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Abstract. Estimating the degree of substitution between energy and non-energy inputs is crucial for any evaluation of environmental and energy policies. Yet, given the large variety of substitution elasticities, the central question arises as to which measure would be most appropriate. Apparently, ALLEN's elasticities of substitution have been the most-used measures in applied production analysis. Using data of the U. S. primary metals sector (1958-1996), this paper empirically illustrates that cross-price elasticities are preferable for many practical purposes. This conclusion is based on a survey of classical substitution measures such as those from ALLEN, MORISHIMA, and MCFADDEN. The survey highlights the fact that cross-price elasticities are their essential ingredients.

JEL classification: C3, D2.

Key words: Substitution Elasticities, Translog Cost Function.

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1 Introduction

Estimating the degree of substitution between production factors such as energy and non-energy inputs is crucial for a host of issues, including environmental and energy policies such as trading greenhouse gas emission allowances, recycling energy tax revenues to reduce output or non-energy factor taxes, and the step-by-step increase of fuel taxes. Another example is the impact of fuel efficiency gains on energy use, which is also largely driven by the ease of factor substitution (SAUNDERS 2008, 1992).

Yet, despite the fact that a large number of empirical studies have appeared since the first energy crisis in the 1970's, there seems to be little consensus on the degree and even the direction of energy substitution. For instance, ever since BERNDT and WOOD's (1975) finding that the energy aggregate complements capital, and GRIFFIN and GREGORY's (1976) results indicating that both factors are substitutes, the energy-capital debate has remained unresolved – for surveys, see KINTIS and PANAS (1989), APOSTOLAKIS (1990), and FRONDEL and SCHMIDT (2002).

Although there are other important causes of divergent results, such as the industries and regions under study, we would like to focus on one important source: the large variety of distinct measures of substitution. Since HICKS (1932) originally defined the unique substitution measure σ for the case of only two inputs, often called "*the* elasticity of substitution", many different generalizations of this fundamental concept up to an arbitrary number of inputs have been provided – see ALLEN and HICKS (1934), ALLEN (1938), UZAWA (1962), MCFADDEN (1963), MORISHIMA (1967), BLACKORBY and RUSSELL (1989). Facing such a variety of measures and given the variation in perspectives and interpretations among substitution elasticities, the central question arises as to which substitution measure would be most appropriate in an empirical study.

Apparently, ALLEN's partial elasticities of substitution (AES) have played a dominant role and have been the most-used measures of substitution in the production literature – see e. g. HAMERMESH (1993:35), FRONDEL and SCHMIDT (2002, 2003). AES, however, has been criticized in the production analysis literature as only being interpretable in terms of cross-price elasticities (CHAMBERS 1988:95). AES is thus argued to add no more information to that already contained in cross-price elasticities (BLACKORBY and RUSSELL 1989:883).

Along the lines of FRONDEL (2004), who focuses on the classical cross-price elasticities when measuring the ease of substitution among energy and non-energy inputs, this paper argues that analysts are frequently better served by appealing to cross-price elasticities. This argument is supported here by a survey of classical substitution measures including ALLENS's partial elasticities of substitution (AES), MORISHIMA's partial elasticities of substitution (MES), and MCFADDEN's shadow elasticities of substitution (SES). The survey illustrates that all these standard measures are founded on cross-price elasticities. Drawing on time-series data for the U. S. primary metals sector (1958-1996), one of the most energy-intensive industries, and the frequently-employed standard translog approach, we then empirically illustrate why cross-price elasticities are preferable for many practical purposes.

This article's main contribution relies on demonstrating that analysts must take great care in interpreting the standard substitution elasticities commonly employed. Whenever one draws conclusions from empirical studies on the degree and direction of substitutability of production factors, it is indispensable to, first, clearly indicate the particular measure employed to denote two inputs as substitutes and, second, to interpret empirical results accordingly in order to avoid harmful policy recommendations. Ultimately, it becomes obvious that there cannot be a universally applicable substitution elasticity. Instead, the selection of a particular measure critically depends on the concrete application and question asked, a conclusion that can be traced to MUNDLAK (1968:234).

