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Yiquan Gu

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Wage and employment effects of workplace representation: a "Right To Co-Manage" model*

Yiquan Gu†

Technische Universität Dortmund

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Abstract

This paper introduces a two-stage union-oligopoly-council model of wage and employment determination wherein at the first stage wage is negotiated through collective bargaining and at the second stage employment in each firm is co-determined by the employer and its works council. We provide a full characterization of the model outcome for all parameter values of bargaining power and co-determination power. In particular, works councils always increase employment while their impact on wage can be non-monotonic. Overall, individual works councils' pursuit of own workers' interests may well harm the workers as a union.

Keywords: Workplace representation, Union-oligopoly bargaining, Firm-council co-determination

JEL-Classification: J50; J54; L13

1 Introduction

Characteristic of many industries in many countries is that workers are not only organized as a union at the industry level but also represented at workplaces. Institutions of employee participation at workplaces may appear under different names including works councils (many

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†Department of Economics and Social Science, Technische Universität Dortmund, Vogelpothsweg 87, 44227 Dortmund, Germany. Email: yiquan.gu@uni-dortmund.de

European countries), company unions, enterprise unions (Japan), ”shop committees” or ”joint consultative committees” (Britain). Arguably, the most extensively researched are European works councils, in particular, Germany’s *Betriebsräte,* but there is also a ”heightened American interest in cooperation/participation at the workplace” (Addison et al., 1997).[1]

As an example, we briefly introduce German works councils in the context of Germany’s dual system of worker representation. In many German industries, notably material and manufacturing, there is a nation-wide trade union of workers. The union negotiates with the firms who are represented by an employers’ association to reach a general labor agreement. Based on this general agreement, at the firm level, employers and their workers decide on specifics.[2] Normally, wage bargaining is the most important part of union-industry negotiation while firm level adjustments usually do not include wage renegotiation.

A works council is a firm level organization which represents a firm’s employees. In Germany, the right to form a works council in firms having five or more employees is granted by the Works Constitution Act.[3] In addition to information and consultation rights, a works council is also endowed co-determination rights with respect to a range of issues related to employees. Of special interest to the current paper is its co-determination (or veto) rights on overtime working and layoffs which directly influence the host firm’s employment level. A works council, however, is not allowed to renegotiate on the wage rate that has been set in the general agreement unless it is explicitly stated otherwise. Although workers’ interests may include various financial or non-financial benefits, we would like to focus on the influence of a works council on firm level employment. In this paper, we assume that a works council primarily seeks to increase the employment level of its host firm either directly through its co-determination rights on layoffs and overtime working or indirectly through their other rights.

Given that the wage is determined in the collective bargaining between the employers and the trade union, it seems natural to view that a works council’s main concern is more jobs. Firstly, the growth of workforce will make current workers’ jobs more secure, especially when seniority plays a role in the process of workforce downsizing triggered by product market situations. Elected by current workers, a works council shall reflect such a preference. Secondly, by representing more workers, a works council normally becomes more powerful. Thirdly, but more importantly, a recent empirical study by Jirjahn (2009) finds that works councils are ”more likely to be adopted in establishments with a very poor sales situation and poor employment growth”. This is indicative of a works council’s main objective being promoting more jobs.

Related to this dual system of industry-labor practice, we introduce a two-stage wage and employment determination model featuring a trade union, N firms and their respective works

[1]Supporting this view, Addison et al. (1997) cited the July 1996 special issue of *Industrial Relations*.

[2]Firms that opted not to join the employers’ association are not covered by the general agreement.

[3]In Germany, a works council in firms that reach this threshold size is mandatory but not automatic. The fine difference is as follows. The law grants the workforce of such firms the right to form a works council but the employees have to initiate the process of formation and to elect the councilors. The associated costs, however, shall be covered by the employer, according to the law.

councils. At the first stage the union bargains with the firms who are represented by an employers' association over the wage that will be applied to all firms in the industry. At the second stage each firm and its own works council co-determine the firm level employment. While the first stage wage negotiation is modeled by a generalized Nash bargaining of industry-wide workers' interest and industry profit, we highlight works councils' co-determination rights at the second stage by making a firm and its works council jointly maximize a convex combination of firm profit and firm level workers' interest. The relative weight in such a combination will be the measure of a works council's co-determination power. Hence, the differences in the power of workplace representations across firms as well as across different industries and countries can be captured.

Compared to using generalized Nash bargaining at the second stage again, such a modeling strategy leads to tractable solutions and allows us to directly relate our model to the strategic delegation literature. The wage bargaining and employment co-determination take place in an environment where firms compete in quantity in a homogenous product market with a linear demand function. With capital being fixed, a firm's employment level dictates its output. Therefore, a convex combination of profit and employment resembles a financial incentive scheme for a manager that includes both firm profit and product sales.

Our main research questions center on workers' interests, namely the wage bargaining outcome and the employment levels. Although union density (or the organizational degree of labor force) varies, in this paper we assume that the union cares about the total employment of the industry for simplicity. We do not expect our main results to change qualitatively when other meaningful exogenous union densities are considered. As a works council is officially independent of the union and is elected in a firm, it cares about its host firm's employment only. Since both the union and works councils are concerned with workers' benefits, it seems that with the union taking care of the wage and additionally works councils demanding more jobs in firms, workers have to be better off than in a setting where only the wage is bargained over. But, will works councils' quests for more jobs at the second stage backfire at the first stage wage negotiation? The argument is, if profits become too low as a consequence of elevated outputs under the influence of works councils, the firms will only agree on a low wage rate.

To have a complete picture of wage and employment outcomes, we provide a full characterization of wage and employment determination in this union-oligopoly-council model for all parameter values of union's bargaining power and works councils' co-determination power. It is found when the union's bargaining power is sufficiently strong, wage will first increase then decrease in works councils' co-determination power. Although stronger councils will always increase firms' labor demand even under endogenous wage, a similar pattern applies to union payoff defined as the product of wage and number of jobs in the industry. The switch point of wage, however, occurs before that of union payoff since workers as a union might be better off having more jobs at a slightly lowered wage. When the union has an intermediate level of bargaining power, wage will only decrease in council power while union payoff may initially increase when council power's positive impact on jobs dominates but it will eventually decrease when the wage becomes too low. When the union is weak, both wage and union payoff will decrease in council power because against powerful employers, not only jobs will only come at the cost of a lower wage, but also wage reduction will be too large to justify any job growth. It is also shown that an increase in the number of firms will push these thresholds of union power upwards. So in an industry that is populated by many firms, works councils are more likely to decrease the wage rate as well as to harm the workers as a union.

In conclusion, an increase in councils' co-determination power will always result in more jobs but if the union has a very strong bargaining power and if the councils are not overly powerful, wage too can be raised. As to the question whether an increase in council power is good for the workers as a union, the affirmative answer also requires the union to be sufficiently powerful and the councils not overly strong. Otherwise, works councils' co-determination power will backfire in the wage bargaining stage leading to a lowered level of union payoff.

From the perspective of an individual works council, a stronger council is always desirable since the wage is fixed at the second stage and a stronger council will bring about more jobs. This individually rational pursuit of workers' interest, however, will make product market competition fiercer and lead to a lower industry profit at any given wage. When the union does not have a sufficiently strong wage bargaining power, firms will be able to shift the losses onto the workers by demanding a lower wage. Therefore, for the workers as a union, strong councils can have an adverse impact. When there are many firms in the product market, councils' influence is amplified and adverse effects emerge quite easily. Ironically, a similar insight also applies to the firms. Since the firms have identical wage costs, in equilibrium a firm with a strong council will make a higher profit than those with less strong councils because it produces more. In other words, a strong council will give its host firm a relative advantage over those with weak councils. However, the stronger the councils are on average the lower the industry profit will be. This holds true even when the wage is endogenously determined.

The rest of the paper proceeds as follows. In Section 2 we relate our model to the literature of works councils and to that of unionized oligopoly. The model is laid out and explained in Section 3. Section 4 analyzes the second stage product market competition and Section 5 the first stage wage bargaining. To derive our main results, we start in Section 6 with the limiting cases of bargaining power and co-determination power both because they help to build intuition and because they deserve a special attention. Section 7 presents our full characterization of works councils' impact on wage, employment and union payoff. Additional results with respect to works councils' impact on firms' profits are derived in Section 8. Our analysis is concluded and further discussed in Section 9.

2 Relation to the Literature

There is an extensive literature on workplace representations of workers, in particular works councils, focusing on industrial relations, more specifically, the relation between management and employees. Frege (2002) provides a critical assessment of the research on German works councils. For the ontology and practice of works councils her piece and the references cited therein are conclusive. Disagreeing with Frege (2002) on her assessment of economic analyses, Addison et al. (2004) review more extensively the empirical economic research on works councils and offer a more optimistic outlook. The main concern of this strand of empirical research is the impact of works councils on firm performance. The results seem to be mixed in identifying the two contrasting views about works councils: productivity enhancing via an improved cooperation/communication between managers and workers, and rent seeking/protection for the workers.[4]

Despite of the vast interest in works councils, the large literature within industrial relations and the smaller yet still sizeable empirical economic literature, theoretical studies remain scarce, in particular with respect to wage and employment determination. Freeman and Lazear (1995) explain how in several different ways works councils can enhance productivity. The theory part of Hübler and Jirjahn (2003) investigates when a works council will be more likely to be engaged in productivity enhancing activities and when in rent seeking activities.

In this paper, we rather wish to abstract from this debate and focus on the consequences that works councils can bring about to firms and in particular to workers, when they set out to promote workers' interests at workplaces. The reason is two-fold. First, on the one hand, often it is the case that only firm profits not levels of productivity are empirically observable. On the other hand, activities that are purely motivated by workers' own interests can also increase firm profit. As one example, in our model, by demanding more jobs a works council may help its host firm to gain an advantage over other firms as well as may increase firm profit. It would be hard to differentiate whether an increase in firm performance is a result of enhanced productivity or merely a side-effect of rent seeking activities. Second, being an organization of workers, a works council is primarily after workers' interests by definition. Whether its employer will be better off or productivity will be enhanced in the end depends on situations at hand. Put differently, productivity enhancing might be the means but will not be the end for works councils. Therefore, in the current paper, we do not consider productivity.

The present paper is also related to the union oligopoly literature and more broadly to the research on labor unions. With employment negotiation included in the bargaining agenda, in an "efficient bargaining" model (McDonald and Solow, 1981) unions and firms bargain off the "contract curve". In contrast, in a "Right-To-Manage" (RTM) model (Nickell and Andrews, 1983) unions and firms bargain over wages alone while leaving employment levels to the discretion

[4]Arguably, these two views need not be mutually exclusive.

of the firms. In practice, labor agreements between unions and firms that also include employment levels are rare, while empirical evidence suggests that employment levels are usually off, and to the right of, firms' labor demand curves, i.e., not consistent with the RTM model. (See, for example, MaCurdy and Pencavel, 1986; Christofides, 1990.)

This puzzle led to the contributions of Bughin (1995a; 1995b) who, in a unionized oligopoly setting which takes into account product market imperfection (Dowrick, 1989), uses the strategic delegation argument of Fershtman (1985) and Fershtman and Judd (1987) to explain this inconsistency between the RTM model and the empirical evidence. One of the insights from the strategic delegation literature is that in Cournot competition a firm may find it advantageous to commit itself to an objective function that also admits sales instead of being a pure profit maximizer. By the same reasoning, assuming wage has been determined, Bughin (1995a) shows that firms have an incentive to strategically concede some power to the union with respect to employment level. Therefore, in Bughin (1995a; 1995b) the additional employment compared to a usual labor demand curve arises from a firm's more aggressive behavior in quantity competition.

