Essays on Industrial and Societal Organization: Certification, Variety and Concern for Face

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Abstract

In this dissertation I report three doctoral research projects: the application of imperfect certification in markets with asymmetric information, the impact of elastic demand on market supplied product variety in differentiated product markets and a microeconomic analysis of gift giving when individuals are concerned with social approval (face). It consists of six chapters including a general introduction, four research papers and an outlook for further projects.

Chapter 2 proposes a model for a certification market with an imperfect testing technology. Such a technology only assures that whenever two products are tested the higher quality product is more likely to pass than the lower quality one. When only one certifier with such testing technology is present in the market, it is found that this monopoly certifier can be completely ignored in equilibrium, in contrast to the prediction of a model with perfect testing technology. A separating equilibrium is also supported in which only relatively high quality types (products) choose to pay for the certification service. It is true that in such an equilibrium having a certificate is better than not. The exact value of a certificate, however, depends both on the prior distribution of product quality and the nature of the testing technology. Welfare accounting shows that the monopoly certifier's profit maximizing conduct can lead to under or over supply of certification service depending on model specification. Socially optimal certification fee is always positive and such that it makes all positive types choose to test. In the case of two competing certifiers with identical testing technologies, the intuition of Bertrand competition does not necessarily hold. Segmentation equilibrium wherein higher seller types choose the more expensive certification service and not so high types choose the less expensive service can be supported. As an application, we argue that

the fee differentiation between major and non-major auditing firms need not be a result of any differences in their auditing technologies.

Chapter 3 revisits the excess entry theorem in spatial models à la Vickrey (1964) and Salop (1979) while relaxing the assumption of inelastic demand. Using a demand function with a constant demand elasticity, we show that the number of firms that enter a market decreases with the degree of demand elasticity. We find that the excess entry theorem does only hold when demand is sufficiently inelastic. Otherwise, there is insufficient entry. In the limiting case of unit elastic demand, the market is monopolized. We point out when and how a public policy can be desirable and broaden our results with a more general transportation cost function. Chapter 4 generalizes on Chapter 3. We introduce consumers with a generic quasi-linear utility function in the framework of the Salop (1979) model. In addition to the results found in Chapter 3, we are able to pin down conditions for efficient variety in entry cost and transportation cost. A proof for the existence and uniqueness of symmetric equilibrium when price elasticity of demand is increasing in price is also provided.

Chapter 5 studies further into the *warm-glow* that donors may benefit from their act of giving. Within the framework of concern for social approval, we emphasize an individual's relative position in social network and introduce the concept of *face*. When individuals are concerned with face, the wealthier will need to contribute more than the poorer in order to gain an equal level of social approval. In aggregate, other things being equal, the more individuals are concerned with face, the more they tend to donate. While this approach is proposed in the context of social acceptance, it is also applicable in morally motivated situations.

Publications and Presentations

Publications

Earlier versions of three chapters of this dissertation appeared in *Ruhr Economic Papers* series as Number 78 , 33 and 92. Chapter 3, "A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand", coauthored with Tobias Wenzel, is now published in the ***International Journal of Industrial Organization***.[1]

Presentations

Presentations based on various chapters were given in the Brown Bag Seminar series in Dortmund from 2007 to 2009. The following is a summary of various national and international conference presentations.

Imperfect Certification

The second chapter on "Imperfect Certification" was presented at

- the 7th Annual International Industrial Organization Conference in Boston, U.S.A. (2009),
- the 35th Annual Conference of the European Association for Research in Industrial Economics (EARIE) in Toulouse, France (2008),
- the XIII. Spring Meeting of Young Economists in Lille, France (2008),
- and the Doctoral Workshop on Game Theory in Konstanz (2008),
- and will be presented at the 2009 Econometric Society European Meeting (ESEM) in Barcelona, Spain.

[1] Gu and Wenzel (2009a) in the Bibliography

A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand

I presented the third chapter on “A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand” at

- the Jahrestagung 2008 des Vereins für Socialpolitik (VfS) in Graz, Austria (2008),
- and the All China Economics (ACE) International Conference in Hong Kong, China (2007).

Product Variety, Price Elasticity of Demand and Fixed Cost in Spatial Models

I presented the forth chapter on “Product Variety, Price Elasticity of Demand and Fixed Cost in Spatial Models” at

- the 2009 Econometric Society Australasian Meeting (ESAM) in Canberra, Australia,
- and will present it at the 36th Annual Conference of the European Association for Research in Industrial Economics (EARIE) in Ljubljana, Slovenia (2009).

Gift Giving and Concern for Face

Finally, the fifth chapter on “Gift Giving and Concern for Face” was presented at

- the PGPPE (Public Goods, Public Projects, Externalities) Workshop in Bonn (2008)

which was organized by the Max-Planck-Institut zur Erforschung von Gemeinschaftsgütern in Bonn.

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Contents

Chapter 1

Introduction

This dissertation centers on three research topics: the application of imperfect certification in markets with asymmetric information, the impact of elastic demand on market supplied product variety in differentiated product markets and a microeconomic analysis of gift giving when individuals are concerned with social approval.

1.1 Asymmetric information and imperfect certification

1.1.1 Asymmetric information and information intermediaries

The problem of asymmetric information in product quality was first brought to attention by George Akerlof (1970)'s classic paper of "The Market for 'Lemons' ". Since then this topic has grown into a large literature in economics.[1] Akerlof's paper points out that when buyers have less knowledge of product quality than sellers do, for example in a used car market, because buyers are only willing to pay up to the value of the expected quality of a product, sellers of high quality (reservation price) will opt out from trading. Following this logic, in the end only low quality products

[1] According to Wikipedia, this paper has been citied more than 4,800 times in academic papers as of October 2008. This data was retrieved from Google Scholar search on October 24, 2008.

will remain in the market and if no one finds low quality products that are worth buying, the market then breaks down.

This phenomenon at that time posed an enormous challenge to the classic theory of general equilibrium which assumes full information and perfect competition. Subsequent periods have therefore witnessed a change of style in which researches in economics are conducted: more papers started looking at specific markets at hand. Observing that to varying extents information asymmetry exists in virtually every market yet markets are still functioning, several explanations have been suggested in the literature. There are models that allow sellers to signal product quality via price, to build reputation in the long run, to provide quality insurance through warranty, etc. Some others feature intermediaries. Particularly related to Chapter 2 are information intermediaries. In this type of models, product quality in principle can be tested by a third party possibly with costs. Depending on market characteristics, quality testing may mitigate information asymmetry and improve market efficiency.

1.1.2 Perfect and imperfect certification

Several papers have studied markets with the presence of information intermediaries, specifically, certifiers.[2] Many of them, however, do not investigate certifiers' behavior. For instance, they are modeled as a public authority providing quality tests for free. Such tests, however, can be and in many cases are provided by private organizations. Therefore, the investigation of certifiers' incentives and conducts and the efficiency of both product and certification markets will be the theme of Chapter 2. The model that I am proposing is based on Lizzeri (1999). The difference and a contribution of my work is the modeling of imperfect testing technologies. Previous literature on strategic certifiers has been mainly interested in certifiers with perfect testing technologies. However, tests that are prone to mistakes are not only more realistic but also have consequential impacts on certifiers and market performance.

With respect to the nature of certification results, I assume that a product can either be certified or not. This is a simplification of the observation that real life certification outcomes are normally discrete signals. Product

[2] A detailed literature review is provided in Chapter 2.

quality, however, more often takes a value from a continuous interval. An imperfect testing technology in this model is proposed in a way such that it approves a higher quality product with a higher probability. It captures the idea that certifiers with certain abilities or experiences in differentiating product quality inevitably make honest but poor judgments. The strategic aspect of imperfect certification comes in when certifiers try to set a profit maximizing certification fee. Therefore, in the model there is an endogenous price formation for both the product and the certification service.

If we maintained everything in Chapter 2's model except that certifiers are assumed to have access to a perfect testing technology, i.e. they are able to know the exact quality of a tested product, a summary of market outcome would be as follows: In the monopoly certification service case, the certifier only certifies products of positive qualities and charges a price equal to the expected value of a certified product. Since buyers are paying the same expected value to the sellers, the certifier, by exerting its power of monopoly and ability of perfect testing, is able to obtain the entire surplus generated in the product market and leave sellers and buyers indifferent between trading and not trading. This result is completely reverted when there are more than one certifier in the certification service market. Competition between certifiers will drive market certification fee to the marginal cost of testing or zero in this model. In the end, all positive quality products get traded and sellers now enjoy the surplus from product trading.

1.1.3 Main results in imperfect certification

Results when imperfect testing is introduced are no longer as extreme as in the case of perfect testing. The main message from Chapter 2 is that a little noise in the testing technology changes the certifier's behavior dramatically. Starting from a technology that only assures whenever two products are tested, the higher quality product is more likely to pass than the lower quality one, it is found that a monopoly certifier can be completely ignored in equilibrium, in contrast to the enormous power a perfect testing technology monopoly certifier has. It is also shown that a separating equilibrium is supported wherein only relatively high quality types (products) choose to pay for the certification service. Hence, such an imperfect testing technology can be useful in reducing information asymmetry.

It is true that in a separating equilibrium having a certificate is better than not. The exact value of a certificate, however, depends both on the prior distribution of product quality and the nature of the testing technology. With respect to market efficiency, analysis shows that the monopolistic certifier's profit maximizing conduct may lead to under- or oversupply of certification service depending on model specification. A socially optimal certification fee is always positive and such that it makes all positive types choose to test.

In the case of two competing certifiers with identical testing technologies, the intuition of Bertrand competition does not necessarily hold. A segmentation equilibrium is found wherein higher seller types choose the more expensive certification service and not so high types choose the less expensive service can be supported. Finally, we apply this finding to the financial auditing industry and argue that the fee differentiation between major and non-major auditing firms need not be a result of any differences in their auditing technologies. Our theoretical model sheds light on the puzzle that quality difference in auditing services between high fee firms and others is hard to identify empirically.

The model provided in Chapter 2 is fairly general yet it nevertheless leads us to several concrete predictions. Arguably every test is imperfect and it is realistic that anyone who looks at a quality certificate would have doubts about the accuracy of the signal and sometimes even have a hard time in understanding what such a certificate means. By constructing such a model, I would like to emphasize the power of strategic thinking and equilibrium analysis. By putting themselves in sellers' shoes, consumers know in a separating equilibrium that it does not pay for low quality products to be tested. Therefore, a possibility of an imperfect certification will at least exclude the really bad quality products. Certifiers, however, understand that they need to keep certification price high to deter low quality products yet are aware of the negative impact of a high price, i.e. a low demand for certification service. Hence, only in equilibrium will consumers and indeed certifiers have a refined knowledge of product quality. It is my hope that my model will help us understand more about how tests that do not provide clear cut results and are prone to mistakes have such a widespread use in real life. Its prediction in the duopoly case is surprising yet reasonable. It offers a new perspective when we look at the certification industry.

1.2 Production differentiation and the excess entry theorem

1.2.1 Production differentiation

Equally important as market provided product quality is product variety. Products that serve a common purpose can be very different in details.[3] In general, differentiated products are provided in the market as a result of producers catering to consumers with heterogeneous preferences. For example, in the automobile market, two cars can be identical except one is black and the other is red. Some consumers like the black car better than the red one. Others have the opposite preference. There are, however, many more colors in the color spectrum that some consumers may find preferable to both black and red. A natural question to ask is how many different colors (varieties) the market will provide. What are the most important parameters that determine the market provided level of variety? And of classic economic interests is to compare this level to the socially efficient benchmark of product differentiation.

To answer the above questions and many more, three different modeling approaches have been suggested in the literature. There are representative consumer models by Spence (1976) and Dixit and Stiglitz (1977), and discrete choice models by Anderson, de Palma, and Thisse (1989). The former relies on a representative consumer whose utility function encompasses all provided varieties. The latter takes a viewpoint of the producers and model consumer choice of differentiated products as a random process. With respect to the above mentioned questions, both approaches pin down several variables including variables related to consumer pref-

[3] Commonly accepted in competition policy literature is that whether two products belong to a single market is an empirical question of cross product price elasticities. In this thesis, we follow the more traditional theoretical approach of product differentiation.

erences and production costs. As expected, both market provided variety and socially optimal level depend on model parameters and in general either of these two may take a higher level.

There is another widely used approach of product differentiation with an even longer history. Originally as a remedy to the instability of Bertrand price competition, Hotelling (1929) first suggested a model with two firms located in different positions on a line where consumers are uniformly distributed. Because almost all consumers have to incur transportation costs when purchasing a product, a slight price variation only translates into a small demand variation faced by the firms. As exemplified by many subsequent papers, physical locations and transportation costs can be interpreted as product characteristics and disutilities in consuming a less preferred variety. In this sense, chocolates with different cacao levels can be seen as if they are located at different locations along the line between the lowest and highest cacao levels. Consumers may have different preferences over chocolates with different cacao levels and consuming a less favorite level incurs some disutility in taste.[4] This approach is known as location or address models.

In his 1979 article, Salop presented the ingenious idea of transforming the unit Hotelling line into a unit circle to avoid boundary complexities. This framework quickly found its power in analyzing firms' entry decisions, a topic the original Hotelling model finds difficult to address. In this model, the basic ingredients of the Hotelling model remain except when there are more than two firms in the market, firms located closest to the two ends of the Hotelling line are now in principle no more different than any other firms. If we look at a Salop circle with uniformly distributed consumers and equidistantly located single variety firms, for a given number of firms, we can calculate their profits in the price equilibrium. A firm then only enters if the profit it expects to earn outweighs its cost of entry. Applying zero profit condition under free entry, we will then have an endogenous market provided level of product variety.

[4]Here, an important assumption with respect to consumer preference is its unimodularity. That is, if a consumer has his most preferred variety at location x_m, then to the left of x_m he prefers x_2 over x_1 as long as $x_1 < x_2 < x_m$ and to the right of x_m he prefers x_3 over x_4 as long as $x_m < x_3 < x_4$.

1.2.2 The excess entry theorem

Determinants of the market level of product variety are the parameters that represent consumer disutility in consuming a less preferred variety, also known as the transportation cost, and firms' entry cost, commonly modeled as the fixed cost of establishing a new business. As the only two major exogenous parameters in the original Salop model, the socially efficient level of product variety depends only on consumer transportation cost and the fixed cost of entry. As shown in Tirole (1988), in this model market provided product variety is always larger than the socially efficient level. In other words, there is always an excess of entry into the market. A similar point was raised by Vickrey (1964). The intuition of this result is that competition between firms are localized and firms will not stop entering until even with their local monopoly power they can only make a profit just to cover their fixed cost of entry. This also explains why the other two approaches are able to produce insufficient entry as in these models competition is global.[5]

Several papers have checked the robustness of this excess entry theorem. Already shown in Anderson, de Palma, and Thisse (1992) is that the excess entry result is quite robust against different functional forms of the transportation cost. For example, it holds under power transportation cost. With respect to the production cost, Matsumura and Okamura (2006) find the result holds for quite general cost structures. Given its robustness, it seems that if a researcher decides to use a Salop model, he/she also "decides" that there is excess entry. Are spatial models then incapable of conducting welfare analyses?

1.2.3 Elastic demand in spatial models

Chapter 3 and 4, coauthored with Tobias Wenzel, weigh in on this long established theorem of excess entry. We argue that it is inadequate to represent consumer preferences only by transportation costs, a seemingly trivial point. By focusing only on transportation cost, one assumes each consumer only demands a fixed amount of the differentiated product, no matter what the price is. But for many products, consumer demanded

[5]See, for instance, Anderson and de Palma (2000).

quantity responses to price changes, hence, demand is generally elastic.[6] Examples include chocolates, beer and many other consumer products. Therefore, we revisit the classic spatial model by introducing elastic consumer demand. The main impact of elastic demand is on market competition of firms, that is, on their pricing behavior. In turn, it impacts on firms' profits and ultimately their entry decisions.

To implement this idea, in chapter 3 we first propose a demand function with constant elasticity. This allows us to bring in one more parameter into the model in a tractable manner. With elastic demand, when firms choose their product price, they not only compete for a larger market share but also have to consider their own customers' individual demands. A low price then increases both a firm's market share and its customers' individual demands. We found that in the price equilibrium of any given number of competing firms, each firm makes a lower profit than it would have under inelastic demand. Hence under free entry, there are less firms in the market. Indeed, the higher the demand elasticity is, the lower the equilibrium number of firms in the market. However, since the socially optimal number of firms under the first best benchmark is independent of price elasticity, it remains unchanged.[7] In consequence, it is shown that there exists a threshold level of demand elasticity below which there is excess entry in the market while above which there is insufficient entry. When the demand elasticity approaches zero, we then of course go back to the classic Salop model and as expected there is always excess entry.

We believe that the insight in chapter 3 also applies in much more general cases. Constant elasticity is a very unrealistic and restrictive assumption and, in principle, demand elasticity should be found in price equilibria for general demand functions. Thus, we are interested in finding out what really determines market entry without assuming an exogenously given demand elasticity. Chapter 4 does exactly that. We start with a very general demand function and identify price equilibrium demand elasticity

[6]For lacking of a better term, by "elastic" we mean consumer demand varies in product price instead of being fixed. We are aware that commonly in industrial organization literature, a level of demand is called elastic if the demand elasticity evaluated at this level is found to be larger than 1 and inelastic when it is less than 1.

[7]Under first best benchmark, the optimal level of entry is found when a regulator can also control product price besides the number of firms. We also do a second best benchmark comparison in which the regulator can only control market entry leaving product price endogenously determined in a price equilibrium.

and the associated firms' profits by implicit functions. We then pin down the endogenous equilibrium number of firms under free entry by transportation cost and fixed cost. We show that there are cases in which when the fixed cost is low enough there is excess entry and when high enough there is insufficient entry. Reformulated in terms of transportation cost, there is excess entry when it is high enough and insufficient entry when it is low enough. These results are quite intuitive but previously eluded researchers.

As we have shown, once elastic demand is considered, market entry or market provided product varieties can be either excessive or insufficient, depending on model parameters. This finding closes the gap between spatial models and the other two approaches in the literature of product differentiation when efficient level variety is considered. Our model in chapter 4 also provides a framework for researchers in search of a spatial model suitable for the market at hand and who would like to investigate welfare issues. As we notice in the literature, the traditional Salop model is used in several welfare analyses, we would like to call for more attention to consumer demand structure before such analyses are carried out.[8]

1.3 Charitable giving and concern for face

1.3.1 Motivations of charitable giving

Departing from topics in industrial economics, Chapter 5 covers a subject that has relevance both in economics and sociology: charitable giving. There are several theories offered in the economics literature on voluntary contribution to charities. When charity is viewed as a public good, some individuals may have a preference on the level of the good that is provided. As long as the amount of provided public good remains unchanged, they may not care who contributes how much. Along with this pure altruism theory, there is the impure altruism theory in which individuals also care about whether he himself has contributed or not. Within this impure altruism theory, a distinction of "prestige benefit" versus "intrinsic benefit" of one's own act of giving, based on whether such

[8] Reference to previous welfare analyses in spatial models is given in chapter 4.

an act is visible to others or not, has been proposed in recent works. Intuitively, if one is after the "prestige benefit", other individuals should at least be able to know about his donation. If visibility does not matter for a donor, then the motivation behind his impure altruistic behavior is more likely to be the "intrinsic benefit" from donating. Various empirical findings based on statical, survey or experiment data have supported the hypothesis of impure altruism, although most of them do not differentiate between "prestige benefit" and "intrinsic benefit". A more comprehensive literature review is provided in the introduction section of chapter 5.

Although most researchers agree that individuals' enjoyment of "joy of giving" is an important incentive to make voluntary donations, the nature of this "joy" or "warm glow" is relatively underinvestigated. Using MRI scans of subjects' brains, Harbaugh, Mayr, and Burghart (2007) find that voluntary financial transfers to public goods increase neural activities in areas linked to reward processing. Compared to a consumption of a physical good, there is less understanding of such a consumption of "voluntary donation" that triggers reward process in a brain. More likely, this reward process is influenced by many more factors than a reward process triggered by a consumption of gourmet food or narcotics. Many of these factors are very subjective. What is the minimal level of donation that would trigger such a process? By how much more donation a certain measure of such activity in a brain will be increased? How about information? Will we observe a higher level of activity when the subject is told he is the most generous donor than otherwise?

1.3.2 Concern for face

Chapter 5 presents a theory of how different donations are translated into individual utilities and gives predictions on human behavior based on equilibrium analysis. I introduce the concept of "face" from sociology literature which, in economics terms, is a case of interdependent preference. Each individual according to his ranking of wealth occupies a relative position in his social network. At a given position, the more he donates the more he enjoys "joy of giving". A key point is, nominal donations from different positions have different "exchange rates" or "prices" for subjective hedonic enjoyment. The same amount of donation by a poor individual gives him a higher level of "reward" than it gives a wealthy person. Presumably, neural activities in a brain is influenced by this person's information of own wealth. Another important point is that such a production of "joy of giving" is also interdependent. When others are donating generously, the same amount of donation gives an individual at a given position less enjoyment than when others' donations are smaller. This idea is represented by an average donation/income ratio which is determined in equilibrium. In summary, in a model of individuals with concern for face, the average donation/income ratio functions as a reference point but is adjusted by individuals' relative positions in the social network. In equilibrium, richer individuals donate more in terms of absolute amount and have higher donation/income ratio since they are already expected to donate more.

The negative externality of one individual's donation to others' enjoyment of "warm glow" was studied in Glazer and Konrad (1996) in a signaling model in which a higher donation level can be seen as a signal of higher income in equilibrium. Therefore, when others are donating generously, a rich individual needs to donate even more to signal his income. In the current model, externality comes from individuals' concern for face. When others are donating more, it will make one look bad or lose face. This negative externality explains the model predictions on government subsidy for donation expenses, for instance via a tax refund. It is found in chapter 5 a government subsidy will increase the aggregate amount of donation more than the cost to the government. In this case, real cost of donation for individuals decreases so every one donates more. But higher individual donation also generates negative externalities to others and in the end every one donates much more. Individuals will also have a lower utility level because of a much lower level of other consumptions. By a similar reasoning, government tax of individual donation will increase their utility.[9]

Chapter 5 is a new explanation of charitable giving with a special interest on the interactions of individual giving. With a few exceptions, the literature so far is mainly interested in modeling, theorizing, confirming and

[9]In the model, individual utilities from aggregate supply of donated public goods are absent. Therefore, these results with respect to public policies should be interpreted with caution.

estimating the "demand" for "warm glow". My model attempts to provide us with a better understanding of the "supply" side of "warm glow", hence to have a better understanding of individuals' charitable behavior.

After the above introduced chapters, this thesis concludes with a chapter on an outlook for future research projects.

Chapter 2

Imperfect Certification

2.1 Introduction

Consider a market in which sellers know more about product quality than buyers do as in Akerlof (1970). It is well understood that serious consequences including market breakdown may result from information asymmetry in this fashion. Other than building up reputation (Klein and Leffler, 1981) and providing warranty (Grossman, 1981), sellers sometimes resort to third-party intermediaries. This chapter studies such markets featuring one type of pure information intermediaries known as certifiers.[1] By using a testing technology certifiers normally are able to assess the quality of tested products. After the assessment, a certifier decides whether to grant the tested product a certificate. With the additional information of a product's certification status, buyers should then know more about its quality. Examples of such certification services are numerous. Laboratories test and certify consumer products; credit rating agencies assign credit ratings to issuers of debt obligations; universities issue diploma to students who meet their graduation criteria; educational testing services carry out tests evaluating testees' scholastic aptitudes;[2] many

[1]Intermediaries who buy and sell products may also improve buyers' information on product quality. This point is studied in Biglaiser (1993) and Biglaiser and Friedman (1994).

[2]The Educational Testing Service (ETS) is, of course, one of such institutions.

software solution companies also run certification programs of technical expertise through which job applicants can obtain relevant credentials.[3]

In studies of certification markets, more significantly so in those with strategic certifiers, it is often assumed that a perfect testing technology is available to the certifiers. That is, they are able to know the exact quality of each tested product without a single mistake. Though this simplification is helpful to many other research topics, it is of both practical and theoretical interest to see how certifiers set prices and how markets perform when testing technologies are imperfect. Justifications for imperfectness in testing technologies are as many as the applications. Laboratories make honest mistakes in certifying consumer products; credit rating agencies only have imperfect knowledge about debt issuers' credit worthiness; there are cases that students fail to graduate because of non-productivity related factors; and luck plays a role in any expertise certification process. Yet, real life experiences indicate that those certification services are helpful in reducing information asymmetry. For example, a university degree usually is a good signal of a worker's ability although some students may have obtained their degrees just out of luck and some high ability students failed to graduate.

Many certification services are imperfect but effective in differentiating products of different qualities. This chapter attempts to model such certification technologies in a general way. Our main assumption is the following: tested by such a technology, a product may or may not pass but for any two products the higher quality one has a higher probability than the lower quality one to pass. In the context of education, it amounts to say that a student may or may not graduate from a university but for any two students the one of the higher ability is more likely to succeed in earning a diploma than the other. As shown in the following, when utilized, such a testing technology is sufficient to render a certification service informative although only to a limited extent.

[3] Currently Microsoft runs four such certification programs: Microsoft Certified Technology Specialist (MCTS), Professional Developer (MCPD), IT Professional (MCITP) and Architect (MAC). Many other software companies such as Sun, Cisco, Oracle, etc., provide their own certification service.

2.1.1 Main results

The deviation from perfect certification generates new results. For example, a monopoly certifier with an imperfect technology can now be completely ignored, in contrast to the prediction of a model with perfect testing technology. A certificate is informative in a separating equilibrium in which only relatively high quality types (products) choose to pay for the certification service. Though having a certificate is preferable, the exact value of a certificate depends both on the product quality distribution and the nature of the testing technology. Welfare accounting shows that the monopolistic certifier's profit maximizing conduct can lead to under or over supply of certification service depending on model specification. Optimal certification fee is always positive and such that it makes all positive types choose to test.

