 1 - \sqrt{f_1}$; P_{true} connected line



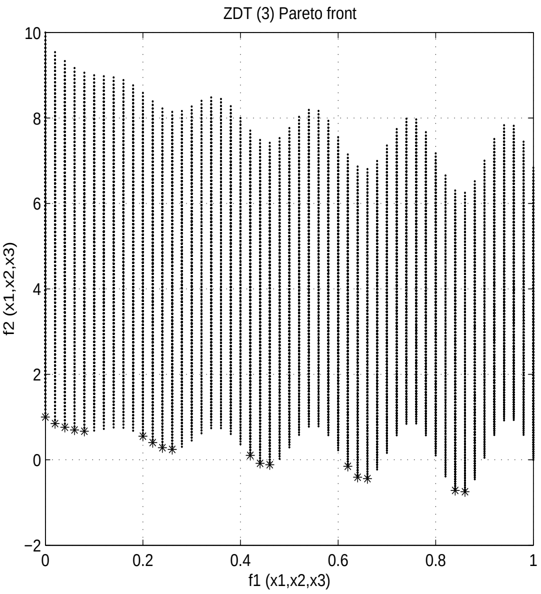


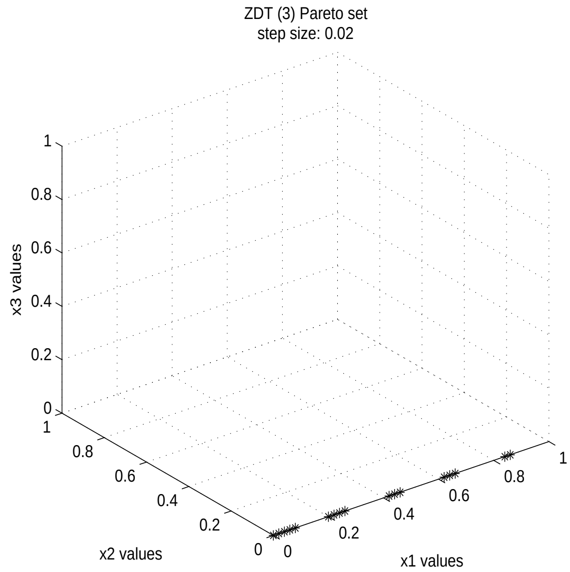
name of problem / authors	definition and restrictions	characteristics
ZDT (2) [35] E. Zitzler, K. Deb, L. Thiele.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= x_1, \\ f_2(\mathbf{x}) &= g(\mathbf{x}) h(f_1, g), \\ g(\mathbf{x}) &= 1 + \frac{9}{n-1} \sum_{i=2}^{n} x_i, \\ h(f_1, g) &= 1 - \left(\frac{f_1}{g} \right)^2 \end{aligned}$$ restrictions: $0 \leq x_i \leq 1$ and $n = 30$	following the scheme of [35] PF_{true} concave curve; P_{true} connected line