The following section provides a summary of classical substitution elasticities. In Section 3, we will use the empirical example of the U. S. primary metals sector (1958-1996) to illustrate the argument that, in many cases, cross-price elasticities are preferable. The last section summarizes and concludes.

2 A Survey of Classical Substitution Elasticities

When discussing elasticities of substitution, it is convenient and intuitive to commence with *the* elasticity of substitution σ, originally introduced by HICKS (1932) for the analysis of only two factors. σ measures the relative change in the factor proportion x_1/x_2 due to the relative change in the marginal rate of technical substitution f_{x_2}/f_{x_1} while output Y is held constant:

$$\sigma = \frac{d\ln\left(\frac{x_1}{x_2}\right)}{d\ln\left(\frac{f_{x_2}}{f_{x_1}}\right)}. \tag{1}$$

With more than two factors being flexible, the marginal rate of technical substitution f_{x_2}/f_{x_1} would not be determined uniquely. To avoid such ambiguities in a multi-factor setting, further assumptions are necessary, leading to an alternative definition of σ in the two-dimensional case that BLACKORBY and RUSSELL (1989) call the HICKS' elasticity of substitution (HES). Under the assumptions of perfect competition and profit maximization, f_{x_2}/f_{x_1} equals relative factor prices p_2/p_1 and hence

$$\text{HES} = \frac{d\ln\left(\frac{x_1}{x_2}\right)}{d\ln\left(\frac{p_2}{p_1}\right)}. \tag{2}$$

It is this definition (2) that serves as a basis for all generalizations of σ for a multi-factor setting. Since output is assumed to be constant, the following generalizations inherit this property.

The literature's consensus of an ideal concept of multi-factor substitution is to report optimal adjustment in relative inputs x_i/x_j when the relative input price of two arbitrary factors i and j changes, with all inputs being flexible and cost minimized for fixed output. This measure is often called HICKS-ALLEN elasticity of substitution (HAES), where

$$\text{HAES}_{ij} = \frac{\partial \ln\left(\frac{x_i}{x_j}\right)}{\partial \ln\left(\frac{p_j}{p_i}\right)} = \frac{\partial \ln x_i}{\partial \ln\left(\frac{p_j}{p_i}\right)} - \frac{\partial \ln x_j}{\partial \ln\left(\frac{p_j}{p_i}\right)}, \tag{3}$$

and only the relative price of two factors i and j changes. If apart from i and j all other factors are assumed to be constant, HAES_{ij} is in fact HICKS' elasticity of substitution HES.[1]

While HAES_{ij} measures the relative change of the input proportion x_i/x_j, and therefore may be termed a measure of relative substitutability, the cross-price elasticity

$$\eta_{x_i p_j} := \frac{\partial \ln x_i}{\partial \ln p_j} \tag{4}$$

may be termed a measure of absolute substitutability, because it focuses merely on the relative change of a single factor i due to a sole change of the price of factor j, with output and all other prices being fixed. Thus, according to MUNDLAK's (1968) classification, $\eta_{x_i p_j}$ is a *one-price-one-factor* elasticity of substitution.

It is now shown that cross-price elasticities are the common basis of AES, MES, and SES. First, AES is – see e. g. FRONDEL and SCHMIDT (2004:220) – related to $\eta_{x_i p_j}$ by

$$\text{AES}_{ij} = \frac{\eta_{x_i p_j}}{s_j}, \tag{5}$$

where $s_j = \frac{x_j p_j}{C}$ denotes the cost share of factor j. According to CHAMBERS (1988:95), expression (5) is the "most compelling argument for ignoring the Allen measure in applied analysis ... The interesting measure is [$\eta_{x_i p_j}$] – why disguise it by dividing by a cost share? This question becomes all the more pointed when

[1]The most general measure of substitution on the basis of (2) would be a concept of *total substitution*, where besides p_i and p_j all other prices are flexible as well. According to MUNDLAK (1996:232), however, "[a]s a concept it may have little to contribute".

the best reason for doing so is that it yields a measure that can only be interpreted intuitively in terms of [$\eta_{x_i p_j}$]". Nevertheless, AES has been the most extensively used elasticity of substitution in empirical studies – see e. g. HAMERMESH (1993:35).

