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Article

Experimental Noise, Idealizations and the Classical-Quantum Relation

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Abstract: Classical dynamics and the classical concept of space-time reality is based on the assumption that the objects of physics can be observed continuously. The discrete structure of matter (atoms) and fields (quanta), however, implies that the process of observation is quantized. In this paper we discuss the consequences of this paradigm change from continuity to discreteness and from determinism to chance. By taking into account the quantized structure of observation, we are led to the conclusion that the classical concept of reality has to be replaced by a quantum concept of observability. It implies that quantum dynamics is not a generalization of classical dynamics. Rather, the two theories apply to complementary idealizations of nature representing opposite extremes on a scale of observability. They are related by correspondence rules. Both theories disregard experimental noise. Statistical and thermal noise provide the experimental foundation for statistical physics.

Keywords: Observability, Complementarity, Correspondence, Determinism, Chance

1 Introduction

Physics confronts us with both dynamic and statistical laws. The dynamic laws represent the determinism, but the statistical laws the influence of chance in nature. Often the dynamic theories are considered as the fundamental ones. These are mechanics and electrodynamics in classical physics and quantum dynamics in modern physics.

However, these dynamic theories have a fundamental deficiency. They fail to explain dissipation and irreversibility of the processes observed in nature. Dissipation and irreversibility are related to the laws of chance (v. Weizsäcker 1985, Ch. 4).

In classical physics the laws of chance are introduced in thermodynamics. They are based on the hypothesis that matter consists of atoms. But in spite of the fact that chance and determinism are opposing concepts, the introduction of the laws of chance in thermodynamics is usually not considered as a change in the fundamental concepts of physics. Rather, the statistical concept is justified in many text-books simply by referring to the enormous number of particles constituting the thermodynamic system and by the lack of detailed information about the motion of the particles. This motion is still assumed to be measurable exactly in principle. But this assumption disagrees with the experimental situation, as has been revealed by the discovery of experimental noise.

In quantum physics the laws of chance appear in connection with the act of measurement. The state of a quantum object can change in two ways (v. Neumann 1932). There is the dynamic evolution of the state vector governed by a Schrödinger equation. This evolution is deterministic and reversible. But there are also quantum jumps triggering elementary events, which can be detected. These quantum jumps occur spontaneously and the elementary events are irreversible processes. As in classical physics, irreversibility in quantum physics is related to the concept of chance. The events of detection cannot be predicted exactly. Only probability distributions can be calculated.

Obviously, chance is a fundamental concept of theoretical physics, which has to be introduced in both classical and quantum physics to account for the irreversibility of physical processes. In both cases, the introduction of the concepts of chance is possible due to the discrete structure of matter and fields. Chance in theory has its correspondence in experiment, where the influence of chance has also to be taken into account in every measurement. Indeed, physical quantities cannot be measured exactly. The measured values have an experimental uncertainty. Even if systematic errors in measurement are avoided, the accuracy is limited by thermal and statistical noise. This noise is governed by the laws of chance.

Usually, the concept of chance is introduced in theoretical physics first, and only later on the advanced theory is used to explain the phenomenon of noise (Reif 1965). But physics is based on experiments. Therefore, it is more appropriate to start with an analysis of the experimental situation. Indeed, the experimental foundation of physics changes dramatically with the discovery of noise. This change has hardly been recognized yet. As has been emphasized already in v. Oppen (2007), statistical and thermal noise is the '*wall flower of physics*'. Noise does not get the attention, which it deserves, and is disregarded in most standard introductory text-books of physics (see e.g. Kerr 1973; Feynman/Leighton/Sands 1964). However, statistical and thermal noise must not be considered as a nuisance, which principally can be avoided, but as an experimental evidence for the fact that nature does not evolve purely deterministically. Laplace's demon does not have any chance to predict the future exactly (due to the influence of chance on the evolution of nature).

In this paper we focus our attention on experimental noise. These principally uncontrollable fluctuations of measured signals arise from the quantized structure of the process of observation. The quantized structure is obvious in all measurements on free atomic particles, where discrete and spontaneously occurring elementary events are detected. But it appears also in every high-resolution experiment on macroscopic bodies. Since physical quantities cannot be measured exactly, they may even loose their physical significance under extreme experimental conditions. But the quantized structure of the process of observation opens also a new perspective on the foundations of physics. In particular, it allows for a classification of the objects of physics according to their *observability*, a quantity introduced in v. Oppen (1996). The objects of classical physics can be observed continuously, whereas the objects of quantum physics have a discrete observability. The ideal object of quantum *dynamics* is unobservable. The act of observation cannot be described dynamically, because the processes taking place are irreversible.

The quantity *observability* considered here is defined by referring to the experimental basis of physics. It must not be confused with the concept of observability often discussed in the literature on the philosophy of science (Kosso 1989). Here we consider the *experimental* foundation of physics. The discreteness and spontaneity of elementary events detected, for example, by using a Geiger counter is an experimental fact that has been mostly disregarded by theory and at least not considered as a fundamental feature of nature so far. Since the philosophy of science is also predominantly focused on theory, even when experiments are discussed (Hacking 1983), it has also not been treated in this field.

Based on the concept of observability of physical objects introduced in section 4, we shall defend the following thesis: Quantum dynamics is not a generalization of classical dynamics, but applies to an idealization of nature opposite to the classical ideal (v. Oppen 2007). Contrary to this claim, quantum dynamics is usually considered as a theory, which is universally applicable and which contains classical dynamics as a limiting case. This point of view seems to have become a *dogma of physics*. According to this dogma, most physicists *believe* that quantum dynamics does not only describe the world of elementary particles, atoms and molecules, but also the macroscopic world (Greenberger/Hentschel/Weinert 2009). This dogma has to be questioned. Atomic particles are not just microscopic bodies, as usually assumed, but are fundamentally different from macroscopic bodies due to their discrete observability.

