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# Liquidity, interest rates and optimal monetary policy

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# Chapter 1

# Introduction

Macroeconomics is commonly distinguished into the analysis of short-run business cycle fluctuations and the study of economic growth, which takes a medium to long term perspective. The main goal of the business cycle literature is to identify sources of business cycle fluctuations, their implications for welfare and the role of monetary and fiscal policy in these fluctuations. This dissertation contains both positive and normative analyses of monetary policy. At the heart of the normative analysis is the characterization of optimal monetary policy and the evaluation of simple policy rules, which are applied in central banks around the globe. Positive analysis is aimed at describing the mechanisms behind real-world phenomena, which in the present context consists of analyzing the impact of monetary policy on the business cycle.

This dissertation makes ample use of theoretical macroeconomic models, building on an extensive literature on business cycle fluctuations. This literature is outlined in the following in order to classify the research presented in this dissertation and to illuminate its contribution to the literature. Models employed by researchers in business cycle analysis have evolved hugely over the last decades. Macroeconomists until the 1970s used models which were partly based on plausible ad hoc assumptions and empirical evidence (see Blanchard (2000)). This approach was criticized by Lucas (1976) because of its inability to assess the impact of policy changes. When policy changes, the argument goes, economic agents may change their behavior in a fundamental way, so that relationships derived from past data fail to capture agents' behavior after the policy change. The Lucas critique led to

the rational expectations revolution, which fundamentally changed macroeconomic modeling. Mainstream macroeconomic models are now dynamic stochastic general equilibrium (DSGE) models based on intertemporally optimizing agents under rational expectations.

Two main strands of literature have developed after the rational expectations revolution. The main idea of the real business cycle literature is that business cycles are not caused by market imperfections but are the result of variation in productivity. Thus, aggregate fluctuations arise from changes in technology so that policy should refrain from stabilizing them (see Prescott (1986) and Long and Plosser (1983)). The second strand of literature builds on the contributions of – to name but a few – Mankiw (1985), Blanchard and Kiyotaki (1987) and Calvo (1983) and has integrated market imperfections such as price stickiness and monopolistic competition into DSGE models, leading to a dynamic variant of the IS-LM model. These models are called New Keynesian because monetary policy has an effect on real activity in the short run. However, they also contain neoclassical ideas, such as long-run neutrality of money and the importance of credibility in policy. Therefore, Goodfriend and King (1997) coin the term New Neoclassical Synthesis, alluding to the neoclassical synthesis incorporated in the IS-LM model. Clarida, Galí, and Gertler (1999) provide a coherent description of the new workhorse model in monetary policy analysis. Notably, while monetary policy is its main field of application, money does not play a role in the canonical New Keynesian model, which considers a cashless economy. Rather, the instrument used by monetary policy is the short-run interest rate.

Researchers have used New Keynesian models to evaluate a broad set of questions related to monetary policy. The main object of normative studies has been the characterization of optimal monetary policy and the evaluation of simple policy rules, which according to Taylor (1993) provide an accurate description of U.S. monetary policy. With respect to both questions, the literature has reached a broad consensus. Woodford (2004), in his survey of the literature states that "it is not a bad first approximation to say that the goal of monetary policy should be price stability". Further, it is generally acknowledged that simple policy rules, as long as they respect the Taylor principle, which demands the nominal interest rate to react more than one for one to deviations of inflation from its target, are a close substitute for optimal policy. This is shown for instance in Schmitt-Grohé and

Uribe (2007) and Erceg, Henderson, and Levin (2000). Further, the literature has emphasized the importance of credibility in conducting monetary policy both in normal and in crisis times. Research has largely contributed to the development and widespread application of Inflation Targeting, a policy strategy which clearly communicates a medium-term policy goal to the public and uses its policy instruments to achieve this target (see Svensson and Woodford, 2004). According to Eggertsson and Woodford (2004) and Bernanke, Reinhart, and Sack (2004), the importance of credibility increases even more at the zero lower bound: When policy cannot raise demand by reducing current nominal interest rates, shaping expectations by announcing future expansionary policy is the only option to stimulate the economy.

Moreover, the literature has extended the baseline New Keynesian model to the analysis of open economies. The setup by Galí and Monacelli (2005) has become the workhorse model in this field. Apart from normative studies, which analyze in particular the desirability of stabilizing the exchange rate (see Galí and Monacelli (2005) and Corsetti and Pesenti (2005)), a literature evaluating the quantitative predictions of New Keynesian open economy models has developed. In the closed economy, the work by Smets and Wouters (2007) sets the standard, demonstrating that an estimated model's out-of-sample predictions can compete with purely statistical methods. Studies analyzing estimated open economy models point to deficits, in particular in modeling exchange rates. Adolfson, Laseen, Linde, and Villani (2007)) estimate an open economy version of Christiano, Eichenbaum, and Evans (2005) and find that the model can reproduce inflation and real exchange rate dynamics if one is willing to include a variety of exogenous shocks which represent unexplained deviations from central model equations, such as uncovered interest rate parity. Justiniano and Preston (2010) find that an estimated small open economy model cannot account for the observed co-movement of U.S. and Canadian business cycles. The authors suggest that the model's failure to endogenously explain real exchange rate dynamics is at the heart of this problem.

This dissertation contributes to a wide variety of normative and positive research questions and uses different theoretical models to answer these questions. The second and the fourth chapters present research conducted jointly with my coauthor and supervisor Prof. Dr. Andreas Schabert. Chapters 2 and 3 ask normative questions regarding the nature of optimal

policy and the performance of simple rules, considering small departures from the canonical New Keynesian model in policy (chapter 2) and in the model environment (chapter 3). The second chapter compares the welfare implications of active policy rules to those of an interest rate peg and discusses how a peg can be implemented without generating equilibrium indeterminacy. Building on empirical evidence by Angeloni, Kashyap, Mojon, and Terlizzese (2003), which documents that monetary transmission affects output largely through its impact on investment, the third chapter analyzes how capital accumulation influences optimal policy, interest rate dynamics and the performance of simple policy rules. From a methodological point of view, the models used in these two chapters are powerful for the type of normative research questions pursued, despite their simplicity. The reason is that rational expectations imply that changes in the model structure and in policy are perfectly understood by economic agents, so that an evaluation of alternative policies respects the Lucas critique. Further, the rigorous microfoundation of New Keynesian models permits analyzing welfare based on household utility instead of imposing an ad-hoc loss function, as was common before the rational expectations revolution.

The nature of the research questions posed in chapters 4 and 5 is different. Both chapters contain positive analyses which aim at explaining the impact of monetary policy on the economy, though in different contexts. Motivated by the policy response to the 2007-2009 financial crisis, the third chapter analyzes the impact of unconventional monetary policy, distinguishing between quantitative and qualitative easing. Chapter 4 analyzes the implications of the U.S. dollar's status as the key currency in the international monetary system. This analysis focuses on interest and exchange rates, asking if key currency effects can explain observed deviations from uncovered interest rate parity. Chapters 4 and 5 stress the importance of liquidity in understanding the effects of monetary policy on the economy and consequently use a common modeling framework, which deviates from the cashless, single interest rate model used in former chapters: Both chapters build on Reynard and Schabert (2009), who demonstrate that modeling the supply of liquidity in terms of open market operations fundamentally changes monetary transmission and can align observed interest rates and their theoretical counterparts.

The research conducted in this dissertation makes a significant contribution to the literature and has interesting implications for policy makers.

The second chapter is novel for its result that an interest rate peg, which is usually ignored due to its violation of the Taylor principle, can raise welfare compared to a Taylor rule. In a similar vein, the third chapter demonstrates that the performance of Taylor rules can deteriorate when taking into account the effects of endogenous capital accumulation on the central bank's trade-off. Because Taylor rules are widely used both in central banks and in academic research, these results are interesting to anyone working in the field. Further, the second chapter demonstrates that endogenous fluctuations associated with an interest rate peg can be ruled out by adopting an autoregressive policy rule. Apart from preventing the adverse impact of these fluctuations on welfare, this is important for carrying out constant interest rate projections, a tool used by policy makers in many central banks.

The fourth chapter makes both a methodological contribution and is highly relevant to policy makers. The literature on unconventional monetary policy, with some recent exceptions such as Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010), has mainly employed models in which the central bank's balance sheet does not play a role above its impact on "the" short term interest rate and the long-run price level, as for instance in Eggertsson and Woodford (2003). Thus, by their structure these models must predict that open market operations are ineffective once the interest rate is zero, except for their impact on expected future money supply. In developing a framework in which both the composition and the size of the central bank's balance sheet are relevant for the allocation even at the zero lower bound, the fourth chapter constitutes an important methodological advance, which is in part owed to the innovative contribution by Reynard and Schabert (2009). Further, the idea of focusing on the impact of unconventional monetary policy measures on liquidity is appealing to policy makers who have stated that ensuring liquidity and lowering the cost of private credit was the main aim of unconventional monetary policy measures implemented during the financial crisis of 2007-2009 (see Yellen (2009)). The results of this analysis are twofold. First, quantitative easing can stimulate the economy at the zero lower bound. In contrast to the literature, this stimulus does not rely on a commitment to ease future policy, but arises because households can be cash constrained even at the zero lower bound as long as liquidity premiums, and thus interest rates on illiquid assets, are positive. Further, the analysis isolates a qualitative

easing effect, which stimulates the economy by reducing firms' borrowing cost without extending the central bank's balance sheet.

Chapter 5 can be considered a first step toward improving the quantitative performance of New Keynesian open economy models. It focuses on a weak spot in open economy models: The uncovered interest rate parity condition is known to be grossly inconsistent with the data. Similar to Canzoneri, Cumby, and Diba (2007), it starts from the intuitively appealing idea that the relatively low interest rates on U.S. government bonds are caused by liquidity premiums related to the leading status of the U.S. dollar in the international monetary system. The chapter presents a two country open economy model, where one country's currency is exclusively used in international trade. Demand and supply of the key currency are modeled as in Reynard and Schabert (2009). This permits an analysis of the international transmission of monetary policy shocks from a rigorously microfounded model, implying a methodological advance compared to the existing literature, such as Canzoneri, Cumby, and Diba (2007). Further, the results are promising. Chapter 5 demonstrates that key currency effects can explain the observed response of exchange rates to monetary policy shocks, which has become known as the delayed overshooting puzzle. Understanding the effects of monetary policy on exchange rates is important to policy makers in many central banks. Interest in exchange rate dynamics can arise due to political pressure by firms engaged in international trade or because a central bank's objective function requires stabilizing the terms of trade, as in Corsetti and Pesenti (2005).

A few words regarding the organization of this dissertation are in order. I refrain from giving a more extensive literature review in this introduction. Rather, each chapter contains an introduction which summarizes the relevant literature and discusses its contribution in more detail. I offer brief concluding remarks at the end of each chapter. Further, the appendices to each chapter can be found at the very end of this dissertation.

# Chapter 2

# An interest rate peg might be better than you think

Coauthor: Prof. Dr. Andreas Schabert[*]

## 2.1 Introduction

Recent macroeconomic research on monetary policy, which is based on New Keynesian models, has led to a simple advice for central bankers: Interest rates should be set in an active way. Though this device for interest rate setting is not exactly implied by welfare-maximization, it is commonly viewed as a useful short-cut for the latter. By raising the nominal interest rate by more than one for one in case inflation is (expected to be) increasing, the real interest rate rises, causing agents to save more and to consume less such that aggregate demand and firms' costs decline. By applying this strategy, monetary policy can stabilize prices, which reduces welfare cost of imperfect price adjustments.

Theoretical analysis of monetary policy has further shown that this is not the main virtue of an active policy: It rules out the possibility of multiple equilibria and thereby endogenous fluctuations (see Benhabib, Schmitt-Grohe, and Uribe (2001), or Woodford (2003)). Due to this property, an active interest rate setting is widely viewed as a prerequisite for macroeco-

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[*]This chapter is based on Hörmann and Schabert (2009a). Part of it has been published in Hörmann and Schabert (2009b).

nomic stability. Consequently, passive policies are usually dismissed given that they in principle allow for self-fulfilling expectations, or, sunspot equilibria. However, this view on active vs. passive policies is not necessarily justified on welfare grounds, since both types of policies are not derived from a welfare maximizing policy plan.

This chapter takes a closer look at the welfare effects of simple policy rules. Thereby, we consider a prominent (passive) monetary policy regime, namely a peg, which is banned from the recent literature, probably due to its failure to guarantee equilibrium uniqueness.[1] A welfare comparison in the workhorse macro model (a standard New Keynesian model) surprisingly shows that a peg can outperform a simple active interest rate rule. This result holds for the minimum state variable solution under a peg as well as for an autoregressive solution, where lagged inflation rates serve as an endogenous state variable. Finally, we demonstrate that a peg can be implemented by the central bank in a way that ensures the existence of a unique solution, i.e. it can *uniquely* implement the autoregressive solution or the minimum state variable solution under a peg. While the main purpose of the chapter is to demonstrate that some simple rules are better (or worse) than one commonly thinks, the analysis also contributes to the debate on the alleged problems associated with constant interest rate projections (see Honkapohja and Mitra (2005a), and Galí (2008)).

This chapter is organized as follows. Section 2.2 presents the framework, introduces different policy specifications, and derives welfare effects. Section 2.3 demonstrates how an equilibrium under a peg can be implemented in a unique way. Section 2.4 concludes.

## 2.2   A consensus model

Consider the following simple New Keynesian model, which can be derived from a microfounded sticky price framework and is for example also applied in Clarida, Galí, and Gertler (1999):

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t, \tag{2.1}$$

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma}\hat{R}_t + \frac{1}{\sigma}E_t\hat{\pi}_{t+1},$$

$$u_t = \rho u_{t-1} + \varepsilon_t, \quad \rho \in (0,1),$$

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[1]An exception is the recent discussion on the usefulness of constant interest rate projections (see Honkapohja and Mitra (2005b) and Galí (2008)).

where $\hat{\pi}_t$ denotes the gross inflation rate, $\hat{x}_t$ the output-gap, $\hat{R}_t$ the gross interest rate, and $u_t$ an autoregressive cost push shock. $\varepsilon_t$ is i.i.d. with $E_{t-1}\varepsilon_t = 0$ and a constant variance $\sigma_\varepsilon^2$. All variables are expressed in terms of percentage deviations from their respective values at an efficient steady state (see Woodford (2003)). The composite coefficient $\kappa$ of the Phillips curve is defined as $\kappa = (1-\phi)(1-\phi\beta)(\sigma+\eta)/\phi$ where $\beta$ is the household's constant discount factor, $1/\sigma$ the elasticity of intertemporal substitution, $1/\eta$ the Frisch labor supply elasticity, and $\phi$ the fraction of firms that do not adjust their prices in each period.

Monetary policy is specified in form of simple feedback rules for the nominal interest rate. We thus refrain from deriving an optimal monetary policy. In particular, we consider three different simple rules for monetary policy: $i$.) The rule proposed by Taylor (1993), $ii$.) an active interest rate policy that is consistent with monetary policy acting under discretion, and $iii$.) an interest rate peg.

## 2.2.1 Solutions under different simple rules

Following common practice, we restrict our attention to convergent equilibrium sequences. The model is simple enough to derive closed form solutions.

**An active interest rate policy**   can be described with interest rate rules of the form

$$\hat{R}_t = w_\pi \hat{\pi}_t + w_x \hat{x}_t, \tag{2.2}$$

where $w_\pi > 1$ and $w_x \geq 0$. Specifically, the feedback coefficients equal $w_\pi = 1.5$ and $w_x = 0.5$ in case of the Taylor rule, and $w_\pi = \rho + (1-\rho)\sigma\varepsilon$ and $w_x = 0$ in the case of a simple active rule consistent with discretionary policy (see Appendix A.1). In both cases, the model given in (2.1) can be reduced to the two-dimensional system

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t$$
$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{w_\pi}{\sigma}\hat{\pi}_t - \frac{w_x}{\sigma}\hat{x}_t + \frac{1}{\sigma}E_t\hat{\pi}_{t+1}$$

It is well-known that this system exhibits exactly one solution when $w_\pi > 1$ and $w_x \geq 0$ (since the system then exhibits two unstable eigenvalues) which allows for convergent equilibrium sequences only if the solution exhibits no history dependence (see for instance Woodford (2001)). Thus, we know

that both simple rules will lead to a linear solution of the following form

$$\hat{\pi}_t = a_u u_t, \quad \hat{x}_t = b_u u_t.$$

Using the method of undetermined coefficients the coefficients $a_u$ and $b_u$ of this minimum state variable (MSV) solution can easily be derived:

$$a_u = \frac{\sigma \left(1 - \rho + \frac{w_x}{\sigma}\right)}{\sigma \left(1 - \beta \rho\right) \left(1 - \rho + \frac{w_x}{\sigma}\right) + \kappa \left(w_\pi - \rho\right)}, \qquad b_u = -\frac{a_u}{\sigma} \frac{w_\pi - \rho}{1 - \rho + \frac{w_x}{\sigma}}.$$

**An interest rate peg,** i.e. a policy that keeps the nominal interest rate at its long-run efficient level ($\bar{R} = 1/\beta > 1$), $\hat{R}_t = 0$, leads to the following conditions for the equilibrium sequences of inflation and the output-gap:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t,$$
$$\hat{x}_t = E_t \hat{x}_{t+1} + \frac{1}{\sigma} E_t \hat{\pi}_{t+1}.$$

It is well known that this policy gives rise to multiple equilibrium solutions. Precisely, equilibrium conditions can be solved by other solutions than the MSV solution, since under a peg there exist one stable and one unstable eigenvalue. Hence, there exist additional stable solutions that exhibit endogenous state variables.

One type of solution features artificial state variables (like past expectations of today's non-predetermined variables, $E_{t-1} \hat{\pi}_t$ or $E_{t-1} \hat{x}_t$). These solutions are well-known to support the existence of sunspot-equilibria, where arbitrary changes in expectations (non-fundamental shocks) can affect macroeconomic variables. Here, we disregard these types of solutions and exclusively apply "well-behaved" solutions, namely, the MSV solution and an autoregressive (AR) solution. By construction, both *cannot* support sunspot equilibria, i.e. welfare reducing endogenous fluctuations.

1. <u>Minimum state solution</u>: As the first type of solution we consider the MSV solution under the peg, which takes the form

$$\hat{\pi}_t = a_u^{peg} u_t, \quad \hat{x}_t = b_u^{peg} u_t.$$

Applying the method of undetermined coefficients delivers

$$a_u^{peg} = \frac{1 - \rho}{\kappa} \left[ (1 - \rho)(1 - \beta \rho)\frac{1}{\kappa} - \frac{\rho}{\sigma} \right]^{-1}, \qquad (2.3)$$
$$b_u^{peg} = \frac{a_u}{\kappa}(1 - \beta \rho) - \frac{1}{\kappa}.$$

2. <u>Autoregressive solution</u>: As the second type of solution we consider an AR solution, where lagged inflation serves as an additional state variable. The solution form is given by

$$\hat{\pi}_t = a_\pi \hat{\pi}_{t-1} + a_u^{peg,AR} u_t, \tag{2.4}$$
$$\hat{x}_t = b_\pi \hat{\pi}_{t-1} + b_u^{peg,AR} u_t,$$

where the method of undetermined coefficients yields

$$a_\pi = \frac{1 + \frac{\kappa}{\sigma}}{2\beta} + \frac{1}{2} - \sqrt{\left(\frac{1 + \frac{\kappa}{\sigma}}{2\beta} + \frac{1}{2}\right)^2 - \frac{1}{\beta}}, \quad b_\pi = \frac{a_\pi}{\kappa} - \frac{\beta}{\kappa} a_\pi^2, \tag{2.5}$$

$$a_u^{peg,AR} = \frac{1-\rho}{\kappa} \left[(1-\rho)(1-\beta\rho)\frac{1}{\kappa} - \frac{\beta(1-\rho)a_\pi}{\kappa} - \frac{\rho}{\sigma} - \frac{a_\pi}{\sigma} - b_\pi\right]^{-1},$$

$$b_u^{peg,AR} = \frac{a_u}{\kappa}(1-\beta\rho) - \frac{\beta}{\kappa} a_\pi a_u - \frac{1}{\kappa}.$$

Note that one of the two solutions for $a_\pi$ lies inside the unit circle, while the other lies outside the unit circle. A stable solution requires picking the stable solution for $a_\pi$, which – since $\frac{1 + \frac{\kappa}{\sigma}}{2\beta} > \frac{1}{2}$ – *must* contain the root with a negative sign.

## 2.2.2    Welfare effects

In this section we compute welfare effects under the alternative policies and solutions. Following large parts of the literature, we apply a second order approximation of household welfare of the underlying model with optimizing agents. In particular, we adopt Woodford's (2003) approach, leading to a quadratic loss function that measures the welfare loss of deviations from an efficient steady state (where long-run distortions are eliminated by fiscal transfer and long-run price stability is ensured by an inflation target equal to one):

$$L = -E_0 \sum_{t=0}^{\infty} \beta^t (\hat{\pi}_t^2 + \lambda \hat{x}_t^2) = -\frac{1}{1-\beta} \left(Var\ \hat{\pi} + \lambda Var\ \hat{x}\right), \tag{2.6}$$

where we assumed that the economy is initially in its steady state and we used that the equilibrium sequences under all solutions are covariance stationary. $Var\ \hat{\pi}$ then denotes the unconditional variance (here, conditional

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| $\sigma$ | $\eta$ | $\beta$ | $\phi$ | $\epsilon$ | $\rho$ | $\sigma_\varepsilon^2$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 0.99 | 0.75 | 6 | 0.9 | 1 |

Table 2.1: Benchmark parameter values

on the information available at the beginning of period 0) of $\hat{\pi}_t$. The weight on output gap fluctuations satisfies $\lambda = \kappa/\epsilon$, where $\epsilon$ is the price elasticity that price setting firms face (see Woodford (2003)). The unconditional variances are given by

$$Var\ \hat{\pi} = \frac{1}{1 - a_\pi^2}\left[a_u^2 Var\ u + 2a_\pi a_u Cov(u, \hat{\pi})\right],$$

$$Var\ \hat{x} = b_\pi^2 Var\hat{\pi} + b_u^2 Var\ u + 2b_\pi b_u Cov(u, \hat{\pi}),$$

where $Var\ u = \frac{1}{1-\rho}\sigma_\varepsilon^2$ and $Cov(u, \hat{\pi}) = a_u\frac{\rho}{(1-\rho^2)(1-a_\pi\rho)}\sigma_\varepsilon^2$ (see Appendix A.2). For the computation of the variances we apply a set of standard parameter values given in Table 2.1, which lead to $\kappa = 0.1717$ and $\lambda = 0.029$. The normalization of $\sigma_\varepsilon^2$ does evidently not affect the relative welfare effects. These parameter values lead to a policy rule under discretionary optimization that is characterized by $w_\pi = 1.5$ (and $w_x = 0$). For this benchmark calibration we obtain the following results for the solutions and the variances under different rules:

<u>Taylor rule</u>: $\hat{\pi}_t = 3.563u_t$, $\ \hat{x}_t = -3.563u_t$,

$\quad Var\ \hat{\pi} = 66.814$, $\ \ Var\ \hat{x} = 66.814$, and $\ \ L^{Taylor} = -6872.52$.

<u>Policy under discretion, $w_\pi = 1.5$</u>: $\hat{\pi}_t = 0.878u_t$, $\quad \hat{x}_t = -5.268u_t$,

$\quad Var\ \hat{\pi} = 4.057$, $\ \ Var\ \hat{x} = 146.05$ ; and $\ \ L^{active} = -823.56$.

<u>Interest rate peg MSV</u>: $\hat{\pi}_t = 0.696u_t$, $\quad \hat{x}_t = -6.267u_t$,

$\quad Var\ \hat{\pi} = 2.552$, $\ \ Var\ \hat{x} = 206.739$, and $\ \ L^{peg,MSV} = -846.74$.

<u>Interest rate peg AR</u>: $\hat{\pi}_t = 0.665\hat{\pi}_{t-1} - 0.182u_t$, $\ \ \hat{x}_t = 1.323\hat{\pi}_{t-1} - 5.244u_t$,

$\quad Var\ \hat{\pi} = 1.241$, $\ \ Var\ \hat{x} = 176.639$, and $\ \ L^{peg,AR} = -629.45$.

The results show that the Taylor rule yields the worst welfare result, which is due to the most effective output gap stabilization that comes at the cost of the highest inflation variance. Evidently, an active policy under discretionary optimization performs much better than the Taylor rule. The MSV solution under the peg leads to an even lower inflation variance, but a slightly higher welfare loss caused by a less stabilized output gap. Notably, the AR solution under the peg clearly leads to the lowest welfare loss, which is mainly due to the smallest inflation variance. The latter property is hardly surprising, since inflation under the AR solution exhibits inertia that helps to smooth inflation fluctuations.[2]

Yet, this welfare ranking is by far not robust to changes of parameter values, as can be seen in Table 2.2. Most of all, the degree of autocorrelation of the exogenous state (i.e. the cost push shock) matters for the relative welfare effects. As argued above, an additional state variable ($\hat{\pi}_{t-1}$) can contribute to less volatile sequences of macroeconomic variables (here, in particular, inflation). However, additional state variables extend the state space and thus the support of the variables, which tends to raise unconditional variances. As long as the autocorrelation $\rho$ of the exogenous state is large, the latter effect will be less important. But, for smaller values of $\rho$, here $\rho \leq 0.8$ (see Table 2.2), the welfare loss can be higher for the AR solution than for the active policy under discretion ($w_\pi = \rho + (1 - \rho)\sigma\varepsilon$ and $w_x = 0$). For a high value of $\rho$ ($\rho = 0.95$), both solutions under the peg outperform both active policies. Finally, variations of the elasticity of intertemporal substitution and of the degree of price stickiness show that the relative performance of the solutions also depends on the relative welfare cost of output and inflation fluctuations. Lowering the fraction of non-price-adjusting firms ($\phi = 0.7$) tends to lower the welfare loss in general. Then, aggressive (active) responses to changes in inflation are less desirable, such that even the MSV solution under the peg outperforms the active discretionary policy for $\rho = 0.9$. If the private sector is less willing to substitute consumption intertemporally ($\sigma = 2$), central bank passiveness

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[2]Under both solutions to the peg, a cost push shock causes output and inflation to decrease. This outcome can differ from the transmission of cost push shocks under solutions with artificial state variables, where inflation rises and – due to a lower real interest rate – output too. Such a solution is known to support self-fulfilling inflation expectations.

| | Peg (AR) | | Peg (MSV) | | Discretion | | Taylor | |
|---|---|---|---|---|---|---|---|---|
| | Welfare loss | | Welfare loss | | Welfare loss | | Welfare loss | |
| | $\mathrm{Var}\hat{\pi}$ | $\mathrm{Var}\hat{x}$ | $\mathrm{Var}\hat{\pi}$ | $\mathrm{Var}\hat{x}$ | $\mathrm{Var}\hat{\pi}$ | $\mathrm{Var}\hat{x}$ | $\mathrm{Var}\hat{\pi}$ | $\mathrm{Var}\hat{x}$ |
| $\rho = 0.75$ | -452.81 | | -4334.86 | | -279.91 | | -1276.50 | |
| | 2.36 | 75.86 | 34.47 | 310.25 | 1.38 | 49.64 | 12.41 | 12.41 |
| $\rho = 0.8$ | -464.98 | | -1767.35 | | -367.92 | | -1982.18 | |
| | 2.02 | 91.95 | 12.12 | 193.98 | 1.81 | 65.25 | 19.27 | 19.27 |
| $\rho = 0.85$ | -508.73 | | -1042.81 | | -517.89 | | -3400.34 | |
| | 1.66 | 119.85 | 5.43 | 174.52 | 2.55 | 91.84 | 33.06 | 33.06 |
| $\rho = 0.9$ | **-629.45** | | **-846.74** | | **-823.56** | | **-6872.52** | |
| | 1.24 | 176.64 | 2.55 | 206.74 | 4.06 | 146.05 | 66.81 | 66.81 |
| $\rho = 0.95$ | -1065.44 | | -1133.14 | | -1754.03 | | -19742.23 | |
| | 0.71 | 347.40 | 1.00 | 361.09 | 8.64 | 311.06 | 191.93 | 191.93 |
| $\phi = 0.7^*$ | -392.31 | | -469.40 | | -476.43 | | -3967.05 | |
| | 0.59 | 75.92 | 1.03 | 83.52 | 1.85 | 66.51 | 38.00 | 38.00 |
| $\sigma = 2^{**}$ | -535.00 | | -892.69 | | -489.62 | | -4994.05 | |
| | 2.01 | 77.74 | 4.78 | 96.72 | 1.92 | 69.26 | 48.41 | 35.57 |
| Notes: We set $\sigma = 1$, $\phi = 0.75$, except for * and ** where $\rho = 0.9$. | | | | | | | | |

Table 2.2: Unconditional variances and welfare for various parameter values

tends to lead to higher inflation and a higher welfare loss.[3]

## 2.3 Uniqueness under an interest rate peg

As argued above, the two solutions analyzed for the interest rate peg are not the only possible solutions. In particular, non-fundamental solutions (with artificial state variables) are possible that might lead to endogenous fluctuations. A policy regime that facilitates the latter is evidently not desirable. Thus, we aim at designing policy rules for the central bank that implement a peg in a way that renders multiple solutions impossible. In particular, we show that both solutions to the peg can be implemented by an appropriately designed interest rate rule.

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[3]The unconditional variances tend to rise with $\rho$ under active policies, whereas $\rho$ exerts an ambiguous effect on the variances under the peg.

### 2.3.1 Minimum state solution

The MSV solution under a peg, which is characterized by $\hat{\pi}_t = a_u^{peg} u_t$ and $\hat{x}_t = b_u^{peg} u_t$, can be implemented by a rule of the form (2.2). To implement a peg, $\hat{R}_t = 0$, the coefficients have to satisfy $\frac{w_x}{w_\pi} = -\frac{\hat{\pi}_t}{\hat{x}_t}$. Using that the MSV solution implies $\frac{\hat{\pi}_t}{\hat{x}_t} = \frac{a_u^{peg}}{b_u^{peg}}$, we get the following conditions for the policy rule coefficients

$$w_\pi = \alpha \text{ and } w_x = -\alpha \left( a_u^{peg} / b_u^{peg} \right),$$

where $\alpha$ is an arbitrary constant. We can easily assess the equilibrium determinacy conditions of the New Keynesian model closed with this interest rate rule. The model in matrix form is given by

$$E_t y_{t+1} = \mathbf{A} y_t + B u_t \text{ with } y_t = (\hat{x}_t, \hat{\pi}_t)' \text{ and } \mathbf{A} = \begin{pmatrix} 0 & \beta \\ \sigma & 1 \end{pmatrix}^{-1} \begin{pmatrix} -\kappa & 1 \\ \sigma + w_x & w_\pi \end{pmatrix}.$$

To ensure determinacy, the matrix A must exhibit two unstable eigenvalues which is guaranteed by the three following conditions

$$\det(\mathbf{A}) > 1 \Leftrightarrow \frac{\sigma + w_x + \kappa w_\pi}{\sigma \beta} > 1,$$

$$\det(\mathbf{A}) - \text{trace}(\mathbf{A}) > -1 \Leftrightarrow w_\pi + w_x \frac{1 - \beta}{\kappa} > 1,$$

$$\det(\mathbf{A}) + \text{trace}(\mathbf{A}) > -1 \Leftrightarrow \kappa (1 + w_\pi) + (2\sigma + w_x)(1 + \beta) > 0.$$

Under the benchmark calibration of Table 2.1 and, for instance, $\alpha = 1.5$, these conditions are fulfilled and the MSV solution is the unique solution. The interest rate rule then reads $\hat{R}_t = 1.5\hat{\pi}_t - 0.1667\hat{x}_t$. This rule uniquely implements sequences $\left\{ \hat{R}_t, \hat{x}_t, \hat{\pi}_t \right\}_{t=0}^{\infty}$ satisfying $\hat{\pi}_t = a_u^{peg} u_t$, $\hat{x}_t = b_u^{peg} u_t$, and $\hat{R}_t = 0$.

### 2.3.2 Autoregressive solution

Similarly, the autoregressive solution can uniquely be implemented by an interest rate rule of the form[4]

$$\hat{R}_t = r_x \hat{x}_t + r_\pi \hat{\pi}_t + r_l \hat{\pi}_{t-1}. \tag{2.7}$$

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[4]Galí (2008) proceeds in a similar way to implement a peg in a unique way: He induces equilibrium determinacy in a New Keynesian model by implementing a peg with a rule, where the central bank reacts to inflation and current and lagged output.

We set the parameters in (2.7) so as to implement the autoregressive solution. Eliminating $\hat{\pi}_t$ in (2.7) by $\hat{\pi}_t = a_\pi \hat{\pi}_{t-1} + a_u^{peg,AR} u_t$ yields $\hat{R}_t = r_x \hat{x}_t + (r_l + r_\pi a_\pi) \hat{\pi}_{t-1} + r_\pi a_u^{peg,AR} u_t$, which further has to imply a sequence of constant interest rates, $\hat{R}_t = 0$. For this, we use the output solution, $\hat{x}_t = b_\pi \hat{\pi}_{t-1} + b_u^{peg,AR} u_t \Leftrightarrow \alpha(-\hat{x}_t + b_\pi \hat{\pi}_{t-1} + b_u^{peg,AR} u_t) = 0$ for an arbitrary $\alpha \neq 0$, which implies $\hat{R}_t = r_x \hat{x}_t + (r_l + r_\pi a_\pi) \hat{\pi}_{t-1} + r_\pi a_u^{peg,AR} u_t = 0$ if $r_x = -\alpha$; $r_l + r_\pi a_\pi = \alpha b_\pi$; and $r_\pi a_u^{peg,AR} = \alpha b_u^{peg,AR}$. Thus, (2.7) implements an interest rate peg if (but not only if) the policy rule coefficients satisfy

$$r_x = -\alpha, \qquad r_l = \alpha \left( b_\pi - a_\pi \varpi \right), \qquad r_\pi = \alpha \varpi, \qquad (2.8)$$

where $\varpi = b_u^{peg,AR}/a_u^{peg,AR}$. Setting $\alpha = 0.25$, for instance and applying the parameter values in Table 2.1, leads to $r_x = -0.25$, $r_l = -4.47$, and $r_\pi = 7.22$. Eliminating the interest rate in (2.1), by a policy rule satisfying (2.7) and (2.8) leads to a $3 \times 3$ system in $\hat{x}_t$, $\hat{\pi}_t$, and $\hat{\pi}_{t-1}$. Since the latter is relevant for monetary policy, the equilibrium solution takes the form (2.4). Determinacy then requires that there are two eigenvalues outside the unit circle and one stable eigenvalue. For $\alpha = 0.25$ and the parameter values in Table 2.1, the eigenvalues under (2.8) are given by $\lambda_1 = 0.665$; $\lambda_2 = \lambda_3 = 1.080$ (modulus), ensuring that the AR solution is the unique solution.[5] The single stable eigenvalue 0.665, resembles the autoregressive coefficient in the inflation solution: $\hat{\pi}_t = 0.665 \hat{\pi}_{t-1} - 0.182 u_t$. Thus, the central bank can construct an interest rate rule in a way, which implies 1) a constant interest rate (i.e. $\hat{R}_t = 0$) and 2) unique equilibrium sequences that are identical to the equilibrium sequences under the autoregressive solution.

## 2.4    Conclusion

This chapter shows that a popular monetary policy device, namely, an active interest rate policy (or a Taylor rule), can easily be outperformed by an even simpler monetary policy strategy: an interest rate peg. While we do not seek to identify optimal policies, we want to demonstrate that central bankers should not apply recipes just because of their appealing simplicity. Once a policy maker departs from a fully optimal (commitment) strategy, it is ex-ante not clear which kind of simple rule most closely resembles the outcome under the commitment plan. To our surprise, even an interest

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[5]Further details on the determinacy conditions are available from the authors upon request.

rate peg can be more desirable (in welfare terms) than an active interest rate policy consistent with an optimal plan under discretion. Finally, it is shown that the common argument in favor of active interest rate policies, i.e. the absence of endogenous fluctuations, is not incompatible with a constant interest rate.

# Chapter 3

# Should central banks care about investment?

## 3.1 Introduction

A paradigm in the literature on optimal monetary policy in New Keynesian models is the prescription of Taylor rules: Schmitt-Grohé and Uribe (2007) argue that these simple policy rules, which require the central bank to adjust the nominal interest rate more than one for one in response to deviations in inflation from its target, resolve the central bank's trade-off in a near-optimal way. Intuitively, an increasing real interest rate in face of inflationary pressure will curb economic activity (and *vice versa*) and thereby dampen fluctuations in inflation. The good performance of Taylor rules, which are also known as active policy rules, is closely related to the fact that most New Keynesian models imply that stabilizing inflation should be given priority over the goal of output stabilization.[6] Woodford (2004), in his survey of the literature, finds that "it is not a bad first approximation to say that the goal of monetary policy should be price stability". The goal of this chapter is to analyze how the inclusion of endogenous capital accumulation quantitatively affects interest rates, the central bank's policy trade-off and the performance of simple policy rules. In particular, I present the Ramsey optimal policy for a model with endogenous capital formation and compare it to Ramsey optimal policy in the standard model with an exogenous capital stock. I further ask to what extent simple rules

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[6]See Schmitt-Grohé and Uribe (2007) and Rotemberg and Woodford (1997).

can mimic the allocation achieved by Ramsey optimal policy when capital accumulation is endogenous.

In the canonical New Keynesian model, as presented for instance in Clarida, Galí, and Gertler (1999), the capital stock is assumed to be fixed, so that the central bank has an impact on aggregate activity exclusively due to its influence on agents' intertemporal consumption decision. This implies that the interest rate set by monetary policy has no impact on the intertemporal allocation of resources: The central bank can reduce aggregate demand by increasing the interest rate without having an impact on future allocations. With endogenous capital accumulation, an increase in the real interest rate triggers a reduction of investment which affects future marginal cost and production. Empirical evidence demonstrates that the investment channel is quantitatively important. Christiano, Eichenbaum, and Evans (2005) find that investment reacts to monetary policy shocks in a hump-shaped way. In the peak, which occurs after around two years, investment declines by one percent. Angeloni, Kashyap, Mojon, and Terlizzese (2003) use vector autoregressions (VARs) to identify the relative contribution of consumption and investment in the response of private sector domestic demand to monetary policy shocks. On a two-year horizon, they estimate the contribution of investment to slightly below 50% for the United States. In the Euro Area, the response of investment is quantitatively even more important than consumption movements, accounting for a fraction of around 70%.

I present two main results in this chapter. First, I find that when capital can be accumulated at no cost, optimal policy reacts to a cost push shock by persistently *reducing* nominal and real interest rates. However, this does not reflect a systematically changed trade-off: As in the model with fixed capital, optimal policy allows for temporary inflation by persistently reducing the real interest rate below its natural rate in order to stabilize output. Rather, it is the path of the natural rate of interest, i.e. the real interest rate in absence of sticky prices, which is affected by capital accumulation: The natural interest rate falls in response to a cost push shock. The reason is that, due to the decline in the capital stock, scarcity of consumption goods is highest around two years after the shock. Thus, households *ceteris paribus* increase borrowing, driving down the equilibrium real interest rate. Second, I find that the performance of simple rules worsens with increasing flexibility of the capital stock. When capital can be accumulated

at no cost, the optimal simple rule induces a welfare loss exceeding that under Ramsey policy by 9.2%. The reason is that the optimal simple rule stabilizes inflation by strongly active policy. However, endogenous capital accumulation magnifies the output cost of this policy: By stabilizing inflation, policy permits the decline in labor demand to lead to a decline in investment, thus reducing future labor demand and output. Under positive investment adjustment cost, the excess welfare cost of simple rules falls to 2.8%. The reason is that adjustment cost mitigate the output fluctuations implied by endogenous capital accumulation.

McCallum and Nelson (1999) argue that fluctuations in the capital stock over the business cycle can be neglected because capital and output movements are not strongly correlated at cyclical frequencies. However, the authors themselves name analytical simplicity as the main justification for the fixed capital assumption. Many studies have analyzed New Keynesian models with endogenous capital accumulation, but none has exposed its impact on optimal monetary policy explicitly. The existing literature generally considers two different assumptions. Under the rental market assumption households accumulate capital and rent it to firms. Another possibility is to assume firm-specific capital. As demonstrated by Galí, Gertler, and López-Salido (2001), this implies that marginal cost differ across firms. Carlstrom and Fuerst (2005) contribute to the discussion on endogenous fluctuations generated by monetary policy. They find that in a rental-market model, forward-looking interest rate rules are likely to generate equilibrium indeterminacy. Sveen and Weinke (2005, 2007) focus on the determinacy properties of simple interest rate rules in a model with firm-specific capital accumulation. They find that firm-specific capital generates endogenous price stickiness which implies that the Taylor principle can be insufficient to guarantee equilibrium determinacy.[7] Sveen and Weinke (2006) analyze the welfare implications of optimized simple interest rate rules in a model with sticky wages. They find that a central bank which ignores the endogenous price stickiness implied by firm-specific capital puts too less weight on price inflation and too much weight on wage

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[7]Altig, Christiano, Eichenbaum, and Linde (2005) show that the endogenous price stickiness implied by models with firm-specific capital helps to reconcile the different estimates of price stickiness from macro and micro models: To match the data, macro models need large price stickiness implying that prices are adjusted on average every six quarters. However, microeconomic estimations suggest much more frequent price adjustment.

inflation. However, Sveen and Weinke (2006) confine to analyzing simple rules and do not compute Ramsey optimal policy. Further, the large-scale models estimated by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) include capital. However, analyzing optimal monetary policy is outside the scope of these contributions.

In the literature on optimal monetary policy, the setup of Schmitt-Grohé and Uribe (2007) is closest to the present chapter. Following them, I analyze Ramsey optimal policy in a sticky price economy where households invest into capital and rent it to firms. Schmitt-Grohé and Uribe evaluate the welfare loss of different policies under shocks to government expenditures and factor productivity, and generate a trade-off for monetary policy by assuming a transaction friction which implies that positive interest rates distort households' cash holding decision. In contrast, the model analyzed in this chapter is a cashless economy under cost push shocks. These are modeled as wage markup shocks arising from imperfect substitutability of different labor types. A rising markup implies an inefficient increase in wages and a reduction of economic activity.[8] Such shocks imply that the central bank faces a different trade-off compared to the analysis of Schmitt-Grohé and Uribe (2007). Many empirical contributions to the literature document variation in markups. Markup shocks play an important role in estimated models, such as Smets and Wouters (2007). Further, Galí, Gertler, and López-Salido (2007) measure an inefficiency gap in the labor market, which is defined as the gap between the marginal product of labor and the marginal rate of substitution between leisure and consumption. They find that this gap can be decomposed into a wage and a price markup and find the wage markup to be the predominant source of fluctuations in the gap. According to Chari, Kehoe, and McGrattan (2009), this inefficiency gap, which they call the labor wedge, accounts for an important share of business cycle fluctuations. I calibrate the wage markup shock to the inefficiency gap measured by Galí, Gertler, and López-Salido (2007).[9]

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[8]The wage increase is inefficient in the sense that it does not reflect a change in the social cost of labor, i.e. the wage rises while the marginal rate of substitution between leisure and consumption is unchanged. Therefore, a social planner would not change the allocation in response to such a shock, as will be made explicit in section 3.3.1.

[9]Note that there is debate on the microfoundation of these shocks and to what extent they represent inefficient fluctuations: Chari, Kehoe, and McGrattan (2009) argue that New Keynesian models are not yet ready for policy analysis because there is no microeconomic evidence on the nature of many shocks required to generate quantitatively

This chapter is structured as follows. I start by presenting the model in section 3.2. Section 3.3 presents the first-best allocation, which provides a useful benchmark and exposes distortions in the competitive equilibrium of the model economy. In 3.3, I further derive the problem of the Ramsey planner and present the welfare measure. Section 3.4 shows the impulse responses to a wage markup shock under different policies in both the model with fixed and variable capital. Moreover, I evaluate welfare under alternative policies and characterize their stabilization properties with reference to the distortions identified in the analysis of the first-best allocation. Following Casares and McCallum (2006), who argue that a model with frictionless capital accumulation yields an implausible variability of investment, I introduce investment adjustment cost into the model in the section 3.5. Further, I conduct a robustness check by assuming different calibrations and specifications of adjustment cost. Section 3.6 concludes.

## 3.2 The model

The model is a standard New Keynesian model with Calvo-type sticky prices, flexible wages and monopolistically competitive firms. I introduce a cost push shock by allowing for imperfect substitutability of labor supply. For purposes of illustration, I will here expose the model in absence of adjustment cost. I introduce investment adjustment cost in section 3.5.1 and, for robustness, .

### 3.2.1 Households

There exists a continuum of households with total mass 1, indexed over $j$. Households derive utility from consuming $c_{j,t}$ and enjoying leisure $1 - n_{j,t}$, with the utility function given by

$$u(c_{j,t}, n_{j,t}) = \frac{c_{j,t}^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{n_{j,t}^{1+\eta}}{1 + \eta}. \tag{3.1}$$

Future utility is discounted with a constant discount factor $\beta$. Households further invest into capital $k_{j,t}$, which firms rent at the (real) rental rate $r_t^k$. In nominal terms, letting $P_t$ denote the price level, their revenue is $P_t r_t^k$.

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reasonable dynamics. Therefore, in principle, the observed inefficiency gap might for instance be the result of a fluctuating utility value of leisure. Such a shock would imply efficient fluctuations in the sense that a social planner would accommodate the shock.

The capital stock evolves according to $k_{j,t+1} = (1 - \delta) k_{j,t} + i_{j,t}$, where $i_{j,t}$ denotes investment and $\delta$ the depreciation rate. Households can buy both investment and consumption goods from retailers at an identical price $P_t$, so that a household's expenditures for consumption and investment goods amount to $P_t c_{j,t} + P_t i_{j,t}$. Households further have access to nominally state-contingent claims which deliver a certain payoff of one unit of currency in a particular state. Let $D_{j,t}$ denote the amount of claims purchased by a household $j$ in period $t$. $\varphi_{t,t+1}$ denotes the stochastic discount factor, i.e. the period t price of such a claim for a particular state divided by the probability of occurrence of that state conditional on time t information, so that $E_t \varphi_{t,t+1} D_{j,t}$ are a household's expenditures for acquiring claims. Note that this setup implies incomplete markets because households do not have access to *real* state-contingent claims. Further, households receive profits earned by firms $P_t \Psi_{j,t}$ and need to pay a lump sum tax $P_t \tau_{j,t}$. Households further supply a differentiated type of labor $n_{j,t}$ for which they earn the real wage $w_{j,t}$. To eliminate the wage markup in the steady state, the wage is subsidized by the factor $\nu^W$. Thus, a household's budget constraint is given by

$$D_{j,t-1} + \nu^W P_t w_{j,t} n_{j,t} + k_{j,t} P_t r_t^k + P_t \Psi_{j,t} = E_t \varphi_{t,t+1} D_{j,t} + c_{j,t} P_t + P_t i_{j,t} + P_t \tau_{j,t}.$$

Defining $d_{j,t} = \frac{D_{j,t}}{P_t}$, so that $E_t \varphi_{t,t+1} d_{j,t}$ denotes real expenditures on state-contingent claims, the budget constraint can be expressed in real terms,

$$\frac{d_{j,t-1}}{\pi_t} + \nu^W w_{j,t} n_{j,t} + k_{j,t} \left( r_t^k + 1 - \delta \right) + \Psi_{j,t} = E_t \varphi_{t,t+1} d_{j,t} + c_{j,t} + k_{j,t+1} + \tau_{j,t},$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ denotes inflation and where I have used the law of motion for the capital stock. Before describing optimal behavior, the structure of the labor market, which resembles Erceg, Henderson, and Levin (2000) and Smets and Wouters (2007), will be explained. Households supply labor not directly to firms but to labor packers who pay the real wage $w_{j,t}$. Each household supplies a differentiated type of labor which is aggregated to a composite labor good by the labor packers using the production function

$$n_t = \left[ \int_0^1 n_{j,t}^{\psi_t} dj \right]^{\frac{1}{\psi_t}}.$$

The labor packers sell the aggregate labor good $n_t$ to intermediate firms at the real wage $w_t$ in a perfectly competitive market. Thus, the labor

packers' demand for the good $n_{j,t}$ is given by

$$n_{j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\zeta_t} n_t,$$

where $\zeta_t = \frac{1}{1-\psi_t}$. I assume that the log of the substitution elasticity $\zeta_t$ follows an AR(1) process with mean $\bar{\zeta}$ and autocorrelation $\rho_\zeta$. Because labor packers are perfectly competitive, I can use the zero-profit condition to obtain the aggregate wage as

$$w_t = \left[\int_0^1 w_{j,t}^{\frac{\psi_t}{\psi_t-1}} dj\right]^{\frac{\psi_t-1}{\psi_t}}.$$

I can now to derive the household's optimal behavior. The first order conditions for consumption, capital and state contingent claims read

$$\lambda_{j,t} = c_{j,t}^{-\sigma}, \tag{3.2}$$

$$c_{j,t}^{-\sigma} = \beta E_t \left[c_{j,t+1}^{-\sigma}(r_{t+1}^k + 1 - \delta)\right], \tag{3.3}$$

$$\varphi_{t,t+1} = \beta \frac{\lambda_{j,t+1}}{\lambda_{j,t}\pi_{t+1}}. \tag{3.4}$$

The transversality conditions read $\lim_{s\to\infty} E_t\varphi_{t,t+s}k_{j,t+s+1} = 0$ and $\lim_{s\to\infty} E_t\varphi_{t,t+s}d_{j,t+s} = 0$, where $\varphi_{t,t+s} = \prod_{i=0}^{s-1} \varphi_{t+i,t+i+1}$. Further, optimal wage setting requires

$$w_{j,t} = \frac{\zeta_t}{\zeta_t - 1}\frac{1}{\nu^W}\chi n_{j,t}^\eta c_{j,t}^\sigma.$$

This implies that households charge a markup over their marginal rate of substitution between leisure and consumption. The government subsidy $\nu^W$ is set so that the steady state is efficient (see section 3.2.3).[10] To simplify notation, I denote the net wage markup as

$$\mu_t^W = \frac{\zeta_t}{\zeta_t - 1}\frac{1}{\nu^W}.$$

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[10]With capital accumulation, a simpler setup with just one subsidy at the firm level is not possible. The reason is that a firm subsidy affects firms' demand for labor and capital. Thus, by eliminating the long-run distortion in the labor market, a distortion of capital accumulation would result.

