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German stock market behavior and the IFO business climate index: a copula-based Markov approach

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Abstract

This paper investigates the driving force for German stock market behavior - stock market confidence. By using monthly new $VDAX$ closing prices and a copula-based Markov approach, a proxy for German stock market confidence is derived. It can be shown that confidence responds to expected output changes in terms of differences of the IFO business climate index and to US confidence changes. Furthermore, German stock market behavior seems to be sticky in comparison to the United States and reduces the marginal effects of the remaining adjustment factors.

JEL Classification: C12, C22, E44
Keywords: Stock market uncertainty, VDAX-New, temporal dependence

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1 Introduction

Almost every economic transaction is based on confidence of the contractual partners. This fundamental economic issue becomes important in the public debate on the latest financial crisis, which can be described as a confidence crisis. Especially the behavior of stock market investors attracted attention due to the deep impact on the economy. According to stock market confidence of a market participant an individual investment strategy is selected and determines on an aggregated level stock market uncertainty. Uncertainty shocks lead to a drop in employment and output (see Bloom (2009))[1] and clarify the link between stock market confidence and macroeconomics. Triggered by the latest financial turbulences various regulatory interventions are discussed, which may suggests that stock market participants are only speculative and are not guided by economic fundamentals. In order to understand stock market behavior, the underlying reason - stock market confidence - has to be analyzed in more detail. The behavioral characterizations could be used for the development of new investment strategies. Hence, the aim of this paper is the derivation of a stock market proxy for Germany, the description of its economic determinants and the characterization of German stock market participants.

The well-established IFO business climate index approximates the effective business climate and considers the actual and expected business situation in Germany. As a leading indicator for economic output the index is generally accepted. If stock market confidence responds to the index, it implies that market participants behave dependent on expectations concerning the real economy. This behavior is consistent with neoclassical theory (see Diamond (1967)). Even in case that output expectations - reflected by the index - deviate from actual output, market participants would behave in accordance to available economic information, if they consider the index for their confidence fixing. If the market participants behave independently from the business climate, the investment behavior is rather speculative and leads to rather unpredictable macroeconomic consequences.

In this paper a proxy for German stock market confidence is derived on the basis of stock market uncertainty. Generally, a canonical and typically used proxy for stock market uncertainty is expected stock market variability. Those uncertainty proxies are constructed by using options of specific underlying stock market indices. Prominent examples are the VIX for the S&P 500 in the USA, $VFTSE$ for the FTSE 100 in the UK and the $VDAX$[2] for

[1] Moreover, empirical evidence from the USA underlines the effect of stock market uncertainty on monetary policy (see Jovanović and Zimmermann (2010)).

[2] In order to ease the notion, the new $VDAX$ is labelled $VDAX$.

the DAX in Germany. The $VDAX$ was introduced by the Deutsche Börse AG in April 2005 and is backward projected until January 1992.

Based on an economic model the temporal dependence of the $VDAX$ serves as a proxy for stock market confidence, where copula-based Markov models are the methodological framework. By the theorem of Sklar (1959) any multivariate distribution can be expressed in terms of its marginal distributions and its copula function. A copula function is a multivariate distribution function with standard uniform marginals, which captures the scale-free dependence structure of the multivariate distribution function. The copula-based approach has the advantage of separating the information about the marginal distributions from the scale-free dependence structure. Darsow et al. (1992) extent this approach to Markov processes. By coupling different marginal distributions with different copula functions, copula-based time series models are able to model a wide variety of marginal behaviors (such as skewness and fat tails) and dependence properties (such as clusters, positive or negative tail dependence). Chen and Fan (2006) develop a two-step estimation procedure for parametric copula functions and make this methodological approach usable for economic applications.

The rest of the paper is organized as follows. Section 2 reviews the methodological concept of copula-based Markov processes. Section 3 presents an economic model which deals with stock market confidence. The preceding section 4 is concerned with the empirical investigation in Germany. Section 5 draws a conclusion. Some tables and technical details are relegated to the Appendix.

2 Methodology

Let $\{Y_t\}$ be a stationary first-order Markov process with continuous state space. Then the joint distribution function $H(y_{t-1}, y_t) = P(Y_{t-1} \leq y_{t-1}, Y_t \leq y_t)$, $(y_{t-1}, y_t) \in \mathbb{R}^2$, of Y_{t-1} and Y_t completely determines the stochastic properties of $\{Y_t\}$. Due to Sklar's theorem, it is possible to express $H(y_{t-1}, y_t)$ in terms of the marginal distribution $G(y_t) = P(Y_t \leq y_t)$, $y_t \in \mathbb{R}$, of Y_t and the dependence function of Y_{t-1} and Y_t. This dependence function