Actually, it is well known that the transition from quantum to classical dynamics poses delicate problems. Though one usually claims that one has to distinguish between macroscopic and microscopic systems for deciding whether classical dynamics can be or quantum dynamics has to be applied, a distinctive criterion for what is macroscopic and what is microscopic is missing. In the contribution *Quasi-Classical Limit* by M. P. Landsman in Greenberger/Hentschel/Weinert (2009), "the appearance of the classical world from quantum theory" (p. 626) is discussed and found to be "a very deep and largely unsolved problem" (p. 628). This statement is also confirmed in the contribution on *Semi-Classical Models* in Greenberger/Hentschel/Weinert (2009) by M. Arndt with the following remark:

> "It has been proven in countless experiments that quantum physics is the correct theory for describing the world of elementary particles,

> atoms and molecules. It is also widely believed that quantum theory is equally correct in the macroscopic world." (p. 697).

But in spite of the fact that the quantum-classical transition is obviously a largely unsolved problem, the belief in the applicability of quantum dynamics to macroscopic objects has rarely been doubted, at least not in Greenberger/Hentschel/Weinert (2009).

Here we claim that the dynamic theories apply to idealizations of nature. Between the two extreme idealizations of quantum and classical dynamics are the objects investigated experimentally. They are not only subject to dynamic laws, but are influenced also by spontaneous interactions with the environment due to their observability. This influence of chance limits the accuracy of measurements. It gives rise to the experimental uncertainties due to thermal and statistical noise. The influence of chance is disregarded in the dynamic theories, but provides the experimental foundation for statistical thermodynamics. However, statistical thermodynamics also applies to an idealization of nature. The ideal of statistical physics is the thermodynamic equilibrium state.

In contrast to these conclusions, physics so far pretends to describe a space-time reality. But the space-time concept itself is based on the assumption that the objects of physics can be observed continuously. Therefore, the quantized structure of the process of observation necessitates a fundamental revision of the concepts of physics. The *classical concept of reality* has to be replaced by the *quantum concept of observability*. The objects of physics must not be classified by their extension in space as microscopic or macroscopic, but primarily by their observability, which can be continuous or based on discrete quantum jumps. This paradigm change does not only affect the fundamental concepts of physics, but has also decisive consequences for the feasibility of high-precision measurements.

Before introducing the quantum concept of observability in section 4, we shall briefly reconsider the transition from the continuity concept of classical dynamics to the discreteness of matter (atoms) and fields (quanta) in modern physics. However, this transition has not been finished. Modern physics is still based on both the idea of continuity and the idea of discrete structures. This (unreflected) coexistence of continuity and discreteness still gives rise to conceptual difficulties. An important consequence of the discrete structure of matter and fields is the appearance of noise in all precision measurements as discussed in section 3. But most important is the consequence that the objects of physics cannot be observed continuously as tacitly assumed in classical dynamics. The quantized structure of the process of observation discussed in section 4 allows for introducing a scale of observability and a distinction between quantum and classical objects. Based on the scale of observability, the objects of physics can be classified. The different types of the objects of physics are discussed in some detail in the final sections.

2 From continuity to discreteness

The world of classical dynamics is continuous. The bodies and fields of classical dynamics are stretched out continuously in space and move or evolve continuously in time. Both bodies and fields are assumed to be observable continuously. This concept of continuity is fundamental also for relativistic dynamics. According to the concept of continuity, material bodies are described by continuously varying density distributions interacting with fields of force also varying continuously in space and time. Point masses and point charges are idealizations. They seem to be discrete, but they are artefacts in the world of classical dynamics. In spite of their infinitesimal extension, they are assumed to be observable continuously.

This classical world of continuity was questioned, when the atomic hypothesis was introduced in chemistry about two hundred years ago to account, in particular, for the laws of constant and multiple proportions. According to the atomic hypothesis, matter consists of atoms and, hence, has a discrete structure. The discrete structure of matter is the basis, on which the laws of chance could be introduced in physics and the theory of statistical mechanics could be established. A prominent example is the kinetic theory of gases based on the assumption that atoms follow the dynamic laws of mechanics as long as they move freely in space, but are scattered randomly when colliding with each other.

This combination of dynamic and statistical laws opened a new field of physics conceptually different from classical dynamics. According to the theory of statistical mechanics, the motion of atomic particles is not completely determined by dynamic laws. There is also the influence of chance giving rise to a *random walk*. Based on this idea, equilibration processes and the state of thermal equilibrium could be explained. By recognizing the discrete structure of matter and combining the time-reversal invariant dynamic laws with statistical laws, the irreversibility of the processes observed in nature became amenable to theory.

However, the atomic hypothesis prepared only the first part of the way from continuity to discreteness. Only matter attained a discrete structure, but fields remained continuous. This asymmetry between the theory of matter and the theory of fields was removed by postulating the quantum and the photon (or light quantum) hypothesis. Indeed, due to the quantum postulate, M. Planck could interpret the thermal equilibrium state of the electromagnetic radiation field, and by introducing the light quantum hypothesis, A. Einstein revealed the relation between black-body radiation and ideal gases. Due to the photon hypothesis, the laws of statistics could be applied also to fields.

The quantum postulate opened the door to the impressively fruitful field of quantum physics. But also quantum physics, if considered as a theory of microscopic objects (Messiah 1970), confronts us with both continuity and discreteness. Though both matter and field have been quantized, there is still the continuity of space and time. This marriage of continuity and discreteness still gives rise to conceptual difficulties.