Note that the only reason for household heterogeneity is differentiated labor supply. However, wages are perfectly flexible and households have identical market power, so that all households will charge the same wage given that their marginal rates of substitution do not differ. There is no reason for such a difference and thus households will behave identically and I can work with a representative household. This further implies that there is no dispersion across households' labor supply. To summarize, the representative household's first order conditions are given by

$$w_t = \mu_t^W \chi n_t^\eta c_t^\sigma, \tag{3.5}$$

$$c_t^{-\sigma} = \beta E_t \left[ c_{t+1}^{-\sigma} (r_{t+1}^k + 1 - \delta) \right], \tag{3.6}$$

$$\varphi_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}. \tag{3.7}$$

I can further use the asset pricing equation to construct a (nominally) risk-free bond which pays an interest of $R_t = \frac{1}{E_t \varphi_{t,t+1}}$. Using this, (3.7) implies a standard Euler equation

$$c_t^{-\sigma} = \beta E_t \left[ c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right]. \tag{3.8}$$

I can use (3.8) to derive an arbitrage-freeness condition between investment into state-contingent claims and capital, which reads

$$E_t c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} = E_t \left[ c_{t+1}^{-\sigma} (r_{t+1}^k + 1 - \delta) \right]. \tag{3.9}$$

This equation describes one of the central mechanisms considered in this chapter. The central bank, who sets the nominal interest rate $R_t$, has an impact on capital accumulation: An increase in the ex ante real interest rate $E_t R_t / \pi_{t+1}$ on nominally risk-free bonds leads households to adapt investment into capital until it promises an expected return equal to the expected real return on bonds. This link is substantially weakened by the introduction of investment adjustment cost in section 3.5.1. A second, important feature of the model is the time-varying markup of labor supply, which generates cost push effects.

### 3.2.2 Firms

There are two types of firms, producers and retailers. Monopolistically competitive producers rent labor and capital to produce the final good $y_{it}$. Retailers assemble these goods to the final good $y_t$ according to the production function

$$y_t = \left[ \int_0^1 y_{it}^q di \right]^{1/q},$$

where $0 < q < 1$ is a function of the elasticity of substitution $\varepsilon$ between two input goods, $q = \frac{\varepsilon - 1}{\varepsilon}$. In contrast to the labor sector, firms' market power is not time-varying.

### Retailers

Retail firms operate in a perfectly competitive market and maximize profits given a price level $P_t$ and aggregate demand $y_t$,

$$\max_{y_{it}} P_t \left[ \int_0^1 y_{it}^q di \right]^{1/q} - \int_0^1 P_{it} y_{it} di.$$

Analogue to the labor sector, retailers demand the following quantity from each individual producer

$$y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t.$$

With perfect competition in the retail sector, the retailers' profits are zero and the equilibrium price level is given by

$$P_t = \left( \int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

### Producers

There are infinitely many producers indexed over $i \in (0,1)$. Producers face two decisions: They minimize cost given a certain level of production. Further, they choose optimal prices, which determine the demand for their product.

**Cost minimization** Producers minimize cost given demand $y_{it}$, taking as given the factor prices $w_t$ and $r_t^k$. Their production function is given by $y_{it} = n_{it}^\alpha k_{it}^{1-\alpha}$. Note that the multiplier on the constraint equals nominal marginal cost, $MC_{it}$. Further, I denote real marginal cost by $mc_{it} = MC_{it}/P_t$. Producers thus solve

$$\min_{n,k} \ P_t w_t n_{it} + P_t r_t^k k_{it} + MC_{it}(y_{it} - n_{it}^\alpha k_{it}^{1-\alpha}).$$

The producers' first order conditions read

$$P_t w_t = MC_{it} f_{n,t} \iff w_t = mc_{it}\alpha \left(\frac{k_{it}}{n_{it}}\right)^{1-\alpha},$$

$$P_t r_t^k = MC_{it} f_{k,t} \iff r_t^k = mc_{it}(1-\alpha)\left(\frac{n_{it}}{k_{it}}\right)^\alpha.$$

These equations can be combined to yield expressions for the optimal capital-labor ratio and real marginal cost

$$\frac{w_t}{r_t^k} = \frac{\alpha}{(1-\alpha)}\frac{k_{it}}{n_{it}}, \tag{3.10}$$

$$mc_{it} = \left(\frac{r_t^k}{1-\alpha}\right)^{1-\alpha}\left(\frac{w_t}{\alpha}\right)^\alpha. \tag{3.11}$$

Note that the capital-labor ratio in equilibrium is identical across firms as all have access to the same factor market. Thus, marginal cost are equal across firms as well, $mc_{it} = mc_t$.

**Pricing decision** I assume Calvo (1983) pricing where each producer faces a constant probability $\phi$ that he cannot reset his price in a given period. Thus, a fraction $(1 - \phi)$ set its price according to $P_{it}^{NA} = P_{it-1}$. Price adjusting producers maximize expected (nominal) profits subject to Calvo pricing scheme. Producers receive a subsidy $\nu^P$ which is designed to eliminate the steady state markup. Using that demand is given by $y_{it} = (P_{it}/P_t)^{-\varepsilon} y_t$ and that due to constant returns to scale, (3.10) and (3.11) imply $w_t n_{it} + r_t^k k_{it} = mc_t y_{it}$, nominal profits are given by

$$P_t \Psi_{it} = \nu^P P_{it} y_{it} - mc_t P_t y_{it} = \nu^P \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t P_t - mc_t P_t \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t.$$

Thus, price adjusting producers solve

$$\max_{Z_t} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \phi^s \left[ \nu^P \left( \frac{Z_t}{P_{t+s}} \right)^{1-\varepsilon} P_{t+s} y_{t+s} - mc_{t+s} \left( \frac{Z_t}{P_{t+s}} \right)^{-\varepsilon} P_{t+s} y_{t+s} \right].$$

With a nominal discount factor of $\Lambda_{t,t+s} = \beta^s \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+s}}$, this gives forward-looking price setting

$$Z_t = \frac{\varepsilon}{\varepsilon - 1} \frac{1}{\nu^P} \frac{E_t \sum_s (\beta\phi)^s c_{t+s}^{-\sigma} y_{t+s} P_{t+s}^{\varepsilon} mc_{t+s}}{E_t \sum_s (\beta\phi)^s c_{t+s}^{-\sigma} y_{t+s} P_{t+s}^{\varepsilon-1}}.$$

Prices are thus set as a markup on a weighted sum of expected future marginal cost, reflecting the positive probability of not being able to adjust prices in future. The price level can be eliminated by rewriting forward-looking price setting in terms of the real price $\tilde{Z}_t = Z_t/P_t$. I can then rewrite the optimal pricing condition and obtain a system of three equations,

$$\tilde{Z}_t = \frac{Z_t^1}{Z_t^2},$$
$$Z_t^1 = c_t^{-\sigma} y_t mc_t + \beta\phi E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1, \tag{3.12}$$
$$Z_t^2 = c_t^{-\sigma} y_t + \beta\phi E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2, \tag{3.13}$$

using that the government subsidy is set to $\nu^P = \frac{\varepsilon}{\varepsilon-1}$. The aggregate price level depends on the optimal price set by adjusters and non-adjusters, $P_t = \left[ \int_0^1 P_{it}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$. As is shown in Appendix B.1.1, the price index can be rewritten in terms of the inflation rate $\pi_t = P_t/P_{t-1}$ and the real price of adjusters $Z_t^1/Z_t^2$

$$1 = (1-\phi) \left( \frac{Z_t^1}{Z_t^2} \right)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}. \tag{3.14}$$

This equation, together with (3.12) and (3.13) summarize optimal price setting. In linear terms, one gets the New Keynesian Phillips curve.

**Price dispersion and aggregate production**    In this chapter, I evaluate welfare under alternative policies. Thus, it has to be taken into account that the production of final goods is affected by price dispersion which implies that capital and labor are inefficiently allocated across firms. To capture this effect, I define intermediate output $IO_t$ as the sum of produced

intermediate goods,

$$IO_t \equiv \int_0^1 y_{it} di = \int_0^1 k_{it} \left(\frac{n_{it}}{k_{it}}\right)^\alpha di = k_t \left(\frac{n_t}{k_t}\right)^\alpha. \tag{3.15}$$

where the last equality uses that the capital labor ratio is constant across firms, as implied by (3.10). Further, demand for varieties is given by $y_{it} = (P_{it}/P_t)^{-\varepsilon} y_t$ and thus

$$IO_t = \int_0^1 y_{it} di = y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di. \tag{3.16}$$

An inefficient distribution of production across firms implies $IO_t > y_t$. Following Schmitt-Grohé and Uribe (2004b), a recursive representation of $s_t \equiv \int_0^1 (P_{it}/P_t)^{-\varepsilon} di$ can be derived. I show in Appendix B.1.1 that $s_t$ evolves according to

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^\varepsilon, \tag{3.17}$$

with $s_t \geq 1$. Combining (3.15) and (3.16), aggregate production is given by

$$y_t = \frac{k_t^{1-\alpha} n_t^\alpha}{s_t}. \tag{3.18}$$

Thus, price dispersion implies an inefficient allocation of resources.

### 3.2.3 Government

I assume that the government does not issue bonds, which implies that its budget must be balanced in every period. The government collects lump sum taxes $\tau_t$ to finance the expenditures for the production subsidy to firms $\left(\tau_t^P\right)$ and the wage subsidy to workers $\left(\tau_t^W\right)$, so that its budget constraint reads

$$\tau_t = \tau_t^P + \tau_t^W.$$

I assume the government to set the production subsidy to $\nu^P = \frac{\varepsilon}{\varepsilon-1}$ which implies that in the steady state, firms produce efficiently at real marginal cost of unity, $mc = 1$.[11] Similarly, the wage subsidy is set to $\nu^W = \frac{\zeta}{\zeta-1}$, so

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[11]Note that the subsidies $\nu^W$ and $\nu^P$ are fixed but expenditures vary depending on economic activity. For details, see Appendix B.1.1.

that the labor supply decision is not distorted in steady state.[12] Monetary policy is given by the Ramsey optimal policy or in form of an interest rate rule

$$R_t = \frac{\pi}{\beta} \left(\frac{\pi_t}{\pi}\right)^{w_\pi} \left(\frac{y_t}{y}\right)^{w_y}, \tag{3.19}$$

where $w_y > 0$, $w_\pi > 1$ and where $\pi$ denotes steady state inflation. This closes the description of the model.

### 3.2.4   Equilibrium

In equilibrium markets clear, i.e. $n_t = \int_0^1 n_{it} di$, $k_t = \int_0^1 k_{it} di$. Further, because state contingent claims are traded only among households, in aggregate, $d_t = \int_0^1 d_{jt} dj = 0$. I show in Appendx B.1.2 that the aggregate resource constraint is given by

$$k_t^{1-\alpha} n_t^\alpha = [c_t + k_{t+1} - (1-\delta)k_t] s_t. \tag{3.20}$$

A rational expectations equilibrium is a set of sequences $\{c_t, n_t, k_t, w_t, r_t^k, R_t, mc_t, Z_t^1, Z_t^2, \pi_t, s_t\}_{t=0}^\infty$ satisfying the households' and firms' first order conditions (3.5) - (3.7), (3.10) - (3.14) and the transversality conditions as well as (3.17), (3.20) and a monetary policy, given an exogenous sequence for the wage markup $\mu_t^W$. For convenience, the model's equilibrium conditions are listed in Appendix B.1.3. Appendix B.3 further contains a derivation of the steady state of this model.

## 3.3   Optimal monetary policy

As a benchmark, this section presents the first best allocation of the model economy. This exposes the potential distortions in a competitive equilibrium and thus characterizes the trade-off faced by monetary policy. This section thus provides a framework for the subsequent analysis of the second-best Ramsey allocation.

### 3.3.1   The social planner allocation

The social planner maximizes household utility subject to the economy's technological restrictions. As the labor aggregator $n_t = \left[\int_0^1 n_{j,t}^{\psi_t} dj\right]^{1/\psi_t}$ is a

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[12]For a full derivation of the steady state, see Appendix B.3.

concave function, and all households' preferences are identical, it is optimal for each type of labor to be used to the same extent. Thus, for any pair of households $(j, k)$, optimality requires $n_{j,t} = n_{k,t}$. I now derive the remaining optimality conditions. Because households' preferences are identical, I can work with a representative agent. Thus, the social planner's problem reads

$$\max_{\{c,n,n_i,k,k_i,y,y_i\}} G = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right] + \lambda_t^1 \left[ y_t - \left( \int_0^1 y_{it}^q di \right)^{1/q} \right] \right.$$

$$+ \lambda_t^2 \left[ n_{it}^\alpha k_{it}^{1-\alpha} - y_{it} \right] + \lambda_t^3 \left[ y_t - c_t - k_{t+1} + (1-\delta)k_t \right]$$

$$\left. + \lambda_t^4 \left[ k_t - \int_0^1 k_{it} di \right] + \lambda_t^5 \left[ n_t - \int_0^1 n_{it+1} di \right] \right\}.$$

Denoting the marginal products of labor and capital as $mpn_t$ and $mpk_t$, the first-order conditions to this problem are given by

$$u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( mpk_{t+1} + 1 - \delta \right) \right], \tag{3.21}$$

$$mpn_t = -\frac{u_{n,t}}{u_{c,t}}, \tag{3.22}$$

$$y_{it} = y_t, \tag{3.23}$$

$$\frac{k_{it}}{n_{it}} = \frac{k_t}{n_t}, \tag{3.24}$$

and the aggregate resource constraint $y_t = c_t + k_{t+1} - (1-\delta)k_t$. (3.21) and (3.22) imply that the social planner employs both production factors up to the point where their marginal products equal their social cost. (3.23) and (3.24) combined require $n_{it} = n_{kt}$ and $y_{it} = y_{kt}$ for any two firms $i, k$. For a detailed derivation, see Appendix B.2.1.

### The nature of the wage markup shock

The social planner's first order conditions imply that the wage markup shock has no impact on the first-best equilibrium. Given that the wage markup shock is the only shock, the social planner would thus choose constant paths for all model's variables. Further, as is evident from the household's first order condition (3.5), a competitive equilibrium is in general affected by the wage markup shock. Moreover, the shock also affects the competitive equilibrium under flexible prices. Thus, it is not necessarily optimal for the central bank to stabilize output at its natural rate, i.e. the level of output that would prevail under flexible prices.

### 3.3.2 Competitive equilibrium

The social planner's optimality conditions allow identifying the potential distortions in the competitive equilibrium of the sticky price economy. These are used to illustrate the trade-off faced by monetary policy. To be clear, I use the term "distortion" in the sense that, whenever an equation characterizing the first best allocation is violated, this constitutes a distortion.[13]

**Distortions in the competitive equilibrium**

I now analyze the potential distortions in turn. As shown in section 3.2.2, there is no dispersion in the capital labor ratio across firms in a competitive equilibrium, so that (3.24) holds in all economies analyzed in this chapter. Equation (3.23) refers to the well-known distortion caused by price dispersion: Individual firms' demand is given by $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t$ so that (3.23) is satisfied only if there is no price dispersion, i.e. if $P_{it} = P_t \; \forall i$. Thus, by the definition of price dispersion, $s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$, the social planner's optimality condition (3.23) requires $s_t = 1$. It can be shown that this requires zero inflation, $\pi_t = 1$.[14] The second well-established distortion concerns deviations from the optimality condition (3.22). I here give this distortion an interpretation in terms of the economy's total markup, defined as $\mu_t^T = \frac{mpn_t}{mrs_t}$, where $mrs_t = -\frac{u_{n,t}}{u_{c,t}}$ is the marginal rate of substitution between leisure and consumption. (3.22) requires absence of markups, $\mu_t^T = 1$. As in Galí (2008), define the firms' price markup as $\mu_t^P = \frac{P_t}{MC_t} = \frac{1}{mc_t}$. Further, (3.5) implies that households' wage markup can be written as $\mu_t^W = \frac{w_t}{mrs_t}$. Using that firms' labor demand is governed by $mc_t = mpn_t/w_t$, the total

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[13]Note that a distortion in this sense does not imply that the allocation in the competitive equilibrium necessarily differs from the one a social planner would choose. The reason is that the impact of a distortion on the competitive equilibrium can be counteracted by another distortion. Rather, my terminology implies that capital accumulation is distorted whenever, *all other things equal*, the representative household accumulates an amount of capital different from that implied by the social planner's optimality condition.

[14]As is common in the literature, the proof sketched here is based on the assumption of zero initial price dispersion, $s_{t-1} = 1$. (3.14) implies that $\frac{Z_t^1}{Z_t^2} = \left(\frac{1-\phi\pi_t^{\varepsilon-1}}{1-\phi}\right)^{\frac{1}{1-\varepsilon}}$. From (3.17), $s_t = 1$ requires $1 - \phi\pi_t^\varepsilon = (1-\phi)^{\frac{1}{1-\varepsilon}} \left(1 - \phi\pi_t^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}}$, which is only satisfied by $\pi_t = 1$.

markup can be written as

$$\mu_t^T = \frac{mpn_t}{mrs_t} = \frac{w_t}{mc_t} \frac{\mu_t^W}{w_t} = \mu_t^W \mu_t^P.$$

Intuitively, whenever $\mu_t^T > 1$ labor at the aggregate level is inefficiently low. Thus, this distortion calls for stabilizing labor demand at its efficient level. Under a fixed capital stock, this is identical to stabilizing output at its first-best level.

The two distortions described so far are also present in the standard model with a fixed capital stock. In the model with endogenous capital accumulation, (3.21) exposes a potential third distortion related to capital accumulation. In the model economy, where intermediate firms' capital demand satisfies $r_t^k = mc_t mpk_t$, the household's first-order condition for capital accumulation reads

$$u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( \frac{mpk_{t+1}}{\mu_{t+1}^P} + 1 - \delta \right) \right]. \tag{3.25}$$

Firms apply a price markup and thus do not rent capital up to its marginal product. This influences households' capital accumulation: When households expect a decline in the price markup, $E_t \mu_{t+1}^P < 1$, the expected rental rate on capital is above its expected marginal product. Thus, households will, *ceteris paribus*, increase capital accumulation.[15]

**The central bank's trade-off**

The distortions of the competitive equilibrium can be summarized by the total markup $\mu_t^T$ and inflation $\pi_t$, as in the model with fixed capital. However, endogenous capital accumulation can have an impact on the central bank's trade-off by rendering stabilization of one variable relatively more important. This is one of the main questions of this chapter. I develop an intuition in the following.

In absence of wage markup shocks, the social planner allocation can be implemented by completely stabilizing the total markup $\mu_t^T$ and inflation $\pi_t$ at their steady state values of unity.[16] Achieving first best is not possible

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[15]Note that $\mu_t^P < 1$ is possible in the present model without firms having an incentive to close down. The reason is that the subsidy makes firms profitable as long as $\mu_t^P > \frac{\varepsilon-1}{\varepsilon}$.

[16]It is shown in Appendix B.3 that unity is the steady state value of both the total markup and inflation if the central bank targets zero long run inflation, $\pi = 1$, so that the first best allocation is achieved.

when a wage markup shock hits the economy. In principle, the impact of increases in the wage markup on the total markup can be eliminated by a decline in the price markup. However, a decline in the price markup, $mc_t > 1$, implies $\pi_t > 1$ and thus leads to price dispersion, $s_t > 1$. Thus, whenever $\mu_t^W \neq 1$, monetary policy cannot achieve $\pi_t = 1$ and $\mu_t^T = 1$ simultaneously. Intuitively, as a reaction to a positive wage markup shock, the central bank can induce a decline in the price markup $\mu_t^P < 1$ by reducing the real interest rate below its natural rate, which is defined as the real interest rate under flexible prices. This stimulates consumption demand, so that output exceeds its natural rate and marginal cost rise above unity.[17] Such a policy stabilizes output at the expense of increased inflation. This is the familiar trade-off of a central bank in a sticky price model with monopolistic competition.[18]

When the capital stock is variable, the central bank influences the economy also by its impact on capital accumulation. By generating a decline in the price markup, $E_t \mu_{t+1}^P < 1$, it can stimulate capital accumulation through (3.25). The intuition is that endogenous capital accumulation changes the central bank's trade-off due to the interactions between capital and labor, which are substitutes in the production function: When the capital stock is variable, the wage markup shock leads to a decline in labor demand, which reduces the marginal product of capital, so that investment is reduced. This leads to a further decline in labor demand in subsequent periods. Through this mechanism, capital accumulation can magnify the fluctuations in labor demand and output caused by markup shocks. Because these fluctuations are inefficient, optimal monetary policy will aim at eliminating them. This suggests that when the capital stock is variable, optimal policy will shift emphasis toward stabilizing labor demand and output at the cost of larger fluctuations in inflation.

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[17]To be exact, it is the path of all future real interest rates that matters for consumption demand.

[18]Note that under flexible prices, policy cannot influence aggregate demand because the price markup is constant. Rather, as in Adao, Correia, and Teles (2003), sticky prices equip the central bank with an additional instrument that can be used to improve upon the flexible price allocation and stabilize output closer to its efficient steady state level.

### 3.3.3 The Ramsey problem

By analyzing Ramsey optimal policy, I assume that the central bank can credibly commit to its monetary policy and thus influence agents' expectations. I set up a Ramsey problem and derive its first-order conditions.[19] Due to the complexity of the model, it is not possible to condense the model's competitive equilibrium into a single implementability constraint. Thus, a dual Ramsey approach is applied: I derive an intertemporal budget constraint by iterating forward the household's budget constraint. This ensures that the transversality conditions for capital and state-contingent bonds hold. Further, I eliminate as many variables as possible in the model's equilibrium conditions. The Ramsey planner then maximizes household utility subject to the intertemporal budget constraint and the condensed version of the model's equilibrium conditions.

**Deriving the intertemporal budget constraint**

I will here focus on the main steps while Appendix B.2.2 contains the detailed derivation. The representative household's budget constraint can be written as

$$w_t n_t + \frac{d_{t-1}}{\pi_t} + y_t \left(1 - mc_t s_t\right) + k_t(r_t^k + 1 - \delta) = c_t + k_{t+1} + E_t r_{t,t+1}^d d_t,$$

where I use that the government collects the subsidy to workers by raising lump sum taxes, so that $y_t \left(1 - mc_t s_t\right)$ is the residual left from firm profits after paying the part of the lump-sum tax which finances the production subsidy, $\tau_t^P$. Iterating forward and using the household's first order conditions, including the transversality conditions, the intertemporal budget constraint in period t can be written as

$$c_t^{-\sigma} \left[\frac{d_{t-1}}{\pi_t} + k_t(r_t^k + 1 - \delta)\right]$$
$$= E_t \sum_{s=0}^{\infty} \beta^t c_{t+s}^{-\sigma} \left(c_{t+s} - w_{t+s} n_{t+s} - y_{t+s} \left(1 - mc_{t+s} s_{t+s}\right)\right),$$

---

[19]The solution to the resulting system of optimality conditions will be solved by second-order approximation methods using the software dynare. Appendix B.3 derives the steady state analytically, where I use that in the long run, all policies analyzed in this chapter implement first best by setting $\pi = 1$. The steady state values of the Ramsey problem's Lagrange multipliers are derived by numerical methods.

Eliminating the wage and the capital rental rate, the intertemporal budget constraint in period t becomes

$$A_t = E_t \sum_{s=0}^{\infty} \beta^s \left[ c_{t+s}^{1-\sigma} + \left( \frac{1-\alpha}{\alpha} \right) \chi n_{t+s}^{1+\eta} \mu_{t+s}^{W} - c_{t+s}^{-\sigma} \frac{IO_{t+s}}{s_{t+s}} \right], \qquad (3.26)$$

where $A_t = c_t^{-\sigma} \frac{d_{t-1}}{\pi_t} + \frac{(1-\alpha)\chi}{\alpha} n_t^{\eta+1} \mu_t^{W} + c_t^{-\sigma} k_t (1-\delta)$.

**Writing the intertemporal budget constraint recursively**

The model presented in 3.2 implies incomplete markets because households do not have access to a set of real state-contingent claims. Under incomplete markets, it is not possible to derive a single intertemporal budget constraint, as in Schmitt-Grohé and Uribe (2004a). Rather, the Ramsey planner faces an intertemporal budget constraint every period. The reason is that shocks affect the expected future surplus. Thus, an allocation satisfying a time-zero intertemporal budget constraint must not necessarily satisfy the transversality conditions at all dates under all contingencies. Therefore, it has to be guaranteed in every period that the allocation satisfies the intertemporal budget constraint. Complete markets would allow reducing the set of intertemporal budget constraints to a single constraint. The reason is that real state-contingent claims ensure that the intertemporal budget constraint holds independent of the realization of shocks, so that if a period-zero intertemporal budget constraint holds, the transversality conditions hold at all dates and across all possible states. However, under the present incomplete markets setup, it is possible to simplify the sequence of intertemporal budget constraints by writing them recursively, as in Ljungqvist and Sargent (2004), yielding

$$E_0 \sum_{t=0}^{\infty} \beta^t \theta_t \left\{ E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[ c_{t+j}^{1-\sigma} + \left( \frac{1-\alpha}{\alpha} \right) \chi n_{t+j}^{1+\eta} \mu_{t+j}^{W} - c_{t+j}^{-\sigma} \frac{IO_{t+j}}{s_{t+l}} \right] - A_t \right\}$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Theta_t \left[ c_t^{1-\sigma} + \left( \frac{1-\alpha}{\alpha} \right) \chi n_t^{1+\eta} \mu_t^{W} - c_t^{-\sigma} \frac{IO_t}{s_t} \right] - (\Theta_t - \Theta_{t-1}) A_t \right\}.$$

where $\theta_t$ denotes the multiplier on the intertemporal budget constraint and $\Theta_t = \Theta_{t-1} + \theta_t$, with $\Theta_{-1} = 0$.

**The Ramsey problem**

The system of equilibrium conditions, which are summarized in Appendix B.1.3 for convenience, can be reduced by eliminating $w_t$, $r_t^k$, and $mc_t$, yielding a system of 7 equations. The Ramsey planner maximizes household utility subject to the intertemporal budget constraint and the remaining equilibrium conditions, so that the Ramsey planner's problem reads

$$\max_{\{c,n,k,R,s,\pi,\tilde{Z},Z^1,Z^2\}} J = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right] \tag{3.27}$$

$$+ \Theta_t \left[ c_t^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_t^{1+\eta} \mu_t^W - c_t^{-\sigma} \frac{k_t^{1-\alpha} n_t^{\alpha}}{s_t} \right]$$

$$- (\Theta_t - \Theta_{t-1}) \left[ \frac{(1-\alpha)\chi}{\alpha} n_t^{\eta+1} \mu_t^W + k_t c_t^{-\sigma} (1-\delta) \right]$$

$$+ \lambda_t^1 \left[ c_t^{-\sigma} - E_t \beta c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right]$$

$$+ \lambda_t^2 \left[ c_t^{-\sigma} - E_t \beta (1-\alpha) \frac{\chi}{\alpha} \frac{n_{t+1}^{\eta+1}}{k_{t+1}} \mu_{t+1}^W - E_t \beta (1-\delta) c_{t+1}^{-\sigma} \right]$$

$$+ \lambda_t^3 \left[ \tilde{Z}_t - \frac{Z_t^1}{Z_t^2} \right]$$

$$+ \lambda_t^4 \left[ \pi_t - \left( \frac{1 + (\phi-1)\tilde{Z}_t^{1-\varepsilon}}{\phi} \right)^{\frac{1}{\varepsilon-1}} \right]$$

$$+ \lambda_t^5 \left[ s_t - (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi \pi_t^{\varepsilon} s_{t-1} \right]$$

$$+ \lambda_t^6 \left[ Z_t^1 - \frac{n_t^{1+\eta}}{s_t} \frac{\chi}{\alpha} \mu_t^W - \phi \beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1 \right]$$

$$+ \lambda_t^7 \left[ Z_t^2 - c_t^{-\sigma} \frac{k_t^{1-\alpha} n_t^{\alpha}}{s_t} - \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2 \right] .$$

The first order conditions to this problem can be found in Appendix B.2.2. These are not straightforward to interpret, so that I resort to numerical methods to solve a calibrated version of the model. In the following, the welfare measure is presented.

### 3.3.4 Welfare measure

The welfare measure is based on the representative household's expected lifetime utility, $W = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$. Following Schmitt-Grohé and Uribe (2006), welfare is measured conditional on the initial state being the deterministic steady state, which is identical across all policies I analyze, as shown in Appendix B.3. The central idea of Schmitt-Grohé and Uribe (2006) is to use the policy function of $V_t = E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}, n_{t+s})$ and to evaluate it at the initial state.[20] Let $V(x_t, \omega)$ denote the policy function of $V_t$, where $x_t$ denotes the vector of states and $\omega$ is a parameter scaling the degree of uncertainty in the economy. With the deterministic steady state being characterized by $(x_t, \omega) = (x, 0)$, what we seek is $V(x, 0)$. Because the wage markup shock is the single shock considered here, $\omega$ equals its standard deviation. As is common in the literature, welfare is measured in percentage points of consumption that would leave the representative household indifferent between the superior policy $A$ and the inferior policy $B$. Denoting this welfare measure with $\gamma$ and letting superscripts refer to a particular policy, $\gamma$ is implicitly defined by

$$W^B = E_0 \sum_{t=0}^{\infty} \beta^t u\left((1-\gamma) c_t^A, n_t^A\right).$$

Given that only the case of $\sigma = 1$ is analyzed in the calibrated model, the utility function (3.1) collapses to $u(c_t, n_t) = \ln(c_t) - \chi \frac{n_t^{1+\eta}}{1+\eta}$. I use this in the following. Schmitt-Grohé and Uribe (2006) show that $\gamma$, which is a function $\Lambda(x_t, \omega)$ of states $x_t$ and uncertainty $\omega$, can be computed accurately up to second order from the second-order approximated policy functions for $V$ around the deterministic steady state,

$$
\begin{aligned}
\gamma &\approx \Lambda(x, 0) + \Lambda_\omega(x, 0)\omega + \Lambda_{\omega\omega}(x, 0)\frac{\omega^2}{2} \\
&\Longleftrightarrow \\
\gamma &\approx (1-\beta)\left[V_{\omega\omega}^A(x, 0) - V_{\omega\omega}^B(x, 0)\right]\frac{\omega^2}{2}.
\end{aligned}
$$

---

[20]If the welfare measure were based on the average value of $V_t$ obtained from a simulation of the model, it would actually evaluate unconditional welfare and ignore the transition path involved after a policy switch. Here, such transition paths are not caused by different deterministic steady states but result from different stochastic steady states around which the model's variables fluctuate in a simulation. For a detailed description of the welfare measure, see Appendix B.2.3.

Here, $V_{\omega\omega}^A(x,0)$ and $V_{\omega\omega}^B(x,0)$ are steady state values of the second derivatives of the policy function $V$ under policy A (or B) in a second-order approximation around the model's deterministic steady state $(x,0)$,

$$V(x_t,\omega) \approx V(x,0) + V_x(x,0)(x_t - x) + V_\omega(x,0)\omega + V_{\omega x}(x,0)\omega(x_t - x)$$
$$+ \frac{1}{2}V_{xx}(x,0)(x_t - x)^2 + \frac{1}{2}V_{\omega\omega}(x,0)\omega^2.$$

In the following analysis, I evaluate the welfare loss of each policy relative to the deterministic steady state, which is identical to the social planner equilibrium. Moreover, I consider the excess welfare loss implied by simple rules, expressed as percentages of the welfare loss under Ramsey optimal policy, $\gamma^{excess} = \gamma^{simple}/\gamma^{Ramsey} - 1$. For the purpose of comparing simple rules to Ramsey optimal policy, I prefer using the latter, relative measure. The reason is that it is independent of the level of welfare cost of business cycle fluctuations implied by macroeconomic models, which are controversially discussed since Lucas (1987). Moreover, it is independent of the shock variance, which differs considerably across studies, as is discussed in the following section.

## 3.4 Impulse responses to wage markup shocks

This section analyzes impulse response to wage markup shocks, focusing on interest rate dynamics and the resolution of the central bank's trade-off under different monetary policies in models with fixed and variable capital. Further, I quantify the welfare loss of simple rules relative to those implied by Ramsey optimal policy. Because the complexity of the model does not allow for analytical results, numerical methods are applied. I calibrate the model's parameters and solve the model using a second-order approximation at the deterministic steady state. Before proceeding to the results, the calibration is summarized.

### 3.4.1 Calibration

I here confine to a description of the single shock, the wage markup shock. The estimated volatility of wage markup shocks differs considerably across studies. Galí, Gertler, and López-Salido (2007) measure an inefficiency gap, defined as $\log \frac{mrs}{mpn}$, which they decompose into a price and a wage markup. The wage markup they measure has a standard deviation of 5.4%, so that

a shock of one standard deviation increases the markup by 1.59%. In contrast Smets and Wouters (2007), who estimate a large scale New Keynesian model, estimate this figure to 24%. I prefer to choose the volatility of the shock in line with the estimates by Galí, Gertler, and López-Salido (2007) because these authors define the marginal rate of substitution and the marginal product of labor as in the model presented in this chapter, whereas Smets and Wouters (2007) assume habit formation. The wage markup shock is calibrated to match the standard deviation estimated by Galí, Gertler, and López-Salido (2007). In setting the autocorrelation to $\rho^\varsigma = 0.9$, I deviate from the high estimate of 0.95 by Galí, Gertler, and López-Salido (2007) in order not to obtain results particular to very persistent shocks. The entire calibration can be found in Appendix B.4.

### 3.4.2 Ramsey optimal policy under fixed and variable capital

I now compare Ramsey optimal policy in the model with variable capital to the model with a fixed capital stock. The latter can be found for example in Woodford (2003). Note that all variables are given in terms of percentage deviations from their steady state, with $\widehat{z}_t = 100 \left[ \log(z_t) - \log(z) \right]$.[21] Variables with superscript $n$ denote a variable's natural rate, i.e. its value under flexible prices. Further, to simplify interpretation, I define the ex ante real interest rate, $r_t = R_t / E_t \pi_{t+1}$. It is compared to the natural rate of interest $r_t^n$, which equals the real interest rate in the respective models with flexible prices. Figure 3.1 shows the responses of both the model with fixed and the one with variable capital to a wage markup shock under Ramsey optimal policy. In both models, the shock induces a rise in real wages so that labor demand and production decline. Naturally, when capital is variable, households reduce investment to smooth consumption. Thus, the consumption decline is less strong on impact but more persistent in this case.

Ramsey optimal policy in the model with fixed capital reduces the nom-

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[21]All figures show deviations from steady state in percentage points, $\hat{z}_t$, except for interest rates and markups which are given in terms of absolute deviations from steady state, $\hat{R}_t = 100 * (R_t - R)$ and can thus be interpreted as quarterly percentage points: For instance $\hat{R}_t = -0.15$ implies a 15 basis point decline in the (quarterly) interest rate, and $\hat{\mu}_t^T = 1.5$ implies that the total markup increases from its steady state of $\mu^T = 100\%$ to 101.5%.
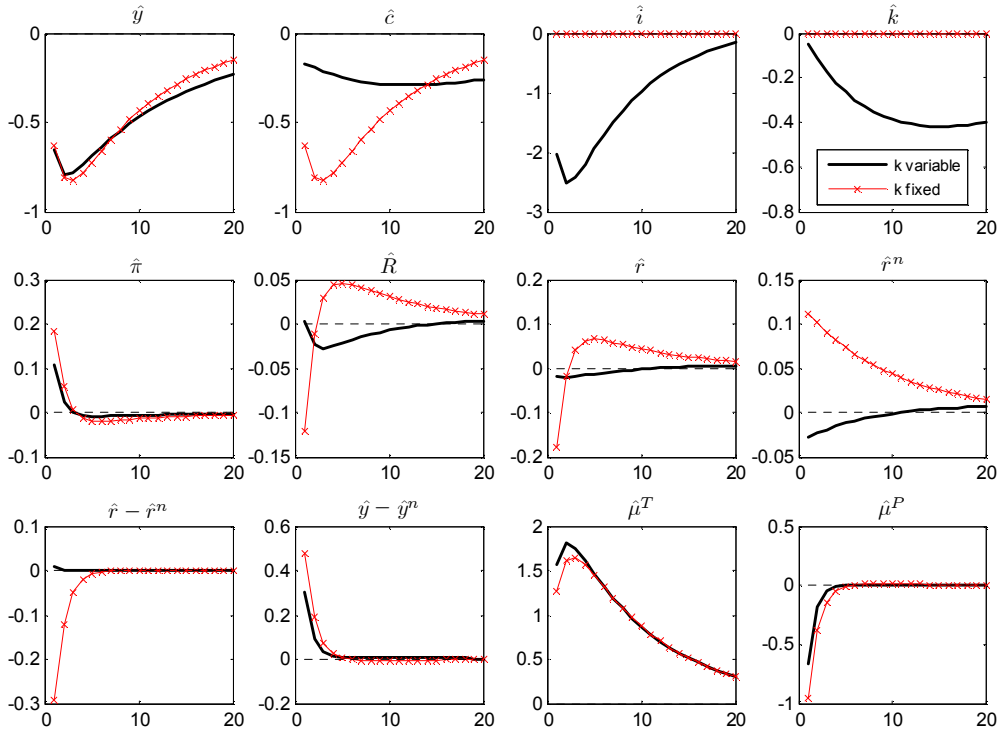
Figure 3.1: Impulse responses to a wage markup shock for fixed and variable capital

inal interest rate on impact, so that the real interest rate falls below the natural rate of interest. This policy generates an increase in output above the natural rate, thus stabilizing output closer to its steady state level. The cost of this is a temporary rise in inflation. In the terms introduced in section 3.3.1, monetary policy reduces the total markup by inducing a decline in the price markup, as shown in Figure 3.1. After two periods, monetary policy focuses on curbing inflation, which is achieved by an increase in the nominal interest rate. The reason why the central bank pursues expansionary policy only for 1-2 periods is that inflation in later periods, due to forward looking price setting, transmits to higher inflation in earlier periods. Thus, the cost of stabilizing output is lowest just after the shock hits the economy. This is the reason for the hump-shaped response of consumption and output in the model with fixed capital.

| Volatility of markups and inflation | $sd(\mu^T)$ | $sd(\mu^P)$ | $sd(\mu^W)$ | $sd(\hat{\pi})$ |
|---|---|---|---|---|
| Fixed capital stock | 0.0456 | 0.0104 | 0.0510 | 0.0020 |
| Variable capital stock | 0.0477 | 0.0068 | 0.0510 | 0.0114 |
| Note: Standard deviations refer to variables defined as $\tilde{\mu}_t = \log(\mu_t/\bar{\mu})$. | | | | |

Table 3.1: Comparison of Ramsey allocations across models

Ramsey optimal policy in the model with variable capital resolves its trade-off similarly, allowing a temporary rise in output above its natural rate at the cost of temporary inflation. Table 3.1 demonstrates this: In both models, optimal policy generates a price markup which reduces the volatility of the total markup below that of the exogenous wage markup. However, in contrast to the model with fixed capital, monetary policy reduces the nominal interest rate persistently below its steady state. This is surprising and not in line with the intuition of central banks increasing interest rates in order to curb inflation associated with a cost push shock. However, it can be explained by the response of the natural interest rate. Given the decline in the capital stock induced by consumption-smoothing households, scarcity of consumption goods is highest ten quarters after the shock. This implies that households on impact, *ceteris paribus*, increase savings so that in equilibrium the natural interest rate falls.[22] Further, observe that Ramsey optimal policy implies that the real interest rate closely follows the natural rate of interest, in contrast to the model with fixed capital. The reason is that investment is more sensitive to changes in the real interest rate than consumption. Thus, when households cannot smooth consumption by adjusting investment, the central bank has to reduce the real interest rate rather extensively for households to increase consumption demand.

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[22]To make the argument more explicit, first nominal and real interest rates follow each other closely because monetary policy allows only moderate inflation. Second, Ramsey optimal policy in the model with capital implies that the real interest rate is set close to the natural rate of interest. This implies that the nominal interest rate has to follow the natural interest rate.

### 3.4.3 Ramsey policy versus simple rules

This section compares simple interest rate rules of the form (3.19) to Ramsey optimal policy in the model with variable capital. Taylor rules, which are characterized by $w_\pi > 1$, are considered quasi-optimal, in the sense that they mimic second-best Ramsey policy closely by Schmitt-Grohé and Uribe (2007). The reason is that the Taylor principle implies that the real interest rate increases when inflation rises, so that inflation is stabilized. I consider "optimal simple rules" by optimizing numerically for the inflation reaction coefficients in the Taylor rule $w_\pi$ and $w_y$ instead of using standard parameter values.[23] I first use the impulse responses to illustrate qualitatively how Ramsey optimal policy improves upon simple rules. Second, I quantify the welfare gains achieved by Ramsey optimal policy. Figure 3.2 shows the impulse response to a wage markup shock under Ramsey optimal policy and the optimal simple rule, which is characterized by an aggressive response to inflation, $w_\pi = 8$ and no reaction to output fluctuations, $w_y = 0$.[24] The optimal simple rule puts a strong emphasis on inflation stabilization and thus achieves an allocation virtually identical to the flexible price economy. Therefore, the output gap is close to zero at all times and the real interest rate closely follows the natural interest rate. In other words, fluctuations in the price markup are negligible so that the total markup approximately equals the exogenous wage markup.

In contrast, as mentioned in the preceding section, Ramsey optimal policy increases output above its natural rate at the cost of temporary inflation. The real interest rate is set close to its natural rate but is reduced persistently from the third quarter after the shock, so that consumption demand rises compared to the simple rule. With respect to capital accumulation, the decline in the price markup triggered by Ramsey policy *ceteris paribus* increases the capital rental rate, so that investment rises according to (3.25). Because the price markup only falls for two periods,

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[23]Optimization is carried out over a grid ranging from $w_\pi = 1.1$ to $w_\pi = 8$ and $w_y = -0.5$ to $w_y = 4$. At every grid point, I solve the model by using second-order approximation methods in dynare. Then, the welfare measure is applied to find the optimal simple rule.

[24]The optimal coefficient on output is virtually, but not exactly identical to zero. Schmitt-Grohé and Uribe (2004a) also find that optimal simple rules do not react to changes in output. Further, allowing for coefficients $w_\pi > 8$ does not improve welfare substantially. For instance, at $w_\pi = 60$, welfare is improved by less than 0.001% relative to $w_\pi = 8$.
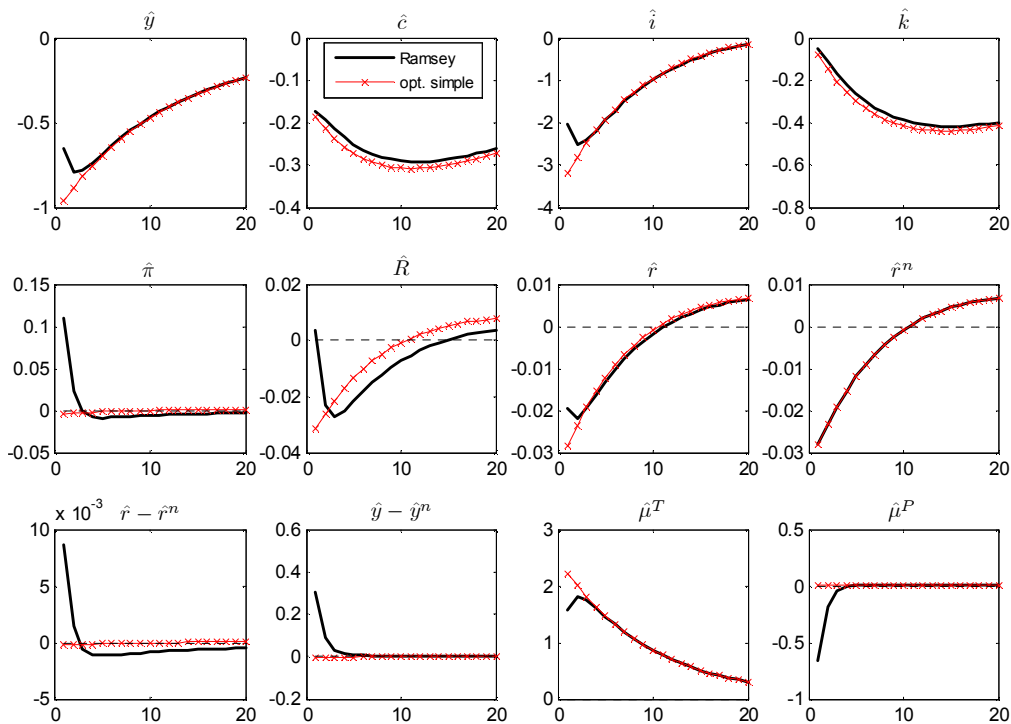
Figure 3.2: Impulse responses to a wage markup shock

households increase investment above its counterpart under the simple rule for two periods. This implies that the capital stock is better stabilized than under the simple rule, allowing households to realize a higher consumption path. Thus, by stimulating investment in the first periods, Ramsey optimal policy leads to a smoother consumption profile.

Table 3.2 demonstrates the differences between both policies quantitatively. The table demonstrates that inflation fluctuations are almost absent under the simple rule. In comparison, Ramsey policy allows higher inflation fluctuations but induces a smaller welfare loss by stabilizing the total markup, consumption, capital and output more successfully. The welfare loss implied by the optimal simple rule exceeds that of the Ramsey policy by 9.19%. This is called the excess welfare loss of the optimal simple rule.

| | Welfare loss $\gamma$ | $sd(\mu^T)$ | $sd(\mu^P)$ | $sd(\pi)$ | $sd(y)$ | $sd(k)$ | $sd(c)$ |
|---|---|---|---|---|---|---|---|
| Optimal simple rule | 0.0682 | 0.0511 | 0.0001 | 0.0085 | 0.0256 | 0.0234 | 0.0162 |
| Ramsey policy | 0.0625 | 0.0477 | 0.0068 | 0.1142 | 0.0239 | 0.0223 | 0.0155 |
| Excess welfare loss | 9.19% | | | | | | |
| Notes: The welfare loss is given in % of st.st. consumption relative to first best. Standard deviations refer to variables defined as $\tilde{\mu}_t = \log(\mu_t/\bar{\mu})$. The optimal simple rule is characterized by $w_\pi = 8$, $w_y = 0$. | | | | | | | |

Table 3.2: Welfare comparison of Ramsey policy to simple rules

## 3.5 Adjustment cost

Intuition suggests that capital accumulation is likely to entail adjustment cost: When a firm wishes to increase its capital stock, there is a cost of installing the purchased investment goods. Such cost lead the capital stock to behave in a more sluggish manner, which is considered empirically realistic. For instance, Casares and McCallum (2006) argue that adjustment cost are needed in order for models with endogenous capital accumulation to match cyclical data. The literature distinguishes two types of adjustment cost, capital adjustment cost as in Chari, Kehoe, and McGrattan (2007) and investment adjustment cost, used by Christiano, Eichenbaum, and Evans (2005). Here, both versions are considered. First, I present impulse responses to a wage markup shock for a baseline calibration of investment adjustment cost. Subsequently, I evaluate the performance of optimal simple rules relative to Ramsey policy across different calibrations for both types of adjustment cost. In order to evaluate how capital accumulation affects the trade-off of monetary policy quantitatively, the values obtained in this analysis are compared to the excess welfare cost of optimal simple rules in the model without adjustment cost.

### 3.5.1 Investment adjustment cost

This section analyzes Ramsey optimal policy and the performance of simple rules under investment adjustment cost. This specification is used in many large-scale models such as Christiano, Eichenbaum, and Evans (2005).[25] Under investment adjustment cost, the law of motion of the capital stock takes the following form

$$k_{t+1} = (1 - \delta) k_t + i_t S \left( \frac{i_t}{i_{t-1}} \right), \qquad (3.28)$$

with $S(i_t/i_{t-1}) = \frac{\kappa}{2} (i_t/i_{t-1} - 1)^2$, where $\kappa$ measures the degree of adjustment cost. Thus, households' first order conditions with respect to investment and capital read

$$q_t = \beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left[ q_{t+1}(1 - \delta) + r_{t+1}^k \right], \qquad (3.29)$$

$$1 = q_t \left[ S_t(\circ) + \frac{i_t}{i_{t-1}} S'_t(\circ) \right] - \beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} q_{t+1} \left[ \left( \frac{i_{t+1}}{i_t} \right)^2 S'_{t+1}(\circ) \right] \quad (3.30)$$

where $S_t(\circ) = S(i_t/i_{t-1})$ and $S'_t(\circ) = \frac{\partial S(i_t/i_{t-1})}{\partial (i_t/i_{t-1})}$ and $q_t$ is the value of capital.[26] Analogue to (3.9), an arbitrage freeness equation can be constructed by equating (3.29) and (3.8), which gives

$$E_t c_{t+1}^{-\sigma} \left[ (1 - \delta) q_{t+1}/q_t + r_{t+1}^k q_t^{-1} \right] = E_t c_{t+1}^{-\sigma} R_t \pi_{t+1}^{-1}.$$

Thus, in contrast to (3.9), the household has to take into account that the market value of installed capital can fluctuate. With respect to the calibration, there is considerable disagreement in the literature, which is discussed further in the appendix. I here analyze as a baseline case the value estimated by Christiano, Eichenbaum, and Evans (2005), $\kappa = 2.48$, which generates empirically reasonable impulse responses to a monetary policy shock in their model.