In the duopoly case, the intuition of Bertrand competition between two identical suppliers (of certificates) need not hold. Facing two certifiers with identical but imperfect testing technologies, higher seller types may choose the certifier who charges the higher fee and not so high types choose the other. In such a segmentation equilibrium, neither the lower fee certifier nor the higher fee one monopolizes the entire market of testing. Moreover, lowering one's certification fee does not necessarily generate a higher demand nor a higher profit. This observation suggests the possibility of positive profits for both certifiers even when their testing technologies are essentially identical. Consequently, competition need not drive the certification fee to zero which would be the case if both certifiers had perfect testing technologies (see Lizzeri 1999). Applied to the case of financial auditing services, we cannot rule out the possibility that auditors charging vastly different fees may have similar auditing abilities.

The rest of the chapter is organized as follows. Section 2.2 reviews the related literature and section 2.3 sets up the model. Section 2.4, 2.5 and 2.6 investigate the monopoly case and section 2.7 the duopoly case. Section 2.8 concludes. All proofs are relegated to the Appendix.

2.2 Related literature

There are a few studies of strategic certifiers, but mostly with perfect testing technologies. Lizzeri (1999) builds up a canonical model of certifiers upon which our model is constructed. In that paper the model is used to study certifiers' strategic behavior in information revelation assuming that they are able to know the exact value of every tested product's quality. Based on a similar model, Albano and Lizzeri (2001) investigate sellers' incentive in quality provision when the possibility of certification is available and the certifier may reveal the quality information in a strategic way. Strausz (2005) studies another important aspect of certification service, namely the credibility of certifiers. Our model on the other hand, focuses on certifier's testing technology. We propose a general representation of imperfect testing technology that only requires a few basic assumptions. By constructing our model on Lizzeri (1999)'s perfect testing model, we'll be able to do a direct comparison of respective results and highlight the implication of imperfectness in testing technologies.[4]

Imperfect testing technology is studied in some other papers of certification markets. In this strand of literature, however, certifiers do not strategically set their prices and there are normally only two possible levels of product quality, either high or low. These papers include, for example, Heinkel (1981), De and Nabar (1991), and Mason and Sterbenz (1994). Heinkel (1981) investigates sellers' incentive in improving product quality in a setup with exogenously provided imperfect tests. Mason and Sterbenz (1994) analyze how imperfect test affects market size. Compared to De and Nabar's (1991) paper, which like ours also studies the equilibria of certification markets with imperfect testing technologies, we introduce strategic certifiers and allow product quality to be drawn from a continuous interval. A shortcoming of limiting quality space to a binary set in modeling imperfect certification is that in an information-revealing separating equilibrium the testing technology becomes "perfect".

Hvide (2005) models strategic certifiers and introduces a zero-mean, normally distributed error term into testing technology. When a product is tested by this technology, a certifier observes the sum of its true quality

[4] It has to be noted that in this chapter we are mainly interested in testing technologies. We do not model certifier's strategic behavior in information revelation.

and the realization of a white noise. If this reading exceeds the certifier's passing score, the tested product will be awarded a certificate. Modeled in this way, as it is in Hvide (2005), for any given passing score such a technology exhibits the property of our approach, namely, the higher the tested product quality is, the more likely it passes. This "measurement error" approach hence amounts to a special case of our modeling of imperfect testing technology.[5]

In a setting of rating agencies, Boom (2001) assumes an investment project's probability of getting a favorable rating is the same as its success probability.[6] With this rating technology, she shows that in a market with a monopolistic rating agency there can be over or under supply of rating services compared to the socially optimal level. Though differing in details, our work shows that both market provision and socially optimal level of certification service depend on product quality distribution and the testing technology; we also establish a necessary condition for market equilibrium to be socially optimal and show that when this condition is not satisfied market either undersupplies or oversupplies certification service depending on model specification.

To explain the significant fee differentiation between major and non-major auditing firms in financial service market, Hvide (2005) argues major auditing firms adopt stricter test standards (higher passing scores in the "measurement error" approach) than non-major auditing firms. With the help of the stricter standards, major auditing firms are then able to charge higher auditing fees and make higher profits. In this chapter we provide an alternative explanation. In our model, we need not assume differences in their auditing processes. Even with identical standards, i.e., identical tests, Bertrand Competition need not happen and segmentation equilibrium may be supported in which firms charge different prices.

[5]Note that the reading gives the expected quality of the tested product. The certifiers have incentive to reveal more information than just the certificate. For instance, revealing the reading itself can attract testees. In our current model, however, this information is not available to the certifiers.

[6]It will become clear in the following that this is also a special case of our modeling of imperfect testing technology, namely $G(t) = t$. See Equation (2.1) in Section 2.3.

2.3 The model

Following the setup of Lizzeri (1999), we analyze the market situation as a non-cooperative game with incomplete information.

2.3.1 Players

We have four players in the model, one seller, one certifier and two buyers. **The seller** wants to sell a product to the buyers. The product has a value equal to its quality t (type) to both of the buyers but is worth nothing to the seller and the certifier. The type t is originally only known to the seller; the buyers and the certifier, however, know the prior distribution of t represented by cumulative distribution function, $F(t)$. $F(t)$ is assumed to be continuous, differentiable and strictly increasing on interval $[a, b]$, where $a < 0 < b$.[7] The associated density function is denoted $f(t)$. The seller has the possibility to get the product tested by the certifier.

The certifier has a testing technology. When it is used to test the product, it prints out a certificate (C) with probability

$$\Pr(C \mid t) = G(t), \tag{2.1}$$

conditional on t. $G(t)$ is also assumed to be continuous, differentiable and strictly increasing on $[a, b]$ with first derivative denoted $g(t)$. Tested by this technology, the higher a product's quality is the higher its probability of receiving a certificate will be. Naturally the probability of no certificate (NC) is $\Pr(NC \mid t) = 1 - G(t)$. This setup requires function $G(t)$ to be bounded below by 0 and above by 1. For convenience, we assume $G(a) = 0$ and $G(b) = 1$, i.e., it is not possible for the lowest type to pass the test while the highest type always passes when tested.[8] It is also assumed that the certifier does not manipulate the test result produced by the technology. The certifier can charge a certification fee P for the test and the cost associated with testing is normalized to zero.

Both buyers observe whether a product possesses a certificate or not and bid simultaneously based on their beliefs. They, however, cannot distin-

[7]When product quality is negative, consumption of such goods harms the buyers.
[8]This assumption does not change our results qualitatively.

guish the event that the product was not tested from the event that the product failed the test. That is, they observe if a product has a certificate, $\theta : \theta \in \{C, NC\}$, but not what the seller did.

2.3.2 Timing

Stage 1 The certifier announces its certification fee, P, for the test.

Stage 2 At the beginning, the seller learns his type t (chosen by nature according to $F(\cdot)$) and the announced certification fee, P; the seller then decides whether or not to get the product tested by paying the certifier the certification fee.

Stage 3 If the seller chooses to test, then the certifier employs the testing technology and the seller receives a certificate with probability $G(t)$, receives no certificate with probability $1 - G(t)$.

Stage 4 Both buyers observe P and if the product has a certificate or not.

Stage 5 Buyers bid independently and simultaneously for the product. The product is sold to the buyer who bids higher than the other at the price of the winning bid. Buyers get the product equally likely in case of a tie. When both bids are zero, the product is not sold.

2.3.3 Strategies

The certifier's strategy is simply the choice of certification fee, $P \in \mathbb{R}_+$.

The seller's strategy specifies his decision for all combinations of own quality type and certification fee level. Namely, it is a function $\rho(P, t)$, from $\mathbb{R}_+ \times [a, b]$ to $\{TS, NTS\}$, that maps the vector (P, t) into a set of two elements, *to test* or *not to test*.

A strategy for a buyer is a function $\beta(P, \theta)$, from $\mathbb{R}_+ \times \{C, NC\}$ to $\mathbb{R}_+$, that maps the announced certification fee and the product's certification status to a bid for that product. Buyers' beliefs are denoted by $\mu(t \mid C, P)$ for a certified product and $\mu(t \mid NC, P)$ for a non-certified product. Since buyers have identical information, when beliefs are Bayesian updatable they are identical. Note that competition will make them both bid up to their common belief. Therefore, no subscripts are used for individual buyers.

2.3.4 Payoffs

All players are assumed to be risk neutral. Hence, they maximize their payoffs in expected terms.

A buyer's payoff function, in the following three types of outcomes, reads

$$U(t,\beta) = \begin{cases} t - \beta(P,NC) & \text{when the buyer gets a non-certified product,} \\ t - \beta(P,C) & \text{when the buyer gets a certified product,} \\ 0 & \text{when the buyer does not get the product.} \end{cases}$$

The seller receives buyers' bids for a non-certified product when the product is not tested. If the seller chooses to test, he has a probability of $G(t)$ getting a certificate and receiving buyers' bids for a certified product. In other cases $(1 - G(t))$, he does not get the certificate and receives bids for a non-certified product. Taking the certification fee into account, the seller's payoff is

$$V(\rho,t,P,\beta) = \begin{cases} \beta(P,NC) & \text{not to test,} \\ [1 - G(t)]\beta(P,NC) + G(t)\beta(P,C) - P & \text{to test.} \end{cases}$$

The certifier's payoff is the product of the certification fee and the demand for the certification service, i.e.,

$$\Pi(P,\rho) = P \cdot \Pr(\text{the event that the seller tests}),$$

or

$$\Pi(P,\rho) = P \cdot \int_T dF(t), \quad \text{where } T = \{t \mid \rho(P,t) = TS\}.$$

2.3.5 Equilibrium notion

The equilibrium notion employed in this chapter is Perfect Bayesian Equilibrium. As we argued before competition between the buyers will force them bid identically up to their common belief, we have

$$\beta^*(P,\theta) = \begin{cases} \mu(t \mid \theta, P) & \text{if } \mu(t \mid \theta, P) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2.2}$$

Bayesian perfectness requires their expectations should be consistent with equilibrium outcome. Hence, for both buyers, when their beliefs are Bayesian updatable,

$$\mu(t \mid \theta, P) = E(t \mid \theta, P), \forall \theta \in \{C, NC\}, \forall P \in \mathbb{R}_+, \tag{2.3}$$

where E is the mathematical expectation operator. We also need that the seller not to have incentive in deviating from equilibrium strategy after knowing his quality type. The seller's strategy choice should be, for each type, his best response to the announced certification fee and buyers' biding strategies. Therefore, for any given combination of certification fee P and buyers bidding function β, we need

$$V(\rho^*, t, P, \beta) \geq V(\rho', t, P, \beta), \forall t \in [a, b], \text{ where } \rho' = \{TS, NTS\} \setminus \{\rho^*\}. \tag{2.4}$$

The certifier's fee should then be chosen to maximize his expected payoff,

$$P^* = \arg\max \left\{ P \cdot \int_{\{t \mid \rho(P,t) = TS\}} dF(t) \right\}. \tag{2.5}$$

Formally we define the equilibrium notion as the following.

Definition 2.1. *A strategy profile* $\{P^*, \rho^*(P,t), \beta^*(P,\theta)\}$ *and buyers' belief* $\mu(t \mid \theta, P)$*, constitute a* Perfect Bayesian Equilibrium *of the game, if and only if conditions (2.2), (2.3), (2.4) and (2.5) hold.*

2.3.6 Discussion

The testing technology (2.1) essentially only requires whenever two products get tested, the product that is of the higher quality has a higher probability than the other to pass. It doesn't specify any functional form.

2.4 Monopoly: bypassing

In the situation depicted in section 2.3, without certification service information asymmetry leads to market breakdown when the prior expectation of product quality is below zero, $E(t) < 0$. When $E(t) > 0$, however, the product is traded with probability one. From social welfare point of view, there is over-trading since there are cases trading results in a loss to the society.[9]

With perfect testing technology, for example, as in Lizzeri (1999), it is found that a monopoly certifier will only certify non-negative seller types; hence, only those certified types will be traded in equilibrium. This is an efficient outcome since all positive types are traded while none of the negative types will be. It is also shown that the mere existence of this perfect testing possibility grants the certifier the power to take away the entire market surplus leaving the seller a payoff of zero. Consequently, the monopolist's interest is coincident with social welfare.[10] This explains why the monopolist's profit maximizing conduct is also socially optimal.

When the testing technology is imperfect, however, the game changes dramatically with respect to both the monopoly certifier's power and the market outcome. Although with perfect testing technology the certifier can always guarantee itself the demand for certification service by offering to the seller that it will reveal the exact quality type of a tested product, when testing technology is imperfect the certifier may even be completely bypassed.

Proposition 2.1. *Any of the following strategy profiles, such that,*

1. *for all levels of the certification fee, all seller types choose not to test,*
2. *for all levels of certification fee, buyers bid for a non-certified product either the ex ante expected quality when it is positive or zero when non-positive, bid for a certified product either the belief for a certified product when it is positive or zero when non-positive,*
3. *the certifier charges any non-negative fee,*

[9] The lowest type a is assumed to be less than 0. Therefore, some negative types will be traded. When $a \geq 0$, full trading is efficient.

[10] Note that buyers always end up with zero payoff because they engage in Bertrand bidding competition.

4. *and the buyers' belief being that the quality of a certified product is* no higher *than the ex ante expected quality,*

constitutes an equilibrium. That is,

$$
\begin{aligned}
P^* &= P \in \mathbb{R}_+ \\
\rho^*(P,t) &= NTS,\ \forall t \in [a,b],\ \forall P \in \mathbb{R}_+ \\
\beta^*(P,NC) &= \max\{E(t),0\},\ \forall P \in \mathbb{R}_+ \\
\beta^*(P,C) &= \max\{\mu(t \mid C,P),0\},\ \forall P \in \mathbb{R}_+ \\
\mu(t \mid NC,P) &= E(t),\ \forall P \in \mathbb{R}_+ \\
\mu(t \mid C,P) &= \mu \in (a,E(t)],\ \forall P \in \mathbb{R}_+.
\end{aligned}
$$

Proof. See Appendix. □

One direct implication of Proposition 2.1 is the following remark.

Remark 2.1. *When testing technology is imperfect, it's possible for the seller to bypass the monopoly certifier.*

The main underlying reason for this result is the strictly positive probability that lower types may pass the test. This leaves the buyers the scope of forming the beliefs that are required for the equilibria in Proposition 2.1. In the perfect testing technology case, such beliefs cannot be supported; consequently, bypassing is not possible.

This difference between perfect and imperfect testing technology is not only of theoretical interest but also of practical importance. Consider "a" seller in the literal sense. Before nature's draw, there are collective interests among seller types. We can think of a monopoly seller or an industry in aggregation. From this perspective, when $E(t) \leq 0$, it is not in the seller's interest to bypass the certification service because there would then be no trading. When $E(t) > 0$, however, the seller makes maximal profit $E(t)$ without the certification service. Given that the testing technology is imperfect, it's at least possible for the seller to bypass the certifier.

We are aware that buyers' belief in Proposition 2.1 seems irregular. It essentially says that a certificate does not serve a signal of high quality even though buyers know that when tested higher types are more likely to obtain a certificate than lower types. First of all, when the certification service is not used, the beliefs stated in Proposition 2.1 are not exactly irrational. Second, the reason we present Proposition 2.1 in this chapter is to show the difference in feasible equilibria when testing technology is perfect versus when it is imperfect. Although we can put more restrictions on buyers' beliefs by adopting other equilibrium notions, this possibility result signifies the decrease of certifier's power caused by imperfectness in testing technology.

2.5 Monopoly: separating equilibrium

In the following we search out those equilibria in which there is a positive measure of seller types paying for the test. This is of particularly importance when $E(t) \leq 0$ since in this case the market would break down if there were no certification service available. To focus on this issue and to simplify the analysis, we assume the prior expected product quality to be negative.[11]

Assumption 2.1. *The prior expected product quality is less than zero, i.e.,* $E(t) \leq 0$.

As an example, consider the labor market for IT specialists. If there are no other signals available and the average potential worker does not qualify, then a certificate for such expertise would be crucial both to job applicants and to employers. Yet, we need to find out for a given imperfect testing technology what a certificate could mean and how the market for the certification service performs.

We solve the game by investigating first the subgames induced by different certification fees. Not surprisingly, when the certification fee is set too high, it does not pay for the seller to get the product tested. The following proposition states.

Proposition 2.2. *In subgames induced by the certifier's fee setting* P*, it is true that:*

[11] Again, this assumption does not change the result on separating equilibrium qualitatively.

1. *if the certifier charges a fee higher than the highest type, then any strategy profile such that all seller types choosing not to test, buyers bidding zero for a non-certified, bidding for a certified product the belief for such a product when it is positive or zero when non-positive, and buyers' beliefs for a certified product being no higher than b, constitutes an equilibrium in the subgame induced by P; that is, in subgames where* $P > b$,

$$\begin{aligned}
\rho^*(t \mid P > b) &= NTS, \ \forall t \in [a, b] \\
\beta^*(NC \mid P > b) &= 0 \\
\beta^*(C \mid P > b) &= \max\{\mu(t \mid C, P > b), 0\} \\
\mu(t \mid NC, P > b) &= E(t) \\
\mu(t \mid C, P > b) &= \mu \in (a, b];
\end{aligned}$$

2. *if the certifier charges a fee equal to the highest type, there is only one equilibrium in the subgame other than bypassing, in which only the highest seller type chooses to test and buyers bid the value of the highest type for a certified product, zero for a non-certified product and buyers' beliefs being the ex ante expectation for a non-certified product and b for a certified product; that is, in the subgame where* $P = b$,

$$\begin{aligned}
\rho^*(t = b \mid P = b) = TS \textit{ and } \rho^*(t \mid P = b) = NTS, \ \forall t \in [a, b) \\
\beta^*(C \mid P = b) = b \textit{ and } \beta^*(NC \mid P = b) = 0 \\
\mu(C \mid P = b) = b \textit{ and } \mu(NC \mid P = b) = E(t).
\end{aligned}$$

Proof. See the Appendix. □

This result can be interpreted as the following. When the price for test is too high, there is intuitively not much demand for it. As a preparation for solving the whole game, we establish the following corollary with respect to the certifier's profit. The result is immediate from Proposition 2.2.

Corollary 2.1. *The certifier makes zero profit by setting* $P \geq b$, *or* $P = 0$.

2.5.1 Separating equilibrium

We now turn to the more interesting subgames induced by intermediate certification fees. Before proceeding to the result, the following definition is useful in simplifying notation.

Definition 2.2. *Denote*

$$\Omega(m,n) = \frac{\int_m^n tG(t)dF(t)}{\int_m^n G(t)dF(t)} \text{ for } a \leq m < n \leq b.$$

Function $\Omega(m,n)$ gives type expectation of a product with a certificate if and only if all types from the interval $(m,n]$ (or (m,n), $[m,n)$, $[m,n]$) choose to test.

Further we introduce the following tie-breaking rule.

Assumption 2.2. *When a seller type is indifferent between to test and not to test, we assume he chooses to test.*

Proposition 2.3 (Separating). *In each subgame induced by $0 < P < b$, there is a* unique *subgame equilibrium other than bypassing the certifier completely. Moreover, the set of seller types, which strictly prefer testing, is of the form $(x,b]$ and type x is indifferent between testing and not testing, where x solves $G(x)\Omega(x,b) = P$. Buyers bid $\beta(P,C) = \Omega(x,b)$ for a certified product and $\beta(P,NC) = 0$ for a non-certified product. That is,*

$$\text{the seller's strategies: } \begin{cases} \rho^*(t \mid P) = TS, \forall t \in [x,b], \\ \rho^*(t \mid P) = NTS, \forall t \in [a,x), \end{cases}$$

$$\text{buyer's strategies: } \begin{cases} \beta^*(C \mid P) = \Omega(x,b), \\ \beta^*(NC \mid P) = 0, \end{cases}$$

$$\text{and buyer's expectation: } \begin{cases} \mu(t,C \mid P) = \Omega(x,b), \\ \mu(t,NC \mid P) < 0. \end{cases}$$

constitute the equilibrium in the subgame induced by $P \in (0,b)$.

Proof. See Appendix. □

This result states that for each positive certification fee that is less than the highest quality type, there is a unique subgame equilibrium in which those relatively high types choose to test by paying the certification fee while relatively low types choose not to.[12] Since only those higher types choose to test, after taking the imperfectness in the testing technology into account, buyers still bid more for a product that has a certificate. This bidding difference justifies the fee that high seller types pay for the test. The probability of a type passing the test is critical to the type's willingness to pay. Even high types have a certain probability failing a test. But the nature of the testing technology ensures that in expected terms higher types are better off by paying for the test while lower types are better off by choosing not to test.

For ease of exposition and motivated by the proof of Proposition 2.3 in Appendix 2.9.3, we introduce the next definition.

Definition 2.3. *Denote $\kappa(P) = x$ such that $G(x)\Omega(x, b) = P$ where $0 < P < b$. For a given P, $\kappa(P)$ gives the unique type who is indifferent between to test and not to test in the equilibrium identified in Proposition 2.3.*

Proposition 2.3 states that in equilibrium all types higher than $\kappa(P)$ prefer paying for the test and playing the certification lottery over not to test. The difference for any type t between these two options can be represented by function $\Gamma(t)$, [13]

$$\Gamma(t) = G(t)\Omega\left(\kappa(P), b\right) - P.$$

While $\Gamma(\kappa(P)) = 0$,

$$\begin{aligned} \Gamma(t \mid t > \kappa(P)) &= G(t \mid t > \kappa(P))\Omega(\kappa(P), b) - P \\ &> G(\kappa(P))\Omega(\kappa(P), b) - P = \Gamma(\kappa(P)) = 0. \end{aligned}$$

This explains that the set of the seller types who pay for the test is always connected. Whenever a certain type finds it worthwhile paying for the test, any type above would find it so as well. For the same fee, a higher type gets a better lottery than a lower type. On the other hand, this guarantees the existence of the separating equilibrium by preventing lower types from applying the test. A certification service provides a device by which relatively high seller types can separate themselves from

[12] Note that bypassing is still possible but in this section we focus on the cases when the certification service is used.

[13] See also Equation (2.17) in 2.9.3.

relatively low types. They also need to pool together to induce buyers to form a quality expectation that is positive. In the case of perfect testing technology, however, pooling is not necessarily needed since a certifier can certify a seller's true type. From the perspective of the seller, we have the following remark.

Remark 2.2. *1. When there is no testing technology, seller types' interests are all pooled together without choice;*

2. when there is a perfect testing technology, an individual seller type has the opportunity to perfectly identify itself unilaterally;

3. when there is an imperfect testing technology, seller types depend on each other to a certain degree.

Recall that in the case of perfect testing technology the certifier is able to make all tested types indifferent between testing and not testing and take away the entire market surplus. The certifier chooses a minimum quality standard, say $\kappa' = 0$, and charges $P' = E(t \mid t \geq 0)$ for the test. It turns out that types above 0 are all indifferent between testing and not testing. Note that even though each seller type is left with zero surplus, this is the unique equilibrium when perfect testing technology is available in the monopoly certifier case.[14]

Suppose that a certifier with an imperfect technology wants to employ such a strategy. The certifier claims that all types higher than κ' will pass the test while all types below will not. Since the certifier is unable to make sure that every low type does not pass and every high type passes, the expected quality of a certified product is not assured to be at $E(t \mid t \geq \kappa')$. Therefore, buyers will not bid as much as $E(t \mid t \geq \kappa')$ and neither will the seller types pay as much for the test. So it is clear that when testing technology is imperfect, a monopoly certifier cannot take away the entire market surplus. Indeed most of the testing seller types derive strictly positive payoff in a separating equilibrium. The following remark summaries.

[14] For a formal reasoning, the reader is referred to Lizzeri (1999). This situation resembles the observation that in the unique subgame perfect equilibrium of a 2-player *Ultimatum game*, the proposer gets all and the other gets nothing even though she can reject.

Remark 2.3. *When imperfect certification service is used in equilibrium, the monopoly certifier's power in taking up market surplus against the seller is limited compared to the case in which a perfect testing technology is available.*

2.5.2 Value of a certificate

It is worth noting how buyers form their expectations towards a certified product. Without equilibrium analysis a certificate does not give a definitive meaning in terms of product quality. Proposition 2.3, however, says only types higher than or equal to $\kappa(P)$ go to the certifier in equilibrium at the cost of a positive fee. By successfully attracting a positive measure of seller types, the certification service practically blocks away types lower than $\kappa(P)$ in the original population and filters the remaining into a new population of those with a certificate. The new population is distributed on $[\kappa(P), b]$ with density $\frac{G(t)f(t)}{\int_{\kappa(P)}^{b} G(t)dF(t)}$ where $f(t)$ is the density function of the original distribution. Thus buyers form their expectations of a certified product as

$$\frac{\int_{\kappa(P)}^{b} tG(t)dF(t)}{\int_{\kappa(P)}^{b} G(t)dF(t)} = \Omega\left(\kappa(P), b\right).$$

First, this observation further emphasizes the idea that buyers are only able to attribute a value to a certificate for equilibrium outcomes but not for off-equilibrium incidences. Second, in an equilibrium of the form stated in Proposition 2.3, the value of a certificate directly depends both on the population of the seller types who choose to test and on the nature of the testing technology. This implies that to be able to assess a certificate, a buyer first needs to understand what types of products are likely to choose to test and how difficult it is to pass such a test. Third, note that the value of the certificate $\Omega\left(\kappa(P), b\right)$ for a given type distribution and a given testing technology is a function of the certification fee P. Hence, when the certification fee changes, the value of the certificate also changes.

Compared to the case in which a perfect testing technology is available, the dependence on the test takers' population is crucial in imperfect testing. In the former case, a certifier can always identify the type when a product is tested. The meaning of such a test can be made independent of the seller's type distribution. In our imperfect testing case, the certifier has to rely on a positive measure of seller types to make the certificate meaningful. This dependence is responsible for the limited ability of the certifier both in ensuring demand for the test (Remark 2.1) and in taking up market surplus against the seller (Remark 2.3).

2.5.3 Free certification

There is one subgame yet to be discussed, the one induced by $P = 0$. It is of additional importance because we are also interested in the case when tests are provided for free to the seller, for instance, through a public policy program.

Proposition 2.4. *In the subgame induced by $P = 0$, buyers make positive bids for a certified product if and only if $\Omega(a, b) > 0$.*

Proof. See Appendix. □

Free certification produces two contrasting outcomes with respect to trading probabilities. It gives the maximum probability of $\int_a^b G(t)dF(t)$ when $\Omega(a, b) > 0$ since all seller types have already chosen to test and there is no other way to increase the probability of having a certified product. If $\Omega(a, b) < 0$, the product will for sure not be traded. However, neither of these two is necessarily desirable compared to the socially optimal level discussed in subsection 2.6.3 below.