$$C(G(y_{t-1}), G(y_t)) = H(y_{t-1}, y_t) \tag{1}$$

is known as "copula". Hence, $C(u_{t-1}, u_t) = P(U_{t-1} \leq u_{t-1}, U_t \leq u_t)$, $(u_{t-1}, u_t) \in [0,1]^2$, is the joint distribution function of the two random variables $U_{t-1} = G(Y_{t-1})$ and $U_t = G(Y_t)$. $h(\cdot,\cdot)$, $c(\cdot,\cdot)$ and $g(\cdot)$ are the associated (joint) density functions. We will consider in this paper three frequently used copulas (Gauss, Clayton, Frank) and one rarely used copula (Fang). For details see the Appendix. One obvious feature of the copula-based time series approach is the possibility to seperate the time dependence structure from the marginal distribution. Especially in Economics this issue becomes important, due to plenty economic information reflected by the marginal distribution.[3] We consider the following set of assumptions:

(A1) $\{Y_t\}_{t=1}^n$ is a sample of a stationary first-order Markov process generated from the true marginal distribution $G(\cdot)$ - which is invariant and absolutely continuous with respect to the Lebesgue measure on the real line - and the true parametric copula $C(\cdot,\cdot;\alpha)$ - which is absolutely continuous with respect to the Lebesgue measure on $[0,1]^2$.
(A2) $G(\cdot)$ and the d-dimensional copula parameter $\alpha \in \mathbb{R}^d$ are unknown.
(A3) $C(\cdot,\cdot;\alpha)$ is neither the Fréchet-Hoeffding upper bound ($C(u_{t-1},u_t) = \min(u_{t-1},u_t)$) nor the lower bound ($C(u_{t-1},u_t) = \max(u_{t-1}+u_t-1,0)$).

If (A3) would not be true, it is well-known that Y_t would be almost surely a monotonic function of Y_{t-1}. Therefore, the resulting time series would be deterministic and in case of stationarity, $Y_t = Y_{t-1}$ for the upper bound and $Y_t = G^{-1}(1-G(Y_{t-1}))$ for the lower bound would follow. We abandon from these cases to focus on stochastic samples of stationary first-order Markov processes. Due to Sklar's Theorem of equation (1) the copula density function $c(u_{t-1},u_t;\alpha) = \frac{\partial^2 C(u_{t-1},u_t;\alpha)}{\partial u_{t-1}\partial u_t}$ equals $\frac{h(y_{t-1},y_t)}{g(y_{t-1})\cdot g(y_t)}$. Hence, the conditional density of y_t given $y_{t-1},\ldots,y_1$ is

$$h(y_t|y_{t-1}) = g(y_t)c(G(y_{t-1}),G(y_t);\alpha)\ . \tag{2}$$

As far as the conditional density is a function of the copula and the marginal, the v_t-th, $v_t \in [0,1]$, conditional quantile Q_{v_t} of y_t given y_{t-1} is a function of the copula and the marginal,

$$Q_{v_t}(y_t|y_{t-1}) = G^{-1}\left(C_{t|t-1}^{-1}[v_t|G(y_{t-1});\alpha]\right)\ . \tag{3}$$

$C_{t|t-1}(u_t|u_{t-1};\alpha) = P(U_t \le u_t|U_{t-1} = u_{t-1}) = \frac{\partial C(u_{t-1},u_t;\alpha)}{\partial u_{t-1}}$ denotes the conditional distribution of U_t given $U_{t-1} = u_{t-1}$, which we assume to exist. Therefore, $C_{t|t-1}^{-1}[v_t|G(y_{t-1});\alpha]$ is the v_t-th conditional quantile of u_t given u_{t-1}. Considering assumption (A2) the unknown marginal distribution $G(\cdot)$ and the unknown copula parameter vector α have to be estimated. Chen

[3]Furthermore, the temporal dependence structure is invariant concerning monotonic transformations by the invariance theorem of copulas . Hence, temporal dependence of the $VDAX$ equals the temporal dependence structure of the frequently used transformations $VDAX^2$ and $\ln VDAX$.

and Fan (2006)[4] derive the following semiparametric two-step procedure:

Step 1: Estimate $G(y)$ by the rescaled empirical distribution

$$\widehat{G}(y) = \frac{1}{n+1} \sum_{t=1}^{n} 1\{Y_t \leq y\} \ . \tag{4}$$

Step 2: Estimate the copula parameter vector by

$$\widehat{\alpha} = \arg\max_{\alpha} \frac{1}{n} \sum_{t=2}^{n} \log c(\widehat{G}(Y_{t-1}), \widehat{G}(Y_t); \alpha) \ . \tag{5}$$

$\widehat{\alpha}$ is root-n consistent and has approximately a normal distribution.