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[25]Beaudry and Portier (2006) and Burnside, Eichenbaum, and Fisher (2004) employ this specification as well. For a survey of models assuming investment adjustment cost, see Groth and Khan (2006).

[26]The equilibrium definition in section 3.2.4 is thus changed as follows: The two first order conditions (3.29)-(3.30) replace (3.6). Further, the aggregate resource constraint (3.20) is replaced by $(n_t^\alpha k_t^{1-\alpha})/s_t = c_t + i_t$ and the new law of motion for capital, $k_{t+1} = (1 - \delta) k_t + i_t S(i_t/i_{t-1})$, is added to the set of equilibrium conditions. Further, the variables $q_t$ and $i_t$ become part of the equilibrium sequences defined in section 3.2.4.
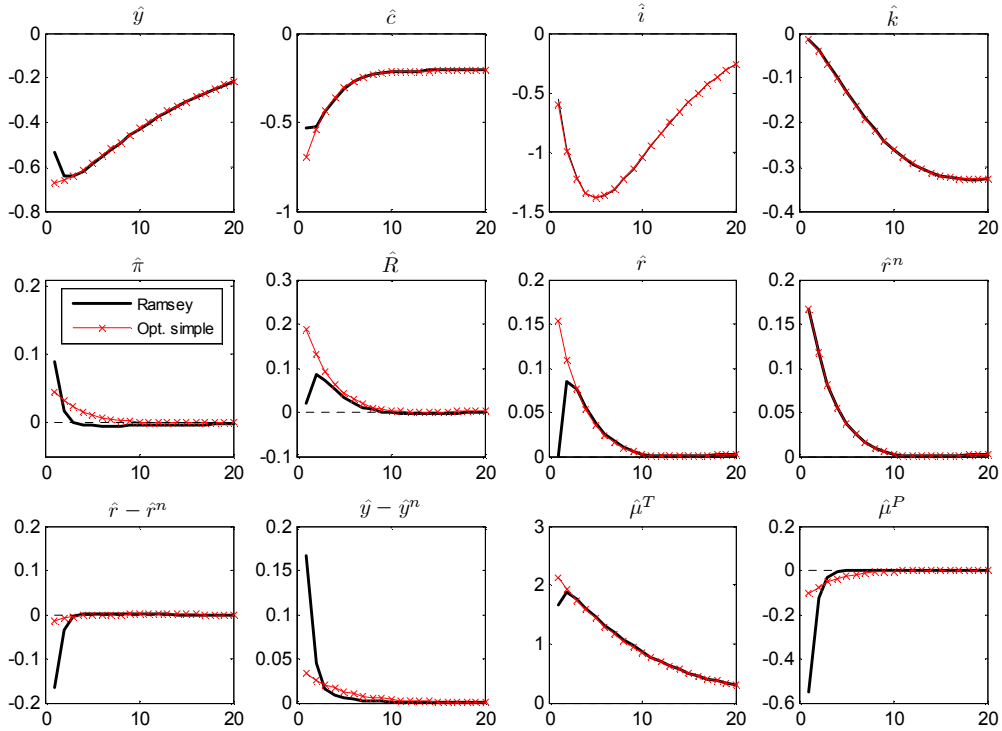
Figure 3.3: Impulse responses to a wage markup shock with positive adjustment cost

## Ramsey policy versus simple rules

Figure 3.3 shows the responses to a wage markup shock with positive investment adjustment cost, both for Ramsey policy and the optimal simple rule. The wage markup shock increases the wage and thus temporarily reduces labor demand and production. Thus, households reduce investment in order to smooth consumption. However, because adjusting investment now entails cost, consumption cannot be perfectly smoothed and drops by more than 0.5% on impact, more than twice the response in the preceding analysis. Further, observe that the nominal interest rate is increased under both the simple rule and Ramsey policy. As before, the behavior of the nominal interest rate is linked to the natural rate of interest, which increases because consumption goods are scarcest in the first period.

|  | Welfare loss $\gamma$ | $sd(\mu^T)$ | $sd(\mu^P)$ | $sd(\pi)$ | $sd(y)$ | $sd(k)$ | $sd(c)$ |
|---|---|---|---|---|---|---|---|
| Optimal simple rule | 0.0581 | 0.0497 | 0.0015 | 0.0648 | 0.0213 | 0.0177 | 0.0162 |
| Ramsey policy | 0.0565 | 0.0482 | 0.0057 | 0.0931 | 0.0210 | 0.0178 | 0.0156 |
| Excess welfare loss | 2.83% | | | | | | |
| Notes: The welfare loss is given in % of st.st. consumption relative to first best. Standard deviations refer to variables defined as $\tilde{\mu}_t = \log(\mu_t/\bar{\mu})$. The optimal simple rule is characterized by $w_\pi = 4$, $w_y = 0$. | | | | | | | |

Table 3.3: Welfare comparison of Ramsey policy to simple rules under investment adjustment cost

The optimal simple rule, which is characterized by $w_\pi = 4$ and $w_y = 0$, stabilizes output slightly above its natural level by reducing the real interest rate below its natural rate. Ramsey optimal policy reduces the real interest rate even more, inducing higher inflation and a larger output gap compared to the simple rule. However, under investment adjustment cost households use the additional output to increase consumption instead of investment. The opposite was the case in section 3.4.3, where households used the additional output to smooth consumption by increasing investment. Table 3.3 quantitatively compares the optimal simple rule to Ramsey optimal policy. The familiar pattern of Ramsey policy stabilizing the total markup by allowing a price markup re-emerges. The benefit of this policy are reduced fluctuations in consumption, while the capital stock is even slightly more volatile under Ramsey optimal policy. The excess welfare loss of the optimal simple rule is 2.83%. Thus, active policy achieves an allocation close to the Ramsey policy under positive investment adjustment cost.

With respect to the trade-off of monetary policy, the improved performance of simple rules under adjustment cost suggests that stabilizing inflation receives a higher weight in the central bank's trade-off when adjustment cost are present.[27] The reason is that investment adjustment cost

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[27]In principle, the excess welfare cost of the optimal simple rule could also decline

prevent the wage markup shock to lead to large reductions in investment, so that optimal policy becomes less concerned with stabilizing investment and focuses on inflation stabilization. However, when capital can be accumulated at no cost, it becomes more important to stabilize labour demand and output by stimulating investment. This view is also consistent with the decline in the welfare loss of both second-best and simple rule policy when adjustment cost are introduced. Notably, welfare improves although adjustment cost are social costs. The reason was already given above: Adjustment cost mitigate the impact of the wage markup shock on investment and thus prevent a reduction in investment to magnify the inefficient fluctuations in labor demand caused by the shock. When adjustment cost are absent, the central bank should prevent the decline in investment by inducing a procyclical price markup.

### 3.5.2 Robustness

This section analyzes the robustness of the previous results regarding the performance of simple policy rules with respect to the calibration and specification of adjustment cost. I here additionally consider capital adjustment cost, which imply that the capital stock evolves according to $k_{t+1} = (1 - \delta)k_t + i_t - F(i_t/k_t) k_t$, where $F(i_t/k_t) = \frac{\psi}{2}(i_t/k_t - \delta)^2$ and $\psi$ measures the extent of adjustment cost. Thus, changes in the level of the capital stock are costly. Under this specification, the households' first order conditions are given by

$$q_t = \beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left\{ r_{t+1}^k + q_{t+1} \left[ (1 - \delta) - F_{t+1}(\circ) + \frac{i_{t+1}}{k_{t+1}} F_{t+1}'(\circ) \right] \right\} \tag{3.31}$$

$$q_t = \left[ 1 - F_t'(\circ) \right]^{-1}, \tag{3.32}$$

where $F_t(\circ) = F(i_t/k_t)$ and $F_t'(\circ) = \frac{\partial F(i_t/k_t)}{\partial (i_t/k_t)}$ and $q_t$ represents the value of capital relative to the consumption good, as before.[28]

Further, acknowledging that empirical estimates of adjustment cost are very heterogeneous, I repeat the above welfare evaluation of simple rules

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because the simple rule achieves a better stabilization of the total markup compared to the model with zero adjustment cost. However, even the excess welfare loss of a simple rule characterized by $\omega_\pi = 8$ and $w_y = 0$, which leads to near complete inflation stabilization, amounts to 3.9% which is well below that welfare cost of an identically

| Excess welfare loss of the optimal simple rule | |
| --- | --- |
| No adjustment cost | 9.19% |
| Low capital adjustment cost ($\psi = 6.7$) | 7.23% |
| High capital adjustment cost ($\psi = 100$) | 5.57% |
| Low investment adjustment cost ($\kappa = 0.17$) | 2.84% |
| High investment adjustment cost ($\kappa = 2.48$) | 2.83% |
| Notes: The excess welfare loss is given by $\gamma^{\text{simple}}/\gamma^{\text{Ramsey}}-1$. The optimal simple rules are characterized by $w_y = 0$ for all cases and $w_\pi = 8$ under both calibrations of capital adjustment cost and are given by $w_\pi = 3$ ($w_\pi = 4$) under low (high) investment adjustment cost. | |

Table 3.4: Welfare evaluation of simple policy rules for varying adjustment cost

under two alternative calibrations, a scenario of high adjustment cost and one implying low adjustment cost. Under both specifications of adjustment cost, the high cost scenario is calibrated to an elasticity of investment with respect to Tobin's q of $\eta_{i,q} = 0.4$, which is the value estimated by Christiano, Eichenbaum, and Evans (2005) from aggregate data. This estimate is also used in section 3.5.1. A second, low adjustment cost scenario is calibrated to the results of Groth and Khan (2006), who estimate industry-specific adjustment cost and show that these imply an elasticity of aggregate investment with respect to the market value of capital amounting to $\eta_{i,q} = 6$. More details on the calibration of adjustment cost can be found

aggressive rule when adjustment cost are absent (see Table 3.2).

[28]The equilibrium definition in (3.2.4) is thus changed as follows: (3.31) and (3.32) replace (3.6). Further, the aggregate resource constraint (3.20) is replaced by $\left(n_t^\alpha k_t^{1-\alpha}\right)/s_t = c_t + i_t$ and the new law of motion for capital, $k_{t+1} = (1 - \delta)k_t + i_t - F\left(i_t/k_t\right)k_t$ is added to the set of equilibrium conditions. Further, the variables $q_t$ and $i_t$ become part of the equilibrium sequences defined in (3.2.4).

in Appendix B.4.

Table 3.4 shows the excess welfare loss of the optimal simple rule (relative to Ramsey policy) for each scenario. In comparison to the case where adjustment cost are absent, introducing positive adjustment cost lowers the excess welfare loss of the optimal simple rule across all specifications and calibrations. Further, the performance of the optimal simple rule improves with the degree of adjustment cost, although the effect is small in case of investment adjustment cost. Both observations confirm the previous result that it becomes the more desirable to deviate from simple rules and stabilize output by allowing temporary inflation, the more flexible the capital stock can be adjusted. Comparing the two specifications, the performance of optimal simple rules worsens under capital adjustment cost relative to investment adjustment cost. In the scenario of low capital adjustment cost, the excess welfare loss amounts to 7.2%, implying that second-best policy can improve considerably upon the optimal simple rule. Thus, simple rules imitate optimal policy more closely under investment adjustment cost, which is the preferred specification in quantitative macroeconomic models, such as Smets and Wouters (2007).

## 3.6   Conclusion

This chapter analyzes how endogenous capital formation affects interest rate dynamics, the central bank's trade-off and the performance of simple policy rules. I introduce a trade-off in form of a wage markup shock calibrated to match the volatility of the inefficiency gap measured by Galí, Gertler, and López-Salido (2007). This shock represents a distortion of the competitive equilibrium and is inefficient in the sense that it leaves the first-best allocation unaffected.

I obtain two main results. First, introducing capital accumulation does not change optimal monetary policy in the following sense: Independent of whether the capital stock is fixed or variable, optimal policy induces similar patterns of inflation and the output gap. In both cases, it is optimal to allow temporary inflation. This creates a procyclical price markup which stabilizes output and mitigates the impact of the wage markup shock on the total markup. However, endogenous capital formation changes the instrument path required to achieve this pattern. The reason is that, in absence of adjustment cost, the natural rate of interest falls for ten quarters. Thus,

in response to a wage markup shock, optimal policy reduces the nominal interest rate for ten quarters so as to lift output above its inefficiently low natural rate. The second result is that the excess welfare loss of simple rules is the higher the more flexible the capital stock can be adjusted. When adjustment cost are absent, the welfare cost of the simple rule exceed those of the Ramsey policy by 9.2%. With positive adjustment cost, these range between 2.8% and 7.2%. The reason is that, by construction, Taylor rules focus on stabilizing inflation. However, when capital can be accumulated at no cost, the wage markup shock reduces labor demand, triggering a decline in investment which amplifies reductions in worked hours and output in subsequent periods. Monetary policy can and should prevent this by inducing a decline in the price markup, which increases the capital rental rate and thereby stimulates investment.

The analysis in this chapter implies that the degree of adjustment cost contains important information for monetary policy. First, the specification and calibration of adjustment cost have an influence on the trade-off of monetary policy and the performance of simple policy rules. Second, adjustment cost affect the behavior of the natural interest rate: In the polar cases of a fixed (flexible) capital stock, the natural interest rate rises (falls) in response to a cost push shock. Monetary policy decisions based on an estimate of the natural rate of interest are thus not robust to large variations in adjustment cost. This supports the results by Orphanides and Williams (2002), who document that the welfare cost of ignoring uncertainty in estimates of the natural rate of interest can be large, providing an argument against the practical usefulness of such estimates in monetary policy. Due to both the importance of adjustment cost for optimal policy and the natural rate of interest, further research on the nature and degree of adjustment cost, in particular in light of the diverging estimates implied by the empirical literature, is warranted.

# Chapter 4

# Macroeconomic Effects of Unconventional Monetary Policy

Coauthor: Prof. Dr. Andreas Schabert

## 4.1 Introduction

Central banks in many industrialized countries have responded to the recent financial crisis with unconventional monetary policy measures. By introducing various newly created lending facilities as well as direct asset purchases, the Federal Reserve for instance doubled its balance sheet in the three months after the climax of the crisis in September 2008. This policy of monetary easing has been aimed at ensuring the functioning of the interbank market and to stabilize stressed credit markets (see Yellen (2009)).[29] However, unconventional monetary policies have been implemented with only little theoretical guidance available. In addition to the open question of the short-run effectiveness of these measures, their medium- and long-run impact on inflation has been subject to debate. While many observers were concerned with a debt-deflation spiral, others were more anxious about the effects of a soaring monetary base on price stability. This chapter aims

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[29]For instance, reacting to the markets' flight to liquidity, the Federal Reserve set up the Treasury Securities Lending Facility (TSLF), which provides Treasury securities in exchange for other securities such as mortgage-backed securities and commercial paper.

to contribute to these questions by analyzing the long- and short-run effects of unconventional monetary policy in a quantitative dynamic general equilibrium model.

Bernanke, Reinhart, and Sack (2004) summarize the literature on unconventional monetary policy and distinguish three types of policies available at the zero lower bound: shaping expectations, quantitative easing, and qualitative easing. Shaping expectations implies reducing expected future (short-term) nominal interest rates or increasing expected future inflation with the goal of reducing current real interest rates and thereby stimulating current spending. Quantitative easing (QnE) involves the purchase of securities such as (long-term) government bonds with central bank reserves. If money and government bonds are imperfect substitutes, this should lead to downward pressure on long-term interest rates. Qualitative easing (QlE) refers to changes in the composition of the central bank's balance sheet without creating additional reserves. The idea is that by purchasing and selling assets with different characteristics or maturities, this policy should influence asset prices.

The consensus view in macroeconomic theory is that the only reliable monetary policy option at the zero lower bound is to shape the expectations, whereas the effects of quantitative or qualitative easing are until now not well understood (see "related literature"). As emphasized by Walsh (2009) there is no robust quantitative evidence on the effects of unconventional open market operations. This chapter aims to fill this gap by providing an analysis of unconventional policy measures (apart from shaping expectations) in a macroeconomic model. We explicitly account for the collateral requirements in open market operations and the role of assets' liquidity, which distinguishing effects from quantitative and qualitative easing. Thus, our analysis of unconventional monetary policy focuses on the role of liquidity provision and liquidity premiums, which accords to Buiter's (2008) argument that central banks in crisis times should provide the "public good of liquidity in the amounts required to eliminate (most of) the liquidity risk premiums at the maturities that matter".

Our model is based on Reynard and Schabert (2009), where multiple assets are considered that differ with regard to their ability to serve as collateral in open market operations. Private agents rely on money for goods market purchases, while money is supplied only in exchange for eligible securities - in particular for short-term government bonds. This requirement

creates an equilibrium spread between the interest rate on non-eligible and eligible assets, i.e. a liquidity premium. It implies that interest rates on non-eligible securities are positive, even if the policy rate is at the zero lower bound. This property accords to the empirical observations that interest rates on many assets, even those not associated with default risk, are non-zero even if the policy rate hits the lower bound. It further facilitates the analysis of monetary policy options at the lower bound, since the opportunity cost of money holdings remain positive, leading to a well-defined money demand.[30]

We augment Reynard and Schabert's (2009) model by inducing firms to demand loans to finance production, via a working capital assumption. Due to the associated borrowing cost, higher loan rates increase marginal cost and thereby exert downward pressure on production. As long as loans, which are supplied by private agents, are not eligible in open market operations, the loan rate exceed the interest rate on eligible government bonds and the policy rate. By declaring loans as eligible for open market operations, the central bank can stimulate the economy via two effects: 1.) a QnE-effect via an increase in the total amount of eligible securities, and 2.) a QlE-effect via a decrease in the liquidity premium and in the loan rate. By increasing the fraction of liquid (eligible) assets the central bank eases (the QnE-effects) the households' access to cash and their willingness to spend, which acts like a conventional money injection (above the ZLB). Moreover, firms' borrowing cost are reduced (the QlE-effects), which stimulates the economy via a standard cost channel.[31]

Our main results can be summarized as follows. In the *long-run*, monetary policy is non-neutral due to a standard inflation tax and due to its impact on firms' borrowing cost. In particular, the central bank can in-

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[30]The model further avoids running into indeterminacy problems even when the policy rate is pegged (see Reynard and Schabert, 2009), which simplifies the analysis at the ZLB.

[31]Throughout the analysis, we disregard default risk and focus on liquidity premiums, for which empirical evidence suggests a significant magnitude. The corporate bond credit spread puzzle (see Christensen (2008)) refers to the empirical observation that interest rate spreads between corporate and government bonds can only partly be attributed to default risk. Longstaff, Mithal, and Neis (2005), for example, attribute around half of the 105 bp spread of *AAA* corporate bonds to default risk. The other half of the spread is shown to be highly correlated to various indicators of market liquidity. Other studies attribute even lower shares of this spread to default risk, see Collin-Dufresne (2001).

crease output and consumption in the long-run if it reduces the long-run loan rate by raising the share of eligible loans or by lowering the policy rate. In the *short-run*, an unconventional monetary policy can exert substantial QnE-effects, but only small QlE-effects. We further offer a tentative discussion of how our model could be applied to the recent financial crisis.

The chapter is organized as follows. Section 4.2 presents the model. In section 4.3, we examine the long-run implications of monetary policy. In section 4.4, we present the short-run effects of conventional and unconventional monetary policy measures both in a qualitative and a quantitative way. Section 4.5 concludes.

### Related literature

There is a large literature on monetary policy options at the zero lower bound.[32] Most of them advocate the possibility of providing monetary stimulus at the zero lower bound through *shaping interest-rate expectations*. The basic idea is that a monetary expansion, if perceived as permanent, can stimulate the economy by creating expected inflation and reducing the real rate of interest (see Krugman (1998)). Applying a microfounded macro model, Eggertsson and Woodford (2003, 2004) show that a commitment to keep nominal interest rates low in future can indeed provide an effective way of escaping a liquidity trap. Jung, Teranishi, and Watanabe (2005) as well as Eggertsson (2006) derive optimal policy under the non-negativity constraint for the interest rate and obtain the same conclusion. Levin, López-Salido, Nelson, and Yun (2010) examine large, persistent shocks and find that a policy relying on shaping interest rate expectations might not be sufficient to stabilize the economy. Auerbach and Obstfeld (2005) analyze open market purchases of government bonds and find that this policy can exert beneficial effects by reducing government debt service and the burden associated with distortionary taxes. Under sticky prices, higher inflation associated with the money injection further produces a temporary boom, which lifts the economy out of the liquidity trap. Thus, this effect is

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[32] A different strand of the literature not reviewed here examines how a liquidity trap can be prevented. For example, Benhabib, Schmitt-Grohe, and Uribe (2002) and Evans, Guse, and Honkapohja (2008) demonstrate the possibility of deflationary equilibria caused by arbitrary changes in expectations, and consider monetary and fiscal policies to prevent such equilibria. Earlier contributions by Fuhrer and Madigan (1997) and Summers (1991) demonstrate that an increase in the central bank's inflation target can reduce the likelihood of being constrained by the ZLB. The possibility of negative nominal interest rates is further discussed in Goodfriend (2000) and Bassetto (2004).

rather caused by the credible commitment to a permanent increase in the monetary base than by quantitative easing. Svensson (2001) and McCallum (2000) recommend using the exchange rate channel of monetary policy to deal with the liquidity trap. They argue that a currency devaluation and the corresponding expectation of a real appreciation will reduce the long-run real interest rate according to uncovered interest rate parity.

The common view on *quantitative easing* (QnE) is that lump-sum injections of money such as helicopter drops are ineffective at the zero lower bound (see Krugman (1998) and McCallum (2006)). The reason is that these authors consider single interest rate frameworks. Therfore, households are fully satiated with money once the policy rate reaches the zero lower bound. In case of a helicopter drop, and when Ricardian equivalence applies, households will just hold the additional money injected to pay for the associated future tax burden. Assuming positive population growth (such that Ricardian equivalence fails), Ireland (2005) shows that lump-sum money injections create a positive wealth effect for the current population. Goodfriend (2000) describes how open market purchases of long bonds could stimulate spending via the portfolio-balance and credit channels. When assets cannot equally be converted into cash, and liquidity services of assets depend negatively on the aggregate stock of monetary assets, open market purchases of long-term bonds can cause households to shift their portfolio toward less liquid assets such as durables or physical capital. This raises asset prices and lowers long-term interest rates, providing stimulus to the economy. A rise in asset prices can have further stimulating effects by reducing the external finance premium, thereby causing lower borrowing rates. Coenen and Wieland (2003) consider a similar portfolio-balance effect and assume that the supply of base money has an impact on the exchange rate aside its effect on interest rates. Purchasing foreign currency with domestic money then produces an exchange rate depreciation, which will stimulate export demand and induce inflation, thereby lowering the real interest rate.

Spurred by the recent events in financial markets, which have led interest rate spreads of many kinds peak, a literature on qualitative easing (QlE) is now developing. Freedman, Johnson, Kamenik, and Laxton (2009) analyze the impact of reductions in risk premiums in the IMF model GPM, where the volume of the central bank's actions is related to the size of changes in risk premiums. We are aware of three micro-founded studies on

unconventional monetary policy, which mainly focus on credit frictions.[33] The first study is Reis (2010), who analyzes various forms of credit policy such as equity injections into banks and purchases of securities. He finds that providing firms that trade asset-backed securities with loans is the most effective way to intervene in financial markets. The second one is Gertler and Karadi (2010), who analyze credit policies in a financial accelerator model where financial intermediaries need collateral in order to attract deposits. When financial institutions need to deleverage due to a decline in asset prices, central bank interventions such as purchasing assets − i.e. borrowing directly to firms − can be a powerful tool. Under moderate efficiency cost of interventions, asset purchases can improve welfare substantially. When efficiency cost are high, the central bank should rather provide equity injections, exploiting the information advantage of financial intermediaries, while bearing the cost implied by agency problems. Cúrdia and Woodford (2010) analyze unconventional monetary policy in a model with imperfect financial intermediation and find that credit policy (direct central bank lending to private agents) can ease financial market distress. However, neither the mere size of credit spread nor the fact that the policy rate has reached the lower bound are sufficient conditions for such a policy to yield welfare gains. They further identify the conditions under which unconventional monetary policy is effective and conclude that these are not likely to hold.

The paper by Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010) is most closely related to our setup. They extend the model by Kiyotaki and Moore (2008) and assume that entrepreneurs are constrained by a resaleability constraint preventing them from selling their equity holdings. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010) embed this friction into a model along the lines of Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005) and show that a negative shock to resaleability, calibrated to match the movement in the share of liquid assets in the U.S. in late 2008, can generate a deep recession. When the central bank intervenes by purchasing illiquid assets in exchange for liquid Treasury bonds, this reduces entrepreneurs' borrowing cost and stimulates the economy. Given an intervention of 1 trillion USD, their model predicts an output decline similar to the one observed in the United States. This chapter differs from this setup in various respects. Most of all, we explic-

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[33]Thus, our paper provides an analysis that is complementary to these studies.

itly model the asset exchange in the form of open market operations, while Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010) assume that the central bank intervention exchanges liquid for illiquid assets at market prices. This chapter further addresses the effects of QnE beside those of QlE.

## 4.2 The model

The model is based on Reynard and Schabert (2009), where money is supplied by the central bank in exchange for eligible securities. We augment their setup by introducing a standard working capital constraint, which induces identical and perfectly competitive intermediate goods producing firms to demand loans. To keep the exposition simple, we assume that these firms are owned by households and that their problem is static, such that they borrow funds by issuing debt in form of intra-period loans.

Usually, the central bank only accepts government bonds in exchange for money in open market operations. However, it might also declare other assets, i.e. corporate debt, as eligible. To facilitate the analysis of such a quantitative (and qualitative) easing policy, we assume that it randomly selects a share $\kappa_t$ of loans issued by identical firms to be eligible. This selection is made not before the market for federal funds is opened, such that at the time of lending all intra-period loans exhibit the same price.

### 4.2.1 Timing of events

Households enter the period with money, government bonds, and household debt, $M_{i,t-1}^H + B_{i,t-1} + D_{i,t-1}$, and with a time-invariant time endowment. They supply labor to intermediate goods producing firms. These firms do not hold any financial wealth. At the beginning of the period productivity shocks and shocks to the monetary policy rate are realized. Further, monetary policy sets its policy instrument $\kappa_t$.

1. The labor market opens, where a perfectly competitive intermediate goods producing firm $j$ hires labor $n_{j,t}$ at the real wage rate $w_t$. In order to produce it has to pay workers a fraction $\theta$ of their payroll in advance. Since it does not hold any financial wealth, it has to borrow liquid funds. The firm $j$ thus faces the liquidity constraint

$$L_{j,t}/R_t^L \geq \theta P_t w_t n_{j,t}, \tag{4.1}$$

where $L_{j,t}/R_t^L$ denotes the amount received by the borrowing firm and $L_{j,t}$ the amount to be repaid at the end of the period. Hence, $R_t^L$ is the risk-free interest rate on intra-period loans. Further, $P_t$ denotes a price index of consumption goods, see section 4.2.2.

2. A households $i$ can lend funds to the firms using money carried over from the previous period $M_{i,t-1}^H \geq L_{i,t}/R_t^L$, where $\int L_{i,t} di = \int L_{j,t} dj$. The cash received by firms is used by intermediate goods producing firms to pay out the fraction $\theta$ of the wage bill. Cash holdings of a household $i$ then equal $M_{i,t-1}^H - \left(L_{i,t}/R_t^L\right) + \theta P_t w_t n_{i,t}$.

3. The money market opens and households can exchange short-term government bonds for money at the discount rate $R_t^m$. Usually, only government bonds are eligible for open market operations. In addition, the central bank may decide to allow a fraction $\kappa_t$ of corporate debt to be eligible for open market trades. While bonds can be traded in repurchase agreements and outright, intra-period loans can only be used in repurchase agreements. The total amount of money $I_{i,t}$ household $i$ can get in open market operations is therefore constrained by

$$I_{i,t} \leq (B_{i,t-1}/R_t^m) + \kappa_t \left(L_{i,t}/R_t^m\right). \tag{4.2}$$

A household's holdings of money, bonds and corporate debt are now $M_{i,t-1}^H + I_{i,t} - L_{i,t}/R_t^L + \theta P_t w_t n_{i,t}$, $B_{i,t-1} - B_{i,t}^{CB}$, and $L_{i,t} - L_{i,t}^R$, where $\Delta B_{i,t}^c$ are bonds received by the central bank and $L_{i,t}^R$ are loans held under repos, $I_{i,t} = \left(\Delta B_{i,t}^c/R_t^m\right) + \kappa_t \left(L_{i,t}^R/R_t^m\right)$.[34]

4. Households enter the goods market where consumption goods $c_{i,t}$ can be bought with money only. Thus, household $i$ faces the cash-in-advance constraint

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H - \left(L_{i,t}/R_t^L\right) + \theta P_t w_t n_{i,t}. \tag{4.3}$$

After goods are sold, households receive dividends $P_t \delta_{i,t}$ and the remaining fraction of wage income $(1-\theta) P_t w_t n_{i,t}$ in cash.

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[34]In the analysis of qualitative easing, we introduce a factor $\kappa_t^B$, so that the open market constraint reads $I_{i,t} \leq (\kappa_t^B B_{i,t-1} + \kappa_t^L L_{i,t})/R_t^m$. This is necessary to conduct experiments where the central bank holds constant the total volume of collateral eligible for open market operations. Note that, apart from affecing the interest rate on government bonds (and the monetary base), $\kappa_t^B$ does not affect the equilibrium allocation.

5. Before the asset market opens, repurchase agreements are settled, i.e. household $i$ uses money to buy back government bonds $B_{i,t}^R$ and corporate debt $L_{i,t}^R$ from the central bank. Then, loans are repaid such that households' money holdings before entering the asset market are given by

$$\tilde{M}_{i,t} = M_{i,t-1}^H + I_{i,t} - \left(L_{i,t}/R_t^L\right) + P_t w_t n_{i,t} - P_t c_{i,t} + P_t \delta_{i,t} - M_{i,t}^R + L_{i,t},$$
$$\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B_{i,t}^c + B_{i,t}^R,$$

where $M_{i,t}^R = B_{i,t}^R + L_{i,t}^R$ denotes a households' total participation in repurchase operations.

6. In the asset market, the government issues new bonds and households receive payoffs from maturing assets as well as government transfers $\tau_{i,t}$. They can carry wealth into the next period by purchasing government bonds, state-contingent claims or by holding money. Thus, their asset market constraint reads

$$(B_{i,t}/R_t) + E_t[\varphi_{t,t+1} D_{i,t}] + M_{i,t}^H \leq \widetilde{B}_{i,t} + D_{i,t-1} + \widetilde{M}_{i,t} + P_t \tau_{i,t}, \quad (4.4)$$

where $\varphi_{t,t+1}$ denotes the stochastic discount factor and $R_t$ the interest rate on government bonds. Further, the central bank reinvests its payoffs from maturing bonds in new bonds and does not change money supply. Since money cannot be issued by the private sector, $\int \widetilde{M}_{i,t} di = \int M_{i,t}^H di$ holds.

### 4.2.2 Firms

There are intermediate firms who are perfectly competitive and sell their goods $y_{j,t}$ to monopolistically competitive retailers. These sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology.

**Intermediate Firms**  There is a continuum of intermediate goods producing firms indexed with $j \in [0,1]$. They are perfectly competitive and owned by the households. In each period a firm $j$ distributes its profits to the owners and rents the production factors, specifically, it hires labor $n_{j,t}$. We assume that a fraction $\theta \geq 1/2$ of the wage has to be paid in advance, i.e. before all its goods are sold. For this it borrows cash $L_{j,t}$ from

households at the price $1/R_t^L$ and repays the loan at the end of the period. Hence, firm $j$ faces the working capital constraint (4.1).

It then produces an intermediate good according to the production function $IO_{j,t} = n_{j,t}^\alpha$ and sells it to retailers who pay them in cash (after these have received the households' money for goods) the price. With these revenues, it repays intra-period loans and finances the remaining wages $(1 - \theta) w_t n_{j,t}$. The problem of the firm $j$ then reads

$$\max \left( Z_t/P_t \right) n_{j,t}^\alpha - w_t n_{j,t} - l_{j,t} \left( R_t^L - 1 \right) / R_t^L, \quad s.t. \ (4.1),$$

where $l_{j,t} = L_{j,t}/P_t$ and $Z_t$ is the sales price of intermediate firms. The first order conditions to this problem are given by

$$\left( Z_t/P_t \right) \alpha n_{j,t}^{\alpha-1} = w_t + \mu_{j,t} \theta w_t,$$
$$R_t^L - 1 = \mu_{j,t},$$

and $\mu_{j,t}[(l_{j,t}/R_t^L) - \theta w_t n_{j,t}] = 0$ and $\mu_{j,t} \geq 0$. We restrict our attention to the case, where intermediate goods producing firms borrow not more then required to pay the fraction of wages $\theta w_t n_{j,t}$, which will be satisfied throughout the analysis (see below): $R_t^L > 1 \Rightarrow \mu_{j,t} > 0$. Ruling out $R_t^L < 1$, the following conditions determine intermediate firms' labor demand as well as the volume of debt they issue

$$\left( Z_t/P_t \right) \alpha n_{j,t}^{\alpha-1} = w_t \left[ 1 + \theta \left( R_t^L - 1 \right) \right], \tag{4.5}$$
$$l_{j,t}/R_t^L = \theta w_t n_{j,t}. \tag{4.6}$$

The first condition states that firms demand labor up to the point where marginal revenues equal marginal cost. The working capital constraint distorts labor demand since $\theta > 0$.

**Retailers** Retail firms buy goods $IO_{j,t}$ from the intermediate firms at the price $Z_t$. Retailer $k$ relabels this good to $y_{k,t}$ and sells it to bundlers at the price $P_{k,t}$. With Dixit-Stiglitz technology the bundlers' demand function is given by $y_{k,t} = \left( P_{k,t}/P_t \right)^{-\varepsilon} y_t$. The retailer's only problem is to set his price subject to the Calvo constraint to maximize his profits, which yields a standard Phillips curve. Under Calvo type sticky prices, the first order condition for a retailer yields (where we use that $Z_t/P_t$ are real marginal cost, $mc_t$):

$$P_{k,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_s \left( \phi \beta \right)^s c_{t+s}^{-\sigma} y_{t+s} P_{t+s}^\varepsilon mc_{t+s}}{\sum_s \left( \phi \beta \right)^s c_{t+s}^{-\sigma} y_{t+s} P_{t+s}^{\varepsilon-1}}.$$

Defining $\tilde{Z}_t = P_{k,t}/P_t$ and writing both the denominator and numerator in a recursive way, this can be expressed as $\tilde{Z}_t = \frac{\varepsilon}{\varepsilon-1} Z_t^1/Z_t^2$, where $Z_t^1 = c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1$ and $Z_t^2 = c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2$. We define the price index as the household's consumption expenditures for a particular bundle at given prices, $P_t y_t = \int_0^1 P_{kt} y_{kt} dk$. Using the demand constraint $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$, we obtain a law of motion for inflation depending on the firms' pricing decision $\tilde{Z}_t$, $1 = (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$.

Further, we have to track price dispersion and its impact on output. Intermediate output is produced efficiently, leading to $IO_t = n_t^{\alpha}$ as every intermediate firm hires an identical amount of labor. However, there is a production inefficiency due to price dispersion across retailers. The market clearing condition in the intermediate goods market, $IO_t = \int_0^1 y_{k,t} dk$, gives $n_t^{\alpha} = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} y_t dk \Leftrightarrow$

$$y_t = n_t^{\alpha}/s_t,$$

where $s_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ and $s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}$ (see Schmitt-Grohé and Uribe (2004b)) given $s_{-1}$.

### 4.2.3 Households

There is a continuum of infinitely lived households indexed with $j \in [0,1]$. Households have identical asset endowments and preferences. Household $j$ maximizes the expected sum of a discounted stream of instantaneous utilities

$$E\sum_{t=0}^{\infty} \beta^t \left[ c_{i,t}^{1-\sigma} (1-\sigma)^{-1} - \chi n_{i,t}^{1+\eta} (1+\eta)^{-1} \right], \qquad (4.7)$$

where $E_0$ is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor.

A household $i$ is initially endowed with money $M_{i,-1}^H$, government bonds $B_{i,-1}$, and privately issued debt $D_{i,-1}$. In each period it supplies labor, consumes a final good, lends out funds to intermediate goods producing firms, trades assets with the central bank in open market operations, and can reinvest in assets. At the beginning of the period it might lend out cash to firms at the price $1/R_t^L$, using money carried over from the previous period, $M_{i,t-1}^H \geq L_{i,t}/R_t^L$. Before it enters the goods market where it needs money as the only accepted means of payment, it can get additional money in open market operations in exchange for government bonds and, eventually,

a fraction of intra-period loans. The decision whether and which loans are eligible is made before the market for federal funds is opened. The household faces the open market constraint (4.2). Throughout this chapter, we will restrict our attention to the case where money holdings will be large enough to ensure $M_{i,t-1}^H > L_{i,t}/R_t^L$, while at the same time the central bank will not withdraw money from the private sector $I_{i,t} \geq 0$, by choosing a suited relation between money supplied under repurchase agreements and under outright sales/purchases (see Appendix C.2).

In the goods market, household $i$ can use the fraction $\theta$ of its wage, money holdings net of lending, and additional cash from current period open market operations for its consumption expenditures (see 4.3). Before the asset market opens it receives repayments from intra-period loans. In the asset market, it further receives pay-offs from maturing assets, it can buy bonds from the government and trade all assets with other households, and it can borrow and lend using a full set of nominally state contingent claims. Dividing the period $t$ price of one unit of nominal wealth in a particular state of period $t+1$ by the period $t$ probability of that state gives the stochastic discount factor $\varphi_{t,t+1}$. The period $t$ price of a payoff $D_{i,t}$ in period $t+1$ is then given by $E_t[\varphi_{t,t+1}D_{i,t}]$. Substituting out the stock of bonds and money held before the asset market opens, $\widetilde{B}_{i,t}$ and $\widetilde{M}_{i,t}$, in (4.4), the asset market constraint of household $i$ can be written as

$$M_{i,t-1}^H + B_{i,t-1} + \frac{L_{i,t}}{R_t^L}\left(R_t^L - 1\right) + P_t w_t n_{i,t} + D_{i,t-1} + P_t \delta_{i,t} + P_t \tau_{i,t} \quad (4.8)$$
$$\leq M_{i,t}^H + \frac{B_{i,t}}{R_t} + E_t[\varphi_{t,t+1}D_{i,t}] + I_{i,t}\left(R_t^m - 1\right) + P_t c_{i,t},$$

where household $i's$ borrowing is restricted by the following no-Ponzi game condition

$$\lim_{s \to \infty} E_t \varphi_{t,t+s} D_{i,t+s} \geq 0, \quad (4.9)$$

as well as $M_{i,t}^H \geq 0$ and $B_{i,t} \geq 0$. The term $\left(R_t^m - 1\right) I_{i,t}$ measures the cost of money acquired in open market operations: The households receive new cash $I_{i,t}$ in exchange for $R_t^m I_{i,t}$ bonds.

Maximizing the objective (4.7) subject to the open market constraint (4.2), the goods market constraint (4.3), the asset market constraints (4.8) and (4.9), for given initial values $M_{i,-1}$, $B_{i,-1}$, and $D_{i,-1}$ leads to the following first order conditions for working time, consumption, federal funds

and loans

$$c_{i,t}^{-\sigma} = \lambda_{i,t} + \psi_{i,t}, \tag{4.10}$$

$$\chi n_{i,t}^{\eta} = w_t \left( \lambda_{i,t} + \theta \psi_{i,t} \right), \tag{4.11}$$

$$\psi_{i,t} = \left( R_t^m - 1 \right) \lambda_{i,t} + R_t^m \eta_{i,t}, \tag{4.12}$$

$$R_t^m \left( \lambda_{i,t} + \eta_{i,t} \right) = R_t^L \left( \lambda_{i,t} + \eta_{i,t} \kappa_t \right), \tag{4.13}$$

as well as for holdings of contingent claims, government bonds and money

$$\lambda_{i,t} = \beta R_t E_t \frac{\lambda_{i,t+1} + \eta_{i,t+1}}{\pi_{t+1}}, \tag{4.14}$$

$$\lambda_{i,t} = \beta E_t \frac{\lambda_{i,t+1} + \psi_{i,t+1}}{\pi_{t+1}}, \tag{4.15}$$

$$\varphi_{t,t+1} = \frac{\beta}{\pi_{t+1}} \frac{\lambda_{i,t+1}}{\lambda_{i,t}}, \tag{4.16}$$

the associated complementary slackness conditions and the transversality conditions for money, bonds, and household debt. The debt rate is defined as follows

$$E_t \varphi_{t,t+1} = 1/R_t^D.$$

Further, $\eta_t$ denotes the multiplier on the open market constraints, and $\psi_t$ is the multiplier on the cash-in-advance constraint for consumption. Combining the optimality conditions for investment in money and government bonds, we obtain

$$R_t E_t \left( \lambda_{i,t+1}/\pi_{t+1} \right) = E_t \left[ R_{t+1}^m \left( \lambda_{i,t+1}/\pi_{t+1} \right) \right]. \tag{4.17}$$

Thus, the interest rate on government bonds closely follows next period's expected policy rate. This condition states that households are indifferent between investing into money vs. investing into government bonds and converting these into cash in the next period at the discount $R_{t+1}^m$. Condition (4.13) shows that when the open market constraint is binding, $\eta_t > 0$, which is the case we analyze, the loan rate depends on the fraction of firm bonds eligible as collateral in open market operations, $\kappa_t$.[35] When loans are

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[35] As shown in the appendix, the cash-in-advance constraint as well as the open market constraint bind in steady state when monetary policy sets its policy rate below the Euler rate, $R^m < R^D = \pi/\beta$ .

not eligible, $\kappa_t = 0$, there can be a spread between the policy rate and the loan rate, which is a *liquidity premium*. When all intra-period loans are eligible as collateral in open market operations $\kappa_t = 1$, the interest rate on corporate debt compensates exactly for the discount, i.e. $R_t^L = R_t^m$.

Further, we observe that in the special case of $\theta = 1$, there is no distortion of the households' labor supply decision as the first orders in that case imply $u_{c,t} = -u_{n,t}$ (see 4.10 and 4.11) The reason is that only the fraction of wages obtained before goods market closure $\theta$ can be used for contemporaneous consumption purchases.

### 4.2.4 Public sector

The central bank transfers seigniorage revenues $P_t \tau_t^m$ to the Treasury, which emits one-period bonds and pays a transfer $P_t \tau_t$ to households. Government bonds grow at a constant rate, $B_t^T = \Gamma B_{t-1}^T$. The Treasury's budget constraint reads

$$B_t^T / R_t + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t, \tag{4.18}$$

where government bonds $B_t^T$ are either held by households, $B_t$, or the central bank, $B_t^{CB} : B_t^T = B_t + B_t^{CB}$. This setup does not require $B^T$ to measure total public debt, rather it is a measure of short-term government bonds which are eligible for open market operations.[36] To avoid further effects of fiscal policy, we assume that the government has access to lump-sum taxes, which adjust to balance the budget. Thus, introducing long-term government bonds as a means of financing government expenditures would not have any consequences for the analysis conducted in this chapter.

The central bank invests its wealth exclusively into new government bonds. It transfers interest earnings on government bonds to the Treasury at the end of the period, $P_t \tau_t^m = B_t^{CB} - B_t^{CB} / R_t$. Its budget constraint reads

$$\left( B_t^{CB} / R_t \right) + P_t \tau_t^m = B_{t-1}^{CB} + R_t^m I_t - M_t^R,$$

where the amount of money withdrawn by repos is given by $M_t^R = B_t^R + L_t^R$. Substituting out transfers, the bond holdings of the central bank evolve

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[36]Note that, because the central bank can decide on the eligibility of collateral, it can choose a long-run inflation rate independent of the growth rate of government debt. To implement such a policy in the current setup, it can adjust the fraction of government debt eligible in open market operations, $\kappa_t^B$.

according to

$$B_t^{CB} = B_{t-1}^{CB} + I_t R_t^m - M_t^R. \tag{4.19}$$

We assume that the central bank sets the repo rate either according to a feedback rule (see section 4.4.1) or according to a peg at the zero lower bound, $R_t^m = 1$. Further, the central bank sets the inflation target $\pi$ and decides on eligible collateral for repos by setting $\kappa \in [0, 1]$. Finally, it can control whether money is supplied in exchange for bonds in repos or outright (while loans are only traded under repos). We assume that it controls the total amount of money supplied under repos, defined as

$$M_t^R = \Omega M_t^H + (\kappa_t - \kappa) L_t / R_t^m,$$

by setting $\Omega \geq 0$. We assume that the central bank will reduce repos in government bonds if it accepts corporate debt eligible as collateral in the long-run equilibrium. Thus, it chooses to repurchase government bonds according to $B_t^R = \Omega M_t^H - \kappa_t L_t / R_t^m$. Intra-period loans are used in repos only, i.e. $L_t^R = \kappa_t L_t / R_t^m$. Thus, the steady state ratio of outright purchases to money holdings is constant and given by $\Omega$ (see Appendix C.2).

## 4.2.5 Equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{jt} dj = \int_0^1 n_{it} di$, $y_t = \int_0^1 y_{jt} dj = \int_0^1 c_{it} di = c_t$, and $\int_0^1 L_{i,t} di = \int_0^1 L_{j,t} dj$. Households will not behave differently and aggregate asset holdings satisfy $\forall t \geq 0 : \int_0^1 D_{i,t} di = 0$, $\int_0^1 M_{i,t}^H di = \int_0^1 \widetilde{M}_{i,t} di = M_t^H$, $\int_0^1 M_{i,t}^R di = M_t^R$, $\int_0^1 B_{i,t} di = B_t$, $\int_0^1 B_{i,t}^c di = B_t^c$, $\int_0^1 I_{i,t} di = I_t = M_t^H - M_{t-1}^H + M_t^R$, and $B_t^T = B_t + B_t^c$. Household bond holdings further satisfy

$$B_t - B_{t-1} = (\Gamma - 1)B_{t-1}^T - R_t^m \left( M_t^H - M_{t-1}^H + M_t^R \right) + M_t^R. \tag{4.20}$$

A rational expectations equilibrium is defined as follows: In a rational expectations equilibrium the firms' first order conditions and the production technology, the households' first order conditions, the goods market constraint, the open market constraint, (4.20), $\Gamma = B_t^T / B_{t-1}^T$, and the transversality conditions are satisfied, for a monetary policy and given initial asset endowments. The full set of equilibrium conditions can be found in Appendix C.1.

## 4.3 Long-run effects of monetary policy

This section intends to clarify the model's mechanisms by exposing central long-run properties of the economy as well as long-run effects of monetary policy. A detailed analysis of the steady state can be found in Appendix C.2. Here, we summarize the main steady state properties of the model.

In the steady state, the debt rate $R^D$ satisfies $R^D = \pi/\beta$, and is thus independent of the target policy rate $R^m$. The steady state interest rate on government bonds is linked to the monetary policy rate, $R = R^m$ (by 4.17). The reason is arbitrage between investment into money (which gives a safe payoff of one unit of currency tomorrow per unit of currency invested today) vs. investment into government bonds and trading them for money in open market operations. The latter option pays off a return $R$ but implies a cost reducing the yield by $1/R^m$. As shown in Appendix C.2, the goods market constraint (4.3) will be binding in a steady state if the inflation target exceeds $\beta$, such that $R^D = \pi/\beta > 1$. Then, households can earn interest by investing in other stores of value (here household debt), such that they economize on money holdings and use money only for goods market expenditures. Notably, this result holds even if the policy rate is at the zero lower bound, $R^m = 1$. Further, the open market constraint (4.2) will be binding in a steady state if the policy rate target of the central bank satisfies $1 \leq R^m < R^D$, where $R^m = R$ and $R^D = \pi/\beta$. Thus, when interest paid on bonds is smaller than on debt, $R < R^D$, households economize on bond holdings and use bonds only to obtain money in open market operations. These results are summarized in the following proposition.