2.6 Monopoly: market performance

2.6.1 Equilibrium of the game

After having investigated all subgames, we are now ready to solve the game in its entirety. At the first stage, the certifier chooses the certification fee for the test, $P \in \mathbb{R}_+$. Since we put aside bypassing equilibria, the next result follows.

Proposition 2.5. *In equilibrium, a monopoly certifier sets P to maximize profit $\Pi(P) = P[1 - F(\kappa(P))]$. That is,*

$$P^* = \arg \max_{P \in (0,b)} P[1 - F(\kappa(P))]. \tag{2.6}$$

It can also be represented as to choose the indifferent type x, such that it maximizes the certifier's profit. Formally,

$$x^* = \arg \max_{x \in (a,b)} G(x)\Omega(x,b)[1 - F(x)]. \tag{2.7}$$

Proof. See Appendix. □

The monopoly certifier's trade-off resembles that of many other monopoly producers who face a downward sloping demand curve. Demand decreases when the fee (price) increases. The difference, however, is that while the negative slope of the demand function of consumer products is normally a result of consumers' descending willingness to pay for the unit-by-unit-identical product, here the value of the certificate that is being offered is actually evolving along with participating seller types. The value of a certificate deteriorates in the participation of lower seller types. When a certifier lowers its certification fee, it lowers the value of its certificate too.

2.6.2 An example

To have a better understanding of the equilibrium outcome, we present a fully specified numerical example.

Example 2.1. *Suppose seller types are uniformly distributed on the interval $[-2, 1]$, that is, $F(t) = \frac{t+2}{3}$. The testing technology $G(t)$ follows a power function, $G(t) = \left(\frac{t+2}{3}\right)^2$ on $[-2, 1]$. Under this model specification, as stated in Equation (2.22), the monopoly certifier solves the following problem,*

$$\max_{-2<x<1} \left(1 - \frac{x+2}{3}\right)\left(\frac{x+2}{3}\right)^2 \frac{\int_x^1 t\frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt}{\int_x^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt}.$$

The solution to this problem is $x = 0.3154$. This means the fee the certifier charges is

$$P = G(x)\Omega(x,1) = \left(\frac{0.3154+2}{3}\right)^2 \frac{\int_{0.3154}^{1} t\frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt}{\int_{0.3154}^{1} \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt} = 0.4092.$$

It turns out that seller types in $[0.3154, 1]$ choose to test while the rest choose not to. Buyers bid

$$\beta(C \mid P = 0.4092) = \Omega(0.3154, 1) = \frac{\int_{0.3154}^{1} t\frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt}{\int_{0.3154}^{1} \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt} = 0.6870$$

for a certified product and 0 for a non-certified. The expected profit the certifier makes is

$$\Pi(0.4092) = P(1 - F(x))) = 0.4092 \int_{0.3154}^{1} \frac{1}{3} dt = 0.0934,$$

which is less than the amount it would have made,

$$\Pi' = \int_0^1 \frac{1}{3} t dt = 0.1667,$$

if a perfect testing technology were available.[15] *This point can indeed be generalized.*

Remark 2.4. *A monopoly certifier with an imperfect testing technology makes a smaller profit than a monopoly certifier with a perfect testing technology under otherwise identical circumstances.*

The explanation is the following. With perfect testing technology, a certifier is able to take away the entire trading surplus in the market leaving nothing to the seller. Consequently, the certifier will seek to reach the highest possible market surplus. In contrast, with imperfect testing technology, the surplus generated in the product market is shared between the certifier and the seller.[16] From the perspective of the certifier, with perfect testing technology it achieves first best outcome; while in the case

[15] The profit under perfect testing technology is found when the certifier only certifies types above zero and charges $E(t \mid t \geq 0)$.

[16] Note that the set of seller types who strictly prefer paying for the test obtain positive expected payoffs. See subsection 2.5.1.

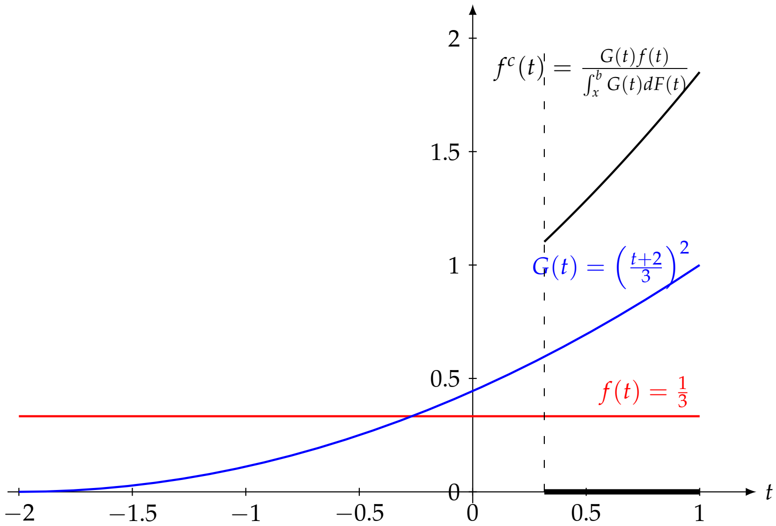

Figure 2.1: A case of an uniformly distributed type population ($f(t) = \frac{1}{3}$) and a power testing technology ($G(t) = \left(\frac{t+2}{3}\right)^2$); types to the right of the dashed line, $[0.3154, 1]$, pay for the test in equilibrium; the curve in the upper right represents the type density function of a certified product.

of imperfect testing technology, not only the certifier's share is less than 1 but also the total level of generated surplus can be well below maximum.

An interesting question concerns the type distribution of a certified product in equilibrium. The type distribution of a certified product has the support of $[0.3154, 1]$. Its density function is a transformation of part of the original density function via the testing technology. Denote $f^c(t)$ the new probability density function of a certified product; $f^c(t)$ can be written as the following.

$$f^c(t) = \frac{G(t)f(t)}{\int_x^b G(t)dF(t)} = \frac{\frac{1}{3}\left(\frac{t+2}{3}\right)^2}{\int_{0.3154}^{1} \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt} = 0.20566(t+2)^2.$$

Figure 2.1 gives a graphical representation of the original distribution, the testing technology and the transformed type distribution of a certified product.

2.6.3 Welfare

An important issue in markets with asymmetric information is market performance in terms of social welfare. The next result gives the condition for welfare maximization.

Proposition 2.6. *In the separating equilibrium of subgames induced by* $0 < P < b$*, market surplus is represented by* $\int_{\kappa(P)}^{b} tG(t)dF(t)$*. It is maximized when* $\kappa(P^{**}) = 0$*, i.e., when type* 0 *is made indifferent between testing and not testing. Therefore, the welfare maximizing certification fee is* $P^{**} = G(0)\Omega(0, b)$*.*

Proof. See Appendix. □

The intuition is the following. For a product to be traded in a separating equilibrium, it has to obtain a certificate. Note that trading of positive types increases while trading of negative types decreases social welfare. So the ideal outcome is that all positive types obtain a certificate while all negative types are uncertified. But given the nature of the imperfect testing technology, this is not achievable. Also note that once a give type decides to test, the probability of getting a certificate is governed by the testing technology. The second best is then to set the certification fee to a level such that it is low enough for all positive types to pay for the test while it is still high enough to discourage negative types from using the test. Hence, the optimal certification fee should make type 0 the indifferent type. Note that $G(0)\Omega(0, b)$ is strictly positive, we emphasize the result as a corollary to Proposition 2.6.

Corollary 2.2. *The social welfare maximizing certification fee* P^{**} *is strictly positive.*

Apparently, free certification under imperfect testing technology is not an optimal policy. Because of the inability of the testing technology in blocking negative types from getting a certificate, we need a positive certification fee to function as a self-selection mechanism.

We can also see the difference between social welfare and the certifier's profit in a comparison of the following two expressions.

$$
\begin{aligned}
\text{Social welfare} \quad &: \quad \int_{\kappa(P)}^{b} tG(t)dF(t) \\
\text{Certifier's profit} \quad &: \quad P[1-F(\kappa(P))] \\
&= \quad [1-F(\kappa(P))]G(\kappa(P))\Omega(\kappa(P),b) \\
&= \quad \left\{ \frac{\int_{\kappa(P)}^{b} G(\kappa(P))dF(t)}{\int_{\kappa(P)}^{b} G(t)dF(t)} \right\} \int_{\kappa(P)}^{b} tG(t)dF(t). \qquad (2.8)
\end{aligned}
$$

They differ by the part in the curly brackets in equation (2.8). Note that $G(t \mid t > \kappa(P)) > G(\kappa(P))$, the part in the curly brackets is less than 1. Hence, not all of the total market surplus is taken by the certifier. Part of it is shared by the seller. But for a certifier equipped with a perfect testing technology, $G(t \mid t \geq \kappa')$ could be set to 1 and $G(t \mid t < \kappa')$ to 0. The part in the curly brackets hence vanishes and the monopoly certifier's profit is equal to the entire social surplus. When such a certifier maximizes its profit it as well maximizes social welfare. This comparison tells us that the inability of taking up all market surplus leads to a lower level of social welfare, i.e., inefficiency.

Boom (2001) shows that in a market with a monopolistic rating agency there can be over or under supply of rating services in equilibrium compared to socially optimal level. In the next proposition we establish the necessary condition for profit maximizing conduct to be welfare maximizing. When this condition does not hold, market either oversupplies or undersupplies certification service depending on model specification.

Proposition 2.7. *A necessary condition for the profit maximizing certifier to set the welfare maximizing certification fee* $P^{**} = G(0)\Omega(0,b)$ *is,*

$$\frac{f(0)}{1-F(0)} = \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}. \qquad (2.9)$$

Moreover, when $P[1-F(\kappa(P))]$ *is concave for* $P \in (0,b)$*, there is oversupply (undersupply) of certification service if*

$$\frac{f(0)}{1-F(0)} > (<) \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}. \qquad (2.10)$$

Proof. See the Appendix. □

This necessary condition requires the Hazard rate of the original type distribution when evaluated at type 0 has to be equal to the sum of a value related to the testing technology ($G(t)$) and certified product's density at type 0. When condition (2.9) doesn't hold, socially optimal certification fee will not be achieved by profit maximizing monopoly certifier.

Further, with additional information of certifier's profit function concavity, we can identify conditions for over and under supply of certification service. When

$$\frac{f(0)}{1-F(0)} < \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}, \tag{2.11}$$

the first derivative of profit is positive at type 0. Therefore, the certifier will have an incentive to raise the certification fee from the socially optimal level $P^{**} = G(0)\Omega(0,b)$ and the indifferent type will be strictly higher than type 0. Because there are strictly positive types find the certification fee too high and do not apply the test, there is under utilization of the certification service. Social welfare could be improved by lowering the certification fee. Similarly, when the reverse of condition (2.11) holds, the indifferent type will be strictly lower than 0 and some negative types will be traded. Hence there will be oversupply of certification service.

2.6.4 Example 2.1 continued

In the above numerical example, the indifferent type is 0.3154. Social welfare would be higher if types in $[0, 0.3154]$ applied the test. Hence, the certification fee 0.4092 is too high. By lowering the fee, more seller types will use the certification service and the product will have a higher probability to be traded. To be exact, the socially optimal fee is

$$P^{**} = G(0)\Omega(0,1) = \left(\frac{2}{3}\right)^2 \frac{\int_0^1 t\frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt}{\int_0^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt} = 0.2515.$$

So that types in $[0,1]$ choose to test while types in $[-2,0]$ choose not to.

In Table 2.1 we compare social welfare and the product's trading probability in example 2.1 under three different scenarios: perfect testing tech-

	Social welfare $\int_x^b tG(t)dF(t)$	Trading probability $\int_x^b G(t)dF(t)$
Perfect testing	$\int_0^1 \frac{1}{3}tdt = \frac{1}{6} = 0.1667$	$\int_0^1 \frac{1}{3}dt = \frac{1}{3} = 0.3333$
Imperfect (Social)	$\int_0^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1327$	$\int_0^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.2346$
Imperfect (Profit)	$\int_{0.3154}^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt$ $= 0.1237$	$\int_{0.3154}^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt$ $= 0.1801$

Table 2.1: Welfare under (im)perfect testing in example 2.1.

nology, imperfect testing technology used to maximize social welfare and imperfect testing technology used to maximize the certifier's profit. According to the original type distribution, the mean of all positive types is $1/6$ which is the entire surplus that can be generated from trading. Since with perfect testing technology, all positive types get a certificate, the probability of trading is $1/3$. With imperfect testing technology, under welfare maximization all positive types should at least be tested. For the given imperfect testing technology $G(t) = \left(\frac{t+2}{3}\right)^2$, the probability that the product gets a certificate is only $\int_0^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.2346$. The surplus generated is $\int_0^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1327$. When the certifier maximizes profit, certification fee is higher and less types apply the test. The probability that the product gets a certificate now is $\int_{0.3154}^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.1801$. The generated surplus is $\int_{0.3154}^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1237$ which is less than the optimal level. So the efficiency of the market is reduced both by the imperfectness in testing technology and by the certifier's profit maximizing conduct.[17]

Generally, profit maximizing monopoly certifier does not set the certification fee to the socially optimal level. But even when the service is run by the public sector and the certification fee is optimally set such that all positive types apply the test and all negative types do not, inefficiency remains because some positive types will fail the test and will not be traded. However, compared to the market breakdown outcome without certification service, there at least will be some trading in a separating equilibrium. The next remark summarizes.

[17]Note that in perfect testing case, the certifier's profit is coincident with social welfare. One may argue the efficiency loss is entirely caused by testing technology imperfectness.

Remark 2.5. *An imperfect testing technology solves the asymmetric information problem imperfectly. The market is not as efficient as it is with perfect testing technology but it does improve buyers' information on product quality in equilibrium.*

2.7 Duopoly

In this section we investigate a market with two certifiers. The main purpose of this section is to provide a new perspective for the study of competing certifiers. To this aim, we are interested in market behavior with given certification fees. The seller now can choose which certifier to go for a test or not to test at all. We do not consider the possibility that a seller type applies both tests. Hence, the seller's decision ρ maps $\mathbb{R}_+^2 \times [a, b]$ to $\{TS_1, TS_2, NTS\}$. TS_1 is to test at Certifier 1 and TS_2 is to test at Certifier 2. When a seller type fails a test, the type is pooled with those who do not test. For buyers, β is now a function from $\mathbb{R}_+^2 \times \{C_1, C_2, NC\}$ to $\mathbb{R}_+$, which specifies their bids for a product conditional on which certificate it has or none at all. Here, C_1 stands for a certificate from Certifier 1 and C_2 a certificate from Certifier 2. As a tie-breaking rule, in the analysis of equilibrium strategies, when a seller type is indifferent between two options, he makes the same decision as the type slightly higher than he is.

2.7.1 Segmentation in identical tests

We consider a case in which these two certifiers employ identical testing technologies. Formally, we have $G_1(t) = G_2(t) = G(t)$ for all $t \in [a, b]$. This setup is to say these two certifiers are providing identical tests and they are identical except that they charge different certification fees. The next result reveals that the usual intuition of Bertrand competition between certifiers need not hold. Even with different certification fees, both certifiers can attract positive measures of seller types in equilibrium.

Proposition 2.8 (Segmentation). *Assume two certifiers charge different certification fees and, without loss of generality, the certifier who charges the higher fee is named Certifier* 1 *and the one charges the lower fee, Certifier* 2, $0 < P_2 < P_1 < b$.

If there exist x_1 and x_2 such that $a < x_2 < x_1 < b$ and

$$P_1 - P_2 = G(x_1)[\Omega(x_1, b) - \Omega(x_2, x_1)] \quad (2.12)$$

$$P_2 = G(x_2)\Omega(x_2, x_1), \quad (2.13)$$

then x_1 and x_2 identify a subgame equilibrium in which types in $(x_1, b]$ strictly prefer testing at Certifier 1, type x_1 is indifferent between testing at either of these two certifiers, types in (x_2, x_1) strictly prefer testing at Certifier 2, type x_2 is indifferent between testing at Certifier 2 and not to test at all, types below x_2 strictly prefer not to test, buyers bid $\Omega(x_1, b)$ for a product with Certificate 1, $\Omega(x_2, x_1)$ for a product with Certificate 2 and 0 for a non-certified product. That is,

$$\rho^*(t \mid P_1, P_2) = TS_1, \forall t \in [x_1, b]$$
$$\rho^*(t \mid P_1, P_2) = TS_2, \forall t \in [x_2, x_1)$$
$$\rho^*(t \mid P_1, P_2) = NTS, \forall t \in [a, x_2)$$
$$\beta^*(C_1 \mid P_1, P_2) = \mu(C_1 \mid P_1, P_2) = \Omega(x_1, b)$$
$$\beta^*(C_2 \mid P_1, P_2) = \mu(C_2 \mid P_1, P_2) = \Omega(x_2, x_1)$$
$$\beta^*(NC \mid P_1, P_2) = 0, \ \mu(NC \mid P_1, P_2) < 0.$$

Proof. See appendix. □

When the equilibrium identified in Proposition 2.8 exists, for instance in our example in subsection 2.7.2, we call such equilibrium *segmentation* equilibrium. The existence of segmentation equilibrium suggests that it is possible for both certifiers to attract positive measures of seller types while charging different fees. Since the testing technologies are identical, they are providing supposedly identical certification service. One may expect that the lower fee certifier takes up entire market demand for the certification service and competition would drive the certification fee to marginal cost as in normal Bertrand competition. In the current setup, this means free certification service.[18] Proposition 2.8, however, shows this line of reasoning need not hold. When segmentation equilibrium exists, certifiers need not engage in Bertrand competition because lowering one's certification fee does not necessarily increase the demand for its

[18] Proposition 2.4 finds free certification is generally not socially optimal.

certification service nor its profit. Being a higher fee certifier does not mean having zero demand either.

This result can be understood in light of the endogeneity of a certificate's value. (Subsection 2.5.2) When the certifiers charge different fees, their certificates have different values in a segmentation equilibrium. Hence, although they have identical testing processes, their end products (certificates) are differentiated.

In the monopoly certifier case, a certification service provides a device that higher types can differentiate themselves from lower types by paying for the test. With two certifiers providing imperfect certification services, those really high types choose the higher fee certifier to differentiate themselves from moderate types.

Remark 2.6. *1. A higher certification fee can serve as a signal of higher product quality.*

2. Even with identical imperfect testing technology, duopoly certifiers need not to engage in Bertrand Competition.

2.7.2 An example in duopoly

We work through an example to verify the existence of segmentation equilibrium.

Example 2.2. *Suppose seller types are distributed on the interval* $[-1, 1]$ *following a power function* $F(t) = \left(\frac{t+1}{2}\right)^{\frac{1}{2}}$. *The testing technology* $G(t)$ *is represented by this power distribution function as well,* $G(t) = F(t) = \left(\frac{t+1}{2}\right)^{\frac{1}{2}}$ *on* $[-1, 1]$.

The type expectation function $\Omega(m, n)$ *is, after simple algebra, simply* $\frac{m+n}{2}$. *Equations (2.12) and (2.13) then read*

$$\begin{aligned} P_1 - P_2 &= \left(\frac{x_1 + 1}{2}\right)^{\frac{1}{2}} \frac{1 - x_2}{2} \text{ and} \\ P_2 &= \left(\frac{x_2 + 1}{2}\right)^{\frac{1}{2}} \frac{x_1 + x_2}{2}. \end{aligned}$$

Suppose Certifier 1 charges $P_1 = 0.6$ *and Certifier 2 charges* $P_2 = 0.1$. *In this case, the above system obtains a unique solution,* $x_1 = 0.4742, x_2 = -0.1648$.

Seller types in $[0.4742, 1]$ *choose Certifier 1, types in* $[-0.1648, 0.4742)$ *choose Certifier 2, types in* $[-1, -0.1648)$ *choose not to test. Type* 0.4742 *is indeed indifferent between choosing either of these two certifiers and type* -0.1648 *is indifferent between choosing Certifier 2 or not to test at all. Buyers in this case bid* $\Omega(0.4742, 1) = (0.4742 + 1)/2 = 0.7371$ *for a product with Certificate 1, bid* $\Omega(-0.1648, 0.4742) = (-0.1648 + 0.4742)/2 = 0.1547$ *for a product with Certificate 2 and bid zero for a non-certified product.*

The profits the certifiers make are

$$\Pi_1(P_1 = 0.6, P_2 = 0.1) = P_1 (1 - F(x_1)) = 0.084873$$

and

$$\Pi_2(P_2 = 0.1, P_1 = 0.6) = P_2 (F(x_1) - F(x_2)) = 0.021233.$$

So in this example the higher fee certifier earns a higher profit than the lower fee certifier.

In the perfect testing case studied in Lizzeri (1999), competition of certifiers will drive the certification fee to zero. When testing technology is imperfect, even if both certifiers provide identical testing technology, the current analysis shows fee differentiation is possible and Bertrand Competition is not guaranteed. The point is that when certifiers charge different fees, there can be subgame equilibria in which high seller types choose the high fee certifier to signal their type. Hence certifiers need not to lower their certification fee to the marginal cost level. In example 2.2, each certifier has a positive profit and lowering one's certification fee doesn't necessarily increase one's demand nor profit.

Remark 2.7. *Although imperfect testing technology limits certifiers' power in collecting generated surplus from the seller, it does help to soften competition among certifiers.*

2.7.3 An alternative explanation to auditing fee differences

The significant fee differentiation between major and non-major auditing firms has long been documented in the accounting literature (e.g., Simunic (1980)). See also more recent evidence like Hay, Knechel, and Wong

(2006).[19] It is also known that in Initial Public Offerings and debt financing, firms audited by major auditors generally receive more favorable bids than those audited by other auditors. Evidences include Teoh and Wong (1993) and Mansi, Maxwell, and Miller (2004) among others. The empirical observation here is, in other words, the positive correlation between auditing fees and bids received.

DeAngelo (1981), Titman and Trueman (1986) and in a context similar to our work, Hvide (2005), suggest that the differences in auditors' auditing qualities or standards are responsible for this observation.[20] Yet, as acknowledged in Hay, Knechel, and Wong (2006), differences in auditing qualities are hard to identify. Here we suggest a new perspective to this question, namely identical imperfect testing technology. We show in Example 2.2 that even two identical testing technologies can support fee differentiation in equilibrium and those who choose the higher fee certifier receive higher bids from the buyers. Applied to the auditing context, those major auditing firms (Certifier 1 in Proposition 2.8) may have exactly the same ability in identifying audited companies' financial soundness as other auditing firms (Certifier 2 in Proposition 2.8). If segmentation equilibrium is supported, by paying a higher audition fee, a company of higher quality receives higher bids in equilibrium. Audited by a non-major auditing firm, however, signals a lower quality. Note also that moderate quality companies will not try major auditing firms since those are too expensive and they are very likely to get unfavorable auditing reports. They try non-major firms nevertheless since the fee is low enough to justify their relatively small probability of getting favorable auditing reports. To apply the above analysis, we only need to assume that auditing processes are imperfect, that is, auditing firms are not able to know exactly the financial situation of each audited firm and yet are able to ensure better companies have a higher probability receiving favorable financial reports.

That major auditing firms make more profits than the rest is also predicted in Example 2.2. Though we have argued that different certification fees

[19] Major auditing firms here refer to the few largest auditing firms. The exact number varies from time to time.

[20] Additional references on this topic can be found in Hvide (2005).

P_1, P_2 are possible in equilibrium, we leave solving the entire duopoly game to future research.

2.8 Conclusion

In this chapter, we propose a general model of imperfect testing technology in certification services. The main assumption of our suggested model is that whenever two products get tested the higher quality product is more likely to pass than the lower quality one. The model also admits continuous quality types and strategic certifiers.

The analysis provided in this chapter aims to improve our understanding of imperfect certification. It's not always clear what a certificate means in real life. Yet, we have seen a large number of successful certification services that are of practical uses. This chapter takes a formal theoretical approach and proves that when a certification service can ensure that higher quality products stand a better chance obtaining a certificate than lower quality products, such certification service can reduce information asymmetry and facilitate trading.

Monopoly certifiers with imperfect testing technologies are not as powerful as they would be if perfect testing technologies were available. According to the analysis, a certifier with an imperfect technology can be completely bypassed. This is in sharp contrast to the case of perfect testing technology.

A separating equilibrium is also supported in which only high quality seller types (products) utilize the certification service. By paying the certification fee a seller type in principle obtains the right to play a lottery. The lottery, however, is type dependent and is in favor of higher types since higher types are more likely to get a certificate for the same certification fee. The value of a certificate is determined jointly by the type distribution and the nature of the testing technology. Welfare accounting shows that the monopolistic certifier's profit maximizing conduct can lead to under or over supply of certification service depending on model specification. The welfare maximizing certification fee is always positive and such that it makes all positive types choose to test. Hence, free certification is not recommended under imperfect testing technology.

When there are two certifiers with identical testing technologies offering certification services in the market, intuition suggests Bertrand competition of the certifiers. While this is true in the perfect testing case studied in Lizzeri (1999), the arguments for Bertrand competition are not valid in imperfect testing cases. Segmentation equilibrium in which higher seller types choose the more expensive certification service and not so high types choose the less expensive service can be supported. In this case, keeping on lowering one's certification fee is not necessarily the best response. In the context of auditing industry, we show that to explain the fee differentiation between major and non-major auditing firms we do not have to assume differences in auditing processes.

2.9 Appendix

2.9.1 Proof of Proposition 2.1

Proof. If no seller types choose to get the product tested, the type population of a non-certified product is exactly the original one. Hence, it is optimal for the buyers to bid $\max\{E(t), 0\}$ for a non-certified product. As long as the buyers believe the type of a certified product $\mu(t \mid C, P) \leq E(t)$, that is, it is not above the population mean, any bid

$$\beta(P, C) = \max\{\mu(t \mid C, P), 0\}$$

for a certified product is one of the best responses (Condition 2.2).