According to Chen and Fan (2006) the following generalized semiparametric regression transformation model exists:

$$\Lambda_1(G(Y_t)) = \Lambda_2(G(Y_{t-1})) + \nu_t \ , \ \ E(\nu_t|Y_{t-1}) = 0 \ , \tag{6}$$

with a parametric increasing function $\Lambda_1(\cdot)$ of U_t, $\Lambda_2(u_{t-1}) := E(\Lambda_1(U_t)|U_{t-1} = u_{t-1})$, and the conditional density of ν_t given $U_{t-1} = u_{t-1}$ is

$$f_{\nu_t|U_{t-1}=u_{t-1}}(\nu_t) = \frac{c(u_{t-1}, \Lambda_1^{-1}(\nu_t + \Lambda_2(u_{t-1})); \alpha)}{\frac{d\Lambda_1(\nu_t + \Lambda_2(u_{t-1}))}{d\nu_t}} \ . \tag{7}$$

It follows in general

$$\Lambda_2(u_{t-1}) = E(\Lambda_1(U_t)|U_{t-1} = u_{t-1}) = \int_0^1 \Lambda_1(u_t) c(u_{t-1}, u_t; \alpha) du_t \tag{8}$$

and for the special case of identity mapping $\Lambda_1(u_t) = u_t$

$$\Lambda_2(u_{t-1}) = E(U_t|U_{t-1} = u_{t-1}) = 1 - \int_0^1 C_{t|t-1}(u_t|u_{t-1}; \alpha) du_t \ . \tag{9}$$

Therefore, without loss of generalization the identity mapping case yields to the autoregressive process[5]

$$u_t = \Lambda_2(u_{t-1}) + \nu_t \ . \tag{10}$$

[4]Instead of using the rescaled empirical distribution function, one could use an adequate kernel estimator of the distribution function. Furthermore, they offer an appropriate bootstrap method to construct statistical inference procedures for the estimated quantiles.
[5]Strictly speaking the process is an autoregressive quantile process, whereas the quantile treatment can be simply interpreted as a stabilizing transformation.

Contrary to the traditional linear case, $|\alpha| < 1$,

$$u_t = \alpha u_{t-1} + \epsilon_t \tag{11}$$

with an iid error ϵ_t, $E(\epsilon_t|u_{t-1}) = 0$, the copula-based approach allows for nonlinear temporal dependence structures. In order to calculate a proxy for the systematic temporal dependence between u_{t-1} and u_t substitute the theoretical quantile u_t by its nonparametric estimate of the empirical distribution $\widehat{u}_t = \frac{n+1}{n}\widehat{G}(y)$ and name the ascended sorted empirical quantiles $\widehat{u}_t$ by $\widehat{u}_t^{\star\star}$. The systematic projection of the expected quantile in the linear case is $\widehat{u}_t^\star = \widehat{\alpha} \cdot \widehat{u}_{t-1}^{\star\star}$ and leads to a constant strength of temporal dependence calculated by $\Delta\widehat{u}_t^\star = \widehat{\alpha}/n$. In the generalized case the systematic projection of the expected quantile is

$$\widehat{u}_t^\star = C_{t|t-1}^{-1}(0.5|\widehat{u}_{t-1}^{\star\star}; \widehat{\alpha}) \tag{12}$$

and can be used to calculate the proxy for the strength of temporal dependence $\Delta\widehat{u}_t^\star = \widehat{u}_t^\star - \widehat{u}_{t-1}^\star$. This generalized version of temporal dependence allows also for nonlinear dependencies conditional on the level of $\widehat{u}_{t-1}^\star$ and the copula C. Therefore, the following definition of conditional temporal dependence will be considered:

Definition 1 *The proxy for conditional temporal dependence between the random variables U_{t-1} and U_t given $\widehat{u}_{t-1}^\star$ and a copula C is defined by:*

$$dep(U_{t-1}, U_t|\widehat{u}_{t-1}^\star, C) := \Delta\widehat{u}_t^\star$$

Once the values for $\Delta\widehat{u}_t^\star$ are calculated, every $\widehat{u}_t$ can be uniquely related to a value y_t and $\Delta\widehat{u}_t^\star$ and leads to a time series of conditional temporal dependencies $dep(Y_{t-1}, Y_t|y_{t-1}, C)$ which correspond to the values of y_t. Although the copula parameters - which can be transformed to the correlation coefficient according to Kendall or Spearman - are treated as time invariant (α and not α_t) the copula itself allows for a variation of temporal dependence conditional on the quantile level.

3 Economic model of stock market confidence

3.1 Time series derivation

Consider the random variable $Y_{i,t}^\star$ which stands for stock market uncertainty of investor $i = 1, \ldots, m$ at the end of the last trading day of month

$t = 1, \ldots, n$ and its realization $y^{\star}_{i,t}$.[6] According to Aoki and Yoshikawa (2007) about 95 percent of the total market participants belong to the two largest subgroups of agents by types. With two largest clusters, there are two regimes; one with a cluster of investors with strategy 1 as the largest share, and the other with a cluster of investors using strategy 2 as the largest share. Namely, fundamentalists dominate the market in regime 1 and chartists dominate the market in regime 2. We postulate that the decision at the end of period t of a market participant i being a fundamentalist ($y_{i,t} = 1$) or a chartist ($y_{i,t} = 0$) is determined by individual stock market uncertainty $y^{\star}_{i,t}$ and an individual threshold φ_i for being a chartist or a fundamentalist.

$$y_{i,t} = \begin{cases} 0 & , \text{ if } \; y^{\star}_{i,t} \geq \varphi_i \\ 1 & , \text{ if } \; y^{\star}_{i,t} < \varphi_i \end{cases}$$

This decision rule implies that individuals make their strategy decision monthly and know their own threshold φ_i.