**Proposition 4.1** *The steady state interest rates are characterized by $R^D = \pi/\beta$ and $R = R^m$. In the steady state, the goods market constraint is binding if the inflation target satisfies $\pi > \beta$. The working capital constraint and the open market constraint are binding, even if the policy rate is at the zero lower bound $R^m = 1$, as long as the policy rate target satisfies $R^m < \pi/\beta$ and $\kappa \in [0, 1)$.*

**Proof.** See Appendix C.2 ■

In contrast to the model of Reynard and Schabert (2009), steady state consumption can be affected by the collateral requirements in open market operations, i.e. by the share of loans eligible in open market operations $\kappa$. The reason is that, first, the loan rate raises the firms' marginal cost and,

second, the level of the loan rate is affected by the central bank decision to accept loans in exchange for money. The latter property is highlighted by the steady state property

$$R^L = \left[ \kappa \frac{1}{R^m} + (1 - \kappa) \frac{1}{R^D} \right]^{-1}, \qquad (4.21)$$

see Appendix C.2. As long as loans are not eligible, $\kappa = 0$, (4.21) implies the loan rate to equal the debt rate $R^L = R^D$. For the case where the open market constraint is binding, $R^m < R^D$, this result does not hold for $\kappa > 0$. By increasing the fraction of eligible loans $\kappa$ (for a given policy rate) or lowering the policy rate (for a given $\kappa$) the central bank can induce $R^L$ to fall in the steady state. Condition (4.21) further reveals that $R^L > 1$, if $R^m < R^D$, given that $R^m \geq 1$. It is further shown in Appendix C.2 that steady state output and thus consumption decrease with the loan rate (via reduced real marginal cost) and with the inflation target (via the inflation tax induced by the cash-in-advance constraint): $\partial c / \partial R^L < 0$ and $\partial c / \partial \pi < 0$. This result is summarized in the following proposition.

**Proposition 4.2** *If the central bank sets its targets according to $\pi < \beta$ and $R^m \in [1, \pi/\beta)$, it can choose the inflation target $\pi$ independently of the share of loans eligible for open market operations $\kappa$ and the policy rate target $R^m$. Steady state consumption then increases with $\kappa$ and decreases with $R^m$ as well as with $\pi$, while the steady state price level sequence shifts upward for a higher $\kappa$.*

**Proof.** See Appendix C.2 ∎

Hence, monetary policy can affect consumption and thus welfare in the long run via the inflation target $\pi$, the share of eligible loans $\kappa$, and the policy rate $R^m$, where the latter two alter the loan rate (see 4.21). It should be noted that the inflation target $\pi$ and the share of eligible loans $\kappa$ can be chosen independently. Put differently, a change in $\kappa$ (e.g. an increase) will not affect the long-run inflation rate. However, the steady state price level sequence shifts upward for a higher $\kappa$, which is associated with a decline in the real value of bonds.

## 4.4 Numerical results

In this section we examine the short-run dynamics of the model. In particular, we analyze the impact of monetary policy on macroeconomic aggregates. The model will be solved by applying local approximation methods. The percent deviation of a generic variable $z_t$ from its steady state value $z$ is then denoted by $\widehat{z}_t : \widehat{z}_t = 100\left[\log(z_t) - \log(z)\right]$.

Throughout the analysis we assume that the central bank sets its targets according to $\pi > \beta$ and $1 \leq R^m < \pi/\beta$, which implies that the open market constraint and the goods market constraint are binding in the steady state (see Proposition 4.1). We further assume that shocks are sufficiently small for the economy to remain in the neighborhood of this steady state. Due to the binding open market constraint, private sector holdings of real government bonds will be relevant for the allocation and act as a relevant state. The central bank can - by lowering the policy rate $R^m$ (a conventional monetary policy measure) or by raising the share of eligible loans $\kappa_t$ (an *un*conventional monetary policy measure) - ease households' access to money in open market operations. Given that the goods market constraint, $M_t^R + M_t^H = P_t c_t$, is also binding, nominal consumption is then stimulated.

### 4.4.1 Calibration

For the quantitative analysis of the macroeconomic effects of monetary policy we calibrate the model, using standard parameter values as far as possible. The parameters of the utility function equal $\sigma = 1$ and $\eta = 0$, the labor share equals $\alpha = 0.66$, the steady state markup $1/mc = 11\%$ ($\varepsilon = 10$), steady state working time $n = 1/3$ (implying a value of 1.77 for utility function parameter $\chi$), and the fraction of non-optimally price adjusting firms $\phi = 0.75$. For the fraction $\theta$ of pre-paid loans, we follow Rabanal (2007), who estimates that around half of the wage sum has been pre-financed in the period 1983-2004, and set $\theta = 0.5$.[37] The target inflation rate is set to $\pi = 1.00575$, which equals the average of U.S. inflation in the last 20 years. We further set the steady state share of repurchase operations to outright purchases to $\Omega = 1.5$, which accords to the value in Reynard and Schabert

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[37]His point estimate is $\theta = 0.56$. Note that in our setup, $\theta$ determines only the "location" of the distortion between labor supply and labor demand. When $\theta = 1$, only the firms' decision to hire labor is distorted, while a value of $\theta = 0$ implies that only the households' labor supply decision is distorted.

(2009) based on data of Federal Reserve open market operations. For the benchmark case, the policy rate target $R^m$ is set equal to its 20-year average $R^m = 1.0105$ (or 4.28% in terms of annualized rates), while the policy rate is set according to $R_t^m = \left(R_{t-1}^m\right)^\rho R_m^{1-\rho} \left(\pi_t/\pi\right)^{w_\pi(1-\rho)} \left(y_t/y\right)^{w_y(1-\rho)} \exp \varepsilon_t$, where $w_\pi = 1.92$, $w_y = 0.1$, and $\rho = 0.8$ (see Justiniano and Primiceri (2008)). Alternatively, we will consider a peg at the zero lower bound $R^m = 1$ (see section 4.4.3). For the numerical analysis, we further restrict our attention of the case where the central bank does not trade corporate bonds in open market operations in the steady state, i.e. $\kappa = 0$.

The spread between the policy rate and the loan rate (which equals $R^D = \pi/\beta$ in a steady state with $\kappa = 0$, see 4.21) crucially affects the size of monetary policy effects. To calibrate this spread, we account for the fact that our model does not imply any kind of default risk and focus on the part of the spread that can be attributed to a liquidity premium. According to the the "corporate bond credit spread puzzle", only a small share of the yield spread between Treasury securities and corporate bonds can actually be explained by default risk. Collin-Dufresne (2001), for example, can explain only 25% of the variation in credit spread changes across 688 corporate bonds. Huang and Huang (2002) report that around 20% of corporate credit spreads can be explained through default risk. For our calibration of the spread between corporate bonds and Treasury securities, we apply the results by Longstaff, Mithal, and Neis (2005), which lead to more conservative estimates of the liquidity premium. Specifically, they report that, for $AAA$ rated corporate bonds, 51% of the credit spread can be explained by default risk.[38] Given that the average short-term spread among $AAA$ corporate bonds equals 104 basis points at annualized rates (see Longstaff, Mithal, and Neis (2005)), we consider a liquidity premium of $(1 + 49\% \cdot 0.0104)^{1/4} = 13$ basis points (in terms of quarterly rates), for which we choose the discount factor to equal $\beta = \frac{\pi}{R^m + 13 \cdot 10^{-4}} = 0.9940$.
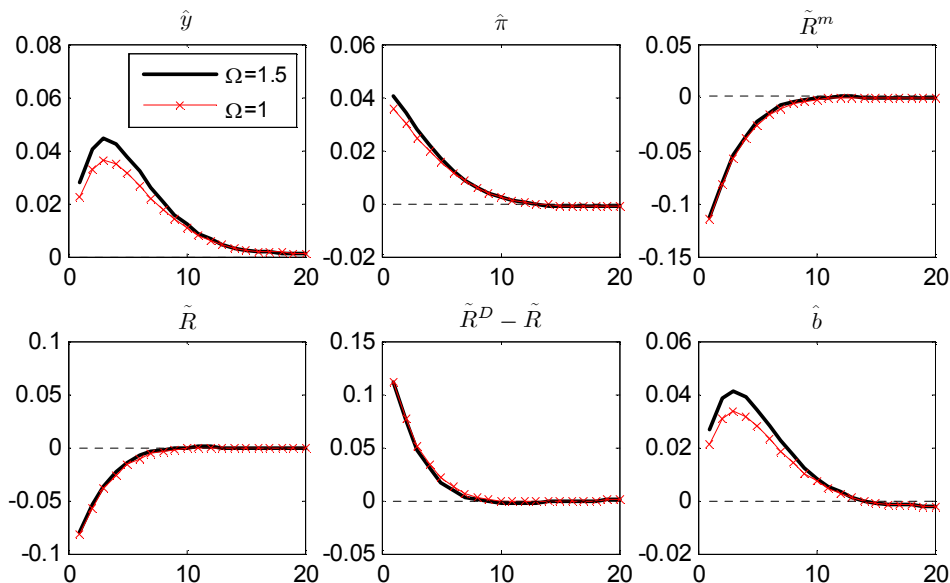
Figure 4.1: Responses to a policy rate shock

## 4.4.2 Effects of conventional monetary policy

We use the calibrated version of our model to analyze the effects of monetary policy in normal times, when $R_t^m > 1$. Figure 4.1 shows responses to a decline the policy instrument $R_t^m$ by 12.5 basis points, i.e. 50 basis points in terms of annualized rates.[39] A decline in the policy rate raises output (and thus consumption and real balances). The output response displays a hump-shape, which is induced by endogenous changes in the distribution of government bonds between the central bank and households, as shown in Reynard and Schabert (2009). Inflation also increases and returns monotonically to steady state. The bond rate closely follows the

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[38]To be more precise, they consider two competing explanations of a non-default component in credit spreads: Differential tax treatment and liquidity considerations. They find both that the cross-sectional and the longitudinal variation in the non-default component are strongly correlated to measures of market liquidity while the differential tax treatment does not have an impact.

[39]All figures show deviations from steady state in percentage points, $\hat{z}_t$, except for interest rates which are given in terms of absolute deviations from steady state, $\tilde{R}_t = 100 * (R_t - R)$ and can thus be interpreted as quarterly percentage points: For instance $\tilde{R}_t^m = -0.12$ implies a 12 basis point decline in the (quarterly) policy rate.

policy rate (see 4.17), while the liquidity premium between non-eligible assets and eligible bonds increases. The reason is that bonds become more liquid (i.e. closer substitutes for money) when the policy rate falls. Hence, a monetary easing leads to an increase in the cost of external funds, which tends to amplify the inflation response. Figure 4.1 shows that the output response depends on the share of repurchase operations in money holdings, $\Omega$. A higher repo share increases the output effect because it implies that a larger share of households' money holdings is affected by the shock to the policy rate. Thus, in accordance with the conventional view on monetary policy effects, a decline in the policy rate is able to stimulate the economy, i.e. to lead to an increase in consumption and employment, at the cost of increasing the inflation rate. This policy option is evidently only available if the policy rate exceeds the zero lower bound, $R_t^m > 1$.

### 4.4.3 Effects of *un*conventional monetary policy

When the policy rate reaches the zero lower bound, a conventional way of stimulating the economy is evidently not possible. However, Japan's liquidity trap and the measures taken by the Federal Reserve to mitigate the recent financial crisis show that central banks do not consider themselves disarmed but resort to unconventional policies. In contrast to standard models, where money is supplied in an unrestricted way, our model allows an analysis of such policy actions. In particular, we assume that the central bank raises the share $\kappa_t$ of loans that are eligible in open market operations above its zero steady state value, $\kappa = 0$. This policy will affect the economy through two effects:

1. a quantitative easing (QnE) effect, i.e. an increase in the amount of eligible securities allowing households to acquire more money,

2. a qualitative easing (QlE) effect, i.e. a decrease in the liquidity premium and thus in the loan rate, which reduces firms' borrowing cost.

In the first part of the analysis, we consider a temporary rise of $\kappa_t$ and examine the joint impact of both effects on macroeconomic variables. In the second part of the analysis, an increase of $\kappa_t$ will be associated with a decline in the share of government bonds that are accepted in open market operations, which allows to keep the total amount of eligible securities constant and thus to isolate the QlE effect.
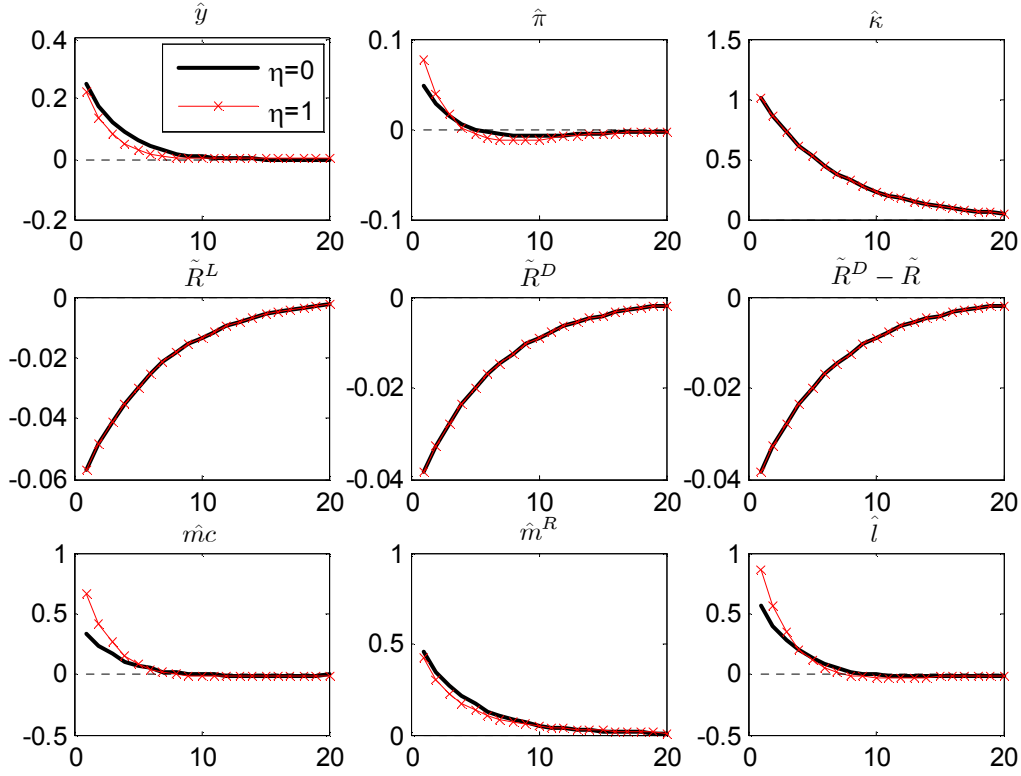
Figure 4.2: Responses to an increase in $\kappa$ $(QlE + QnE)$ at the ZLB

As mentioned in Proposition 4.1, a policy rate at the zero lower bound does not imply money demand to be unbounded. Moreover, we can even implement an interest rate peg without running into local indeterminacy problems, as pointed out by Reynard and Schabert (2009). To simplify the analysis of monetary policy at the zero lower bound, we assume that the central bank sets $R_t^m = 1$. Due to the (il-)liquidity premium on non-eligible assets, the debt rate and the loan rate will then be strictly positive, which has also been empirically observed during Japan's liquidity trap period and since the Fed lowered the federal funds rate (almost) to zero in the recent past. By increasing the share of eligible loans $\kappa_t$, the loan rate will decline due to a fall in the liquidity premium. Only if the share were set to equal one, $\kappa_t = 1$, the loan rate would be identical to the policy rate (see 4.13).
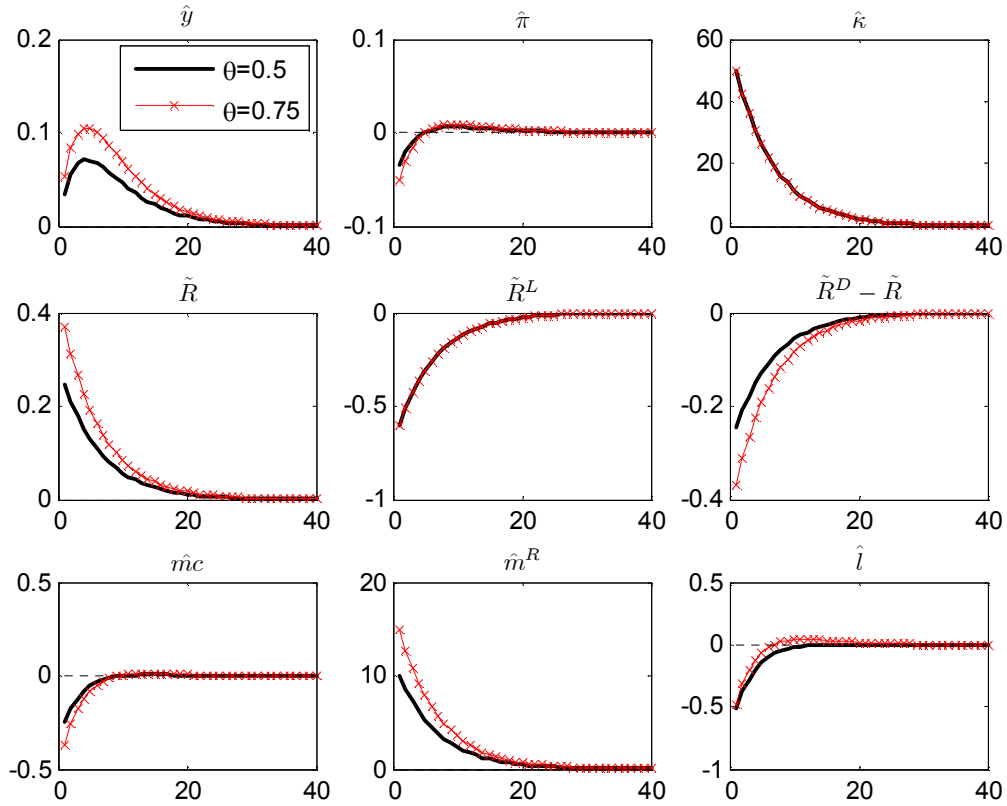
Figure 4.3: Responses to a neutralized increase in $\kappa$ ($QlE$) at the ZLB

Figure 4.2 shows the impulse response to a shock raising $\kappa_t$ from 0 to 1% (with an autocorrelation coefficient of 0.85). By increasing the amount of collateral available in open market operations, households can acquire additional money which they use to increase consumption spending, given that the goods market constraint is binding. In our benchmark calibration ($\eta = 0$), output increases by 0.25%. Similar to a conventional monetary expansion, inflation increases. This effect is only moderate with inflation rising by around five basis points. Figure 4.2 further reveals that the loan rate falls, since loans are now partially eligible in open market operations. This QlE effect is considerably small for the current change in $\kappa$. Further, Figure 4.2 demonstrates that the output effect declines when labor is inelastically supplied ($\eta = 1$). In that case, increasing labor demand by

firms puts further upward pressure on wages, so that the inflation increase is larger. This reduces the real value of the additional injections and thus reduces the output effect.[40]

To examine the effectiveness of QlE, we further increase $\kappa_t$ from 0 to 50%. In order to isolate the QlE effect from the effects from the expansion in the amount of collateral (QnE), we reduce the fraction of eligible bonds accordingly, $\kappa_t^B B_{t-1}$, such that total injections, $\kappa_t L_t + \kappa_t^B B_{t-1}$, remain constant. In particular, we set $\kappa_t^B = \kappa^B - (l/b\pi)(\kappa_t - \kappa)$, where $\kappa^B = 1$ and $\kappa = 0$. Figure 4.3 shows the impulse response to a shock which raises $\kappa_t$ from zero to 0.5 (with an autocorrelation coefficient of 0.85). As revealed by the output response, this policy has a stimulating impact on the economy, though the central bank has not increased the amount of collateral. However, the impact on output and consumption is small, i.e. a maximum deviation of 0.07% from the steady state value in the benchmark calibration ($\theta = 0.5$). As explained above, these effects are due to the reduction of the loan rate, which declines by around 60 basis points (in terms of quarterly rates), thereby reducing production cost. At the same time the bonds rate increases, since less bonds are now eligible, thus lowering the liquidity premium $R_t^D - R_t$. Figure 4.3 further shows that the effectiveness of qualitative easing increases with the degree of the working capital friction. When the fraction of loans paid in advance rises from $\theta = 0.5$ to $\theta = 0.75$, both the output and inflation effects increase (in absolute value), despite an identical decline in the loan rate. The reason is that a larger share of labor cost is now affected by the decline in the loan rate.

### 4.4.4   Effectiveness of monetary easing

This section compares conventional and unconventional policies quantitatively with respect to their output and inflation effects. In addition to the policies analyzed above, we consider a policy of pure quantitative easing ("QnE"), which consists of an increase in $\kappa_t^B$ only. We further consider the interventions analyzed above, where "QnE & QlE" describes an increase in $\kappa_t$ and "QlE" an increase in $\kappa_t$ neutralized by a decline in $\kappa_t^B$. To facilitate comparability between the different policy experiments, we calibrate the interventions "QnE" and "QnE &  QlE" such that they lead to a change in

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[40]Note that the liquidity premium in both cases evolves identically because it measures the value of a nominal unit of currency.

the monetary base that equals the impact of a policy rate shock on the monetary base. In particular, we consider an increase in the policy rate by 12.5 basis points ($\triangle R_t^m = 0.00125$), which leads to a 7.5 basis point decline in the monetary base. Table 4.1 presents the impact and cumulative effects of different monetary policies on output and inflation at the zero lower bound, where the elasticities are multiplied with changes in the policy instrument that lead to identical monetary base effects. QlE does not imply a change in the monetary base so that we scale our QlE intervention differently (see below). All policy interventions are assumed to have an identical autocorrelation, $\rho = 0.85$. We report the negative effects of a positive shock to an exogenously set policy rate, and the effects of expansionary unconventional monetary policy.[41]

Table 4.1 shows that the policy rate shock has a small positive impact on output and inflation. The reason is that our model implies a relatively weak monetary transmission channel.[42] Comparing conventional monetary policy to QnE, we observe that the output effect of conventional policy is smaller on impact, while its cumulative output effect is larger. However, the cumulative output effect of a policy rate shock in the first three years is only 0.18%, compared to 0.20% under QnE. Figure 4.4 compares the response of key variables under both a policy rate shock and QnE. In explaining the different transmission, the response of households' bond holdings is central: When the policy rate declines, seigniorage revenues fall, so that households' bond holdings increase (see 4.20). This increases collateral available for future open market operations and thus magnifies the positive output effect of the initial shock. In contrast, under quantitative easing, households transfer more bonds to the central bank, so that their bond holdings decline. This mitigates the expansionary impact of the shock in future periods, so that the expansion is less persistent.

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[41]The monetary base is defined as $MB_t = M_{t-1}^H + I_t = M_{t-1}^H + \kappa_t^B B_{t-1}/R_t^m + \kappa_t L_t/R_t^m$. We assume a steady state characterized by $\kappa = 0$ and $\kappa^B = 0.99$, and where the policy rate is fixed at $R^m = 1$. The "QnE" and "QnE & QlE" interventions are thus scaled as follows $\triangle \kappa_t^B = \kappa^B/R^m \triangle R_t^m$ and $\triangle \kappa_t = \triangle R_t^m \frac{\kappa^B b}{R^m \pi l}$. We compute the semielasticities $\partial \hat{y}_t/\partial R_t^m$, $\partial \hat{\pi}_t/\partial R_t^m$ and $\partial \hat{y}_t/\partial \kappa_t^B$, $\partial \hat{\pi}_t/\partial \kappa_t^B$ as well as $\partial \hat{y}_t/\partial \kappa_t$, $\partial \hat{\pi}_t/\partial \kappa_t$ (the latter is computed twice, one for each of the policy experiments "QnE & QlE" and "QlE") both as an impact effect and a cumulative effect, $\partial \hat{y}_t^{cum}/\partial R_t^m$ where $\hat{y}_t^{cum} = \lim_{l \to \infty} E_t \sum_{s=0}^{l} \hat{y}_{t+l}$, from a numerical simulation using the software dynare.

[42]For comparison, the output effects of a one s.d. shock to labor productivity are around ten times larger. For details on this, see Reynard and Schabert (2009).

| | Output effects | | Inflation effects | |
|---|---|---|---|---|
| Policy experiment | Impact | Cumulative | Impact | Cumulative |
| Policy rate shock (negative effects) | 0.011% | 0.928% | 0.065% | 0.000% |
| QnE & QlE | 0.063% | 0.206% | 0.012% | 0.000% |
| QnE | 0.063% | 0.201% | 0.012% | 0.000% |
| QlE | 0.017% | 0.434% | -0.017% | 0.000% |
| Note: The intervention sizes are given by $\triangle R_t^m = 12.5$b.p., $\triangle\kappa_t = 0.0025$ (QnE&QlE), $\triangle\kappa_t^B = 0.0012$ (QnE) and $\triangle\kappa_t = 0.25$, $\triangle\kappa_t^B = -0.12$ (QlE), respectively. | | | | |

Table 4.1: Output and inflation effects of conventional and unconventional monetary policy

This difference in transmission between conventional monetary policy and QnE has further implications for the responses of output and inflation. The inflation effect under QnE is smaller than that of the policy rate shock for more than the first five years. With forward-looking price setting, this implies that the impact effect on inflation is significantly lower under QnE. Given an equivalent increase in the (nominal) monetary base, this leads the impact effect of QnE on output to be more than five times larger than that implied by conventional policy.[43] Moreover, Figure 4.4 demonstrates that the expansionary impact of QnE does not arise from a commitment to raise future inflation: The largest increase in inflation occurs on impact and thus unexpectedly. After that, the cumulative inflation effect is neg-

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[43]Note that the persistence of the effects of a policy rate shock depends on the form of the interest rate rule. If we replace the exogeneity assumption and instead use a Taylor rule, the effects described here are mitigated. However, the impact output response is still larger under QnE and the inflation effects of conventional policy in this case exceed those of QnE for 4 years.
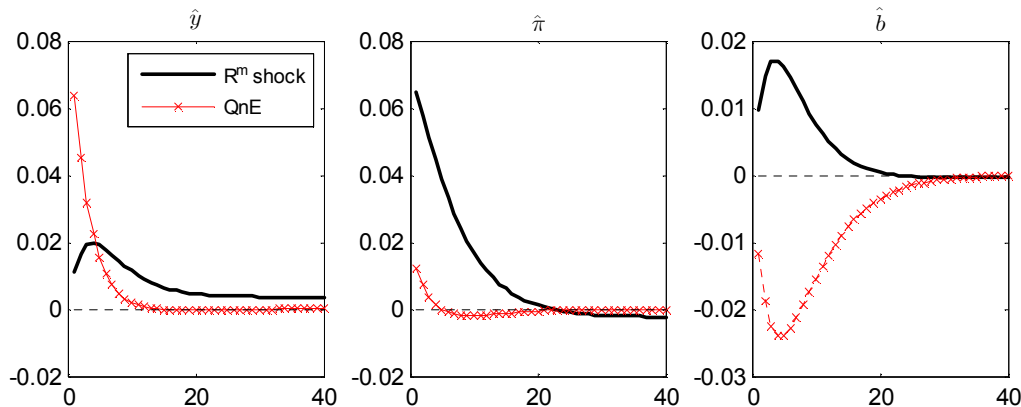
Figure 4.4: Comparing transmission of conventional and unconventional monetary policy

ative, amounting to –0.063%.[44] Further, output is stimulated even in the fourth quarter after the shock, although inflation falls below steady state in all periods thereafter. This result is in contrast to the existing literature on quantitative easing, which finds that QnE can only be effective by "committing to being irresponsible", as argued by Eggertsson (2006).

Further, comparing "QnE" and "Qne & QlE" in Table 4.1 shows that conducting quantitative easing by accepting bonds or loans leads to quantitatively similar results. However, the small size of the intervention hides the impact of qualitative easing: We isolate the impact of QlE on output and inflation by analyzing the case of a neutralized intervention of the size $\triangle \kappa_t = 0.25$. The scale of this intervention is such that the increase in central bank lending against firm loans exceeds that of the "QnE & QlE" experiment by factor 100. Still, QlE leads to a smaller impact effect on output. As mentioned in section 4.4.3, qualitative easing allows for an increase in real activity despite constant injections by reducing firms' production cost. This leads to a decline in inflation, which increases the real value of money and bonds held by households. As in the case of the policy

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[44]This can be easily calculated because cumulative inflation is zero across all policies we analyze. The reason for this is that, in the long run, eligible collateral and the policy rate are not changed by either of the policies we analyze. Thus, the price level has to return to its previous steady state path for real bond holdings, and thus consumption, to return to steady state as well.

rate shock, the increase in households' bond holdings eases access to future liquidity. Therefore, the cumulative output effect of qualitative easing is large relative to its impact effect, amounting to 0.33% after three years.

To conclude, quantitative easing has positive output effects that can be large in case of a sizable intervention. In comparison to conventional monetary policy, QnE more quickly develops its positive output effect. At the same time, inflationary pressure is lower than under conventional policy. Qualitative easing can lead to non-negligible output effects if conducted on a large scale, and at the same time reduces inflation.

### 4.4.5 Further applications

This section relates the unconventional monetary policies analyzed above to disruptions in financial markets, such as those observed during 2007-08. Especially in late 2008, there was considerable uncertainty about banks' creditworthiness, so that banks mistrusted each other. This led to a collapse of interbank lending. To insure themselves from increasing illiquidity risks associated with the disruptions in interbank lending, banks started to hoard liquid assets, triggering a decline in the money multiplier. In our model, we do not distinguish between high-powered money and broader monetary aggregates. However, one might think of modeling such financial market disruptions by allowing for a money multiplier in equation (4.3). Our model implies that quantitative easing is a powerful instrument to counter the effects of a decline in the multiplier: By increasing injections contingent on the decline in the multiplier, the central bank could mitigate or even prevent the recession triggered by such a shock.

Another important element in the financial crisis of 2007-08 was increasing credit risk. In our model, an increase in credit risk premiums would have contractionary effects by increasing firms' borrowing cost. Preventing such an increase was one of the aims of the Federal Reserve in setting up liquidity facilities such as the Commercial Paper Funding Facility. In our model, monetary policy disposes of an instrument capable of influencing firms' borrowing cost without causing inflationary pressure: By implementing a policy of qualitative easing, the central bank can counter the effects of credit risk shocks. It is outside the scope of this chapter to analyze these issues in more detail. However, we think that an application of our model to the recent financial crisis should proceed along these lines.

## 4.5 Conclusion

We develop a model which can readily be used to analyze the impact of unconventional monetary policy tools, such as those introduced by the Federal Reserve and other central banks around the globe in late 2008. Modeling open market operations explicitly allows an analysis of policy at the zero-lower bound as well as an assessment of the impact of qualitative and quantitative easing. We find that qua*n*titative easing, i.e. an expansion of the monetary base against conventional collateral, is an effective policy tool that can stimulate consumption. The reason for a positive output effect of quantitative easing is that we take into account that households' goods demand can be cash constrained even when the policy rate has reached the zero lower bound: When interest rates on illiquid assets are positive, the opportunity cost of money holdings is positive and households are not satiated with money.

We further analyze the effects of qua*l*itative easing, i.e. changes in the composition of the central bank's assets. We find that qualitative easing has a stimulating impact on the economy, even absent any changes in the monetary base. This effect operates through a reduction in the loan rate, which arises due to the liquidity services provided by loans when these are eligible in open market operations. This lowers firms' cost of production and stimulates the economy through the supply side. Although the reduction in the loan rate is sizable, the output effect is found to be relatively small.

Comparing the effectiveness of policies quantitatively, we find that conventional monetary easing (via lower policy rates) and a policy of quantitative easing - designed to generate an identical increase in the monetary base - have different output and inflation effects. Conventional monetary policy has a more than five times lower impact output effect and triggers higher inflation. This is mainly due to a difference in monetary transmission which implies that conventional monetary policy has more persistent output and inflation effects. Further, in contrast to the existing literature, we find that the stimulus of quantitative easing does not rely on a commitment to raise future inflation.

# Chapter 5

# Key currency pricing, liquidity and exchange rates

## 5.1   Introduction

Empirical studies reject uncovered interest rate parity (UIP), which states that a currency is expected to depreciate relative to another country's currency when the interest rate difference to that country is positive. One aspect of empirical failure of UIP is that exchange rates do not react to interest rate shocks as predicted by Dornbusch's (1976) overshooting but are characterized by delayed overshooting, as documented by Eichenbaum and Evans (1995). This chapter takes the evidence against UIP as a starting point and develops a model in which there is a spread between interest rates paid on assets eligible for central bank's open market operations and those paid on ineligible assets, i.e. a liquidity premium. The model further allows for a key currency which is required to participate in international trade. Therefore, assets allowing access to key currency liquidity are held by agents around the globe. This chapter shows that the liquidity premiums implied by this setup generate deviations from UIP and offer an explanation for delayed overshooting. Moreover, it analyzes how the international transmission of shocks is affected by modeling key currency liquidity.

Empirical failure of UIP is documented by various types of evidence including forward premium regressions, vector autoregressions (VARs) and model estimations. Testing UIP by applying regression analysis is difficult because expectations cannot be measured. However, as pointed out

by Chinn (2006), UIP can be tested jointly with the assumption of rational expectations. In the forward premium regression, empirical studies regress realized exchange rate changes on the interest rate difference (the forward premium) between two countries. Under rational expectations and risk neutrality, UIP predicts this regression to yield a positive coefficient of unity. Froot (1990) finds that the average estimate of this coefficient across 75 published studies is -0.88 with only a few estimates above zero and none greater than unity. The finding of a negative coefficient in the forward premium regression has become known as the forward premium puzzle. It implies that the forward premium predicts exchange rate movements inconsistent with theory not only in magnitude, but also in terms of the direction of the movement.[45] When investors are risk averse, the UIP condition allows for risk premiums, which are positive when an asset's domestic currency return is positively correlated to consumption growth. However, it is consensus in the literature that risk premiums cannot explain the negative coefficient in the forward premium regression, so that empirical UIP failure has become a stylized fact (see Backus, Foresi, and Telmer (2001)). Recent improvements in data availability have spurred a re-evaluation of these results with respect to maturities and countries. Chinn (2006) and Bansal and Dahlquist (2000) confirm the forward premium puzzle for short maturities in developed economies, but find evidence supportive of UIP with respect to long horizons and for emerging economies.

A second type of evidence documents the empirical failure of uncovered interest rate parity: Eichenbaum and Evans (1995) estimate a VAR to analyze the impact of monetary policy shocks on exchange rates. Their conclusion is known as the delayed overshooting puzzle: In contrast to Dornbusch's (1976) overshooting, which is based on UIP, they find that a contractionary U.S. monetary policy shock leads the dollar to appreciate continuously until it peaks after around three years. Some studies question the identification assumptions made by Eichenbaum and Evans (1995) and find evidence in line with Dornbusch's overshooting (see Kim and Roubini (2000) and Faust and Rogers (2003)). However, Scholl and Uhlig (2008) reconfirm the delayed overshooting result and find that the exchange rate peaks around one or two years after a monetary shock.

A third type of evidence stems from estimations of small open econ-

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[45]Surveys of this literature include Froot and Thaler (1984), Engel (1996) and Taylor (1995).

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omy models, which commonly include a UIP condition. Justiniano and Preston (2010) find that their model cannot account for the observed co-movement of Canadian and U.S. business cycles. Further, volatility in the real exchange rate is virtually entirely caused by shocks to an ad-hoc risk premium, so that the authors find an extreme version of exchange rate disconnect.[46] Justiniano and Preston (2010) suggest that the failure of the model to associate movements of exchange rates with fundamentals is related to its poor performance. Thus, improving the exchange rate predictions of economic models is a promising avenue to enhance the quantitative performance of open economy models.

The literature has analyzed various possibilities to reconcile theory and evidence, with a focus on risk premiums. Fama (1984) shows that a negative coefficient in the forward premium regression implies that the risk premium would have to be negatively correlated to, and more volatile than, the expected exchange rate change. There is consensus that the volatility of the risk premium implied by Fama's conditions is too high for any reasonable risk premium (see Froot and Thaler (1984) and Backus, Foresi, and Telmer (2001)). Some authors challenge this view: Lustig and Verdelhan (2007) find that high-interest rate currencies depreciate on average when consumption growth is low, so that a consumption based risk premium can explain excess returns if one is willing to assume large coefficients of risk aversion. Alvarez, Atkeson, and Kehoe (2009) build a model where asset markets are segmented, so that the investor's marginal utility varies more than indicated by fluctuations in aggregate consumption. This can increase the fluctuations of the risk premium. Other studies that aim to explain the forward premium puzzle follow the research agenda set out by Fama (1984) and construct volatile risk premiums by assuming changes in preferences such as habits (Campbell and Cochrane (1999) and Verdelhan (2010)), deviations from rationality (Gourinchas and Tornell (2004)) or multiple equilibria (Sarantis (2006)).

This chapter does not deal with risk premiums but combines two features, *liquidity* and *key currency pricing*: First, as is conveyed in anecdotal evidence - for instance about recurring flight to quality and flight to liquidity episodes - and in empirical studies, interest rates on assets vary not only according to their risk but also as a function of their liquidity. For

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[46]Lubik and Schorfheide (2006) obtain a qualitatively identical result.

instance, Longstaff (2004) shows that U.S. Treasury bonds pay lower interest rates than Refcorp bonds, which are backed by the Treasury, and finds that the premium is related to indicators of liquidity preferences.[47] In a closed economy, Reynard and Schabert (2009) show that taking into account liquidity premiums by modeling open market operations can align observed interest rates and their theoretical counterparts. Further, they demonstrate that monetary transmission is fundamentally affected. This suggests that the international transmission of shocks can be improved by a model analyzing the impact of liquidity on interest and exchange rates. The second observation relates to the leading role of the U.S. dollar in the international monetary system. Canzoneri, Cumby, and Diba (2007) coin the term key currency pricing, which states that a large share of international trade is conducted in dollars. Key currency pricing implies that importers and exporters find it convenient to hold dollar assets to facilitate their transactions. Canzoneri, Cumby, and Diba (2007) argue that such liquidity services provided by key currency bonds are the driving force behind relatively low U.S. interest rates, which imply an "exorbitant privilege" for the United States.[48]

This chapter combines these two observations and analyzes the impact of key currency pricing and liquidity on exchange rate dynamics. I develop a two-country open economy model which explicitly models open market operations in the foreign country, i.e. the key currency country, as in Reynard and Schabert (2009): The foreign central bank supplies cash in exchange for foreign government bonds, so that these pay lower interest rates compared to assets not eligible for open market operations. Liquidity demand is motivated from households' demand for goods purchases, which require cash. Due to the assumption of key currency pricing, households in the home economy require foreign currency to purchase import goods and hold foreign government bonds despite their low interest rates. I analyze how this setup affects uncovered interest rate parity and exchange rate movements, in particular in response to monetary policy shocks. The goal is to answer

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[47]Further evidence documenting liquidity premiums is given by Longstaff, Mithal, and Neis (2005) and Krishnamurthy and Vissing-Jorgensen (2007) who find that the supply of Treasury debt (relative to GDP) is negatively correlated to the spread between corporate and Treasury bond yields, even when controlling for default risk.

[48]This quote is attributed to Charles de Gaulle but stems from Valery Giscard d'Estaing, who was French finance minister at the time of the statement. See Canzoneri, Cumby, and Diba (2007).

the following questions: Can liquidity premiums generate deviations from uncovered interest rate parity? Can key currency effects reconcile theory and empirical evidence, for instance with respect to delayed overshooting? Does modeling key currency liquidity affect the international transmission of shocks in a fundamental way?

In the literature, the present work is most closely related to Canzoneri, Cumby, and Diba (2007). Like them, this chapter stresses the importance of the U.S. dollar in international trade and models liquidity services provided by government bonds. However, both the setup and goal of this chapter are different. The model in this chapter builds on Reynard and Schabert (2009), so that liquidity premiums in the model analyzed in this chapter are microfounded and endogenously derived from households' demand for cash. In contrast, Canzoneri, Cumby, and Diba (2007) employ an ad hoc specification for the liquidity services provided by government bonds. Further, these authors analyze the implications of their setup for a variety of macroeconomic issues and focus on asymmetries in fiscal and monetary policy transmission between countries. Further, Canzoneri, Cumby, and Diba (2007) analyze the quantitative implications of the UIP deviations implied by their setup by evaluating the correlation of expected exchange rate changes and the interest rate difference. They find that the liquidity premiums implied by their setup reduce this correlation. However, the correlation remains positive so that they cannot resolve the forward premium puzzle. In contrast, this chapter conducts a more detailed analysis of the impact of monetary policy on exchange rates using a microfounded model.

The results of this analysis are the following. I show that modeling key currency liquidity generates deviations from UIP and demonstrate that the key currency model predicts delayed overshooting of the nominal exchange rate, as in Eichenbaum and Evans (1995). The reason is that a rising foreign monetary policy rate increases the interest rate on foreign government bonds but reduces liquidity premiums overproportionately. This reduces the marginal benefit of investing in foreign government bonds, so that the foreign currency is expected to appreciate. I find an exchange rate peak after seven quarters, in line with the empirical evidence. Moreover, I demonstrate that the international transmission of shocks is fundamentally affected by considering key currency effects. For instance, when the home country pegs its exchange rate to the foreign currency, a demand shock in the foreign economy induces upward pressure on home country interest

rates even absent any change in the foreign monetary policy rate, triggering a recession in the home economy. The reason is that liquidity premiums on foreign assets increase due to rising foreign goods demand, which increases demand for the key currency.

This chapter is structured as follows. The model is presented in section 5.2. It gives rise to a modified uncovered interest parity condition which contains a liquidity premium. This is demonstrated in section 5.3. Building on these results, section 5.4 analyzes the response of interest and exchange rates to monetary policy shocks. Further, section 5.4 analyzes the transmission of a foreign demand shock when the home country pegs implements an exchange rate peg. Section 5.5 concludes

## 5.2 The model

### 5.2.1 Setup and timing of events

I model a small open economy (SOE) and its interactions to a large foreign economy, say the United States, which is explicitly modeled so that the impact of shocks to the foreign economy on the small home country can be analyzed. In the domestic and foreign economies, there is a continuum of infinitely lived households. I assume that households in both economies have identical asset endowments and preferences, so that I can consider a representative household in each country. As in Canzoneri, Cumby, and Diba (2007), I assume key currency pricing: International goods trade is carried out in terms of the foreign economy's currency, while domestic goods are purchased with local currency. This is equivalent to assuming producer currency pricing for large economy exports and local currency pricing for small economy exports. Moreover, it is assumed that the law of one price holds, so that exchange rate pass-through is perfect. Further, I analyze a foreign economy which is relatively large compared to the home economy, so that the home economy does not influence the foreign allocation of resources and prices. However, I take into account the impact of home holdings of foreign assets on asset stocks in the foreign economy.

In the following, the timing of events is described. The representative household in the home economy enters the period with holdings of foreign currency $M_{F,t-1}$, domestic and foreign private debt $D_{t-1}$, $D_{F,t-1}$ and for-

eign government bonds $B_{F,t-1}$.[49] Foreign households enter the period with holdings of foreign currency $M_{F,t-1}^*$, foreign government bonds $B_{F,t-1}^*$ and foreign private debt $D_{F,t-1}^*$. For simplicity, I neglect domestic government bonds and assume that domestic households hold currency only within periods. Further, it is assumed that firms in each country are owned by local households.

1. At the beginning of the period, shocks realize, households supply labor $n_t$ and $n_t^*$ and firms produce goods.

2. The foreign money market opens and both domestic and foreign households can exchange foreign government bonds $B_{F,t-1}$ and $B_{F,t-1}^*$ for money at the policy rate $R_t^{m*}$. The amounts $I_{F,t}$ and $I_{F,t}^*$ of foreign currency which home and foreign households can obtain in open market operations are therefore constrained by

$$I_{F,t} \leq \frac{B_{F,t-1}}{R_t^{m*}}, \tag{5.1}$$

$$I_{F,t}^* \leq \frac{B_{F,t-1}^*}{R_t^{m*}}. \tag{5.2}$$

   With respect to the home money market, I assume abundant supply of collateral, so that home households can obtain cash $M_t$ at the opportunity cost $R_t - 1$. More explicitly, at the beginning of the period, households in the small open economy can exchange their holdings of private debt against cash at a discount identical to the interest rate on private debt, $R_t$. As private debt can be created by households at no cost, this constraint does not bind in equilibrium. I further assume that home households can engage in repurchase operations only, so that they will not hold domestic money across periods. Seigniorage is transferred back to households via a lump sum transfer.

3. Households in both countries enter the goods markets, where goods can be bought with currency only. Key currency pricing requires

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[49]Throughout the paper, the subscripts $H$ and $F$ refer to home and foreign origin of goods and assets. An asterisk denotes variables decided upon by foreign agents. Verbally, I distinguish between both economies by using the terms "foreign" or "large" economy versus "home" or "small" economy. The terms "local" and "domestic" can refer to either economy, depending on the context. Further, upper case letters refer to nominal variables while lower case letters denominate real variables.

import goods in both countries to be purchased with foreign currency only. Further, households in both economies purchase domestic goods with their domestic currencies. Thus, households in the small open economy are constrained by

$$P_{F,t}^* c_{F,t} \leq I_{F,t} + M_{F,t-1}, \tag{5.3}$$

$$P_{H,t} c_{H,t} \leq M_t, \tag{5.4}$$

where $c_{F,t}$ and $c_{H,t}$ denote home consumption of foreign and, respectively, domestic goods and where $P_{H,t}$ is the price of home goods in home currency and $P_{F,t}^*$ is the price of foreign goods in terms of foreign currency. Households in the large economy require foreign currency for their entire goods purchases and are thus constrained by

$$P_{F,t}^* c_t^* \leq I_{F,t}^* + M_{F,t-1}^*. \tag{5.5}$$

4. Before the asset markets open, households in both countries receive dividends $P_t \delta_t$ and wages $P_t w_t n_t$ as well as government transfers $\tau_t$ and $\tau_t^*$. Further, repurchase agreements are settled. I assume that the foreign central bank conducts repo operations amounting to $M_{F,t}^{R*} + M_{F,t}^R = \Omega \left( M_{F,t}^* + M_{F,t} \right)$, where $M_{F,t}^{R*}$ and $M_{F,t}^R$ is the amount of money repurchased from foreign and, respectively, home households.

5. The asset markets open. Home households can carry wealth into the next period by purchasing domestic private debt $D_t$, foreign government bonds $B_{F,t}$ and foreign currency $M_{F,t}$. Foreign households invest into foreign assets only and acquire government bonds $B_{F,t}^*$, money $M_{F,t}^*$ as well as private debt $D_{F,t}^*$. The interest rates on domestic private debt, foreign government bonds and foreign private debt are given by $R_t$, $R_t^*$ and $R_t^{D*}$.

### 5.2.2   The home economy

**Households**

Households maximize the expected sum of the discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u \left( c_t, n_t \right) \right], \tag{5.6}$$

where $u$ is separable in its arguments, increasing, twice continuously differentiable, strictly concave and satisfies the Inada conditions, $\beta$ is the households' discount factor and $n_t$ is the share of his time endowment a household spends working. Home households' consumption is a composite good of foreign and domestic goods

$$c_t = \gamma c_{H,t}^{1-\eta} c_{F,t}^\eta, \tag{5.7}$$

where $\gamma^{-1} = \eta^n (1-\eta)^{1-\eta}$ and $\eta$ provides an openness measure of the home country. Households maximize utility subject to the asset market constraint,

$$S_t \left[ M_{F,t} - M_{F,t-1} + \frac{B_{F,t}}{R_t^*} - B_{F,t-1} + \frac{D_{F,t}}{R_t^{D*}} - D_{F,t-1} + P_t^* I_{F,t} (R_t^{m*} - 1) \right]$$

$$\leq P_t w_t n_t + P_t \delta_t + P_t \tau_t - M_t (R_t - 1) - \frac{D_t}{R_t} + D_{t-1} - P_{H,t} c_{H,t} - S_t P_{F,t}^* c_{F,t},$$

where $S_t$ refers to the nominal exchange rate, i.e. the price of a unit of foreign currency in terms of domestic currency, the cash in advance constraints for imported and domestic goods, (5.3)-(5.4), the open market constraint (5.1) and the non-negativity constraints $M_{F,t}, M_t, B_{F,t} \geq 0$ as well as the no-Ponzi game condition $\lim_{s\to\infty} E_t \prod_{i=0}^{s} D_{F,t+s}/R_{t+i} \geq 0$. The first order conditions for working time $n_t$, domestic and foreign consumption $c_{H,t}$ and $c_{F,t}$, open market operations $I_{F,t}$, holdings of domestic and foreign money, and investment into home and foreign private debt as well

as foreign government bonds are given by

$$\lambda_t w_t = -u_{n,t}, \tag{5.8}$$

$$\left(\lambda_t + \psi_{H,t}\right) \frac{P_{H,t}}{P_t} = u_{c,t}\gamma\left(1-\eta\right)\left(\frac{c_{F,t}}{c_{H,t}}\right)^{\eta}, \tag{5.9}$$

$$\left(\psi_{F,t} + \lambda_t\right) q_t = u_{c,t}\gamma\eta\left(\frac{c_{F,t}}{c_{H,t}}\right)^{\eta-1}, \tag{5.10}$$

$$\left(\psi_{F,t} + \lambda_t\right) q_t = \left(\lambda_t + \mu_t\right) R_t^m q_t, \tag{5.11}$$

$$\psi_{H,t} = \lambda_t\left(R_t - 1\right), \tag{5.12}$$

$$\lambda_t q_t = \beta E_t q_{t+1} \frac{\lambda_{t+1} + \psi_{F,t+1}}{\pi_{t+1}^*}, \tag{5.13}$$

$$\lambda_t = \beta E_t \frac{\lambda_{t+1} R_t}{\pi_{t+1}}, \tag{5.14}$$

$$\lambda_t q_t = \beta E_t q_{t+1} \frac{\lambda_{t+1} R_t^{D*}}{\pi_{t+1}^*}, \tag{5.15}$$

$$\lambda_t q_t = \beta E_t q_{t+1} \frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}^*} R_t^*, \tag{5.16}$$

where $\psi_{H,t}$, $\psi_{F,t}$, $\mu_t$ and $\lambda_t$ are the respective Lagrange multipliers on the cash, open market, and asset market constraints, $q_t = S_t P_t^*/P_t$ is the real exchange rate and $\pi_t^* = P_t^*/P_{t-1}^*$ and $\pi_t = P_t/P_{t-1}$ are foreign and domestic (CPI) inflation. The budget constraint binds in equilibrium, $\lambda_t > 0$, because the disutility of working is strictly negative, $u_{n,t} < 0$. The complementary slackness conditions are given by

$$\psi_{H,t} \geq 0, \quad M_t - P_{H,t}c_{H,t} \geq 0, \quad \psi_{H,t}\left(M_t - P_{H,t}c_{H,t}\right) = 0,$$
$$\psi_{F,t} \geq 0, \quad M_{F,t} + I_{F,t} - P_{F,t}^*c_{F,t} \geq 0, \quad \psi_{F,t}\left(M_{F,t} + I_{F,t} - P_{F,t}^*c_{F,t}\right) = 0,$$
$$\mu_t \geq 0, \quad I_{F,t} - B_{F,t-1}/R_t^{m*} \geq 0, \quad \mu_t\left(I_{F,t} - B_{F,t-1}/R_t^{m*}\right) = 0,$$

and the transversality condition requires $\lim_{s\to\infty} E_t \prod_{i=0}^{s} D_{F,t+s}/R_{t+i} = 0$.