An early example for these conceptual difficulties arising already in thermodynamics is Gibbs' paradox (Reif 1965). Gibbs considered a container divided by a partition in two equal parts filled with gases at the same temperature T and the same pressure P. If the partition is removed, the atoms of the gases can diffuse freely from one part to the other. Thermodynamics leads to an apparently paradoxical conclusion. If the atoms of the gases differ by some whatsoever insignificant property, one can extract mechanical work from the diffusion process, for the entropy of the two gases increases. However, an extraction of work is impossible, if the atoms of the two gases are identical, because in this case the entropy stays constant. This result is incomprehensible and utterly inconsistent with the classical idea of space-time reality. The paradox is solved by quantum dynamics, where identical atoms are assumed to be indistinguishable. They are indistinguishable not only like a couple of twins, who still have separate identities, but indistinguishable in principle. How is that possible?

According to the classical idea of space-time reality, two particles (or a couple of twins) can be distinguished, independently of whether they are identical or not, by tracing the motion of the particles along their trajectories. However, this option is based on the assumption that the particles can be observed continuously. This assumption is certainly justified for macroscopic bodies, but not for free atoms. As a consequence of the discrete structure of matter and fields, *the process of measurement is basically not continuous.* The discontinuity becomes relevant in measurements on macroscopic bodies only, if high-resolution experiments are performed. But it is obvious in every measurement on *free* atomic particles. They are detected by discrete *elementary events* (v. Oppen/Melchert 2005), triggered by quantum jumps, which occur spontaneously. The events can be counted as, for example, the current pulses of a photo-multiplier triggered by the photons created with the quantum jumps.

This *quantized structure of the process of observation* changes the experimental foundation of physics dramatically. Regarding the indistinguishability of atomic particles, it provides an obvious explanation (Bergmann/Schaefer 2008). Due to the discrete structure of the process of observation, it is impossible to keep track of atomic particles and, therefore, it is principally impossible to distinguish identical atomic particles experimentally. The indistinguishability of atoms, which had to be introduced in theory, is in full accordance with the experimental situation, though it is not in accord with the classical idea of a continuous space-time reality. This consideration gives rise to the conclusion that "*we have to abandon the description of atomic events as happenings in space and time*", as conjectured by Einstein/Infeld (1938), p. 313, and focuses our attention on the quantized structure of the process of observation.

3 Noise

The quantized structure of the process of observation significantly influences every precision measurement. The experimental accuracy is limited due to statistical and thermal noise. The resulting experimental uncertainties are neglected in classical dynamics, which is based on the assumption that the objects of physics can be observed continuously and experimental parameters can be measured exactly, at least in principle. However, the process of observation is quantized and exact measurements are principally not possible. Therefore, classical dynamics (including relativistic dynamics) describes an idealization of nature, where the quantized structure of the process of observation is disregarded.

The quantized structure of the process of observation and its consequences for the experimental accuracy are most obvious, if the energy E of the quanta triggering the detected elementary events is sufficiently large in comparison with the thermal energy kT of the environment including the detection device. In that case, the elementary events can be counted. A prominent example is the detection of radioactivity using a Geiger counter. Measurements with a Geiger counter revealed that the counting rate fluctuates according to the laws of chance. In particular, the experimental uncertainty ΔN of the number N of events counted within a time interval τ increases with $\sqrt{N}$, and the relative uncertainty $\Delta N/N$ decreases with $\sqrt{N}$. These fluctuations are called *shot noise* or statistical noise and determine the experimental uncertainty. Since the elementary events are subject to the laws of chance, one concludes that the events occur *spontaneously*.

Shot noise gives evidence of the fact that observation is a process inherently governed by the laws of chance. Therefore, the objects of physics principally do not only follow the deterministic laws of the dynamic theories, but are also subject to the laws of chance, for the objects of physics must be observable. Otherwise they cannot be studied experimentally. Classical dynamics has to be supplemented by the statistical approach to thermodynamics and quantum dynamics is valid only for unobservable objects. Quantum jumps triggering observable and, hence, irreversible events occur spontaneously and have to be treated separately. They principally cannot be described dynamically.

The quantized structure of the process of observation is less obvious, if the energy E of the quanta of detection exchanged between object and environment is comparable with or smaller than the thermal energy kT of the environment. $E \ll kT$ is usually the case, if an object is observable due to direct contact between object and measuring device. In this case, the process of observation is based on an exchange of phonons. Though phonons with $E \ll kT$ cannot be counted and, hence, the quantized structure of the process of observation is not obvious, the assumption is justified that observation is principally based on quantized processes. The irreversibility of the process of observation is a strong argument in favor of this assumption. The quantized structure becomes detectable only, if the temperature T of the measuring device is lowered to $kT < E$.

Due to the quantized structure of the process of observation, the observable objects of physics are exposed to an influence of chance. A manifestation of the influence of chance on matter is the Brownian motion of small particles suspended in a liquid. These particles are still macroscopic, that is they can be observed continuously. Therefore, their motion can be related continuously to some bodies of reference in the environment and, hence, described classically by trajectories as functions in space and time. However, the motion of the suspended particles is not determined by the laws of classical dynamics and cannot be predicted in detail. Only statistical predictions are possible. The suspended particles perform a random walk. Experimental evidence for the randomness is the fact that, according to the law of Einstein and v. Smoluchowski, the mean square $\langle \Delta x^2 \rangle$ of the distance Δx travelled by the particles in a time τ increases linearly with τ. A theoretical explanation of the randomness is provided by the fact that the motion is driven by collisions with atomic particles, which are not observable continuously.

The Brownian motion is an example for the *thermal noise* affecting all measurements. It is related to the temperature of the liquid, which may be considered as a measure for the influence of chance on the evolution of the liquid. Massive particles have a mean kinetic energy $\langle E_{kin} \rangle = (3/2)kT$, and the radiative spectral emission power $P(\nu)$ of a resistor per connection line is given by the Nyquist formula $P(\nu) = kT$. Thermal noise usually dominates, if $E \ll kT$.