The main argument for this decision rule is the attempt of the investors to maximize their expected profits. Referring to this reasoning consider stock market uncertainty in the conventional sense as expected stock market variability. Hence, $y^{\star}_{i,t}$ can be substituted by $E_{i,t}(\sigma_{t+1})$, where σ_{t+1} stands for stock market variability during the month $t + 1$. As Aoki and Yoshikawa (2007) show, a market structure dominated by chartists leads to higher stock market variability σ than a market structure dominated by fundamentalists. Corresponding to Fama (1970) the market structure dominated by chartists reflects inefficient markets and the market structure dominated by fundamentalists reflects weak efficient markets. It is therefore conceivable that investors conclude from variability to market efficiency. This behavioral assumption allows for the link between $E_{i,t}(\sigma_{t+1})$ and $E_{i,t}(\text{market efficiency}_{t+1})$. In case of inefficient markets asset prices do not reflect historical price information and it is possible to earn excess returns r by being a chartist. On the other hand, if the market is rather weak efficient, asset prices reflect historical price information and it is possible to achieve excess returns by being a fundamentalist. Consequently, $y^{\star}_{i,t} \geq \varphi_i$ implies $E_{i,t}(r_{t+1}|y_{i,t} = 0) > E_{i,t}(r_{t+1}|y_{i,t} = 1)$ and $y^{\star}_{i,t} < \varphi_i$ implies $E_{i,t}(r_{t+1}|y_{i,t} = 1) > E_{i,t}(r_{t+1}|y_{i,t} = 0)$. Hence, the investment decision is motivated by expected profits and follows the expected market structure. High uncertainty leads by the decision rule to an investment strategy which causes higher stock market variability (Aoki and Yoshikawa (2007)). Straightfor-

[6] In fact $Y^{\star}_{i,t}$ symbolizes the quantile of stock market uncertainty. In order to avoid a burdensome notation the economic argumentation neglects this transformation without loss of generalization in this chapter.

ward, the decision rule acts like an accelerator for financial market instability and equals a complementary game, which induces nonlinearities on a macroeconomic level (see Cooper (1999)).

To construct a proxy for stock market confidence it is necessary to formulate an expectation formation mechanism of the expectation $E_{i,t}(\sigma_{t+1}) = y^{\star}_{i,t}$ in the decision rule. The following rule is motivated by Keynes (1936) and explains expectations by a projection of the existing situation and expected changes. Adopting this general approach in a time series context the projection of the existing situation is $\Lambda_3(E_{i,t-1}(\sigma_t))$ with a function Λ_3 determined by a copula. The expected changes are $E_{i,t}(\sigma_{t+1}|\sigma^-_t) - E^-_t(\sigma_{t+1}) = \epsilon^{\star}_{i,t}$ with a projection of realized variability $E_{i,t}(\sigma_{t+1}|\sigma^-_t)$ conditional on information concerning realized variability σ^-_t up to the day prior the last trading day and information concerning market variability expectations $E^-_t(\sigma_{t+1})$ until the day before the last trading day. According to this thoughts we receive the individual expectation formation

$$y^{\star}_{i,t} = \Lambda_3(y^{\star}_{i,t-1}) + \epsilon^{\star}_{i,t} \tag{13}$$

with $E(\epsilon^{\star}_{i,t}|y^{\star}_{i,t-1}) = 0$. The variability $\sqrt{V(\epsilon^{\star}_{i,t}|y^{\star}_{i,t-1})} = |\epsilon^{\star}_{i,t}|$ of $\epsilon^{\star}_{i,t}$ corresponds to the absolute deviation of individual realized variability expectations and market variability expectations. Following Keynes (1936) "confidence" is defined by the relevance - or equivalently weight - of the systematic expectation argument and causes the expectation. Regarding equation (13) the systematic component $y^{\star}_{i,t-1}$ is weighted by the function Λ_3. If the relevance of $y^{\star}_{i,t-1}$ for $y^{\star}_{i,t}$ is high, the confidence of the expectation argument is high and vice versa. This mechanism implies in connection with the decision rule that in case of high confidence the development of expectations show more persistence and with it more persistence of the development of investment strategies. The market participants have less incentive to change their strategy in face of high confidence. As long as the expectations are linked to stock market variability it is reasonable to equate expectation confidence with stock market confidence. Hence, the correct specification of Λ_3 in the copula-based Markov approach of (13) allows for a description of stock market confidence dependent on the level of stock market uncertainty. In line with Definition 1 individual stock market confidence is then measurable by the temporal dependence between $Y^{\star}_{i,t-1}$ and $Y^{\star}_{i,t}$. Leaving the individual level by aggregating individual investment decisions leads to the market structure $S_{t+1} = m^{-1}\sum_{i=1}^{m} y_{i,t}$ with $0 \leq S_{t+1} \leq 1$ and the market uncertainty $E_t(\sigma_{t+1}) = y^{\star}_t = m^{-1}\sum_{i=1}^{m} y^{\star}_{i,t}$, which can be described analog to (13) by

