On Nonparametric Bayesian Analysis under Shape Constraints with Applications in Biostatistics

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Abstract

Shape constraints are a way of incorporating geometric or structural prior information into a statistical model. An advantage of shape constraints over other structural modelling assumptions is the fact that shape constraints are usually directly motivated from the science underlying the studied application, while other modelling assumptions (e.g. parametric assumptions) often lack such a motivation. Compared to plain nonparametric inference, shape constrained nonparametric inference has the advantage that shape constraints, when adequate, can substantially improve the efficiency of inference, as they reduce the effective modelling complexity (by restricting the model). Examples of shape constraints which are covered in this thesis are monotonicity, convexity and concavity constraints, when modelling a functional relationship and a stochastic ordering constraint when modelling probability distributions.

The thesis starts with a short introduction to the Bayesian world of thought from the nonparametric perspective and a motivation for shape constraints from three concrete applications. In Chapter 2 we then present an overview of nonparametric (i.e. infinite dimensional) Bayesian inference. We first review prior distributions for probability measures and functions, and then present a brief summary of an asymptotic analysis of Bayesian nonparametrics. The focus of the review is on methods, which are needed in the following chapters. In Chapter 3, 4 and 5 we then develop novel Bayesian nonparametric models for particular shape constrained problems. In Chapter 3 a Bayesian nonparametric model for monotone regression is developed. For this purpose the monotone function is modelled as a mixture of shifted and scaled parametric probability distribution functions, and a general random probability measure is assumed as the prior for the mixing distribution. We investigate the theoretical properties of the model and illustrate it on two practical examples (dose-response analysis and growth curve analysis). Chapter 4 then extends the model developed in Chapter 3 to the case of more general constraints on derivatives of the modelled function, such as, for example, convexity and monotone convexity. Beside a practical illustration of the model, we also derive a consistency result in this chapter. In Chapter 5 we finally introduce a model for estimating a set of stochastically ordered densities, where the ordering is assumed with respect to multivariate continuous covariates. For this purpose the residual density is modelled as a mixture of normal distributions and the stochastic ordering is induced by assuming multivariate monotone functions as component specific means. We investigate the support properties of the so formed prior distribution and illustrate the method on an epidemiologic data set. The thesis ends with a summary and an outlook of possible future work.

Keywords:
Bayesian Statistics, Monotone Nonparametric Regression, Convex Nonparametric Regression, Derivative Constraints, Stochastic Order