Once firms started to strategically concede power to unions, however, they invited themselves into a prisoner's dilemma. In equilibrium, each firm earns a profit that is strictly less than in the case of all firms each being a pure profit maximizer. From the viewpoint of anti-trust research, all firms would legally collude on zero power concession, especially when the number of firms is small. In this paper, it is the work councils who are pushing employment levels to the right of firms' labor demand curves. As a works council would like to have as many jobs as possible, its host firm no longer enjoys the discretion of to what extent firm employment enters into the objective function. There was also no need in Bughin (1995a) for a complete bargaining model that includes wage negotiation as a stage that precedes the power concession stage since in a symmetric equilibrium duopoly model, wage determination is quite straightforward. Bughin (1995b) solves a unionized duopoly model that incorporates wage bargaining (with endogenous symmetric power concession decisions) and compares the outcomes to a standard RTM model. He finds that decentralized "company unions" would agree with the firms on maximizing an objective function that also includes employment. The present paper, therefore, is different from Bughin's (1995a; 1995b) contributions also in the regard that we have an industry-wide union bargaining with N firms over wages in a collective fashion while each firm deals with its own works council with respect to the employment level in the firm. Our union-oligopoly-council model hence can be seen as a "Right To Co-Manage" (RTCM) model that addresses the inconsistency between the RTM model and the empirical evidence.[5]

In the unionized oligopoly literature, there is a distinction between centralized/collective bargaining and decentralized bargaining. In the former case, an industry-wide trade union and the firms engage either in "efficient bargaining" or RTM bargaining. In the latter case, each firm has its own labor union and each pair separately engages in either one of the two modes of

[5] Workplace representation therefore also offers an alternative explanation to the empirical observation that firms are not always pure profit maximizers.

bargaining. Our RTCM model amounts to be the first model wherein wage and employment determination take place at different levels of centralization in a sequential fashion.[6]

One difficulty related to centralized bargaining is the distribution of jobs over the firms. When the number of jobs–either directly bargained over or implied by the wage outcome in a RTM model–falls short of union membership, standard in the literature is to assume each member gets an equal probability of being employed (see Oswald, 1985). But even when there are enough jobs, workers in individual firms would still like to see more jobs available in their own firm because of their concerns of job security. In this regard, works councils offer an insight: co-determination with employers at workplaces is a way in which workers of individual firms non-cooperatively divide industry employment, parallel to the firms competing for their market shares.

3 The model and notations

Consider an industry in which $N \geq 2$ firms, indexed $i = 1, 2 \ldots N$, compete in quantities (Cournot) on the product market. Market demand is linear and product price (p) is negatively related to total industry output ($\sum_{i=1}^{N} q_i$):

$$p = A - \sum_{i=1}^{N} q_i \tag{1}$$

where A, the market size, is a strictly positive constant and for the moment is assumed to be large enough to accommodate the N firms. When wage is endogenously determined, we will see market size merely works as a scaling factor. To highlight labor issues, one unit of labor (l_i) is assumed to produce one unit of output (q_i) with capital being fixed:

$$q_i = l_i. \tag{2}$$

Labor is supplied by workers who are represented at the industry level by a trade union. In each individual firm, workers also form a works council that promotes the interests of the workers of its host firm.

3.1 Union-oligopoly wage bargaining

There are two stages. At the first stage, the union and the firms who are represented by an employers' association bargain over wages to maximize the Generalized Nash Product (G). In this collective wage bargaining, the negotiated wage (w) will be applied to all N firms. We adopt a

[6]The sequentiality part alone, however, is not exactly new. For instance, Manning (1987) explores the topic of sequential bargaining in a union-firm 2-player setting, i.e., in the absence of the oligopolistic structure of the current paper.

simplified version of utilitarian union preference, namely, the industry wage bill, $w\sum_{i=1}^{N} l_i$.[7] The employers' association is concerned about industry total profit, $\sum_{i=1}^{N}\pi_i$. Measuring the union's wage bargaining power relative to the firms' is the parameter $\alpha \in (0,1)$. In case of disagreement, workers obtain the non-industry specific competitive wage while the firm owners may invest in the capital market. After normalizing both fall-back payoffs to zero, the Generalized Nash Product reads

$$G = \left(w\sum_{i=1}^{N} l_i\right)^{\alpha}\left(\sum_{i=1}^{N}\pi_i\right)^{1-\alpha}. \tag{3}$$

3.2 Firm-Council employment co-determination

At the second stage, with the wage having been determined in the first stage, firms compete on the product market in quantity. Normally, a firm tries to maximize its profit. A works council, however, promotes workers' interests. In the spirit of co-determination, we view the firm-council entity as making a joint decision to maximize a convex combination of the firm's profit and the plant level wage bill:

$$F_i = (1-\beta_i)\,\pi_i + \beta_i w l_i \tag{4}$$

where $\beta_i \in [0, 1/2)$ is the measure of the co-determination power of firm i's works council. If $\beta_i = 0$, this stage becomes a standard Cournot model and the two stages combined become a (collective wage bargaining) RTM model. As the council's power, β_i, increases, more weight is allocated to the employees' interest, which is in more jobs for any given wage. We need council power to be less than $1/2$ to have an economically meaningful analysis. After all, a firm owner's interest should be more important than a works council's concern of employment.

Since firm profit is $(p-w)\,q_i$, by treating $q_i = l_i$ as binding, i.e., firms cannot hire workers without letting them produce, there is only one decision variable in the objective function (4), say q_i. Allowing q_i to be any non-negative real number, this second stage is an N-person non-cooperative game with payoffs defined in (4). This payoff function clearly resembles that of a strategic delegation game where product sales instead of plant wage bill would have entered. Note, however, β_i is not a choice variable in our model. Rather, it is exogenously given and represents the co-determination power of a firm's works council.

[7] From a general utilitarian form to the wage bill preference one assumes risk-neutrality and that membership equals employment. See Oswald (1985).

4 Product market competition

We solve the model backwards. The second stage of a firm-council pair's quantity or employment decision is analyzed in this section. For a given price p, the objective function (4) is

$$\begin{aligned} F_i &= (1-\beta_i)(p-w)\,q_i + \beta_i w q_i \\ \Longrightarrow \frac{F_i}{1-\beta_i} &= \left(p - \frac{1-2\beta_i}{1-\beta_i} w\right) q_i. \end{aligned} \tag{5}$$

Let $\tilde{F}_i := F_i / (1-\beta_i)$ and more importantly we define our *reverse measure of council power* as

$$r_i := \frac{1-2\beta_i}{1-\beta_i}.$$

The objective function (5) can then be seen as

$$\tilde{F}_i = (p - r_i w)\, q_i. \tag{6}$$

Since r_i decreases in β_i, a more powerful works council makes the firm as if it had a more competitive labor cost. Note also for $0 \leq \beta_i < 1/2$, $0 < r_i \leq 1$. *In the following, our results will be derived in r. When we say r_i is increasing, we mean firm i's council's co-determination power is decreasing or, equivalently, it becomes weaker!*

After taking into account market demand (1), for given outputs of other firms, the objective functions (6) are, for all i,

$$\tilde{F}_i = \left(A - \sum_{j=1, j\neq i}^{N} q_j - q_i - r_i w\right) q_i.$$

Individual best response functions are

$$q_i = \frac{1}{2}\left(A - \sum_{j=1, j\neq i}^{N} q_j - r_i w\right). \tag{7}$$

Summing up these N best response functions and looking for Nash equilibrium quantities, industry total output (Q) is

$$Q^* = \sum_{i=1}^{N} q_i^* = \frac{NA - w\sum_{i=1}^{N} r_i}{N+1}, \tag{8}$$

where $*$ denotes equilibrium value. Because of our production technology, Q^* can also be interpreted as industry total employment. Making use of this total output, we can find market price by the market demand (1) and each firm's equilibrium output or employment by the best response functions (7),

$$p^* = \frac{A + w\sum_{i=1}^{N} r_i}{N+1}, \tag{9}$$

$$q_i^* = \frac{A + w\sum_{j=1, j\neq i}^{N} r_j - N r_i w}{N+1}. \tag{10}$$

In Section 3, we assumed A to be large enough to accommodate all N firms, i.e., no firm will make a loss. We now can be more specific on this assumption. Essentially, we need the market price to be high enough to compensate firms' labor costs,

$$\frac{A + w\sum_{i=1}^{N} r_i}{N+1} \geq w, \quad i = 1, 2, \dots N,$$

which is equivalent to

$$A \geq \left(N + 1 - \sum_{i=1}^{N} r_i\right) w. \tag{11}$$

On the one extreme of zero council power in all firms ($\forall i, r_i = 1$), A only needs to be larger than the wage w. On the other, when all councils are extremely powerful ($\forall i, r_i \to 0$), firms become very aggressive and the market becomes extremely competitive. In this case, we require A to be no less than $(N+1)\, w$ to accommodate all firms. Note that condition (11) only depends on the aggregate level of r. If one firm survives, all firms will survive. When the wage is endogenously determined, industry profit will be non-negative for all $A > 0$ (because the firms have their fall-back payoffs of 0) so this condition will always be satisfied.

4.1 Equilibrium output (employment) and product price

Here we can identify a few results for given wages. Recall that a more powerful works council translates into a lower r_i. Compared to the case of zero council power ($\forall i$, $r_i = 1$), Equations (8) and (9) imply that works councils increase the total supplied quantity and total employment, and decrease the product market price.

Turning to an individual firm and holding all other firms' council power constant, the stronger a firm's works council is the more it produces (employs). Since quantities in a Cournot model are strategic substitutes, other firms produce (employ) less. To verify this, note that $-dq_i^*/dr_i = wN/\left(N+1\right)$ while $-dQ^*/dr_i = w/\left(N+1\right)$. When the number of firms is relatively small, this (equilibrium) externality to each of the other firms can be significant. E.g., in the case of duopoly, the impact of one firm's council power on its own employment is of the order of $2w/3$, while the industry employment only increases in the order of $w/3$. The other firm whose council power remained unchanged absorbs the difference by reducing employment accordingly. This is also evident from the fact that $-dq_i^*/dr_j = -w/\left(N+1\right)$.

Lemma 1 *For a given wage, an increase in firm i's council power increases its own and decreases other firms' output and employment. Total output and employment will increase and product price will decrease.*

Remark 1 *A works council promotes its own workers' interest at the cost of workers in other firms.*

As long as firms do not make losses, an additional entry should drive down the product market price. To compare our model to a standard Cournot model, we first select an N firm case and subsequently let one more firm, indexed $N+1$, join the group. Adjusting condition (11) accordingly, market price does not increase if

$$\begin{aligned} & \frac{A + w\sum_{i=1}^{N} r_i + w r_{N+1}}{N+2} \le \frac{A + w\sum_{i=1}^{N} r_i}{N+1} \\ \iff \quad & r_{N+1} w \le \frac{A + w\sum_{i=1}^{N} r_i}{N+1}, \text{the pre-entry price.} \end{aligned} \tag{12}$$

The right hand side of condition (12) is just the market price in the N firm case. Since w is no more than market price and $r_{N+1} \le 1$, condition (12) always holds. Moreover, because market price is related to the average of council powers, $\left(\sum_{i=1}^{N} r_i + r_{N+1}\right) / (N+2)$, an entrant with a strong works council lowers the product market price further.

4.2 Firm profit

Let $R_{-i} := \sum_{j=1, j\neq i}^{N} r_j$, the sum of all firms' reverse measure of council power except that of firm i. Firm i's profit in equilibrium then is

$$\pi_i^* = (p^* - w)\, q_i^* = \left(\frac{A + wR_{-i} + r_i w}{N+1} - w\right)\left(\frac{A + wR_{-i} - N r_i w}{N+1}\right). \tag{13}$$

Of particular interest to the industrial relation research is the impact of works council on firm profitability. From (13) it is clear that firm i's profit decreases when R_{-i} decreases which means when other firms' council powers increase, firm i makes less profit.