$$y^{\star}_t = \Lambda(y^{\star}_{t-1}) + \epsilon^{\star}_t \tag{14}$$

with $E(\epsilon_t^\star|y_{t-1}^\star) = 0$. The market wide stock market confidence proxy is the temporal dependence of the market wide stock market uncertainty.

3.2 Structural explanation

Human behavior in terms of confidence is persistent. It takes time to provide confidence and to lose confidence. In order to investigate the dynamics of German stock market confidence κ consider $\Delta\kappa_t^\star$, which induces a persistence neutral systematic adjustment of stock market confidence in the following framework:

$$\kappa_t = (1-\rho)\Delta\kappa_t^\star + \rho\kappa_{t-1} + \nu_t \quad , 0 \leq \rho < 1 \tag{15}$$

ν_t is an independent and identically distributed error with $E(\nu_t) = 0$, $V(\nu_t) > 0$ and $E(|\nu_t|^n) < \infty$ for all $n = 1, 2, \ldots$. Generally, in the case $\Delta\kappa_t^\star > 0$ positive confidence adjustments and in the case $\Delta\kappa_t^\star < 0$ negative confidence adjustments follow. Dependent on the behavioral persistence - expressed in the value for ρ - confidence adjustment varies. If ρ is high, adjustment processes are low and vice versa. To concretize equation (15) $\Delta\kappa_t^\star$ has to be specified.

According to neoclassical theory the price p_t of a specific stock equals the expected discounted future profits $x_{t+\tau}$, $\tau = 1, 2, \ldots$, and is therefore caused by expected future output of the real economy (see Diamond (1967)). For a constant discount rate i the present value model leads to

$$p_t = \sum_{\tau=1}^{n}(1+i)^{-\tau}E_t(x_{t+\tau}) \text{ and } p_{t-1} = \sum_{\tau=1}^{n+1}(1+i)^{-\tau}E_{t-1}(x_{t+\tau-1}) \, . \tag{16}$$

Consequently,

$$\Delta p_t = \sum_{\tau=1}^{n}(1+i)^{-\tau}\Delta E_t(x_{t+\tau}) - (1+i)^{-(n+1)}E_{t-1}(x_{t+n}) \tag{17}$$

and

$$\lim_{n\to\infty}\Delta p_t = \sum_{\tau=1}^{\infty}(1+i)^{-\tau}\Delta E_t(x_{t+\tau}) \tag{18}$$

follows. In order to ease the notation, "$\lim_{n\to\infty}$" in equation (18) will be neglected in the following. The commonly used unconditional estimate $\widehat{\sigma}_t$ of the return variability σ_t is $\sqrt{\Delta p_t^2} = |\Delta p_t|$ and leads by equation (18) and the triangle inequality for sums to

$$\widehat{\sigma}_t \leq \sum_{\tau=1}^{\infty}(1+i)^{-\tau}|\Delta E_t(x_{t+\tau})| \, . \tag{19}$$

Hence, neoclassical theory implies an estimate for stock market variability, which includes an upper bound determined by expectation changes concerning future profits. Stock market uncertainty in terms of expected future stock market variability, i.e. $E_t(\sigma_{t+1})$, considers behavioral assumptions concerning expectation formation. Assuming worst case expectations and consequently substituting σ by the upper bound for $\widehat{\sigma}$ in the definition for stock market uncertainty, leads to the neoclassical explanation of uncertainty

$$E_t(\sigma_{t+1}) \approx \sum_{\tau=1}^{\infty}(1+i)^{-\tau}|\Delta E_t(x_{t+\tau+1})| \ . \qquad (20)$$

Therefore, according to neoclassical theory stock market uncertainty is determined by absolute values of expectation changes concerning future profits. In turn, as previously discussed stock market confidence κ_t is the driving factor of stock market uncertainty. For this reason it is reasonable to conclude that expectation changes concerning profits cause confidence adjustments and is an element of $\Delta\kappa_t^{\star}$.[7] For the aggregated stock market it is quite unrealistic that individuals discount infinite future changes of profit expectations, which is implied by the present value model. Therefore, the expectation effect on confidence changes is reduced to a feasible and empirically justified term $\Delta E_t(x_{t+1})$ (see section 4). The second element of $\Delta\kappa_t^{\star}$ is the change of US stock market confidence $\Delta\kappa_{us,t}$ and reflects the fact of US dependence of German stock market behavior. According to this considerations German stock market confidence adjustments are systematically driven by

$$\Delta\kappa_t^{\star} = \beta\Delta E_t(x_{t+1}) + \gamma\Delta\kappa_{us,t} \ . \qquad (21)$$