Second, MES is most generally defined by

$$\mathrm{MES}_{x_i p_j} := \frac{\partial \ln(x_i/x_j)}{\partial \ln p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} - \frac{\partial \ln x_j}{\partial \ln p_j} = \eta_{x_i p_j} - \eta_{x_j p_j} \tag{6}$$

and is a *two-factor-one-price* elasticity, where solely the price of factor j is flexible, again with all other prices being fixed (BLACKORBY and RUSSELL 1989). Similar to cross-price elasticities, but unlike AES, MES is asymmetric: In general, $\mathrm{MES}_{x_i p_j} \neq \mathrm{MES}_{x_j p_i}$. It becomes transparent from definition (6) that if one were to classify two factors using MES, one would more frequently conclude that these factors are substitutes than if one were using AES or cross-price elasticities. The reason is that even if $\eta_{x_i p_j}$ is negative and thus factor i and j are termed complements, $\mathrm{MES}_{x_i p_j}$ may be positive, hence indicating substitutability, if the magnitude of the always negative own-price elasticity is sufficiently large.

In line with FRONDEL (2004), it is argued here that for many practical purposes, cross-price elasticities should be favored over MES. The reason is that it is frequently more interesting to get to know how the use of factor i is changing due to an exogenous increase in the price p_j of factor j, rather than to learn something about the change of the input proportion x_i/x_j, as would be measured by $\mathrm{MES}_{x_i p_j}$. If, for instance, oil prices are soaring, politicians would much rather want to know how much of a detrimental impact the high prices will have on the labor input of the economy alone than to know how the labor-energy input proportion changes and whether the use of either labor or energy is more reduced due the increase in oil prices. Hence, notwithstanding the significance of MES as the sole true generalization of HICKS' σ, estimating cross-price elasticities, rather than any substitution measure involving input ratios, frequently appears to be more appropriate in empirical studies on issues such as the consequences of energy price policies.

Third, the *two-factor-two-price* elasticity HAES_{ij} is a weighted average of $\mathrm{MES}_{x_ip_j}$ and $\mathrm{MES}_{x_jp_i}$.

Proof: Using the chain rule, we have

$$\frac{\partial \ln x_i}{\partial \ln(\dfrac{p_j}{p_i})} = \frac{\partial \ln x_i}{\partial \ln p_i} \cdot \frac{\partial \ln p_i}{\partial \ln(\dfrac{p_j}{p_i})} + \frac{\partial \ln x_i}{\partial \ln p_j} \cdot \frac{\partial \ln p_j}{\partial \ln(\dfrac{p_j}{p_i})} = \eta_{x_ip_i} \frac{\partial \ln p_i}{\partial \ln(\dfrac{p_j}{p_i})} + \eta_{x_ip_j} \frac{\partial \ln p_j}{\partial \ln(\dfrac{p_j}{p_i})} \quad (7)$$

and

$$\frac{\partial \ln x_j}{\partial \ln(\dfrac{p_j}{p_i})} = \eta_{x_jp_i} \frac{\partial \ln p_i}{\partial \ln(\dfrac{p_j}{p_i})} + \eta_{x_jp_j} \frac{\partial \ln p_j}{\partial \ln(\dfrac{p_j}{p_i})}, \quad (8)$$

because merely the prices p_i and p_j are flexible. Hence,

$$\begin{aligned} \mathrm{HAES}_{ij} &= \frac{\partial \ln(\dfrac{x_i}{x_j})}{\partial \ln(\dfrac{p_j}{p_i})} = \frac{\partial \ln x_i}{\partial \ln(\dfrac{p_j}{p_i})} - \frac{\partial \ln x_j}{\partial \ln(\dfrac{p_j}{p_i})} \\ &= \underbrace{(\eta_{x_ip_j} - \eta_{x_jp_j})}_{\mathrm{MES}_{x_ip_j}} \frac{\partial \ln p_j}{\partial \ln(\dfrac{p_j}{p_i})} - \underbrace{(\eta_{x_jp_i} - \eta_{x_ip_i})}_{\mathrm{MES}_{x_jp_i}} \frac{\partial \ln p_i}{\partial \ln(\dfrac{p_j}{p_i})}, \end{aligned} \quad (9)$$