4.2.1 Firm-Council relations

The impact of own works council's power, however, is not obvious. To investigate this, we take a look at the derivative of a firm's equilibrium profit with respect to its own works council's co-determination power measured by r.

$$\begin{aligned} \frac{d\pi_i^*}{dr_i} &= q_i^* \frac{dp^*}{dr_i} + (p^* - w)\frac{dq_i^*}{dr_i} \\ &= \frac{w}{N+1}\left(q_i^* - Np^* + Nw\right). \end{aligned}$$

Therefore, the sign of own council power's impact on profit is given by

$$
\begin{aligned}
-\frac{d\pi_i^*}{dr_i} \gtreqless 0 \quad &\Longleftrightarrow \quad q_i^* - Np^* + Nw \lesseqgtr 0 \\
&\Longleftrightarrow \quad r_i \gtreqless \frac{N+1}{2} - \frac{(N-1)\,(A + wR_{-i})}{2Nw}. \qquad (14)
\end{aligned}
$$

To check if condition (14) has any discriminative power, we set the value of A to $(N+1-R_{-i})\,w$ which clearly satisfies the lower bound condition (11). Condition (14) then reads,

$$
r_i \gtreqless \frac{N+1}{2N}. \qquad (15)
$$

The right hand side (RHS) of (15) ranges from $1/2$ when N is extremely large to $3/4$ in the duopoly case. Since r_i is in the interval of $(0,1]$, we know that own council power's impact on profit can be in either direction.

What we can learn from condition (14) is that for a given industry environment, an increase in its work council's co-determination power may have different directions of impact on the firm's profitability. It can work in favor of the firm owner when its council currently is sufficiently weak (for a sufficiently high r_i, $-d\pi_i^*/dr_i > 0$). It may also work against the firm owner when its council is already quite strong (for a sufficiently low r_i, $-d\pi_i^*/dr_i < 0$.) On the other hand, a works council always wants to be more powerful so that firm employment would be higher (Lemma 1). This highlights the relation between employers and their works councils. They are neither in pure conflict nor in perfect harmony.

Proposition 1 *When a firm's works council is sufficiently weak, an increase in council power can increase the firm's profit; when it is sufficiently strong, an increase of it can decrease the firm's profit.*

An increase in council power has two effects to the employer's profit: it increases firm output which has a positive impact when product price is given; it, however, decreases product price because output will be increased. The overall effect depends on the market condition as well as councils in other firms. In the example of $A = (N+1-R_{-i})\,w$, when a council is weak ($r_i > (N+1)/2N$), its firm benefits from an increase in council power because the positive quantity effect dominates the negative price effect. When the council is strong already, a further increase in council power harms the employer because the negative price effect dominates.

Proposition 1 is formulated from the perspective of absolute payoffs. In a product market equilibrium, firms have identical price-cost margins, and hence firm profits may differ only in their produced quantities. The stronger a firm's council is, the more the firm produces. This is because from equation (10), one derives $q_i \geq q_j \iff r_i \leq r_j$. Therefore, the firm that has the strongest council will have the highest profit.[8] Of empirical relevance is that firms that host

[8] This observation does not contradict Proposition 1, because that result is in terms of absolute profits not relative payoffs.

stronger councils will have higher profits due to their larger sizes of workforce and output.

Proposition 2 *In a product market equilibrium, the stronger a firm's works council is, the larger its size of employment and the higher its profit will be.*

The equality part of condition (14) gives a firm owner's most preferred level of council power in the sense that it yields the highest profit for any given other councils' power (R_{-i}) and any given wage cost (w). Excluding corner solutions, the RHS of (14) decreases in R_{-i} meaning when other councils become weaker, it pays for firm i to have a stronger council. This observation of strategic substitution follows from Cournot competition. The RHS of (14) also decreases in A meaning that a larger market makes the firms prefer stronger councils.

Remark 2 *A firm prefers a stronger council when other councils become weaker and when the market grows.*

4.2.2 Dummy works councils?

If council power were a strategic variable of the firms, we could solve the N conditions in (14) for equilibrium levels of council power. In general, for a given environment, a firm would prefer letting its council to have some power which is well understood in the strategic delegation context. For instance, it is optimal for a firm's owner to condition its manager's financial compensation partly on the sales volume. See also Bughin (1995a; 1995b).

In the context of works councils, we do not think firms are able to freely choose values for this parameter. First, as it is evident from our analysis, a works council always wants to have more power while its firm does not want it to be excessively powerful. On the other hand, in many countries there are laws protecting the rights of work councils. It is not at all clear how extensive a firm can influence its works council's power. Second, as in strategic delegations, although each firm would prefer some council power, once that door is open, they invited themselves into a "prisoner's dilemma". With powers strategically conceded to works councils, all firms make less profits than without. In the view of competition policy research, especially when the number of firms is small, firms would "legally collude" on zero council power. We therefore in the following analysis continue to treat councils' co-determination power as exogenous and proceed to a fully characterization of model outcome.[9]

4.2.3 The limiting case of competitive market

The standard Cournot model has the desirable feature of its equilibrium converging to the competitive market equilibrium as the number of firms goes to infinity. What can be said about this

[9] It may be argued that council power is only one-sided rigid. It might be easy for a firm to increase its council's power but not the other way around. This is an interesting aspect to be explored in future research.

issue in this model of firm-council co-determination? To address this, we disregard the lower bound condition of market size. Rather, we need $A < \infty$ to have a meaningful argument. The limit of product market price when N goes to infinity is:

$$\lim_{N \to +\infty} p_N^* = \lim_{N \to +\infty} \frac{A + w \sum_{i=1}^N r_i}{N+1} = w \lim_{N \to +\infty} \frac{\sum_{i=1}^N r_i}{N}.$$

Market price instead of converging to the wage rate (marginal cost) it converges to the "average council power" corrected wage. For the industry to survive at all, firms cannot have any council power, at least on average. Hence, there seems to be an economic argument for Germany's Works Constitution Act not granting the right of establishing a works council to workers in firms that have less than 5 employees.

5 Union-Oligopoly wage bargaining

The first stage wage bargaining problem between the firms and the union is analyzed in this section. Observe that for product price and market total output, only the average council power is relevant; the composition is not. Let r be the average (reverse) council power $\left(\sum_{i=1}^N r_i\right)/N$, $r \in (0, 1]$.

5.1 Industry profit and union payoff

When the firms and the union bargain over wages, they anticipate the outcomes of the second stage product market competition. The industry total profit is

$$\begin{aligned} \Pi &\equiv \sum_{i=1}^N \pi_i = \sum_{i=1}^N (p-w)\, q_i = (p-w)\, Q \\ &= \frac{N\left[A - (N+1-Nr)\, w\right](A - rw)}{(N+1)^2}. \end{aligned} \tag{16}$$

On the total profit of the pooled firms, a higher wage has one direct negative effect resulting from a higher production cost ($-w$). It also has two indirect effects. One comes from the output (Q) and the other from the market price (p). The former is negative while the latter is positive. From (16), however, it is clear that the negative effects dominate and a higher wage always reduces industry profit.

In the last section, we analyzed the impact of a works council's co-determination power on its host firm's profit. The results were mixed. Here, we have a look at the industry level. Take the derivative of industry profit against the average (reverse) council power and make use of the lower bound of A (11), then

$$-\frac{d\Pi}{dr} = -\frac{Nw\left[(N-1)A + (N+1-2Nr)w\right]}{(N+1)^2}$$
$$\leq -\frac{N^2 w^2 (1-r)}{N+1} \leq 0.$$

Note that for any given non-zero feasible wage, the equality holds only when $r = 1$. Therefore, industry profit will unequivocally decrease in council power making the pooled firms dislike the councils even though individually, they may appreciate a certain degree of council power.

Lemma 2 *For any given feasible non-zero wage, the stronger the councils are, the lower the industry profit is.*

The union is interested in the industry wage bill which is the product of wage and industry total employment,

$$wQ = w\frac{N(A-rw)}{N+1}. \tag{17}$$

For the union, the direct effect of a higher wage is obvious. There is, however, a negative indirect effect which comes from the lower employment since a higher wage will have an adverse impact on hiring. If the union has all of the bargaining power, it will act like a monopolist in the labor market (under of course the constraint that the firms will at least be able to break even). Indeed, equation (17) resembles the "profit" function of a zero cost[10] monopoly facing a linear downward sloping demand function with the "optimal" wage being $A/2r$. In this case, a lower r (councils are stronger on average) translates into a higher labor demand at any given wage rate which seems to work in favor of the union. However, because stronger councils will reduce industry profit (Lemma 2), will it backfire for the union in the wage bargaining process? We will answer this question in the next sections.

5.2 Generalized Nash bargaining

As specified in (3), the firms and the union are to reach an agreement on the wage rate that maximizes the Generalized Nash product with α being the measure of the trade union's bargaining power relative to the firms,

$$G = (wQ)^{\alpha}\left[(p-w)Q\right]^{1-\alpha} = Qw^{\alpha}(p-w)^{1-\alpha} \tag{18}$$
$$= \frac{N(A-rw)w^{\alpha}\left[A-(N+1-Nr)w\right]^{1-\alpha}}{(N+1)^{2-\alpha}}. \tag{19}$$

Before deriving the bargaining solution, we emphasize the important observation that the composition of individual council power does not appear in the bargaining problem. Nevertheless,

[10] This is, of course, an artifact of fall-back payoff normalization.

it is a works council's co-determination power relative to those of other councils that determines the employment level of its host firm, see equation (10). The average council power determines the industry employment via the following chain. For any given wage, it influences industry output and industry profit. Ultimately, it influences the wage outcome in the bargaining process. After the wage has been determined, total output and total employment are determined. The distribution of the total employment over individual firms, however, is determined by the composition of individual council powers. While workers of different firms may exert direct influences on the union management, another way of demanding more jobs is to exercise co-determination rights at the firm level. We argue this might be an important function of works councils: *distributing employment within the union over the firms*.

Remark 3 *While for any given number of firms, market size and the union's wage bargaining power, only the average of council powers influences the wage outcome and the industry total employment, it is the composition of the council powers that determines the distribution of the total employment over individual firms.*

Remark 4 *Workers in individual firms by engaging in firm level co-determination demand a larger share of the industry total employment. The relative powers of individual councils dictate the distribution of industry total employment.*

5.3 The wage solution

In Appendix A.1, the wage bargaining outcome is derived by maximizing equation (19) with respect to wage. First, we scale the wage solution in equation (46) by the constant $4/A$. This is equivalent to setting A to 4. As has been noted before, market size in this endogenous wage determination setting is merely a scaling factor. Let

$$M := N + 1 - Nr,$$

and adjust $\tilde{\Delta}$ defined in equation (45) accordingly.

Proposition 3 *The solution to the wage bargaining problem (18) is*

$$w = \frac{1}{r} + \frac{1+\alpha}{M} - \sqrt{\Delta} \tag{20}$$

$$\text{where} \qquad \Delta = \left(\frac{1}{r} - \frac{1+\alpha}{M}\right)^2 + \frac{4\,(1-\alpha)}{rM}. \tag{21}$$

Proof. *See Appendix A.1.* ■

We briefly discuss a few points related to this wage solution. First and importantly, this wage solution is continuous because the component functions are continuous. It is also differentiable everywhere because for all parameters $\alpha \in (0,1)$ and $r \in (0,1]$, Δ is strictly larger than 0. If the boundaries of α were included, the wage solution would not have been differentiable when $\alpha = 1$ and $r = (N+1)/(N+2)$.[11] But when $\alpha = 1$, the bargaining problem is not well defined because the firms' fall-back option cannot be guaranteed.

Second, two parts in the solution stand out, $1/r$ and $1/M$. The first part is obviously related to the average council power and the second one is coming from firms' profit margin. As we have noted in Subsection 5.1, the ideal wage for the union without restrictions would be $A/2r = 2/r$. The firms, on the other hand, always want to have a lower wage to increase profits. The wage bargaining solution thus will naturally include these two ingredients.

Third, note that

$$\sqrt{\Delta} > \left| \frac{1}{r} - \frac{1+\alpha}{M} \right|,$$

and

$$\frac{1}{r} \geq \frac{1+\alpha}{M} \iff r \leq \frac{N+1}{N+1+\alpha}. \tag{22}$$

We can identify the following upper bound on the wage solution.

$$\begin{aligned} w &< \frac{1}{r} + \frac{1+\alpha}{M} - \left| \frac{1}{r} - \frac{1+\alpha}{M} \right| \\ &< 2 \min \left\{ \frac{1}{r}, \frac{1+\alpha}{M} \right\} \qquad (23) \\ &\leq 2 \frac{N+1+\alpha}{N+1}, \qquad (24) \end{aligned}$$

where in the last step we used (22) to find out the lower value in (23) and then applied (22) again. The result that $w < 2/r$ will prove useful in the analyses that follow.