From (5.9) and (5.10), I observe that both imported and domestically produced goods are subject to a cash credit friction. This implies that households' optimal allocation of consumption good spending depends not only on the relative prices of foreign and domestic goods, but also on foreign and domestic interest rates. Using (5.9)-(5.11) and (5.7), I demonstrate in

Appendix D.1.1 that demand for foreign and domestic goods is given by

$$c_{F,t} = \frac{\eta u_{c,t}}{(\lambda_t + \mu_t) R_t^{m*}} q_t^{-1} c_t, \qquad (5.17)$$

$$c_{H,t} = \frac{(1 - \eta) u_{c,t}}{(\lambda_t + \psi_{H,t})} \left( \frac{P_{H,t}}{P_t} \right)^{-1} c_t. \qquad (5.18)$$

Further, (5.15) and (5.16) show that households are willing to hold foreign government bonds at an interest rate below that on foreign private debt whenever the open market constraint (5.1) is binding. Further, as shown in Appendix D.1.1, the consumer price index $P_t$ is given by

$$P_t = \frac{[(\lambda_t + \mu_t) R_t^m]^\eta (\lambda_t + \psi_{H,t})^{1-\eta}}{c_t^{-\sigma}} P_{F,t}^\eta P_{H,t}^{1-\eta}, \qquad (5.19)$$

where $P_{F,t}$ is the price of foreign goods in terms of the domestic currency. This implies that the cash distortion influences the price index. The reason is that the households take into account the cash credit friction into their optimal choice of consumption goods.

**Firms**

There is a continuum of monopolistically competitive firms indexed with $j \in [0, 1]$. Firms rent labor at the nominal wage $P_t w_t$ and produce a differentiated good using a linear technology,

$$y_{H,t}(j) = n_t(j).$$

Cost minimization implies that marginal cost in real (PPI) terms, $mc_t$, are constant across firms and given by

$$mc_t = w_t \frac{P_t}{P_{H,t}}. \qquad (5.20)$$

Firms produce varieties which are aggregated to a final good by competitive retailers according to

$$y_{H,t} = \left[ \int_0^1 y_{H,t}^{\frac{\varepsilon-1}{\varepsilon}}(j) dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

so that firms face the demand constraint $y_{H,t}(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} y_{H,t}$. Following Calvo (1983), every firm reoptimizes its price in a given period

with probability $\phi$. Firms who do not reoptimize prices are assumed to increase prices with the steady state PPI inflation rate $\pi_H$, as in Ascari (2004). Denoting with $Z_t$ the price of firms which reoptimize their price in period $t$, optimal forward looking price setting is given by

$$Z_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_s (\phi\beta)^s u_{c,t+s} y_{H,t+s} P_{H,t+s}^\varepsilon mc_{t+s}}{\sum_s (\phi\beta)^s u_{c,t+s} y_{H,t+s} P_{H,t+s}^{\varepsilon-1}}. \tag{5.21}$$

The optimal price setting condition can be rewritten recursively as

$$Z_t^1 = \varepsilon/(\varepsilon - 1) u_{c,t} y_{H,t} mc_t + \phi\beta\pi_H^{-\varepsilon} E_t \pi_{H,t+1}^\varepsilon Z_{t+1}^1, \tag{5.22}$$

$$Z_t^1 = u_{c,t} y_{H,t} + \phi\beta\pi_H^{1-\varepsilon} E_t \pi_{H,t+1}^{\varepsilon-1} Z_{t+1}^2, \tag{5.23}$$

where $\tilde{Z}_t = Z_t/P_{H,t} = Z_t^1/Z_t^2$. To determine the PPI inflation rate $\pi_{H,t}$, I use that the price index for home goods satisfies $P_{H,t} y_{H,t} = \int_0^1 P_{H,t}(j) y_{H,t}(j) \, dj$. Using the firms' demand constraint, $y_{H,t}(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} y_{H,t}$, this yields

$$1 = (1 - \phi)\left(Z_t^1/Z_t^2\right)^{1-\varepsilon} + \phi\pi_H^{1-\varepsilon}\pi_{H,t}^{\varepsilon-1}. \tag{5.24}$$

Further, the impact of price dispersion on output is given by

$$y_{H,t} = \frac{n_t}{s_t}, \tag{5.25}$$

where $s_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$ captures price dispersion and evolves according to

$$s_t = (1 - \phi)\left(Z_t^1/Z_t^2\right)^{-\varepsilon} + \phi\pi_H^{-\varepsilon}\pi_{H,t}^\varepsilon s_{t-1}. \tag{5.26}$$

For derivations of (5.20)-(5.26), see Appendix D.1.2.

### Public sector

The public sector in the home economy has a balanced budget. Thus, seigniorage earnings on domestic cash holdings are redistributed as a lump sum transfer $P_t \tau_t$ to domestic households, so that the public budget constraint reads

$$P_t \tau_t = M_t (R_t - 1).$$

Further, monetary policy is either given by an exchange rate peg, $S_t = S_{t-1}$ or by the interest rate rule

$$R_t = R^{(1-\rho_R)} R_{t-1}^{\rho_R} \left(\pi_{H,t}/\pi_H\right)^{w_\pi(1-\rho_R)} \left(y_{H,t}/y_H\right)^{w_y(1-\rho_R)}, \tag{5.27}$$

where $\rho_R$ governs interest rate inertia and $w_\pi$ ($w_y$) describes the central bank's reaction to deviations of producer price inflation (domestic output) from steady state. This rule is a simplified version of Justiniano and Preston (2010).

### 5.2.3 The foreign economy

In modeling the foreign economy, I closely follow Reynard and Schabert (2009). The only difference is that households import goods from the small economy. However, it is assumed that the foreign economy is large compared to the home economy, so that neither the allocation nor the price system in the small open economy influences the foreign economy. However, the impact of changes in domestic holdings of foreign government bonds on asset stocks in the foreign economy is taken into account.

**Households**

Foreign households consume an aggregate of foreign and home goods, $c_t^* = \gamma^* \left( c_{F,t}^* \right)^{1-\eta^*} \left( c_{H,t}^* \right)^{\eta^*}$ where $\gamma^* = \left[ \eta^{*\eta^*} (1-\eta^*)^{1-\eta^*} \right]^{-1}$. As is standard in the literature, the large open economy is treated as approximately closed, i.e. I analyze the case of $\eta^* \to 0$ so that foreign consumption and the price index are approximately given by $c_t^* = c_{F,t}^*$ and $P_t^* = P_{F,t}^*$. However, the demand function for import goods is relevant for the small open economy and given by $c_{H,t}^* = \eta^* \left( \frac{P_{H,t}/S_t}{P_t^*} \right)^{-1} c_t^*$. I assume that foreign households' discount factor is identical to that applied by households in the small economy. Foreign households maximize the expected sum of a discounted stream of instantaneous utilities which are separable in consumption and labor,

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left( c_t^*, n_t^* \right).$$

subject to the asset market constraint

$$M_{F,t-1}^* + B_{F,t-1}^* + P_t^* w_t^* n_t^* + D_{F,t-1}^* + P_t^* \delta_t^* + P_t^* \tau_t^*$$
$$\leq M_{F,t}^* + \frac{B_{F,t}^*}{R_t^*} + \frac{D_{F,t}^*}{R_t^{D*}} + P_t^* c_t^* + (R_t^{m*} - 1) I_{F,t}^*, \tag{5.28}$$

the open market constraint

$$I_{F,t}^* R_t^{m*} \leq B_{F,t-1}^*, \tag{5.29}$$

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the cash in advance constraint

$$P_t^* c_t^* \leq I_{F,t}^* + M_{t-1}^*, \tag{5.30}$$

and non-negativity conditions $M_{F,t}^* \geq 0$ and $B_{F,t}^* \geq 0$ as well as the no Ponzi game condition $\lim_{s \to \infty} E_t \prod_{i=0}^{s} D_{F,t+s}^* / R_{t+i}^{D*} \geq 0$. The first order conditions with respect to working time, consumption, open market operations and holdings of private as well as government debt and money are given by

$$-\frac{u_{n,t}^*}{w_t^*} = \lambda_t^*, \tag{5.31}$$

$$u_{c,t}^* = \lambda_t^* + \psi_t^*, \tag{5.32}$$

$$R_t^{m*} \left( \lambda_t^* + \mu_t^* \right) = \lambda_t^* + \psi_t^*, \tag{5.33}$$

$$\lambda_t^* = \beta^* E_t \frac{\lambda_{t+1}^*}{\pi_{t+1}^*} R_t^{D*}, \tag{5.34}$$

$$\lambda_t^* = \beta^* E_t \frac{\lambda_{t+1}^* + \mu_{t+1}^*}{\pi_{t+1}^*} R_t^*, \tag{5.35}$$

$$\lambda_t^* = \beta^* E_t \frac{\lambda_{t+1}^* + \psi_{t+1}^*}{\pi_{t+1}^*}, \tag{5.36}$$

where $\lambda_t^*$, $\mu_t^*$ and $\psi_t^*$ are the Lagrange multipliers on the budget, open market and cash in advance constraints. The complementary slackness conditions are given by

$$\psi_t^* \geq 0, \quad M_t^* + I_{F,t}^* - P_t^* c_t^* \geq 0, \quad \psi_t^* \left( M_t^* + I_{F,t}^* - P_t^* c_t^* \right) = 0,$$
$$\mu_t^* \geq 0, \quad I_{F,t}^* - B_{F,t-1}^* / R_t^{m*} \geq 0, \quad \mu_t^* \left( I_{F,t}^* - B_{F,t-1}^* / R_t^{m*} \right) = 0.$$

Further, the transversality condition, $\lim_{s \to \infty} E_t \prod_{i=0}^{s} D_{F,t+s}^* / R_{t+i}^{D*} = 0$ has to be satisfied.

### Firms

The setup of the firm sector is identical to the home economy: A continuum of firms indexed over $k$ rents labor and produces intermediate goods with a linear technology, given exogenous and constant total factor productivity $A^*$. Intermediate goods are aggregated like in the home economy,

$y_t^* = \left[ \int_0^1 \left( y_t^*(k) \right)^{\frac{\varepsilon-1}{\varepsilon}} dk \right]^{\frac{\varepsilon}{\varepsilon-1}}$, where I assume an identical elasticity of substitution, $\varepsilon^* = \varepsilon$. This yields the following equilibrium conditions

$$w_t^* = mc_t^* A^*, \tag{5.37}$$
$$Z_t^{1*} = \varepsilon / \left( \varepsilon - 1 \right) u_{c,t}^* y_t^* mc_t^* + \phi^* \beta \pi^{*-\varepsilon} E_t \pi_{t+1}^{*\varepsilon} Z_{t+1}^{1*}, \tag{5.38}$$
$$Z_t^{2*} = u_{c,t}^* y_t^* + \phi^* \beta \pi^{*1-\varepsilon} E_t \pi_{t+1}^{*\varepsilon-1} Z_{t+1}^{2*}, \tag{5.39}$$
$$1 = (1 - \phi) \left( Z_t^{1*} / Z_t^{2*} \right)^{1-\varepsilon} + \phi \pi^{*1-\varepsilon} \pi_t^{*\varepsilon-1}. \tag{5.40}$$

As in the home economy, price dispersion is defined as $s_t^* = \int_0^1 \left( \frac{P_t^*(k)}{P_t^*} \right)^{-\varepsilon} dk$, so that aggregate resources are inefficiently employed whenever $s_t^* > 1$. Aggregate production and price dispersion are given by

$$y_t^* = A^* n_t^* / s_t^*, \tag{5.41}$$
$$s_t^* = (1 - \phi) \left( Z_t^{1*} / Z_t^{2*} \right)^{-\varepsilon} + \phi s_{t-1}^* \pi_t^{*\varepsilon}. \tag{5.42}$$

**Public sector**

The public sector is identical to that in Reynard and Schabert (2009) with the exception that I take into account the impact of holdings of foreign government bonds in the home economy on asset stocks in the foreign economy. Given a constant growth rate of the volume of Treasury bonds, which evolve according to

$$B_t^{T*} = \Gamma B_{t-1}^{T*}, \tag{5.43}$$

the Treasury's budget constraint is given by

$$\frac{B_t^{T*}}{R_t^*} + P_t^* \tau_t^{m*} = B_{t-1}^{T*} + P_t^* \tau_t^*, \tag{5.44}$$

where $P_t^* \tau_t^{m*}$ are seigniorage revenues and $P_t^* \tau_t^*$ lump-sum transfers to households. The central bank's bond holdings thus evolve according to

$$\frac{B_t^{CB*}}{R_t^*} + P_t^* \tau_t^{m*} = B_{t-1}^{CB*} + R_t^{m*} I_t^* - M_{F,t}^{R*}, \tag{5.45}$$

where $I_t^* = I_{F,t}^* + I_{F,t}$ denotes total injections and $M_{F,t}^{R*} + M_{F,t}^R$ are repo operations in both countries. Seigniorage is defined as interest earnings on

government bonds held at period end, $P_t^* \tau_t^{m*} = \frac{B_t^{CB*}}{R_t^*} - B_t^{CB*}$. Thus, the central bank's bond holdings evolve according to

$$B_t^{CB*} - B_{t-1}^{CB*} = R_t^m I_t^* - M_{F,t}^{R*} - M_{F,t}^R.$$

Foreign households' bond holdings can now be derived residually from $B_{F,t}^* = B_t^T - B_{F,t} - B_t^{CB}$, which in differences reads

$$B_{F,t}^* - B_{F,t-1}^* = B_t^{T*} - B_{t-1}^{T*} - (B_{F,t} - B_{F,t-1}) - \left(B_t^{CB*} - B_{t-1}^{CB*}\right).$$

Plugging in central bank bond holdings (5.45) yields

$$B_{F,t}^* - B_{F,t-1}^* = (\Gamma - 1) B_{t-1}^{T*} - (B_{F,t} - B_{F,t-1}) - \left(R_t^m I_t^* - M_{F,t}^{R*} - M_{F,t}^R\right). \tag{5.46}$$

Monetary policy is assumed to conduct repurchase operations amounting to, $M_{F,t}^{R*} + M_{F,t}^R = \Omega \left(M_{F,t}^* + M_{F,t}\right)$. Further, the foreign policy rate follows an interest rate rule similar to that in the home economy

$$R_t^{m*} = R^{m*(1-\rho)} \left(R_{t-1}^{m*}\right)^\rho \left(\frac{\pi_t^*}{\pi^*}\right)^{w_\pi^*(1-\rho)} \left(\frac{y_t^*}{y^*}\right)^{w_y(1-\rho_R)} \exp(\varepsilon_t^*)^\rho, \tag{5.47}$$

where $\varepsilon_t^*$ is independently identically distributed. This closes the description of the foreign economy.

## 5.2.4 Equilibrium

In equilibrium markets clear, i.e. $n_t = \int_0^1 n_t(j)dj$, $y_{H,t} = c_{H,t} + c_{H,t}^*$, and for the foreign economy $n_t^* = \int_0^1 n_t^*(k)dk$ and $y_t^* = c_{F,t}^* = c_t^*$ because the home economy's imports $c_{F,t}$ are considered quantitatively negligible for the foreign economy. Further, private debt in both economies is in zero net supply, so that $D_{F,t} = -D_{F,t}^*$ and $D_t = 0$, because foreign households do not invest into home private debt. Throughout, I assume that the central banks in both countries set their instruments so that the cash in advance constraints (5.3), (5.4) and (5.30) bind ($\psi_{H,t}, \psi_{F,t}, \psi_t^* > 0$). I further assume that the share of repurchase agreements in money holdings is identical in both economies, so that the amounts of bonds repurchased by home and foreign households are given by $M_{F,t}^R = \Omega M_{F,t}$ and $M_{F,t}^{R*} = \Omega M_{F,t}^*$. Therefore, home households' holdings of foreign money are given by

$$M_{F,t} = M_{F,t-1} + I_{F,t} - P_{F,t}^* c_{F,t} + P_{H,t} c_{H,t}^*/S_t - M_{F,t}^R$$
$$= P_{H,t} c_{H,t}^*/S_t - M_{F,t}^R, \tag{5.48}$$

where the second equality uses the binding cash in advance constraint for the home economy's imports.[50] Further, when (5.4) binds, households in the small economy hold domestic currency amounting to

$$M_t = P_{H,t} c_{H,t}. \tag{5.49}$$

Foreign households' currency holdings are given by $M_{F,t}^* = M_{F,t-1}^* + I_{F,t}^* - P_{F,t}^* c_{F,t}^* + P_t^* w_t^* n_t^* + P_t^* \delta_t^* - M_{F,t}^{R*}$. Using that foreign firms distribute their revenues entirely to foreign households, this simplifies to

$$M_{F,t}^* = M_{F,t-1}^* + I_{F,t}^* - \Omega M_{F,t}^*. \tag{5.50}$$

**Capital account, trade balance and the real exchange rate**  The evolution of net foreign asset holdings can be determined by using the home household's budget constraint, which reads

$$M_t (R_t - 1) + S_t \frac{B_{F,t}}{R_t^*} + S_t \frac{D_{F,t}}{R_t^{D*}} + P_{H,t} c_{H,t} + S_t P_{F,t}^* c_{F,t} + S_t I_{F,t} (R_t^{m*} - 1)$$
$$= S_t M_{F,t-1} + S_t B_{F,t-1} + S_t D_{F,t-1} + P_t w_t n_t + P_t \delta_t + P_t \tau_t - S_t M_{F,t},$$

using that home private debt is in zero net supply. Further, home firms distribute all revenues as dividends and wages to home households, leading to $P_t w_t n_t + P_t \delta_t = P_{H,t} y_{H,t} = P_{H,t} c_{H,t} + P_{H,t} c_{H,t}^*$. Using this and the public sector's budget constraint (5.44) yields

$$S_t \frac{B_{F,t}}{R_t^*} - S_t B_{F,t-1} + S_t \frac{D_{F,t}}{R_t^{D*}} - S_t D_{F,t-1} + S_t M_{F,t} - S_t M_{F,t-1} \tag{5.51}$$
$$= P_{H,t} c_{H,t}^* - S_t P_{F,t}^* c_{F,t} - S_t I_{F,t} (R_t^{m*} - 1).$$

Thus, the foreign country receives interest payments from home households' participation in open market operations. Except for this, the capital account is standard: The change in net foreign asset holdings of domestic households equals the current account, which consists of interest rate payments and the trade balance. The trade balance in terms of the home consumption basket is given by

$$tb_t = \frac{P_{H,t}}{P_t} c_{H,t}^* - \frac{S_t P_{F,t}^* c_{F,t}}{P_t} = q_t (\eta^* c_t^* - c_{F,t}). \tag{5.52}$$

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[50]The reason why exports appear is that they are paid for in foreign currency. Thus, households in the small open economy receive a share of dividends and wages in foreign currency. This share is given by $P_{H,t} c_{H,t}^*/S_t$. The remaining amount $P_t w_t n_t + P_t \delta_t - P_{H,t} c_{H,t}^*$ is received in domestic currency.

Further, (5.19) can be rewritten by using the law of one price and the assumption of a large foreign economy, which implies that $P_{F,t} = S_t P_{F,t}^* = S_t P_t^*$. Using the definition of the real exchange rate,

$$q_t = \frac{S_t P_t^*}{P_t} = \frac{P_{F,t}}{P_t}. \qquad (5.53)$$

Thus, (5.19) can be rewritten as $\frac{P_{H,t}}{P_t} = \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}}$, which in differences reads

$$\pi_t = \pi_{H,t} \left( \frac{\Phi_t}{\Phi_{t-1}} \right)^{\frac{1}{1-\eta}} \left( \frac{q_t}{q_{t-1}} \right)^{\frac{\eta}{1-\eta}}, \qquad (5.54)$$

where $\Phi_t = \frac{[(\lambda_t + \mu_t) R_t^m]^\eta \left( \lambda_t + \psi_{H,t} \right)^{1-\eta}}{c_t^{-\sigma}}$.

**Binding cash and open market constraints** With the exception of section 5.3.1, I only consider equilibria where the open market constraints in both economies bind. In steady state, this is guaranteed by $R^{m*} < \frac{\pi^*}{\beta^*}$.[51] This implies that money injections are given by households' holdings of foreign government bonds,

$$I_{F,t} = \frac{B_{F,t-1}}{R_t^{m*}}, \qquad (5.55)$$

$$I_{F,t}^* = \frac{B_{F,t-1}^*}{R_t^{m*}}. \qquad (5.56)$$

Further, binding open market constraints in both economies imply that total injections are given by $I_t^* = \frac{B_{F,t-1}^* + B_{F,t-1}}{R_t^{m*}}$, so that foreign households' bond holdings evolve according to

$$B_{F,t}^* = (\Gamma - 1) B_{t-1}^{T*} - B_{F,t} + \Omega M_{F,t}^* + \Omega M_{F,t}. \qquad (5.57)$$

A rational expectations equilibrium is a set of sequences $\{ c_t, c_{F,t}, c_{H,t}, n_t, P_{H,t}, P_t, M_t, S_t, q_t, M_{F,t}, I_{F,t}, D_{F,t}, B_{F,t}, w_t, \lambda_t, \psi_{H,t}, \psi_{F,t}, \mu_t, y_{H,t}, mc_t, Z_t^1, Z_t^2, s_t, R_t, c_t^*, c_{H,t}^*, n_t^*, P_t^*, \lambda_t^*, \psi_t^*, \mu_t^*, M_{F,t}^*, I_{F,t}^*, B_{F,t}^*, B_t^{T*}, R_t^{m*}, R_t^{D*}, R_t^*, w_t^*, mc_t^*, y_t^*, Z_t^{1*}, Z_t^{2*}, s_t^* \}_{t=0}^{\infty}$ satisfying the households' and firms' first order conditions including the transversality conditions, the open market constraints (5.1) and (5.2), binding cash in advance constraints (5.3), (5.4) and (5.5),

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[51] For a derivation of this property, see Appendix D.3.

the households' holdings of foreign and home currency and foreign bonds, (5.48), (5.50) and (5.46), the capital account (5.51), the definition of the real exchange rate (5.53) and the home CPI (5.54) and PPI (5.24), aggregate production $y_{H,t} = c_{H,t} + c_{H,t}^* = n_t/s_t$ and $y_t^* = c_t^* = A^* n_t^*/s_t^*$ with price dispersion (5.26) and (5.42), export demand $c_{H,t}^* = \eta^* P_t^* S_t/P_{H,t} c_t^*$ and monetary policy rules (5.27) and (5.47) as well as the supply of foreign government bonds (5.43) for given $A^*$ and initial values $M_{F,-1}, M_{F,-1}^* \geq 0$, $B_{F,-1}, B_{F,-1}^*, B_{-1}^{T*} > 0$ and $D_{F,-1} = -D_{F,-1}^*$, and $P_{-1}, P_{H,-1}, P_{-1}^*, S_{-1} > 0$. A summary of equilibrium conditions for the case of binding open market constraints is given in Appendix D.2.

## 5.3 Uncovered interest rate parity

In this section, I derive the uncovered interest rate parity conditions implied by the model economy. When open market constraints bind, the model gives rise to a modified UIP condition, which contains a liquidity premium. This condition collapses to the standard UIP condition when open market constraints do not bind.

### 5.3.1 A standard UIP condition

Assume that $\mu_t = \mu_t^* = 0$ so that the open market constraints in both economies, (5.1) and (5.2), do not bind. In steady state, this is the case if foreign monetary policy sets the long-run policy rate to $R^{m*} = \pi^*/\beta^*$. The foreign households' first order conditions (5.34)-(5.35) imply that in this case, there is no spread between interest rates on private and government debt, which must then equal the policy rate, $R_t^{D*} = R_t^* = R_t^{m*}$. Thus, there are no liquidity premiums when open market constraints do not bind. Consider the home households' first order conditions for investment in domestic private debt and foreign government bonds, (5.14) and (5.16) which are repeated here for convenience

$$\lambda_t = \beta E_t \frac{\lambda_{t+1} R_t}{\pi_{t+1}},$$

$$\lambda_t = \beta E_t \frac{S_{t+1}}{S_t} \frac{\lambda_{t+1}}{\pi_{t+1}} R_t^*,$$

and where I have used the definition of the real exchange rate (5.53). Combining the two equations yields

$$R_t \left[ E_t \lambda_{t+1} E_t \pi_{t+1}^{-1} + Cov \left( \lambda_{t+1}, \pi_{t+1}^{-1} \right) \right]$$
$$= \frac{R_t^*}{S_t} \left\{ Cov \left( \lambda_{t+1} S_{t+1}, \pi_{t+1}^{-1} \right) + E_t \pi_{t+1}^{-1} \left[ Cov \left( \lambda_{t+1}, S_{t+1} \right) + E_t S_{t+1} E_t \lambda_{t+1} \right] \right\}.$$

This can be rewritten as

$$E_t \frac{S_{t+1}}{S_t} = \frac{R_t}{R_t^*} + \Upsilon_t, \tag{5.58}$$

using that the Inada conditions imply $\lambda_t > 0 \; \forall t$ and where terms of order higher than one are summarized in $\Upsilon_t = \frac{1}{R_t^* E_t \lambda_{t+1} E_t \pi_{t+1}^{-1}} \left\{ R_t Cov \left( \lambda_{t+1}, \pi_{t+1}^{-1} \right) \right.$ $\left. - \frac{R_t^*}{S_t} Cov \left( \lambda_{t+1} S_{t+1}, \pi_{t+1}^{-1} \right) - \frac{R_t^*}{S_t} E_t \lambda_{t+1} Cov \left( \lambda_{t+1}, S_{t+1} \right) \right\}$. I am not interested in effects of order two and above and thus ignore covariance terms in the analysis in this and the following sections. Equation (5.58) is a standard uncovered interest rate parity condition, which can be found in many small open economy models, such as Galí and Monacelli (2005). It requires the expected nominal depreciation to be equal to the interest difference between the home and foreign economies.

## 5.3.2 A modified UIP condition

When open market constraints bind, $\mu_t, \mu_t^* > 0$, foreign government bonds will pay a lower interest rate compared to foreign private debt. The reason is that foreign government bonds can be exchanged into cash, which domestic households need to purchase internationally traded goods. Consider the domestic households' optimality conditions for investment into domestic and foreign private debt (5.14) and (5.15)

$$\lambda_t = \beta E_t \frac{\lambda_{t+1} R_t}{\pi_{t+1}},$$
$$\lambda_t = \beta E_t \frac{S_{t+1}}{S_t} \frac{\lambda_{t+1}}{\pi_{t+1}} R_t^{D*}, \tag{5.59}$$

where I have used the definition of the real exchange rate. Combining these two equations, summarizing terms of higher order in $\Upsilon_t'$ and using that $\lambda_t > 0$ yields

$$E_t \frac{S_{t+1}}{S_t} = \frac{R_t}{R_t^{D*}} + \Upsilon_t', \tag{5.60}$$

where $\Upsilon'_t = \frac{1}{R_t^{D*}E_t\lambda_{t+1}E_t\pi_{t+1}^{-1}}\left\{R_tCov\left(\lambda_{t+1},\pi_{t+1}^{-1}\right) - \frac{R_t^{D*}}{S_t}\left[Cov\left(\lambda_{t+1}S_{t+1},\pi_{t+1}^{-1}\right)\right.\right.$
$\left.\left.+E_t\pi_{t+1}^{-1}Cov\left(\lambda_{t+1},S_{t+1}\right)\right]\right\}$. Thus, a standard UIP condition holds with respect to the interest rate difference in terms of the foreign debt rate $R_t^{D*}$. This rate is usually not observable. To obtain a UIP condition in the observable interest rate difference of home to foreign government bonds, I use the domestic households' optimality condition for investment into foreign government bonds, $\lambda_t = \beta E_t\frac{S_{t+1}}{S_t}\frac{\lambda_{t+1}+\mu_{t+1}}{\pi_{t+1}}R_t^*$. Combining this with (5.59) gives

$$R_tE_t\frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{R_t^*}{S_t}E_tS_{t+1}\frac{\theta_{t+1}}{\pi_{t+1}},$$

which can be written as

$$R_t\left[E_t\lambda_{t+1}E_t\pi_{t+1}^{-1} + Cov\left(\lambda_{t+1},\pi_{t+1}^{-1}\right)\right]$$
$$=\frac{R_t^*}{S_t}\left\{Cov\left(\pi_{t+1}^{-1},S_{t+1}\theta_{t+1}\right) + E_t\pi_{t+1}^{-1}\left[Cov\left(S_{t+1},\theta_{t+1}\right) + E_tS_{t+1}E_t\theta_{t+1}\right]\right\}.$$

This yields a modified UIP condition,

$$E_t\frac{S_{t+1}}{S_t} = \frac{R_t}{R_t^*\theta_t} + \Upsilon''_t, \tag{5.61}$$

where $\theta_t = \left(1 + E_t\frac{\mu_{t+1}}{\lambda_{t+1}}\right)$ and with terms of higher order summarized in $\Upsilon''_t = \frac{1}{R_t^*\theta_tE_t\lambda_{t+1}E_t\pi_{t+1}^{-1}}\left\{R_tCov\left(\lambda_{t+1},\pi_{t+1}^{-1}\right) - \frac{R_t^*}{S_t}Cov\left(\pi_{t+1}^{-1},S_{t+1}\theta_{t+1}\right)\right.$
$\left.-\frac{R_t^*}{S_t}E_t\pi_{t+1}^{-1}Cov\left(S_{t+1},\theta_{t+1}\right)\right\}$. Thus, the interest rate difference between home and foreign government bonds is not the only determinant of exchange rate behavior. When the open market constraint in the home economy binds, $\mu_t > 0$, the term $\theta_t$ exceeds unity, reflecting the liquidity value of foreign government bonds. (5.60) and (5.61) imply that

$$\theta_t = \frac{R_t^{D*}}{R_t^*}\left(1 + \Upsilon'''_t\right), \tag{5.62}$$

where $\Upsilon'''_t = (\Upsilon'_t - \Upsilon''_t)$ summarizes higher order terms. The interest rate spread $R_t^{D*}/R_t^*$ represents the opportunity cost of holding foreign government bonds, which in equilibrium, up to first order, will be equal to the premium $\theta_t$. This premium captures the marginal liquidity value of holding foreign government bonds and will thus be called a liquidity premium.

## 5.4 Monetary policy and exchange rates

The goal of this section is to analyze the response of the exchange rate to a foreign monetary policy shock when open market constraints bind, so that a non-standard UIP condition holds. Further, I analyze a log-linear approximation to the equilibrium conditions around the model's steady state, which is derived in Appendix B.3. Let $\hat{x}_t = 100 \log(x_t/x)$ denote the percentage deviation of $x_t$ from its steady state $x$. The linearized version of (5.61) then reads

$$E_t \hat{S}_{t+1} - \hat{S}_t = \hat{R}_t - \hat{R}_t^* - \hat{\theta}_t, \qquad (5.63)$$

where $\hat{\theta}_t = \hat{R}_t^{D*} - \hat{R}_t^*$. The liquidity premium can be reexpressed as a function of the policy rate using that (5.35) and (5.36) imply $\hat{R}_t^* = E_t \hat{R}_{t+1}^{m*}$, so that

$$\hat{\theta}_t = \hat{R}_t^{D*} - E_t \hat{R}_{t+1}^{m*}. \qquad (5.64)$$

Because a closed form solution for the general model version cannot be derived, I analyze a simplified model version.

### 5.4.1 Flexible prices

Assume flexible prices in the foreign economy, so that (5.37) becomes $w_t^* = A^*$ and (5.38)-(5.40) become redundant. Further, assume a utility function of the form $u(c_t^*, n_t^*) = \log c_t^* - \chi^* n_t^*$ and an exogenous instrument rule for the foreign policy rate, $R_t^{m*} = (R^{m*})^{1-\rho^*} (R_{t-1}^{m*})^{\rho^*} \exp \varepsilon_t^*$.[52] Moreover, nominal growth of foreign government debt is given by $\Gamma^* = 1$, and the central bank targets zero steady state inflation, $\pi^* = 1$.[53] Further, I assume

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[52]Note that the model does not imply equilibrium indeterminacy under an interest rate peg, which would be the case in a standard small open economy model. The reason is that the supply of collateral determines the price level path in the long run and thus prevents indeterminacy.

[53]Existence of a steady state then requires a long-run policy rate of $R^{m*} = 1$ because a positive policy rate in the steady state would imply that the central bank in every period acquires a share of households' bond holdings. With a constant supply of bonds, this would imply that foreign households' holdings of foreign government bonds, and thus foreign consumption, would converge to zero. Note that in principle, the central bank could also target an inflation rate different from zero, as long as $\pi > \beta^*$ so that the cash constraints in both economies continue to bind. For a steady state to exist, the policy rate then must satisfy $R^{m*} = (\Omega/(\Omega\pi^* + \pi^* - 1))$. For details, see Appendix D.3.2.

that the impact of home households' holdings of foreign government bonds on foreign households' holdings $B_{F,t}^*$ is negligible, so that (5.57) collapses to $B_{F,t}^* = \Omega M_{F,t}^*$. This implies that the foreign allocation and price system are independent from the home economy.

It can be shown that a shock to the foreign policy rate $R_t^{m*}$ leads to an increase in the interest rate on foreign government debt which is more than compensated by a decline in the liquidity premium. Intuitively, the rising foreign policy rate makes it more costly to exchange government bonds for cash, so that the marginal liquidity value of holding foreign government bonds declines. This result is summarized in the following proposition.

**Proposition 5.1** *Consider the simplified model version. A foreign monetary policy shock then leads to a decline in the liquidity premium which is larger than the rise in the interest rate on foreign government bonds, $\hat{\theta}_t > \hat{R}_t^*$.*

**Proof.** See Appendix D.4. ∎

I now turn to exchange rate dynamics. Proposition 5.1 shows that in response to a contractionary foreign policy shock, the liquidity premium declines and overcompensates the rise in the government bond interest rate. Thus, at a constant home interest rate, the expected rate of depreciation $E_t \hat{S}_{t+1} - \hat{S}_t$ increases in order to compensate for the lower marginal benefit of investing into foreign government bonds. This result is in stark contrast to standard UIP conditions, which predict that a rise in the foreign interest rate (which in a standard model is identical to the foreign policy rate) leads to a decline in the expected rate of depreciation. This result is summarized in the following:

**Corollary 5.2** *Consider the effect of a rise in the foreign policy rate on exchange rates given a constant home interest rate in the simplified model version. When the open market constraints do not bind, a rise in $R_t^*$ leads to a decline in the expected rate of appreciation of the home currency, $E_t \hat{S}_{t+1} - \hat{S}_t < 0$. Under binding open market constraints, a positive shock to the foreign policy rate implies that the expected rate of depreciation is positive, $E_t \hat{S}_{t+1} - \hat{S}_t > 0$.*

To summarize, the liquidity premium leads to a modified UIP condition. Using a simplified version of the model presented in 5.2, I have shown that

endogenous movements in the liquidity premium can alter exchange rate dynamics to an extent that the sign of the exchange rate change can switch. This is in line with the empirical evidence by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008), who find that a foreign monetary shock lets the home currency depreciate for several quarters. Because it is difficult to derive analytical results for the full version of the model, I analyze a calibrated version in the next section.

## 5.4.2 Sticky prices

This section analyzes a calibrated version of the model economy with sticky prices in both economies, using a first-order approximation of the model's equilibrium conditions around the steady state.[54] Foreign monetary policy is assumed to set the long-run policy rate according to $R^{m*} < \frac{\pi^*}{\beta}$ and targets long-run inflation $\pi^* > \beta^*$ , so that the the open market and cash constraints in the home and the foreign economy bind in steady state (see Appendix D.3). I analyze the model in a local neighborhood of the steady state where shocks are sufficiently small so that open market and cash constraints continue to bind. Households in both economies are assumed to maximize utility functions of the form

$$u\left(c_t, n_t\right) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{n_t^{1+\omega}}{1 + \omega}, \tag{5.65}$$

$$u\left(c_t^*, n_t^*\right) = \upsilon_t^{c*} \frac{c_t^{*1-\sigma^*} - 1}{1 - \sigma^*} - \chi^* \frac{n_t^{*1+\omega^*}}{1 + \omega^*}, \tag{5.66}$$

where $\upsilon_t^{c*}$ is a shock to the marginal utility of consumption with a steady state value of unity.

### Calibration

Table 5.1 summarizes the calibration. With respect to the intertemporal substitution elasticity of consumption goods and the Frisch elasticity of labor supply, I choose $\sigma = \sigma^* = 1.5$ and $\omega = \omega^* = 1$, which I consider a reasonable trade-off between diverging estimates resulting from microeconomic and macroeconomic data: Card (1994) suggests a range of 0.2 to 0.5 for the Frisch elasticity while Smets and Wouters (2007) estimate it to

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[54]The full set of (non-linearised) equilibrium conditions can be found in Appendix D.2.

| | |
|---|---|
| Discount Factor | $\beta = \beta^* = 0.9889$ |
| Inverse of intertemporal substitution elasticity | $\sigma = \sigma^* = 1.5$ |
| Inverse of Frisch elasticity of labor supply | $\omega = \omega^* = 1$ |
| Openness home economy | $\eta = 0.27$ |
| Openness foreign economy | $\eta^* = 0.01$ |
| Subst. elasticity home and foreign varieties | $\varepsilon = \varepsilon^* = 10$ |
| Calvo price stickiness | $\phi = 0.85; \phi^* = 0.75$ |
| Taylor rule coefficients - Inflation | $w_\pi = w_\pi^* = 2$ |
| Taylor rule coefficients - Output | $w_y = 0.2, w_y^* = 0.1$ |
| Interest rate inertia | $\rho = 0.88; \rho^* = 0.80$ |
| Share of repos to outright purchases | $\Omega = 1.5$ |
| Steady state inflation | $\Gamma = 1.00575 = \pi^* = \pi$ |
| Steady state foreign policy rate | $R^{m*} = 1.0105$ |
| Steady state labor supply | $n = n^* = 0.33$ |
| Foreign labor productivity | $A^* = 10$ |
| Home net foreign asset position relative to imports (steady steady) | $\frac{b_F + d_F + m_F}{c_F} = -1$ |

Table 5.1: Paramater calibration

$\omega = 1.92$. With respect to the intertemporal substitutability of consumption, Barsky, Kimball, Juster, and Shapiro (1997) estimate an elasticity of 0.18 using micro data, implying a value of around 5 for $\sigma$. Macroeconomic data generally implies lower estimates, e.g. Smets and Wouters (2007) estimate $\sigma = 1.39$. I further choose $\chi$ and $\chi^*$ to calibrate working time in both economies to $n = n^* = 0.33$. Foreign labor productivity is set to $A^* = 10$, so that the relative size of the economies matches the ratio of Canadian to U.S. gross domestic product. I follow Justiniano and Preston's (2010) estimate of openness and price stickiness for Canada, $\eta = 0.27$ and $\phi = 0.85$. With respect to the foreign economy, I choose $\phi^* = 0.75$ as a compromise between the estimates of Smets and Wouters (2007), Justiniano and Primiceri (2008) and Justiniano and Preston (2010) for the United States, which range between 0.65 and 0.90. Monetary policy in both countries sets the interest rate according to a Taylor rule, where home policy is calibrated to $w_\pi = 2$, $w_y = 0.2$ and $\rho = 0.88$, as estimated by Justiniano and Preston (2010) for the Canadian economy. In the foreign economy, monetary policy is characterized by $w_\pi^* = 2$, $w_y^* = 0.1$ and $\rho^* = 0.80$, which is in line

with Smets and Wouters (2007) and Justiniano and Primiceri (2008), who estimate models with Bayesian techniques using U.S. data. The parameter $\Omega$ is chosen to match the observed share of reserves supplied in repurchase operations to total reserves, as in Reynard and Schabert (2009). The long-run inflation rate and the policy rate in the foreign economy are set to the 20-year averages of U.S. consumer price inflation and, respectively, the Federal Funds rate, $\pi^* = 1.00575$ and $R^{m*} = 1.0105$. The home central bank is assumed to adopt an identical long-run inflation target, $\pi = \pi^*$. The discount factor is assumed to be equal across both countries and calibrated to the liquidity premium, i.e. the spread between the debt rate $R^{D*}$ and the rate on foreign government bonds $R^*$. The debt rate is the interest rate on a safe but illiquid bond. I follow Canzoneri, Cumby, and Diba (2007) and calibrate the spread to 65 basis points, which equals the difference between the interest rate faced by high-quality (AAA) borrowers and the interest rate on 3 months Treasury bills. Because there is no asset without any liquidity value, it is likely that this figure underestimates the true liquidity premium. Thus, the discount factor is set to $\beta = \frac{\pi}{R^m + 65 \cdot 10^{-4}} = 0.9889$. Further, the home economy is assumed to be a net debtor in steady state, with debt equivalent to 100% of the home country's quarterly imports, $\frac{b_F + d_F + m_F}{c_F} = -1$. This is in line with the ratio of Canadian foreign debt to average imports over the past 20 years and leads to a ratio of debt to domestic absorption of 9%, as in Bouakez and Rebei (2008).[55]

## Responses to a shock to the foreign policy rate

This section analyzes the impact of a foreign monetary policy shock. Figure 5.1 shows the impact of a 12.5 basis point innovation to $R_t^{m*}$ on the foreign economy. All variables are in per cent deviations from steady state, $\widehat{z}_t = 100 \left[ \log(z_t) - \log(z) \right]$, except for interest rates and inflation, which are given in absolute deviations, $\hat{R}_t^* = 100 * (R_t^* - R^*)$. The increase in the foreign policy rate induces a decline in foreign consumption and a reduction in inflation in the foreign economy. Consumption responds in a hump-shaped way because a rising policy rate increases seigniorage and thus reduces households' bond holdings, which implies that consumption declines with a lag. Further, the increase in the policy rate reduces the liquidity value of government bonds, so that the interest rate on these rises. The nominal

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[55]Data on imports and net foreign debt were taken from Statistics Canada, Publications 67-202-X and 13-019-X.
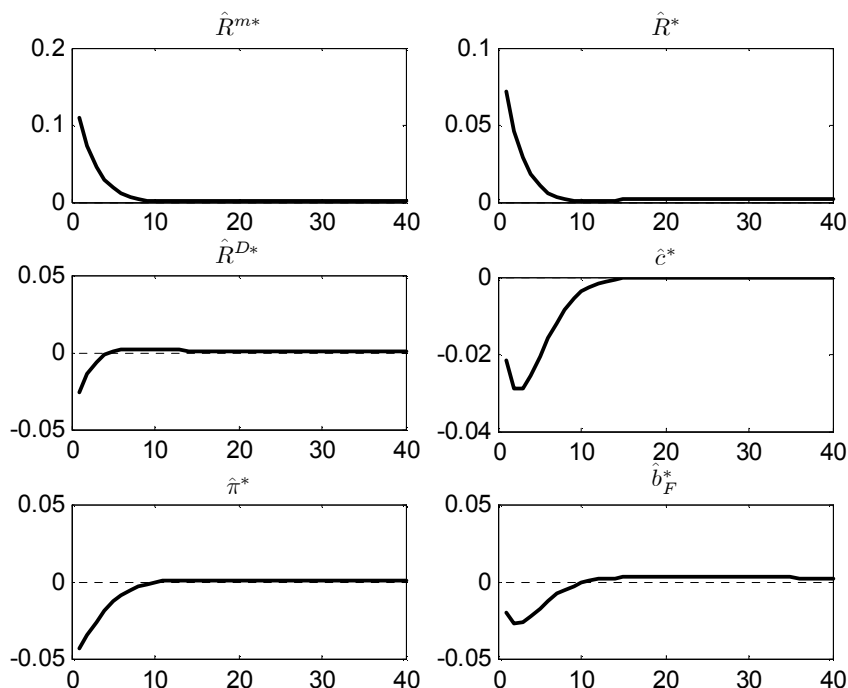
Figure 5.1: Responses to a foreign monetary policy shock in the foreign economy

interest rate on private debt declines because inflation falls.

Figure 5.2 shows the responses of the home economy. The foreign interest rate shock affects the home economy through different channels. First, it renders imports more expensive because foreign currency becomes more costly. Further, the decline in foreign consumption reduces export demand and implies that the home currency devalues both in nominal and real terms. This makes imports even more expensive for domestic households, who reduce consumption and increase worked hours, so that production rises. Turning attention to the exchange rate, a pattern different from that implied by standard models is observed: The nominal exchange rate depreciates on impact, and continues to depreciate until it peaks in the seventh quarter, consistent with Corollary 5.2. Thus, the model predicts delayed overshooting in line with the analysis by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008). The driving force behind delayed overshooting is
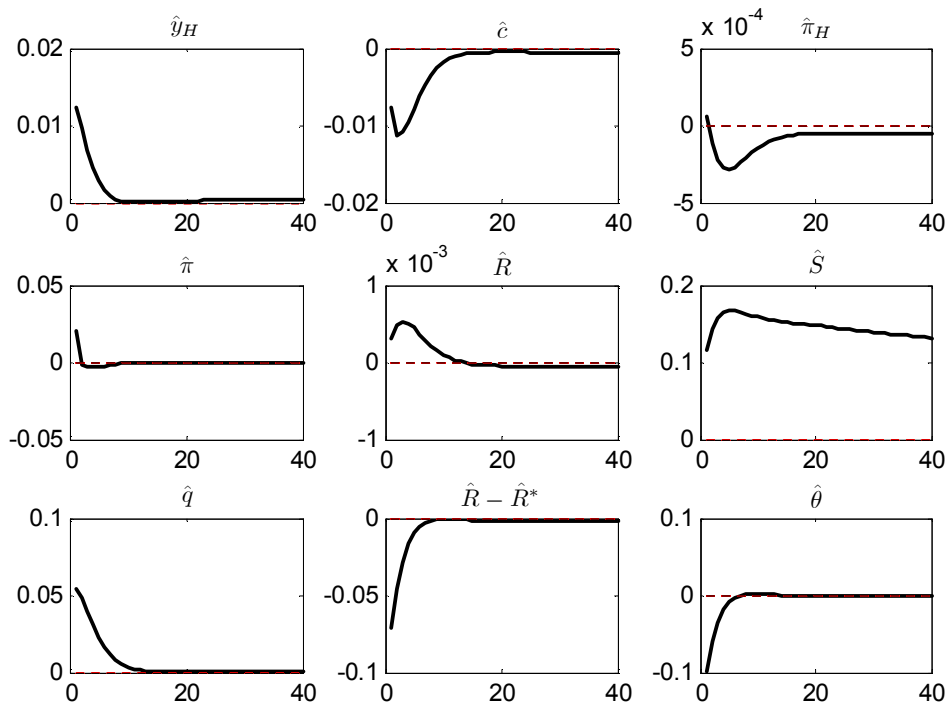
Figure 5.2: Responses to a foreign monetary policy shock in the home economy

the liquidity premium. A rising foreign policy rate implies that government bonds become less liquid, so that the liquidity premium declines. As in Proposition 5.1, the decline in the liquidity premium exceeds the increase in the foreign government bond interest rate.

With respect to the real exchange rate, the model does not predict delayed overshooting: In real terms, the domestic currency depreciates on impact, peaks in the shock period and then appreciates gradually back toward its steady state. The reason for the divergence between nominal and real exchange rates is the persistent decline in foreign inflation, which implies that the real rate of depreciation is negative while the rate of nominal depreciation is positive in the shock period. In line with the high observed correlation between real and nominal exchange rates, the VAR evidence quoted above predicts delayed overshooting for both the nominal and the real exchange rate. Although the key currency model does not predict

delayed overshooting for the real exchange rate, the liquidity premium increases the rate of real appreciation, so that real exchange rate movements are closer to the pattern observed by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008), as predicted by standard UIP.