Because a certificate is an off-equilibrium incidence and any type except type a could get a certificate with a strictly positive probability, buyers' beliefs for a certified product can be supported (Condition 2.3).[21,22]

If buyers' bids for a certified product are no higher than those for a non-certified product, no seller types choose to test. Note also that a single

[21]Given that there is a positive probability for low types to pass, buyers' belief are not irrational. For perfect Bayesian equilibrium, any not exactly impossible off-equilibrium belief will do. In other words, there is no prior to be updated.

[22]Here buyers can hold different beliefs so long as they satisfy the specified conditions, i.e., their beliefs for a non-certified product are both no higher than the ex ante type expectation.

type choosing to test does not convince the buyers to bid higher, so the seller will not pay for the test after learning his own type (Condition 2.4).

Given the strategies of the seller and the buyers, the certifier's action is irrelevant (Condition 2.5). □

2.9.2 Proof of Proposition 2.2

Proof. With respect to the certification fee P, we have the following two cases.

$P > b$: It is obvious that in no cases buyers will bid above b. All seller types will make a loss by paying for the test. Since $E(t) \leq 0$, buyers bid zero for a non-certified and up to their belief for a certified product. So any of the stated strategy pair constitutes an equilibrium in these subgames. Note that buyers's out of equilibrium belief $a < \mu(t \mid C, P > b) \leq b$ can be supported.

$P = b$: Note that any combination of seller types other than type b alone choosing to test will result buyers' belief for a certified product being less than b, $\mu(t \mid C) < b$. In turn their bids $\beta(C \mid P) < b$. Choosing to test makes a loss for all seller types in such a situation.

When type b alone chooses to test, however, we have $\mu(t \mid C) = b$. Because type b for sure gets the certificate by choosing to test, type b is indifferent between testing

$$\beta(C \mid P) - P = b - b = 0,$$

and not testing (also 0). Types other than b has a strictly positive possibility of getting no certificate. Consequently, if choose to test, seller types $t < b$ will receive a negative payoff $G(t)b - b < 0$. The only equilibrium other than bypassing when $P = b$ is then the one in which type b alone chooses to test and all others not to. The buyers then bid b for a certified product and 0 for a non-certified product in this equilibrium. Since type b alone is of zero measure, buyers' belief for a non-certified product remains to be the product's prior expectation $E(t)$ which is less than zero.

□

2.9.3 Proof of Proposition 2.3

Proof. The logic of the proof is the following. First, we investigate the properties of equilibrium strategies in subgames induced by $P \in (0, b)$ with some seller types choosing to test, when such equilibrium exists. Second, we prove the existence by constructing strategies that fulfill all such properties. The uniqueness of the equilibrium is then shown by examination of an equivalent mathematical system.

Step 1 is to show that in such equilibria buyers bid more for a certified product and the lowest seller type does not choose to test in equilibrium. In the subgames induced by $0 < P < b$, suppose there exist a set of seller types who choose to test by paying the testing fee P in equilibrium. Denote such a set $\Psi(P)$. That is,

$$\Psi(P) \equiv \{t \mid \rho^*(t \mid P) = TS\}.$$

For all seller types in $\Psi(P)$, the expected payoff from testing has to be no less than what they could get by not to test. We have, $\forall t \in \Psi(P)$,

$$G(t)\beta(C \mid P) + (1 - G(t))\beta(NC \mid P) - P \geq \beta(NC \mid P). \tag{2.14}$$

After rearranging, $\forall t \in \Psi(P)$,

$$G(t)[\beta(C \mid P) - \beta(NC \mid P)] \geq P. \tag{2.15}$$

Since $P > 0$ by assumption, $\forall t \in \Psi(P)$

$$G(t)[\beta(C \mid P) - \beta(NC \mid P)] > 0.$$

Note that $\forall t \in [a, b], G(t) \geq 0$, so *both* $G(t \mid t \in \Psi(P))$ and $\beta(C \mid P) - \beta(NC \mid P)$ have to be strictly larger than zero. That is,

$$a \notin \Psi(P) \wedge \beta(C \mid P) > \beta(NC \mid P). \tag{2.16}$$

So we showed that when there exist a set of seller types who choose to test by paying a strictly positive fee in equilibrium, buyers bid more for a certified product and the lowest seller type a does not test.

Step 2 is to prove when buyers bid more for a certified product the set of seller types that pay for the test exists and is of the form $[x, b]$.
Let's denote $\Gamma(t)$ the difference in expected payoffs for type t between to test and not to.

$$\Gamma(t) \equiv G(t)[\beta(C \mid P) - \beta(NC \mid P)] - P. \tag{2.17}$$

Apparently, $t \in \Psi(P)$ if and only if $\Gamma(t) \geq 0$. Note that for any given P and β such that $0 < P < b$ and $\beta(C \mid P) > \beta(NC \mid P)$, $\Gamma(t)$ is continuous and strictly increasing in t; $\Gamma(b) \geq \Gamma(t)\ \forall t \in [a, b]$. Hence, if any types choose to test, type b must be one of them, $b \in \Psi(P)$.

1. Suppose type b is the only element of $\Psi(P)$, that is $\Psi = \{b\}$. From Proposition 2.2, $\beta(C \mid P) = b$ and $\beta(NC \mid P) = 0$. Therefore, combined with $G(b) = 1$ and $P < b$, we have $\Gamma(b) = G(b)b - P > 0$. Solving the equation $G(\bar{t})b - P = 0$, we have $\bar{t} = G^{-1}(P/b)$ where G^{-1} is the inverse of G. Because G(t) is strictly increasing, for the types $t \in (G^{-1}(P/b), b)$, their expected payoff of testing $G(t)b - P$ is strictly larger than zero. These types will also choose to test. Hence we prove that when $0 < P < b$, the supposition that $\Psi(P)$ has only one element is false.

2. Now we know $\Psi(P)$, when it exists, contains more elements than just type b alone. Note also $G(t)$ is strictly increasing and $\beta(C \mid P) > \beta(NC \mid P)$. Therefore, if a type t' other than b is in $\Psi(P)$, that is, if the expression (2.15) holds for t', it also must hold with strict inequality for any $t > t'$. Hence, all t such that $t > t'$ should be in $\Psi(P)$ as well. Moreover, these types strictly prefer testing. In equilibrium, the set of seller types strictly prefer testing must be of the form $(x, b]$ or $[x, b]$ for some $x < b$.

3. For type b, we have

$$\Gamma(b) = G(b)[\beta(C \mid P) - \beta(NC \mid P)] - P > 0.$$

This inequality holds strictly because type b obtains a higher payoff than type $\inf \Psi(P)$. For type a, $G(a) = 0$,

$$\Gamma(a) = -P < 0.$$

> By the continuity and monotonicity of function $\Gamma(t)$, there is a unique solution for $\Gamma(t) = 0$ in the domain of (a, b). Suppose $x = \Gamma^{-1}(0)$, for type x, it is indifferent between to test and not to test. For $t > x$, $\Gamma(t) > 0$. Consequently, when buyers bid more for a certified product the set of seller types that pay for the test exists in each subgame induced by $0 < P < b$ and, by the tie-breaking rule, is of the form $[x, b]$.

Step 3 is to construct the required buyers' optimal bids.
In this part we search out compatible buyers' strategies, $\beta(\cdot \mid P)$ that will satisfy

$$\beta(C \mid P) > \beta(NC \mid P) \geq 0.$$

Buyers bid positively for a certified product ($\beta(C \mid P) > 0$), only when their beliefs for a certified product is positive ($\mu(t \mid C) > 0$). In equilibrium, $\mu(t \mid C)$ requires to be consistent with rational expectation,

$$\mu(t \mid C) = E(t \mid C).$$

Further, by the following identity

$$\Pr(C)E(t \mid C) + (1 - \Pr(C))E(t \mid NC) \equiv E(t) < 0, \tag{2.18}$$

it cannot be true that both conditional expectations are non-negative. Hence, to have $E(t \mid C) > 0$, $E(t \mid NC)$ has to be less than zero. In turn, $\mu(t \mid NC) < 0$ and $\beta(NC \mid P) = 0$. Since the set of seller types that choose to test is of the form $[x, b]$, the buyers' Bayesian updated belief should be,

$$E(t \mid C) = \frac{\int_x^b tG(t)dF(t)}{\int_x^b G(t)dF(t)} = \Omega(x, b). \tag{2.19}$$

The bid for a certified product is, therefore, $\beta(C \mid P) = E(t \mid C) = \Omega(x, b)$.
To find indifferent type x, we need to solve

$$G(x)[\Omega(x, b) - 0] = P.$$

The existence and uniqueness of the solution is established in the next step. Note that if $G(x)\Omega(x, b) = P$ holds, then $\Omega(x, b) = \frac{P}{G(x)}$. Since both P and $G(t), \forall t \in (a, b]$ are larger than zero, $\Omega(x, b)$ is also large than zero. Hence we constructed feasible buyers' strategies and their beliefs. For $0 < P < b$, buyers bid

$$\beta(C \mid P) = \mu(t \mid C) = E(t \mid C) = \Omega(x, b)$$

and $\beta(NC \mid P) = 0$ with belief $\mu(t \mid NC) < 0$. These bidding strategies are compatible to the seller's strategy.

Step 4 is to prove the existence and uniqueness of the indifferent type x for each $0 < P < b$.
The existence and uniqueness of the equilibrium in the subgames boils down to the existence and uniqueness of the solution to $\Gamma(t) = 0$ or

$$G(x)\Omega(x, b) = P. \tag{2.20}$$

Note that $\Omega(x, b)$ is bounded, it is clear that

$$\begin{aligned}\lim_{x \to a} G(x)\Omega(x, b) &= 0 \text{ and} \\ \lim_{x \to b} G(x)\Omega(x, b) &= b.\end{aligned}$$

Note also function $\Omega(x, b)$ and $G(x)\Omega(x, b)$ are continuous,[23] $G(x)\Omega(x, b) = P$ obtains at least one solution when $0 < P < b$.

To prove the uniqueness, we first derive the derivative of the function $\Omega(x, b)$,

$$\frac{d\Omega(x, b)}{dx} = \frac{G(x)f(x) \int_x^b (t - x)G(t)dF(t)}{\left(\int_x^b G(t)dF(t)\right)^2}.$$

It's easy to verify that all parts in the right hand side are positive. Hence $\frac{d\Omega(x,b)}{dx} > 0$ and $\Omega(x, b)$ increases in x.

According to the value of $\Omega(a, b)$, we discuss two cases.

1. When $\Omega(a, b) \geq 0$, then $\Omega(x, b) \geq 0, \forall x \in (a, b]$. Since the derivative of the function $G(x)\Omega(x, b)$ is as the following:

$$\frac{d\left(G(x)\Omega(x, b)\right)}{dx} = g(x)\Omega(x, b) + G(x)\frac{d\Omega(x, b)}{dx}. \tag{2.21}$$

[23]The continuity of $\Omega(x, b)$ follows from the theorem that the quotient of two continuous functions is continuous. That the divisor $\int_x^b G(t)dF(t)$ is non-zero for $x \in (a, b)$ is checked.

All parts are positive and $G(x)\Omega(x,b)$ increases monotonically form 0 to b. Hence Equation 2.20 only obtains one solution when $0 < P < b$.

2. When $\Omega(a,b) < 0$, because of continuity and monotonicity of $\Omega(x,b)$, we first find $\bar{x}$ such that $\Omega(\bar{x},b) = 0$. For any $x < \bar{x}$, $\Omega(x,b) < 0$ hence Equation (2.20) has no solution. Within the interval of $[\bar{x},b]$, $G(x)\Omega(x,b)$ increases monotonically form 0 to b. Hence Equation (2.20) only obtains one solution in $[\bar{x},b]$ when $0 < P < b$.

This proves the existence and uniqueness of the indifferent type x for each $0 < P < b$. Together with above steps, all conditions required by equilibrium notion (Definition 2.1) for the subgames are satisfied and we have established uniqueness. □

2.9.4 Proof of Proposition 2.4

Proof.

1. If $\Omega(a,b) > 0$, it's easy to verify that all types choose to test and buyers bid $\Omega(a,b)$ for a certified product and zero for a non-certified product is an equilibrium.

2. On the other hand, if buyers make positive bids for a certified product, all types above a will choose to test. This is because there is simply no cost involved in testing for the seller and there is a certain probability receiving positive bids. Hence, to test is the dominant strategy except for the lowest type. Suppose $\Omega(a,b) \leq 0$, then buyers' belief for a certified product is non-positive and consequently will bid zero for a certified product. This contradicts the supposition that buyers make positive bids. Hence when buyers make positive bids, $\Omega(a,b) > 0$.

□

2.9.5 Proof of Proposition 2.5

Proof. According to Corollary 2.1, if $P \geq b$ or $P = 0$ the seller's profit will be zero. Note as well that according to the proof of the uniqueness of the subgame equilibrium when $0 < P < b$, $G(t)\Omega(t,b)$ is a continuous and strictly increasing function in (a, b) or $(\overline{x}, b)$ where $\overline{x}$ is find by solving $\Omega(\overline{x}, b) = 0$ when $\Omega(a, b) < 0$.[24] Hence, its inverse function $\kappa(P)$ from $(0, b)$ to (a, b) or $(\overline{x}, b)$ is also strictly increasing in $(0, b)$. Consequently, the certifier can also maximize his profit by optimally choosing the indifferent type x. The certification fee P is then $G(x)\Omega(x, b)$. From Proposition 2.3, the demand for certification service will be $1 - F(x)$. The product of these two components give the profit,[25]

$$\Pi(x) = (1 - F(x))G(x)\Omega(x, b), x \in (a, b). \tag{2.22}$$

Since the extreme points in Corollary 2.1 are dominated, the maximum *is obtained* inside the interval. The certifier's best response to the equilibrium strategies of the seller and the buyers is hence P^* defined in Equation (2.6). This, together with Proposition 2.3, concludes the proof. □

2.9.6 Proof of Proposition 2.6

Proof. Because buyer always bid up to the expected value of a certified product, they do not derive positive gains. Social welfare is then the sum of the payoff of the certifier and the payoff of the seller. Moreover, the sum is exactly what buyers pay for the product in equilibrium, because this is the only source for the revenues of both the certifier and the seller.

Since buyers bid zero for a non-certified product, trading only takes place when the product has a certificate. The total surplus is then, for a given certification fee, the result of multiplying buyers' bid for a certified product and the probability of the product getting a certificate,

$$\Omega(\kappa(P), b) \int_{\kappa(P)}^{b} G(t)dF(t) = \int_{\kappa(P)}^{b} tG(t)dF(t).$$

Taking derivative of this expression gives us,

$$\frac{d\left(\int_{\kappa(P)}^{b} tG(t)dF(t)\right)}{d\left(\kappa(P)\right)} = -\kappa(P)G(\kappa(P))f(\kappa(P)). \tag{2.23}$$

[24]See 2.9.3, especially Step 4 and Equation (2.21).

[25]Note that when $\Omega(x, b) < 0$ when $x \in (a, \overline{x})$, $\Pi(x) < 0$ on this interval too. This allows us to represent the problem as Equation (2.22) without explicitly write the case for $(\overline{x}, b)$.

It is then obvious that the right hand side of equation (2.23) is strictly negative when $\kappa(P) > 0$, strictly positive when $\kappa(P) < 0$ and equal to zero when $\kappa(P) = 0$. Maximization of $\int_{\kappa(P)}^{b} tG(t)dF(t)$ with $a < \kappa(P) < b$ requires $\kappa(P) = 0$. The welfare maximizing certification fee is hence $P^{**} = G(0)\Omega(0,b)$. □

2.9.7 Proof of Proposition 2.7

Proof. In the proof of Proposition 2.5 we show that the certifier can set the indifferent type x to maximize profit. The first order derivative of $\Pi(x) = G(x)\Omega(x,b)[1 - F(x)]$ is

$$\begin{aligned} & g(x)\Omega(x,b)[1-F(x)] + G(x)[1-F(x)]\frac{d\Omega(x,b)}{dx} - G(x)\Omega(x,b)f(x) \\ =& g(x)\Omega(x,b)\left([1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) - \frac{G(x)}{g(x)}f(x)\right). \end{aligned} \tag{2.24}$$

Since $g(x) > 0$ and $\Omega(x,b) > 0$, when $0 < P < 0$ a necessary condition for profit maximization is

$$\begin{aligned} & [1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) - \frac{G(x)}{g(x)}f(x) = 0 \\ \Rightarrow\ & [1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) = \frac{G(x)}{g(x)}f(x) \\ \Rightarrow\ & \frac{f(x)}{1-F(x)} = \frac{g(x)}{G(x)} + \frac{d\Omega(x,b)}{dx}\frac{1}{\Omega(x,b)}. \end{aligned} \tag{2.25}$$

Hence if profit maximizing x is socially optimal, i.e., $x^* = 0$, the next condition has to hold,

$$\frac{f(0)}{1-F(0)} = \frac{g(0)}{G(0)} + \frac{1}{\Omega(0,b)}\frac{d\Omega(x,b)}{dx}\Big|_{x=0}.$$

Note that

$$\begin{aligned} \frac{d\Omega(x,b)}{dx} &= \frac{d}{dx}\left(\frac{\int_x^b tG(t)f(t)dt}{\int_x^b G(t)f(t)dt}\right) \\ &= \frac{G(x)f(x)\int_x^b tG(t)f(t)dt - xG(x)f(x)\int_x^b G(t)f(t)dt}{\left(\int_x^b G(t)f(t)dt\right)^2} \end{aligned}$$

hence

$$\frac{d\Omega(x,b)}{dx}\Big|_{x=0} = \frac{G(0)f(0)\int_0^b tG(t)f(t)dt}{\left(\int_0^b G(t)f(t)dt\right)^2}.$$

Consequently,

$$\begin{aligned}
\frac{f(0)}{1-F(0)} &= \frac{g(0)}{G(0)} + \frac{1}{\Omega(0,b)}\frac{d\Omega(x,b)}{dx}\Big|_{x=0} \\
&= \frac{g(0)}{G(0)} + \left(\frac{\int_0^b G(t)f(t)dt}{\int_0^b tG(t)f(t)dt}\right)\frac{G(0)f(0)\int_0^b tG(t)f(t)dt}{\left(\int_0^b G(t)f(t)dt\right)^2} \\
&= \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}.
\end{aligned}$$

This proves the first part of Proposition 2.7. With the additional condition of profit function concavity, we know the second derivative is negative and the first order condition (2.25) becomes sufficient for profit maximization. However, we are interested in the value of the first derivative (2.24) at $x = 0$. When it is larger than 0, the monopoly certifier will increase P in order to increase x and because of the profit function concavity the profit maximizing x^* is larger than 0. Consequently, some positive types find it too expensive to test and the certification service is under supplied. Hence, the condition for undersupply is

$$\begin{aligned}
& g(x)\Omega(x,b)\left([1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) - \frac{G(x)}{g(x)}f(x)\right)\Big|_{x=0} > 0 \\
\Rightarrow\ & \left([1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) - \frac{G(x)}{g(x)}f(x)\right)\Big|_{x=0} > 0 \\
\Rightarrow\ & \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt} > \frac{f(0)}{1-F(0)}.
\end{aligned}$$

Likewise, when

$$\frac{f(0)}{1-F(0)} > \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}$$

there is oversupply of certification service. So we proved the second part of Proposition 2.7. □

2.9.8 Proof of Proposition 2.8

Proof. The following is to prove when $a < x_2 < x_1 < b$ solve the system of equations (2.12) and (2.13), we claim the strategies profile in Proposition 2.8 constitutes a perfect Bayesian equilibrium. This is done in the following steps.

1. First, for given $0 < P_2 < P_1 < b$ when $a < x_2 < x_1 < b$ solves

$$\begin{aligned} P_1 - P_2 &= G(x_1)[\ \Omega(x_1, b) - \Omega(x_2, x_1)] \\ P_2 &= G(x_2)\Omega(x_2, x_1), \end{aligned}$$

   we have $\Omega(x_1, b) > \Omega(x_2, x_1) > 0$. This is because $G(t) > 0, \forall t > a$.

2. Suppose types in $[x_1, b]$ choose Certifier 1, types in $[x_2, x_1)$ choose Certifier 2 and types in $[a, x_2)$ chooses not to test, then buyers expectation for a product certified by Certifier 1 $\mu(C_1 \mid P_1, P_2) = E(C_1 \mid P_1, P_2)$ is $\Omega(x_1, b)$ and for a product certified by Certifier 2 $\mu(C_2 \mid P_1, P_2) = E(C_2 \mid P_1, P_2)$ is $\Omega(x_2, x_1)$. Because the prior expectation of the product is negative, the expectation for a none certified product $\mu(NC \mid P_1, P_2)$ is less than zero.

3. Then buyers bids are $\beta(C_1 \mid P_1, P_2) = \Omega(x_1, b)$ for a product certified by Certifier 1, $\beta(C_2 \mid P_1, P_2) = \Omega(x_2, x_1)$ for a product certified by Certifier 2 and 0 for a non-certified product.

4. Since $P_1 - P_2 = G(x_1)[\ \Omega(x_1, b) - \Omega(x_2, x_1)]$, $P_2 = G(x_2)\Omega(x_2, x_1)$ and $G(t)$ strictly increases in t, we have for all $x_1 < t \leq b$,

$$\begin{aligned} G(t)[\ \Omega(x_1, b) - \Omega(x_2, x_1)] &> P_1 - P_2 \\ G(t)\Omega(x_2, x_1) &> P_2 \\ \Longrightarrow G(t)\ \Omega(x_1, b) - P_1 &> G(t)\ \Omega(x_2, x_1) - P_2 > 0; \end{aligned}$$

   for all $x_2 < t < x_1$,

$$\begin{aligned} G(t)[\ \Omega(x_1, b) - \Omega(x_2, x_1)] &< P_1 - P_2 \\ G(t)\Omega(x_2, x_1) &> P_2 \\ \Longrightarrow G(t)\ \Omega(x_2, x_1) - P_2 &> G(t)\ \Omega(x_1, b) - P_1 \\ G(t)\ \Omega(x_2, x_1) - P_2 &> 0; \end{aligned}$$

for all $a \leq t < x_2$,

$$\begin{aligned} G(t)[\ \Omega(x_1,b) - \Omega(x_2,x_1)] &< P_1 - P_2 \\ G(t)\Omega(x_2,x_1) &< P_2 \\ \Longrightarrow 0 > G(t)\ \Omega(x_2,x_1) &> P_2.G(t)\ \Omega(x_1,b) - P_1. \end{aligned}$$

Hence we compared the expected payoffs for different choices for types in $[a, b]$. Employing also the tie break rule, we conclude that it is true that types in $[x_1, b]$ choose Certifier 1, types in $[x_2, x_1)$ choose Certifier 2 and types in $[a, x_2)$ choose not to test.

5. In summary, if there exist such x_1, x_2 that satisfy $a < x_2 < x_1 < b$ and solve the system of equations (2.12) and (2.13), the above construction proves that the strategy combinations in Proposition 2.8 constitute an equilibrium for the given P_1, P_2.

□

Chapter 3

A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand[1]

3.1 Introduction

Three main frameworks have been widely used to study product differentiation and monopolistic competition: representative consumer, discrete choice and spatial models. In representative consumer and discrete choice models, it is understood that equilibrium product variety could either be excessive or insufficient or optimal depending on the model configuration.[2] In spatial models such as Vickrey (1964) and Salop (1979), however, analysis shows that there is always excessive entry. This result became known as the excess entry theorem. Matsumura and Okamura (2006) extend this result for a large set of transportation costs and production technologies.[3]

One drawback of standard spatial models such as Hotelling (1929) and Salop (1979) is that consumer demand is completely inelastic. Each con-

[1]This chapter is coauthored with Tobias Wenzel and has been published in the *International Journal of Industrial Organization*.

[2]See, for example, Dixit and Stiglitz (1977), Pettengill (1979), Lancaster (1975), Sattinger (1984), Hart (1985) among many others.

[3]They do point out that there are also some situations in which entry can be insufficient.

sumer demands a single unit of a differentiated product.[4] In this chapter we lift this restrictive assumption in the context of the Salop model and investigates the implications of price-dependent demand for the excess entry theorem.

To this aim, we incorporate a demand function with a constant elasticity into the Salop framework. We find that the number of entrants in a free-entry equilibrium is the lower the more elastic demand is. We also find that only when demand is sufficiently inelastic, there is excess entry. Otherwise, entry is insufficient. In the limiting case when the demand elasticity approaches unity, the market becomes a monopoly. Thus, the excess entry theorem is only valid for sufficiently inelastic demand and hence, the assumption of inelastic demand, typically employed, is not an innocuous one. This result is independent of whether we use a first-best or a second-best welfare benchmark. As a consequence of our welfare analysis we point out when and how a public policy can be desirable. In an extension, we broaden our result with a more general transportation cost function.

Our model setup is closely related to Anderson and de Palma (2000). The purpose of their paper is to develop a model that integrates features of spatial models where competition is localized and representative consumer models where competition between firms is global. The formulation of the individual demand function is the same as in Anderson and de Palma (2000).[5] They also consider a constant elasticity demand function. However, the difference lies in the perspectives of the works. Their focus is on the interaction between local and global competition, while the present work focuses on the implications of price-dependent demand on the excess entry result in spatial models.

Other approaches to introduce price-dependent demand into spatial models are Boeckem (1994), Rath and Zhao (2001) and Peitz (2002).[6] The first

[4]The assumption of inelastic demand can be a realistic one in the case of some durable goods, e.g. houses, etc. However, in case of nondurables, e.g. groceries, etc, the assumption seems less plausible.

[5]Our model is the special case of Anderson and de Palma (2000) when eliminating the taste component in their utility function. Thus, in this chapter we consider a pure spatial model, while Anderson and de Palma (2000) analyze a model that has features of spatial and representative consumer models.