The next three sections present our main results based on this wage solution.

6 Powers of labor: the union, and the councils in the limiting cases

The understanding of the impact of works councils' co-determination power on wage and industry employment requires an understanding of the effects that the union's bargaining power has on wage and employment. We will also investigate in detail the limiting case of $\alpha \to 1$, that is, when the union has almost all of the bargaining power. This analysis will prove helpful for

[11] Found by letting $\Delta = 0$.

the understanding of the general case studied in Section 7. This limiting case also deserves a special attention as we will see some effects that are present generally may vanish in this limit.

6.1 The effects of union power

The impact of α is easy to evaluate. First, Δ as in equation (21) is decreasing in α. This is because

$$\frac{d\Delta}{d\alpha} = -\frac{2\left[3\left(1-r\right)N + 3 - \left(1+\alpha\right)r\right]}{rM^2}$$

and $3 > (1+\alpha)\,r$ for $\alpha \in (0,1)$ and $r \in (0,1]$. Then, the wage solution (20) can be easily verified to be strictly increasing in α. Therefore, we know that the lower and the upper bound on wage, for a given r, are attained when $\alpha \to 0$ and 1 respectively. The wage solution then falls into the following range.

$$0 < w < \min\left\{\frac{2}{r}, \frac{4}{M}\right\} \leq \frac{2}{r}. \tag{25}$$

An increase in the union's bargaining power will increase the wage, $dw/d\alpha > 0$. Industry total employment, however, is decreasing in wage. Nevertheless, the union's interest, i.e., the industry wage bill should be increasing in wage bargaining power; otherwise, the union would not have asked for a higher wage. To gain some more intuition, we have a look at the derivative of the union payoff with respect to the wage for a fixed r:

$$\frac{d\left(wQ\right)}{dw} = N\frac{4-2rw}{N+1}$$

Since $w < 2/r$ for all $0 < \alpha < 1$, for any given r, an increase in wage strictly increases union payoff. Another way of understanding this is that the wage solution is always no higher than the union's ideal wage ($2/r$) for any given r. Therefore, although a higher wage causes a lower employment level, moving closer to the "monopoly" wage, the resulting job loss will be over compensated by the direct wage increase effect. The overall effect of the union's bargaining power therefore is positive:

$$\frac{d\left(wQ\right)}{d\alpha} = \frac{d\left(wQ\right)}{dw}\frac{dw}{d\alpha} > 0.$$

This is to say, to the full extent of its power, the union wants to raise the wage rate. Of course, it also means that an increase in firms' bargaining power will strictly reduce the wage.

Lemma 3 *Both wage and industry wage bill strictly increase in the union's wage bargaining power, α.*

Let us look closer to the special case of zero council power ($r = 1$). When $r = 1$, the model becomes a (collective wage bargaining) "Right To Manage" model. The wage in this case is simply

$$w = 2 + \alpha - \sqrt{\alpha^2 - 4\alpha + 4} = 2 + \alpha - (2 - \alpha) = 2\alpha.$$

Note if the union has all of the bargaining power ($\alpha \to 1$), a wage of 2 is the union's "monopoly price" of labor with product market price being $2(N+2)/(N+1)$ and total output or employment $2N/(N+1)$. Product market price is higher than the wage since the firms will mark up (double marginalization in Cournot competition). The mark up is decreasing in N. Because the union cannot fully internalize the revenue generated in the product market, the product price and the output are "inefficiently" high and low, respectively. Product market revenue maximization requires both product price and output to be 2.

With zero council power, the wage 2α is a convex combination of the union's ideal wage (which is 2) and the firms' ideal wage (which is 0) weighted by the measure of their relative bargaining power. In terms of the sum of their respective payoffs, when will this be an efficient solution for the two parties involved? The sum of the union payoff and industry profit, i.e., the entire revenue generated in the product market is maximized if the wage is set to $2(N-1)/N$, i.e., the wage that gives rise to the price of 2 in the product market. Therefore, if $\alpha = (N-1)/N$, the pie that the union and the firms are dividing is maximized. When N is large, this requires a powerful union. In the case of duopoly though, if the union and the firms are equally powerful, they indeed can maximize the pie and then divide it through a wage of 1. When $\alpha < (N-1)/N$, i.e., a less powerful union, product price will be lower than 2 meaning market supply will be larger than the product market monopoly output. When $\alpha > (N-1)/N$, product price will be higher than 2 as we know from the limiting case of $\alpha \to 1$. Hence, the "efficient" outcome in this RTM model requires a level of balance in the parties' relative bargaining power.

6.2 The impact of works councils when there is an upstream monopoly of labor

The analysis of works councils' co-determination power is much more involved. We start with the limiting case of $\alpha \to 1$. One point should be noted is that this is not the case wherein the union totally controls the firms as in vertical integration. Here we have a limiting case of the union's bargaining power such that in the limit the firms view the wage being unilaterally set by the union and as given. The implication is that the firms can always revert to their fall-back option if the wage is too high. The limiting case of the firms having all of the bargaining power is less illuminating since the wage will be set close to 0, the level to which the union's outside option has been normalized.

6.2.1 Wage solution

When $\alpha \to 1$,

$$\sqrt{\Delta} \to \left| \frac{1}{r} - \frac{2}{M} \right|.$$

To determine the wage, we need to differentiate two cases. First, if

$$r \leq \frac{N+1}{N+2} \Longrightarrow \frac{1}{r} \geq \frac{2}{M}, \tag{26}$$

the wage solution (20) in the limit of $\alpha \to 1$ approaches $4/M = 4/\left(N+1-Nr\right)$. It is straightforward to see that in this case the wage increases in r, meaning a weaker council, will increase the wage. Since condition (26) requires the councils to be sufficiently strong, this finding says overly powerful councils will indeed force the union to reduce the wage. The intuition is that when the councils are quite strong, firms will make losses if they produce at high wages. To keep them producing, the union cannot ask for its ideal wage $2/r$. Instead, it sets a wage that makes them just break even. To see this, note that the profit margin, $p - w$, when the wage is $4/M$ becomes zero.

Second, if $r > \left(N+1\right)/\left(N+2\right)$, the wage solution approaches $2/r$ in the limit of $\alpha \to 1$. Hence, the union obtains its ideal wage, that is, the solution to the "unconstrained" maximization problem of (17). This wage decreases in r which means when councils are sufficiently weak, an increase in councils' co-determination power will actually increase the wage. The rationale is as follows. With zero council power ($r = 1$), the wage $2/r = 2$ is optimal for the union. In this case, the firms will make a profit of $4N/\left(N+1\right)^2$. Even though the union could further slightly increase the wage without worrying about the firms making losses, this wage increase will be over weighted by the resulting reduction in employment. When council power is increased, however, the union would like to increase the wage ultimately because now labor demand elasticity is decreased. To see this,

$$-\frac{dQ}{dw}\frac{w}{Q} = \frac{rw}{A - rw} \tag{27}$$

which decreases when councils get stronger.

For all $r > \left(N+1\right)/\left(N+2\right)$, the wage $2/r$ is available. The product market price is always

$$p = \frac{A + Nrw}{N+1} = \frac{2\left(N+2\right)}{N+1}$$

which means the total employment stays at $2N/\left(N+1\right)$ and the revenue generated stays at $4N\left(N+2\right)/\left(N+1\right)^2$. Therefore, when the councils get stronger, the wage increases, the employment stays constant, the industry wage bill increases, the product market revenue stays constant and industry profit decreases. When r goes down to $\left(N+1\right)/\left(N+2\right)$, the wage rises to the product market price and the firms just break even. When r goes further below, $2/r$ becomes so high that the firms will not product under this wage. The union therefore, sets the wage to $4/M$.

Let us try it in another direction. Starting from r being close to 0 (really strong councils), when r increases, wage first increases because the wage that makes the firms break even increases. When r hits $\left(N+1\right)/\left(N+2\right)$, $2/r$ becomes feasible and from this point on the union opts for this option. With r continuing to rise, $2/r$ decreases, the wage decreases.

Lemma 4 *When $\alpha \to 1$, wage first increases then decreases in r. The switching point is $(N+1)/(N+2)$.*

6.2.2 Industry employment and union payoff

The union cares about employment in addition to the wage. As we have explained, when $r > (N+1)/(N+2)$ the total output or employment remains at $2N/(N+1)$. When $r \leq (N+1)/(N+2)$, however, the break even condition is binding and the industry employment level

$$Q = \frac{4N(1-r)}{N+1-Nr} \tag{28}$$

changes in r. The derivative of (28) reads $-4N/(N+1-Nr)^2$ which means employment will decrease in r, or will increase in councils' co-determination power.

Lemma 5 *When $\alpha \to 1$, industry employment decreases in r when $r \leq (N+1)/(N+2)$, and remains constant when $r > (N+1)/(N+2)$.*

We now check the union payoff which is measured by the industry wage bill. When the wage is $4/M$, i.e., when $r \leq (N+1)/(N+2)$, it is

$$wQ = \frac{16N(1-r)}{(N+1-Nr)^2},$$

with its derivative against r being

$$\frac{d(wQ)}{dr} = \frac{N16(N-1-Nr)}{(N+1-Nr)^3}. \tag{29}$$

Therefore, the industry wage bill is increasing in r if $r \leq (N-1)/N$. It means that when councils are strong enough such that $r \leq (N-1)/N$ more powerful councils will not only decrease the wage level, they also decrease the industry wage bill. On the one hand, this is surprising because councils' pursuit of workers' interests will actually harm the workers as a union. On the other hand, this is an example of negative externalities. By demanding more jobs, councils left the union no choice but to lower the wage since the latter has to keep the firms active. If the councils collectively restrain from exercising too much power of co-determination, the workers could have been better off as a union. In this case, council power backfires. When $(N-1)/N < r \leq (N+1)/(N+2)$, the RHS of (29) is negative which means the industry wage bill will decrease in r. In other words, in this region stronger councils will increase the industry wage bill even though the wage will decrease.

When the wage is $2/r$, i.e., when $r > (N+1)/(N+2)$, union payoff is $4N/[(N+1)r]$ which is decreasing in r so that for sufficiently weak councils, an increase in co-determination power will

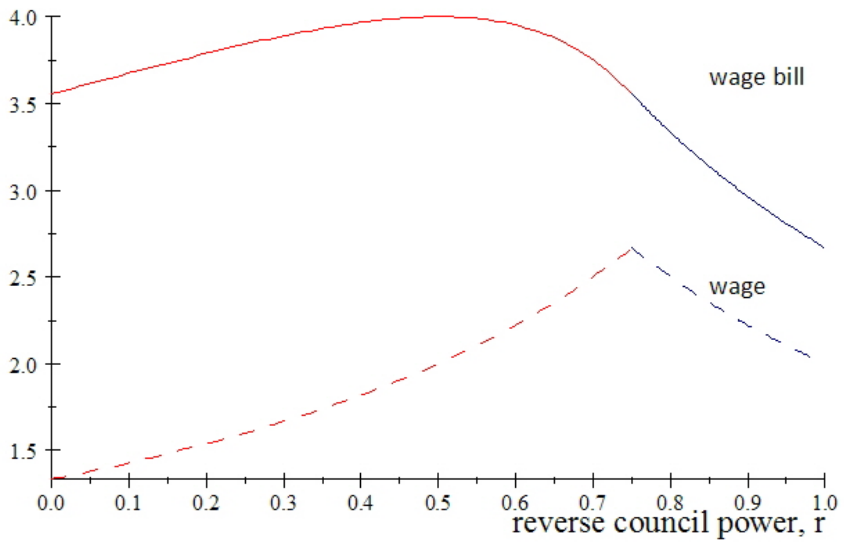

Figure 1: Wage and industry wage bill in the duopoly case when $\alpha \to 1$

increase workers' interest as a union. The intuition behind this result has been offered before. When councils are sufficiently weak, the union gets its ideal wage $2/r$ without worrying about the firms not being active. In this case, both the wage and the industry wage bill increase in council power. We summarize the above analysis in the following proposition.