**Comparing exchange rate dynamics to standard UIP**

This section compares exchange rate dynamics to those predicted by a standard UIP condition. In principle, the model without binding open market constraints is characterized by such a standard UIP. However, analyzing the impact of a shock to the foreign policy rate within the model without binding open market constraints would imply that, apart from the different UIP condition, general equilibrium effects would affect exchange rate movements. For instance, the reaction of inflation in the foreign economy would be different due to differences in monetary transmission. Thus, I construct a counterfactual scenario which shows how exchange rates would behave under a standard UIP condition, all other things equal.[56] Denoting ex ante real interest rates as $\hat{r}_t = \hat{R}_t - E_t\hat{\pi}_{t+1}$ and $\hat{r}_t^* = \hat{R}_t^* - E_t\hat{\pi}_{t+1}^*$, time series for the expected nominal and real exchange rates are constructed from standard UIP conditions

$$E_t\hat{S}_{t+1} - \hat{S}_t = \hat{R}_t - \hat{R}_t^*,$$
$$E_t\hat{q}_{t+1} - \hat{q}_t = \hat{r}_t - \hat{r}_t^*,$$

where the series for $\hat{R}_t - \hat{R}_t^*$ and $\hat{r}_t - \hat{r}_t^*$ are given by the responses to a foreign policy rate shock in the model with liquidity premiums. These are compared to the exchange rate movements which result when taking into account the liquidity premium, which are identical to those presented in Figure 5.2. Figure 5.3 shows the results of this analysis.

Under a conventional UIP, a rise in the foreign interest rate leads to an impact nominal depreciation, followed by a persistent appreciation. This is Dornbusch's (1976) famous "overshooting" result: The nominal exchange rate jumps on impact after a monetary shock and overshoots its new long-run equilibrium value. Given that the decline in the nominal interest rate

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[56]"All other things" also refers to the long-run equilibrium values for the nominal and real exchange rates. In other words, I assume that in the counterfactual scenario, the nominal and real exchange rates converge to long-run equilibrium values identical to those in the model with liquidity premiums. This assumption is required to compute the impact response of the exchange rates in the counterfactual scenario.
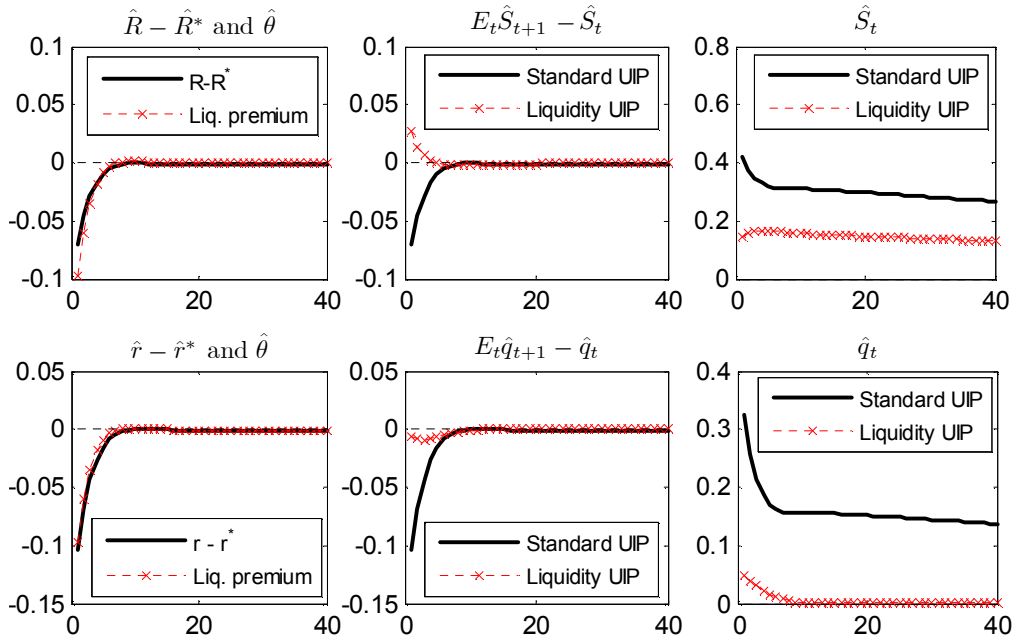
Figure 5.3: Comparison of exchange rate dynamics under standard and modified UIP

on foreign government bonds under sticky prices implies a decline in the real interest rate, the standard UIP condition predicts overshooting for the real exchange rate as well.

Taking into account movements of the liquidity premium fundamentally affects exchange rate dynamics: An increase in the foreign policy rate reduces the liquidity premium and leads to an impact depreciation of the domestic currency, as before. However, because the liquidity premium falls more strongly than the interest rate difference for the first seven quarters, in nominal terms the domestic currency continues to depreciate for seven quarters. Thus, the liquidity premium reverses the sign of the expected rate of nominal depreciation, compared to a standard UIP. Apart from the pattern of the response, also the timing of the peak, which occurs in the seventh quarter is in line with the estimates by Scholl and Uhlig (2008), who find that the median of the peak in the exchange rates of the U.S. dollar to the currencies of Germany, the U.K., Japan and a G7 basket occurs

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after 17-36 months.

The response of the real exchange rate under the modified UIP condition depends on real interest rates in both countries and the liquidity premium. The foreign monetary policy shock leads to a persistent decline in foreign inflation, which implies that the foreign real interest rate (on government bonds) increases more strongly than its nominal counterpart. Figure 4 shows that this leads to a decline in the real interest rate difference which slightly exceeds the decline in the liquidity premium, so that the real exchange rate will appreciate and return toward its steady state after its peak in the first period. Thus, the pattern of the real exchange rate's response to a foreign monetary policy shock under the modified UIP condition is similar to standard UIP. However, the decline in the liquidity premium moderates the appreciation after the peak, so that the predictions of the modified UIP condition become closer to the empirical evidence, which finds delayed overshooting for nominal and real exchange rates. To summarize, the nominal exchange rate predictions of the key currency model are in line with the empirical evidence. However, with respect to the real exchange rate, it cannot reconcile theory and evidence.

### 5.4.3   An exchange rate peg

This section asks how the modified UIP condition derived in this chapter affects the international transmission of foreign shocks when monetary policy in the small home economy is characterized by an exchange rate peg. This implies that the home interest rate follows

$$\hat{R}_t = \hat{R}_t^* + \hat{\theta}_t, \tag{5.67}$$

which characterizes arbitrage freeness between investment into home and foreign assets. (5.67) implies that the interest rate on foreign government bonds is not a sufficient indicator for the stance of monetary policy imposed upon the peg country. Rather, apart from the foreign interest rate, movements in the liquidity premium have an impact on home interest rates as well. The implications of this setup are examined more closely by analyzing the response of the home economy to a foreign demand shock. To examine how modeling key currency liquidity affects the international transmission of shocks, I compare these results to the impact of an increase in foreign consumption in a simplified version of the small open economy model by

Galí and Monacelli (2005). I rely on linear approximations of both models around the steady state.

Appendix D.5 shows that, in a simplified two-country version of Galí and Monacelli (2005), denoted GM in the following, a shock to foreign consumption leads to a real appreciation of the home currency, an increase in home consumption and a decline in production and producer price inflation in the home country. Rising foreign consumption increases export demand in the small open economy, triggering a real appreciation of the home currency. This leads to an expenditure switching effect, which implies that home households raise imports and reduce consumption of domestically produced goods. In the home economy, the decline in domestic demand outweighs the increase in export demand, so that production falls. However, the increase in imports due to the real appreciation permits households in the small open economy to increase total consumption.

Now, I turn to the impact of a shock to foreign households' valuation of consumption in the key currency model. I assume that $v_t^{c*}$ is given by the exogenous process $v_t^{c*} = \left(v_{t-1}^{c*}\right)^{\rho_c} \exp\left(\varepsilon_t^c\right)$ with an autocorrelation of $\rho_c = 0.75$ and where $\varepsilon_t^c$ is i.i.d. with $E_{t-1}\varepsilon_t = 0$ and a constant standard deviation $\sigma^c = 0.083$, as estimated by Christensen and Dib (2008) for the United States.[57] It is further assumed that foreign monetary policy is given by an interest rate peg, $R_t^{m*} = R^{m*}$. Figure 5.4 shows the impulse responses to a foreign demand shock which increases the marginal utility of consumption. In response to the shock, foreign households increase labor supply, so that wages and inflation decline. In the home country, the exchange rate peg leads to an increase in the interest rate. The reason is that rising demand in the foreign economy increases demand for foreign currency, so that the liquidity premium rises. Thus, despite a constant interest rate on foreign government bonds, which follows the pegged policy rate, monetary policy in the home economy is tightened to prevent a nominal depreciation of the home currency. This policy induces a decline in domestic demand

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[57]Note that this shock conceptually differs from the shock to foreign consumption analyzed above. The latter represents an exogenous increase in consumption, whose origin is not explained. In contrast, the shock to the foreign valuation of consumption is a change in preferences, which in equilibrium leads to an increase in foreign consumption. This shock also affects other variables in the foreign economy. However, the solution of the GM model derived in the appendix does not require any restrictions on foreign variables except for the foreign nominal interest rate, which is assumed to be constant in line with the pegged policy rate in the key currency model.
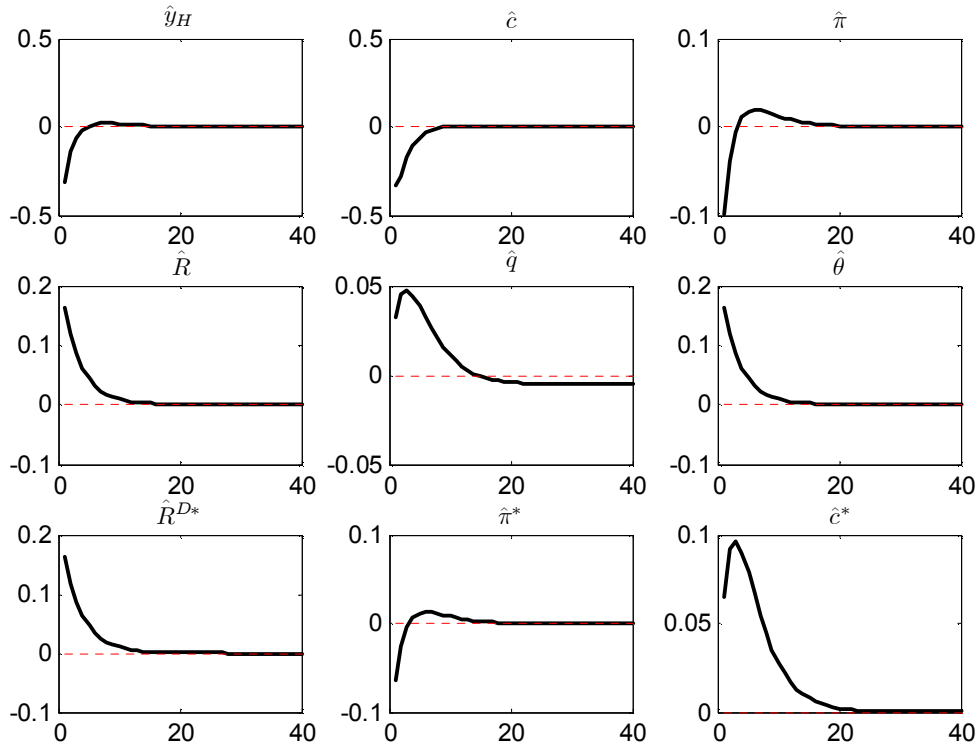
Figure 5.4: Impulse responses to a foreign demand shock

with the result that both production and consumption in the small open economy fall. This causes a decline in PPI and CPI inflation which is more pronounced than the decline in foreign inflation. Thus, the home currency devalues in real terms.

This result is in stark contrast to the GM model. The main differences lie in the exchange rate dynamics implied by both models and in the extent of expenditure switching they permit. In the GM model, increased export demand leads to a real appreciation, which induces domestic households to substitute toward foreign goods, triggering an increase in imports. However, when the supply of key currency is collateral constrained, home consumption of foreign goods cannot increase at the same time that foreign consumption rises. Increased scarcity of the key currency is reflected in a rising liquidity premium, which leads to a real depreciation of the home

currency. This mechanism prevents the expenditure switching observed in the GM model, so that imports decline. This effect, combined with tight domestic monetary policy implies that in the key currency model, domestic demand for both foreign and home goods, and thus aggregate domestic consumption, decline.

To summarize, key currency pricing and liquidity premiums can fundamentally affect the international transmission of shocks. This is most obvious under an exchange rate peg, where movements in the liquidity premium are transmitted directly to the home interest rate. However, this influence is also present under a flexible exchange rate regime, where the liquidity premium influences exchange rate dynamics and thus the relative price of international goods according to the modified UIP condition (5.61). In both cases, the international transmission of shocks is changed. In particular, the responses of interest rates and exchange rates are affected.

## 5.5    Conclusion

This chapter asks if the leading role of the U.S. dollar in international trade can explain observed deviations from uncovered interest rate parity, focusing on the impact of monetary policy shocks on exchange rates. The model implies that U.S. government bonds trade at a premium because they serve as collateral in the Federal Reserve's open market operations. I augment the setup of Reynard and Schabert (2009) and show that U.S. government bonds are held by agents outside the United States despite lower interest rates because they facilitate access to liquid dollar funds, which are required to carry out purchases of internationally traded goods.

This chapter shows that key currency effects give rise to liquidity premiums which imply deviations from uncovered interest rate parity. In line with the empirical evidence by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008), the model predicts delayed overshooting of the exchange rate in response to a monetary policy shock: When the Federal Reserve increases its policy rate, the interest rate on U.S. government bonds rises. However, the liquidity value of U.S. government bonds declines because it becomes more costly to obtain liquidity in exchange for these bonds. It is shown both analytically and in a calibrated version of the model economy that this decline in the liquidity value more than compensates the increase in the government bond interest rate, so that the marginal value of invest-

ing into U.S. government bonds falls. This implies that the expected rate of appreciation of the dollar increases. The calibrated model predicts an exchange rate peak after 7 quarters, which is in line with the findings of Eichenbaum and Evans (1995) and Scholl and Uhlig (2008). With respect to the real exchange rate, the model does not predict delayed overshooting. The reason is that inflation in the United States declines in response to the contractionary monetary policy, which turns the expected real rate of appreciation of the dollar negative. Features that reduce the speed of price adjustment, such as sticky import prices in the home economy, might contribute to further align evidence and theory with respect to real exchange rates.

This chapter further demonstrates the implications of modeling key currency liquidity for the transmission of international shocks, assuming for purposes of illustration that the central bank in the small open economy implements an exchange rate peg. A demand shock in the foreign economy then induces upward pressure on interest rates in the home country, even absent any change in the foreign policy rate, triggering a recession in the home economy. The reason is that increasing foreign demand leads to increased scarcity of key currency liquidity, implying rising liquidity premiums. This implies that the setup considered in this chapter fundamentally changes the international transmission of shocks. In particular, interest rates and exchange rates are affected by key currency effects. This is important because Justiniano and Preston (2010) attribute deficits of estimated New Keynesian models in explaining the international transmission of shocks to their problems in capturing exchange rate movements. It is outside the scope of this chapter to answer the question if estimating a model along the lines of this chapter can remove this deficit. However, I consider research into this direction promising because the liquidity premium considered in the present chapter has interesting quantitative properties: It fundamentally changes the predictions of standard uncovered interest rate parity in response to a monetary policy shock and can account for a hump-shaped response of consumption, as demonstrated by Reynard and Schabert (2009).

# Chapter 6

# Conclusion

This dissertation consists of applications and extensions of New Keynesian models which address a variety of research questions, proceeding from normative analysis of monetary policy in the second and third chapters to positive analysis in the latter chapters. Broadly understood, chapters 2 and 3 can be considered as a statement of caution toward policy makers and researchers which take the current consensus regarding the performance of Taylor rules as an irrevocable truth. Chapters 4 and 5 demonstrate the potential of the new neoclassical synthesis. Drawing on the basic idea of Reynard and Schabert (2009), they present applications of New Keynesian models which model monetary policy more carefully. The wide applicability of the framework by Reynard and Schabert (2009) is illustrated by this research, which covers topics ranging from unconventional monetary policy to the impact of monetary policy shocks on exchange rates. Apart from such positive analyses, research on the optimality of using the additional instruments available in this setup is promising. Its strict microfoundation makes a thorough welfare evaluation of monetary and fiscal policy in this model possible.

# Bibliography

ADAO, B., I. CORREIA, AND P. TELES (2003): "Gaps and Triangles," *Review of Economic Studies*, 70(4), 699–713.

ADOLFSON, M., S. LASEEN, J. LINDE, AND M. VILLANI (2007): "Bayesian estimation of an open economy DSGE model with incomplete pass-through," *Journal of International Economics*, 72(2), 481–511.

ALTIG, D., L. CHRISTIANO, M. EICHENBAUM, AND J. LINDE (2005): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," NBER Working Paper 11034.

ALVAREZ, F., A. ATKESON, AND P. J. KEHOE (2009): "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium," *Review of Economic Studies*, 76(3), 851–878.

ÁLVAREZ, L. J., E. DHYNE, M. HOEBERICHTS, C. KWAPIL, H. L. BIHAN, P. LÜNNEMANN, F. MARTINS, R. SABBATINI, H. STAHL, P. VERMEULEN, AND J. VILMUNEN (2006): "Sticky Prices in the Euro Area: A Summary of New Micro-Evidence," *Journal of the European Economic Association*, 4(2-3), 575–584.

ANGELONI, I., A. K. KASHYAP, B. MOJON, AND D. TERLIZZESE (2003): "The Output Composition Puzzle: A Difference in the Monetary Transmission Mechanism in the Euro Area and United States," *Journal of Money, Credit and Banking*, 35(6), 1265–1306.

ASCARI, G. (2004): "Staggered Prices and Trend Inflation: Some Nuisances," *Review of Economic Dynamics*, 7(3), 642–667.

AUERBACH, A. J., AND M. OBSTFELD (2005): "The Case for Open-Market Purchases in a Liquidity Trap," *American Economic Review*, 95(1), 110–137.

BACKUS, D. K., S. FORESI, AND C. I. TELMER (2001): "Affine Term Structure Models and the Forward Premium Anomaly," *Journal of Finance*, 56(1), 279–304.

BANSAL, R., AND M. DAHLQUIST (2000): "The forward premium puzzle: different tales from developed and emerging economies," *Journal of International Economics*, 51(1), 115–144.

BARSKY, R. B., M. S. KIMBALL, F. T. JUSTER, AND M. D. SHAPIRO (1997): "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *The Quarterly Journal of Economics*, 112(2), 537–79.

BASSETTO, M. (2004): "Negative Nominal Interest Rates," *American Economic Review*, 94(2), 104–108.

BEAUDRY, P., AND F. PORTIER (2006): "Stock Prices, News, and Economic Fluctuations," *American Economic Review*, 96(4), 1293–1307.

BENHABIB, J., S. SCHMITT-GROHE, AND M. URIBE (2001): "The Perils of Taylor Rules," *Journal of Economic Theory*, 96(1-2), 40–69.

——— (2002): "Avoiding Liquidity Traps," *Journal of Political Economy*, 110(3), 535–563.

BERNANKE, B. S., V. R. REINHART, AND B. P. SACK (2004): "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," *Brookings Papers on Economic Activity*, 35(2), 1–100.

BLANCHARD, O. (2000): "What Do We Know About Macroeconomics That Fisher And Wicksell Did Not?," *The Quarterly Journal of Economics*, 115(4), 1375–1409.

BLANCHARD, O. J., AND N. KIYOTAKI (1987): "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, 77(4), 647–66.

BOUAKEZ, H., AND N. REBEI (2008): "Has exchange rate pass-through really declined? Evidence from Canada," *Journal of International Economics*, 75(2), 249–267.

BUITER, W. (2008): "Central banks and financial crises," Discussion paper, prepared for the Jackson Hole symposium on Maintaining Stability in a Changing Financial System, August 21-23, 2008.

BURNSIDE, C., M. EICHENBAUM, AND J. D. M. FISHER (2004): "Fiscal shocks and their consequences," *Journal of Economic Theory*, 115(1), 89–117.

CALVO, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.

CAMPBELL, J. Y., AND J. COCHRANE (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107(2), 205–251.

CANZONERI, M. B., R. E. CUMBY, AND B. T. DIBA (2007): "Euler equations and money market interest rates: A challenge for monetary policy models," *Journal of Monetary Economics*, 54(7), 1863–1881.

CARD, D. (1994): "Intertemporal Labor Supply: An Assessment," in *Advances in Econometrics: Sixth World Congress*, ed. by C. Sims, vol. 2, pp. 49–78. Cambridge University Press.

CARLSTROM, C. T., AND T. S. FUERST (2005): "Investment and interest rate policy: a discrete time analysis," *Journal of Economic Theory*, 123(1), 4–20.

CASARES, M., AND B. T. MCCALLUM (2006): "An optimizing IS-LM framework with endogenous investment," *Journal of Macroeconomics*, 28(4), 621–644.

CHARI, V. V., P. J. KEHOE, AND E. R. MCGRATTAN (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.

———— (2009): "New Keynesian Models: Not Yet Useful for Policy Analysis," *American Economic Journal: Macroeconomics*, 1(1), 242–66.

CHINN, M. D. (2006): "The (partial) rehabilitation of interest rate parity in the floating rate era: Longer horizons, alternative expectations, and emerging markets," *Journal of International Money and Finance*, 25(1), 7–21.

CHRISTENSEN, I., AND A. DIB (2008): "The Financial Accelerator in an Estimated New Keynesian Model," *Review of Economic Dynamics*, 11(1), 155–178.

CHRISTENSEN, J. (2008): "The corporate bond credit spread puzzle," *FRBSF Economic Letter*, 2008(10).

CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

CHRISTIANO, L. J., AND J. FISHER (1995): "Tobin's q and Asset Returns: Implications for Business Cycle Analysis," NBER Working Paper 5292.

CLARIDA, R., J. GALÍ, AND M. GERTLER (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37(4), 1661–1707.

COENEN, G., AND V. WIELAND (2003): "The zero-interest-rate bound and the role of the exchange rate for monetary policy in Japan," *Journal of Monetary Economics*, 50(5), 1071–1101.

COLLIN-DUFRESNE, P. (2001): "The Determinants of Credit Spread Changes," *Journal of Finance*, 56(6), 2177–2207.

CORSETTI, G., AND P. PESENTI (2005): "International dimensions of optimal monetary policy," *Journal of Monetary Economics*, 52(2), 281–305.

CÚRDIA, V., AND M. WOODFORD (2010): "The Central-Bank Balance Sheet as an Instrument of Monetary Policy," NBER Working Paper 16208.

DEL NEGRO, M., G. EGGERTSSON, A. FERRERO, AND N. KIYOTAKI (2010): "The Great Escape? A Quantitative Evaluation of the Fed's Non-Standard Policies," Discussion paper, Federal Reserve Bank of San

Francisco Conference on Financial Market Imperfections and Macroeconomics, March 5, 2010.

DORNBUSCH, R. (1976): "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*, 84(6), 1161–76.

EGGERTSSON, G. B. (2006): "The Deflation Bias and Committing to Being Irresponsible," *Journal of Money, Credit and Banking*, 38(2), 283–321.

EGGERTSSON, G. B., AND M. WOODFORD (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34(1), 139–235.

———— (2004): "Policy Options in a Liquidity Trap," *American Economic Review*, 94(2), 76–79.

EICHENBAUM, M., AND C. L. EVANS (1995): "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *The Quarterly Journal of Economics*, 110(4), 975–1009.

ENGEL, C. (1996): "The forward discount anomaly and the risk premium: A survey of recent evidence," *Journal of Empirical Finance*, 3(2), 123–192.

ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, 46(2), 281–313.

EVANS, G. W., E. GUSE, AND S. HONKAPOHJA (2008): "Liquidity traps, learning and stagnation," *European Economic Review*, 52(8), 1438–1463.

FAMA, E. F. (1984): "Forward and spot exchange rates," *Journal of Monetary Economics*, 14(3), 319–338.

FAUST, J., AND J. H. ROGERS (2003): "Monetary policy's role in exchange rate behavior," *Journal of Monetary Economics*, 50(7), 1403–1424.

FREEDMAN, C., M. JOHNSON, O. KAMENIK, AND D. LAXTON (2009): "The implications of non-conventional monetary easing and fiscal deficit for interest rates and the economy: Some scenarios based on GPM and GIMF," Discussion paper, IMF Fiscal Affairs and Research Departments, Conference on Fiscal Policy, June 2, 2009.

FROOT, K. A. (1990): "Short Rates and Expected Asset Returns," NBER Working Paper 3247.

FROOT, K. A., AND R. THALER (1984): "Anomalies: foreign exchange," *Journal of Economic Perspectives*, 4(3), 179–192.

FUHRER, J. C., AND B. F. MADIGAN (1997): "Monetary Policy When Interest Rates Are Bounded At Zero," *The Review of Economics and Statistics*, 79(4), 573–585.

GALÍ, J. (2008): *Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

GALÍ, J., M. GERTLER, AND J. D. LÓPEZ-SALIDO (2001): "European inflation dynamics," *European Economic Review*, 45(7), 1237–1270.

——— (2007): "Markups, Gaps, and the Welfare Costs of Business Fluctuations," *The Review of Economics and Statistics*, 89(1), 44–59.

GALÍ, J., J. D. LÓPEZ-SALIDO, AND J. VALLÉS (2004): "Rule-of-Thumb Consumers and the Design of Interest Rate Rules," *Journal of Money, Credit, and Banking*, 36(4), 739–763.

GALÍ, J., AND T. MONACELLI (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72(3), 707–734.

GERTLER, M., AND P. KARADI (2010): "A Model of Unconventional Monetary Policy," Manuscript, New York University.

GOODFRIEND, M. (2000): "Overcoming the Zero Bound on Interest Rate Policy," *Journal of Money, Credit and Banking*, 32(4), 1007–35.

GOODFRIEND, M., AND R. KING (1997): "The New Neoclassical Synthesis and the Role of Monetary Policy," in *NBER Macroeconomics Annual 1997, Volume 12*, pp. 231–296.

GOURINCHAS, P.-O., AND A. TORNELL (2004): "Exchange rate puzzles and distorted beliefs," *Journal of International Economics*, 64(2), 303–333.

GROTH, C., AND H. KHAN (2006): "Investment Adjustment Costs: An Empirical Assessment," Carleton Economic Papers 07-08, Carleton University, Department of Economics.

——— (2007): "Investment adjustment costs: evidence from UK and US industries," Bank of England working paper 332.

HALL, R. E. (2004): "Measuring Factor Adjustment Costs," *The Quarterly Journal of Economics*, 119(3), 899–927.

HONKAPOHJA, S., AND K. MITRA (2005a): "Performance of inflation targeting based on constant interest rate projections," *Journal of Economic Dynamics and Control*, 29(11), 1867–1892.

——— (2005b): "Performance of monetary policy with internal central bank forecasting," *Journal of Economic Dynamics and Control*, 29(4), 627–658.

HÖRMANN, M., AND A. SCHABERT (2009a): "An interest rate peg might be better than you think," Ruhr Economic Paper 115, TU Dortmund University.

——— (2009b): "An interest rate peg might be better than you think," *Economics Letters*, 105(2), 156–158.

HUANG, J.-Z., AND M. HUANG (2002): "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?," Working Paper FIN 02-40, New York University.

IRELAND, P. N. (2005): "The Liquidity Trap, The Real Balance Effect, And The Friedman Rule," *International Economic Review*, 46(4), 1271–1301.

JUNG, T., Y. TERANISHI, AND T. WATANABE (2005): "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit and Banking*, 37(5), 813–35.

JUSTINIANO, A., AND B. PRESTON (2010): "Can structural small open-economy models account for the influence of foreign disturbances?," *Journal of International Economics*, 81(1), 61–74.

JUSTINIANO, A., AND G. E. PRIMICERI (2008): "The Time-Varying Volatility of Macroeconomic Fluctuations," *American Economic Review*, 98(3), 604–41.

KIM, H., J. KIM, E. SCHAUMBURG, AND C. A. SIMS (2008): "Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models," *Journal of Economic Dynamics and Control*, 32(11), 3397–3414.

KIM, S., AND N. ROUBINI (2000): "Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach," *Journal of Monetary Economics*, 45(3), 561–586.

KIYOTAKI, N., AND J. MOORE (2008): "Liquidity, Monetary Policy and Business Cycles," Mimeo, Princeton University.

KRISHNAMURTHY, A., AND A. VISSING-JORGENSEN (2007): "The Demand for Treasury Debt," NBER Working Paper 12881.

KRUGMAN, P. R. (1998): "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 29(2), 137–206.

LEVIN, A., D. LÓPEZ-SALIDO, E. NELSON, AND Y. YUN (2010): "Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound," *International Journal of Central Banking*, 6(1), 143–189.

LJUNGQVIST, L., AND T. J. SARGENT (2004): *Recursive macroeconomic theory*. MIT Press, second edn.

LONG, JOHN B, J., AND C. I. PLOSSER (1983): "Real Business Cycles," *Journal of Political Economy*, 91(1), 39–69.

LONGSTAFF, F. A. (2004): "The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices," *Journal of Business*, 77(3), 511–526.

LONGSTAFF, F. A., S. MITHAL, AND E. NEIS (2005): "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *Journal of Finance*, 60(5), 2213–2253.

LUBIK, T., AND F. SCHORFHEIDE (2006): "A Bayesian Look at the New Open Economy Macroeconomics," in *NBER Macroeconomics Annual 2005, Volume 20*, pp. 313–382.

LUCAS, R. E. (1976): "Econometric policy evaluation: A critique," *Carnegie-Rochester Conference Series on Public Policy*, 1(1), 19–46.

——— (1987): *Models of Business Cycles*. Basil Blackwell, New York.

LUSTIG, H., AND A. VERDELHAN (2007): "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk," *American Economic Review*, 97(1), 89–117.

MANKIW, N. G. (1985): "Small Menu Costs and Large Business Cycles: A Macroeconomic Model," *The Quarterly Journal of Economics*, 100(2), 529–38.

MCCALLUM, B. T. (2000): "Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*, 32(4), 870–904.

MCCALLUM, B. T. (2006): "Misconceptions Regarding the Zero Lower Bound on Interest Rates," *Monetary and Economic Studies*, 24(S1), 13–26.

MCCALLUM, B. T., AND E. NELSON (1999): "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis," *Journal of Money, Credit and Banking*, 31(3), 296–316.

ORPHANIDES, A., AND J. C. WILLIAMS (2002): "Robust Monetary Policy Rules with Unknown Natural Rates," *Brookings Papers on Economic Activity*, 33(2), 63–118.

PRESCOTT, E. C. (1986): "Theory ahead of business-cycle measurement," *Carnegie-Rochester Conference Series on Public Policy*, 25(1), 11–44.

RABANAL, P. (2007): "Does inflation increase after a monetary policy tightening? Answers based on an estimated DSGE model," *Journal of Economic Dynamics and Control*, 31(3), 906–937.

REIS, R. (2010): "Interpreting the Unconventional U.S. Monetary Policy of 2007-09," NBER Working Paper 15662.

REYNARD, S., AND A. SCHABERT (2009): "Modeling Monetary Policy," Tinbergen Institute Discussion Papers 09-094/2, Tinbergen Institute.

ROTEMBERG, J., AND M. WOODFORD (1997): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual 1997, Volume 12*, pp. 297–361.

SARANTIS, N. (2006): "Testing the uncovered interest parity using traded volatility, a time-varying risk premium and heterogeneous expectations," *Journal of International Money and Finance*, 25(7), 1168–1186.

SCHMITT-GROHÉ, S., AND M. URIBE (2004a): "Optimal fiscal and monetary policy under sticky prices," *Journal of Economic Theory*, 114(2), 198–230.

SCHMITT-GROHÉ, S., AND M. URIBE (2004b): "Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle," NBER Working Paper 10724.

SCHMITT-GROHÉ, S., AND M. URIBE (2004c): "Solving dynamic general equilibrium models using a second-order approximation to the policy function," *Journal of Economic Dynamics and Control*, 28(4), 755–775.

SCHMITT-GROHÉ, S., AND M. URIBE (2006): "Optimal Simple and Implementable Monetary and Fiscal Rules: Expanded Version," NBER Working Paper 12402.

——— (2007): "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54(6), 1702–1725.

SCHOLL, A., AND H. UHLIG (2008): "New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates," *Journal of International Economics*, 76(1), 1–13.

SMETS, F., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

SUMMERS, L. (1991): "How Should Long-Term Monetary Policy Be Determined? Panel Discussion," *Journal of Money, Credit and Banking*, 23(3), 625–31.

SVEEN, T., AND L. WEINKE (2005): "New perspectives on capital, sticky prices, and the Taylor principle," *Journal of Economic Theory*, 123(1), 21–39.

——— (2006): "Firm-specific capital and welfare," Working Paper 2006/04, Norges Bank.

——— (2007): "Firm-specific capital, nominal rigidities, and the Taylor principle," *Journal of Economic Theory*, 136(1), 729–737.

SVENSSON, L. E. (2001): "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap," *Monetary and Economic Studies*, 19(S1), 277–312.

SVENSSON, L. E., AND M. WOODFORD (2004): "Implementing Optimal Policy through Inflation-Forecast Targeting," in *The Inflation-Targeting Debate*, pp. 19–92. Ben S. Bernanke and Michael Woodford.

TAYLOR, J. B. (1993): "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.

TAYLOR, M. P. (1995): "The Economics of Exchange Rates," *Journal of Economic Literature*, 33(1), 13–47.

VERDELHAN, A. (2010): "A Habit-Based Explanation of the Exchange Rate Risk Premium," *Journal of Finance*, 65(1), 123–146.

WALSH, C. (2009): "Using Monetary Policy to Stabilize Economic Activity," Discussion paper, prepared for the Jackson Hole symposium on Financial Stability and Macroeconomic Policy, 2009.

WOODFORD, M. (2001): "The Taylor Rule and Optimal Monetary Policy," *American Economic Review*, 91(2), 232–237.

——— (2003): *Interest and Prices*. Princeton University Press.

——— (2004): "Inflation targeting and optimal monetary policy," *Federal Reserve Bank of St. Louis Review*, pp. 15–42.

YELLEN, J. L. (2009): "U.S. monetary policy objectives in the short and long run," *FRBSF Economic Letter*, 2009(01-02).

# Appendix A

# Appendix to Chapter 2

## A.1 Discretionary policy

Under discretion, the CB minimizes its loss function w.r.t. $\hat{\pi}_t$ and $\hat{x}_t$, treating expectations as given: $\min_{\hat{\pi}_t,\hat{x}_t} \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \hat{\pi}_{t+i}^2 + \lambda \hat{x}_{t+i}^2 \right\}$, subject to $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t$. It is well established that the first order conditions for $\hat{\pi}_t$ and $\hat{x}_t$ under discretion lead to the targeting rule $\hat{\pi}_t = -\frac{\lambda}{\kappa}\hat{x}_t$ (see Woodford (2003)). Using the generic solution form $\hat{\pi}_t = a_u u_t$ and $\hat{x}_t = b_u u_t$ and plugging it into the Euler equation leads to $\hat{R}_t = [b_u \sigma(\rho - 1) + \rho a_u] u_t$. Further using $u_t = \frac{\hat{\pi}_t}{a_u}$ and that the target rule implies $\frac{a_u}{b_u} = -\frac{\lambda}{\kappa}$ yields $\hat{R}_t = \left( \frac{\kappa}{\lambda}\sigma(1-\rho) + \rho \right) \hat{\pi}_t$, which with $\frac{\kappa}{\lambda} = \epsilon$ leads to $\hat{R}_t = [\epsilon\sigma(1-\rho) + \rho] \hat{\pi}_t$ which is the policy used in section 2.2.1.

## A.2 Unconditional moments

Given the state space form for $\hat{\pi}_t$ in (2.4), the unconditional variance of $\hat{\pi}_t$, $Var\hat{\pi}_t$, can be computed by

$$Var\hat{\pi}_t = a_\pi^2 Var\hat{\pi}_{t-1} + a_u^2 Var\ u_t + 2a_\pi a_u Cov(u_t, \hat{\pi}_{t-1}).$$

We use that the unconditional expectation of $u_t$ satisfies $E(u_t) = 0$, so that $Cov(u_t, \hat{\pi}_{t-1}) = E(u_t \hat{\pi}_{t-1})$. We thus need to derive $E(u_t \hat{\pi}_{t-1})$. First iterate

$u_t$ and $\hat{\pi}_{t-1}$ backwards to obtain

$$u_t = \rho^t u_0 + \sum_{s=0}^{t-1} \rho^s \varepsilon_{t-s},$$

$$\hat{\pi}_{t-1} = a_\pi^t \hat{\pi}_{-1} + a_\pi^{t-1} a_u u_0 + \sum_{s=0}^{t-2} a_\pi^s a_u \left( \rho^{t-1-s} u_0 + \sum_{j=0}^{t-2-s} \rho^j \varepsilon_{t-1-s-j} \right).$$

Computing $E(u_t \hat{\pi}_{t-1})$ from these expressions and using $E(\varepsilon_n \varepsilon_m) = 0 \; \forall \; n \neq m$ yields

$$E(u_t \hat{\pi}_{t-1}) = \rho^t a_\pi^t E(u_0 \hat{\pi}_{-1}) + \rho^t a_\pi^{t-1} a_u E\left(u_0^2\right) + \rho^{2t-1} a_u \frac{1 - (a_\pi/\rho)^{t-1}}{1 - (a_\pi/\rho)} E\left(u_0^2\right)$$

$$+ E \sum_{s=0}^{t-2} a_\pi^s a_u \sum_{j=0}^{t-2-s} \varepsilon_{t-1-s-j}^2 \rho^{s+1+2j}. \tag{A.1}$$

We now simplify the last term in (A.1) by applying the formula for finite geometric sums. This yields

$$\sum_{s=0}^{t-2} a_\pi^s a_u \sum_{j=0}^{t-2-s} \varepsilon_{t-1-s-j}^2 \rho^{s+1+2j} = \left( a_u \rho \sum_{s=0}^{t-2} (a_\pi \rho)^s \sum_{j=0}^{t-2-s} \rho^{2j} \right) \sigma_\varepsilon^2$$

$$= \sigma_\varepsilon^2 a_u \rho \sum_{s=0}^{t-2} (a_\pi \rho)^s \frac{1 - \rho^{2(t-1-s)}}{1 - \rho^2}$$

$$= \sigma_\varepsilon^2 \frac{a_u \rho}{1 - \rho^2} \left( \frac{1 - (a_\pi \rho)^{t-1}}{1 - a_\pi \rho} - \rho^{2t-2} \frac{1 - \left(\frac{a_\pi}{\rho}\right)^{t-1}}{1 - \frac{a_\pi}{\rho}} \right).$$

where we used that the unconditional expectation $E\left(\varepsilon_t^2\right) = Var\; \varepsilon = \sigma_\varepsilon^2$. Substituting out the sums in (A.1) and using that stationarity of $u$ and $\hat{\pi}$ implies $E(u_0 \hat{\pi}_{-1}) = E(u_t \hat{\pi}_{t-1})$, we obtain

$$E(u_t \hat{\pi}_{t-1}) = \frac{1}{1 - \rho^t a_\pi^t} \left\{ \left[ \frac{\rho^t a_\pi^{t-1} a_u}{1 - \rho^2} \sigma_\varepsilon^2 + \frac{\rho^{2t-1} a_u}{1 - \rho^2} \frac{1 - \left(\frac{a_\pi}{\rho}\right)^{t-1}}{1 - \left(\frac{a_\pi}{\rho}\right)} \sigma_\varepsilon^2 \right] \right.$$

$$\left. + \left[ \sigma_\varepsilon^2 \frac{a_u \rho}{1 - \rho^2} \left( \frac{1 - (a_\pi \rho)^{t-1}}{1 - a_\pi \rho} - \rho^{2t-2} \frac{1 - \left(\frac{a_\pi}{\rho}\right)^{t-1}}{1 - \frac{a_\pi}{\rho}} \right) \right] \right\},$$

where we used $E\left(u_0^2\right) = Var\ u = \frac{\sigma_\varepsilon^2}{1-\rho^2}$. Cancelling identical terms and expanding the first term by $(1 - a_\pi \rho)$ produces

$$E(u_t \hat{\pi}_{t-1}) = \frac{a_u \rho}{1 - \rho^t a_\pi^t} \left[ \frac{(\rho a_\pi)^{t-1} - (\rho a_\pi)^t}{1 - \rho^2} \frac{1}{1 - a_\pi \rho} + \frac{1}{1 - \rho^2} \frac{1 - (a_\pi \rho)^{t-1}}{1 - a_\pi \rho} \right] \sigma_\varepsilon^2$$

$$= \frac{1}{1 - \rho^t a_\pi^t} \frac{a_u \rho}{(1 - \rho^2)(1 - a_\pi \rho)} \left\{ \left[ 1 - \rho^t a_\pi^t \right] \sigma_\varepsilon^2 \right\}.$$

This expression can be simplified to yield

$$Cov(u, \hat{\pi}) = a_u \frac{\rho}{(1 - \rho^2)(1 - a_\pi \rho)} \sigma_\varepsilon^2, \quad \text{where } Cov(u, \hat{\pi}) = E(u_t \hat{\pi}_{t-1}).$$

Using this and that stationarity of $\hat{\pi}$ implies $Var\ \hat{\pi}_t = Var\ \hat{\pi}_{t-1} = Var\ \hat{\pi}$, we can compute the unconditional variance of inflation as $Var\ \hat{\pi} = \frac{1}{1-a_\pi^2} \left[ a_u^2 Var\ u + 2 a_\pi a_u Cov(u, \hat{\pi}) \right]$.

# Appendix B

# Appendix to Chapter 3

## B.1  Equilibrium conditions

This section contains derivations of the model's equilibrium conditions concerning inflation and price dispersion. It further presents the model's fiscal policy, derives the aggregate resource constraint and closes with a summary of the model.

### B.1.1  Inflation, price dispersion and fiscal policy

**Inflation**

Inflation can be derived from the definition of the price index, which is given by $P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di$. To show this, use that reoptimizing firms all set identical prices $Z_t$. Further, the share of firms which were allowed to reoptimize $k$ periods ago is $\phi^k(1-\phi)$ which allows to write the price index as

$$
\begin{aligned}
P_t^{1-\varepsilon} &= \int_0^1 P_{it}^{1-\varepsilon} di \\
&= (1-\phi) \left[ Z_t^{1-\varepsilon} + \phi Z_{t-1}^{1-\varepsilon} + \phi^2 Z_{t-2}^{1-\varepsilon} + ... \right].
\end{aligned}
$$

Lagging this expression by one period and pre-multiplying with $\phi$ gives

$$
\phi P_{t-1}^{1-\varepsilon} = (1-\phi) \left[ \phi Z_{t-1}^{1-\varepsilon} + \phi^2 Z_{t-2}^{1-\varepsilon} + \phi^3 Z_{t-3}^{1-\varepsilon} + ... \right],
$$

so that the price index follows

$$
P_t^{1-\varepsilon} = (1-\phi) Z_t^{1-\varepsilon} + \phi P_{t-1}^{1-\varepsilon}.
$$

Dividing by $P_t^{1-\varepsilon}$, this implies

$$1 = (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}.$$

## Price dispersion

It is possible to derive a recursive representation of price dispersion $s_t$ so that it is not necessary to track individual prices of firms (following Schmitt-Grohé and Uribe (2004b)). I again use that $Z_t$ is identical across all firms which set a new price:

$$
\begin{aligned}
s_t &\equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di \\
&= (1-\phi)\left(\frac{Z_t}{P_t}\right)^{-\varepsilon} + (1-\phi)\phi\left(\frac{Z_{t-1}}{P_t}\right)^{-\varepsilon} + (1-\phi)\phi^2\left(\frac{Z_{t-2}}{P_t}\right)^{-\varepsilon} + \dots \\
&= (1-\phi)\tilde{Z}_t^{-\varepsilon} + (1-\phi)\phi\tilde{Z}_{t-1}^{-\varepsilon}\pi_t^\varepsilon + (1-\phi)\phi^2\tilde{Z}_{t-2}^{-\varepsilon}\pi_t^\varepsilon\pi_{t-1}^\varepsilon + \dots \\
&= (1-\phi)\sum_{j=0}^\infty \phi^j \tilde{Z}_{t-j}^{-\varepsilon} \prod_{s=1}^j \pi_{t+1-s}^\varepsilon.
\end{aligned}
$$

Lagging this expression by one period gives

$$
\begin{aligned}
s_{t-1} &= (1-\phi)\sum_{j=0}^\infty \phi^j \tilde{Z}_{t-1-j}^{-\varepsilon} \prod_{s=1}^j \pi_{t-s}^\varepsilon \\
&= (1-\phi)\tilde{Z}_{t-1}^{-\varepsilon} + (1-\phi)\phi\tilde{Z}_{t-2}^{-\varepsilon}\pi_{t-1}^\varepsilon + (1-\phi)\phi^2\tilde{Z}_{t-3}^{-\varepsilon}\pi_{t-1}^\varepsilon\pi_{t-2}^\varepsilon + \dots
\end{aligned}
$$

Multiplying this by $\pi_t^\varepsilon\phi$ yields

$$
\begin{aligned}
s_{t-1}\pi_t^\varepsilon\phi &= (1-\phi)\phi\tilde{Z}_{t-1}^{-\varepsilon}\pi_t^\varepsilon + (1-\phi)\phi^2\tilde{Z}_{t-2}^{-\varepsilon}\pi_t^\varepsilon\pi_{t-1}^\varepsilon + (1-\phi)\phi^3\tilde{Z}_{t-3}^{-\varepsilon}\pi_t^\varepsilon\pi_{t-1}^\varepsilon\pi_{t-2}^\varepsilon \\
&= (1-\phi)\sum_{j=0}^\infty \phi^{j+1} \tilde{Z}_{t-1-j}^{-\varepsilon} \prod_{s=0}^j \pi_{t-s}^\varepsilon \\
&= (1-\phi)\sum_{j=1}^\infty \phi^j \tilde{Z}_{t-j}^{-\varepsilon} \prod_{s=1}^j \pi_{t+1-s}^\varepsilon.
\end{aligned}
$$

Thus, I can eliminate all lagged $Z$ terms by taking the difference

$$
\begin{aligned}
s_t - \phi s_{t-1}\pi_t^\varepsilon &= (1-\phi)\tilde{Z}_t^{-\varepsilon} \\
&\Longleftrightarrow \\
s_t &= (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^\varepsilon,
\end{aligned}
$$

which is the equation used in chapter 3, (3.17). Schmitt-Grohé and Uribe (2004b) show that $s_t$ is limited below by 1: Define $\xi_{it} = \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}$. From the definition of the price index, $\left(\int_0^1 \xi_{it} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1$. Given that $\varepsilon/(\varepsilon-1) > 1$, Jensen's inequality implies that $s_t = \int_0^1 \xi_{it}^{\frac{\varepsilon}{\varepsilon-1}} di \geq \left(\int_0^1 \xi_{it} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1$.

**Fiscal policy**

Expenditures for the production subsidy $\nu^P = \frac{\varepsilon}{\varepsilon-1}$ amount to

$$
\begin{aligned}
\tau_t^P &= \int_0^1 \left(\nu^P - 1\right) \frac{P_{it}}{P_t} y_{it} di \\
&= \left(\nu^P - 1\right) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t di \\
&= \left(\nu^P - 1\right) y_t,
\end{aligned}
$$

where I used the demand condition of firms, $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t$ and the definition of the price index, $\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} = 1$. Expenditures for the subsidy to workers are given by

$$
\tau_t^W = \left(\nu^W - 1\right) w_t n_t.
$$

Using constant returns to scale, the sum of firm profits $\Psi_t$ is given by

$$
\begin{aligned}
\Psi_t &= \int_0^1 \nu^P \frac{P_{it}}{P_t} y_{it} - mc_t y_{it} di \\
&= y_t \left[\nu^P \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} di - mc_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di\right] \\
&= y_t \left(\nu^P - mc_t s_t\right).
\end{aligned}
$$

Therefore, the difference of firm profits minus lump sum taxes (in real terms) equals

$$
\begin{aligned}
\Psi_t - \tau_t^W - \tau_t^P &= y_t \left(\nu^P - mc_t s_t\right) - \left(\nu^P - 1\right) y_t - \left(\nu^W - 1\right) w_t n_t \\
&= y_t \left(1 - mc_t s_t\right) - \left(\nu^W - 1\right) w_t n_t.
\end{aligned}
$$

## B.1.2   Aggregate resources

In the following the aggregate resource constraint is derived from the representative household's budget constraint, which reads

$$w_t n_t + \frac{d_{t-1}}{\pi_t} + y_t \left(1 - mc_t s_t\right) + k_t(r_t^k + 1 - \delta) = c_t + k_{t+1} + E_t r_{t,t+1}^d d_{j,t},$$

where I have used that $\nu^W w_t n_t + \Psi_t - \tau_t = w_t n_t + y_t \left(1 - mc_t s_t\right)$. In other words, because the government collects lump-sum taxes to finance the subsidy to firms and workers, the net revenue the household gains from receiving firm profits and the labor subsidy after paying the lump-sum tax equals $y_t \left(1 - mc_t s_t\right)$. Because the representative household cannot hold state-contingent claims which are traded only among households, in aggregate $d_t = 0 \ \forall \ t$. Thus, the constraint simplifies to

$$y_t \left(1 - mc_t s_t\right) + w_t n_t + k_t r_t^k = c_t + k_{t+1} - k_t(1 - \delta).$$

Using that the producers' first order conditions are given by $w_t = mc_t mpn_t$ and $r_t^k = mc_t mpk_t$, which implies that $w_t n_t + k_t r_t^k = mc_t IO_t$, this simplifies to

$$y_t \left(1 - mc_t s_t\right) + mc_t IO_t = c_t + k_{t+1} - k_t(1 - \delta).$$

With $y_t = \frac{IO_t}{s_t}$, this becomes

$$\frac{IO_t}{s_t} = c_t + k_{t+1} - k_t(1 - \delta),$$

which by using the definition of intermediate output $IO_t$ is equivalent to

$$k_t^{1-\alpha} n_t^\alpha = [c_t + k_{t+1} - (1 - \delta)k_t] \, s_t.$$

### B.1.3 Summary of equilibrium conditions

This section summarizes the equilibrium conditions for the case of zero adjustment cost. They are given by the behavior of households,

$$w_t = \chi n_t^{\eta} c_t^{\sigma} \mu_t^{W}, \tag{B.1}$$

$$c_t^{-\sigma} = \beta E_t \left[ c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right], \tag{B.2}$$

$$c_t^{-\sigma} = \beta E_t \left[ c_{t+1}^{-\sigma} (r_{t+1}^k + 1 - \delta) \right], \tag{B.3}$$

firms,

$$r_t^k = \frac{(1-\alpha)}{\alpha} \frac{w_t n_t}{k_t}, \tag{B.4}$$

$$mc_t = \left( \frac{r_t^k}{1-\alpha} \right)^{1-\alpha} \left( \frac{w_t}{\alpha} \right)^{\alpha}, \tag{B.5}$$

$$Z_t^1 = c_t^{-\sigma} y_t mc_t + \beta \phi E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1, \tag{B.6}$$

$$Z_t^2 = c_t^{-\sigma} y_t + \beta \phi E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2, \tag{B.7}$$

$$1 = (1-\phi) \left( \frac{Z_t^1}{Z_t^2} \right)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{B.8}$$

and aggregate resources including price dispersion,

$$k_t^{1-\alpha} n_t^{\alpha} = \left[ c_t + k_{t+1} - (1-\delta)k_t \right] s_t, \tag{B.9}$$

$$s_t = (1-\phi) \left( \frac{Z_t^1}{Z_t^2} \right)^{-\varepsilon} + \phi \pi_t^{\varepsilon} s_{t-1}. \tag{B.10}$$

To close the model, monetary policy is specified either by a Taylor rule or by Ramsey optimal policy.