[6]A recent paper by Peng and Tabuchi (2007) combines a model of spatial competition with taste for variety in the spirit of Dixit and Stiglitz (1977). In their setup, the quantity

two papers consider variants of the Hotelling framework. Boeckem (1994) introduces heterogenous consumers with respect to reservation prices. Depending on the price charged by firms some consumers choose not to buy a product. The paper by Rath and Zhao (2001) introduces elastic demand in the Hotelling framework by assuming that the quantity demanded by each consumer depends on the price charged. The authors propose a utility function that is quadratic in the quantity of the differentiated product leading to a linear demand function. In contrast to those two models we build on the Salop model as we are interested in the relationship between price-dependent demand and entry into the market. Our approach is closer to Rath and Zhao (2001) as we also assume that each consumer has a downward sloping demand for the differentiated good. However, our demand function takes on a different functional form which has the advantage of yielding tractable results. Peitz (2002) features unit-elastic demand both in Hotelling and Salop settings but focuses on conditions for existence of Nash equilibrium in prices. He does not consider entry decisions.

This chapter is organized as follows. Section 3.2 sets up the model. Section 3.3 presents the analysis of the model. Section 3.4 analyzes the welfare outcome and policy implications. An extension with more general transportation cost functions is provided in section 3.5. Section 3.6 summarizes.

3.2 The model

There is a unit mass of consumers who are located on a circle with circumference one. The location of a consumers is denoted by x. In contrast to Salop (1979), consumers are not limited to buy a single unit of the differentiated good. The amount they purchase depends on the price. We propose the following utility function which leads to a demand function with a constant elasticity of ϵ. We assume that this utility function is identical for all consumers:

demanded also depends on the price. However, their focus is a different one. They study the incentives of how much variety to offer and how many stores to establish. A paper by Hamilton, Klein, Sheshinski, and Slutsky (1994) analyzes elastic demand in a model with quantity competition. In contrast to the present note the authors employ a transportation costs per unit of quantity purchased.

$$U = \begin{cases} \left(V - \frac{\epsilon}{1-\epsilon} q_d^{\frac{\epsilon-1}{\epsilon}} - t * dist\right) + q_h & \text{if consumes the differentiated product} \\ q_h & \text{otherwise.} \end{cases} \tag{3.1}$$

The utility derived by the consumption of the differentiated good consists of three parts. There is a gross utility for consuming this good (V). The second utility component depends on the quantity consumed (q_d). The parameter ϵ —which lies between (0,1)—will later turn out to be the demand elasticity. Finally, consumers have to incur transportation costs if the product's attributes do not match consumers' locations. We assume that transportation costs do not depend on the quantity consumed. Furthermore, we assume that transportation costs are linear in distance.[7] In section 3.5, we will lift this assumption and cover a broader class of transportation cost functions, namely power transportation costs. The variable q_h denotes the quantity of a homogenous good which serves as a numeraire good. The utility is linear in this commodity. Additionally, we make the assumption that the gross utility of the differentiated good (V) is large enough such that no consumers abstains from buying the differentiated product.[8]

Each consumer has an exogenous income of Y which he can divide between the consumption of the differentiated good and the numeraire good. The price of the differentiated good is p_d, while the price of the numeraire is normalized to one. This leads to the following budget constraint:

$$Y = p_d * q_d + q_h. \tag{3.2}$$

Consumers maximize their utility (3.1) under their budget constraint (3.2). Then, demand for the differentiated product and the numeraire is:

$$\hat{q_d} = p_d^{-\epsilon}, \tag{3.3}$$

[7]This allows a direct comparison to Salop (1979) model because the transportation cost is linear in that paper as well.

[8]This helps us to avoid situations in which a firm could be a local monopoly, hence the kink in the firm's demand curve.

$$\hat{q}_h = Y - p_d^{1-\epsilon}. \tag{3.4}$$

The demand for the differentiated good exhibits a constant demand elasticity of ϵ. A higher value of ϵ corresponds to more elastic demand. The limit case of $\epsilon \to 0$ corresponds to completely inelastic demand. Inserting these demand functions into equation (3.1) gives the indirect utility a consumer derives from consuming the differentiated product from a certain firm:

$$\hat{U} = V + Y - \frac{1}{1-\epsilon} p_d^{1-\epsilon} - t * dist. \tag{3.5}$$

There are n firms that offer the differentiated product. We assume that these firms are located equidistantly on the circle. Hence, the distance between two neighboring firms is $\frac{1}{n}$. Consumers choose to buy the differentiated product from the firm which offers them the highest utility. Given the symmetric structure of the model, we seek for a symmetric equilibrium. Therefore we derive demand of a representative firm i. The marginal consumer is the consumer who is indifferent between choosing firm i and an adjacent firm. When firm i charges a price p_i while the remaining firms charge a price p, the marginal consumer is implicitly given by

$$V + Y - \frac{1}{1-\epsilon} p_i^{1-\epsilon} - t\bar{x} = V + Y - \frac{1}{1-\epsilon} p^{1-\epsilon} - t\left(\frac{1}{n} - \bar{x}\right), \tag{3.6}$$

or explicitly by

$$\bar{x} = \frac{1}{2n} + \frac{p^{1-\epsilon} - p_i^{1-\epsilon}}{2(1-\epsilon)t}. \tag{3.7}$$

As each firm faces two adjacent firms, the number of consumers choosing to buy from firm i is $2\bar{x}$. According to equation (3.3), each consumer buys an amount of $\hat{q}_i = p_i^{-\epsilon}$. Hence total demand at firm i is:

$$D_i = 2\bar{x} * p_i^{-\epsilon}. \tag{3.8}$$

In contrast to the Salop model, total demand consists now of two parts: market share and quantity per consumer.

3.3 Analysis

This section analyzes the equilibrium. We start by deriving equilibrium prices for a given number of firms in the market. In a second step, we seek to determine the number of firms that enter.

3.3.1 Price equilibrium

We look for a symmetric equilibrium in which all firms charge the same price. Assuming zero production costs, the profit of a representative firm i when this firm charges a price p_i and all remaining firms charge a price p is given by:

$$\Pi_i = \left[\frac{1}{n} + \frac{p^{1-\epsilon} - p_i^{1-\epsilon}}{(1-\epsilon)t}\right] p_i^{-\epsilon} p_i. \tag{3.9}$$

Maximizing profits with respect to the price p_i and assuming symmetry among all firms leads to the following equilibrium price:[9]

$$p^* = \left[(1-\epsilon)\frac{t}{n}\right]^{\frac{1}{1-\epsilon}}. \tag{3.10}$$

The corresponding quantity purchased by each consumer then is

$$q^* = \left[(1-\epsilon)\frac{t}{n}\right]^{-\frac{\epsilon}{1-\epsilon}}. \tag{3.11}$$

As in the Salop model, the equilibrium price increases in transportation costs and decreases in the number of firms in the market. Conversely, the quantity purchased by each consumer rises with the number of firms and decreases with transportation costs. More interesting is the impact of the

[9]For the proof of the existence of a symmetric price equilibrium, the reader is referred to Anderson and de Palma (2000).

demand elasticity on the equilibrium price and quantity. Differentiation with respect to ϵ yields:

$$\frac{\partial p^*}{\partial \epsilon} \gtreqless 0 \Leftrightarrow \frac{(1-\epsilon)t}{n} \gtreqless e, \tag{3.12}$$

$$\frac{\partial q^*}{\partial \epsilon} \lesseqgtr 0 \Leftrightarrow \frac{(1-\epsilon)t}{n} \gtreqless e^{\epsilon}. \tag{3.13}$$

where e denotes the Euler number. A higher demand elasticity has an ambiguous impact on equilibrium price and quantity. It can lead to a higher price as well as to a lower price. The intuition behind this result lies in the fact that firms can attract additional demand in two ways, via a larger market share and a larger quantity per consumer. Note, however, that the revenue per customer $p^*q^* = \frac{(1-\epsilon)t}{n}$ decreases in the price elasticity. In the limiting case of $\epsilon \to 1$, revenue per customer approaches zero.

In the equilibrium with a given number of firms in the market, each firm makes a profit of

$$\Pi^* = \frac{t(1-\epsilon)}{n^2}. \tag{3.14}$$

The impact of the demand elasticity on firms' profits is unambiguous. A larger demand elasticity reduces profits. This is due to the result that revenue per customer decreases with the demand elasticity and that the market share is constant at $\frac{1}{n}$ in equilibrium. Hence, product market competition is tougher as consumers react stronger to price changes. Higher transportation costs and a smaller number of active firms increase profits.

Result 3.1. *For a given number of firms, profits decrease with the demand elasticity.*

3.3.2 Entry

Until now the analysis treated the number of firms which offer differentiated products as exogenously given. We now investigate the number of active firms when it is endogenously determined by the zero profit condition. We assume that to enter, a firm has to incur an entry cost or fixed cost of f. Additionally, we treat the number of entrants as a continuous variable. Setting equation (3.14) equal to f and solving for n yields the number of entrants:

$$n^c = \sqrt{\frac{t(1-\epsilon)}{f}}. \tag{3.15}$$

The comparative static results concerning transportation costs and fixed costs are as expected. Higher transportation costs lead to more entry while higher fixed costs to less entry. The interesting result concerns the impact of the demand elasticity:

Result 3.2. *The number of entrants decreases in the demand elasticity.*

A larger demand elasticity leads to less entry into the market. The reason for this result is that a higher elasticity leads to lower profits for any given number of firms (see result 3.1).

Corresponding price and quantity in a free-entry equilibrium are:

$$p^c = \left[\sqrt{1-\epsilon}\sqrt{tf}\right]^{\frac{1}{1-\epsilon}}, \tag{3.16}$$

$$q^c = \left[\sqrt{1-\epsilon}\sqrt{tf}\right]^{-\frac{\epsilon}{1-\epsilon}}. \tag{3.17}$$

Higher transportation costs and higher fixed costs lead to higher prices and to lower quantities. As in the equilibrium for a given number of firms, the impact of the demand elasticity on price and quantity is ambiguous. More elastic demand may lead to higher or lower prices and quantities.

The model has interesting results in the limiting cases.

Result 3.3. *i) With $\epsilon \to 0$, the model reduces to the Salop model. ii) As $\epsilon \to 1$, the market is monopolized.*

When demand is completely inelastic, $\epsilon \to 0$, the model reduces to the Salop model. Thus that model is a special case of the present one. As the demand elasticity approaches unity, a monopoly is the outcome. Competition in the market is so tough that as soon as more than one firm enters the market profits are driven to zero (see equation (3.14)).

3.4 Welfare

This section considers the welfare and policy implications. We ask whether there is excess entry into the market as it is the case in models with inelastic demand.

In contrast to models with inelastic demand, we have to consider prices in our welfare analysis as they have an impact on the quantity purchased and hence on welfare. We define social welfare as the sum of consumer utility and industry profits:

$$W = \underbrace{V + Y - \frac{1}{1-\epsilon} p^{1-\epsilon} - 2n \int_0^{\frac{1}{2n}} tx \; dx}_{\text{Consumer welfare}} + \underbrace{p^{1-\epsilon} - fn}_{\text{Industry profits}} . \tag{3.18}$$

We consider two different welfare benchmarks, a first-best benchmark in which the social planner chooses both the level of entry and the price charged by firms, and a second-best benchmark in which the social planner can only control the level of entry, but not prices. Our result are qualitatively independent of the choice of the welfare benchmark.

3.4.1 First-best benchmark

In the first-best benchmark, the social planner can control prices and level of entry, that is he maximizes total welfare with respect to p and n. From equation (3.18), we see that the optimal price set by the regulator is equal to marginal cost, in this case, $p = 0$. Inserting this into equation (3.18) yields

$$W = V + Y - 2n \int_0^{\frac{1}{2n}} tx \; dx - fn. \tag{3.19}$$

The problem for the social planner is then identical to the case with inelastic demand, hence reduced to a trade-off between transportation costs and fixed costs. The optimal number of entrants is

$$n^f = \sqrt{\frac{t}{4f}}. \tag{3.20}$$

Comparison with the free-entry level, n^c, leads to the following result:

Result 3.4. *Compared to the first-best benchmark, there is excess entry when* $\epsilon < \frac{3}{4}$*, insufficient entry when* $\epsilon > \frac{3}{4}$*, and optimal entry when* $\epsilon = \frac{3}{4}$.

The previous result shows that the result of excess entry in the Salop model does not hold when demand is elastic. In the model with elastic demand whether there is too much entry or not enough depends on the demand elasticity. Whenever demand is sufficiently inelastic, there is excess entry as is the case in the Salop model ($\epsilon \to 0$). However, if the demand elasticity exceeds $\frac{3}{4}$, there is insufficient entry into the market. Only when $\epsilon = \frac{3}{4}$, entry coincides with the socially optimal number. Thus, the excess entry theorem in spatial models depends crucially on the assumption of inelastic demand.

3.4.2 Second-best benchmark

Here we derive the welfare-maximizing number of firms given their pricing behavior after entry. Inserting equation (3.10) into (3.18) gives

$$W = V + Y - \frac{t}{n} - 2n \int_0^{\frac{1}{2n}} tx \; dx + \frac{t(1-\epsilon)}{n} - fn. \qquad (3.21)$$

Maximizing total welfare (3.21) with respect to n yields the optimal number of firms:[10]

$$n^s = \sqrt{\frac{t(1+4\epsilon)}{4f}}. \qquad (3.22)$$

Comparing the optimal number of firms, n^s, with the outcome under free entry, n^c, the following result can be established:

Result 3.5. *Compared to the second-best benchmark, there is excess entry when* $\epsilon < \frac{3}{8}$*, insufficient entry when* $\epsilon > \frac{3}{8}$*, and optimal entry when* $\epsilon = \frac{3}{8}$.

Using the second-best benchmark, our result has the same structure as with the first-best benchmark. For sufficiently inelastic demand, we get excess entry and for sufficiently elastic demand, we get insufficient entry.

[10]The second-order condition for maximization is satisfied: $-\frac{t(1+4\epsilon)}{2n^3} < 0$.

3.4.3 Policy implications

Here we derive some policy implications of our welfare analysis focusing on the case of the second-best welfare benchmark. Suppose that a government agency may either charge a fee against or grant a subsidy to each entry, e.g. license fee or start-up funds, respectively. Let s denote the value of such a transfer. When $s < 0$ we call it a subsidy, and when $s > 0$ we call it an entry fee.

Hence the number of firms under such an otherwise "Free Entry" policy now is:

$$n^{c'} = \sqrt{\frac{t(1-\epsilon)}{f+s}}. \tag{3.23}$$

This, of course, follows directly from equation (3.15) by adjusting the fixed cost term accordingly. By setting equation (3.23) equal to (3.22), we can determine the value of s that induces optimal entry into the market. This value is

$$s = f\frac{3-8\epsilon}{1+4\epsilon}. \tag{3.24}$$

The following corollary then immediately follows from result 3.5.

Corollary 3.1. *i) When $\epsilon < \frac{3}{8}$, a government agency should charge an entry fee to reduce excess entry; ii) when $\epsilon > \frac{3}{8}$, a government agency should subsidize entry.*

By such a transfer scheme, a government agency could effectively influence the number of active firms.

3.5 Power transportation costs

This section reconsiders the analysis assuming a more general transportation cost function. Instead of linear transportation costs, we now assume power transportation costs tx^{β} with $\beta \geq 1$. This functional form is also considered by Anderson, de Palma, and Thisse (1992) and Matsumura and Okamura (2006) which both show that the excess entry theorem always holds in the case of inelastic demand.[11] Our analysis will show that their result depends very much on the assumption of inelastic demand.

Following the same steps as in section 3.3, we can derive the number of entrants in a free-entry equilibrium and the socially optimal number. The derivation of these results is given in appendix 3.7.

The number of entrants in a free-entry equilibrium is

$$n^c = \left[\frac{(1-\epsilon)t\beta 2^{1-\beta}}{f} \right]^{\frac{1}{1+\beta}}, \tag{3.25}$$

and the optimal number of firms—using the second-best welfare benchmark—is

$$n^w = \left[\frac{t\beta 2^{-\beta}(2\beta\epsilon + \frac{1}{1+\beta})}{f} \right]^{\frac{1}{1+\beta}}. \tag{3.26}$$

We denote by $\bar{\epsilon} = \frac{1+2\beta}{2(1+\beta)^2}$ the demand elasticity such that optimal and competitive entry coincides. This leads to the following result:

Result 3.6. *Suppose that transportation costs are of the power function form tx^β. Then we have that i) there is excess entry if $\epsilon < \bar{\epsilon}(\beta)$ and insufficient entry if $\epsilon > \bar{\epsilon}(\beta)$, and ii) $\bar{\epsilon}(\beta)$ decreases in β.*

The first part of the result generalizes result 3.5 for the case of a more general transportation cost function. It states that as long as demand is sufficiently inelastic the excess entry theorem still holds. Otherwise it does not hold. The second part of the result, follows directly as $\frac{\partial \bar{\epsilon}}{\partial \beta} = -\frac{\beta}{(1+\beta)^3} < 0$. It says that the interval of demand elasticities for which the excess entry theorem holds shrinks with β.

3.6 Conclusion

In this chapter, we introduce elastic demand in the Salop (1979) model and investigate if the excess entry theorem still holds. We feature a utility

[11] Note that existence of price equilibrium is not ensured if β is too high. See Anderson, de Palma, and Thisse (1992, Ch. 6).

function that leads to a demand function with constant elasticity. We find that a larger demand elasticity leads to less entry into the market. This is a hypothesis that can be tested empirically. Markets with higher demand elasticity should offer less product variety. In the limiting case of a unit demand elasticity the market outcome is a monopoly. Turning to welfare analysis, we show that when demand is sufficiently inelastic there is excess entry. However, when demand is sufficiently elastic the number of entrants is lower than the socially optimal number. Further, we provide conditions on when and how a government intervention can be desirable. We also show that our results hold with more general transportation cost functions.

3.7 Appendix

Here we provide the derivation of the results for the model with power transportation costs. The derivation follows Anderson, de Palma, and Thisse (1992, Ch. 6), but extended to price-dependent demand.

With power transportation costs, the marginal consumer is implicitly given by

$$-\frac{1}{1-\epsilon}p_i^{1-\epsilon} - t\bar{x}^{\beta} = -\frac{1}{1-\epsilon}p^{1-\epsilon} - t\left(\frac{1}{n} - \bar{x}\right)^{\beta}. \tag{3.27}$$

In contrast to the case of linear transportation costs, it is not possible to give a closed form for the marginal consumer. However, by total differentiation it is possible to calculate the impact of a price change on the marginal consumer, which is

$$\frac{d\bar{x}}{dp_i} = -\frac{p_i^{-\epsilon}}{t\beta(\bar{x}^{\beta-1} + (\frac{1}{n} - \bar{x})^{\beta-1})}. \tag{3.28}$$

As we are interested in a symmetric equilibrium we can evaluate this expression at the symmetric equilibrium, that is at $\bar{x} = \frac{1}{2n}$. Then, we get

$$\frac{d\bar{x}}{dp_i}\Big|_{\bar{x}=\frac{1}{2n}} = -\frac{p_i^{-\epsilon}}{2t\beta(\frac{1}{2n})^{\beta-1}}. \tag{3.29}$$

Profits for the representative firm i is $\Pi_i = 2\bar{x}p_i^{1-\epsilon}$. The first-order condition for profit maximization and assuming symmetry gives the following equilibrium prices for a given number of firms in the market:

$$p = \left[(1-\epsilon)\frac{t\beta 2^{1-\beta}}{n^{\beta}}\right]^{\frac{1}{1-\epsilon}}. \tag{3.30}$$

For $\beta = 1$, this gives the results of our base model, and for $\epsilon = 0$, we get the results of Anderson, de Palma, and Thisse (1992, Ch. 6). Each firm earns a profit of

$$\frac{(1-\epsilon)t\beta 2^{1-\beta}}{n^{\beta+1}} - f. \tag{3.31}$$

The number of firms that enter in a free-entry equilibrium is determined via the zero-profit condition. This leads to the following number of entrants:

$$n^c = \left[\frac{(1-\epsilon)t\beta 2^{1-\beta}}{f}\right]^{\frac{1}{1+\beta}}. \tag{3.32}$$

With power transportation costs the second-best welfare benchmark can be expressed as:

$$W = V + Y - \frac{t\beta 2^{1-\beta}}{n^{\beta}} - \frac{t}{(1+\beta)n^{\beta}2^{\beta}} + \frac{(1-\epsilon)t\beta 2^{1-\beta}}{n^{\beta}} - fn. \tag{3.33}$$

The number of firms that maximizes total welfare is then

$$n^w = \left[\frac{t\beta 2^{-\beta}(2\beta\epsilon + \frac{1}{1+\beta})}{f}\right]^{\frac{1}{1+\beta}}. \tag{3.34}$$

Comparison with the number of firms in a free-entry equilibrium shows that there is excess entry if $\epsilon < \frac{1+2\beta}{2(1+\beta)^2}$.

Chapter 4

Product Variety, Price Elasticity of Demand and Fixed Cost in Spatial Models[1]

4.1 Introduction

Spatial models of product differentiation in the spirit of Hotelling (1929) and Salop (1979) have been a popular tool in Industrial Organization and Regional Science. They have been used to study competition in a large variety of markets and issues.[2] Typically, the Hotelling model has been used to study location decisions by firms while the Salop model has been used to study entry decisions and market structure. Concerning the Salop model, one prominent result is the so-called excess entry theorem. It states that in a free-entry equilibrium, there are always more firms entering into the market than would be desirable from a welfare point of view. That is, there is excessive entry into the market. As firms are usually assumed to be single product firms, the result can also be interpreted as an excess of product variety provided in the market.[3]

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[1]This chapter is coauthored with Tobias Wenzel. An earlier version of this chapter is Gu and Wenzel (2009b).

[2]E.g., Anderson and Coate (2005) on media markets, Friedman and Thisse (1993) on collusion, Armstrong (2006) for a study on two-sided markets, and many more.

[3]With respect to variants of the standard Salop model, Matsumura and Okamura (2006) find this excess entry result holds for a broad class of transport and production cost functions.

However, one underlying, and quite restrictive assumption in the Salop model, is that consumer demand does not depend on the price of the product. Each consumer demands a single unit of a differentiated product. In consequence, the price then constitutes a mere transfer between consumers and firms and thus has no impact on total welfare. It is the aim of the present chapter to lift this assumption of completely inelastic demand and investigate the consequences of this modification on the validity of the excess entry theorem. In contrast to a version of the model with completely inelastic demand, the price of the differentiated product is no longer a mere transfer between consumer and firm, but has a real welfare impact by influencing the quantity the consumer demands of the differentiated product. A higher price now leads to a lower quantity of the differentiated product and to lower total welfare.

The present chapter explores the relationship between price-dependent demand and the excess entry theorem in a quite general setting. We study a setup in which consumer preferences can be represented by a quasi-linear utility function. We make a mild restriction on the resulting consumer demand function for the differentiated product, namely we assume that the price elasticity is increasing in the price. This assumption is satisfied by many demand functions, for instance, linear demand functions.[4] In this setup, we establish existence and uniqueness of a symmetric price equilibrium.

Our main objective is to characterize the welfare properties of the free-entry equilibrium. We show that unlike the standard Salop model with completely inelastic demand, the free-entry equilibrium may exhibit excessive, insufficient or optimal entry. The intuition behind this result is the following. When setting the price of the product firms have to take two effects into account. An increase in the price reduces the market share as well as the quantity sold to each consumer. This second effect—not present in the standard Salop model with completely inelastic demand—makes firms more careful when setting the price, and hence leads to a lower equilibrium price than in the standard model. This, in turn, leads to lower profits and reduces the incentives to enter the market. Thus, considering price-dependent demand leads to a downward correction of

[4] It is also a common assumption in the business literature. See, e.g., Lariviere and Porteus (2001) and Ziya, Ayhan, and Foley (2003).

the number of firms which are active in the market. Whether we obtain excessive or insufficient entry now depends on the strength of consumer reactions to a price increase. If consumers react only mildly to a price increase, the downward correction of the equilibrium price and profit is small, and hence, we still get excessive entry. On the other hand, if consumers react strongly on a price increase the downward correction is large, and equilibrium price and profit are largely reduced and, in consequence, we obtain insufficient entry. This chapter obtains conditions for each of the possible welfare outcomes.

The central message of this chapter is that considering price-dependent demand is a decisive factor for the welfare results. The excess entry result may not hold when consumers react to prices by adjusting the quantities they demand. As the Salop is used as a building block in many applications, we think one should be careful in interpreting these welfare results which rely on the assumption of completely inelastic demand. When considering price-dependent demand these results may change.

In Chapter 3, we provided an example of the issue by considering a specific functional form for the consumer demand. We employ a demand function with a constant demand elasticity which enables us to express equilibrium solutions in closed form. This has the advantage to provide a simple formulation that is suitable for use in applications. In contrast, the present chapter aims to study the issue at a general level without relying on specific forms of consumer demand.

Our result also closes, at least partially, the gap between different approaches of modeling competition in differentiated product markets. In representative consumer models, such as Dixit and Stiglitz (1977), or in discrete choice models of product differentiation, for instance see the overview in Anderson, de Palma, and Thisse (1992), equilibrium entry can be excessive, insufficient, or optimal depending on the exact model configuration.

This chapter contributes to a recent literature that introduces price-dependent demand into spatial models. Related contributions are Boeckem (1994), Rath and Zhao (2001), Peitz (2002) and Anderson and de Palma (2000). The first two papers consider variants of the Hotelling framework. Boeckem (1994) introduces heterogenous consumers with respect to reservation prices. Depending on the price charged by firms some consumers choose not to buy a product. The paper by Rath and Zhao (2001) introduces elastic demand in the Hotelling framework by assuming that the quantity demanded by each consumer depends on the price charged. The authors propose a utility function that is quadratic in the quantity of the differentiated product leading to a linear demand function. In contrast to those two models we build on the Salop model as we are interested in the relationship between price-dependent demand and entry into the market. Our approach is closer to Rath and Zhao (2001) as we also assume that each consumer has a downward sloping demand for the differentiated good although we do not postulate a specific functional form. Peitz (2002) features unit-elastic demand both in Hotelling and Salop settings but focuses on conditions for existence of Nash equilibrium in prices. He does not consider entry decisions. Anderson and de Palma (2000) propose a model that integrates features of spatial models where competition is localized and representative consumer models where competition is global. In this model, consumer demand is elastic with a constant demand elasticity. The study focuses on the interaction between local and global competition.

The remainder of the chapter is structured as follows. Section 4.2 outlines our model. Section 4.3 establishes the existence and uniqueness of the symmetric price equilibrium and analyzes its properties both for a given number of firms and under free entry condition. In Section 4.4 we compare the market equilibrium with the welfare optimal outcomes. Section 4.6 concludes.