Proposition 4 *When $\alpha \to 1$, more powerful councils will increase wage and union payoff when $r > (N+1)/(N+2)$; decrease wage but increase union payoff when $(N-1)/N < r \leq (N+1)/(N+2)$; decrease wage and union payoff when $r \leq (N-1)/N$.*

Figure 1 is a plot of the wage and the industry wage bill in the duopoly case. We can see wage (the dashed line) is increasing in council power when $r > (N+1)/(N+2) = 3/4$ and decreasing when $r \leq 3/4$. Industry wage bill (the solid line) is increasing in council power until r decreases to $(N-1)/N = 1/2$. There seems to be a kink in the wage graph at $r = 3/4$ resulting from the in-differentiability of the wage solution (20) at this point when $\alpha = 1$. Since we bounded α away from 1, we are not concerned about this.

Note also that both of the two threshold levels in Proposition 4 are increasing in N and approaching 1 when N goes to infinity. Therefore, when N is large, to have any positive impact on either wage or union payoff, r needs to be very close to 1.

Remark 5 *The more firms are there in the product market, the more likely works councils' co-determination power will be harmful to the workers as a union.*

6.2.3 Comments on vertical industrial relations

We have argued that firm-council co-determination resembles managerial incentive contracts in the strategic delegation literature. What we have presented in this limiting case of union bargaining power can also be interpreted as how an upstream monopoly could optimally set prices for its product when it faces N oligopolistic downstream firms who use this product as an input, especially when the firms in the downstream practice either delegation or co-determination. In particular, when sales related managerial incentives are very strong, the upstream monopoly cannot charge a very high price because then the downstream firms will not produce. When sales related incentives are low, the upstream firm might be able to take advantage of the downstream delegation practice and hence set higher prices and extract more end product market revenue from the downstream firms for the same arguments in this section.

7 Powers of labor: a full characterization

We now have understood councils' impact on wage and union payoff in the limiting case of full union power. The central theme is when councils are sufficiently weak an increase in council's co-determination power is beneficial to the workers as a union. When council are sufficiently strong, it would backfire. The main task in this section is to check if and how far we can generalize this insight for the general case of union power.

7.1 On wage determination

To this aim, we carry out a general analysis of wage behavior in Appendix A.2. Here goes the intuition. Recall that in Lemma 4 we show that in the case of $\alpha \to 1$, wage first increases then decreases in r. The turning point occurred at $(N+1)/(N+2)$. What happens if α is slightly lowered? In the zero council power case, $r=1$, the wage is 2α instead of 2. When r decreases, the same arguments in the case of $\alpha \to 1$ apply. When r gets smaller, the elasticity of labor demand (27) decreases and therefore, the union will be able to get a slightly higher wage. The union is able to do so, so long as r is sufficiently large, so that industry profit is no less than the firms' "fair" share according to their bargaining power. Therefore, the wage continues to increase until industry profit cannot be further lowered. After this point, wage decreases to compensate the firms. This line of arguments apply until α is sufficiently low such that industry profit when $r=1$ is already the firms' "fair" share. For an even lower α, industry profit becomes more important, and hence council power would not help to increase the wage.

Indeed, we have the following general result. Let

$$r_w := \frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{(1-\alpha)N}}, \tag{30}$$

where $N+1+\alpha-2\sqrt{(1-\alpha)N}$ can be verified to be positive.

Proposition 5 *Wage is increasing in council power when $r > r_w$, decreasing in council power when $r < r_w$.*

Proof. *See Appendix A.2.* ■

This result says when councils are sufficiently strong ($r < r_w$), decreasing council power will increase the wage. Otherwise, an increase in council power will increase the wage. Therefore, the highest wage is attained when $r = r_w$. Let us check if this condition admits the limit of $\alpha \to 1$ as a special case. In this case, r_w as in (30) is indeed approaching $(N+1)/(N+2)$. Then Proposition 5 says when $r < (N+1)/(N+2)$, wage is increasing in r; otherwise it is decreasing in r. We have obtained this result in Lemma 4.

In general, for the interval $(r_w, 1]$ to be non-empty we need the following condition. Since $N+1+\alpha-2\sqrt{(1-\alpha)N} > 0$,

$$\begin{aligned} & \frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{(1-\alpha)N}} < 1 \\ \iff \quad & \alpha > \frac{N-1}{N}. \end{aligned} \tag{31}$$

It is then evident that the union needs to be sufficiently strong for wage ever to increase in works councils' power. If $\alpha = (N-1)/N$, $r_w = 1$. So when $\alpha < (N-1)/N$ wage will always be decreasing in council power. When $\alpha > (N-1)/N$ wage first increases in council power but eventually it will decrease when r goes below r_w. We know wage will eventually decrease in council power because in Appendix A.3 we show that r_w is decreasing in α, and hence when $\alpha \to 1$, r_w approaches its infimum $(N+1)/(N+2)$. Therefore, for all α, the interval $(0, r_w]$ is non-empty.

Proposition 6 *When $\alpha > (N-1)/N$, the level of council power for the highest wage increases in the union's bargaining power, i.e., $dr_w/d\alpha < 0$. When $\alpha \leq (N-1)/N$, the highest wage is attained when $r = 1$.*

Proof. *See Appendix A.3.* ■

This means, provided that $\alpha > (N-1)/N$, when the union's bargaining power is increased, the switching point r_w will decrease. More specifically, starting from $r = 1$ and $\alpha = (N-1)/N$, when α gradually increases, r_w gradually decreases. When $\alpha \to 1$, r_w goes to $(N+1)/(N+2)$ not further below.

Proposition 6 is also interesting in its own right. It recovers that, union bargaining power and council co-determination power are in a way complements. Only when the union is sufficiently powerful, can it take advantage of a less elastic labor demand. Otherwise, councils by demanding more jobs will bring about lower wages. Moreover, the amount of council power that the union can utilize to increase the wage also depends on the union's own wage bargaining power. The more powerful the union is, the more council power the union can utilize to increase the wage. We will continue on this point when we analyze union payoff. Note, however, that as $N \to +\infty$, $\alpha \leq (N-1)/N$ must hold and hence under perfect competition the highest wage attainable comes with no council power at all.

7.2 On industry total employment

Now let us study industry total employment. In the limiting case of $\alpha \to 1$, employment increases in council power when the councils are sufficiently strong. For weak enough councils, the industry total employment remains constant. In this part, we argue that the constant part of the above finding is rather a special case of limiting behavior. In general, an increase in council power always increases industry total employment even when wage too is increasing.

The impact of council power on endogenous industry total output or employment is

$$\begin{aligned} \frac{dQ}{dr} &= \frac{\partial Q}{\partial r} + \frac{\partial Q}{\partial w}\frac{dw}{dr} \\ &= \frac{-N}{N+1}\left[w + r\frac{dw}{dr}\right]. \end{aligned}$$

For council power to increase total employment, we need

$$-dQ/dr > 0 \iff w + r\frac{dw}{dr} > 0 \iff -\frac{dw}{dr}\frac{r}{w} < 1.$$

Therefore, the council power elasticity of wage needs to be less than 1, i.e., inelastic. Indeed, this elasticity is less than 1 in general and we have the following result.

Proposition 7 *An increase in council power always increases industry total employment although in the limit of $\alpha \to 1$ when $r > (N+1)/(N+2)$ this positive effect vanishes.*

Proof. *See Appendix A.4.* ■

Propositions 5 and 7 together answer the question raised in the title. Stronger works councils will always lead to more jobs in the industry irrespective of their current co-determination power and the union's wage bargaining power. However, the impact on wage may not be monotonic. In fact, when the union has a sufficiently strong wage bargaining power and the councils originally are weak, an increase in council power can increase the wage.

7.3 On union payoff

We have identified the conditions under which an increase in council power will increase or decrease wage. We also know that an increase in council power will always increase output and employment. When councils are weak enough, $r > r_w$, both wage and employment are increasing in council power. But when they are strong, an increase in employment comes at the cost of a lower wage. To the union, it is then a question of which one of the two effects, namely the wage effect and the employment effect, dominates the other. We have seen in the special case of a very powerful union, when council power increases, union payoff keeps on rising after wage hits its maximum level until it is getting so low that employment increase is no longer justifiable.

We now have a look at the councils' impact on union payoff in general,

$$-\frac{d\left(wQ\right)}{dr} = \frac{N}{N+1}\left[-\left(4-2rw\right)\frac{dw}{dr}+w^2\right]. \tag{32}$$

An increase in council power will cause the wage to change as well as increase jobs whose impact has the magnitude of $w^2 N/\left(N+1\right)$ because jobs are gained at the rate of $wN/\left(N+1\right)$ and each job is valued for w.

When wage is increasing in council power, $dw/dr < 0$, the RHS of (32) is certainly positive. This is also because when $r > r_w$, both wage and employment increase in council power. When council power passes the threshold, for (32) to be positive, dw/dr cannot be too positive.

7.3.1 Analysis

Before we carry out the analysis, we establish the following identity.

$$\begin{aligned}
& w+\sqrt{\Delta} = \frac{1}{r}+\frac{1+\alpha}{M} \\
\Longleftrightarrow \quad & w^2+\Delta+2w\sqrt{\Delta} = \left(\frac{1}{r}+\frac{1+\alpha}{M}\right)^2 \\
\Longleftrightarrow \quad & w\sqrt{\Delta} = \frac{4\alpha}{rM}-\frac{1}{2}w^2.
\end{aligned} \tag{33}$$

We look for the condition for union payoff to increase in r.

$$\begin{aligned}
& \frac{d\left(wQ\right)}{dr} \geq 0 \iff \frac{dw}{dr} \geq \frac{w^2}{4-2rw} \iff \\
& \left(2-rw\right)\left[w\left(M-Nr\right)+2\left(N-1-\alpha\right)\right] \geq rM\left(w\sqrt{\Delta}\right) \iff
\end{aligned}$$

$$(3Nr - N - 1)\left(rw^2 - 4w\right) + 4wr(1+\alpha) + 8(N - 1 - \alpha) \geq 8\alpha \tag{34}$$

where dw/dr was substituted by (48) in Appendix A.2 and identity (33) was used to substitute out $\sqrt{\Delta}$.

Because of the similarity between the LHS of (34) and the first order condition (47) which can be written as

$$(N + 1 - Nr)\, rw^2 - 2w(N + 1 - Nr) - 2wr(1+\alpha) + 8\alpha = 0,$$

we add the FOC to (34) to obtain

$$(Nr + N + 1)\, rw^2 - 8Nrw + 8(N-1) \geq 0. \tag{35}$$

Condition (35) is a U shaped quadratic in w. If its discriminant is no larger than zero we then know that this condition will always hold, namely when

$$\begin{aligned} & (8Nr)^2 - 4(Nr + N + 1)\, r \cdot 8(N-1) \leq 0 \\ \iff \quad & r \leq \frac{N-1}{N}. \end{aligned}$$

This is of course only a sufficient condition for an increase in r to increase industry wage bill.

Lemma 6 *When $r \leq (N-1)/N$, union payoff increases in r.*

What happens when r is larger than $(N-1)/N$? Then condition (35) says we need w to be either less than the smaller root or larger than the larger root of (35). Note that condition (35) does not depend on α. Therefore, for a given r, whenever a wage is larger than the larger root, the wage when $\alpha \to 1$ should of course be larger than the larger root. This implies (35) is satisfied for this r when $\alpha \to 1$. However, we have seen in Subsection 6.2 that this is not true for all $r > (N-1)/N$. There we have concluded that union payoff will decrease in r when $r > (N-1)/N$. See Proposition 4. Hence, no wage could be larger than the larger root of (35).