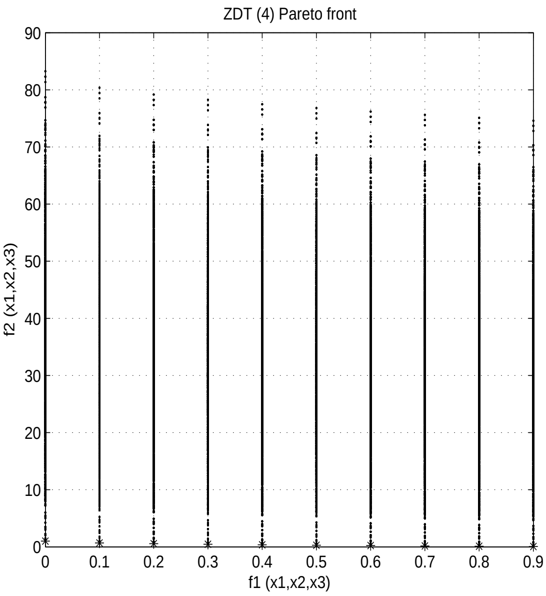


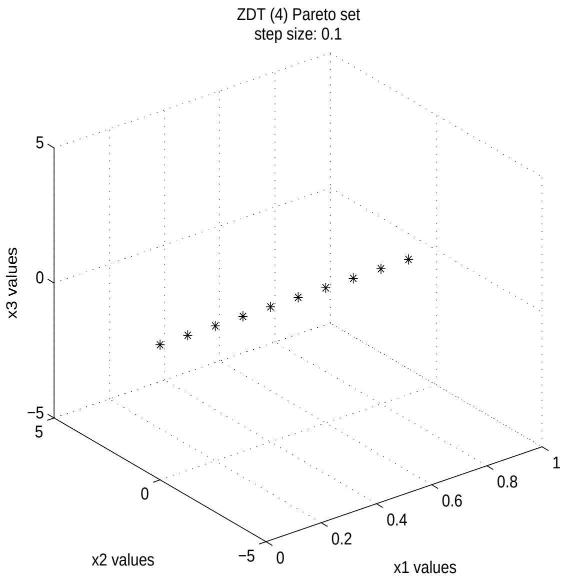
name of problem / authors	definition and restrictions	characteristics
ZDT (3) [35] E. Zitzler, K. Deb, L. Thiele.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= x_1, \\ f_2(\mathbf{x}) &= g(\mathbf{x})h(f_1, g), \\ g(\mathbf{x}) &= 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i, \\ h(f_1, g) &= 1 - \sqrt{\frac{f_1}{g}} - \left(\frac{f_1}{g}\right)\sin(10\pi f_1) \end{aligned}$$ restrictions: $0 \leq x_i \leq 1$ and $n = 30$	following the scheme of [35] PF_{true} not connected curve; P_{true} not connected line



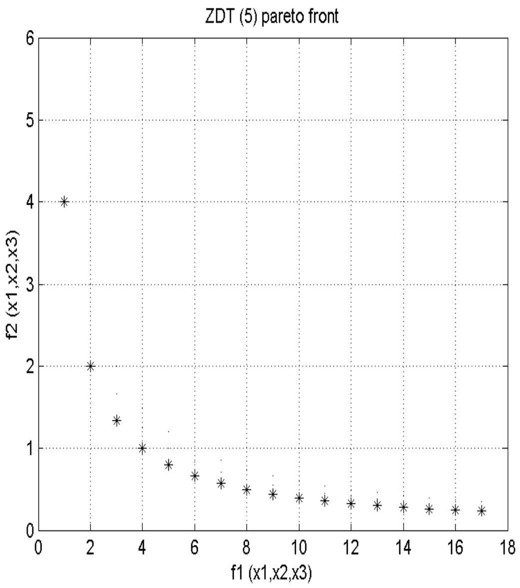


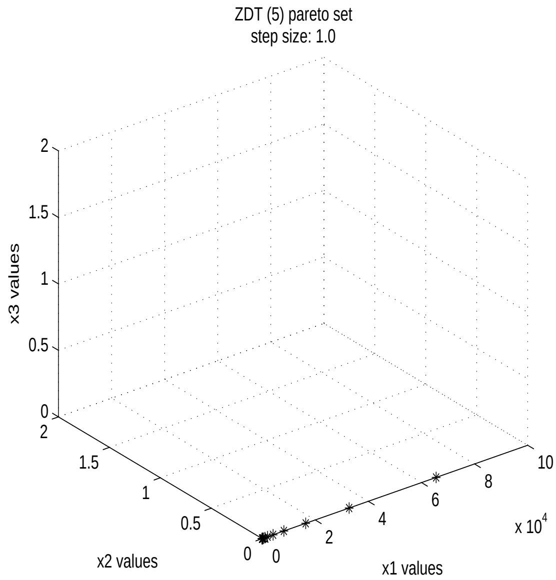
name of problem / authors	definition and restrictions	characteristics
ZDT (4) [35] E. Zitzler, K. Deb, L. Thiele.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $f_1(\mathbf{x}) = x_1$, $f_2(\mathbf{x}) = g(\mathbf{x})h(f_1, g)$, $g(\mathbf{x}) = 1 + 10(n-1) + \sum_{i=2}^{n}(x_i^2 - 10\cos(4\pi x_i))$, $h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}}$ restrictions: $0 \leq x_1 \leq 1$, $-5 \leq x_i \leq 5$ and $n = 10$	following the scheme of [35] PF_{true} convex curve; P_{true} line