The influence of chance on the motion of macroscopic matter can be disregarded within the limits of the experimental uncertainties due to noise, if the bodies of study can be observed with high spatial resolution. In this case, the motion seems to proceed deterministically and obeys the laws of classical dynamics with high, but not infinitely high accuracy. The motion follows deterministic laws only within the limits of the experimental uncertainties. If, however, the spatial resolvability is low and the spatial experimental uncertainty is comparable with the spatial extension of the body, the influence of chance becomes dominant. The microscopic particles performing the Brownian movements are of this size. They are observable just about continuously.

In conclusion, the presence of noise in all measurements gives evidence of the fact that the objects of physics are not only subject to dynamic laws, but are also influenced by chance. The purely dynamic theories apply to idealizations of nature. The influence of chance on the objects of physics cannot be avoided, because physical objects must be observable and observation is based on spontaneous processes governed by the laws of chance. The elementary events can be counted, if they are triggered by quanta with $E \gg kT$ and, therefore, the spontaneity can be investigated experimentally. But the assumption is justified that the process of observation is quantized also, when thermal noise is dominant and the elementary events of observation cannot be counted experimentally. The irreversibility of the process of observation strongly supports this assumption, because irreversibility is intimately related to discreteness and the laws of chance. Only the idealized processes described by the dynamic theories are reversible.

4 The quantum concept of observability

Classical physics is based on the idea that the objects of physics can be described in the continuum of space and time with unlimited precision. However, this classical concept of reality with bodies and fields obeying the laws of classical dynamics is based itself on the assumption that the objects of physics can be observed continuously and, therefore, continuously related to some bodies of reference in the environment. This concept of an objective space-time reality has an experimental foundation, if both the object and the bodies of reference are observable continuously. Actually, due to the experimental noise resulting from the quantized structure of observation, even the motion of macroscopic bodies cannot be represented as motion in space and time precisely. And classical dynamics fails completely if applied to atomic particles, where detection is related to discrete and spontaneously occurring quantum jumps.

The quantized structure of the process of observation necessitates a revision of the classical concept of reality. The objects of physics with a discrete observability are fundamentally different from the objects of classical physics. Due to the discrete observability, free atoms cannot be related continuously to the bodies of reference, and their motion cannot be described by classical trajectories. Neither can they serve as bodies of reference. They have to be described in terms of quantum physics. In particular, the indistinguishability of identical atoms mentioned above is at variance with the concept of an objective space-time reality. If one tries to describe atomic phenomena as classical processes in space and time, the behavior of atomic particles seems "spooky".

Therefore, we abandon the classical concept of an objective space-time reality and replace it by a *quantum concept of observability.* This quantum concept is based on the fact that the objects of physics must be observable. In a first step of a scientific approach to nature, one has to distinguish between an object and the environment. The environment can be assumed to be macroscopic, that is there are macroscopic bodies of reference and macroscopic measuring devices as, for example, scales and clocks, which can be described classically in a first approach. But one has to be aware of the fact that classical dynamics describes only an idealized environment and not some objective reality.

The object of study is embedded in the environment. As an object of physics, it must be observable. For defining it, the object must be sufficiently well separated from the environment. Yet object and environment must interact with each other that the object becomes observable. This interaction is spontaneous and subject to the laws of chance and, hence, can trigger irreversible processes in the environment, which can be detected.

The distinction between object and environment is fundamental. The macroscopically structured environment provides the space-time frame of reference suitable for describing macroscopic processes and roughly localizing the object of study, which is embedded in this environment. The choice of a suitable procedure for describing this object depends on the spontaneous coupling of the object to the environment. If the coupling is continuous, the object can be described classically. However, if the coupling gives rise to a series of discrete macroscopic elementary events, which can be counted, the object has to be described using the language of quantum physics. These objects are called *quantum objects* or (free) *atomic particles.* Between two successive elementary events triggered by quantum jumps of the object, the object is *unobservable.* It is *isolated* from the environment. The evolution of these isolated objects is not influenced by chance and follows the laws of quantum *dynamics.*

This conceptual procedure makes obvious that quantum dynamics is not more fundamental than classical dynamics. Rather, there is a delicate interdependence

of both theories. They are complementary and interrelated by correspondence rules. Since classical dynamics describes only idealized macroscopic objects, quantum dynamics is needed for describing properties of a macroscopic body related to its atomic structure. But also quantum dynamics relies on classical dynamics. In a first step of the conceptual procedure one needs classical dynamics for describing an (idealized) environment with space and time for relating the quantum object to the environment, where the observable events take place, which are to be predicted by calculating probability amplitudes. Only in a second step the quantum objects embedded in the environment can be described using quantum dynamics. The formalism allows for evaluating, in particular, the discrete energy spectra of isolated objects.
The replacement of the classical concept of reality by the quantum concept of observability is a rational answer to the question of Einstein/Podolsky/Rosen (1935):

> "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" (p. 777).

They write:

> "The elements of physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements." (ibid.).

Based on this agreeable requirement, they formulate their criterion of physical reality:

> "*If without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*" (ibid.).

It is exactly this precondition, which cannot be fulfilled experimentally. Due to the experimental uncertainties, a measurable quantity cannot be predicted with certainty. Due to the quantized structure of the process of observation, observable phenomena cannot be predicted exactly. There is always an influence of chance. Therefore, we have to abandon the classical concept of reality and replace it by the quantum concept of observability. With this replacement the transition from continuity to discreteness is completed. Space and time have not to be quantized, but the classical space-time concept has to be replaced by the quantum concept of observability.
According to the new quantum concept, the objects of physics can be ordered on a scale of observability (Fig. 1). So far, we have distinguished only classical objects from quantum objects. Classical objects have a macroscopic structure, which can be observed continuously, whereas quantum objects perform quantum jumps giving rise to discrete elementary events, which can be counted. But these objects are only prominent extremes. Actually, there is a much larger variety of objects, which can roughly be classified by a quantity *observability*. We define

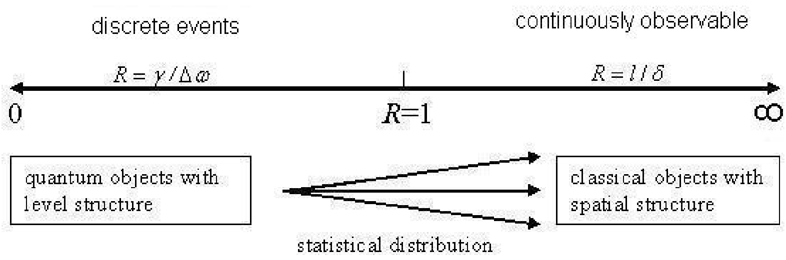

Figure 1: Scale of observability

this quantity R for quantum objects (with a discrete observability) and classical objects (with a continuous observability) separately.