On the other hand, if the wage is smaller than the smaller root of (35), denoted by w_u, then even if $r > (N-1)/N$, union payoff can be increasing in r. Condition (35) then reduces to

$$\begin{aligned} w \quad & \leq \quad \frac{4Nr - 2\sqrt{4N^2r^2 - 2(Nr + N + 1)\, r(N-1)}}{(Nr + N + 1)\, r} \\ & = \quad 2\left(\frac{2N}{Nr + N + 1} - \sqrt{\frac{(2N)^2}{(Nr + N + 1)^2} - \frac{2(N-1)}{(Nr + N + 1)\, r}}\right) \equiv w_u. \end{aligned} \tag{36}$$

Write out our wage solution,

$$\frac{1+\alpha}{N + 1 - Nr} - \sqrt{\left(\frac{1}{r} - \frac{1+\alpha}{N + 1 - Nr}\right)^2 + \frac{4(1-\alpha)}{r(N + 1 - Nr)}}$$

$$\leq \quad \frac{4N}{Nr+N+1} - 2\sqrt{\frac{4N^2}{(Nr+N+1)^2} - \frac{2(N-1)}{(Nr+N+1)r}} - \frac{1}{r}. \tag{37}$$

The LHS of (37) is increasing in α and we know that this condition will not be satisfied when α is large enough and $r > (N-1)/N$. On the other hand, when $\alpha \to 0$, the wage goes to 0 meaning condition (35) is satisfied. We, of course, can check this with respect to condition (36). The RHS of (36) is always larger than zero for $r > (N-1)/N$. This is to say for a really small α, for all r, an increase in r will increase union payoff.

In principle we can explicitly solve this inequality but the solution is not meaningful enough to be reported here. Instead, we first have a look at the special case of $r = 1$. In this case, condition (37) reduces to

$$\alpha \leq \frac{2N - \sqrt{2N+2}}{2N+1} =: \alpha^*, \tag{38}$$

where we have denoted the solution to the equality version of (37) in terms of α in the case of $r = 1$ by α^*. α^* can be verified to be less than $(N-1)/N$. Since when $\alpha \leq \alpha^* < (N-1)/N$, wage is increasing in r, a smaller r leads to a lower wage. Therefore, when $\alpha \leq \alpha^*$, condition (36) is satisfied for all $r \leq 1$.

7.3.2 The arguments for the general case

To understand condition (36) in general, further analyses are needed. Recall that wage is increasing in α and in r when $r < r_w$. We start at $r = (N-1)/N$. In this case all wages for α below 1 satisfy condition (36) because for the wage at $\alpha \to 1$ it is just binding. Now we make r slightly higher. In this case, the wage when $\alpha \to 1$ is larger than w_u. On the other hand, when $\alpha \leq \alpha^*$, wage is low and condition (36) is satisfied. By continuity and monotonicity, there is one and only one $\alpha \in [\alpha^*, 1)$ such that at this α the condition changes state. Above (below) it, union payoff is decreasing (increasing) in r.

Now we switch our perspective. For a given α, we look at the gap between the wage solution and w_u, $w_u - w$. First, in the appendix we show $dw_u/dr < 0$.

Lemma 7 *The smaller root of (35) is decreasing in r, i.e., $dw_u/dr < 0$.*

Proof. *See Appendix A.5.* ■

When $r = (N-1)/N$, condition (36) holds, i.e. $w \leq w_u$. We now differentiate three cases according to a.

1. When $\alpha > (N-1)/N$, at r_w, condition (36) cannot hold, i.e., $w > w_u$. When $r < r_w$ wage is increasing in r, $d(w_u - w)/dr < 0$. Because $(w_u - w)$ is continues and monotonic in r when $r \leq r_w$, we can find the unique solution to $w_u = w$ and denote it as $r^*(\alpha)$. We know $r^*(\alpha) < r_w$.

2. When $\alpha^* < \alpha \leq (N-1)/N$, at $r = 1$ we see from (38) $w > w_u$. Since in this case wage is also increasing in r, $d(w_u - w)/dr < 0$. Again, because $(w_u - w)$ is continues and monotonic in r we can find the unique solution to $w_u = w$. Therefore $r^*(\alpha)$ continuous in this case and we know $r^*(\alpha) < 1$.

3. When $\alpha \leq \alpha^*$, we know condition (36) is satisfied for all $r \leq 1$. Let $r^*(\alpha) = 1$ in this case.

We name the point where r hits $r^*(\alpha)$ as $(r^*(\alpha), \alpha)$. Because whenever $r < r^*(\alpha)$, wage is increasing in r, all points with a smaller r and a smaller α will satisfy condition (36). In other words, the set of points that satisfy condition (36) in $(0,1) \times (0,1]$ is comprehensive. Therefore, α^* as identified in the $r = 1$ case is the highest α that sees $r^*(\alpha)$ being constrained by 1. If not, the assumption that the wage at some $\alpha > \alpha^*$ and $r = 1$ satisfies condition (36) implies the wage at $\alpha = \alpha^*$ and $r = 1$ strictly satisfy condition (36). It thus contradicts with the fact we found α^* by the equality version of (37).

Although $r^*(\alpha)$ is only implicitly given by the solution to the equality version of (37), we do know it is decreasing in α when $\alpha > \alpha^*$. This is because wage is strictly increasing in α, and in r when $r < r_w$ or $\alpha < (N-1)/N$. Meanwhile, w_u is strictly decreasing in r. Start from one $(r^*(\alpha), \alpha)$, say in the case of $\alpha \to 1$, $r^*(\alpha) = (N-1)/N$. When r is slightly increased, the wage is larger than w_u at this r. If we move α downwards, it will meet with w_u at the α that will make $r^*(a)$ equal this slightly increased r. This argument continues until $r^*(a)$ converges to $r^*(a^*)$ which is 1.

Indeed, the derivatives of the two sides of $w = w_u$ are as follows:

$$\begin{aligned} & \frac{dw}{d\alpha}\frac{d\alpha}{dr} + \frac{dw}{dr} \equiv \frac{dw_u}{dr} \\ \iff \quad & \underbrace{\frac{dw}{d\alpha}}_{+}\frac{d\alpha}{dr} = \underbrace{\frac{dw_u}{dr}}_{-} - \underbrace{\frac{dw}{dr}}_{+}. \end{aligned}$$

The sign of dw/dr holds because either $r^*(\alpha) < r_w$ or $\alpha < (N-1)/N$. It is then apparent that $d\alpha/dr^*(a) < 0$, and hence $dr^*(a)/d\alpha < 0$, provided that $\alpha > \alpha^*$.

Proposition 8 *When $\alpha > \alpha^*$, the level of council power optimal for the workers as a union increases in the union's bargaining power, i.e., $dr^*(a)/d\alpha < 0$. When $\alpha \leq \alpha^*$, it is best for the workers as a union to have zero co-determination power.*

We continue our discussion on the complementarity of a union's wage bargaining power and councils' co-determination power. The locus given by $(r^*(\alpha), \alpha)$ which includes the lower part of the $r = 1$ line ($\alpha \leq \alpha^*$) indicates the best "choice" of council power for the workers as a union for any given level of wage bargaining power. When the union is weak, councils' quest for more jobs can only harm the workers as a union because when the firms are more or less in control, they can secure their share of profit. When the union has a sufficiently high bargaining power, some level of council power can be beneficial for the workers. Moreover, the more powerful the union is, the more council power it can take advantage of. Therefore, our discussion on the complementarity of wage bargaining power and co-determination power in wage determination also applies when union payoff is concerned.

7.4 General results

We summarize our results and provide explanations for them in this subsection.

When $\alpha \leq \alpha^*$, we now know that both wage and union payoff always increase in r.

When $\alpha^* < \alpha \leq (N-1)/N$, wage always increases in r. However, somewhere before 1, the unique switching point, $(r^*(\alpha), \alpha)$, appears and union payoff starts to decrease in r. From $r^*(\alpha)$ to 1, even though wage is increasing, the negative effect of employment loss is too high so that union payoff decreases. When $\alpha \to \alpha^*$, $r^*(\alpha)$ converges to 1 .

When $\alpha > (N-1)/N$, there will be cases wherein the wage is decreasing in r, namely when $r > r_w$. At that point, the switching point of union payoff, $(r^*(\alpha), \alpha)$, has already appeared. Therefore, before $r^*(\alpha)$ both wage and union payoff increase in r. From $r^*(\alpha)$ to r_w, wage is increasing but union payoff is decreasing in r. After r_w, both wage and union payoff decrease in r. When $\alpha \to 1$, $r^*(\alpha)$ converges to $(N-1)/N$.

We provide the full characterization of wage and union payoff outcome in Proposition 9. All results are derived for increasing r meaning decreasing council power while holding α and N constant.

Proposition 9 *A decrease in council power (an increase in r) 1) increases both wage and union payoff when* $0 < r \leq r^*(\alpha)$*, 2) increases wage but decreases union payoff when* $r^*(\alpha) < r \leq r_w$*, 3) decreases both wage and union payoff when* $r_w < r < 1$*, where* $r^*(\alpha)$ *is given by the equality version of (37), and when* $0 < \alpha \leq \alpha^*$ *case 2 and 3 do not exist, when* $\alpha^* < \alpha \leq (N-1)/N$ *case 3 does not exist, when* $(N-1)/N < \alpha < 1$ *all three cases apply. This characterization of wage and union payoff behavior is reinterpreted in terms of an increase in council power in Table 1.*

Figure 2 is a graphical representation for the duopoly case where $r^*(\alpha)$ has been plotted using the implicit function $w = w_u$. We have also presented the results in terms of increasing council power. In the upper right corner, the union is powerful in wage bargaining and the councils are relatively weak. In this area we see both wage and wage bill are increasing in council power. The large area of the lower left part is where either the union's bargaining power is weak or the councils are strong. In this area, an increase in council power would decrease both the wage and the wage bill. In the strip in between, wage is decreasing while wage bill is increasing. When the number of firms increases, all threshold levels of α and r converges to 1 meaning the the

The impact of council power on wage and union payoff			
	$0 < r \leq r^*(\alpha)$	$r^*(\alpha) < r \leq \frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{(1-\alpha)N}}$	$\frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{(1-\alpha)N}} < r < 1$
$\frac{N-1}{N} < \alpha < 1$	*Wage* $\downarrow$, *Union Payoff* $\downarrow$	*Wage* $\downarrow$, *Union Payoff* $\uparrow$	*Wage* $\uparrow$, *Union Payoff* $\uparrow$
$\frac{2N-\sqrt{2N+2}}{2N+1} < \alpha \leq \frac{N-1}{N}$	*Wage* $\downarrow$, *Union Payoff* $\downarrow$	*Wage* $\downarrow$, *Union Payoff* $\uparrow$	/
$0 < \alpha \leq \frac{2N-\sqrt{2N+2}}{2N+1}$	*Wage* $\downarrow$, *Union Payoff* $\downarrow$	/	/

Table 1: Impact on wage and union payoff when council power decreases

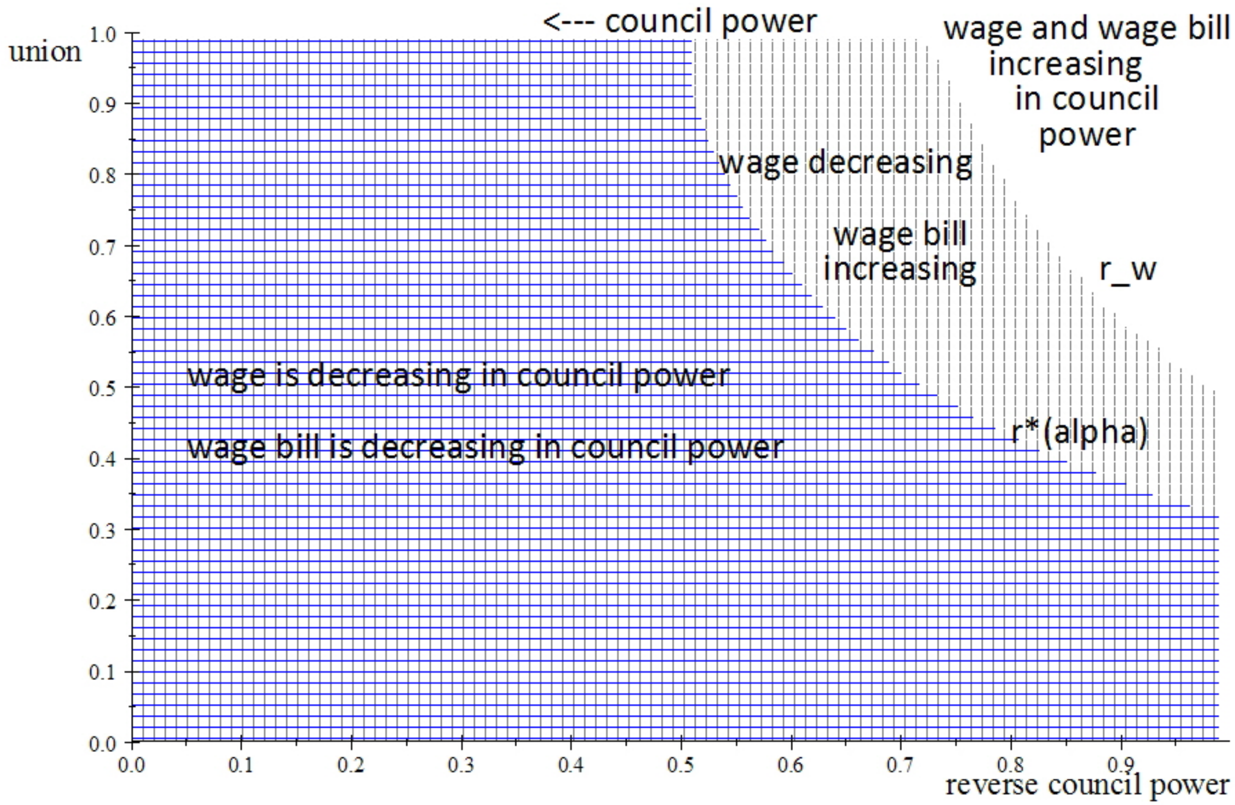

Figure 2: Wage and wage bill outcomes different powers of labor

large area of the lower left part will progress. In the limit of $N \to \infty$, council power could only decrease the wage and harm the workers. Therefore, Remark 5 applies in general.