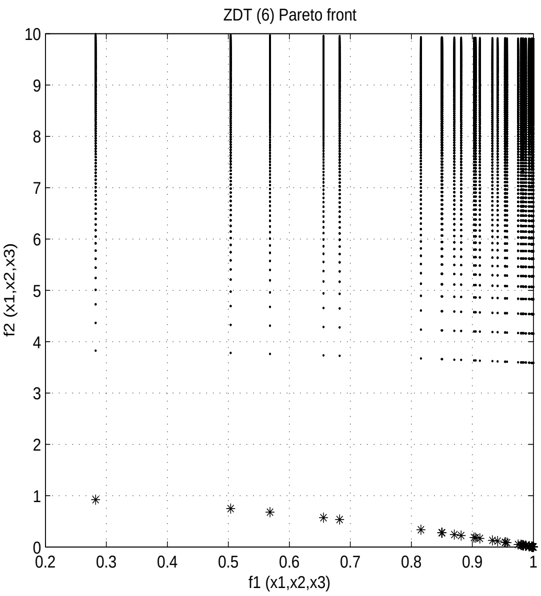


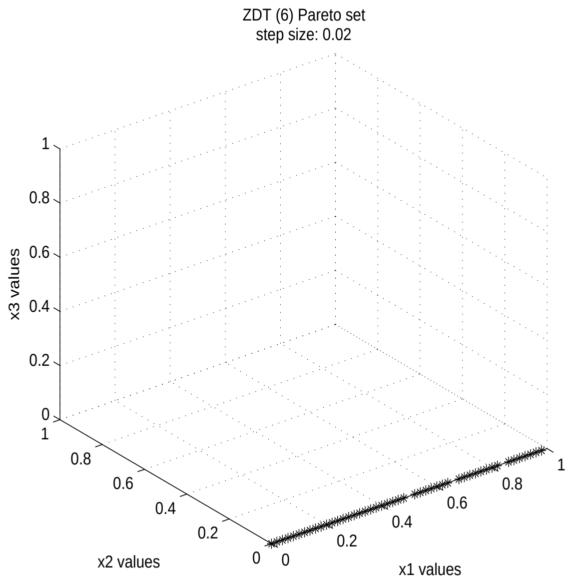
name of problem / authors	definition and restrictions	characteristics
ZDT (5) [35] E. Zitzler, K. Deb, L. Thiele.	Minimiere $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, wo $$\begin{aligned} f_1(\mathbf{x}) &= 1 + u(x_1), \\ f_2(\mathbf{x}) &= g(\mathbf{x})h(f_1, g), \\ g(\mathbf{x}) &= \sum_{i=2}^{n} v(u(x_i)), \\ h(f_1, g) &= \frac{1}{f_1(\mathbf{x})} \end{aligned}$$ $$v(u(x_i)) = \begin{cases} 2 + u(x_i) & \text{if } u(x_i) \leq 5, \\ 1 & \text{if } u(x_i) = 5 \end{cases}$$ $u(x_i)$ is the number of ones in a string that are needed to describe the variable x_i. restrictions: x_1 is a 30-bit string, $x_2, ..., x_{11}$ sind 5-Bit Strings	following the scheme of [35] PF_{true} is a convex curve; irritating problem, because the true front is $g(\mathbf{x}) = 10$, and the best irritating front is reached with $g(\mathbf{x}) = 11$.





name of problem / authors	definition and restrictions	characteristics
ZDT (6) [35] E. Zitzler, K. Deb, L. Thiele.	Minimize $F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$\begin{aligned} f_1(\mathbf{x}) &= 1 - \exp(-4x_1)\sin^6(6\pi x_1), \\ f_2(\mathbf{x}) &= g(\mathbf{x})h(f_1, g), \\ g(\mathbf{x}) &= 1 + 9\left[\frac{\sum_{i=2}^{n} x_i}{n-1}\right]^{0.25}, \\ h(f_1, g) &= 1 - \left(\frac{f_1}{g}\right)^2 \end{aligned}$$ restrictions: $0 \leq x_i \leq 1$ and $n = 10$	following the scheme of [35] PF_{true} convex nonconnected curve; P_{true} line





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