The thesis at hand focuses on nonlinear hyperbolic problems of second order. Two different kinds of nonlinearities are considered: Quasilinear elliptic differential operators in space and contact problems. Static as well as dynamic contact problems are discussed. A posteriori error estimates of the finite element discretisation error are derived, where the error is measured with respect to arbitrary nonlinear functionals of interest. Based on the a posteriori error estimates, adequate adaptive methods are developed to enhance the efficiency of the discretisation. They particularly improve the treatment of the effects caused by the use of dynamic meshes, for instance, the violation of energy conservation.