where the weights add to unity:

$$\frac{\partial \ln p_j}{\partial \ln(p_j/p_i)} + (-\frac{\partial \ln p_i}{\partial \ln(p_j/p_i)}) = \frac{\partial \ln(p_j/p_i)}{\partial \ln(p_j/p_i)} = 1. \quad (10)$$

The weighted sum given in (9) reflects the fact that there is an infinite number of changes of prices p_i and p_j that lead to the same change of price ratio p_j/p_i. There are two polar cases: If only p_j changes and p_i is fixed, HAES_{ij} equals $\mathrm{MES}_{x_ip_j}$, while, vice versa, HAES_{ij} specializes to $\mathrm{MES}_{x_jp_i}$.

To complete the survey, it is proved now that McFadden's *shadow elasticity of substitution* SES, which *additionally* holds cost constant, is both a weighted average of $\mathrm{MES}_{x_ip_j}$ and $\mathrm{MES}_{x_jp_i}$ and a special case of the basic definition (3) as well.

Proof: Since SES fixes cost C and only two prices p_i and p_j are supposed to change, on the basis of Shephard's Lemma, $\frac{\partial C}{\partial p_i} = x_i$, and the chain rule follows:

$$0 = \frac{\partial C}{\partial(\frac{p_j}{p_i})} = \frac{\partial C}{\partial p_i} \cdot \frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + \frac{\partial C}{\partial p_j} \cdot \frac{\partial p_j}{\partial(\frac{p_j}{p_i})} = x_i \cdot \frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + x_j \cdot \frac{\partial p_j}{\partial(\frac{p_j}{p_i})}. \quad (11)$$

By dividing (11) by C, one obtains

$$0 = (\frac{1}{p_i}) \cdot (\frac{p_i x_i}{C}) \cdot \frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + (\frac{1}{p_j}) \cdot (\frac{p_j x_j}{C}) \cdot \frac{\partial p_j}{\partial(\frac{p_j}{p_i})} = s_i \cdot \frac{\partial \ln p_i}{\partial(\frac{p_j}{p_i})} + s_j \cdot \frac{\partial \ln p_j}{\partial(\frac{p_j}{p_i})}. \quad (12)$$

Multiplying by p_j/p_i leads to

$$0 = (\frac{p_j}{p_i}) \cdot s_i \cdot \frac{\partial \ln p_i}{\partial(\frac{p_j}{p_i})} + (\frac{p_j}{p_i}) \cdot s_j \cdot \frac{\partial \ln p_j}{\partial(\frac{p_j}{p_i})} = s_i \cdot \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + s_j \cdot \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})}. \quad (13)$$

Combining equation (10) and the right-hand side of equation (13) yields

$$\frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} = -\frac{s_j}{s_i + s_j} \quad \text{and} \quad \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} = \frac{s_i}{s_i + s_j}. \quad (14)$$

By plugging both derivatives into the right-hand side of (9), we finally get

$$\text{SES}_{ij} = (\frac{s_i}{s_i + s_j})\text{MES}_{x_i p_j} + (\frac{s_j}{s_i + s_j})\text{MES}_{x_j p_i}. \quad (15)$$

The symmetry of this expression indicates that SES is symmetric – like AES.

In sum, two common features of AES, MES, HAES, and SES become apparent in this section. First, all these elasticities ignore output effects and, second, all are mixtures of cross-price elasticities. While HAES is the most general of the presented measures, because it captures factor substitution when two factor prices are flexible, this generality is also the reason for HAES being of minor practical importance: It is simply not possible to obtain from HAES a single substitution estimate for any two factors without specifying how these two factor prices change.