8 Works councils and industry profitability

Important to the industrial relations research is the impact of works councils on firm/industry profitability. While previous studies have approached this question with exogenous costs, in particular wage costs, with the help of the current model we can provide additional answers with endogenously determined wages. In Section 5.1 we learned that for any given feasible wage, industry profit will decrease in council power because strong councils make product market more competitive. Now with the wage endogenously negotiated, we have to consider co-determination power's effect on wage.

In the limiting case of $\alpha \to 1$, we explained that the industry zero profit condition is binding when councils are strong. This is case when the union has to lower the wage to keep the firms active. When the councils are sufficiently week, i.e., $r > (N+1)/(N+2)$, the union obtains its ideal wage $2/r$ and allows the firms to make some profits. The firms benefit from an increased profit margin when the union demands $2/r$ for wage.

In general, industry total profit is

$$\begin{aligned} \Pi &= (p-w)\,Q = \frac{N\,(A-Mw)\,(A-rw)}{(N+1)^2} \\ &= \frac{N}{(N+1)^2}\left[16-4\,(M+r)+rMw^2\right]. \end{aligned}$$

After rewriting and substituting the rMw^2 part by the FOC (47) we have

$$\frac{(N+1)^2}{2N}\Pi = 8-4\alpha-w\,(N+1-Nr-\alpha r+r)\,, \tag{39}$$

which means the derivative $d\Pi/dr$ is

$$\frac{(N+1)^2}{2N}\frac{d\Pi}{dr} = -\left[N\,(1-r)+(1-\alpha r)+r\right]\frac{dw}{dr}+w\,(N+\alpha-1)\,. \tag{40}$$

Now we know that $dw/dr < 0$ when $r > r_w$. In this case, the RHS of (40) is necessarily positive. We therefore have the following result.

Lemma 8 *When $r > r_w$, industry profit increases in r.*

When $r < r_w$, $dw/dr > 0$ so the RHS of (40) might be negative. In the limiting case of $\alpha \to 1$, however, industry profit remains zero for all $r < r_w$. On the other extreme, when $\alpha \to 0$, the wage will tend to be 0 so councils will hardly have any influence on industry profitability. In this case, firms compete with zero marginal cost. Product market price and industry total output are $A/(N+1)$ and $NA/(N+1)$ respectively.

In general, Proposition 7 states endogenous output and employment always increase in council power. One way of investigating industry profit would be to link it to total output. To this end, first note that

$$\begin{aligned} \Pi &= (p-w)\,Q = \frac{A-Mw}{N+1}Q, \\ Q &= \frac{N}{N+1}\,(A-rw)\,. \end{aligned}$$

We can then use these relations to back out Mw and rw. Equation (39), therefore, can be transformed as follows.

$$\begin{aligned} \frac{(N+1)^2}{2N}\Pi &= 8-4\alpha-wM-wr\,(1-\alpha) \\ &= 8-4\alpha-\left(4-\frac{(N+1)\,\Pi}{Q}\right)-(1-\alpha)\left(4-\frac{(N+1)\,Q}{N}\right) \\ &= \frac{(N+1)\,\Pi}{Q}+(1-\alpha)\,\frac{(N+1)\,Q}{N}. \end{aligned}$$

We write out Π in Q.

$$\begin{aligned} & \frac{N+1}{2N}\Pi = \frac{\Pi}{Q}+(1-\alpha)\,\frac{Q}{N} \\ \iff \quad & \Pi = \frac{2\,(1-\alpha)\,Q^2}{(N+1)\,Q-2N}. \end{aligned} \tag{41}$$

The derivative of Π w.r.t. r then is

$$\frac{d\Pi}{dr} = \frac{dQ}{dr}\left\{[(N+1)\,Q-4N]\,\frac{2\,(1-\alpha)\,Q}{[(N+1)\,Q-2N]^2}\right\}. \tag{42}$$

We, however, know that $(N+1)\,Q-4N<0$ because $Q=N\,(A-rw)\,/\,(N+1)$ and

$$N\,(4-rw)-4N<0$$

holds obviously.

According to Proposition 7, $dQ/dr<0$, hence $d\Pi/dr>0$ always holds. We should, however, be cautious in the limit case of $\alpha\to 1$ because equation (42) contains the part $(1-\alpha)$. We discuss this case separately. When $\alpha\to 1$ and when $r>(N+1)\,/\,(N+2)$, Q does not change in r in the limit. The profit margin, however, is increasing in r. For $r\leq (N+1)\,/\,(N+2)$, Q decreases in r but the profit margin remains 0. So industry profit is at least non-decreasing in r in this case.

We generalize Lemma 8 to the following Proposition.

Proposition 10 *Industry profit always increases in r although when $r < (N+1)/(N+2)$ in the limit of $\alpha \to 1$ this effect vanishes.*

In consequence, even when wage is endogenously negotiated, strong councils will always decrease industry profit. On the other hand, as we have discussed in Subsection 4.2, a firm with a strong council makes a higher profit than a firm with a weak council because of its higher output in equilibrium. Thus, a firm with a strong council has a relative advantage over those with weak councils.

9 Conclusion

In this paper we have introduced a two-stage union-oligopoly-council model of wage and employment determination wherein at the first stage wage is negotiated through collective bargaining and at the second stage employment in each firm is co-determined by the employer and its works council. It differs from an "efficient bargaining" model in that wage bargaining precedes employment determination. It differs from a "Right-To-Manage" model in that firm level employment is co-determined.

Although works councils increase employment for any given wage, whether the workers as a union are able to take advantage of it depends critically on the union's wage bargaining power. When the union's bargaining power is sufficiently strong, it is able to negotiate a higher wage with the help of some council power. In this case, workers enjoy both a higher wage and more jobs. When the union is weak, however, councils' quest for more jobs will backfire in the wage bargaining stage and will result in a lower wage. Workers as a union will be worse off even though they have more jobs. In between, there also exists an interval of wage bargaining power where wage decreases in council power but industry wage bill may increase. This is the case when the positive effect of more jobs dominates the negative effect of a lower wage. In all cases, overly powerful works councils could never help the workers as a union because of their strong adverse impact on wage. The beneficial effects of works councils also depend on market structure. The more firms are there in the market, the less likely are these beneficial effects.

From the perspective of workers in individual firms, engaging in co-determination and demanding more jobs are justifiable. This is because at the second stage wage is fixed. Co-determination helps to gain more jobs for their own firm at the cost of workers in other firms. In equilibrium, firms with stronger councils hire more and produce more. This positive impact on firm size allows these firms to make more profits than others. Therefore, in equilibrium, there is a positive correlation between council power and firm profit across firms within the same industry. This correlation is likely to disappear once firm size is controlled for. This advantage of a stronger council is only in relative terms, however. A stronger council does not necessarily maximize its host firm's profit. Moreover, it is found that the stronger the councils are, the lower industry profit will be. Therefore, after controlling for market conditions, there is a negative correlation between average council power and industry profit across different industries.

A Appendix

A.1 Derivation of the wage solution

In the following, we search for the solution to this problem

$$\tilde{G} = \max_{w} (A - rw) w^{\alpha} [A - (N + 1 - Nr) w]^{1-\alpha}.$$

The first order condition (FOC)[12] reads

$$2r (N + 1 - Nr) w^2 - [(N + 1 - Nr) + (1 + \alpha) r] Aw + \alpha A^2 = 0. \tag{43}$$

As a first step, we have a look at the (modified) discriminant of this quadratic,

$$\begin{aligned} \tilde{\Delta} &= \frac{[(N + 1 - Nr) A + (1 + \alpha) rA]^2 - 4 \cdot 2r (N + 1 - Nr) \alpha A^2}{A^2} &(44) \\ &= [(N + 1 - Nr) - (1 + \alpha) r]^2 + 4 (1 - \alpha) r (N + 1 - Nr). &(45) \end{aligned}$$

Since $\alpha \in (0, 1)$, $r \in (0, 1]$ and $N + 1 > Nr$, $\tilde{\Delta}$ is clearly strictly positive. Should the first part of (45) be zero, there still is the strictly positive second part. This proves the existence of real roots to the first order condition. The solution to the quadratic (43) is

$$\begin{aligned} w &= A \frac{(N + 1 - Nr) + (1 + \alpha) r \pm \sqrt{\tilde{\Delta}}}{4r (N + 1 - Nr)} \\ &= \frac{A}{4} \left[\frac{1}{r} + \frac{1 + \alpha}{N + 1 - Nr} \pm \frac{\sqrt{\tilde{\Delta}}}{r (N + 1 - Nr)} \right]. \end{aligned}$$

To identify which of these two roots is our solution, we check the second order derivative of the maximization problem evaluated at the two roots. After some simplification,

$$\frac{d^2 \tilde{G}}{dw^2} |_{FOC=0} = \frac{w^{\alpha-1} \left\{ - [NA + (\alpha - 3) Nrw] - (1 + \alpha) Nr \left[\frac{A}{N+1-Nr} - w \right] \right\}}{(N + 1 - Nr) [A - (N + 1 - Nr) w]^{\alpha}}.$$

[12] Since this is a Generalized Nash bargaining problem, the second order condition is implied.

We want to identify the condition for a negative second order derivative,

$$\begin{aligned}
& \frac{d^2\tilde{G}}{dw^2}\,|_{FOC=0} < 0 \\
\iff \quad & [NA + (\alpha - 3)\,Nrw] + (1+\alpha)\,Nr\left[\frac{A}{N+1-Nr} - w\right] > 0 \\
\iff \quad & w < \frac{A}{4}\left[\frac{1}{r} + \frac{1+\alpha}{N+1-Nr}\right].
\end{aligned}$$

It turns out that the solution to the maximization problem, i.e., the wage determined in the collective bargaining, is the smaller root,

$$w = \frac{A}{4}\left[\frac{1}{r} + \frac{1+\alpha}{N+1-Nr} - \frac{\sqrt{\tilde{\Delta}}}{r\,(N+1-Nr)}\right] \tag{46}$$

where $\tilde{\Delta}$ is the discriminant defined in (45).