1. Quantum objects: If the observability is discrete, the rate γ of elementary events triggered in the environment by quantum jumps of the object is a measure of its observability. In the limit $\gamma \to 0$, the object becomes unobservable and evolves purely dynamically. In this limit it is not influenced by chance. These idealized objects are the objects of quantum dynamics. The quantum objects have an observable level structure, if the ratio $R = \gamma/\Delta\omega$ of the rate γ of observability and the frequency separation $\Delta\omega$ of neighboring energy levels is $R < 1$.

2. Classical objects: If the observability is continuous, the spatial resolvability R of the object is a measure of its observability. Here $R = l/\delta$ is the ratio of the length l characterizing the size of the object and the minimum distance δ within the object, which can be observed continuously. If $R > 1$ is finite, spatial structures can be measured. But the size of these structures is at least of the order $\delta = l/R$. If one tries to measure structures smaller than δ, the process of observation becomes a discrete series of elementary events and a spatial/temporal description is no longer justified under the given experimental conditions. Identical atoms within the range of δ are indistinguishable. The ideal of classical dynamics is reached in the limit $R \to \infty$.

The concept of observability leads to the conclusion that classical dynamics and quantum dynamics have to be considered as theories applying to opposite extremes on the scale of observability. Quantum dynamics is not a generalization of classical dynamics, but describes an idealization of nature, which is complementary to the ideal of classical dynamics. The ideal of classical dynamics are objects which can be observed continuously in every detail ($R \to \infty$), whereas the ideal objects of quantum dynamics are unobservable ($R \to 0$). Real objects, that is the objects observed experimentally, have an observability characterized by a finite rate of elementary events or a finite resolvability ($R \sim 1$). The evolution of these objects is influenced by both determinism and chance and, therefore, cannot be predicted exactly. The influence of chance dominates the evolution of objects, which are approximately in a thermodynamic equilibrium state. The ideal of statistical physics is the thermodynamic equilibrium, where nothing happens on macroscopic scales.

The paradigm change from the classical concept of reality to the quantum concept of observability opens a new perspective on the foundations of physics. The fundamental theories of physics do not image some objective reality, but apply to idealizations of nature. Only idealizations can be described exactly. Based on the classical concept of reality one may expect that a theory unifying all presently known theoretical approaches to nature can be formulated. The quantum concept of observability questions this conclusion. Since according to this concept the idealizations described theoretically are complementary approaches to nature, it is more likely that a unifying theory cannot be conceived. Only correspondence rules interrelating the complementary theories can be formulated.

Beside these philosophical implications also the physical consequences of the paradigm change have to be discussed. In particular, the experimental noise gains significance. The realistic concept of bodies and fields evolving in space and time led to the conclusion that quantum dynamics has to be considered as a generalization of classical dynamics (Messiah 1970). From this point of view, a standard quantum limit (SQL) for the accuracy of measurements has been defined, which results from Heisenberg's uncertainty relations (Caves et al. 1980). However, this limit is valid only for so-called *amplitude-and-phase* measurements, that is for measurements, where non-commuting observables are measured simultaneously. According to the realistic classical concept, a single observable (as, for example, the amplitude) can principally be measured arbitrarily accurately. In particular, the techniques of quantum non-demolition or back-action-evading have been proposed (Caves et al. 1980). From this point of view, noise has been considered as a nuisance, which can principally be avoided (Braginsky/Gorodetsky/Vyatchanin 1999).

The quantum concept of observability leads to different conclusions. According to this concept, one has to distinguish between quantum objects (or free atomic particles) and classical (or macroscopic) objects. Only quantum objects evolve quantum dynamically, as long as they are unobservable. In particular, these objects can be in entangled states. But macroscopic objects are observable continuously. Therefore, they evolve according to classical dynamics and cannot be in an entangled state as has been (from the present point of view) wrongly assumed (Müller-Ebhardt et al. 2008). Macroscopic objects are necessarily subject to an influence of chance. This influence of chance gives rise to experimental uncertainties arising from noise. This noise is not some nuisance resulting from some avoidable disturbance, but is a fundamental feature of nature.

From the present point of view, one has to distinguish between *experimental errors* and *experimental uncertainties*. Only errors can be avoided by experimenting carefully. But the experimental uncertainties are fundamental and unavoidable. They are related to the observability of the object of study. The question arises: What is the ultimate accuracy, which can be reached in measurements on macroscopic bodies? Since both classical and quantum dynamics describe idealizations of nature, where noise is disregarded, these theories are inadequate for analyzing

the accuracy of experimental techniques. Special investigations based on statistical physics and/or experimental data are needed. Though the influence of noise has been discussed in detail in connection with projects for the detection of gravitational waves (Grote 2008), it has been assumed that this influence can principally be eliminated. From the present point of view, this assumption is not justified for measurements on macroscopic objects.
Due to their observability, macroscopic objects cannot be described exactly. They are subject to influences, which cannot be controlled experimentally. Nevertheless, they can often be treated approximately using perturbational approaches based on the exact theories of classical and quantum dynamics or statistical physics. We shall briefly discuss the various theoretical approaches to nature in the following sections.