4.2 The model

Here we set up our model. Our aim is to stay close to the original Salop model as, for instance, outlined in Tirole (1988). The only modification we make is to introduce price-dependent demand.

4.2.1 Model setup

There is a unit mass of consumers who are uniformly located on a circle with circumference one. The location of a consumer is denoted by x.

Consumers derive utility from the consumption of a differentiated product and of a homogenous product, which serves as a numeraire good. The homogenous good is produced in a competitive industry while the differentiated product is produced within an oligopolistic industry. Behavior in the oligopolistic industry is the focus of our analysis.

We assume that consumers' utility is quasi-linear. Then, a consumer, located at x, gains the following utility from consuming a differentiated product with characteristic x_i:

$$U = \begin{cases} V + v(q_D) - t|x - x_i| + q_H & \text{if the differentiated product is consumed} \\ q_H & \text{otherwise,} \end{cases} \tag{4.1}$$

where q_D and q_H are the quantity of the differentiated and homogenous good, respectively. The utility derived by the consumption of the differentiated good consists of three parts. There is a gross utility for consuming this good V. The second utility component depends on the quantity consumed $v(q_D)$; $v(q_D)$ is assumed to be continuous and three times differentiable with $v^{'} > 0$ and $v^{''} < 0$. Finally, consumers have to incur costs of mismatch (transportation cost) if the product's attributes do not match consumers' preferences; these costs are linear in distance and do not depend on the quantity consumed.[5] We assume the gross utility V is large enough so that no consumer abstains from buying the differentiated product. Note also that there is decreasing marginal utility in the quantity of the differentiated product.

Each consumer is endowed with wealth Y which she can spend on the two commodities, the differentiated product and the numeraire good. We restrict consumers to consume only one variant of the differentiated product. Let us denote the price of the differentiated product by p and normalize the price of the numeraire good to 1. Then each consumer faces the following budget constraint:

$$Y = p \cdot q_D + q_H. \tag{4.2}$$

[5]Transport costs are one time costs independent of the quantity. As an interpretation these could be costs for driving to a shopping mall. Alternatively, one could also assume transport costs to depend on the quantity. These would be a plausible assumption if the horizontal dimension is interpreted as a taste dimension.

The differentiated product is offered by an oligopolistic industry with $n \geq 2$ firms each offering a single variant. We are not interested in location patterns. Hence, we assume that these firms are located equidistantly on the unit circle.[6] The distance between two neighboring firms then is $\frac{1}{n}$.

To model competition in this market, we study the following three stage game. In the first stage firms may enter the market. In the second stage, firms compete in prices. In the third stage, consumers choose a supplier of the differentiated product and the quantity.

4.2.2 Demand for the differentiated product

We start by deriving individual demand for the differentiated product. Suppose a consumer has decided to choose a certain supplier i. Then, the quantity she demands is the solution to the following maximization problem:

$$\begin{aligned} \max_{q_D, q_H} u(q_D, q_H) &= V + v(q_D) + q_H \\ \text{s.t. } p \cdot q_D + q_H &= Y \\ q_D, q_H &\geq 0. \end{aligned}$$

A consumer's demand for the differentiated good is determined by maximizing utility (equation (4.1)) under the budget constraint (equation (4.2)). We further assume that Y is sufficiently large such that the demand for the homogenous good is always positive. Then, by solving $v^{'}(q_D) = p$ we get a downward sloping individual demand function for the differentiated product $q(p)$. Since $v(q_D)$ is continuous and three times differentiable, $q(p)$ is continuous and twice differentiable in $(0, Q)$ where $Q < +\infty$ is the up-bound of demand obtained when $p = 0$.

Our assumption of quasi-linearity becomes convenient when expressing indirect utility. The surplus associated with the demand function $q(p)$ when a consumer located at x buys the differentiated product from a firm

[6] See Economides (1989) for the existence of symmetric location equilibria in the model with unit demand.

located at x_i at a price $p_i < \hat{p}$ is

$$\hat{U} = V + Y + \int_{p_i}^{\hat{p}} q\left(p\right) dp - t|x - x_i|, \tag{4.3}$$

where $\hat{p}$ denotes the minimum price where the function $q(p)$ becomes zero.

4.2.3 Marginal consumer and demand

Given the symmetric structure of the model, we seek for a symmetric equilibrium. Therefore we derive demand of a representative firm i which for convenience is designated to be located at zero. Suppose that this firm charges a price of p_i while all remaining firms charge a price of p_o. Then the marginal consumer is the consumer indifferent between choosing to buy from firm i and the neighboring firm located at $\frac{1}{n}$. Using equation (4.3) the marginal consumer ($\bar{x}$) is implicitly given by

$$V + Y + \int_{p_i}^{\hat{p}} q\left(p\right) dp - t\bar{x} = V + Y + \int_{p_o}^{\hat{p}} q\left(p\right) dp - t\left(\frac{1}{n} - \bar{x}\right),$$

or explicitly by

$$\bar{x} = \frac{1}{2n} + \frac{1}{2t}\int_{p_i}^{p_o} q\left(p\right) dp. \tag{4.4}$$

As each firm faces two adjacent firms, the number of consumers choosing to buy from firm i is $2\bar{x}$. According to the demand function, each consumer buys an amount of $q\left(p_i\right)$. Hence total demand at firm i is:

$$D_i = 2\bar{x} \cdot q\left(p_i\right). \tag{4.5}$$

In contrast to the standard model with completely inelastic demand, total demand consists now of two parts: market share and quantity per consumer. When choosing prices firms have to take into account both effects. An increase in price reduces market share as well as the quantity that can be sold to each customer. This second effect is not present in the standard model.

4.3 Analysis

This section analyzes the equilibrium. In a first step we characterize the price equilibrium for a given number of firms and provide conditions for the existence. In a second step, we seek to determine the number of firms that enter.

4.3.1 Price equilibrium

We look for a symmetric equilibrium in which all firms charge the same price. Assuming zero production costs, the profit of a representative firm i when this firm charges a price p_i and all remaining firms charge a price p_o is given by:

$$\Pi_i = D_i \cdot p_i = \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q\,(p)\,dp\right] q\,(p_i)\,p_i. \tag{4.6}$$

To find profit maximizing price p_i, we first derive the first order derivative,

$$\frac{d\Pi_i}{dp_i} = -\frac{1}{t}p_i\,[q\,(p_i)]^2 + \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q\,(p)\,dp\right]\left[q\,(p_i) + p_i\frac{dq\,(p)}{dp}\Big|_{p=p_i}\right]. \tag{4.7}$$

By setting equation (4.7) to zero we obtain the following necessary condition,

$$[q\,(p_i)]^2\,p_i\frac{1}{t} = \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q\,(p)\,dp\right]\left[q\,(p_i) + p_i\frac{dq\,(p)}{dp}\Big|_{p=p_i}\right]. \tag{4.8}$$

For the moment let us suppose that a symmetric price equilibrium exists. Later we will turn to this issue and provide existence conditions. Applying symmetry to the first-order condition, a symmetric price equilibrium is characterized by:

$$q\,(p^*)\,p^* = \frac{t}{n}\left[1 + \frac{p^*}{q\,(p^*)}\frac{dq\,(p)}{dp}\Big|_{p=p^*}\right]. \tag{4.9}$$

Note that the last part of equation (4.9) includes the price elasticity of individual demand evaluated at the equilibrium price. After the following definition, we express this equilibrium condition in terms of price elasticity of demand.

Definition 4.1. *Denote the absolute value of price elasticity of demand ε as*

$$\varepsilon = -\frac{p}{q(p)}\frac{dq(p)}{dp}.$$

Equation (4.9) now can be rewritten as

$$q\left(p^{*}\right)p^{*} = \frac{t}{n}\left[1-\varepsilon^{*}\right]. \tag{4.10}$$

We use this condition to state corresponding equilibrium profits. Inserting equation (4.9) into equation (4.6) we get

$$\Pi^{*} = \frac{t}{n^{2}}\left[1+\frac{p^{*}}{q\left(p^{*}\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p^{*}}\right] = \frac{t}{n^{2}}\left[1-\varepsilon^{*}\right]. \tag{4.11}$$

It can be seen immediately that there is a negative relationship between equilibrium demand elasticity and equilibrium profit.

4.3.2 Equilibrium existence

Now we provide conditions that ensure existence of a symmetric price equilibrium as stated in equation (4.10). We start with a preliminary result:

Lemma 4.1. *In equilibrium, demand is inelastic, that is, $\varepsilon^{*} < 1$.*

Proof. see appendix. □

This result reveals that analogous to a monopolist who sets the price to reach unit elasticity, a firm in the current model will only set price at the inelastic segment of the demand function.

To ensure existence, we have to impose some additional structure on the demand function. We introduce the following assumption:[7]

[7]This assumption is sufficient but not necessary for the existence of a symmetric price equilibrium.

Assumption 4.1. *The absolute value of price elasticity of demand* ε *is strictly increasing in* $p \in (0, \hat{p})$ *and* $\lim_{p \to \hat{p}} \varepsilon(p) = \lim_{p \to \hat{p}} \varepsilon|_p \geq 1$.

When ε is strictly increasing in p, it is shown in the literature that the individual consumer revenue function $R(p) = pq(p)$ is strictly unimodal over the entire interval of strictly positive demand. For a discussion on this point see Ziya, Ayhan, and Foley (2004). Strict unimodality of $R(p)$ means that $pq(p)$ has a unique global maximum $\widetilde{p}$ in $(0, \hat{p})$ and if p_1 and p_2 are two points in $(0, \hat{p})$ such that $p_1 < p_2 < \widetilde{p}$ or $\widetilde{p} < p_1 < p_2$ then $R(p_1) < R(p_2) < R(\widetilde{p})$ or $R(p_2) < R(p_1) < R(\widetilde{p})$, respectively.[8] Apparently a profit maximizing monopolist will set $p = \widetilde{p}$ when no production cost is involved and it's well known that $\varepsilon(\widetilde{p}) = 1$. When price p goes down from $\widetilde{p}$, both price elasticity of the demand ε and product revenue $R(p)$ strictly decrease.

As an example, one functional form that satisfies assumption 4.1 is a linear demand function of the type $q = a - bp$ or a quadratic function of the form $q = a - bp^2$, where both a and b are suitable positive constants.[9]

Under Assumption 1, we are now ready to establish the existence of a unique symmetric price equilibrium given by equation (4.10).

Proposition 4.1. *For any given number of firms* $n \geq 2$, *there exists a unique symmetric price equilibrium identified by condition (4.10), namely,* $q(p^*)\, p^* = \frac{t}{n}\left[1 - \varepsilon^*\right]$.

Proof. see appendix. □

It is relatively straightforward to verify the existence of the symmetric price equilibrium when firms have no incentive to undercut their neighbors and to establish its uniqueness. By constructing an auxiliary demand function, we show that undercutting is not possible. The detailed proof is relegated to the Appendix.

[8] This representation follows Bertsekas (1999) and it is also adopted by Ziya, Ayhan, and Foley (2004).

[9] In both examples, the maximum value of the elasticity is obviously larger than 1.

4.3.3 Properties of price equilibrium

We can now study the properties of the price equilibrium. Lemma 4.2 below states the comparative statics effect of the number of firms which are active in the market and of transportation costs on equilibrium price, equilibrium price elasticity and firm profit.

Lemma 4.2. *Comparative statics.*

1. *Equilibrium price, price elasticity of demand and firm profit decrease in the number of entrants, that is,* $\frac{dp^*}{dn} < 0$, $\frac{d\varepsilon^*}{dn} < 0$ *and* $\frac{d\Pi^*}{dn} < 0$.
2. *Equilibrium price, price elasticity of demand and firm profit increase in transportation costs, that is,* $\frac{dp^*}{dt} > 0$, $\frac{d\varepsilon^*}{dt} > 0$ *and* $\frac{d\Pi^*}{dt} > 0$.

Proof. see appendix. □

Unsurprisingly, the larger the number of firms the lower the price. Profits also decrease with the number of firms in the market. Additionally, the demand elasticity decreases with the number of firms in the market. This follows from our assumption that the demand elasticity increases in the price. The impact of transportation costs on prices and profits is the same as in standard location models. Prices and profits increase with transportation costs.

4.3.4 Entry

Until now the analysis has treated the number of firms which offer differentiated products as exogenously given. We now investigate the number of active firms when it is endogenously determined by a zero profit condition. We assume that to enter, a firm has to incur an entry cost or fixed cost of f. Additionally, we treat the number of entrants as a continuous variable. Setting equation (4.11) equal to f determines implicitly the number of firms that enter. We denote this number by n^c:

$$\frac{t}{(n^c)^2}(1 - \varepsilon^*_{n^c}) = f. \tag{4.12}$$

In general, it is not possible to express the number of entrants explicitly as the equilibrium demand elasticity ($\varepsilon^*_{n^c}$) depends on the number of competitors. In this chapter, we have assumed that the market is viable for at least two firm. So the fixed costs must not be prohibitively high: $f \leq F = \frac{t}{4}(1 - \varepsilon^*_{n=2})$. Thus, we only consider fixed costs in $f \in (0, F)$.[10] We know from Lemma 4.2 that profits decrease monotonically in the number of firms. Hence, we know that a solution to equation (4.12) exists and is unique.

The comparative static results concerning transportation costs and fixed costs are as expected. Higher transportation costs lead to more entry while higher fixed costs to less entry, that is, $\frac{dn^c}{dt} > 0$ and $\frac{dn^c}{df} < 0$. This follows immediately from Lemma 4.2.

Later, it will turn out that equilibrium demand elasticity is a crucial factor for our welfare results. Thus, we are interested in its properties. With endogenous entry, equilibrium demand elasticity is essentially a function of the exogenous variables, fixed costs and transportation costs. When fixed costs are low, a large number of firms enter which decreases equilibrium demand elasticity (as shown in Lemma 4.2). Converse is the impact of transportation costs. High transportation costs lead to a large number of entrants, and hence, to a low demand elasticity. Formally,

Lemma 4.3. *Equilibrium price elasticity increases in fixed costs and decreases in transportation costs, that is,* $\frac{d\varepsilon^*_{n^c}}{df} > 0$ *and* $\frac{d\varepsilon^*_{n^c}}{dt} < 0$.

Proof. $\frac{d\varepsilon^*_{n^c}}{df} = \frac{d\varepsilon^*}{dn}\frac{dn^c}{df} > 0$, as $\frac{d\varepsilon^*}{dn} < 0$ by lemma 4.2 and $\frac{dn^c}{df} < 0$ from above. $\frac{d\varepsilon^*_{n^c}}{dt} = \frac{d\varepsilon^*}{dn}\frac{dn^c}{dt} < 0$, as $\frac{d\varepsilon^*}{dn} < 0$ by lemma 4.2 and $\frac{dn^c}{dt} > 0$ from above. □

Hence, because of these strictly monotone relationships, there is a one-to-one relationship between equilibrium demand elasticity and fixed costs or transportation costs, respectively. For instance, for each value of fixed cost $f \in (0, F)$ we can identify the corresponding equilibrium price elasticity $\varepsilon^*(f) \in (0, \varepsilon^*_{n=2})$, and vice versa. The same applies to transportation costs. We will make use of these relationships when expressing welfare results.

[10] Alternatively, this can be re-stated in terms of transportation costs: $t > \frac{4f}{1-\varepsilon^*_{n=2}} = T$.

4.4 Welfare

This section considers the welfare implications. We ask whether there is excess entry into the market as it is the case in models with completely inelastic demand.

In contrast to models with completely inelastic demand, we have to consider prices in our welfare analysis as they have an impact on the quantity purchased and hence on welfare. We define social welfare as the sum of consumer utility and industry profits:

$$W = \underbrace{V + Y + \int_{p}^{\hat{p}} q\left(p\right) dp - 2n \int_{0}^{\frac{1}{2n}} tx \; dx}_{\text{Consumer welfare}} + \underbrace{\frac{t}{n}\left[1 - \varepsilon_n^*\right] - fn}_{\text{Industry profits}}. \tag{4.13}$$

We consider a first-best benchmark, in which the social planner can control prices and the level of entry, that is, she maximizes total welfare with respect to p and n.[11] From equation (4.13), we see that the optimal price is equal to marginal cost, in this case, $p = 0$. Inserting this into equation (4.13) yields

$$W = V + Y + \int_{0}^{\hat{p}} q\left(p\right) dp - 2n \int_{0}^{\frac{1}{2n}} tx \; dx - fn. \tag{4.14}$$

The problem for the social planner is then identical to the case with completely inelastic demand, hence reduced to a trade-off between transportation costs and fixed costs. The optimal number of entrants is

$$n^f = \sqrt{\frac{t}{4f}}. \tag{4.15}$$

To shape intuition, it is useful to start with a preliminary result:

[11]Our results derived below will not change qualitatively if a second-best benchmark is used, i.e., if the social planner can only control the level of entry but not the price. The reason is, the second-best benchmark level of entry is generally higher than the first-best level since at the first-best level of entry without price regulation there is additional benefit of further entry resulting from the reduced difference between market price and marginal cost. Therefore, if market entry is insufficient compared to the first-best benchmark, this is also true compared to the second-best benchmark.

Lemma 4.4. *There is excess entry if* $\varepsilon^*_{n^c} < \frac{3}{4}$*, insufficient entry if* $\varepsilon^*_{n^c} > \frac{3}{4}$*, and optimal entry if* $\varepsilon^*_{n^c} = \frac{3}{4}$.

Lemma 4.4 can easily be derived by comparing equations (4.12) and (4.15). This lemma provides conditions for the existence of excessive, insufficient, and optimal entry. If the equilibrium demand elasticity is sufficiently low we get excess entry as in the standard model with completely inelastic demand. If, on the other hand, equilibrium demand elasticity exceeds $\frac{3}{4}$, there is insufficient entry into the market. The intuition behind the result can be seen in equation (4.11). The higher equilibrium demand elasticity is, the lower the profits are; and hence the smaller the incentives to enter the market will be.

However, the equilibrium demand elasticity is endogenous in this model. Thus, our aim is to state the welfare result in terms of exogenous variables. Now we can make use of the monotone relationship between equilibrium demand elasticity and fixed costs of entry. Entry is excessive (insufficient) if fixed costs are such that $\varepsilon^*_{n^c} < \frac{3}{4}$ ($\varepsilon^*_{n^c} > \frac{3}{4}$). This leads to:

Proposition 4.2. *Welfare result.*

1. *Suppose* $\varepsilon^*_{n=2} \geq \frac{3}{4}$ *and define* $\overline{f}$ *as the fixed cost level that leads to equilibrium price elasticity* $\varepsilon^*_{n^c} = \frac{3}{4}$*. Then there is excess entry if* $f < \overline{f}$*, insufficient entry if* $f > \overline{f}$ *and optimal entry if* $f = \overline{f}$.
2. *Suppose* $\varepsilon^*_{n=2} < \frac{3}{4}$*, then there is excess entry for all* $f \in (0, F]$.

Proof. see appendix. □

Proposition 4.2 contains the main contribution of the chapter. When accounting for price-dependent demand the excess entry result of the standard model with completely inelastic demand needs not hold. In the proposition we have to consider two cases. First, if the demand function is such that $\varepsilon^*_{n=2} \geq \frac{3}{4}$. Then, if fixed costs of entry are high such that the corresponding equilibrium demand elasticity is high, entry into the market is insufficient. Conversely, if fixed costs are low, the number of firms that enter is high which leads to a low demand elasticity. And hence, entry into the market is excessive. The second case we have to consider is a demand function which has the property such that $\varepsilon^*_{n=2} < \frac{3}{4}$. As $\varepsilon^*_{n^c}$

decreases in n, $\varepsilon^*_{n^c} < \frac{3}{4}$ for all values of fixed costs $(f \in (0, F))$. And thus, there is always excess entry in this case.

Alternatively, it is also possible to restate the welfare result in terms of transportation costs. This is formally done in the appendix. There, we show that insufficient entry is possible if transportation costs are sufficiently low.

4.5 Example

The analysis before was quite general. To illustrate our results, we provide an example using a linear demand function for individual consumer demand for the differentiated product. Suppose that

$$q(p) = \frac{1}{2} - p, \tag{4.16}$$

with an associated (absolute) demand elasticity of

$$\varepsilon = \frac{p}{\frac{1}{2} - p}. \tag{4.17}$$

We set transportation costs equal to one and solve for the free-entry equilibrium. It is not possible to solve it analytically, so we turn to a numerical solution. Figures 4.1 and 4.2 show the results of this numerical analysis.

Figure 4.1 displays the number of entrants in the free-entry equilibrium (solid line) vs. optimal entry (dashed) for different values of fixed costs of entry f and thus illustrates Proposition 4.2. The figure shows that for low levels of fixed cost ($f < \bar{f} = 0.014994$), there is excessive entry into the market and for high levels of fixed costs ($f > \bar{f} = 0.014994$) there is insufficient entry in the market.[12]

Figure 4.2 displays the corresponding equilibrium demand elasticity ($\varepsilon^*_{n^c}$) for given values of f. In line with the previous figure and with Lemma

[12] Proposition 4.2 provides a simple way to check whether insufficient entry is compatible with a certain demand function and transportation costs. It suffices to compute the demand elasticity evaluated at $n = 2$. If the demand elasticity is smaller than $\frac{3}{4}$, there is always excess entry. If, however, the demand elasticity exceeds $\frac{3}{4}$, we know that there exists some level of fixed cost $\bar{f}$ with $f > \bar{f}$ leading to insufficient entry and $f < \bar{f}$ leading to excessive entry.

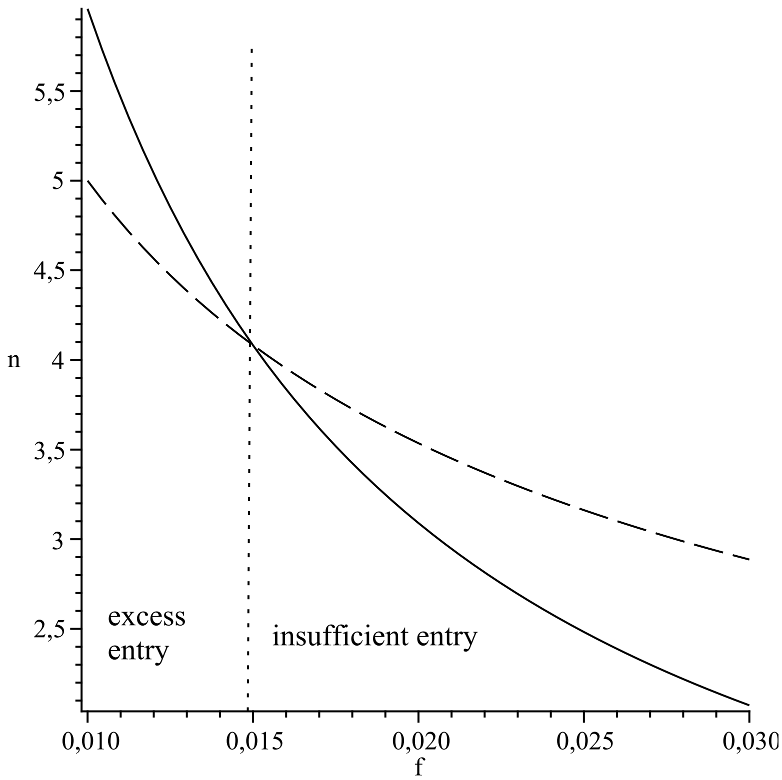

Figure 4.1: Equilibrium (solid curve) vs. optimal entry (dashed curve)

4.4, equilibrium demand elasticity is lower than $\varepsilon^*_{n^c} < \frac{3}{4}$ for $f < \bar{f}$ and is larger than $\frac{3}{4}$ for $f > \bar{f}$.

4.6 Conclusion

This chapter has introduced price-dependent demand into the Salop model. Our analysis focuses on the welfare implications of this generalization of the original model outlined by Salop. While in the model with completely inelastic demand the excess entry result holds, this is no longer true when accounting for price-dependent demand. Results are not that clear-cut anymore. Entry or product variety, respectively, can be excessive, insufficient, or optimal.

As the Salop model is widely used in all sorts of applications, we believe that our results are of some importance. In the light of the present chapter, accounting for price-dependent demand may lead to different welfare conclusions.

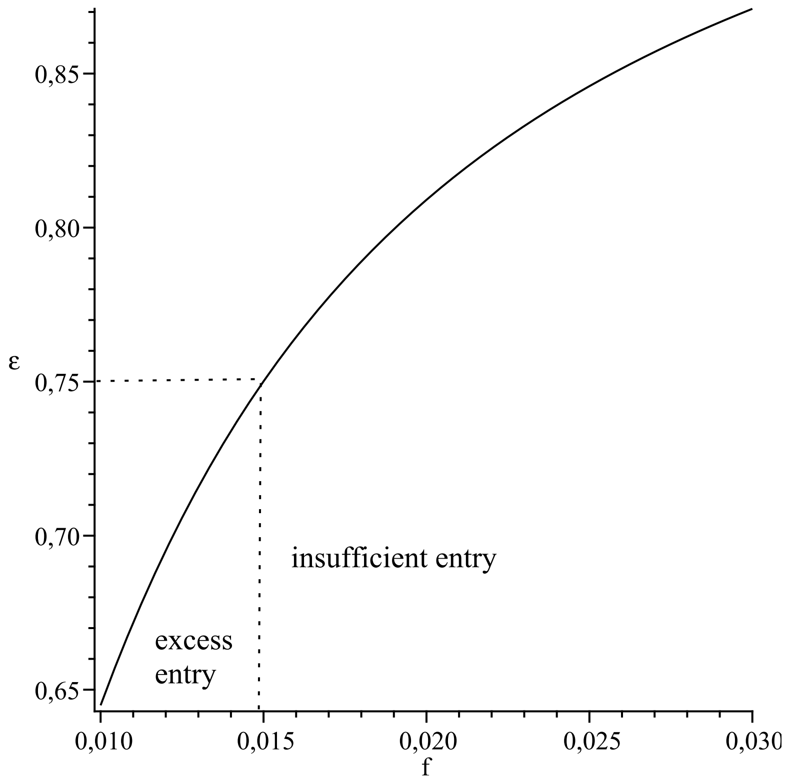

Figure 4.2: Equilibrium demand elasticity

4.7 Appendix

4.7.1 Proof of Lemma 4.1

Proof: Note when $\varepsilon \geq 1$ i.e., $\frac{dq(p)}{dp}\frac{p}{q(p)} \leq -1$, the first order derivative (4.7)

$$\frac{d\Pi_i}{dp_i} = \underbrace{-\left[q\left(p_i\right)\right]^2 p_i \frac{1}{t}}_{\text{negative}} + \underbrace{\left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q\left(p\right)dp\right] q\left(p_i\right)}_{\text{positive}} \underbrace{\left[1 + \frac{p_i}{q\left(p_i\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p_i}\right]}_{\text{non-positive}} \tag{4.18}$$

obtains a strictly negative value. The middle part in the right-hand side of (4.18) is positive because we are interested in symmetric equilibrium $(p_i = p_o)$. With $\frac{d\Pi_i}{dp_i}$ being negative, whenever demand elasticity exceeds or is equal to 1, a firm wants to reduce price in order to boost demand.