From equation (15) and (21) follows

$$\kappa_t = (1-\rho)[\beta\Delta E_t(x_{t+1}) + \gamma\Delta\kappa_{us,t}] + \rho\kappa_{t-1} + \nu_t \ . \qquad (22)$$

As long as equation (22) contains only unobservable variables adequate proxies have to be substituted. The IFO business climate index is a highly respected leading indicator for economic output and contains information about profit expectations. Therefore, $\Delta E_t(x_{t+1})$ will be substituted by the difference of the IFO index, $\Delta\widehat{x}_t$. Furthermore, κ_t will be substituted by the Frank-copula-based confidence proxy $\widehat{\kappa}_t$ and $\kappa_{us,t}$ will be substituted by the Fang-copula-based confidence proxy $\widehat{\kappa}_{us,t}$ (see preceding section). By the appropriate substitutions equation (22) leads to the feasible model

$$\widehat{\kappa}_t = (1-\rho)[\beta\Delta\widehat{x}_t + \gamma\Delta\widehat{\kappa}_{us,t}] + \rho\widehat{\kappa}_{t-1} + e_t \qquad (23)$$

[7] The absolute values of expectation changes are only valid, if the square in Δp_t^2 is considered as a volatility estimate.

with the linear combination of approximation errors and ν_t

$$\begin{aligned} e_t \; &:= \; -(\kappa_t - \widehat{\kappa}_t) - \rho(\kappa_{t-1} - \widehat{\kappa}_{t-1}) \\ &\quad +(1-\rho)\{\beta[\Delta E_t(x_{t+1}) - \Delta\widehat{x}_t] + \gamma[\Delta\kappa_{us,t} - \Delta\widehat{\kappa}_{us,t}]\} + \nu_t \; , \end{aligned} \tag{24}$$

which is assumed to be independent and identically distributed with $E(e_t) = 0$, $V(e_t) > 0$ and $E(|e_t|^n) < \infty$ for all $n = 1, 2, \ldots$ and uncorrelated with the right hand side variables of equation (23). Consequently, the unknown parameter vector (β, γ, ρ) of equation (23) can be consistently estimated by OLS. A simple autoregressive approach is misspecified and accounts not for systematic adjustment processes of confidence.

4 Empirical results

Stock market uncertainty in terms of expected stock market volatility (see e.g. Bloom (2009)) is canonically approximated by the $VDAX$ in the German case and by the VIX in the US case. Based on this index stock market confidence is implied derivable. We use data from Thompson Datastream[8] for the period January 1992 to December 2010. Hence, the number of observed months is $n = 228$. In order to derive a stock market confidence proxy

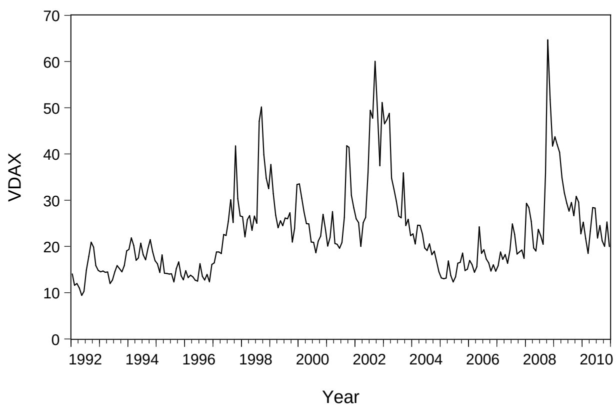

Figure 1: Monthly $VDAX$ closing prices.

four parametric copulas are discussed (Gauss, Clayton, Frank, Fang). For technical details see the copula review and Table 4 of the Appendix. The

[8]The time series code for the daily $VDAX$ closing prices in Euro is "VDAXNEW" and for the VIX closing prices in US dollars is "CBOEVIX".

hypothesis that the Frank copula captures the time dependence structure of the $VDAX$ can not be rejected on any plausible level of significance.[9] To test the correctness of a copula in a first-order Markov framework, consider the following multiple hypothesis test of interest (notation in line with chapter 2):
H_0: $\{Y_t\}$ is a first-order Markov process with copula C

H_0 is equivalent to

H_0' : $V_t = C_{t|t-1}(U_t|U_{t-1}; \alpha)$ is uniformly on $[0, 1]$ distributed and not autocorrelated

We reject H_0 if H_0' is rejected. Table 1 shows the estimation and test results.