## B.2 Optimal policy

### B.2.1 The social planner

The social planner maximizes household utility subject to the economy's technological restrictions. Because households' preferences are identical, the social planner will choose identical levels for consumption and working

time for every household, i.e. $n_{it} = n_t$ and $c_{it} = c_t \; \forall i$. He thus solves

$$\max_{\{c,n,n_i,k,k_i,y\}} G = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\chi n_t^{1+\eta}}{1+\eta} \right] \right.$$
$$+ \lambda_t^1 \left[ y_t - \left( \int_0^1 \left[ n_{it}^\alpha k_{it}^{1-\alpha} \right]^q di \right)^{1/q} \right]$$
$$+ \lambda_t^3 \left[ y_t - c_t - k_{t+1} + (1-\delta)k_t \right]$$
$$+ \lambda_t^4 \left[ k_t - \int_0^1 k_{it} di \right]$$
$$\left. + \lambda_t^5 \left[ n_t - \int_0^1 n_{it+1} di \right] \right\}.$$

The first order conditions are given by

$$u_{c,t} = \lambda_t^3, \tag{B.11}$$
$$u_{n,t} = -\lambda_t^5, \tag{B.12}$$
$$\lambda_t^5 = -\lambda_t^1 \left( \frac{y_t}{y_{it}} \right)^{1-q} \alpha \left( \frac{k_{it}}{n_{it}} \right)^{1-\alpha}, \tag{B.13}$$
$$-\lambda_t^1 = \lambda_t^3, \tag{B.14}$$
$$\lambda_t^3 = \beta E_t \left[ (1-\delta) \lambda_{t+1}^3 + \lambda_{t+1}^4 \right], \tag{B.15}$$
$$\lambda_t^4 = -\lambda_t^1 \left( \frac{y_t}{y_{it}} \right)^{1-q} (1-\alpha) \left( \frac{n_{it}}{k_{it}} \right)^\alpha. \tag{B.16}$$

(B.11)-(B.14) imply

$$-u_{n,t} = u_{c,t} \left( \frac{y_t}{y_{it}} \right)^{1-q} \alpha \left( \frac{k_{it}}{n_{it}} \right)^{1-\alpha}. \tag{B.17}$$

Combining (B.13) and (B.16) yields

$$\alpha \left( \frac{k_{it}}{n_{it}} \right)^{1-\alpha} \left( \lambda_t^5 \right)^{-1} = (1-\alpha) \left( \frac{n_{it}}{k_{it}} \right)^\alpha \left( \lambda_t^4 \right)^{-1}$$
$$\Longleftrightarrow$$
$$\frac{k_{it}}{n_{it}} = \frac{1-\alpha}{\alpha} \frac{\lambda_t^5}{\lambda_t^4}.$$

so that the capital labor ratio of firms is constant. This implies with (B.13) that the Lagrange multiplier on the labor market equilibrium condition is given by

$$\lambda_t^5 = -\lambda_t^1 \left(\frac{y_t}{y_{it}}\right)^{1-q} \alpha \left[\frac{1-\alpha}{\alpha}\frac{\lambda_t^5}{\lambda_t^4}\right]^{1-\alpha},$$

which requires $y_{it} = y_t$. Using this in (B.17) implies that the marginal rate of substitution must equal the marginal productivity of labor,

$$\alpha \left(\frac{k_{it}}{n_{it}}\right)^{1-\alpha} = -\frac{u_{n,t}}{u_{c,t}}.$$

Further, using (B.16) to replace $\lambda_t^4$ in (B.15) implies

$$u_{c,t} = \beta E_t u_{c,t+1} \left[(1-\delta) + (1-\alpha)\left(\frac{n_{it}}{k_{it}}\right)^\alpha\right].$$

## B.2.2   The Ramsey problem

This section derives the first order conditions to the Ramsey problem for the model with zero adjustment cost. The first order conditions for the Ramsey problem for the variants including adjustment cost are obtained in a similar way.

### Deriving the intertemporal budget constraint

Writing states explicitly with $z^t$ denoting the history of states up to period t and denoting the price of a state-contingent claim to a unit of currency with $Q_{t,t+1}(z^{t+1})$, the representative household's budget constraint can be written as

$$w_t(z^t)n_t(z^t) + \frac{d_{t-1}(z^t)}{\pi_t(z^t)} + y_t(z^t)\left(1 - mc_t(z^t)s_t(z^t)\right)$$

$$= c_t(z^t) + k_{t+1}(z^t) - k_t(z^{t-1})(r_t^k(z^t) + 1 - \delta) + \sum_{z^{t+1}} Q_{t,t+1}(z^{t+1})d_t(z^{t+1}),$$

which uses $\nu^W w_t n_t + \Psi_t - \tau_t = w_t n_t + y_t\left(1 - mc_t s_t\right)$, as was derived in section 3.2.3. The intertemporal budget constraint is derived by iterating the households' budget constraint forward for $d$. It will be useful to define

$q_{0,t}(z^t)$ as the "real" price of state contingent claims,

$$q_{0,t}(z^t) = pr(z^t|z^0)\varphi_{0,t}(z^t)\prod_{i=0}^{t-1}\pi_{i+1}(z^{i+1})$$
$$= \beta^t pr(z^t|z^0)\frac{\lambda_t(z^t)}{\lambda_0(z^0)},$$

where $\varphi_{0,t}(z^t) = \beta^t \frac{\lambda_t(z^t)/\lambda_0(z^0)}{\prod_{i=0}^{t-1}\pi_{k+1}(z^{i+1})}$ is the stochastic discount factor and $pr(z^t|z^0)$ is the conditional probability of state history $z^t$. Using the first order condition for state contingent claims $Q_{t,t+1}(z^{t+1}) = \beta\frac{pr(z^{t+1})}{pr(z^t)}\frac{\lambda_{t+1}(z^{t+1})}{\lambda_t(z^t)\pi_{t+1}(z^{t+1})}$, I can express $q_{0,t}(z^t)$ as

$$q_{0,t}(z^t) = \prod_{i=0}^{t-1} Q_{i,i+1}(z^{i+1})\pi_{i+1}(z^{i+1}).$$

Rearranging the budget constraint yields

$$d_{-1}(z^0) = \pi_0(z^0)\Omega_0(z^0) + \pi_0(z^0)\sum_{z^1} Q_{0,1}(z^1)d_0(z^1)$$
$$\Longleftrightarrow$$
$$d_{-1}(z^0) = \pi_0(\Omega_0) + \pi_0\sum_{z^1}\frac{q_{0,1}(z^1)}{\pi_1(z^1)}d_0(z^1). \qquad (B.18)$$

where $\Omega_t(z^t) = c_t(z^t) - w_t(z^t)n_t(z^t) - y_t(z^t)(1 - mc_t(z^t)s_t(z^t)) + k_{t+1}(z^t) - k_t(s^{t-1})(r_t^k(z^t) + 1 - \delta)$. Further, use

$$d_0(z^1) = \pi_1(z^1)\Omega_1(z^1) + \pi_1\sum_{z^2}\frac{q_{1,2}(z^2)}{\pi_2(z^2)}d_1(z^2)$$

to iterate (B.18) forward for $d$,

$$d_{-1}(z^0) = \pi_0(z^0)\Omega_0(z^0) + \pi_0(z^0)\sum_{z^1} q_{0,1}(z^1)\left(\Omega_1(z^1) + \frac{q_{1,2}(z^2)}{\pi_2(z^2)}d_1(z^2)\right)$$
$$= \pi_0(z^0)\Omega_0(z^0) + \pi_0(z^0)\sum_{z^1} q_{0,1}(z^1)\Omega_1(z^1)$$
$$+ \pi_0\sum_{z^1} q_{0,1}(z^1)q_{1,2}(z^2)\Omega_2(z^2) + ...,$$

so that

$$d_{-1}(z^0)/\pi_0(z^0) = \Omega_0(z^0) + \sum_{z^1} q_{0,1}(z^1)\Omega_1(z^1) + \sum_{z^2} q_{0,2}(z^2)\Omega_2(z^2) + \ldots$$

Now I can obtain an arbitrage freeness equation by first rewriting the capital Euler equation and then using $\frac{1}{\beta}\frac{Q_{t,t+1}(z^{t+1})}{p(z^{t+1}|z^t)}c_t^{-\sigma}(z^t)\pi_{t+1}(z^{t+1}) = c_{t+1}^{-\sigma}(z^{t+1})$,

$$c_t^{-\sigma}(z^t) = \beta E_t\left[c_{t+1}^{-\sigma}(q_{t+1}^k + 1 - \delta)\right]$$

$$c_t^{-\sigma}(z^t) = \beta \sum_{z^{t+1}} pr(z^{t+1}|z^t)c_{t+1}^{-\sigma}(z^{t+1})(r_{t+1}^k(z^{t+1}) + 1 - \delta)$$

$$1 = \sum_{z^{t+1}} Q_{t,t+1}(z^{t+1})\pi_{t+1}(z^{t+1})(r_{t+1}^k(z^{t+1}) + 1 - \delta)$$

$$1 = \sum_{z^{t+1}} q_{t,t+1}(z^{t+1})(r_{t+1}^k(z^{t+1}) + 1 - \delta).$$

This arbitrage freeness condition can be used to eliminate capital in the intertemporal budget constraint whose capital terms are

$$k_1(z^0) - k_0(z^{-1})(r_0^k(z^0) + 1 - \delta)$$
$$+ \sum_{z^1} q_{0,1}(z^1)\left[k_2(z^1) - k_1(z^0)(r_1^k(z^1) + 1 - \delta)\right]$$
$$+ \sum_{z^2} q_{0,2}(z^2)\left[k_3(z^2) - k_2(z^2)(r_2^k(z^2) + 1 - \delta)\right] + \ldots$$
$$= -k_0(z^{-1})(r_0^k(z^0) + 1 - \delta) + \lim_{t\longrightarrow\infty}\sum_{z^t} q_{0,t}(z^t)k_{t+1}(z^t),$$

where I have used that $k_1(z^0)$ is independent of $z^1$. The intertemporal budget constraint in period zero thus reads

$$\frac{d_{-1}(z^0)}{\pi_0(z^0)} + k_0(z^{-1})(r_0^k(z^0) + 1 - \delta)$$
$$= E_0 \sum_{t=0}^{\infty}\sum_{z^t} q_{0,t}(z^t)\left(c_t(z^t) - w_t(z^t)n_t(z^t) - y_t(z^t)\left(1 - mc_t(z^t)s_t(z^t)\right)\right)$$
$$+ \lim_{t\longrightarrow\infty}\sum_{z^t} q_{0,t}(z^t)k_{t+1}(z^t) + \lim_{t\longrightarrow\infty}\sum_{z^t} q_{0,t}(z^t)d_t(z^t).$$

The two latter terms are zero by the transversality conditions. The next step is to eliminate $q_{0,t}$ by using $q_{0,t}(z^t) = \beta^t pr(z^t|z^0)\frac{c_t^{-\sigma}(z^t)}{c_0^{-\sigma}(z^0)}$, so that the period zero intertemporal budget constraint is given by

$$c_0^{-\sigma}\left[\frac{d_{-1}}{\pi_0} + k_0(r_0^k + 1 - \delta)\right] = E_0 \sum_{t=0}^{\infty} \beta^t c_t^{-\sigma}\left(c_t - w_t n_t - y_t(1 - mc_t s_t)\right).$$

Now, the wage, capital rental rate and real marginal cost are eliminated, using

$$mc_t = \left(\frac{r_t^k}{1-\alpha}\right)^{1-\alpha}\left(\frac{w_t}{\alpha}\right)^{\alpha},$$

$$r_t^k = \frac{(1-\alpha)}{\alpha}\frac{w_t n_t}{k_t},$$

$$w_t = \chi n_t^{\eta} c_t^{\sigma} \mu_t^W,$$

$$\implies mc_t = \frac{1}{n_t^{\alpha} k_t^{1-\alpha}}\frac{\chi}{\alpha}\left(n_t^{1+\eta} c_t^{\sigma} \mu_t^W\right)$$

Further, using aggregate production $y_t = \frac{n_t^{\alpha} k_t^{1-\alpha}}{s_t}$ implies that

$$c_t^{-\sigma} y_t(1 - mc_t s_t) = c_t^{-\sigma}\frac{n_t^{\alpha} k_t^{1-\alpha}}{s_t} - c_t^{-\sigma} n_t^{\alpha} k_t^{1-\alpha} mc_t$$

$$= c_t^{-\sigma}\frac{n_t^{\alpha} k_t^{1-\alpha}}{s_t} - \frac{\chi}{\alpha} n_t^{1+\eta} \mu_t^W.$$

Thus, the period zero intertemporal budget constraint can be written as

$$A_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t^{1-\sigma} - \chi n_t^{1+\eta} \mu_t^W - c_t^{-\sigma}\frac{n_t^{\alpha} k_t^{1-\alpha}}{s_t} + \frac{\chi}{\alpha} n_t^{1+\eta} \mu_t^W\right]$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t^{1-\sigma} + \frac{1-\alpha}{\alpha}\chi n_t^{1+\eta} \mu_t^W - c_t^{-\sigma}\frac{n_t^{\alpha} k_t^{1-\alpha}}{s_t}\right],$$

where $A_t = c_t^{-\sigma}\left[\frac{d_{t-1}}{\pi_t} + k_t(r_t^k + 1 - \delta)\right]$. Because households trade state-contingent claims among each other, in the aggregate, no state contingent bonds can be held so that in equilibrium $d_t = \int d_{it} di = 0 \; \forall \; t$. Thus, the left hand side simplifies to $A_t = c_t^{-\sigma}\left[k_t(r_t^k + 1 - \delta)\right]$.

**Writing the intertemporal budget constraint recursively**

The Ramsey planner faces an intertemporal budget constraint in every period, as explained in section 3.3.3. Letting $\theta_t$ denote the multiplier on the intertemporal budget constraint, the Ramsey planner's Lagrange takes the form

$$
L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right]
$$
$$
+ \theta_t \left\{ E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[ c_{t+j}^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_{t+j}^{1+\eta} \mu_{t+j}^W - c_{t+j}^{-\sigma} \frac{IO_{t+j}}{s_{t+l}} \right] - A_t \right\}
$$
$$
+ \lambda_t^1 [\circ]
$$
$$
+ ...,
$$

neglecting the other constraints so as to focus on the intertemporal budget constraint. As described in Ljungqvist and Sargent (2004), the sequence of intertemporal budget constraints can be written recursively as

$$
E_0 \sum_{t=0}^{\infty} \beta^t \theta_t \left\{ E_t \sum_{j=0}^{\infty} \beta^{t+j} \left[ c_{t+j}^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_{t+j}^{1+\eta} \mu_{t+j}^W - c_{t+j}^{-\sigma} \frac{IO_{t+j}}{s_{t+l}} \right] - A_t \right\}
$$
$$
= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Theta_t \left[ c_t^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_t^{1+\eta} \mu_t^W - c_t^{-\sigma} \frac{IO_t}{s_t} \right] - (\Theta_t - \Theta_{t-1}) A_t \right\},
$$

where $\Theta_t = \Theta_{t-1} + \theta_t$ and $\Theta_{-1} = 0$.

## First order conditions to the Ramsey problem without adjustment cost

The Ramsey problem (3.27) reads

$$
\max_{\{c,n,k,R,s,\pi,\tilde{Z},Z^1,Z^2\}} J = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} \right]
$$

$$
+ \Theta_t \left[ c_t^{1-\sigma} + \frac{1-\alpha}{\alpha} \chi n_t^{1+\eta} \mu_t^W - c_t^{-\sigma} \frac{k_t^{1-\alpha} n_t^{\alpha}}{s_t} \right]
$$

$$
- (\Theta_t - \Theta_{t-1}) \left[ \frac{(1-\alpha)\chi}{\alpha} n_t^{\eta+1} \mu_t^W + k_t c_t^{-\sigma} (1-\delta) \right]
$$

$$
+ \lambda_t^1 \left[ c_t^{-\sigma} - E_t \beta c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right]
$$

$$
+ \lambda_t^2 \left[ c_t^{-\sigma} - E_t \beta (1-\alpha) \frac{\chi}{\alpha} \frac{n_{t+1}^{\eta+1}}{k_{t+1}} \mu_{t+1}^W - E_t \beta (1-\delta) c_{t+1}^{-\sigma} \right]
$$

$$
+ \lambda_t^3 \left[ \tilde{Z}_t - \frac{Z_t^1}{Z_t^2} \right]
$$

$$
+ \lambda_t^4 \left[ \pi_t - \left( \frac{1 + (\phi-1)\tilde{Z}_t^{1-\varepsilon}}{\phi} \right)^{\frac{1}{\varepsilon-1}} \right]
$$

$$
+ \lambda_t^5 \left[ s_t - (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi \pi_t^{\varepsilon} s_{t-1} \right]
$$

$$
+ \lambda_t^6 \left[ Z_t^1 - \frac{n_t^{1+\eta}}{s_t} \frac{\chi}{\alpha} \mu_t^W - \phi \beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1 \right]
$$

$$
+ \lambda_t^7 \left[ Z_t^2 - c_t^{-\sigma} \frac{k_t^{1-\alpha} n_t^{\alpha}}{s_t} - \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2 \right] .
$$

The first order conditions to this problem are

$$\frac{\partial J}{\Theta_t} = 0 \Leftrightarrow c_t^{1-\sigma} = \frac{(1-\alpha)\chi}{\alpha}\left[c_t^{-\sigma}\frac{IO_t}{s_t} - \beta n_{t+1}^{\eta+1}\mu_{t+1}^W\right] - \beta k_{t+1}c_{t+1}^{-\sigma}(1-\delta)$$

$$\frac{\partial J}{R_t} = 0 \;\Leftrightarrow\; \lambda_t^1 = 0 \;\forall\; t \quad \text{(which is used in all further equations)},$$

$$\frac{\partial J}{c_t} = 0 \;\Leftrightarrow\; 0 = c_t^{-\sigma}\left[1 + \Theta_t(1-\sigma)\right] + \sigma(1-\delta)\lambda_{t-1}^2 c_t^{-\sigma-1}$$

$$+\sigma c_t^{-\sigma-1}\left[\Theta_t\frac{k_t^{1-\alpha}n_t^\alpha}{s_t} + (1-\delta)k_t - \lambda_t^1 + \frac{R_{t-1}}{\pi_t}\lambda_{t-1}^1 - \lambda_t^2 + \lambda_t^7\frac{k_t^{1-\alpha}n_t^\alpha}{s_t}\right]$$

$$\frac{\partial J}{n_t} = 0 \;\Leftrightarrow\; c_t^{-\sigma}\frac{k_t^{1-\alpha}\alpha n_t^{\alpha-1}}{s_t}\left[\Theta_t + \lambda_t^7\right] + \chi n_t^\eta$$

$$= (1+\eta)\frac{\chi}{\alpha}\left[\Theta_{t-1}(1-\alpha)\mu_t^W n_t^\eta - \lambda_{t-1}^2(1-\alpha)\frac{\mu_t^W}{k_t}n_t^\eta - \lambda_t^6\frac{\mu_t^W}{s_t}n_t^\eta\right]$$

$$\frac{\partial J}{k_{t+1}} = 0 \;\Leftrightarrow\; (1-\delta)c_{t+1}^{-\sigma}\left(\Theta_t - \Theta_{t+1}\right) + \frac{(1-\alpha)\chi}{\alpha}\lambda_t^2 E_t\frac{n_{t+1}^{\eta+1}\mu_{t+1}^W}{(k_{t+1})^2}$$

$$= \frac{c_{t+1}^{-\sigma}}{s_{t+1}}(1-\alpha)k_{t+1}^{-\alpha}n_{t+1}^\alpha\left[\Theta_{t+1} + \lambda_{t+1}^7\right],$$

$$\frac{\partial J}{\pi_t} = 0 \;\Leftrightarrow\; \lambda_t^4 = \phi\left[\varepsilon s_{t-1}\lambda_t^5\pi_t^{\varepsilon-1}\varepsilon\lambda_{t-1}^6 Z_t^1\pi_t^{\varepsilon-1} + (\varepsilon-1)\lambda_{t-1}^7 Z_t^2\pi_t^{\varepsilon-2}\right],$$

$$\frac{\partial J}{Z_t^1} = 0 \;\Leftrightarrow\; -\frac{\lambda_t^3}{Z_t^2} + \lambda_t^6 - \lambda_{t-1}^6\phi\pi_t^\varepsilon = 0,$$

$$\frac{\partial J}{Z_t^2} = 0 \;\Leftrightarrow\; \lambda_t^3\frac{Z_t^1}{(Z_t^2)^2} + \lambda_t^7 - \lambda_{t-1}^7\phi\pi_t^{\varepsilon-1} = 0,$$

$$\frac{\partial J}{\tilde{Z}_t} = 0 \;\Leftrightarrow\; \lambda_t^3 + \varepsilon\lambda_t^5(1-\phi)\tilde{Z}_t^{-\varepsilon-1}$$

$$= -\lambda_t^4\left(\frac{1 + (\phi-1)\tilde{Z}_t^{1-\varepsilon}}{\phi}\right)^{\left(\frac{1}{\varepsilon-1}-1\right)}\frac{(\phi-1)}{\phi}(1-\varepsilon)\tilde{Z}_t^{-\varepsilon},$$

$$\frac{\partial J}{s_t} = 0 \;\Leftrightarrow\; \Theta_t c_t^{-\sigma}\frac{k_t^{1-\alpha}n_t^\alpha}{s_t^2} + \lambda_t^5$$

$$= \lambda_{t+1}^5\beta\phi\pi_{t+1}^\varepsilon - \frac{\chi}{\alpha}\lambda_t^5\frac{n_t^{\eta+1}}{s_t^2}\mu_t^W - \lambda_t^5 c_t^{-\sigma}\frac{k_t^{1-\alpha}n_t^\alpha}{s_t^2}.$$

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### B.2.3 Welfare measure

The welfare measure I use is derived in Schmitt-Grohé and Uribe (2006). It is based on the representative household's expected lifetime utility

$$W = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right).$$

**Conditional vs. unconditional welfare**  I will here follow Schmitt-Grohé and Uribe (2006) and evaluate welfare conditional on the initial state being the deterministic steady state.[58]  This is sensible because all models analyzed in the second chapter have the same deterministic steady state independent of policy.  For this reason, I do not face the problem of having to evaluate transition periods from one to another deterministic steady state, which is not straightforward when using local approximation methods.  However, I encounter a similar problem which stems from the fact that the models' stochastic steady states generally depend on monetary policy.  Therefore, an unconditional welfare measure will be biased because it ignores the transition to a particular stochastic steady state.

**Welfare measure**  When comparing two policies, the welfare measure I use is the percentage $\gamma$ of steady state consumption that a household would be willing to give up under the superior policy A so that he can still gain the same utility as under the inferior policy B (equivalent variation). Note that $\gamma$ can only be interpreted as percentage points of steady state consumption when A refers to the deterministic steady state.[59]  Let superscripts A and B denote a variable's value under policy A or B. Thus, I need to solve

$$W^B = E_0 \sum_{t=0}^{\infty} \beta^t u\left((1 - \gamma)\, c_t^A, n_t^A\right)$$

---

[58]This approach is also recommended by Kim, Kim, Schaumburg, and Sims (2008).

[59]This is the reason why I compute the welfare loss separately for each policy and do not directly compare simple rules to optimal policy.

for $\gamma$. Given that $\sigma = 1$, the utility function collapses to $u(c_t, n_t) = \ln(c_t) - \chi \frac{n^{1+\eta}}{1+\eta}$, so that the preceding equation can be rewritten as

$$
\begin{aligned}
W^B &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[(1-\gamma) c_t^A\right] - \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\
&= (1-\gamma) W^A - [1 - (1-\gamma)] E_0 \sum_{t=0}^{\infty} \beta^t \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta}.
\end{aligned}
$$

Rearranging terms yields

$$
\begin{aligned}
W^B &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[(1-\gamma) c_t^A\right] - \chi \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(1-\gamma) + \ln c_t^A - \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\
&= \frac{\ln(1-\gamma)}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t^A - \frac{\left(n_t^A\right)^{1+\eta}}{1+\eta} \right\} \\
&= \frac{\ln(1-\gamma)}{1-\beta} + W^A.
\end{aligned}
$$

Solving for $\gamma$ yields

$$
\begin{aligned}
\ln(1-\gamma) &= \left(W^B - W^A\right)(1-\beta) \\
&\Longleftrightarrow \\
1-\gamma &= \exp\left[\left(W^B - W^A\right)(1-\beta)\right] \\
&\Longleftrightarrow \\
\gamma &= 1 - \exp\left[\left(W^B - W^A\right)(1-\beta)\right].
\end{aligned}
$$

**Evaluating welfare from a second-order approximated policy function** Now define $V_t = E_t \sum_{s=0}^{\infty} \beta^s u\left(c_{t+s}, n_{t+s}\right)$. The idea of Schmitt-Grohé and Uribe (2006) is that by determining the solution of $V_t$ in terms of its policy function, $W$ is equal to the value of this policy function at the state $x_t$, which is assumed to be the deterministic steady state, $x_t = x$. Thus, I use standard methods to solve for the policy function of $V_t$ and evaluate it at the initial state. Schmitt-Grohé and Uribe (2004c) show that the solution

to an equation system which can be written as $E_t f(z_{t+1}, z_t, x_{t+1}, x_t) = 0$ is given by a policy function $z_t = g(x_t, \omega)$. $y$ here denotes non-predetermined variables (such as $V_t$) while $x$ includes endogenous and exogenous state variables. The parameter $\omega$ summarizes uncertainty in the model. As I analyze wage markup shocks only, $\omega$ equals the standard deviation of that shock. As mentioned above, welfare is evaluated conditional on the deterministic steady state. Thus, the policy function of interest is $V$, evaluated at the deterministic steady state $V(x, 0)$. Algebraically, this approach implies

$$W = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) = V(x, 0).$$

**Computing $\gamma$ from the approximated policy function for V**   Schmitt-Grohé and Uribe (2004c) demonstrate the application of perturbation methods to solve for the policy functions. A second-order approximation of the policy function for $V$ around the deterministic steady state yields

$$V(x_t, \omega) \approx V(x, 0) + V_x(x, 0)(x_t - x) + V_\omega(x, 0)\omega + V_{\sigma x}(x, 0)\omega(x_t - x)$$
$$+ \frac{1}{2} V_{xx}(x, 0)(x_t - x)^2 + \frac{1}{2} V_{\omega\omega}(x, 0)\omega^2,$$

using that in the deterministic steady state, $\omega = 0$. I now proceed along the lines of Schmitt-Grohé and Uribe (2006) to show that $\gamma$ can be approximated correctly up to second order from a second-order approximation of the policy function $V$. The welfare measure $\gamma$ is a function $\Lambda$ of states $x_t$ and the uncertainty parameter

$$\Lambda(x_t, \omega) = 1 - \exp\left\{ \left[ V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega) \right] (1 - \beta) \right\}.$$

Because welfare is evaluated conditional on the initial state being the deterministic steady state, it is sufficient to consider the derivatives of $\Lambda$ with respect to $\omega$, so that

$$\gamma \approx \Lambda(x, 0) + \Lambda_\omega(x, 0)\omega + \Lambda_{\omega\omega}(x, 0)\frac{\omega^2}{2}.$$

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Computing the derivatives yields

$$
\Lambda_\omega(x_t, \omega) = -\exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\}
$$
$$
* \left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\},
$$
$$
\Lambda_{\omega\omega}(x_t, \omega) = -\exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\}
$$
$$
* \left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\}^2
$$
$$
-\exp\left\{\left[V_\omega^B(x_t, \omega) - V_\omega^A(x_t, \omega)\right](1-\beta)\right\}
$$
$$
* \left\{\left[V_{\omega\omega}^B(x_t, \omega) - V_{\omega\omega}^A(x_t, \omega)\right](1-\beta)\right\}.
$$

Identical (deterministic) steady states imply $V^A(x,0) = V^B(x,0)$ and thus $\Lambda(x,0) = 0$. From Schmitt-Grohé and Uribe (2004c), $V_\omega^A(x,0) = V_\omega^B(x,0) = 0$, which implies that $\Lambda_\omega(x,0) = 0$. This can be used to simplify the second derivative, evaluated at the steady state, to

$$
\Lambda_{\omega\omega}(x,0) = -\left\{\left[V_{\omega\omega}^B(x,0) - V_{\omega\omega}^A(x,0)\right](1-\beta)\right\}.
$$

Thus, the welfare measure is given by

$$
\gamma \approx (1-\beta)\left[V_{\omega\omega}^A(x,0) - V_{\omega\omega}^B(x,0)\right]\frac{\omega^2}{2}.
$$

The terms $V_{\omega\omega}^A(x,0)\frac{\omega^2}{2}$ and $V_{\omega\omega}^B(x,0)\frac{\omega^2}{2}$ are obtained from the "correction terms" in *dynare*.

**Interpretation**  Because both policies have identical steady states, $V_{\omega\omega}^B(x,0) - V_{\omega\omega}^A(x,0)$ is the only welfare relevant difference between the two policies.[60] Thus, the welfare measure requires comparing the solution to the value functions $V$ under two policies, conditional on the same (deterministic) steady state. In principle, I could use the approximate solutions for the policy functions of $V$ conditional on any other steady state, as long as I condition on the same steady state for both policies. Here, I use the deterministic steady state so that the welfare effects implied by the transition from the deterministic steady state to the (different) stochastic steady states are taken into account.

---

[60] All other terms in the policy functions for $V^A$ and $V^B$ are zero in the deterministic steady state when variables are expressed as deviations from their deterministic steady state - except for the constant term $V^A(x,0)$ which is equal across any two policies with an identical deterministic steady state. Thus, computing $V_{\omega\omega}^B(x,0) - V_{\omega\omega}^A(x,0)$ is equivalent to computing $V^B(x,0) - V^A(x,0)$.

## B.3 Steady state

Here, the steady state of the model without adjustment cost is derived from the equilibrium conditions given in B.1.3. I proceed by assuming existence of a steady state and confirm its existence analytically. Let variables without time index refer to steady state values. (B.2) implies that the long-run nominal interest rate determines long-run inflation, $R = \frac{\pi}{\beta}$. Thus, by setting the long-run nominal interest rate, monetary policy determines long-run inflation. From section 3.3.1, observe that the first-best equilibrium is characterized by $\pi = 1$ and $\mu^T = \mu^W/mc = 1$. By setting $R = \beta^{-1}$, monetary policy ensures a long-run inflation rate of unity, thus implementing the first-best allocation. The reasoning behind this is as follows: In steady state, the wage markup $\mu^W = \frac{\zeta}{\zeta-1}\frac{1}{\nu^W}$ is eliminated by the wage subsidy $\nu^W = \frac{\zeta}{\zeta-1}$, so that $\mu^W = 1$. Further, setting $\pi = 1$ implies by (B.8) that $Z^1/Z^2 = 1$. The steady state versions of (B.6) and (B.7) imply $Z^1/Z^2 = mc$, so that real marginal cost equal unity in steady state if policy sets $\pi = 1$. From (B.10), it then follows that in steady state, price dispersion must be zero, $s = 1$. Thus, the Ramsey planner will achieve first best in the long-run by setting $\pi = 1$. I further assume that the simple interest rate rule given in (3.2.3) targets long-run inflation of unity, $\pi = 1$. Thus, both policies will lead to identical deterministic steady states characterized by $mc = \mu^T = \pi = s = Z^1/Z^2 = 1$ and $R = \beta^{-1}$. This result is useful for evaluating welfare because it permits conditioning welfare on identical steady states when measuring the welfare loss implied by different policies. The steady states of the remaining model variables can be found by using the equilibrium conditions (B.1), (B.3), (B.4), (B.5) and (B.9), which imply

$$r^k = \frac{1}{\beta} - (1-\delta),$$

$$w = \left[\alpha^\alpha (1-\alpha)^{1-\alpha} \left(r^k\right)^{\alpha-1}\right]^{1/\alpha},$$

$$n/k = \frac{\alpha}{1-\alpha}\frac{r^k}{w},$$

$$k = \left(\frac{w}{\chi}\right)^{1/\sigma} \left(\frac{n}{k}\right)^{-\eta/\sigma} \left[\left(\frac{n}{k}\right)^\alpha - \delta\right]^{-\frac{1}{1+\eta/\sigma}},$$

$$c = k\left[(n/k)^\alpha - \delta\right].$$

Further, it is important to note that the model variants with adjustment cost imply an identical steady state. The reason is that in steady state, adjustment cost are zero. Thus, the value of capital goods relative to consumption goods equals unity, $q = 1$ and the investment capital ratio equals the depreciation rate, $i/k = \delta$ under both specifications.

## B.4  Calibration

| Discount Factor | $\beta = 0.9901$ |
|---|---|
| Inverse of intertemporal substitution elasticity | $\sigma = 1$ |
| Inverse of Frisch elasticity of labor supply | $\eta = 1$ |
| Substitution elasticity intermediate goods | $\varepsilon = 6$ |
| Substitution elasticity labor goods (st.st.) | $\bar{\zeta} = 6$ |
| Scale parameter for disutility of labor | $\chi = 10$ |
| Labour share | $\alpha = 0.64$ |
| Depreciation rate | $\delta = 0.025$ |
| Calvo price stickiness | $\phi = 0.7$ |
| Wage markup shock | $\rho_\zeta = 0.9;\ \sigma^\zeta = 0.1112$ |
| *(shock calibrated to match)* | $\sigma(\mu_t^W) = 0.051$ |
| Capital adjustment cost | $\psi = [6.7;\ 100]$ |
| Investment adjustment cost | $\kappa = [0.17;\ 2.48]$ |

Table B.1: Parameter calibration

**Calibration of the wage markup shock**

It is assumed that the substitution elasticity $\zeta_t$ between labor varieties evolves according to

$$\ln \zeta_t = (1 - \rho_\zeta) \ln \bar{\zeta} + \rho^\zeta \ln \zeta_{t-1} - \varepsilon_t^\zeta,$$

where $\varepsilon_t^\zeta \sim iid$ with standard deviation $\sigma_\zeta$. The resulting wage markup is given by $\mu_t^W = \frac{\zeta_t}{\zeta_t - 1} \frac{1}{\nu^W}$, where $\nu^W = \frac{\bar{\zeta}}{\bar{\zeta} - 1}$. A first-order approximation of $\mu_t^W$ around $\bar{\mu}^W = 1$ yields

$$\hat{\mu}_t^W = \frac{1}{\bar{\zeta} - 1} \hat{\zeta}_t,$$

where $\hat{x}_t = \frac{x_t - x}{x}$ denotes percentage deviations from steady state. Thus, the markup inherits the autocorrelation parameter of the stochastic process for the substitution elasticity, $\rho^\varsigma$. This parameter is set to $\rho^\varsigma = 0.9$, slightly below the value of 0.95 estimated by Galí, Gertler, and López-Salido (2007). Concerning volatility, a first-order approximation yields $\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} - \varepsilon_t^\zeta$ which implies that $Var\ \hat{\zeta}_t = \frac{\sigma_\zeta^2}{1-\rho_\zeta^2}$. Therefore, the variance of the markup depends on the variance of the driving shock, $\sigma_\zeta^2$,

$$Var\left(\hat{\mu}_t^W\right) = \frac{\sigma_\zeta^2}{\left(\bar{\zeta} - 1\right)^2 \left(1 - \rho_\zeta^2\right)}.$$

$\sigma_\zeta^2$ is calibrated to match the standard deviation of the inefficiency gap estimated by Galí, Gertler, and López-Salido (2007) at 0.051. Given the parameters in Table B.1, this implies setting $\sigma_\zeta = 0.1112$.

**Calibration of the remaining parameters**

The supply side of the economy is calibrated in a standard way. A model period is taken to represent a quarter. $\alpha$ is calibrated to the labor income share of 0.64, $\delta$ to a (yearly) steady state investment/capital ratio of around 0.1 and $\beta$ is calibrated to a quarterly steady state real interest rate of 1%, i.e. $R = \frac{1}{\beta} = 1.01$. Thus, the steady state capital output ratio is determined by the capital Euler equation

$$1 = \beta\left(r^k + 1 - \delta\right) \iff \frac{k}{y} = \frac{1 - \alpha}{\frac{1}{\beta} - 1 + \delta},$$

which yields $\frac{k}{y} = 10.29$ given the parameter values in Table B.1. Note that this is in line with empirical observations: Annually, $\frac{k}{y}$ is around 2.5-3 and thus its quarterly value is between 7.5 and 12. Following Galí, Gertler, and López-Salido (2007), the inverse of the labor supply elasticity is set to unity, $\eta = 1$. Card (1994) suggests a range of 0.2 to 0.5 based on microeconomic estimates. Further, Smets and Wouters (2007) estimate an elasticity of 1.92 from macroeconomic data. Thus, the value of unity represents a reasonable choice. I also follow Galí, Gertler, and López-Salido (2007) in the choice of $\sigma$, the intertemporal substitution elasticity of consumption goods and set $\sigma = 1$, i.e. utility logarithmic in consumption. Barsky, Kimball, Juster, and Shapiro (1997) estimate an elasticity of 0.18 using micro data, implying

a value of around 5 for $\sigma$. However, the macroeconomic literature tends to use lower risk aversion coefficients: For instance, Smets and Wouters (2007) estimate $\sigma = 1.39$. Thus, $\sigma = 1$ is a choice rather close toward the lower bound of available estimates. The parameter $\chi$ affects disutility from work and is set to $\chi = 10$ so as to generate a reasonable steady state share of time spent working of 0.29.

A Calvo parameter of $\phi = 0.7$ is in line with price stickiness in the euro area, implying that on average prices are reset every 3-4 quarters: For the euro area, Álvarez, Dhyne, Hoeberichts, Kwapil, Bihan, Lünnemann, Martins, Sabbatini, Stahl, Vermeulen, and Vilmunen (2006) estimate a mean duration of prices of 13 months, i.e. 3.25 quarters. For the U.S., average price duration is estimated much lower, at 6.7 months. The steady state substitution elasticities between the differentiated labor and intermediate goods are both set to 6 so that in the steady state, markups of firms and workers are 20%. The price markup is in line with Galí, López-Salido, and Vallés (2004) who assumes $\varepsilon = 6$ and Galí, Gertler, and López-Salido (2007) who suggest steady state price markups of 0.15 to 0.20. Justiniano and Primiceri (2008), in a Bayesian estimation, find posterior means of 0.17 for the wage markup and 0.22 for firm markups on goods prices. Concerning the steady state wage markup, Galí, Gertler, and López-Salido (2007) suggests values around 0.30 to 0.35, i.e. a value of 4 for the steady state substitution elasticity. Sveen and Weinke (2006) use $\bar{\zeta} = 6$ as well. Note that the effects of both markups on the allocation are eliminated by steady state subsidies. The parameter $\varepsilon$ mainly affects the importance of price dispersion (and thus inflation) for welfare: A low $\varepsilon$ implies higher market power and consequently less responsive demand. Thus, a given inflation rate implies less dispersion in factor input across firms, reducing the welfare loss of price dispersion.

**Adjustment cost**

The calibration of adjustment cost is highly controversial. Macroeconomic studies find rather high adjustment cost. For instance, Christiano, Eichenbaum, and Evans (2005) estimate the investment adjustment cost parameter to $\kappa = 2.48$. Using that a first-order approximation to (3.30) yields $\hat{\imath}_t = \frac{1}{1+\beta}\hat{\imath}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\imath}_{t+1} + \frac{1}{(1+\beta)\kappa}\hat{q}_t$, which can be rewritten as $\hat{\imath}_t = \hat{\imath}_{t-1} = \kappa^{-1}\sum_{s=0}^{\infty}\beta^s\hat{q}_{t+s}$, this implies an elasticity of investment with respect to the market value of capital of $\eta_{i,q} = \frac{\partial i}{\partial q}\frac{i}{q} = \frac{1}{\kappa} = 0.4$. Smets and Wouters

(2007) estimate an even higher adjustment cost parameter, implying an elasticity of only $\eta_{i,q} = 0.18$. The q literature estimates this elasticity based on stock market data and finds values between 0.4 and 1.1, which is the range given by Christiano and Fisher (1995). Even lower adjustment cost, i.e. higher elasticities are obtained from estimating factor demand equations. Hall (2004) estimates industry-specific adjustment cost parameters which are close to and not significantly different from zero for almost all industries he considers. Groth and Khan (2007) pursue a similar approach and find evidence in favor of small adjustment cost. They estimate factor demand equations to obtain industry-specific estimates of adjustment cost and aggregate these, yielding an elasticity of $\eta_{i,q} = 6$. Thus, estimates of adjustment cost differ largely between macroeconomic and microeconomic estimation approaches. I acknowledge this high uncertainty by considering a low adjustment cost scenario calibrated to yield a steady state elasticity of $\eta_{i,q} = 6$, which implies setting $\kappa = 0.17$, and a high adjustment cost scenario, which is calibrated according to the elasticity estimated by Christiano, Eichenbaum, and Evans (2005), $\eta_{i,q} = 0.4$ and implies a value of $\kappa = 2.48$.

Capital adjustment cost are calibrated to these two elasticities as well. The first order condition of the representative household with respect to capital (3.32) reads

$$q_t = \left[ 1 - \psi \left( \frac{i_t}{k_t} - \delta \right) \right]^{-1}.$$

Thus, the elasticity of investment with respect to Tobin's q is given by

$$\eta_{i,q} = \frac{\partial i}{\partial q} \frac{q}{i} = \frac{k}{\psi} \frac{q}{i} = (\psi \delta)^{-1}.$$

The two scenarios above thus require setting $\psi = 100$ and, respectively, $\psi = 6.7$.

# Appendix C

# Appendix to Chapter 4

## C.1  Rational expectations equilibrium

A RE equilibrium is a set of sequences $\{c_t, n_t, y_t, \lambda_t, m_t^R, m_t^H, b_t, b_t^T, l_t, w_t, mc_t, \tilde{Z}_t, Z_t^1, Z_t^2, s_t, \pi_t, R_t, R_t^D, R_t^L, R_t^m\}_{t=0}^{\infty}$ satisfying the following conditions summarizing the optimal behavior of households

$$\chi n_t^{\eta} = w_t \left[ \lambda_t (1 - \theta) + \theta c_t^{-\sigma} \right], \tag{C.1}$$

$$R_t^L = \frac{c_t^{-\sigma}}{\lambda_t (1 - \kappa_t) + \kappa_t c_t^{-\sigma}/R_t^m}, \tag{C.2}$$

$$\lambda_t = \beta E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}}, \tag{C.3}$$

$$\lambda_t = \beta R_t E_t \frac{c_{t+1}^{-\sigma}}{R_{t+1}^m \pi_{t+1}}, \tag{C.4}$$

$$\lambda_t = \beta R_t^D E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{C.5}$$

$$m_t^H + m_t^R = c_t, \text{ if } \psi_t = (R_t^m - 1) \lambda_t + \left( c_t^{-\sigma} - R_t^m \lambda_t \right) > 0, \tag{C.6}$$

$$\text{or } m_t^H + m_t^R \geq c_t, \text{ if } \psi_t = 0,$$

$$\frac{\kappa_t l_t + b_{t-1}/\pi_t}{R_t^m} = m_t^H - \frac{m_{t-1}^H}{\pi_t} + m_t^R, \text{ if } \eta_t = \left( c_t^{-\sigma}/R_t^m \right) - \lambda_t > 0, \tag{C.7}$$

$$\text{or } \frac{\kappa_t l_t + b_{t-1}/\pi_t}{R_t^m} \geq m_t^H - \frac{m_{t-1}^H}{\pi_t} + m_t^R, \text{ if } \eta_t = 0,$$

$$b_t = (\Gamma - 1) \frac{b_{t-1}^T}{\pi_t} - \kappa_t l_t + m_t^R, \tag{C.8}$$

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of firms

$$mc_t \alpha n_t^{\alpha-1} = w_t \left[ 1 + \theta \left( R_t^L - 1 \right) \right], \tag{C.9}$$

$$l_t / R_t^L = \theta w_t n_t, \tag{C.10}$$

$$\tilde{Z}_t = \varepsilon \left( \varepsilon - 1 \right)^{-1} Z_t^1 / Z_t^2, \tag{C.11}$$

$$Z_t^1 = c_t^{-\sigma} y_t mc_t + \phi \beta E_t \pi_{t+1}^\varepsilon Z_{t+1}^1, \tag{C.12}$$

$$Z_t^2 = c_t^{-\sigma} y_t + \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2, \tag{C.13}$$

$$1 = (1 - \phi) \left( Z_t^1 / Z_t^2 \right)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{C.14}$$

the public sector

$$m_t^R = \Omega m_t^H + (\kappa_t - \kappa) \, l_t / R_t^m, \tag{C.15}$$

$$R_t^m = \left( R_{t-1}^m \right)^\rho R_m^{(1-\rho)} \left( \pi_t / \pi \right)^{w_\pi (1-\rho)} \left[ (y_t / A_t) / (y/A) \right]^{w_y (1-\rho)} \exp \varepsilon_t, \tag{C.16}$$

$$b_t^T = (\Gamma - 1) \, b_{t-1}^T / \pi_t, \tag{C.17}$$

and aggregate resources

$$y_t = c_t, \tag{C.18}$$

$$y_t = n_t^\alpha / s_t, \tag{C.19}$$

$$s_t = (1 - \phi) \tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \pi_t^\varepsilon, \tag{C.20}$$

the transversality conditions, a monetary policy setting $\{ R_t^m \geq 1, \ \kappa_t \in [0,1] \}_{t=0}^\infty$, $\Omega > 0$ and the inflation target $\pi \geq \beta$, for given sequences of stochastic variables and initial values.

## C.2  Steady state

In this appendix we examine the steady state of the model in detail (steady state variables will not be indexed with a time index). The central bank determines $\kappa \in [0,1]$ and target values for the inflation rate $\pi \geq \beta$ and the policy rate $R^m \geq 1$. In a steady state, all endogenous variables grow with a constant rate. Thus, to be consistent with a long-run equilibrium, the time-invariant policy targets have to be consistent with the steady state. In what follows we examine properties of all other endogenous variables in a steady state.

Given the steady state inflation rate $\pi$, the equilibrium condition (C.14) implies the ratio $Z_t^1 / Z_t^2$ to equal $\left( (1 - \phi \pi^{\varepsilon-1}) / (1 - \phi) \right)^{1/(1-\varepsilon)}$ and thus

to be constant. Condition (C.11) then implies that $\tilde{Z}$ is also constant. The price dispersion term $s_t$ satisfying (C.20), thus converges in the long run to $s = \frac{1-\phi}{1-\phi\pi^\varepsilon}\tilde{Z}^{-\varepsilon} > 0$ if $\phi\pi^\varepsilon < 1 \Leftrightarrow \pi < (1/\phi)^{1/\varepsilon}$. Since $s$ is bounded from below and neither productivity nor labor supply exhibit trend growth, real resources and therefore working time, output, and consumption cannot permanently grow with a non-zero rate in the steady state, $y = c = n^\alpha/s$. Then, (C.13) implies that $Z_t^2$ converges to $Z^2 = y^{1-\sigma}/\left(1 - \phi\beta\pi^{\varepsilon-1}\right)$ if $\phi\beta\pi^{\varepsilon-1} < 1 \Leftrightarrow \pi < [1/(\phi\beta)]^{1/(\varepsilon-1)}$. Given that $Z_t^1/Z_t^2$ and $Z_t^2$ are constant, and that (C.12) implies $Z_t^1 = Z^1 = \frac{c^{1-\sigma}mc}{1-\phi\beta\pi^\varepsilon}$, since $Z_t^1/Z_t^2 = Z^{1,2}$, such that real marginal cost is constant and given by $mc = \tilde{Z}\left(\varepsilon - 1\right)\varepsilon^{-1}\left(1 - \phi\beta\pi^\varepsilon\right)/\left(1 - \phi\beta\pi^{\varepsilon-1}\right)$.