In equilibrium, whenever it exists, however, the F.O.C. (4.9) holds,

$$
\begin{aligned}
& 1+\frac{p^*}{q\left(p^*\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p^*}>0 \\
\Longrightarrow\ & \frac{p^*}{q\left(p^*\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p^*}>-1 \\
\Longrightarrow\ & \varepsilon^*<1.
\end{aligned}
$$

This concludes the proof.

4.7.2 Proof of Proposition 4.1

Proof: The structure of the proof is the following. We first show that the necessary first order condition (4.10) admits a unique solution. Second, we prove that under the condition of symmetric price and without undercutting, firm profit is quasi-concave in the strategy variable p_i. Last we show that firms have no incentive to undercut neighbors when they are in the situation identified by the first order condition.

1) Define $\Delta(p)=q\left(p\right)p-\frac{t}{n}\left[1-\varepsilon(p)\right]$. Because $v(q_D)$ is continuous and three times differentiable, $q(p)$ and $\varepsilon(p)$ are continuous and differentiable. Hence, $\Delta(p)$ is continuous. Note that

$$
\lim_{p\to 0}\Delta(p)=0-\frac{t}{n}\left[1-\lim_{p\to 0}\varepsilon(p)\right]=0-\frac{t}{n}<0
$$

and for the individual consumer ($R(p)=pq(p)$) revenue-maximizing $\widetilde{p}$,

$$
\Delta(\widetilde{p})=q\left(\widetilde{p}\right)\widetilde{p}>0.
$$

Because of continuity, $\Delta(p)=0$ obtains solution(s) for $p\in(0,\widetilde{p})$. Take the derivative of $\Delta(p)$,

$$
\frac{d\Delta(p)}{dp}=\frac{dR(p)}{dp}+\frac{t}{n}\frac{d\varepsilon(p)}{dp}.
$$

Following Assumption 4.1, $\frac{d\varepsilon(p)}{dp}>0$; since $R(p)$ is strictly unimodal, for $p\in(0,\widetilde{p})$, $\frac{dR(p)}{dp}>0$ as well. Hence, we conclude $\frac{d\Delta(p)}{dp}>0$. Because of this monotonicity, $\Delta(p)=0$ obtains a unique solution in $(0,\widetilde{p})$. When $p\in[\widetilde{p},\hat{p})$, we know $\varepsilon(p)\geq 1$ which means $\Delta(p)>0$ for $[\widetilde{p},\hat{p})$. So the solution given by $q(p)\,p = \frac{t}{n}\left[1-\varepsilon(p)\right]$ for $p \in (0, \widetilde{p})$ is the unique solution.

2) Take derivative of the F.O.C. (4.7),

$$\frac{d^2\Pi_i}{dp_i^2} = -\frac{1}{t}\left([q(p_i)]^2 + 2p_i q(p_i)\frac{dq}{dp}\Big|_{p=p_i}\right) - \frac{1-\varepsilon(p_i)}{t}\left[q(p_i)\right]^2 + \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q(p)\,dp\right]\left((1-\varepsilon(p_i))\frac{dq}{dp}\Big|_{p=p_i} - q(p_i)\frac{d\varepsilon}{dp}\Big|_{p=p_i}\right).$$

We know when the first order condition under symmetric price holds, $q(p^*)\,p = \frac{t}{n}\left[1-\varepsilon(p^*)\right]$. Evaluate the second order derivative at p^*,

$$\begin{aligned}\frac{d^2\Pi_i}{dp_i^2}\Big|_{p=p^*} &= -\frac{1}{t}\left([q(p^*)]^2 + 2p^* q(p^*)\frac{dq}{dp}\Big|_{p=p^*}\right) - \frac{np^*q(p^*)}{t^2}\left[q(p^*)\right]^2 \\ &\quad + \left[\frac{1}{n} + \frac{1}{t}\int_{p^*}^{p^*} q(p)\,dp\right]\left(\frac{np^*q(p^*)}{t}\frac{dq}{dp}\Big|_{p=p^*} - q(p^*)\frac{d\varepsilon}{dp}\Big|_{p=p^*}\right) \\ &= -\frac{p^*q(p^*)}{t}\frac{dq}{dp}\Big|_{p=p^*} - \frac{q^2(p^*)}{t} - \frac{np^*q^3(p^*)}{t^2} - \frac{q(p^*)}{n}\frac{d\varepsilon}{dp}\Big|_{p=p^*} \\ &= -\frac{q^2(p^*)}{t}\left(1-\varepsilon(p^*)\right) - \frac{np^*q^3(p^*)}{t^2} - \frac{q(p^*)}{n}\frac{d\varepsilon}{dp}\Big|_{p=p^*}.\end{aligned} \tag{4.19}$$

Since price elasticity is increasing in price $\left(\frac{d\varepsilon}{dp}\Big|_{p=p_i} > 0\right)$ and whenever the first order condition holds $(1-\varepsilon(p^*)) > 0$, the right hand side of equation (4.19) is strictly negative for $\forall p_i \in (0, \widetilde{p})$. In consequence, any firm's profit function is necessarily strictly concave whenever condition (4.10) holds. Hence for all of the firms, firm payoff is strictly quasiconcave in strategy variable p_i.

3) In this step we verify if any firm would have incentive to undercut its neighbors. For a firm to undercut its closest neighbors, the price it sets has to be low enough to attract consumers with a distance further than $\frac{1}{n}$. Using consumer's indirect utility function, for $0 < p_i < p^*$ the following condition has to hold.

$$\begin{aligned} &\int_{p_i}^{\hat{p}} q(p)dp - \frac{t}{n} + Y + V \geq \int_{p^*}^{\hat{p}} q(p)dp + Y + V \\ \Longleftrightarrow &\int_{p_i}^{p^*} q(p)dp \geq \frac{t}{n}. \end{aligned} \tag{4.20}$$

To investigate condition (4.20), we first prepare an additional result (i.e., inequality (4.23) below) for further use. By solving (4.10) we will have equilibrium price p^*, the corresponding demand $q^* = q(p^*)$ and price elasticity $\varepsilon^* = \varepsilon(p^*)$. Define constant

$$\varphi = \frac{q^*}{(p^*)^{-\varepsilon^*}}.$$

We construct an auxiliary demand function with constant elasticity ε^*,

$$q^\dagger(p) = \varphi p^{-\varepsilon^*}$$

which also passes through the point (p^*, q^*). With this demand function we can obtain the following closed form formula for $0 < p_i < p^*$,

$$\int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp = \frac{\varphi}{1-\varepsilon^*} p^{1-\varepsilon^*} \Big|_{p_i}^{p^*}. \tag{4.21}$$

Applying the necessary condition for symmetric equilibrium $1 - \varepsilon^* = \frac{n}{t} p^* q\left(p^*\right)$, equation (4.21) becomes

$$\begin{aligned}\int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp =& \frac{q^*}{(p^*)^{-\varepsilon^*}} \frac{t}{n p^* q^*} \left((p^*)^{1-\varepsilon^*} - (p_i)^{1-\varepsilon^*} \right) \\ =& \frac{t}{n} \frac{(p^*)^{1-\varepsilon^*} - (p_i)^{1-\varepsilon^*}}{(p^*)^{1-\varepsilon^*}} \\ =& \frac{t}{n} \left(1 - \left(\frac{p_i}{p^*} \right)^{1-\varepsilon^*} \right).\end{aligned}$$

Since $0 \leq 1 - \varepsilon^* < 1$ and $0 < p_i < p^*$, we have

$$\int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp < \frac{t}{n}, \; \forall \; p_i \in (0, p^*). \tag{4.22}$$

Note also that $q^\dagger(p) = \varphi p^{-\varepsilon^*}$ has a constant elasticity ε^* while $q(p)$ obtains elasticity ε^* at the point (p^*, q^*) but strictly lower elasticity $\varepsilon < \varepsilon^*$ when price decreases. That is, for the same percentage decrease of price, although $q(p)$ and $q^\dagger(p)$ start out at the same point (p^*, q^*),

$q(p)$ increase less than $q^{\dagger}(p)$ does. Hence,

$$
\begin{aligned}
& q(p) < \varphi p^{-\varepsilon^*}, \text{ for } \forall\ p \in (0, p^*) \\
\Longrightarrow & \int_{p_i}^{p^*} q(p) dp < \int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp, \text{ for } \forall\ p_i \in (0, p^*).
\end{aligned}
$$

By condition (4.22) we have the next result,

$$\int_{p_i}^{p^*} q(p) dp < \frac{t}{n} \text{ for } \forall\ p_i \in (0, p^*). \tag{4.23}$$

Now we are ready to discuss the undercutting strategy for firm i facing consumer demand function $q(p)$. To undercut its neighbors who are charging the symmetric equilibrium price p^*, condition (4.20) has to hold. Because of the result we established in (4.23), there exists no positive price that satisfies condition (4.20). Hence there is no firm who is able to take over neighbor's business without losing money.

4) We have shown that for any $n \geq 2$, there exists a unique solution to condition (4.10). Moreover, the strategy profile characterized by this condition is indeed an equilibrium because firms' payoffs are strictly quasiconcave in own strategy and there is no incentive for firms to undercut neighbors. This concludes the proof.

4.7.3 Proof of Lemma 4.2

Proof:

1. Take total differentiation of equation (4.10) with respect to the number of firms,

$$
\begin{aligned}
& \frac{dq^*}{dp}\frac{dp^*}{dn}p^* + q^*\frac{dp^*}{dn} = \frac{t}{n}\left(-\frac{d\varepsilon^*}{dp}\frac{dp^*}{dn}\right) - (1-\varepsilon^*)\frac{t}{n^2} \\
\Longrightarrow & \left(\frac{dq^*}{dp}p^* + q^*\right)\frac{dp^*}{dn} = -\frac{t}{n}\frac{d\varepsilon^*}{dp}\frac{dp^*}{dn} - \frac{t}{n^2}(1-\varepsilon^*) \\
\Longrightarrow & \frac{dp^*}{dn}\left(q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}\right) = -\frac{t}{n^2}(1-\varepsilon^*) \\
\Longrightarrow & \frac{dp^*}{dn} = \frac{-\frac{t}{n^2}(1-\varepsilon^*)}{q(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}}.
\end{aligned}
$$

Since $(1-\varepsilon^*) > 0$ by Lemma 4.1 and $\frac{d\varepsilon^*}{dp} > 0$, in equilibrium $\frac{dp^*}{dn} < 0$. Also from Assumption 4.1

$$\frac{d\varepsilon^*}{dn} = \frac{d\varepsilon^*}{dp}\frac{dp^*}{dn} < 0.$$

Differentiate equation (4.11) with respect to n:

$$\begin{aligned}
\frac{d\Pi^*}{dn} &= -\frac{2t}{n^3}(1-\varepsilon^*) - \frac{t}{n^2}\frac{d\varepsilon^*}{dn} \\
&= -\frac{t}{n^3}(1-\varepsilon^*)\left[2 - \frac{t}{n}\frac{d\varepsilon^*}{dp}\left(\frac{1}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}}\right)\right] \\
&= -\frac{t}{n^3}(1-\varepsilon^*)\frac{2q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} < 0.
\end{aligned}$$

2. Take total differentiation of equation (4.10) with respect to transportation costs,

$$\begin{aligned}
&\frac{dq^*}{dp}\frac{dp^*}{dt}p^* + q^*\frac{dp^*}{dt} = -\frac{t}{n}\left(\frac{d\varepsilon^*}{dp}\frac{dp^*}{dt}\right) + (1-\varepsilon^*)\frac{1}{n} \\
\Longrightarrow &\frac{dp^*}{dt}\left(q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}\right) = \frac{1}{n}(1-\varepsilon^*) \\
\Longrightarrow &\frac{dp^*}{dt} = \frac{\frac{1}{n}(1-\varepsilon^*)}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} > 0.
\end{aligned}$$

It follows:

$$\begin{aligned}
\frac{d\varepsilon^*}{dt} &= \frac{d\varepsilon}{dp}\frac{dp^*}{dt} \\
&= \frac{d\varepsilon}{dp}\frac{\frac{1}{n}(1-\varepsilon^*)}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} > 0.
\end{aligned}$$

Differentiate equation (4.11) with respect to transportation costs:

$$\begin{aligned}
\frac{d\Pi^*}{dt} &= \frac{1}{n^2}(1-\varepsilon^*) - \frac{t}{n^2}\frac{d\varepsilon^*}{dt} \\
&= \frac{1}{n^2}(1-\varepsilon^*)\left[1 - \frac{t}{n}\frac{d\varepsilon}{dp}\left(\frac{1}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon}{dp}}\right)\right] \\
&= \frac{1}{n^2}(1-\varepsilon^*)\frac{q^*(1-\varepsilon^*)}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} > 0.
\end{aligned}$$

4.7.4 Proof of Proposition 4.2

Proof: From Lemma 4.3, we know that equilibrium elasticity under free entry increases in fixed cost. From Lemma 4.4, we know that $n^c > n^f$ when $\varepsilon^*_{n^c} < \frac{3}{4}$, $n^c = n^f$ when $\varepsilon^*_{n^c} = \frac{3}{4}$, and $n^c < n^f$ when $\varepsilon^*_{n^c} > \frac{3}{4}$.

We have to consider two cases:

1. $\varepsilon^*_{n=2} \geq \frac{3}{4}$. Then, there exists a fixed cost $\bar{f}$ such that the resulting equilibrium demand elasticity is equal to $\frac{3}{4}$. Since $\varepsilon^*_{n^c}$ increases in f, for $f < \bar{f}$, $\varepsilon^*_{n^c} < \frac{3}{4}$ which leads to excessive entry by Lemma 4.4. Conversely, for $f > \bar{f}$, $\varepsilon^*_{n^c} > \frac{3}{4}$ which means insufficient entry.
2. $\varepsilon^*_{n=2} < \frac{3}{4}$. Then, since $\varepsilon^*_{n^c}$ decreases in n, $\varepsilon^*_{n^c} < \frac{3}{4}$ for all values of f. And hence, there is excess entry.

We can also reformulate Proposition 4.2 in terms of transportation costs. What we need first is to show that there is a monotone relationship between equilibrium demand elasticity and transportation costs. This is $\frac{d\varepsilon^*_{n^c}}{dt} = \frac{d\varepsilon}{dn}\frac{dn}{dt} < 0$, as $\frac{d\varepsilon}{dn} < 0$ from Lemma 4.2 and $\frac{dn}{dt} > 0$.

Again, we must distinguish the two cases:

1. $\varepsilon^*_{n=2} \geq \frac{3}{4}$. Then, there exists a transportation cost $\bar{t}$ such that the resulting equilibrium demand elasticity is equal to $\frac{3}{4}$. Since $\varepsilon^*_{n^c}$ decreases in t, for $t > \bar{t}$, $\varepsilon^*_{n^c} < \frac{3}{4}$ which leads to excessive entry by Lemma 4.4. Conversely, for $t < \bar{t}$, $\varepsilon^*_{n^c} > \frac{3}{4}$ which means insufficient entry.
2. $\varepsilon^*_{n=2} < \frac{3}{4}$. Then, since $\varepsilon^*_{n^c}$ decreases in n, $\varepsilon^*_{n^c} < \frac{3}{4}$ for all values of t. And hence, there is excess entry.

We state the result formally as a corollary:

Corollary 4.1. *Welfare result in terms of transportation costs.*

1. *Suppose $\varepsilon^*_{n=2} \geq \frac{3}{4}$ and define $\bar{t}$ as the transportation cost level that leads to equilibrium price elasticity ε^* of $\frac{3}{4}$. Then there is excess entry if $t > \bar{t}$, insufficient entry if $t < \bar{t}$ and optimal entry if $t = \bar{t}$.*
2. *Suppose $\varepsilon^*_{n=2} < \frac{3}{4}$, then there is excess entry for all $t > T$.*

Chapter 5

Gift Giving and Concern for Face

The Widow's Offering

And he called unto him his disciples, and saith unto them, Verily I say unto you, That this poor widow hath cast more in, than all they which have cast into the treasury: For all they did cast in of their abundance; but she of her want did cast in all that she had, even all her living. — Mark 12: 43–44

5.1 Introduction

5.1.1 Theories of gift giving

People voluntarily contribute to public goods, make charitable transfers to the needy and give gifts to others. Explanations for such behavior are not apparent from the point of view of neoclassical economic theory. A reasonable attempt was to admit the total provision of public goods into individual preferences together with private consumptions (see, for example, McGuire (1974), Warr (1982), Roberts (1984) and Bergstrom, Blume, and Varian (1986)). However, Andreoni (1988) shows that such a *pure altruism* model typically leads to prevailing of free-riding in large economies. This framework also predicts that government spending on public goods will crowd out equal amount of private donation while empirical studies find such crowding out is rather small.[1] These issues can be addressed by the inclusion of the *act* of giving into individual preferences. The argument is that private donation may also be motivated by social or psychological concerns such as esteem, prestige, social pressure or simply a feeling of a warm glow. A large body of sociological, psychological, and anthropological research supports this view and important contributions employing this *impure altruism* approach in economic studies have been presented by, for example, Arrow (1972), Margolis (1981), Andreoni (1989, 1990), McClelland (1989) among many others.

5.1.2 Warm glow

Harbaugh (1998a, 1998b) further differentiates two distinctive types of "joy of giving" that may be derived from the act of donation. The first is the "intrinsic benefit" which a donor enjoys because of her own moral concerns and only requires her own knowledge of the act of giving. For instance, a donor is happier after a donation because only then she feels she is entitled to think herself as a good person. The other is the "prestige benefit" which comes from a donor's social concerns and is only obtained when the act of giving is publicly recognized. In this case, a donor's decision is influenced by how others think about himself. See, for example, Deci and Ryan (1985), Brekke, Kverndokk, and Nyborg (2003) for studies of moral motivation and Holländer (1990), Lindbeck (1997) for studies of social acceptance.

Though the literature of warm glow has convincingly argued and extensively studied the relevance of the *demand* for warm glow, what remains less investigated is the *supply* or the production technology of it. In the social concern case obviously, from the same $100 donation to a charity, the "prestige" one enjoys when he is the only donor differs to what one would claim, if any, when he was just one of the one hundred $100-donors. In other words, how much prestige one could enjoy from his own donation depends also on how much others have given. Moreover, same amount of donation also tends to be judged differently when it comes from differ-

[1] Theoretical prediction on crowding-out is shown in, for example, Warr (1982), Roberts (1984), Bernheim (1986) and Bergstrom, Blume, and Varian (1986); empirical evidences have been provided by Abrams and Schmitz (1978), Khanna, Posnett, and Sandler (1995), Payne (1998) and more recently Garrett and Rhine (2007), among others.

ent donors. A $100 donation from a poor person will usually be praised higher than a $100 donation from a wealthy person.

Parallel arguments can also be made even when only internal intrinsic benefit is relevant. In this case, the one who "judges" is not the "society" but one's own moral. It's not uncommon when people make voluntary contributions they acknowledge the reason being "doing my bit". Then the question is, how much is a "bit"? One story could be like this: Suppose there is a way a person can find out the total income and total charitable donation of all of his fellow countrymen, then he will be able to compute the aggregate donation-income ratio.[2] If a person treats such a ratio as a reasonable reference point or the "bit", the higher his own donation-income ratio above this reference point, the better he may look to himself in the mirror. Likewise, if one knows others have donated so much out of their own moderate income, esteem might not be a very significant motivation for his one penny donation.

5.1.3 The concept of face

When donations are visible to the public, each of them is seen differently by the society and comparison of individual donations is inevitable in many situations. The motivation of private donation identified herein falls into the warm glow approach since a donor directly benefits from the act of giving, but it differs from what prestige, honor, esteem, dignity, social status or social norm may refer to. It is, therefore, conceptually appealing to introduce the concept of face to the analysis of current situation although it is arguably more relevant in some communities than others.

> "Face is the respectability and/or deference which a person can claim for himself from others, by virtue of the relative position he occupies in his social network and the degree to which he is judged to have functioned adequately in that position as well as acceptably in his general conduct;......" — Ho (1976), page 883, *American Journal of Sociology, 81(4)*

[2]Such statistics of national income and national charitable donation are published in many countries.

Face is important for an individual to function properly within the community though the degree of the importance varies from one part of the world to another. The idea is that each one has a *face* which can either be improved or lost to a certain extent depending on, for example, whether one has donated sufficiently compared to what ought to be contributed based on the position he occupies in our context. If the relative positions are assigned according to individual's income, individual donations will be judged along with the donor's income level. However, the exact amount that ought to be contributed for each position is not a written rule. It depends on how much others have donated. In this sense, concern for face is not strictly a social norm since it does not prescribe what one has to do (see, Elster (1989)). Indeed, the criteria based on which individual's behavior is judged are endogenously determined. Face is also not status. If an individual "functioned adequately" in his relatively low status, he can still be possessing a sound face.

Though by definition face is a social concept, the approach proposed in this chapter is also capable of addressing entirely morally motivated donations. It therefore does not require that each donation has to be seen by others nor individuals have to care about what others think about themselves so long as they do care whether themselves have functioned adequately according to a certain type of standards. This chapter hence extends and complements studies of both intrinsic and prestige type of benefits by investigating what kind of warm glow it could be and how it influences individual decision making.

Glazer and Konrad (1996)'s signaling explanation for charity is the closest paper to the present work for the fact that both place interests primarily on the impact of one donor's donation on others' enjoyment of warm glow.[3] In their paper, donations are observable and in equilibrium a more "generous" donation signals a higher income level of that donor. Hence the benefit one derives from the act of giving comes from the fact that others get to know the donor's income level and to be known to have high income is important. While their model addresses situations in which individual income is completely unobservable, this chapter refers to

[3]Hopkins and Kornienko (2004) is a more elaborated and general contribution to the problem of game of status and positional consumption. When donation is seen as a special case of positional consumption, their model supports Glazer and Konrad (1996).

situations in which individuals' income is directly or indirectly observable along with individual donations, when the relevant concern is face. In the case of pure moral concerns, both individual income level and donation can be completely unobservable.

The rest of the chapter proceeds as follows. Section 5.2 introduces the model and derives the main implications. Section 5.3 extends the model to the analysis of potential public policies. Finally, some concluding remarks are presented in Section 5.4.

5.2 An economy with concern for face

5.2.1 The model

Suppose a single good economy with individuals $i \in I$ of mass 1. Each individual is initially endowed with w_i amount of that good, $0 < \underline{w} \leq w_i \leq \overline{w}$. Each individual also has the possibility of making a charitable giving $g_i \geq 0$ and is aware that $g_i, \forall i \in I$, will be publicly reported after they *simultaneously* decided their own giving. Moreover, $w_i, \forall i \in I$, is also common knowledge.[4] The utility that one derives from private consumption x_i and donation g_i is

$$u(g_i, x_i) = \left(f + g_i - \frac{G_{-i}}{W_{-i}} w_i \right)^{\alpha} (x_i)^{1-\alpha}, \tag{5.1}$$

where G_{-i} (W_{-i}) represents the total donation from (endowment of) all other individuals except i, i.e.,

$$G_{-i} = \int_{j \in I,\ j \neq i} g_j dj \qquad \left(W_{-i} = \int_{j \in I,\ j \neq i} w_j dj \right).$$

$\alpha \in (0, 1)$ and $f > 0$ (further conditions specified below in inequality (5.8)) are two constants.

[4]Endowment level w_i may be observed directly or inferred in an equilibrium of a signaling game of conspicuous consumption. Further, the complete knowledge assumptions both of individual donation and endowment are stronger than necessary. In the current model, as long as the average donation and average endowment are known after the donation and people who are relevant to one's concern of face know his information, the argument of face will be valid and the results still hold.

The first part of the utility function is an individual's concern for face. It specifies the degree of face individual i can claim from others, judged by whether he has functioned adequately on his relative position (w_i) in his social network. An adequate amount of donation for i is formulated by the product of donation-endowment ratio of all others (G_{-i}/W_{-i}) and i's own endowment (w_i). Therefore, $w_i G_{-i}/W_{-i}$ gives what individual i should do if he appears at least as "generous" as the average.[5] The difference of individual i's donation and his "adequate amount" then determines whether i enjoys a gain of face or suffers from losing it. Initially all individuals are assumed having the same level of "face" f but how much individuals care about face when they make consumption-donation decisions is specified by the parameter α. In an economy, the more individuals care about face, the higher α is. In the extreme case when α goes to 1, individuals almost only care about face.

The second part of the utility function is an individual's private consumption. Note that the utility that an individual might derive from the public goods supplied, $G = \int g_i di$, is completely absent. Evidence has shown that both pure altruistic and warm glow motivations are important for the understanding of private donation.[6] However, since the primary tasks in this chapter are to investigate the direct benefits from the act of giving and to provide alternative explanations for the behavior of voluntary donation, we leave out this pure altruistic motivation.[7]

5.2.2 Analysis

Each individual maximizes utility (5.1) subject to budget constraint. The problem reads,

$$\max_{x_i, g_i} \left(f + g_i - \frac{G_{-i}}{W_{-i}} w_i \right)^{\alpha} (x_i)^{1-\alpha}$$

[5]In this chapter, we do not discuss the relationship between individual behavior and individual character, hence how much one donates in the current context does not help to know his personal quality of "generosity".