Table 1: $VDAX$ results

Copula	Estimate	Estimated autocorrelation						G-o-f
		1	2	3	4	5	6	
Gauss	0.885	-0.10	-0.12	0.02	-0.03	-0.01	0.09	
	(0.031)	(0.14)	(0.08)	(0.73)	(0.63)	(0.93)	(0.16)	(0.17)
Clayton	2.812	0.13	0.08	$0.13^{\star}$	0.11	0.08	$0.18^{\star}$	
	(0.169)	(0.06)	(0.26)	(0.05)	(0.11)	(0.24)	(0.01)	(0.76)
Frank	12.076	-0.05	-0.05	0.02	0.01	0.02	0.06	
	(0.861)	(0.45)	(0.46)	(0.79)	(0.84)	(0.80)	(0.40)	(0.38)
Fang, α	0.186	0.03	-0.01	0.03	0.05	0.03	0.06	
	(0.016)	(0.61)	(0.89)	(0.65)	(0.48)	(0.61)	(0.34)	(0.99)
Fang, β	0.9996							
	(0.0002)							

Sample: 1992:1-2010:12 • Included observations $T = 227$ • Initial value of the one parameter copulas is 1 and of the Fang copula are $\widehat{\alpha}_1 = 0.3$ and $\widehat{\beta}_1 = 1$ • ML-estimates are different from zero at any level of significance (standard errors in brackets) • Spearman's correlation coefficients and p-values of the hypothesis $\rho_s(V_t, V_{t-l}) = 0$, $l = 1, \ldots, 6$, in brackets • * indicates a significant autocorrelation on the 10% overall error rate using Bonferroni's adjustment (see e.g. (Sokal & Rohlf, 1995)) • 2 is the number of tests performed (correlation test up to a specific lag and goodness-of-fit (G-o-f) test) • Finite sample adjustment of the Kolmogorov statistic and corresponding p-values of the hypothesis $V_t \sim U[0, 1]$ in brackets (see e.g. (D'Agostino & Stephens, 1986))

[9]Although, the correctness of the Gauss and Fang copula can also not be rejected the Frank copula is selected, because of its superior fit (see the section concerning copula-specific forecasting errors of the Appendix). In contrast to the German case the Fang copula is the unique identified correct copula in the US case and leads to the conclusion of unambiguous asymmetric tail dispersion.

Summing up the hypothesis tests and the forcasting errors of the Appendix the appropriateness of the Frank copula is indicated. Figure 2 allows for a

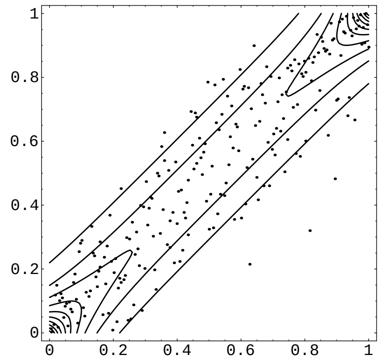

Figure 2: Contour with scatter plot of the empirical $VDAX$ quantiles t and $t-1$ of the Frank copula, $\widehat{\alpha} = 12.076$.

graphical inspection of its density based on the parameter estimates. The Frank copula shows more density mass in the lower and upper tails and obeys symmetric tail dispersion.

Once the correct copula is specified, it is possible to calculate the stock market confidence proxy according to equation (12) and Definition 1. Figure 3 shows on the left side the dependence structure between confidence and un-

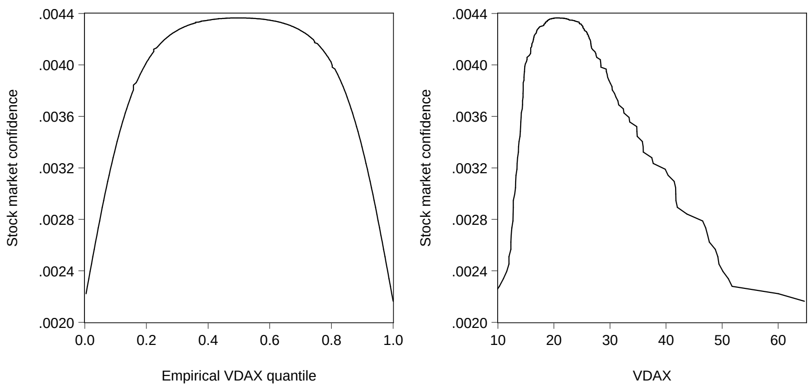

Figure 3: Dependence between German stock market uncertainty and German stock market confidence.