**Proof of Proposition 4.1** Combining (C.3) and (C.4) shows that the government bond rate equals the (constant) long-run policy rate $R = R^m$. Since (C.3) further implies $\lambda = \frac{\beta}{\pi}c^{-\sigma}$, the Euler equation for private debt (C.5) leads to the usual Fischer equation $R^D = \pi/\beta$. Eliminating $\psi_t$ in (4.12) with (4.10), gives $\eta_t = (c^{-\sigma}/R^m) - \lambda$ and thus

$$\eta = c^{-\sigma}\left(R^m - \beta/\pi\right) \geq 0, \tag{C.21}$$

which shows that the money market constraint (4.2) is binding in steady state if monetary policy sets the policy rate below the private debt rate, $R^m < R^D = \frac{\pi}{\beta} \Rightarrow \eta > 0$. Using $\lambda = \frac{\beta}{\pi}$ and (4.10), gives

$$\psi = c^{-\sigma}\left(1 - \beta/\pi\right) \geq 0, \tag{C.22}$$

which shows that the goods market constraint (4.3) is binding in the steady state if the central bank sets an inflation target $\pi > \beta \Rightarrow \psi > 0$. Condition (C.2) further implies the steady state loan rate to satisfy

$$R^L = \left[(1 - \kappa)/R^D + \kappa/R^m\right]^{-1}. \tag{C.23}$$

Given that the loan rate, marginal cost, and working time are constant, (C.9) implies a constant steady state wage rate,

$$w = mc\alpha n^{\alpha-1}\left[1 + \theta\left(R^L - 1\right)\right]^{-1}.$$

The firms' working capital constraint (4.1) is further binding $l = \theta w n R^L$, if $R^m < R^D \Rightarrow R^L > 1$ (see C.23). $\square$

**Proof of Proposition 4.2** Now suppose that the central bank sets its target according to $\pi > \beta$ and $R^m < \pi/\beta$, the constraints in the goods market and in the money market are binding (see C.21 and C.22). Further, (C.23) then implies $R^L > 1$, which implies the working capital constraint to be binding, $l = \theta w n R^L$ (as shown in the proof of proposition 4.1).

Substituting out $\lambda$ and $w$ in (C.1) and eliminating working time with $y = n^\alpha/s$ gives the following steady state output and consumption level

$$y = c = \left[ \frac{\alpha m c s^\sigma}{\chi} \frac{\beta}{\pi} \frac{1 + \theta\left(\frac{\pi}{\beta} - 1\right)}{1 + \theta\left(R^L - 1\right)} \right]^{\frac{\alpha}{1+\eta+\sigma\alpha-\alpha}} s^{-1}, \qquad (C.24)$$

which decreases with the loan rate. Hence, a decline in the loan rate $R^L$, which can be induced by a higher $\kappa$ or a lower $R^m$ (see C.23), leads to an increase of consumption and output in the steady state (see C.24). The binding goods market constraint (C.6) and the central bank's money supply (C.15) further imply that real balances held outright and held under repurchase agreements are constant and satisfy $m^H = c/(1 + \Omega)$ and $m^R = c\Omega/(1 + \Omega)$. Households' money holdings that evolve according to (C.7) then satisfy $m^H + m^R = \frac{m^H}{\pi} + \frac{\kappa l + b_{t-1}/\pi}{R^m}$, which implies that household bond holdings are constant in the steady state

$$b = \pi \left[ R^m m^h \left(1 - \pi^{-1} + \Omega\right) - \kappa l \right]. \qquad (C.25)$$

Since household bond holdings evolve according to (C.8), the real stock of government bonds has to be constant and equal to $b^T = \frac{\pi}{\Gamma-1}\left[b + \kappa l - \Omega m^h\right]$. Consequently, to be consistent with the supply of government bonds (C.17), $b_t^T = \Gamma b_{t-1}^T \pi^{-1}$, the growth rate $\Gamma$ has to equals the long-run inflation rate $\pi = \Gamma$, while the central bank can set the inflation target independently from $\kappa$ and $R^m$. An increase in $\kappa$ tends to decrease the real value of household government bond holdings (see C.25) and the real value of total government bonds, as can be seen from

$$b^T = \pi c \left[ -\kappa \frac{\theta R^L}{1 + \theta\left(R^L - 1\right)} mc\alpha + \frac{R^m + \Omega\frac{\pi R^m - 1}{\pi - 1}}{1 + \Omega} \right], \qquad (C.26)$$

where we substituted out household bonds $b$ using (C.25) and loans by $l = c\theta R^L mc\alpha/[1 + \theta\left(R^L - 1\right)]$. Since the loan rate $R^L$ decreases with $\kappa$

the overall impact on $b^T$ (see C.26) and thus on the aggregate price level is ambiguous. It can easily be shown that the effect of $\kappa$ on $b^T$ depends on the sign of the term $-[1 + (\frac{1}{R^D} - \frac{1}{R^m})\frac{(1-\theta)R^L}{1+\theta(R^L-1)}]$, which is strictly negative if (but not only if) $\theta \geq 0.5$. Since the latter is ensured by assumption (see section 4.2.2), the price level sequence thus shifts upward in the long-run for a higher $\kappa$. $\square$

**Cash holdings and intra-period loans**  We show that household purchases of loans will not be constrained by money holdings in steady state in our analysis, i.e. we examine under what conditions $m^h/\pi > l/R^L$ holds. Given steady state money holdings $m^h = c/(1 + \Omega)$ and demand for loans $l = \theta wnR^L$, this inequality implies $c/(1 + \Omega) > \pi\theta wn$. Using $wn = mc\alpha n^\alpha/R^L$ and $n^\alpha/s = c$, money holdings will exceed demand for loans if the share of money supplied under repos is sufficiently small such that $\Omega < \left(\pi\theta mcs\alpha/R^L\right)^{-1} - 1$. The central bank will then supply a sufficient amount of money outright. For the parameter values (see below) that will be applied in our analysis, this requires $\Omega < 2.34$, while we set $\Omega = 1.5$. Thus, in a small neighborhood of the steady state, aggregate money holdings will exceed demand for loans, so that $M_{t-1}^H \geq L_t/R_t^L$ is not a binding constraint.

## C.3   Parameter values

| | |
|---|---|
| Subjective discount factor | $\beta = 0.9940$ |
| Inverse of intertemporal substitution elasticity | $\sigma = 1$ |
| Inverse of Frisch elasticity of labor supply | $\eta = 0$ |
| Substitution elasticity | $\varepsilon = 10$ |
| Steady state working time | $n = 0.33$ |
| Labour share | $\alpha = 0.66$ |
| Share of working capital | $\theta = 0.5$ |
| Calvo price stickiness | $\phi = 0.75$ |
| Taylor coefficient inflation | $w_\pi = 1.92$ |
| Taylor coefficient output | $w_y = 0.1$ |
| Interest rate smoothing | $\rho = 0.8$ |
| Steady state interest rate | $R^m = 1.0105$ |
| Steady state share of repos to outright purchases | $\Omega = 1.5$ |
| Share of loans eligible in open market operations | $\kappa = 0$ |
| Steady state inflation | $\Gamma = 1.00575$ |

Table C.1: Parameter calibration

# Appendix D

# Appendix to Chapter 5

The appendix contains the derivation of equilibrium conditions of the home economy as well as summaries of home and foreign equilibrium conditions for the case of binding open market constraints, a derivation of the steady states, a proof of proposition 5.1 and responses to a foreign consumption shock in a simplified version of Galí and Monacelli (2005).

## D.1 Home economy equilibrium conditions

### D.1.1 Price index and households' goods demand

**Households' goods demand**

First, I rewrite (5.10) by using (5.11) and $\left(\frac{c_{F,t}}{c_{H,t}}\right)^{1-\eta} = \frac{c_t}{c_{F,t}\gamma}$ to obtain

$$c_{F,t} = \frac{\eta u_{c,t}}{(\lambda_t + \mu_t) R_t^m q_t} c_t.$$  (D.1)

Similarly, rewriting (5.9) by using $\left(\frac{c_{F,t}}{c_{H,t}}\right)^{\eta} = \frac{c_t}{c_{H,t}\gamma}$ implies

$$c_{H,t} = \frac{(1-\eta) u_{c,t}}{\left(\lambda_t + \psi_{H,t}\right) \frac{P_{H,t}}{P_t}} c_t.$$  (D.2)

Using (D.1) and (D.2) in the definition of the price index yields

$$
\begin{aligned}
P_t c_t &= P_{H,t} c_{H,t} + P_{F,t} c_{F,t} \\
&\Longleftrightarrow \\
1 &= (1-\eta) \frac{u_{c,t}}{\lambda_t + \psi_{H,t}} + \eta \frac{u_{c,t}}{(\lambda_t + \mu_t) R_t^m},
\end{aligned}
$$  (D.3)

which characterizes the optimal labor leisure trade-off given that domestic and imported goods are subject to cash credit frictions. Given this choice, (D.1) and (D.2) determine how total consumption is split up into domestic and imported goods.

### Derivation of the price index

I use $c_{H,t} = \left( \frac{c_t}{\gamma c_{F,t}^\eta} \right)^{\frac{1}{1-\eta}}$ to cancel out $c_{H,t}$ in (D.1) and (D.2) and obtain

$$c_{F,t} = \left[ \frac{(1-\eta)\, u_{c,t}}{\left( \lambda_t + \psi_{H,t} \right) P_{H,t}/P_t} \right]^{\frac{\eta-1}{\eta}} c_t \gamma^{\frac{-1}{\eta}}$$

and

$$c_{F,t} = \frac{\eta c_t^{-\sigma}}{\left( \lambda_t + \mu_t \right) R_t^m P_{F,t}/P_t} c_t.$$

Combining these to substitute out $c_{F,t}$ and solving for $P_t$ yields

$$\left[ \frac{(1-\eta)\, u_{c,t}}{\left( \lambda_t + \psi_{H,t} \right) P_{H,t}/P_t} \right]^{\frac{\eta-1}{\eta}} c_t \gamma^{\frac{-1}{\eta}} = \frac{\eta u_{c,t}}{\left( \lambda_t + \mu_t \right) R_t^m P_{F,t}/P_t} c_t,$$

which is equivalent to

$$P_t^{\frac{1}{-\eta}} = \frac{u_{c,t}}{\left( \lambda_t + \mu_t \right) R_t^m P_{F,t}} \eta \left( 1-\eta \right)^{\frac{1-\eta}{\eta}} \gamma^{\frac{1}{\eta}} \left[ \frac{u_{c,t}}{\left( \lambda_t + \psi_{H,t} \right) P_{H,t}} \right]^{\frac{1-\eta}{\eta}},$$

where $\gamma^{1/\eta} = \eta^{-\eta/\eta} \left( 1-\eta \right)^{\frac{\eta-1}{\eta}}$ so that

$$P_t = \frac{\left[ \left( \lambda_t + \mu_t \right) R_t^m \right]^\eta \left( \lambda_t + \psi_{H,t} \right)^{1-\eta}}{u_{c,t}} P_{F,t}^\eta P_{H,t}^{1-\eta}. \qquad (D.4)$$

Thus, the price index takes into account that households' consumption choice is influenced by the cash credit friction. For simplicity, define $\Phi_t = \frac{\left[ \left( \lambda_t + \mu_t \right) R_t^m \right]^\eta \left( \lambda_t + \psi_{H,t} \right)^{1-\eta}}{u_{c,t}}$ which measures the extent of the cash-credit friction. Introducing the real exchange rate $q_t = \frac{S_t P_t^*}{P_t} = \frac{P_{F,t}}{P_t}$ and using $z_t = P_{H,t}/P_{F,t} = \frac{P_{H,t}}{S_t P_t^*} = q_t^{-1} \frac{P_{H,t}}{P_t}$, which implies $\Phi_t \left( \frac{P_{H,t}}{P_{F,t}} \right)^{1-\eta} = \frac{P_t}{P_{F,t}}$, I can rewrite (D.4) as

$$\frac{P_{H,t}}{P_t} = \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}}.$$

In differences,

$$\pi_t = \pi_{H,t} \left( \frac{\Phi_t}{\Phi_{t-1}} \right)^{\frac{1}{1-\eta}} \left( \frac{q_t}{q_{t-1}} \right)^{\frac{\eta}{1-\eta}}. \tag{D.5}$$

## D.1.2 Home firm sector

### Cost minimization

Firms minimize cost for given production, where $MC_t$ denotes nominal marginal cost,

$$\min_{n_t(j)} P_t w_t n_t(j) - MC_t \left[ y_{H,t}(j) - n_t(j) \right],$$

so that firms' marginal cost are given by, $P_t w_t = MC_t \iff \frac{MC_t}{P_t} = w_t$ and are thus independent of the production level. In real (PPI) terms, marginal cost are given by $\frac{MC_t}{P_{H,t}} = mc_t = w_t \frac{P_t}{P_{H,t}}$.

### Price setting

Firms produce varieties which are aggregated according to

$$y_{H,t} = \left[ \int_0^1 y_{H,t}^{\frac{\varepsilon-1}{\varepsilon}}(j) dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

so that they face the demand constraint $y_{H,t}(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} y_{H,t}$. Following Calvo (1983), every firm resets its price in a given period with constant probability $\phi$. Firms who do not reset prices are assumed to raise prices with the steady state PPI inflation rate $\pi_H$. A firm's nominal profits in period t are given by $\Pi_t^{nom}(j) = [P_{H,t}(j) - MC_t] y_{H,t}(j) = [P_{H,t}(j) - MC_t] \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} y_{H,t}$. A firm resetting its price in period $t$ thus maximizes expected profits given the probability that it may not reoptimize prices in future. Denote with $Z_t$ the sales price of firms resetting prices in period $t$, which is determined by the solution of

$$\max_{Z_t} \sum_{s=0}^{\infty} \Lambda_{t,t+s}^{\text{real}} \phi^s \left[ \left( \frac{\pi_H^s Z_t}{P_{H,t+s}} \right)^{1-\varepsilon} - mc_{t+s} \left( \frac{\pi_H^s Z_t}{P_{H,t+s}} \right)^{-\varepsilon} \right] y_{H,t+s},$$

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where $\Lambda_{t,t+s}$ is the discount factor for real profits. Using that $\Lambda_{t,t+s} = \beta^s \frac{u_{c,t+s}}{u_{c,t}}$, the first order condition requires

$$\sum_{s=0}^{\infty} \frac{u_{c,t+s}}{u_{c,t}} \left(\beta\phi\pi_H^{1-\varepsilon}\right)^s P_{H,t+s}^{\varepsilon-1} Z_t y_{H,t+s}$$

$$= \frac{\varepsilon}{\varepsilon-1} \sum_{s=0}^{\infty} \left(\frac{c_{t+s}}{c_t}\right)^{-\sigma} \left(\beta\phi\pi_H^{-\varepsilon}\right)^s mc_{t+s} P_{H,t+s}^{\varepsilon} y_{H,t+s},$$

so that the optimal price is given by

$$Z_t = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{s=0}^{\infty} \left(\beta\phi\pi_H^{-\varepsilon}\right)^s u_{c,t+s} P_{H,t+s}^{\varepsilon} y_{H,t+s} mc_{t+s}}{\sum_{s=0}^{\infty} \left(\beta\phi\pi_H^{1-\varepsilon}\right)^s u_{c,t+s} P_{H,t+s}^{\varepsilon-1} y_{H,t+s}}.$$

Defining $\tilde{Z}_t = Z_t/P_{H,t}$ and writing both the denominator and numerator in a recursive way, this can be expressed as

$$\tilde{Z}_t = Z_t^1 / Z_t^2,$$

where

$$Z_t^1 = \varepsilon/\left(\varepsilon-1\right) u_{c,t} y_t mc_t + \phi\beta E_t \left(\pi_{H,t+1}/\pi_H\right)^{\varepsilon} Z_{t+1}^1,$$
$$Z_t^2 = u_{c,t} y_t + \phi\beta\pi_H E_t \left(\pi_{H,t+1}/\pi_H\right)^{\varepsilon-1} Z_{t+1}^2.$$

**Price index**

The zero profit condition of competitive retailers implies

$$P_{H,t} y_{H,t} = \int_0^1 P_{H,t}\left(j\right) y_{H,t}\left(j\right) dj$$

$$= y_{H,t} \int_0^1 P_{H,t}\left(j\right) \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj.$$

This gives

$$P_{H,t}^{1-\varepsilon} = \int_0^1 P_{H,t}^{1-\varepsilon}(j) dj$$

$$= (1-\phi) \left[Z_t^{1-\varepsilon} + \phi\left(Z_{t-1}\pi_H\right)^{1-\varepsilon} + \phi^2 \left(Z_{t-2}\pi_H^2\right)^{1-\varepsilon} + ...\right]$$

$$= (1-\phi) \left[Z_t^{1-\varepsilon} + \left(\phi\pi_H^{1-\varepsilon}\right) Z_{t-1}^{1-\varepsilon} + \left(\phi\pi_H^{1-\varepsilon}\right)^2 Z_{t-2}^{1-\varepsilon} + ...\right].$$

Lagging this expression by one period and pre-multiplying with $\phi\pi_H^{1-\varepsilon}$ gives

$$\phi\pi_H^{1-\varepsilon}P_{H,t-1}^{1-\varepsilon} = (1-\phi)\left[\phi\pi_H^{1-\varepsilon}Z_{t-1}^{1-\varepsilon} + \left(\phi\pi_H^{1-\varepsilon}\right)^2 Z_{t-2}^{1-\varepsilon} + \left(\phi\pi_H^{1-\varepsilon}\right)^3 Z_{t-3}^{1-\varepsilon} + ...\right].$$

Subtracting both expressions yields the price index

$$P_{H,t}^{1-\varepsilon} = (1-\phi)Z_t^{1-\varepsilon} + \phi\pi_H^{1-\varepsilon}P_{H,t-1}^{1-\varepsilon}.$$

Dividing by $P_{H,t}^{1-\varepsilon}$ gives, with $\tilde{Z}_t = Z_t/P_{H,t}$

$$1 = (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi\pi_H^{1-\varepsilon}\pi_{H,t}^{\varepsilon-1}. \tag{D.6}$$

**Price dispersion**

Further, I compute price dispersion and its impact on output. Define intermediate output $IO_t$ as the sum of produced intermediate goods,

$$IO_t \equiv \int_0^1 y_{H,t}(j)dj = \int_0^1 n_{jt}dj = n_t.$$

Demand for varieties is given by $y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} y_{H,t}$, which implies

$$IO_t = \int_0^1 y_{H,t}(j)dj = y_{H,t}\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj.$$

Let $s_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj$ so that $y_{H,t} = IO_t/s_t$. Thus, $s_t$ evolves according to

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi\pi_H^{-\varepsilon}\pi_{H,t}^{\varepsilon}s_{t-1}. \tag{D.7}$$

Schmitt-Grohé and Uribe (2004b) show that $s_t$ is limited below by 1, $s_t \geq 1$. It increases above unity whenever firms reset their prices, i.e. $\tilde{Z}_t \neq 1$. Thus, any price dispersion implies an inefficient allocation of aggregate resources which is evident by $y_{H,t} = \frac{n_t}{s_t}$.

## D.2 Equilibrium conditions when open market constraints bind

### D.2.1 Home economy

The representative household's first order conditions can be summarized by

$$\lambda_t w_t = -u_{n,t}, \tag{D.8}$$

$$1 = (1 - \eta) \frac{u_{c,t}}{\lambda_t + \psi_{H,t}} + \eta \frac{u_{c,t}}{(\lambda_t + \mu_t) R_t^m}, \tag{D.9}$$

$$c_{F,t} = \frac{\eta u_{c,t}}{(\lambda_t + \mu_t) R_t^{m*} q_t} c_t, \tag{D.10}$$

$$c_{H,t} = \frac{(1 - \eta) u_{c,t}}{(\lambda_t + \psi_{H,t}) \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}}} c_t, \tag{D.11}$$

$$\lambda_t = \beta E_t \frac{\lambda_{t+1} R_t}{\pi_{t+1}}, \tag{D.12}$$

$$\lambda_t q_t = \beta E_t q_{t+1} \frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}^*} R_t^*, \tag{D.13}$$

$$\frac{R_t}{E_t \pi_{t+1}} = E_t \frac{q_{t+1}}{q_t} \frac{R_t^{D*}}{\pi_{t+1}^*}, \tag{D.14}$$

$$\psi_{H,t} = \lambda_t (R_t - 1), \tag{D.15}$$

$$\pi_{H,t} = \pi_t \left( \frac{\Phi_t}{\Phi_{t-1}} \right)^{\frac{1}{\eta-1}} \left( \frac{q_t}{q_{t-1}} \right)^{\frac{\eta}{\eta-1}}, \tag{D.16}$$

where $\Phi_t = \frac{[(\lambda_t + \mu_t) R_t^{m*}]^\eta (\lambda_t + \psi_{H,t})^{1-\eta}}{u_{c,t}}$. The binding cash and open market constraints read

$$c_{F,t} = \frac{b_{F,t-1}}{R_t^{m*} \pi_t^*} + \frac{m_{F,t-1}}{\pi_t^*}, \tag{D.17}$$

$$m_{F,t} = \frac{1}{1 + \Omega} \eta^* c_t^*, \tag{D.18}$$

$$m_t = \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}} c_{H,t}, \tag{D.19}$$

where $m_{F,t} = M_{F,t}/P_t^*$, $b_{F,t} = B_{F,t}/P_t^*$ and $m_t = M_t/P_t$ denote real money and bond holdings. The firms' block of first order conditions is given by

$$mc_t = w_t \Phi_t^{\frac{1}{1-\eta}} q_t^{\frac{\eta}{1-\eta}} \tag{D.20}$$

$$Z_t^1 = u_{c,t} y_{H,t} mc_t + \phi\beta\pi_H^{-\varepsilon} E_t \pi_{H,t+1}^\varepsilon Z_{t+1}^1 \tag{D.21}$$

$$Z_t^2 = u_{c,t} y_{H,t} + \phi\beta\pi_H^{1-\varepsilon} E_t \pi_{H,t+1}^{\varepsilon-1} Z_{t+1}^2 \tag{D.22}$$

$$1 = (1-\phi)\left(Z_t^1/Z_t^2\right)^{1-\varepsilon} + \phi\pi_H^{1-\varepsilon}\pi_{H,t}^{\varepsilon-1} \tag{D.23}$$

The final block of equilibrium conditions contains, among others, the resource constraint, the production function including price dispersion and the evolution of foreign debt,

$$y_{H,t} = c_{H,t} + c_{H,t}^*, \tag{D.24}$$

$$y_{H,t} = n_t^\alpha / s_t, \tag{D.25}$$

$$s_t = (1-\phi)\left(Z_t^1/Z_t^2\right)^{-\varepsilon} + \phi\pi_H^{-\varepsilon}\pi_{H,t}^\varepsilon s_{t-1}, \tag{D.26}$$

$$\Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{1}{\eta-1}} c_{H,t}^* - c_{F,t} = \frac{b_{F,t}}{R_t^*} - \frac{b_{F,t-1}}{R_t^{m*}\pi_t^*} + \frac{d_{F,t}}{R_t^{D*}} - \frac{d_{F,t-1}}{\pi_t^*} + m_{F,t} - \frac{m_{F,t-1}}{\pi_t^*},$$

$$\frac{q_t}{q_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\pi_t^*}{\pi_t}, \tag{D.27}$$

$$c_{H,t}^* = q_t^{\frac{1}{1-\eta}} \Phi_t^{\frac{1}{1-\eta}} \eta^* c_t^*, \tag{D.28}$$

$$tb_t = q_t\left(\eta^* c_t^* - c_{F,t}\right), \tag{D.29}$$

where $d_{F,t} = D_{F,t}/P_t^*$ denotes real holdings of foreign private debt. Monetary policy follows a Taylor rule.

$$R_t = R^{(1-\rho_R)} R_{t-1}^{\rho_R} \left(\pi_{H,t}/\pi_H\right)^{w_\pi(1-\rho_R)} \left(y_{H,t}/y_H\right)^{w_y(1-\rho_R)}, \tag{D.30}$$

where $R = \pi/\beta$ is the steady state interest rate in the home economy.

## D.2.2 Foreign economy

When cash and open market constraints bind, the foreign economy can be
described by the behavior of households,

$$\frac{-u^*_{n,t}}{w^*_t} = \beta^* E_t \frac{u^*_{c,t+1}}{\pi_{t+1^*}}, \tag{D.31}$$

$$E_t \frac{u^*_{c,t+1}}{\pi^*_{t+1}} = R^*_t E_t \frac{u^*_{c,t+1}}{\pi^*_{t+1} R^{m*}_{t+1}}, \tag{D.32}$$

$$\frac{u^*_{n,t}}{w^*_t} = \beta R^{D*}_t \frac{u_{n,t+1^*}}{w^*_{t+1}\pi^*_{t+1}}, \tag{D.33}$$

$$c^*_t = (1 + \Omega)m^*_{F,t}, \tag{D.34}$$

$$m^*_{F,t}(1 + \Omega) = \frac{m^*_{F,t-1}}{\pi^*_t} + \frac{b^*_{F,t-1}/\pi^*_t}{R^{m*}_t}, \tag{D.35}$$

firms,

$$w^*_t = mc^*_t A^*, \tag{D.36}$$

$$Z^{1*}_t = \varepsilon/(\varepsilon - 1)\, u^*_{c,t}y^*_t mc^*_t + \phi^*\beta\pi^{*-\varepsilon}E_t\pi^{*\varepsilon}_{t+1}Z^{1*}_{t+1}, \tag{D.37}$$

$$Z^{2*}_t = u^*_{c,t}y^*_t + \phi^*\beta\pi^{*1-\varepsilon}E_t\pi^{*\varepsilon-1}_{t+1}Z^{2*}_{t+1}, \tag{D.38}$$

$$1 = (1 - \phi)\left(Z^{1*}_t/Z^{2*}_t\right)^{1-\varepsilon} + \phi\pi^{*1-\varepsilon}\pi^{*\varepsilon-1}_t, \tag{D.39}$$

the public sector,

$$b^*_{F,t} = (\Gamma - 1)\, b^{T*}_{t-1}/\pi^*_t - b_{F,t} + m^{R*}_{F,t}, \tag{D.40}$$

$$b^{T*}_t = \Gamma b^{T*}_{t-1}/\pi^*_t, \tag{D.41}$$

$$R^{m*}_t = R^{m*(1-\rho)}\left(R^{m*}_{t-1}\right)^{\rho}(\pi^*_t/\pi^*)^{w^*_\pi(1-\rho)}(y^*_t/y^*)^{w_y(1-\rho_R)}\exp(\varepsilon^*_t)^{\rho}, \tag{D.42}$$

and aggregate resources,

$$y^*_t = c^*_t, \tag{D.43}$$

$$y^*_t = A^*n^*_t/s^*_t, \tag{D.44}$$

$$s^*_t = (1 - \phi^*)\left(Z^{1*}_t/Z^{2*}_t\right)^{-\varepsilon} + \phi^*s^*_{t-1}\pi^{*\varepsilon}_t, \tag{D.45}$$

where $m^*_{F,t} = M^*_{F,t}/P^*$ and $b^*_{F,t} = B^*_{F,t}/P^*$ denote real money and bond
holdings, $A^*$ is exogenous labor productivity and $b^{T*}_t = B^{T*}_t/P^*_t$ denotes
the real stock of foreign bonds in circulation.

## D.3 Steady States under binding open market constraints

This section derives the steady state of the model given binding open market constraints. This is required for the log-linear approximation used in section 5.4.2.

### D.3.1 Home economy

I use that the utility function is given by (5.65), which is repeated here for convenience

$$u\left(c_t, n_t\right) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{n_t^{1+\eta}}{1 + \eta}.$$

The first order conditions for price setting imply

$$Z^1 = \frac{\varepsilon}{\varepsilon - 1} \frac{u_c y_H}{1 - \phi\beta} mc,$$

$$Z^2 = \frac{u_c y_H}{1 - \phi\beta},$$

$$mc = \frac{\varepsilon - 1}{\varepsilon},$$

$$s = \left(\frac{Z^1}{Z^2}\right)^{-\varepsilon} = 1.$$

The steady state inflation rate of home goods, $\pi_H$, can be set by the central bank through the interest rate rule. There is no price dispersion in steady state due to indexation of non-optimized prices to steady state inflation. The domestic Euler rate is given by $R = \pi/\beta$, and the UIP condition implies identical real interest rates $R/\pi = R^{D*}/\pi^*$ and thus identical discount factors, $\beta = \beta^*$. Further, in steady state CPI inflation equals PPI inflation, $\pi = \pi_H$. Moreover, I assume that the home central bank targets an inflation rate identical to foreign inflation, $\pi = \pi^*$, so that $R = R^{D*}$ and the nominal exchange rate is constant, $S_t/S_{t-1} = 1$ but in its level not determined.

Consider the remaining system of equilibrium conditions,

$$\chi = \lambda w n^{-\omega} \tag{D.46}$$

$$\lambda = c^{-\sigma} \left[ \frac{1-\eta}{R^D} + \frac{\eta}{R^{D*}} \right] = c^{-\sigma}/R^D \tag{D.47}$$

$$\lambda = \eta q^{-1} c^{-\sigma} \left( \frac{c}{c_F} \right) \frac{1}{R^{D*}} \tag{D.48}$$

$$\mu = \lambda \left( \frac{R^{D*}}{R^*} - 1 \right) \tag{D.49}$$

$$\psi_F = \lambda \left( R^{D*} - 1 \right) \tag{D.50}$$

$$\psi_H = \lambda \left( R^D - 1 \right) \tag{D.51}$$

$$\Phi = \frac{\lambda}{c^{-\sigma}} \left( R^{D*} \right)^\eta \left( R^D \right)^{1-\eta} = 1 \tag{D.52}$$

$$c_F = \frac{b_F}{R^{m*}\pi^*} + \frac{m_F}{\pi^*} \tag{D.53}$$

$$m_F = \frac{1}{1+\Omega} \eta^* c^* \tag{D.54}$$

$$w = \Phi^{\frac{1}{\eta-1}} q^{\frac{\eta}{\eta-1}} \frac{\varepsilon - 1}{\varepsilon} \tag{D.55}$$

$$n^\alpha = (c/c_F^\eta \gamma)^{\frac{1}{1-\eta}} + \eta^* q^{\frac{1}{1-\eta}} \Phi^{\frac{1}{1-\eta}} c^* \tag{D.56}$$

$$\frac{b_F}{R^*} = \eta^* c^* - d_F \left( \frac{1}{R^{D*}} - \frac{1}{\pi^*} \right) - m_F, \tag{D.57}$$

where the last equation uses $R^m = R^*$ as well as the binding open market constraint. Observe from the multipliers on the cash in advance constraint $(\psi_F, \psi_H)$ and the open market constraint $(\mu)$ that a foreign interest rate policy satisfying $R^{m*} < \frac{\pi^*}{\beta^*}$ and $\pi^* > \beta^*$ as well as a positive domestic interest rate in the long run $(\pi > \beta)$ implies that all cash and open market constraints bind in the long run. Using (D.54), I can rewrite (D.57) as

$$b_F + d_F \left( \frac{R^*}{R^{D*}} - \frac{R^*}{\pi^*} \right) = \eta^* c^* R^* \frac{\Omega}{1+\Omega}$$

and can solve for $b_F$ given a level of total foreign asset holdings relative to imports $\bar{d} = \frac{b_F + d_F + m_F}{c_F}$,

$$b_F + \left( \bar{d} c_F - b_F - m_F \right) \left( \frac{R^*}{R^{D*}} - \frac{R^*}{\pi^*} \right) = \eta^* c^* R^* \frac{\Omega}{1+\Omega},$$

so that

$$b_F = \frac{\eta^* c^* R^* \frac{\Omega}{1+\Omega} + \left(\bar{d}c_F - \frac{\eta^* c^*}{1+\Omega}\right)\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right)}{1 + \frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}}.$$

Using (D.53) to solve for $b_F$ yields

$$
\begin{aligned}
c_F &= \frac{b_F}{R^{m*}\pi^*} + \frac{m_F}{\pi^*} \\
&= \frac{\eta^* c^* R^* \frac{\Omega}{1+\Omega} + \left(\bar{d}c_F - \frac{\eta^* c^*}{1+\Omega}\right)\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right)}{\left(1 + \frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) R^{m*}\pi^*} + \frac{m_F}{\pi^*} \\
&\Longleftrightarrow \\
c_F &= B^{-1}\eta^* c^* R^* \frac{\Omega}{1+\Omega} - \frac{\eta^* c^*}{1+\Omega}\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) \\
&\quad + B^{-1}\left(1 + \frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) R^{m*}\pi^* \frac{m_F}{\pi^*},
\end{aligned}
$$

where $B = \left[\left(R^{m*}\pi^* - \bar{d}\right)\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) + R^{m*}\pi^*\right]$. Then, back out $d_F$ by using holdings of foreign private debt by using $d_F = \bar{d}c_F - b_F - m_F$. Further, with (D.47) and (D.48), $\lambda$ can be eliminated, so that consumption is given by

$$c = q\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right)c_F.$$

To obtain $q$, I use this in (D.56) to replace $c$,

$$
\begin{aligned}
n^\alpha &= \left[\frac{q\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right)c_F}{c_F^\eta \gamma}\right]^{\frac{1}{1-\eta}} + \eta^* q^{\frac{1}{1-\eta}} \Phi^{\frac{1}{1-\eta}} c^* \\
&= q^{\frac{1}{1-\eta}}\left\{\left[\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right)\gamma^{-1}\right]^{\frac{1}{1-\eta}} c_F + \eta^* \Phi^{\frac{1}{1-\eta}} c^*\right\}. \quad \text{(D.58)}
\end{aligned}
$$

Further, I set $n = 0.33$ and use (D.46) to back out $\chi$ after the other steady state variables are determined. Thus, (D.58) can be used to solve for the real exchange rate,

$$q = n^{\alpha(1-\eta)}\left\{\left[\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right)\gamma^{-1}\right]^{\frac{1}{1-\eta}} c_F + \eta^* \Phi^{\frac{1}{1-\eta}} c^*\right\}^{\eta-1}.$$

Thus, home consumption is given by

$$c = q \left( 1 + \frac{R^{D*}}{R^D} \frac{1-\eta}{\eta} \right) c_F.$$

With this result at hand, the remaining variables can be backed out, yielding

$$w = \frac{\varepsilon-1}{\varepsilon} \Phi^{\frac{1}{\eta-1}} q^{\frac{\eta}{\eta-1}}, \qquad\qquad m_F = \frac{\eta^* c^*}{1+\Omega},$$

$$b_F = c_F R^{m*} \pi^* - m_F R^{m*}, \qquad\qquad \lambda = \frac{c^{-\sigma}}{R^D},$$

$$\chi = \lambda w n^{-\omega}, \qquad\qquad \psi_H = \lambda \left( R^D - 1 \right),$$

$$\psi_F = \lambda \left( R^{D*} - 1 \right), \qquad\qquad \mu = \lambda \left( \frac{R^{D*}}{R^*} - 1 \right),$$

$$tb = q \left( \eta^* c^* - c_F \right), \qquad\qquad c_H = \left( \frac{c}{c_F^\eta \gamma} \right)^{\frac{1}{1-\eta}},$$

$$c_H^* = q^{\frac{1}{1-\eta}} \Phi^{\frac{1}{1-\eta}} \eta^* c^*.$$

## D.3.2 Foreign economy

I use that the utility function is given by (5.66). As shown in Reynard and Schabert (2009), steady state inflation is determined by the growth rate of short-term government bonds, $\Gamma^*$. The central bank is assumed to adjust its long-run inflation target to this value, $\pi^* = \Gamma^*$. The households' first order conditions imply that the steady state interest rate on private debt is given by $R^{D*} = \frac{\pi^*}{\beta^*}$. Further, using the first order conditions for money holdings and consumption yields

$$\mu^* = c^{*-\sigma} \left( \frac{1}{R^{m*}} - \frac{1}{R^{D*}} \right).$$

Thus, the open market constraint binds in steady state when policy sets $R^{m*} < R^{D*} = \frac{\pi^*}{\beta}$. Further, the multiplier on the cash in advance constraint is given by $\psi^* = R^{m*} \eta^* + \lambda^* \left( R^{m*} - 1 \right)$ where $\lambda^* = \beta^* \frac{c^{*-\sigma}}{\pi^*}$ implies that $\psi^* = c^{*-\sigma} \left[ 1 - \frac{\beta^*}{\pi^*} \right]$. Thus, the cash in advance constraint binds whenever $\pi^* > \beta^*$, which is assumed to be fulfilled throughout the fourth chapter. Further, attention is restricted to a small neighborhood of the steady state, where the open market and cash in advance constraints bind. The steady state can be derived analytically from the remaining equilibrium conditions. Using the households' and firms' first order conditions (as well as the aggregate

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resource constraint $c^* = \frac{n^*}{s^*}$) gives

$$n^* = \left[ \frac{\varepsilon-1}{\varepsilon} \frac{A^{*1-\sigma} \beta^*}{\chi^* \pi^*} \right]^{\frac{1}{\omega^* + \sigma^*}}, \qquad\qquad w^* = A^* mc^*,$$

$$c^* = \frac{n^*}{s^*}, \qquad\qquad\qquad\qquad Z^{1*} = \frac{\varepsilon}{\varepsilon-1} \frac{c^{*-\sigma} y^*}{1 - \phi\beta} mc^*,$$

$$R^{D*} = \frac{\pi^*}{\beta^*}, \qquad\qquad\qquad\qquad Z^{2*} = \frac{c^{*-\sigma} y_H^*}{1 - \phi\beta},$$

$$R^* = R^{m*}, \qquad\qquad\qquad\qquad Z^{1*}/Z^{2*} = 1 \Longrightarrow mc^* = \frac{\varepsilon-1}{\varepsilon},$$

$$s^* = (Z^{1*}/Z^{2*})^{-\varepsilon} = 1.$$

Further, the cash-in-advance constraint and the households' holdings of money and bonds can be used to obtain the steady state values for $m, b$ and $b^T$,

$$m_F^* = \frac{c^*}{1 + \Omega},$$

$$b_F^* = R^{m*} m_F^* \pi^* \left( 1 + \Omega - \pi^{*-1} \right),$$

$$b^{T*} = \frac{\pi^*}{\Gamma^* - 1} \left[ b_F^* + b_F - \Omega \left( m_F^* + m_F \right) \right].$$

**Steady state under $\Gamma^* = 1$**

Consider the case analyzed in section 5.4.1 where nominal bond growth is zero, $\Gamma^* = 1$. In this case, the foreign economy's equilibrium conditions are fundamentally affected. (D.40) changes to $b_{F,t}^* = m_{F,t}^{R*}$ and thus, the real stock of government bond holdings becomes irrelevant for the equilibrium allocation. Ignoring the influence of foreign asset holdings (as in section 5.4.1), I obtain identical conditions as above, except for the steady state holdings of government bonds. Household money holdings (D.35) require

$$b_F = R^{m*} m_F^* \pi^* \left( 1 + \Omega - \pi^{*-1} \right),$$

while the evolution of households' bond holdings (D.40) requires

$$b_F^* = \Omega m_F^*.$$

A steady state exists only if both equations are satisfied, i.e. if

$$R^{m*} = \frac{\Omega}{\Omega \pi^* + \pi^* - 1}.$$

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Thus, if the central bank targets zero inflation, $\pi^* = 1$, the long-run policy rate has to be zero as well. For $\pi^* = 1$ and $R^{m*} > 1$, the economy has no steady state. The reason is that the central bank acquires bonds every period in its open market operations when $R^m > 1$. Given a nominally constant amount of bonds, and no steady state inflation, households' real bond holdings then must decline.

## D.4 Proof of Proposition 5.1

(5.62) implies that the decline in the liquidity premium is larger than the increase in the interest rate on foreign government bonds if the foreign debt rate falls below its steady state, $\hat{R}_t^{D*} < 0$. Consider the foreign economy under the assumptions in section 5.4.1, i.e. $u(c_t^*, n_t^*) = \log c_t^* - \chi n_t^*$, binding cash and open market constraints, flexible prices, constant nominal foreign government debt, $\Gamma^* = 1$, zero steady state inflation $\pi^* = 1$ as well as a policy rate governed by $R^{m*} = 1$ and $R_t^{m*} = R_{t-1}^{m*\rho^*} \exp\left(\varepsilon_t^R\right)$ and a negligible impact of home households' holdings of foreign government bonds on foreign households' holdings, $b_{F,t}^* = \Omega m_{F,t}^*$. The set of equilibrium conditions describing the foreign economy is then given by the linearized versions of (5.31) - (5.36), (5.41) with zero price dispersion $s_t^* = 1$, (5.43), binding open market and cash constraints (5.29) and (5.30), households' money holdings (5.50), labor demand $w_t^* = A^*$, the resource constraint $y_t^* = c_t^*$ and the policy rule $R_t^{m*} = R_{t-1}^{m*\rho^*} \exp\left(\varepsilon_t^R\right)$. Substituting out Lagrange multipliers in (5.31) - (5.36) yields the following system of linear equilibrium conditions

$$\hat{R}_t^* = E_t \hat{R}_{t+1}^{m*}, \tag{D.59}$$

$$\hat{R}_t^{D*} = E_t \hat{\pi}_{t+1}^*, \tag{D.60}$$

$$-E_t \hat{c}_{t+1}^* = E_t \hat{\pi}_{t+1}^*, \tag{D.61}$$

$$\hat{c}_t^* = \frac{m_F^*}{c^* \pi^*} \hat{m}_{F,t-1}^* + \frac{b_F^*}{c^* \pi^* R^{m*}} \left(\hat{b}_{F,t-1}^* - \hat{R}_t^{m*}\right) - \hat{\pi}_t^*, \tag{D.62}$$

$$\hat{m}_{F,t}^* = \hat{c}_t^*, \tag{D.63}$$

$$\hat{R}_t^{m*} = \rho^* R_{t-1}^{m*} + \varepsilon_t^R, \tag{D.64}$$

$$\hat{b}_{F,t}^* = \Omega \frac{m^*}{b_F^*} \hat{c}_t^*, \tag{D.65}$$

and conditions for the wage, production, injections and real government debt. Applying the expectations operator to (D.62), and using (D.63)

yields,

$$E_t \hat{c}^*_{t+1} = \frac{m^*_F}{c^* \pi^*} \hat{c}^*_t + \frac{b^*_F}{c^* \pi^* R^{m*}} \left( \hat{b}^*_{F,t} - E_t \hat{R}^{m*}_{t+1} \right) - E_t \hat{\pi}^*_{t+1}.$$

Thus, (D.61) can be rewritten as

$$
\begin{aligned}
\hat{c}^*_t &= -\frac{b^*_F}{m^* R^{m*}} \left( \hat{b}^*_{F,t} - E_t \hat{R}^{m*}_{t+1} \right) \\
&= -\pi^* \left( 1 + \Omega - \pi^{*-1} \right) \left( \hat{b}^*_{F,t} - E_t \hat{R}^{m*}_{t+1} \right),
\end{aligned}
$$

where I use the steady state relation $b^*_F / m^*_F = R^{m*} \pi^* \left( 1 + \Omega - \pi^{*-1} \right)$ derived in Appendix D.3.2. Replacing bond holdings by (D.65) yields

$$
\begin{aligned}
\hat{c}^*_t &= -\pi^* \left( 1 + \Omega - \pi^{*-1} \right) \left( \Omega \frac{m^*}{b^*_F} \hat{c}^*_t - E_t \hat{R}^{m*}_{t+1} \right) \\
&= \left( 1 + \Omega / R^{m*} \right)^{-1} \pi^* \left( 1 + \Omega - \pi^{*-1} \right) E_t \hat{R}^{m*}_{t+1} \\
&= \frac{\Omega}{1 + \Omega} E_t \hat{R}^{m*}_{t+1}.
\end{aligned}
$$

The debt rate is given by $\hat{R}^{D*}_t = E_t \hat{\pi}^*_{t+1} = -E_t \hat{c}^*_{t+1}$, so that its solution reads

$$
\begin{aligned}
\hat{R}^{D*}_t &= -\frac{\Omega}{1 + \Omega} E_t \hat{R}^{m*}_{t+2} \\
&= -\rho a_1 \hat{R}^{m*}_{t-1} - a_1 \varepsilon^R_t,
\end{aligned}
$$

where $a_1 = \rho^2 \frac{\Omega}{1+\Omega} > 0$. Thus, a positive foreign policy shock leads to a decrease in the private debt rate, which persists until the shock fades out. $\square$

## D.5 A foreign consumption shock in Galí and Monacelli (2005)

A simplified two-country version of Galí and Monacelli (2005) is characterized by sequences for $\{q_t, \pi_{Ht}, R_t\}$ given a monetary policy and exogenous sequences for $\hat{c}^*_t$ and $\hat{R}^*_t$. The sequences must satisfy a Phillips curve

$$\widehat{\pi}_{H,t} = \kappa \left( \frac{1}{1 - \vartheta} + \eta\psi \right) \hat{q}_t + \kappa \left( \sigma + \eta \right) \hat{c}^*_t + \beta E_t \widehat{\pi}_{H,t+1}, \qquad (D.66)$$

and an Euler equation

$$\frac{1}{1-\vartheta}E_t\left(\hat{q}_{t+1}-\hat{q}_t\right)=\hat{R}_t-E_t\widehat{\pi}_{Ht+1}-\sigma E_t\left(\hat{c}^*_{t+1}-\hat{c}^*_t\right),\qquad\text{(D.67)}$$

where $\kappa=\frac{(1-\phi\beta)(1-\phi)}{\phi}$ where $\phi$ characterizes price stickiness, $\sigma$ is the elasticity of intertemporal substitution of consumption goods, $\eta$ is the inverse of the Frisch elasticity of labor supply, $\psi=\vartheta+\frac{\vartheta}{1-\vartheta}+\frac{1-\vartheta}{\sigma}$ with $\vartheta$ characterizing openness of the home economy. Note that I have used both market clearing $\hat{y}_{H,t}=\psi\hat{q}_t+\hat{c}^*_t$ and the risk sharing condition $\sigma\hat{c}_t=\hat{q}_t+\sigma\hat{c}^*_t$ to derive the model. Monetary policy is given by a nominal exchange rate peg which requires $E_tS_{t+1}-S_t=0$. This is ensured by $\hat{R}_t=\hat{R}^*_t$ where a constant foreign interest rate, $\hat{R}^*_t=0$, is assumed. Thus, (D.67) becomes

$$\frac{1}{1-\vartheta}E_t\left(\hat{q}_{t+1}-\hat{q}_t\right)=-E_t\widehat{\pi}_{Ht+1}-\sigma E_t\left(\hat{c}^*_{t+1}-\hat{c}^*_t\right).\qquad\text{(D.68)}$$

Applying the minimum state variable solution

$$\hat{q}_t=\gamma_1\hat{c}^*_t,$$
$$\widehat{\pi}_{H,t}=\gamma_2\hat{c}^*_t,$$

and using that $\rho_c$ is the autocorrelation of foreign consumption gives

$$\gamma_2=(1-\beta\rho_c)^{-1}\left[\kappa\left(\frac{1}{1-\vartheta}+\eta\psi\right)\gamma_1+\kappa\left(\sigma+\eta\right)\right]$$

from the Phillips curve and

$$\frac{\rho_c-1}{1-\vartheta}\gamma_1 = -\rho_c\gamma_2-\sigma\left(\rho_c-1\right)$$
$$\Longleftrightarrow$$
$$-\rho_c\gamma_2 = \frac{\rho_c-1}{1-\vartheta}\gamma_1+\sigma\left(\rho_c-1\right)$$
$$\Longleftrightarrow$$
$$\gamma_2 = \frac{1-\rho_c}{\rho_c\left(1-\vartheta\right)}\gamma_1+\sigma\frac{1-\rho_c}{\rho_c}$$

from the Euler equation under the peg. Solving for $\gamma_1$ yields

$$\gamma_1=\frac{\kappa\left(\sigma+\eta\right)-\sigma\frac{(1-\beta\rho_c)(1-\rho_c)}{\rho_c}}{\frac{(1-\beta\rho_c)(1-\rho_c)}{\rho_c(1-\vartheta)}-\kappa\left(\frac{1}{1-\vartheta}+\eta\psi\right)},$$

whose sign cannot be determined analytically. The same is true for the sign of $\gamma_2 = \frac{1-\rho_c}{\rho_c(1-\vartheta)}\gamma_1 + \sigma\frac{1-\rho_c}{\rho_c}$. The responses of production and consumption are given by the risk sharing condition $\hat{c}_t = \hat{q}_t/\sigma + \hat{c}_t^* = \gamma_3\hat{c}_t^*$ and market clearing $\hat{y}_{H,t} = \psi\hat{q}_t + \hat{c}_t^* = \gamma_4\hat{c}_t^*$ where

$$\gamma_3 = 1 + \gamma_1/\sigma,$$
$$\gamma_4 = 1 + \psi\gamma_1.$$

Given the parameterization summarized in Table 5.1, $\gamma_1 < 0$, $\gamma_2 < 0$, $\gamma_3 > 0$ and $\gamma_4 < 0$. Thus, an increase in foreign consumption leads to a real appreciation of the home currency, a decline in domestic production and PPI inflation and an increase in consumption in the home economy.

# Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Arbeit selbständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus fremden Quellen direkt oder indirekt entnommenen Gedanken sind als solche kenntlich gemacht. Die Arbeit wurde bisher in gleicher oder ähnlicher Form keiner anderen Prüfungsbehörde vorgelegt und auch nicht veröffentlicht.

Berlin, im Dezember 2010