5 Objects of the dynamic theories

Physics is a science of *observable* objects. Due to their observability, they are inevitably exposed to the influence of chance. The dynamic theories apply to idealizations of nature, where the influence of chance is disregarded.
The dynamic theories of classical physics (mechanics and electrodynamics) are based on the assumption that bodies and fields are observable continuously. The precondition of continuous observability has to be fulfilled, in particular, by the bodies of reference and the measuring devices. Obviously, the classical concept of reality in space and time is based on the presupposition that the environment is macroscopically structured. This presupposition is not self-evidently fulfilled. The early universe traditionally described as quark-gluon plasma was still unstructured. Space and time emerged only with the evolution of the universe. Only the structured environment can be described in terms of classical dynamics with good approximation.
Both classical dynamics and quantum dynamics apply to objects, which are embedded in a classical environment. The measuring devices are part of the environment and have to be described classically, as already N. Bohr often emphasized (Wheeler/Zurek 1983) (though he also considered quantum dynamics to be a *rational generalization* of classical dynamics). The object has to be described separately. Whether one has to apply classical or quantum dynamics, depends on the observability of the object, but not on the size as usually assumed. Depending on the experimental conditions, the same kind of object has to be described once classically and another time quantum dynamically. A molecule, for example, can be investigated under extremely different experimental conditions. A free molecule in a dilute gas is temporarily unobservable and behaves quantum dynamically, whereas a (sufficiently large) molecule embedded in a crystal or a liquid is observable (almost) continuously that classical dynamics can be applied. In the quantum dynamic limit, the molecule has an observable level structure, whereas it has an observable spatial structure in the classical limit.
As quantum dynamics and classical dynamics are complementary theories, also level structure and spatial structure are complementary features of the objects

of physics. Either the level structure or the spatial structure is relevant. In the quantum limit, where the rate γ of observability is smaller than the frequency separations $\Delta E/\hbar$ of the energy levels, only the level structure is measurable and, hence, meaningful, whereas in the classical limit, where the resolvability is large, only the spatial structure is meaningful.

Quantum physics and classical physics are complementary. Since quantum and classical dynamics describe opposite idealizations of nature, there is no continuous transition from quantum to classical dynamics. However, there is a structural and formal correspondence between quantum and classical dynamics. Due to this correspondence, many quantum objects, as for example atoms and nuclei, can be modelled as quasi-classical systems composed of particles considered as more elementary than the total object. The classical Hamilton function of the classical model can then be transformed into a quantum dynamic Hamilton operator. The correspondence rule (Messiah 1970) states that canonically conjugate quantities of the Hamilton function as, for example, position x and momentum p have to be replaced by operators fulfilling the commutator relation $[p, x] = i\hbar$. But this correspondence only allows the transition from the dynamics of non-relativistic classical systems to quantum dynamics. The transition for relativistic dynamics is a more difficult task and will not be treated here.

In spite of this correspondence, the objects of quantum dynamics differ fundamentally from the objects of classical dynamics.

In classical physics the dynamics of large and complex objects results from the dynamics of its parts. Therefore, complex systems are usually decomposed into simply structured components for analyzing the dynamics. Classical physics is compatible with a reductionist philosophy. Quantum physics, however, puts a crucial limit to the reductionist concept. Quantum objects have to be considered as a *single entity*. But they can be modelled as composed systems due to the quantum-classical correspondence. If one tries to visualize a quantum object as a composed system, one usually finds that the state vectors of the components are entangled in the quantum state of the total object. This entanglement represents characteristic features of the total object. Therefore, its dynamic evolution cannot be reduced to the dynamic evolution of its components. This *feature of wholeness* (Wheeler/Zurek 1983) becomes obvious also, if the object interacts spontaneously with the environment and triggers an elementary event. In this case the dynamic evolution breaks down and the object as a whole jumps from an upper energy level to a lower one.

The definition of a quantum object, that is the decision where one should make the cut (Heisenberg 1959, Ch. 3) between the object and the environment, is not only determined by the experimental situation, but is an essential step of the scientific approach to nature. Ultimately, it is the decision of the scientist. The dynamic theories describe only idealizations of nature. Therefore, there is some arbitrariness in the definition of the objects described by quantum dynamics. Note that only the object is described quantum dynamically, but the environment is macroscopic and can be described using classical dynamics in a first approach.

As an example, we consider a free atom in a radiation field. If the field is weak, that is the modes of the field are scarcely occupied with photons, it is reasonable to include the radiation field to the quantum object. In this case, Dirac's theory of radiation applies. Atom and field have to be considered as a single entity, as E. Fermi emphasizes in his famous review article on Dirac's theory (Fermi 1932). However, if the radiation field is strong as, for example, a radio-frequency or laser field, it is reasonable to consider the radiation field as part of the environment and to describe it using classical electrodynamics. It is treated as an *external* field.

As another example we consider collision processes of free atomic particles. One can concentrate on the dynamics of the separate particles and analyze the evolution of these particles in free space or in an external radiation field. But for describing the collisions of two (indistinguishable) atomic particles, the collision system has to be considered as a single entity, which evolves in time. After the collision the particles are in an entangled state and must not be considered as separate objects, as long as the dynamic evolution continues. However, they can be detected separately.