A.2 Council power's impact on the wage solution

Instead of directly investigating the wage solution, we start from the first order condition of the Nash wage bargaining. After new notations are incorporated, the FOC reads,

$$rMw^2 - 2\,[M + (1+\alpha)\,r]\,w + 8\alpha = 0. \tag{47}$$

Take total derivative of the LHS of the FOC,

$$\begin{aligned}
\frac{d\,(FOC)}{dr} &= \frac{\partial FOC}{\partial r} + \frac{\partial FOC}{\partial M}\frac{dM}{dr} + \frac{\partial FOC}{\partial w}\frac{dw}{dr} \\
&= \left(Mw^2 - 2\,(1+\alpha)\,w\right) - N\left(rw^2 - 2w\right) \\
&\quad + \frac{dw}{dr}\left(2rMw - 2M - 2\,(1+\alpha)\,r\right).
\end{aligned}$$

Set this total derivative to zero, dw/dr can be solved and simplified to

$$\begin{aligned}
\frac{dw}{dr} &= \frac{Mw^2 - 2\,(1+\alpha)\,w - Nrw^2 + 2Nw}{2M + 2\,(1+\alpha)\,r - 2rMw} \\
&= \frac{w^2\,(M - Nr) + 2w\,(N - 1 - \alpha)}{2\,[M + (1+\alpha)\,r] - 2rMw}.
\end{aligned} \tag{48}$$

We first investigate the denominator.

$$2\,[M + (1+\alpha)\,r] - 2rMw$$

$$
\begin{aligned}
&= \quad 2Mr\left[\frac{1}{r}+\frac{1+\alpha}{M}-w\right] &(49)\\
&= \quad 2Mr\sqrt{\Delta}>0. &(50)
\end{aligned}
$$

From equation (49) to (50), we have used the fact that we are interested in the smaller root of (47). The sign of dw/dr, therefore, will be the same as the numerator of (48).

We proceed to identify the parameter values that would give a positive impact of council power on the wage rate, namely $dw/dr<0$,

$$
\begin{aligned}
& w^2(M-Nr)+2w(N-1-\alpha)<0\\
\Longleftrightarrow\quad & \frac{M-Nr}{2}\left[\frac{1}{r}+\frac{1+\alpha}{M}-\sqrt{\Delta}\right]+(N-1-\alpha)<0\\
\Longleftrightarrow\quad & (M-Nr)\left[\frac{1}{r}+\frac{1+\alpha}{M}\right]+2(N-1-\alpha)<(M-Nr)\sqrt{\Delta}. \qquad (51)
\end{aligned}
$$

Because we do not know the sign of either side of (51), we have to discuss two possible cases.

Case 1: when councils are strong,

$$
r\leq\frac{N+1}{2N}\iff M-Nr\geq 0.
$$

In this case, both sides of (51) are non-negative. Continue from inequality (51),

$$
\begin{aligned}
\Longleftrightarrow\quad & \left[\frac{(M+Nr)\left[M-(1+\alpha)r\right]}{rM}\right]^2<\left[(M-Nr)\sqrt{\Delta}\right]^2\\
\Longleftrightarrow\quad & \left[M-(1+\alpha)r\right]^2<(M-Nr)^2\,\frac{1-\alpha}{N}.
\end{aligned}
$$

Since $r\leq\frac{N+1}{2N}<\frac{N+1}{N+1+\alpha}\implies M-(1+\alpha)r>0$, the above condition is

$$
\begin{aligned}
\Longleftrightarrow\quad & M-(1+\alpha)r<(M-Nr)\sqrt{\frac{1-\alpha}{N}}\\
\Longleftrightarrow\quad & r>\frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{N(1-\alpha)}}=:r_w. \qquad (52)
\end{aligned}
$$

Because r_w and the case condition $(N+1)/2N$ has the following relation

$$
\begin{aligned}
& \frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{\left(N+1+\alpha-2\sqrt{N(1-\alpha)}\right)}\;\frac{2N}{N+1}\\
= \quad & \frac{N+N-2\sqrt{N(1-\alpha)}}{N+1+\alpha-2\sqrt{N(1-\alpha)}}\;>1, \qquad (53)
\end{aligned}
$$

r_w is larger than or equal to $(N+1)/2N$. Therefore, these two intervals do not cross each other. In consequence, when $r \leq (N+1)/2N$, then $r < r_w$, and hence wage is increasing in r, $dw/dr > 0$.

Case 2: when councils are weak,

$$r > \frac{N+1}{2N} \iff M - Nr < 0.$$

In this case, both sides of (51) need to be negative. From (51) we would need

$$\frac{(M+Nr)(M-(1+\alpha)r)}{rM} < (M-Nr)\sqrt{\Delta}$$

which asks for the necessary condition

$$M - (1+\alpha)r < 0 \iff r > \frac{N+1}{N+1+\alpha}. \tag{54}$$

The above condition continues,

$$\begin{aligned}
&\iff \left[\frac{(M+Nr)[M-(1+\alpha)r]}{rM}\right]^2 > \left[(M-Nr)\sqrt{\Delta}\right]^2 \\
&\iff [(1+\alpha)r - M]^2 > (Nr-M)^2 \frac{1-\alpha}{N} \\
&\iff (1+\alpha)r - M > (Nr-M)\sqrt{\frac{1-\alpha}{N}} \\
&\iff r > \frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{(1-\alpha)N}} = r_w. \qquad (55)
\end{aligned}$$

We now show that r_w is larger than $(N+1)/(N+1+\alpha)$ so that when condition (55) is satisfied, the necessary condition (54) is automatically satisfied. This is the case because the following holds. First, for the use in (57), note that

$$\begin{aligned}
&1 - \frac{2\sqrt{(1-\alpha)N}}{N+1+\alpha} > 0 \qquad (56) \\
\iff\ &[N+(3\alpha-1)]^2 + 8\alpha(1-\alpha) > 0.
\end{aligned}$$

Hence, the quotient of r_w and $(N+1)/(N+1+\alpha)$ is

$$\begin{aligned}
&\frac{(N+1)\left(1-\sqrt{\frac{1-\alpha}{N}}\right)}{N+1+\alpha-2\sqrt{(1-\alpha)N}}\frac{N+1+\alpha}{N+1} > 1 \\
\iff\ &\frac{1-\frac{1}{N}\sqrt{N(1-\alpha)}}{1-\frac{2\sqrt{(1-\alpha)N}}{N+1+\alpha}} > 1 \qquad (57)
\end{aligned}$$

$$
\begin{aligned}
&\iff \quad \frac{1}{N}\sqrt{N\left(1-\alpha\right)} < \frac{2}{N+1+\alpha}\sqrt{\left(1-\alpha\right)N} \\
&\iff \quad 1+\alpha < N.
\end{aligned}
$$

The last step holds trivially.

Note also that r_w is larger than $\left(N+1\right)/2N$ so the case condition is satisfied too. Hence, we only need condition (55) to hold. It is also easy to verify that when $r < r_w$, $dw/dr > 0$ and that only when $r = r_w$, $dw/dr = 0$.

In conclusion, wage is decreasing in r when $r > r_w$; otherwise, the wage is increasing in r.

A.3 Proof of r_w being decreasing in α

In this part, we provide the proof for the result of r_w being decreasing in α. After noticing directly taking derivative of r_w against α can be rather complicated, we perform the log transformation of r_w,

$$
\log r_w = \log\left(N+1\right) + \log\left(1-\sqrt{\frac{1-\alpha}{N}}\right) - \log\left[N+1+\alpha-2\sqrt{N\left(1-\alpha\right)}\right].
$$

The derivative of $\log r_w$ is

$$
\begin{aligned}
\frac{d\left(\log r_w\right)}{d\alpha} &= \frac{\frac{1}{2N}\sqrt{\frac{N}{1-\alpha}}}{1-\sqrt{\frac{1-\alpha}{N}}} - \frac{1+\sqrt{\frac{N}{1-\alpha}}}{N+1+\alpha-2\sqrt{N\left(1-\alpha\right)}} \\
&= \frac{\frac{1}{2N}\sqrt{\frac{N}{1-\alpha}}}{1-\sqrt{\frac{1-\alpha}{N}}} - \frac{\frac{r_w}{N+1}\left(1+\sqrt{\frac{N}{1-\alpha}}\right)}{1-\sqrt{\frac{1-\alpha}{N}}} \\
&= \frac{\left(\frac{1}{2N}-\frac{r_w}{N+1}\right)\sqrt{\frac{N}{1-\alpha}} - \frac{r_w}{N+1}}{1-\sqrt{\frac{1-\alpha}{N}}}.
\end{aligned}
$$

A sufficient condition for $d\left(\log r_w\right)/d\alpha < 0$ is

$$
\begin{aligned}
&\frac{1}{2N} - \frac{r_w}{N+1} < 0 \\
\iff \quad & r_w > \frac{N+1}{2N}.
\end{aligned}
$$

This condition, however, has been established in inequality (53). This proves $dr_w/d\alpha < 0$.

A.4 Council power's impact on industry total employment

In this part, we prove

$$-\frac{dw}{dr}\frac{r}{w} < 1.$$

From (48) and (50), we have

$$\begin{aligned}
& -\frac{w^2 (M - Nr) + 2w (N - 1 - \alpha)}{2\left[M + (1 + \alpha) r\right] - 2rMw}\frac{r}{w} < 1 \\
\iff \quad & -\frac{w (M - Nr) + 2 (N - 1 - \alpha)}{2M\sqrt{\Delta}} < 1 \\
\iff \quad & \frac{M}{r} + (N + 1)\sqrt{\Delta} - Nr\left(\sqrt{\Delta} + w\right) + 2N - (1 + \alpha) > 0 \\
\iff \quad & \frac{M}{r} + (N + 1)\sqrt{\Delta} - \frac{1 + \alpha}{M} Nr + N - (1 + \alpha) > 0 \\
\iff \quad & \frac{M}{r} + N + (N + 1)\left(\frac{1}{r} - w\right) > 0 \\
\iff \quad & \frac{2}{r} > w.
\end{aligned}$$

In the derivation, we used the wage solution several times. The condition in the last step holds strictly when $\alpha < 1$ according to the upper bound derived in (25). Therefore, an increase in council power always increases total employment.

A.5 Proof of Lemma 7

In this part, we show $dw_u/dr < 0$. To this aim, we start from the quadratic (35). Denote

$$S := (Nr + N + 1) rw^2 - 8Nrw + 8 (N - 1).$$

Recall that w_u is the smaller root of this quadratic. Take the total derivative with respect to r and set it to zero,

$$\begin{aligned}
& \frac{dS}{dr} = \frac{\partial S}{\partial r} + \frac{\partial S}{\partial w_u}\frac{dw_u}{dr} \equiv 0 \\
\iff \quad & \frac{dw_u}{dr} = \frac{(2Nr + N + 1) w_u^2 - 8Nw_u}{8Nr - 2rw_u (Nr + N + 1)}.
\end{aligned} \tag{58}$$

Since we are interested in the smaller root,

$$8Nr - 2rw_u (Nr + N + 1) = \sqrt{64N^2r^2 - 32 (N - 1) (Nr + N + 1) r}$$

where

$$64N^2r^2 - 32 (N - 1) (Nr + N + 1) r$$

is strictly larger than zero when $N > (N-1)/N$. So the sign of dw_u/dr is the same as the numerator in (58). Because w_u is always larger than zero we left out one w_u.

$$
\begin{aligned}
& (2Nr + N + 1)\, w_u - 8N < 0 \\
\iff\quad & w_u < \frac{8N}{2Nr + N + 1} \\
\iff\quad & \frac{4N}{Nr + N + 1} - 2\sqrt{\frac{4N^2}{(Nr + N + 1)^2} - \frac{2(N-1)}{(Nr + N + 1)\, r}} < \frac{8N}{2Nr + N + 1} \\
\iff\quad & \frac{2N(2Nr + N + 1)}{Nr + N + 1} - 4N < (2Nr + N + 1)\sqrt{\frac{4N^2}{(Nr + N + 1)^2} - \frac{2(N-1)}{(Nr + N + 1)\, r}} \\
\iff\quad & \frac{-2N(N+1)}{Nr + N + 1} < (2Nr + N + 1)\sqrt{\frac{4N^2}{(Nr + N + 1)^2} - \frac{2(N-1)}{(Nr + N + 1)\, r}}.
\end{aligned}
$$

The last step holds trivially. We, therefore, conclude dw_u/dr is strictly less than zero.

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