By contrast, apart from SES, which also measures substitution relationships when two prices are flexible, yet under the additional, restrictive assumption that output and cost are constant, not just output, all other measures described in this section are based on the assumption that only one factor price alters. It could be argued, however, that in modeling practise one is frequently confronted with counterfactual situations describing what would happen if the price of only a single factor were to drastically increase. In modeling industrial energy consumption, for instance, this is a rather typical situation, since, most importantly, oil prices are highly volatile and are frequently doubling within short periods of time.

In any case, this didactic survey should have demonstrated that whenever one draws conclusions from empirical studies on the degree of substitutability of two inputs, it is indispensable to, first, clearly indicate the particular measure employed to denote these inputs as substitutes and, second, to interpret empirical results accordingly. We now provide an empirical illustration of the differences in both the estimates of all these elasticities and their interpretations in order to explain in greater depth why cross-price elasticities are preferable for many practical purposes.

3 An Empirical Illustration

Similar to JORGENSON and STIROH (2000) and FRONDEL and SCHMIDT (2006), we draw on JORGENSON's time-series data set[2] of U. S. manufacturing (1958-1996). The data set includes the inputs and prices of four production factors: capital (K), labor (L), energy (E), and materials (M). Since our focus is on the substitution relationship between energy and non-energy inputs, we concentrate on time-series data for the primary metals sector, one of the most energy-intensive industries of the available 35 sectors, and apply this data set to the prominent and frequently employed translog cost function approach. Other recent translog contributions are, for instance, RYAN and WALES (2000) and YATCHEW (2000).

Translog cost functions are typically of the following structure (CHRISTENSEN *et al.* 1971:255):

$$\ln C = \ln \beta_0 + \beta_Y \cdot \ln Y + \sum_{i \in F} \beta_i \cdot \ln p_i + \frac{1}{2} \sum_{i,j \in F} \beta_{ij} \ln p_i \ln p_j + \sum_{i \in F} \beta_{iT} \ln p_i \cdot T, \quad (16)$$

[2]This data set is accessible via internet – see Prof. JORGENSON's homepage:

http://post.economics.harvard.edu/faculty/jorgenson/data.html where Y is a given level of output, F denotes a set of inputs, where $F = \{K, L, E, M\}$ in our example, T reflects a linear time trend that is included to capture technological progress, and symmetry of β_{ij} is typically imposed *a priori*.

Linear homogeneity in prices, an inherent feature of any cost function, requires the following conditions:

$$\begin{aligned} \beta_K + \beta_L + \beta_E + \beta_M &= 1\,, && (17) \\ \beta_{Kl} + \beta_{Ll} + \beta_{El} + \beta_{Ml} &= 0 \qquad \text{for all } l \in F = \{K, L, E, M\}, && (18) \\ \beta_{KT} + \beta_{LT} + \beta_{ET} + \beta_{MT} &= 0\,. && (19) \end{aligned}$$

Under the assumptions of optimal behavior and perfect competition, the cost share s_i of any factor $i \in F$ is given by

$$s_i = \frac{x_i \cdot p_i}{C} = \frac{p_i}{C} \cdot \frac{\partial C}{\partial p_i} = \frac{\partial \ln C}{\partial \ln p_i} = \beta_i + \beta_{iT} T + \sum_{l \in F} \beta_{il} \ln p_l, \tag{20}$$

where SHEPHARD's lemma, $x_i = \frac{\partial C}{\partial p_i}$, has been employed. Using $x_i = s_i C / p_i$, the concrete expression of cross-price elasticity $\eta_{x_i p_j}$ resulting from translog approaches such as (16) can be derived as follows:

$$\eta_{x_i p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} = \frac{\partial \ln s_i}{\partial \ln p_j} + \frac{\partial \ln C}{\partial \ln p_j} - \frac{\partial \ln p_i}{\partial \ln p_j} = \frac{1}{s_i} \frac{\partial s_i}{\partial \ln p_j} + s_j = \frac{\beta_{ij}}{s_i} + s_j. \tag{21}$$