[6]See, for instance, evidence from neuroscience reported in Harbaugh, Mayr, and Burghart (2007).

[7]A similar treatment is also used in Glazer and Konrad (1996) and Harbaugh (1998a, 1998b).

$$\text{s.t. } x_i + g_i = w_i$$
$$x_i, g_i \geq 0.$$

Simply substituting x_i by the budget constraint,

$$\max_{g_i} \left(f + g_i - \frac{G_{-i}}{W_{-i}} w_i \right)^{\alpha} (w_i - g_i)^{1-\alpha}$$

and solving the first order condition gives

$$g_i = \left[\alpha + (1-\alpha) \frac{G_{-i}}{W_{-i}} \right] w_i - (1-\alpha) f. \tag{5.2}$$

Because individuals are from a population of mass 1,

$$G_{-i} = \int_{j \in I,\ j \neq i} g_j dj = \int_{j \in I} g_j dj \equiv G$$

which is the total donation. Similarly

$$W_{-i} = \int_{j \in I,\ j \neq i} w_j dj = \int_{j \in I,} w_j dj \equiv W$$

which is the total endowment of the population. Apparently,

$$\frac{G_{-i}}{W_{-i}} = \frac{G}{W}, \quad \forall i \in I.$$

Equation (5.2) can now be written as

$$g_i = \left[\alpha + (1-\alpha) \frac{G}{W} \right] w_i - (1-\alpha) f. \tag{5.3}$$

Integrate equation (5.3) over the population,

$$\begin{aligned} G &= \int_{j \in I} g_j dj \qquad (5.4) \\ &= \int_{j \in I} \left[\alpha + (1-\alpha) \frac{G}{W} \right] w_j dj - \int_{j \in I} (1-\alpha) f dj \\ &= \left[\alpha + (1-\alpha) \frac{G}{W} \right] W - (1-\alpha) f \\ &= \alpha W + (1-\alpha) G - (1-\alpha) f. \end{aligned}$$

Solving it for G we have

$$G = W - \frac{1-\alpha}{\alpha} f$$

and

$$\frac{G}{W} = 1 - \frac{1-\alpha}{\alpha} \frac{f}{W}. \tag{5.5}$$

Substituting equation (5.5) back to individual decision (5.3), we obtain

$$g_i = \left[1 - \frac{(1-\alpha)^2}{\alpha} \frac{f}{W} \right] w_i - (1-\alpha) f \tag{5.6}$$

which pins down individual donation level in equilibrium.

To ensure that each individual chooses a positive and feasible amount of donation in equilibrium, which has been assumed so far, g_i has to fulfill

$$0 < g_i < w_i, \forall i \in I.$$

That is,

$$0 < \left[1 - \frac{(1-\alpha)^2}{\alpha} \frac{f}{W} \right] w_i - (1-\alpha) f < w_i, \ \forall i \in I. \tag{5.7}$$

The second part of inequality (5.7) can be easily verified and the first part can be reduced to

$$f < \left(\frac{(1-\alpha)}{w_i} + \frac{(1-\alpha)^2}{\alpha} \frac{1}{W} \right)^{-1}, \ \forall i \in I.$$

Observe that the right hand side of the inequality increases in w_i, so if

$$0 < f < \left((1-\alpha) \frac{1}{\underline{w}} + \frac{(1-\alpha)^2}{\alpha} \frac{1}{W} \right)^{-1} \tag{5.8}$$

then $0 < g_i < w_i, \ \forall i \in I$.

Proposition 5.1. *In an economy with a mass* 1 *population of individuals whose utility functions are represented by equation* (5.1) *and condition* (5.8) *satisfied,there exists a unique equilibrium wherein individuals' charitable giving and* *private consumptions are,* $\forall i \in I$,

$$g_i^* = \left[1 - \frac{(1-\alpha)^2}{\alpha}\frac{f}{W}\right] w_i - (1-\alpha) f \tag{5.9}$$

and

$$x_i^* = \frac{(1-\alpha)^2}{\alpha}\frac{f}{W} w_i + (1-\alpha) f \tag{5.10}$$

respectively.

Proof. The above construction proves the existence of equilibrium. For the uniqueness, note first that the cases in which only *zero* measure of individuals donate positively do not qualify as equilibrium. Since if that is the case, the donation from the highest w_i can be found by substituting $G = 0$ into equation (5.2).

$$\begin{aligned}
g_{\overline{w}}^* &= \alpha\overline{w} - (1-\alpha) f \\
&> \alpha\overline{w} - (1-\alpha)\left((1-\alpha)\frac{1}{\underline{w}} + \frac{(1-\alpha)^2}{\alpha}\frac{1}{W}\right)^{-1} \\
&= \alpha\overline{w} - \frac{\alpha W \underline{w}}{\alpha W + (1-\alpha)\underline{w}} \\
&= \alpha\left(\frac{\overline{w}\left(\alpha W + (1-\alpha)\underline{w}\right) - W\underline{w}}{\alpha W + (1-\alpha)\underline{w}}\right) \\
&> \alpha\left(\frac{W\underline{w} - W\underline{w}}{\alpha W + (1-\alpha)\underline{w}}\right) = 0.
\end{aligned}$$

Therefore, a positive measure of individuals whose endowments are close to the highest level $\overline{w}$ will have incentive to donate positive amounts, contradicting to the assumption that there are only zero measure of individuals donate positively.

When there is a positive measure of individuals who donate positive amounts, the form of individual strategies can only be $\beta w_i - (1-\alpha) f$ where β is a constant. Since the construction in equation (5.4) admits one and only one solution, there are no other possible strategy combinations that qualify as equilibrium. $\square$

5.2.3 Comparative statics

Individual donation takes the form of equation (5.9). Given the assumption placed on f in inequality (5.8),

$$\beta \equiv 1 - \frac{(1-\alpha)^2}{\alpha} \frac{f}{W}$$

can be easily verified to be strictly positive. Not surprisingly, individual donation increases in the endowment level ($\partial g_i^* / \partial w_i > 0$). The more an individual possesses, the more he is expected to donate and the more he donates. With own endowment level unchanged, one donates more when the average endowment (W) of the population increases ($\partial g_i^* / \partial W > 0$). In this case, when the society gets richer and others donate more, individual i catches up with an increase in donation even when his own income remained constant. The impact of others' donation can be understood intuitively as others' generous donation makes one look bad.

The derivative $\partial g_i^* / \partial \alpha$ is also larger than zero which means individual donation increases in α, the parameter which governs how strongly individuals care about "face". The more they care, the tougher the contest of donation gets. The derivative $\partial g_i^* / \partial f$ is less than 0 so that private donation decreases with initial level of face. In many situations, f is related to the nature of the fund raising's purpose. When resources are raised to feed starving people or to provide relief to victims of major natural disasters, the situations speak for themselves and very likely will put each and every individual in a low level of initial face. In contrast, if some additional money needs to be raised to build a second football stadium of a university, then a high level of f can be expected.

The individual donation endowment ratio, defined as

$$\frac{g_i^*}{w_i} = \left[1 - \frac{(1-\alpha)^2}{\alpha} \frac{f}{W}\right] - \frac{(1-\alpha) f}{w_i},$$

is also increasing in individual wealth. Rich people tend to donate larger proportions of their endowments to charity in our simple model. Technically, this is a result of the constant term f and the structure of the utility function. The intuition is, for poor individuals, private consumption is more important given that there is an initial level of "face", while for

	∂w_i	∂W	$\partial \alpha$	∂f
$\partial g_i^*; \partial \left(\frac{g_i^*}{w_i}\right)$	+	+	+	−
∂x_i^*	+	−	−	+
$\partial G^*; \partial (\frac{G^*}{W})$	/	+	+	−
∂X^*	0	0	−	+

Table 5.1: Comparative Statics: the signs represent the direction of change of a row variable when a column variable increases

richer ones, to increase their overall as well as marginal benefit of private consumption, a larger proportion of additional wealth will go to charitable giving. There are real world observations of wealthy individuals donating most of their wealth.

Both, total donation

$$G^* = W - \frac{1-\alpha}{\alpha} f$$

and total donation endowment ratio

$$\frac{G^*}{W} = 1 - \frac{1-\alpha}{\alpha} \frac{f}{W}$$

increase in total endowment W and the parameter α but decreases in f. The proportion of total wealth that a society donates for charitable purpose increases in total wealth and in the relevance of "face" to individuals utility. Total private consumption

$$X^* = W - G^* = \frac{1-\alpha}{\alpha} f$$

which is surprisingly independent of the level of wealth. In other words, when society is getting richer, increased wealth will be entirely transferred to donation. Above results are summarized in Table 5.1 and Result 5.1.

Result 5.1. Ceteris paribus

i) *When an individual's wealth is increased, his donation level, donation income ratio and private consumption increase.*

ii) *When society's wealth is increased, an individual with original wealth level will increase his donation level which in turn increases his donation income ratio and decreases his private consumption; Society's donation level and donation income ratio will also increase.*

iii) When individuals are more concerned with face, individual donation level and donation income ratio increase and private consumption decreases. Same effects apply to the society level.

iv) When initial face level is increased, individual donation level and donation income ratio decrease and private consumption increases. Same effects apply to the society level.

5.2.4 Wealth, donation and happiness

With increased wealth and increased donation, will individuals be happier? In equilibrium, individual utility is

$$\begin{aligned} u(g_i^*, x_i^*) &= \left(f + g_i^* - \frac{G_{-i}}{W_{-i}} w_i\right)^{\alpha} (x_i^*)^{1-\alpha} \\ &= (1-\alpha) f \left[\frac{\alpha}{(1-\alpha)} + \frac{w_i}{W}\right]^{\alpha} \left[\frac{1-\alpha}{\alpha}\left(\frac{w_i}{W}\right) + 1\right]^{1-\alpha} \\ &= u\left(\frac{w_i}{W}; \alpha, f\right). \end{aligned} \tag{5.11}$$

Consequently, the relative endowment (w_i/W) determines individual utility. If each and every individual is endowed 50% more of wealth, the resulting relative wealth ratio remains the same for all, so does everyone's utility. But in aggregation,

$$G^{*'} = 1.5W - \frac{1-\alpha}{\alpha} f$$

which is a $0.5W$ increase compared to original donation and also is the entire wealth increase. Therefore, they donate more but consume the same and end up with an unchanged utility despite the fact that each has 50% more of endowment. This observation, like results of other models of interdependent preferences, helps to understand the Easterlin (1974) Paradox, namely average happiness seems not to be increasing with average wealth once basic needs (X^*) are fulfilled.

Result 5.2. *i) Individual utility is determined by the ratio of one's own wealth to the average wealth* (5.11).

ii) When each individual's wealth is changed with a same proportion, provided that condition (5.8) *is satisfied, individuals' equilibrium utility remain unchanged.*

5.3 An extension with public policy

We will not discuss government funds' crowding out effect using our much simplified model. Individuals do not derive direct utility from the total supply of public good and their engagement in charity is solely motivated by their own personal reasons i.e., to gain or not to lose face. A direct transfer from government to charity has no impact to the individual utility; equilibrium donation remains the same.

5.3.1 Proportional subsidy

The policy of proportional subsidy, however, is relevant. Suppose for each dollar donated by an individual, the government reimburses a fraction of t, $0 < t < 1$. Then for the same utility function (5.1), the budget constraint now reads

$$x_i + (1-t)g_i = w_i.$$

After similar steps, we obtain the equilibrium donation

$$g_i^*(t) = \left[\frac{1}{1-t} - \frac{(1-\alpha)^2}{\alpha}\frac{f}{W}\right] w_i - (1-\alpha)f. \tag{5.12}$$

Equation (5.12) differs from the original solution (5.9) by an increase in marginal spending on donation.[8] Private consumption in equilibrium now

[8]The condition that has to be put on f to insure interior solution, which we assume is satisfied, is now

$$0 < f < \left[(1-t)\left((1-\alpha)\frac{1}{\underline{w}} + \frac{(1-\alpha)^2}{\alpha}\frac{1}{W}\right)\right]^{-1}.$$

is

$$
\begin{aligned}
x_i^*(t) &= w_i - (1-t)g_i^*(t) \\
&= (1-t)\left[\frac{(1-\alpha)^2}{\alpha}\frac{f}{W}w_i + (1-\alpha)f\right].
\end{aligned}
$$

Clearly, with government subsidy donation becomes "cheaper" and people donate more. However, what it is perhaps surprising is that private consumption is now only a fraction $(1-t)$ of what is consumed without the subsidy. Hence, government subsidy did not help individuals to save on charitable giving instead it increased their donation level.

As a result of increased individual donation, aggregate donation

$$
G^*(t) = \frac{W}{1-t} - \frac{1-\alpha}{\alpha}f
$$

increases with the cost to the government being

$$
tG^*(t) = \frac{Wt}{1-t} - \frac{1-\alpha}{\alpha}ft. \tag{5.13}
$$

Is it well spent from the point of view of the total supply of public goods? The increased donation is

$$
\Delta G(t) = G^*(t) - G^*(t=0) = \frac{Wt}{1-t}
$$

which exceeds the government spending (5.13) by $ft(1-\alpha)/\alpha$. This is exactly the difference between private consumptions with and without the government subsidy since

$$
\begin{aligned}
\Delta X(t) &= X^*(t) - X^*(t=0) \\
&= (1-t)\frac{1-\alpha}{\alpha}f - \frac{1-\alpha}{\alpha}f \\
&= -\frac{1-\alpha}{\alpha}ft.
\end{aligned}
$$

Result 5.3. *A proportional subsidy rate of t costs the government*

$$
\frac{Wt}{1-t} - \frac{1-\alpha}{\alpha}ft
$$

but increases total donation by $Wt/(1-t)$. The difference $ft(1-\alpha)/\alpha$ is covered by a reduction in individuals' private consumption.

Public subsidy for donation seems to be a beneficial policy for individuals because it in effect lowers the price of donation. A closer look reveals the opposite. In equilibrium, individuals donate even more than what they used to donate before the subsidy. The reason is the following. Suppose every one else donates the original amount, an individual will place some of the money saved from the subsidy to donation while still have some left for private consumption. This, however, is not an equilibrium. When every one donates more, which has a negative impact on an individual's level of face, this individual will catch up in donation to equalize his marginal rate of substitution to the price ratio.

In equilibrium, we found that individuals increase the same percentage of individual wealth to donation compared the case without subsidy. In the end, however, they all obtain the same level of face as they do in the original case. One can verify that the final levels are both

$$\alpha f + (1-\alpha)\frac{f}{W}w_i$$

by using the results we obtained before. Therefore, in aggregate, increased individual donations will not bring any one any increase in face. The decrease in private consumption, however, results an unambiguous decrease of utility to all individuals. The utility with subsidy can be written as

$$u(g_i^*, x_i^*; t) = (1-t)^{1-\alpha} u(g_i^*, x_i^*; t = 0). \tag{5.14}$$

So the higher the subsidy is the lower individual utility will be.

Result 5.4. *Excluding the benefits from the provided public goods, a government subsidy for individual donation reduces individual utility.*

To summarize, a government subsidy encourages individuals engaging in face gaining activities. Had the subsidy only applied to one individual, it's true that this individual would have benefited from this subsidy since his budget set would have expanded; he would also have donated more to obtain a higher level of face. But the subsidy applies to all individuals, the negative externalities of donation to each other render individuals' higher level of donation completely ineffective in face improving. The concept of equilibrium also excludes the possibility for the individuals to maintain the original non-subsidized donation level.

5.3.2 A welcomed tax

If the proportional subsidy rate t is set to be negative ($t < 0$), the government then in fact levies on each dollar donated a tax of $-t$. In this case, donation gets more "expensive" and indeed the amount donated will decrease, see equation (5.12). Private consumption will increase since now people save on donation even though they have to pay tax for it. As we can see from individual utility level (5.14), individuals are better off when t decreases (lower subsidy or higher tax). Hence, the more severely the government taxes donation, the higher individual utility will be. Note that the benefit from the supply of total donation is not modeled. If the money raised is wasted, for instance, when the transfers are not donations to charity but contributions to wasteful consumption in the occasion of a festival or the like. Taxation on this type of gift giving is beneficial to all. Individuals enjoy higher utilities while the government can also raise money.

The logic behind this perverse result, namely taxation on private "consumption" of gift giving can lead to higher utilities of the individuals, is in line with Mill (1848)'s argument for taxation of conspicuous consumption.

5.4 Concluding remarks

The argument for face builds on the importance of social acceptance to an individual's all other activities. It is also crucial that individual donations are publicly observable for "face" to be a relevant motivation. Both points have more significant relevance in the East, especially East Asian countries than in the West. Face in some places is such an important concept and being referred to "no face" is an outright insult. The loss of face "makes it impossible for him to function properly within the community" (Ho (1976)). Indeed some researchers suggest that "face" should be accepted as an important theoretical concept in the literature. For example, Kim and Nam (1998) in management literature. To a lesser extent, the idea of "face" is also relevant in Western countries where individualism is more prominent.

Though it's true that donation should be and in many communities is a private conduct. In some other communities where people tend to openly discuss all kinds of information it is practically public. In those communities, individual wealth level is quite often also commonly known both because individuals signal out the information to obtain their relative positions in the social network and because those communities also are more stratified which makes the inference of wealth level easier. Note that as long as people who are relevant to a person know that person's conduct and wealth, the face argument is valid. It does not need all individuals in a society know every donation and wealth level. In the West, theories on conspicuous consumption and status game support that individual income is at least partially observable. Charitable activities that attract public attention are also quite common.

The approach presented here is based on a story of concern for face. It's by nature a social concept. The model, however, can also address purely moral motivations of donation. In this context, individuals compare what they've done to what they "should" have done. An outdoing would generally make one to feel being more than a responsible person and an insufficient conduct would make one less comfortable. The assumptions that each individual ascribes to the same reference rule and shares the same value of $\alpha$and f are admittedly strong. Nevertheless, the "negative externality" of one's donation to others' enjoyment of "being a good person" is robust and the current work maybe helpful for future research.

In an economy with concern for face, a publicly reported charitable fund raising is a big contest.[9] Because when individuals making their decisions they do not take the negative externality to others into account, in equilibrium a large part of donations from each individual does not give anyone any measure of benefit. This observation is similar to the results of status games wherein a large part of spending on positional goods is

[9] In moral context, instead of competing directly with each other, each individual competes with the fictional self whose characteristic is prescribed collectively by all other individuals.

simply wasted. The difference here is, the resources that are raised by a charity can be put into good uses rather than being wasted. The current model does not allow a complete analysis of welfare implications. However, when the gifts that individuals give end up with festivals and unnecessary conspicuous consumptions, taxation on this type of gift giving is favorable.

An interesting finding is the total private consumption of an economy does not change with aggregate income. Such economies consume more when there is no situation that induces a low level of initial face (f) and when individuals care less about face (α). When a "gift" tax is levied, individuals actually consume more and attain higher utility levels than without the tax.

In conclusion, in this chapter we present concern for face as one possible motivation behind visible conducts, especially in charitable donations. The policy analysis should be interpreted with caution since we have not included individuals' preference for the provided public goods. When the visible conduct in question is beneficial to no one, individuals' sacrifice of private consumption is inefficient.

Chapter 6

Further Research

In this chapter, I discuss several possible future research projects for each of the previously presented topics.

6.1 Capturable certifiers and umbrella branding

6.1.1 Capturable certifiers

In Chapter 2, certifiers are assumed to only passively use their testing technology. Therefore, the strategic aspect of manipulating testing results is absent. It is nevertheless an important research topic. In the case of perfect testing technology, Strausz (2005) employs a repeated game framework and finds that a monopoly certifier is less likely to be captured since it has more profit to lose than competitive certifiers. This insight was first put forward in Klein and Leffler (1981)'s seminal work on reputation.

Whether certifiers will honestly reveal testing results and under what circumstances will they do so when their testing technology is imperfect? This will be a natural direction for future research. Presumably, the likelihood that a certifier is captured is linked to the underlying quality type distribution and the nature of the imperfect testing technology. A more subtle point is how to differentiate the case where a certifier manipulates a testing result from the case in which it makes an honest mistake. An idea is to model consumers' trust on a certifier being depreciating only gradually when they observe bad quality products being certified. Consumers will also take the nature of the testing technology into account in such a way that when the testing technology is more precise, they will be less tolerant of certified bad quality products. A more interesting case is when there are multiple certifiers on the market.

6.1.2 Umbrella branding

Chapter 2 is concerned with an asymmetric information model with an adverse selection problem. In many situations, market provision of quality is also a moral hazard problem. Brand name products are often associated with high quality and in consequence are charged with a quality premium. This is an example of mitigating asymmetric information problem by establishing reputation. It is also a common practice to leverage reputation of one product to others via umbrella branding. With emphases being put on consumer belief and punishment strategy, a new research project will explore additional possible yet relevant equilibria other than those already treated in the literature (Cabral 2009, Hakenes and Peitz 2008).

Both certification and umbrella branding can be seen as practices intending to reduce information asymmetry. Hakenes and Peitz (2009) study to which extent umbrella branding can replace outside certification. This paper, however, proposes only a one-period model. Drawing on my researches in certification and umbrella branding, I plan to investigate the interaction of these two practices in a repeated game setting, hoping to uncover the underlying long-run relationship.

6.2 Elastic demand in the Hotelling model and empirical investigation of spatial models

6.2.1 Elastic demand in the Hotelling model

The Salop model has been widely used to address entry issues in product differentiation while the Hotelling model is often used to investigate firms' location decision. Since traditionally both models mainly feature products of unit demand, our initiative of considering general elastic demand can be applied in the Hotelling model as well. Our main interest is to revisit previous results on "minimum" versus "maximum" product differentiation under elastic demand. Rath and Zhao (2001) introduce a linear demand function and quadratic transportation cost in a Hotelling model and find both minimal and maximal differentiation are possible. Our approach in Chapter 4 is more general and we expect our finding will encompass their result as a special case.

6.2.2 Empirical investigation of spatial models

As we have argued, once elastic demand is included efficiency issues can be properly addressed in spatial models. In many situations, however, a real answer to a specific market can only be provided by a carefully conducted empirical investigation. To test whether our work in Chapter 4 provides a good framework for empirical studies of spatial models, I am interested in carrying out an empirical research of bottled and canned beer market.[1] Following the theoretical guidance, an estimation of consumer demand function of beer based on market data will be the first step. Of course, a good estimation of transportation or taste parameter is hard to grasp but data on variety switch caused by price changes might be very helpful in this regard. In the end, I hope I will be able to provider better answers to efficiency questions.

6.3 An experimental investigation of concern for relative social approval (face): a research proposal

In Chapter 5, I presented a theoretical explanation of charitable giving and introduced individuals' concern for face as a case of interdependent preferences. In the following, I present a self contained research proposal of an experiment. I first review the relevant theories then proceed to experiment design.

[1]Beer is an idea example for our theoretical model in Chapter 4 since normally each consumer has a small number of preferred variety of beer or even only one favorite variety of beer.

6.3.1 Theories

Andreoni (1989, 1990) differentiates *pure altruism* and *impure altruism* in private provision of public goods. While the former denotes a donor's preference for the well-being of others, the latter is defined to highlight private benefits of giving. The nature of private benefits or warm-glow is, however, less well understood. Besides internal moral concerns (Brekke, Kverndokk, and Nyborg, 2003), "prestige benefit" (Harbaugh 1998a, 1998b) and signaling motivations (Glazer and Konrad, 1996), Gu (2008b)[2] argues that an individual's concern for social approval is also an important motivation for making positive contributions. This concern therefore, constitutes one form of impure altruism. Differing from other models of social acceptance (Holländer 1990, Lindbeck 1997), Gu (2008b) emphasizes *an individual's relative position* in the social network and introduces the concept of "face" (Ho, 1976). When individuals are concerned with "face", the wealthier will need to contribute more than the poorer in order to gain an equal level of social approval. In the current project, we intend to empirically investigate this relative-position-adjusted concern.

6.3.2 Experiment design

To investigate this concern for relative social approval, we use a linear Public Goods Game (hereafter PGG, for a survey see Ledyard 1995) with two modifications. First, we introduce the opportunity of receiving social approval into the otherwise anonymous PGG. Since Laury, Walker, and Williams (1995), there have been a few studies of social approval using PGGs with subjects' identity observable. Gachter and Fehr (1999) allow subjects to express and receive social approval after the game and Rege and Telle (2004) create a situation in which their subjects present their own contribution level in front of all subjects. The results of these experiments are mixed but we leave detailed analysis to future development of this project. By social approval, we also mean non-pecuniary sanctions. Therefore, we do not discuss the approach of Fehr and Gachter (2000).

Our second modification to the game is on the distribution of endowments. In normal PGGs, subjects are endowed equally. To investigate if

[2] Also Chapter 5 of this thesis

the concern for social approval will make wealthier subjects contribute more, we assign, say, the ten integers from 21 to 30 randomly (without replacement) to ten subjects as their initial endowment levels. The impact of endowment heterogeneity in public goods experiments is studied in Chan, Mestelman, Moir, and Muller (1999), Cherry, Kroll, and Shogren (2005) and Kroll, Cherry, and Shogren (2007).

We intend to run our experiment with the following three treatments:

1. Normal anonymous PGG but with heterogeneous endowments (pure altruism, preference for fairness, intrinsic benefits of giving)

2. Let individual contributions be observable but not endowment levels (social approval, prestige effect)

3. Let both individual contributions and endowment levels be observable (relative social approval/ concern for ”face”)

Cited in parentheses are the related theories. Observed experiment results will also be compared to the findings in normal PGGs, PGGs with endowment heterogeneity (Cherry, Kroll, and Shogren 2005) and PGGs with social approval treatment (Gachter and Fehr 1999, Rege and Telle 2004). The proposed experiment will investigate if the concern for “face” is relevant in individuals’ decision-making processes and will attempt to shed light on both the nature of social approval and its impact on cooperative behavior in communities.

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Erklärung

Ich versichere, dass ich diese Dissertation selbstständig verfasst habe. Bei der Erstellung der Arbeit habe ich mich ausschließlich der angegebenen Hilfsmittel bedient. Die Dissertation ist nicht bereits Gegenstand eines erfolgreich abgeschlossenen Promotions- oder sonstigen Prüfungsverfahrens gewesen.

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