In conclusion, quantum dynamics and classical dynamics are complementary approaches to nature. Both theories apply to an idealization of nature, where the influence of chance is eliminated. The ideal of classical dynamics is the unrestrictedly continuously observable object, and the ideal of quantum dynamics is the isolated object without any spontaneous coupling to the environment. These isolated objects can almost perfectly be prepared experimentally. Outstanding examples are trapped particles as ions (Neuhauser et al. 1980), electrons (Dehmelt 1990) or atoms (Metcalf/Phillips 1986). Ions and atoms are isolated as long as they are in the quantum dynamic ground state. Only after excitation by some laser field, they interact spontaneously with the environment and can be detected. Electrons in a Penning trap interact spontaneously with the cap electrodes of the quadrupole field due to the oscillating motion in z-direction. The spontaneous coupling of this motion is sufficiently strong that the electron can be detected and the oscillating motion can be described classically, whereas spin, cyclotron and magnetron motion are sufficiently decoupled from the environment and can be described quantum dynamically. But not only trapped particles, also the atoms of gases or particles of atomic beams are isolated during the time of free flight. Actual objects investigated experimentally, however, are neither isolated nor observable (unrestrictedly) continuously. Therefore, perturbational approaches based on one of the dynamic theories are needed. On one hand, starting from the ideal of quantum dynamics, the influence of chance is taken into account by introducing the spontaneous quantum jump. On the other hand, starting from classical dynamics, chance is introduced as thermal unrest, the random motion of small particles and the statistical fluctuations of fields. In both limits, the influence of chance gives rise to experimental uncertainties resulting from shot noise and thermal noise, respectively.

The correspondence of the theories of classical and quantum dynamics is the foundation of the visualizations of atomic particles as composed systems with some diffuse geometric structure as, for example, electron clouds. This structure corresponds to the geometric structure appearing with the transition from quantum to classical physics, which takes place when the spontaneous coupling of the object to the environment is increased. Nevertheless, these visualizations of quantum objects must not be considered as some objective reality in space and time. They are needed only for constructing the appropriate Hamiltonian. The identity of objects is not defined by their position and extension in space, but by various quantities fulfilling conservation laws. The same object can give rise to quantum dynamic interference effects and can show classical reality depending on the spontaneous coupling of the object to the environment. Molecules as large as the C_{60} fullerene could be used as quantum objects for quantum dynamic interference experiments (Arndt et al. 1999), but they also show up as classical ball-like spatial structures, when they are strongly coupled to the environment.

6 Objects with finite observability

Between the two extreme idealizations of isolation ($R \to 0$) and continuity ($R \to \infty$) described by quantum and classical dynamics, respectively, is the vast field of objects with finite observability. They are the objects investigated experimentally. Experimental investigations on these objects provide the foundation for the theoretical description of nature. But the experimental foundation of physics has a fundamental deficiency, which puts a limit to the scientific approach to nature.

The ideal scientific experiment should be reproducible exactly. But this requirement cannot be fulfilled. Even if the experimentalist strives for experimental conditions, where all external influences on the object of study are controlled as well as possible and uncontrollable random influences are avoided, the experimental noise puts a fundamental limit to the reproducibility of scientific experiments. Since the objects of physics have to be observable, this influence of chance is unavoidable.

As a consequence of this deficiency on the experimental side, theory cannot image the experimental situation exactly. The fundamental theories apply to idealizations of nature. By disregarding the influence of chance in nature completely, the dynamic theories of classical physics were established based on the concept of an objective reality in space and time. Presently many scientists assume that also the interpretation of quantum physics can be based on the classical concept of reality. This assumption led to the conclusion that quantum dynamics is a theory of microscopic objects as atoms and subatomic particles (Messiah 1970). Often one continues to conclude that with the physics of microscopic objects also the physics of macroscopic objects is understood, at least in principle. Since macroscopic matter is composed of atoms, which fulfil the laws of quantum dynamics, it seems that one simply has to decompose the macroscopic matter for a detailed understanding. The complexity of macroscopic systems seems to be the only problem, which still has to be analyzed (Crick 1994).

The quantum concept of observability leads to a different conclusion. The principle of decomposition has to be abandoned already in quantum dynamics, as emphasized above. Actually the reductionist concept can be used only in classical physics as long as the components are still observable continuously. Principally quantum objects have to be considered as a single entity. Only the quantum-classical correspondence justifies the visualization of quantum objects as composed systems.

From the present point of view, both classical dynamics and quantum dynamics apply to idealizations of nature suitable for an intellectual understanding of experimental observations. But they do not reflect an objective reality. Since experiments cannot be reproduced exactly, theory cannot image nature exactly. Therefore, the vast field of objects with a finite observability is actually a *terra incognita*, which can only be approached from the two extreme idealizations, which can be described dynamically, by combining the dynamic theories with some laws of chance. We shall discuss this procedure briefly in what follows, even though we cannot give more than a rough sketch here. However, it is important to note that these statistical approaches are still inadequate to appropriately describe all phenomena observed in nature. In particular, the phenomena of life cannot be understood by simply combining dynamic and statistical laws. The relation between biology and physics has still to be clarified. But by realizing that the fundamental theories of physics apply only to idealizations of nature, new prospects for clarifying this relation may be opened.

The statistical approaches to nature are based on the dynamic theories. Starting from the idealization of unobservable objects considered in quantum dynamics, one has to introduce the spontaneous quantum jump. The laws of chance become relevant at the interface between quantum object and classical environment. The quantum jump triggers an elementary event in the environment, which is macroscopic and, hence, can be observed and localized in space-time. However, position and time of the elementary event cannot be predicted exactly. Only probability distributions can be calculated (see Fig. 1). These calculations are based on the quantum-classical correspondence.

If we proceed on the scale of observability from the consideration of a singular event to a situation, where the spontaneous coupling gives rise to a series of events, that is from $R = 0$ to $0 < R < 1$, also the quantum object itself is influenced by chance. The quantum object does not evolve purely dynamically, but can be treated using perturbation theory and the density matrix formalism. As a consequence of the spontaneous coupling, the energy levels are broadened and ultimately overlap, if $R \sim 1$. In this case, one has reached the transition region between the realms of quantum and classical physics. Many surprising features of so-called nanoparticles are likely to be related to the fact that nano-physics deals with objects of this transition region.