The derivation of the own-price elasticity $\eta_{x_i p_j}$ follows in a similar way:

$$\eta_{x_i p_i} = \frac{\partial \ln x_i}{\partial \ln p_i} = \frac{\partial \ln s_i}{\partial \ln p_i} + \frac{\partial \ln C}{\partial \ln p_i} - \frac{\partial \ln p_i}{\partial \ln p_i} = \frac{1}{s_i} \frac{\partial s_i}{\partial \ln p_i} + s_i - 1 = \frac{\beta_{ii}}{s_i} + s_i - 1. \tag{22}$$

Moreover, using the expressions for the cross- and own-price elasticities and the definition of MES, we obtain the specific formulae of $\text{MES}_{x_i p_j}$ and $\text{MES}_{x_j p_i}$ that are valid for translog approach (16):

$$\text{MES}_{x_i p_j} = \eta_{x_i p_j} - \eta_{x_j p_j} = \frac{\beta_{ij}}{s_i} + s_j - \frac{\beta_{jj}}{s_j} - s_j + 1 = \frac{\beta_{ij}}{s_i} - \frac{\beta_{jj}}{s_j} + 1, \tag{23}$$

and for symmetry reasons:

$$\text{MES}_{x_j p_i} = \eta_{x_j p_i} - \eta_{x_i p_i} = \frac{\beta_{ij}}{s_j} - \frac{\beta_{ii}}{s_i} + 1, \tag{24}$$

On the basis of these expressions for $\mathrm{MES}_{x_i p_j}$, $\mathrm{MES}_{x_j p_i}$, and the cross-price elasticities $\eta_{x_i p_j}$, we now estimate these measures as well as AES_{ij} and SES_{ij}, for which we use formulae (5) and (15). Yet, basically, the calculation of all these rests on that of cross-price elasticities. The results for those elasticities where energy is involved are reported in Table 1; maximum likelihood (ML) estimates of the translog cost function parameters are displayed in Table A in the appendix.

The unknown parameters might be estimated directly from a stochastic version of (16). Yet, it is widely known in the econometric literature that efficiency gains can be realized by estimating a system of cost-share equations (BERNDT 1991:470). In our example, the stochastic version of the cost-share equation system may read as follows:

$$\begin{aligned} s_K &= \beta_K + \beta_{KK}\ln(\frac{p_K}{p_M}) + \beta_{KL}\ln(\frac{p_L}{p_M}) + \beta_{KE}\ln(\frac{p_E}{p_M}) + \beta_{KT}\cdot T + \varepsilon_K \\ s_L &= \beta_L + \beta_{KL}\ln(\frac{p_K}{p_M}) + \beta_{LL}\ln(\frac{p_L}{p_M}) + \beta_{LE}\ln(\frac{p_E}{p_M}) + \beta_{LT}\cdot T + \varepsilon_L, \\ s_E &= \beta_E + \beta_{KE}\ln(\frac{p_K}{p_M}) + \beta_{LE}\ln(\frac{p_L}{p_M}) + \beta_{EE}\ln(\frac{p_E}{p_M}) + \beta_{ET}\cdot T + \varepsilon_E\ , \end{aligned} \tag{25}$$

where the restrictions (18) and (19) are already imposed and disturbances are denoted by $\varepsilon_K, \varepsilon_L$, and ε_E. In order to avoid the singularity of the disturbance covariance matrix that arises because cost shares always add to unity, the share equation for materials (M) has been dropped arbitrarily. The unknown parameters of the seemingly unrelated regression (SUR) model (25) are preferably estimated using ML methods to ensure that results do not depend upon the choice of which share equation is dropped (BERNDT 1991:473).

Most importantly, the results of Table 1 support the point that qualitative conclusions regarding substitutability crucially rest on the choice of the substitution concept. On the basis of the MES_{Ep_K} estimates, for instance, capital and energy might be denoted as substitutes, whereas the estimates of the cross-price elasticities, $\hat{\eta}_{Kp_E}$ and $\hat{\eta}_{Ep_K}$ indicate that both factors are complements, though not significantly throughout.

These results are to be explained as follows: Since the estimate of the capital-price elasticity of energy, $\widehat{\eta}_{Ep_K}$, lies around -0.12 and $\widehat{\eta}_{Kp_K}$ (not reported in Table 1) is roughly -0.4, $\widehat{\mathrm{MES}}_{Ep_K} \approx -0.12 - (-0.32) = 0.20 > 0$, implying that a 1 % increase in the price of capital approximately leads to a 0.12 % reduction in the use of energy and a 0.32 % reduction for capital holding output constant. Relative to capital, more energy is used when capital gets more expensive. Thus, capital and energy are MES substitutes, though the input of energy in fact shrinks, as indicated by $\widehat{\eta}_{Ep_K} = -0.12$, and, therefore, capital and energy are to be denoted as complements on the basis of cross-price elasticities.

Table 1: Estimates of AES, Cross-Price Elasticities, MES, and SES.

Year	AES_{EK}	η_{Kp_E}	η_{Ep_K}	MES_{Ep_K}	MES_{Kp_E}	SES_{KE}
1958	-1.33 (0.77)	-0.08 (0.04)	-0.12 (0.07)	0.25 (0.12)	-1.24 (0.11)	-0.67 (0.10)
1967	-2.10 (1.01)	-0.08 (0.04)	-0.22 (0.11)	0.21 (0.14)	-2.26 (0.28)	-1.59 (0.24)
1977	-1.65 (0.85)	-0.12 (0.07)	-0.10 (0.05)	0.04 (0.17)	-0.81 (0.10)	-0.34 (0.10)
1987	-0.92 (0.63)	-0.07 (0.05)	-0.07 (0.05)	0.23 (0.11)	-0.66 (0.10)	-0.21 (0.09)
1996	-1.41 (0.77)	-0.09 (0.05)	-0.12 (0.07)	0.21 (0.11)	-1.12 (0.17)	-0.55 (0.14)
Year	AES_{EL}	η_{Lp_E}	η_{Ep_L}	MES_{Ep_L}	MES_{Lp_E}	SES_{LE}
1958	-1.45 (0.55)	-0.08 (0.03)	-0.32 (0.12)	-0.12 (0.13)	-1.25 (0.11)	-1.02 (0.10)
1967	-2.78 (0.75)	-0.11 (0.03)	-0.59 (0.16)	-0.41 (0.17)	-2.29 (0.27)	-2.00 (0.24)
1977	-0.65 (0.34)	-0.05 (0.03)	-0.16 (0.09)	0.07 (0.10)	-0.73 (0.09)	-0.55 (0.07)
1987	-0.55 (0.35)	-0.04 (0.03)	-0.14 (0.09)	0.10 (0.09)	-0.63 (0.08)	-0.45 (0.07)
1996	-1.01 (0.42)	-0.06 (0.03)	-0.25 (0.11)	-0.01 (0.11)	-1.09 (0.22)	-0.88 (0.14)
Year	AES_{EM}	η_{Mp_E}	η_{Ep_M}	MES_{Ep_M}	MES_{Mp_E}	SES_{ME}
1958	-1.15 (0.32)	-0.07 (0.02)	-0.73 (0.20)	-0.59 (0.23)	-1.23 (0.12)	-1.18 (0.12)
1967	-2.13 (0.54)	-0.08 (0.02)	-1.38 (0.35)	-1.25 (0.38)	-2.26 (0.29)	-2.21 (0.29)
1977	-0.69 (0.28)	-0.05 (0.02)	-0.42 (0.17)	-0.28 (0.20)	-0.74 (0.11)	-0.69 (0.12)
1987	-0.64 (0.24)	-0.05 (0.02)	-0.38 (0.14)	-0.22 (0.18)	-0.64 (0.09)	-0.59 (0.09)
1996	-1.09 (0.34)	-0.06 (0.02)	-0.66 (0.21)	-0.51 (0.24)	-1.10 (0.17)	-1.04 (0.17)

Note: Standard errors are given in parentheses and estimated using the delta method. AES and SES estimates require only one column, since both measures are symmetric.