Starting from the idealization of continuously observable objects considered in classical dynamics, the object is considered as an ensemble of atomic particles, that is an ensemble of quantum objects. In accordance with the fundamental concepts of quantum dynamics, the identical atomic particles are considered as principally indistinguishable discrete entities with a finite number f of degrees of freedom. They are *not* tiny classical bodies with infinitely many degrees of freedom, which are observable continuously and can be distinguished. An essential precondition for the thermodynamic approach is the assumption that the atomic particles are sufficiently well separated from each other that they can be considered as quantum objects, that is that quantum dynamics is the appropriate approach for describing them. In particular, the level structure of the particles must be measurable. With regard to the fact that the elementary constituents are not classical, but quantum objects, it is reasonable to call all systems considered in statistical thermodynamics *quantum gases*, though this expression is usually used only for degenerate gases.

Quantum gases confront us again with both quantum and classical dynamics. The whole gas is macroscopic and behaves classically. But its elementary constituents are quantum objects. When considering sub-ensembles, there is a gradual transition from sub-ensembles with discrete observability to sub-ensembles, which can be observed continuously. The former are appropriately described in the language of quantum physics and the latter in classical terms. Again, the concept of chance has to be introduced at the interface, where the elementary quantum objects are related to the space-time structure of the macroscopic ensemble described classically.

Usually one assumes that the motion of particles, in principle, can still be described by classical trajectories and that the statistical approach is appropriate because of lack of information. By recognizing the fundamental difference between quantum and classical objects, we are led to a different conclusion. The motion of quantum objects principally cannot be described classically as motion in space and time. However, under specific experimental conditions, their motion can still be viewed in the sense of classical dynamics, if only average values are considered as physically relevant parameters. This statistical approach to nature is applicable under the experimental condition that the ensemble is in thermal equilibrium. In that case, macroscopic dynamic motions are not possible and, therefore, pure chance determines what happens.

Obviously, as the dynamic theories, also the thermodynamic theory of quantum gases applies to an idealization of nature. Firstly, *a thermodynamic system* must be composed of elementary constituents, which can be treated as quantum objects, and secondly, the quantum gas must be in thermal equilibrium. If the thermodynamic ensemble is only in a local thermal equilibrium, one has to distinguish between the classical motion of macroscopic sub-ensembles and the thermal motion of the elementary constituents. Only the classical motion can be described in detail, but the thermal motion can be treated classically only with respect to a statistical interpretation.

Contrary to the classical point of view, the statistical interpretation of the atomic motion is in accordance with the indistinguishability of atomic particles. Furthermore, the distinction between *macroscopic* motion, which can be well controlled experimentally, and uncontrollable *thermal* motion provides a solid basis for distinguishing work and heat in statistical mechanics. Though work and heat are equivalent with respect to the first law of thermodynamics, they have to be clearly distinguished with respect to the second law. Transfer of work changes macroscopic motion, whereas transfer of heat changes the thermal motion of the atomic particles. The difference between work and heat is obscured in the classical interpretation of statistical mechanics. Therefore, it is extremely satisfying that the new interpretation leads to a well founded distinction between macroscopic and thermal motion and, correspondingly, between work and heat.

7 Conclusions

Physics has to be based on observable quantities. This requirement led W. Heisenberg in 1925 to the formulation of *matrix mechanics*. In this paper we focused the attention on the process of observation. Its quantized structure leads to the conclusion that the classical concept of reality has to be replaced by a quantum concept of observability. According to this concept, classical dynamics and quantum dynamics apply to fundamentally different, but complementary idealizations of nature. Both idealizations are related by correspondence rules. Classical dynamics is based on the assumption that the objects of physics are observable continuously, whereas quantum dynamics applies to unobservable objects. Only the idealized objects of classical and quantum mechanics evolve purely deterministically. They describe processes, which are reversible in time. The objects investigated experimentally, however, have a finite observability and, therefore, are subject also to the influence of chance. This influence of chance is most obvious for free atomic particles, which can be observed only due to discrete and spontaneously occurring quantum jumps triggering elementary events in the detection device.

The paradigm change from the classical concept of physics based on the idea of a reality in space and time to the idealistic quantum concept of observability is not only of theoretical and philosophical interest, but implies also fundamental consequences for experimental physics. The realistic classical space-time concept led to the conclusion that quantum dynamics be a generalization of classical dynamics. In this case Heisenberg's uncertainty relations would determine the ultimately achievable experimental accuracy (at least for non-relativistic problems). From this point of view, physical quantities could be measured with unlimited accuracy, though non-commuting observables cannot be measured exactly simultaneously. From the present point of view, however, one has to take into account the observability of the objects under investigation. The observability implies some influence of chance giving rise to experimental noise. This noise is fundamental and unavoidable and limits the experimental accuracy. This noise is well known quantitatively for measurements on objects, which can be described as quantum gases, that is macroscopic ensembles of quantum objects. In this case, the thermal motion of the elementary quantum objects gives rise to thermal noise, and the spontaneity of the quantum jumps gives rise to statistical noise.

The fact that observable objects do not evolve purely dynamically, but are subject also to an influence of chance provides the experimental foundation of the statis-

tical theory of thermodynamics. Statistical thermodynamics applies to another idealization of nature, namely the state of thermodynamic equilibrium.
Therefore, the statistical approach to nature does not bridge the 'yawning gap' (Kadomtsev 1994) between the quantum and the classical idealization. There remains still a large open field of observable phenomena, which cannot even approximately be explained by present-day physics. In particular, there are the phenomena of life. Living creatures can neither be considered as composite systems in the classical sense, nor as ensembles of quantum objects. Quantum physics leads to the conclusion that the reductionist concept cannot be used unrestrictedly. Since theory cannot image nature exactly, one cannot even hope that the presently known theoretical approaches to nature can be brought together in a unifying theory.

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