Finally, this example demonstrates that comparing the reductions of energy and capital intensiveness due to higher energy prices by MES_{Kp_E} or SES_{KE} might be less interesting than considering the separate impacts of higher energy prices on the input of capital and labor, which are measured by the cross-price elasticities η_{Kp_E} and η_{Lp_E}, respectively. In the end, these empirical results and our theoretical survey demonstrate that whenever empirical researchers want to draw conclusions with respect to the substitution relationship between two factors they are well-advised to, first, estimate a variety of substitution measures starting with cross-price elasticities as the common basis of all the classical measures and, second, to interpret empirical results accordingly.

4 Summary and Conclusion

Estimating the economic impact of energy policies necessarily obliges one to resort to measures of the substitution of energy and non-energy inputs. Given the multitude of generalizations of HICKS' σ, the unique elasticity of substitution for the two-factor case, the central question arises as to which measure would be appropriate to capture energy-non-energy substitution relationships. In a multi-factor setting, ALLEN's elasticities of substitution (AES) apparently have been the most-used measures in applied production analysis. BLACKORBY and RUSSELL (1989:883), however, criticize that AES adds no more information to that already contained in cross-price elasticities.

On the basis of a survey of σ's most prominent generalizations, including AES, HICKS-ALLEN's (HAES), MORISHIMA's (MES), and MCFADDEN's shadow elasticities of substitution (SES), this paper argues that cross-price elasticities play a fundamental role in measuring substitution issues, since they are the common basis for AES, MES, and SES. Moreover, using the example of the U.S. primary metals sector, it has been empirically illustrated that cross-price elasticities are often more relevant in terms of economic content. The ultimate reason for this conclusion is that cross-price elasticities measure the relative change of only one factor due to price changes of another input, whereas HAES, MES, and SES measure the relative change of a *factor ratio* due to price changes of either of these two factors.

While measuring the relative change of a factor ratio appears to be of minor importance for many applications, it is argued that any substitution measure has to match the specific task it is employed for and emphasize FUSS, MCFADDEN and MUNDLAK's (1978:241) conclusion that there "is no unique natural generalization of the two factor definition ... [and that] the selection of a particular definition should depend on the question asked". Hence, a clear understanding of the differences in interpretations and perspectives captured by the variety of substitution measures is indispensable.

Yet, all the presented elasticities solely measure pure substitution effects; that is, they ignore output effects, because constancy of output is the maintained hypothesis underlying these concepts. Oil price shocks, however, indicate that it is frequently problematic to ignore output effects in empirical studies of factor substitution. As it is most likely that output shrinks when the price of a factor such as energy rises, elasticities capturing gross substitution effects – that is, pure substitution and output effects – are preferable in any empirical study. Based on the argument that cross-price elasticities are often more relevant for many practical purposes, a generalization of cross-price elasticities that allows for output variations would be a possible candidate concept.

Appendix

Table A: ML Parameter Estimates of the KLEM Translog Cost Function (16) – U.S. Primary Metals Sector 1958 - 1996.

	Estimates	Std. Error		Estimates	Std. Error
β_K	-1.694382**	0.374253	β_{KT}	0.000893**	0.000189
β_L	3.153601**	0.560144	β_{LT}	-0.001458**	0.000282
β_E	0.982211**	0.381398	β_{ET}	-0.000458**	0.000192
β_M	-1.441419	0.929015	β_{MT}	0.001022**	0.000467
β_{KK}	0.048876**	0.007006	β_{LE}	-0.030666**	0.006724
β_{KL}	-0.031554**	0.007506	β_{LM}	-0.065845**	0.018405
β_{KE}	-0.012164**	0.003872	β_{EE}	0.120199**	0.006724
β_{KM}	-0.005158	0.013368	β_{EM}	-0.077369**	0.012110
β_{LL}	0.128065**	0.012156	β_{MM}	0.148372**	0.034144

Note: * Significant at the 5 %-level. ** Significant at the 1 %-level.

To check whether translog cost function (16) is well-behaved, that is, whether it is, first, non-decreasing and, second, concave in factor prices, we have verified that, first, fitted cost shares (20) are always positive and, second, the Hessian matrix with the components $\frac{\partial^2 C}{\partial p_i \partial p_j}$ is negative semi-definite.

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