Table 2: German stock market confidence

	ρ	β	γ	$\overline{R}^2$	BG	White	ARCH
(a)	0.719	-	-	0.52	{0.59}	{0.02}	{0.01}
	(0.056)	-	-				
(b)	0.709	2.035	0.420	0.55	{0.37}	{0.20}	{0.12}
	(0.057)	(0.751)	(0.176)				

Confidence explanation: $\widehat{\kappa}_t = (1-\rho)[\beta\Delta\widehat{x}_t + \gamma\Delta\widehat{\kappa}_{us,t}] + \rho\widehat{\kappa}_{t-1} + e_t$ with (a) restricted model ($\beta = \gamma = 0$) and (b) unrestricted model • Sample: 1992:1-2010:12 • OLS estimation • Included observations $T = 227$ • White heteroskedasticity-consistent standard errors in parenthesis • p-values in curly brackets • Breusch-Godfrey test (BG) of the lag order 2. The test indicates on the 95% level at any lag uncorrelated residuals • White's heteroskedasticity test (White) with cross terms of the residuals • Test of ARCH(1) effects of the residuals (ARCH) • The test statistics are $T \cdot R^2$.

certainty quantiles[10] and on the right side the dependence between confidence and uncertainty levels. The backward projection from the quantiles to the levels is done by the empirical distribution function. By using copula-based Markov models as the methodological framework the statistical significant and stable relationship between the Keynesian motivated stock market confidence proxy and its dependent stock market uncertainty can be derived. It is interesting to note that the estimated autocorrealation coefficient (lag 1) of German stock market confidence (0.72) exhibits the US value of 0.42 by wide margins and leads to the conclusion of more persistence of stock market behavior in Germany.

The estimation results of the structural model of German stock market confidence (see equation (23)) and of the autoregressive approach are summarized in Table 2. Hereby the IFO business climate index is received from Thompson Datastream.[11] If on the one hand stock market confidence is explained autoregressive, unexplained systematic variability is relegated to the residuals and leads to ARCH(1) heteroskedasticity. If on the other side the autoregressive approach is augmented by expectation changes concerning the real economy and US confidence changes, the residuals show no systematic pattern anymore. Estimating the simple autoregressive model leads to

[10]Obvious outliers are substituted by local means.

[11]The time series code for the monthly index is "BDCNFBUSQ". All time series of the regression approach are normalized with the arithmetic mean of 0 and the sample variance of 1.

misspecification.[12]

5 Conclusions

In this paper German stock market confidence is approximated by temporal dependence of the monthly $VDAX$ closing prices in a Frank copula-based Markov model. The derived time series proxy for confidence is additionally explained by an autoregressive structural equation. Neoclassical theory implies the relevance of expected output changes for confidence adjustments. Based on this theory differences of the IFO business climate index are used as proxies for expected output changes. Due to the highly significant confidence response to IFO index changes, German stock market behavior is systematically driven by expectations on the real economy. Even in case of irrational expectations market behavior is systematically influenced by these expectations. Hence, regardless of the rationality of the expectations German stock market behavior can be characterized as fundamental due to the systematic output focus.

Because of the empirical evidence of the IFO business climate relevance for German stock market behavior, further forecasting research concerning the predictability of out-sample stock market uncertainty or uncertainty change-points is indicated. From a macroeconomic perspective the predictability of stock market uncertainty is important with respect to financial stability and due to risk premiums essential for interest rates. From a finance perspective the predictability of stock market uncertainty is directly important for volatility derivatives, where the payoff of an asset is dependent on an uncertainty index like the $VDAX$. Furthermore, improved uncertainty predictions could be potentially useful for the development of new investment strategies.

US confidence changes also influence German stock market confidence adjustments significantly. Although the international response of German stock market confidence is much less than domestic expected output changes, the US dependence of the German stock market is empirically verifiable.

German stock market behavior is also characterized by high persistence and influences the effect of output and US confidence changes. High persistence reduces the marginal effects and, therefore, German confidence adjustments seem to be sticky. Compared to US stock market confidence it takes more time to increase or to decrease stock market confidence in Germany and leads to the conclusion of a less fluctuating German stock market behavior.

[12]Especially out-sample predictions could potentially benefit from the augmented specification.

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Appendix

Copula review:

Bivariate tail dependence is one way to focus on variability of temporal dependence. This concept relates to the amount of dependence in the lower-quadrant tail or the upper-quadrant tail of a bivariate distribution (see e.g. (Joe, 1997)) and is relevant for dependence in extreme values. A copula has lower tail dependence if $\lambda_L \in (0,1]$, where $\lambda_L = \lim_{u\to 0} P(U_{t-1} \leq u | U_t \leq u)$, and no lower tail dependence if $\lambda_L = 0$. Similarly, a copula has upper tail dependence if $\lambda_U \in (0,1]$, where $\lambda_U = \lim_{u\to 1} P(U_{t-1} > u | U_t > u)$, and no upper tail dependence if $\lambda_U = 0$.

I. The Gauss copula (e.g. Joe (1997))

$$C(u_{t-1}, u_t; \alpha) = \Phi_\alpha[\Phi^{-1}(u_{t-1}), \Phi^{-1}(u_t)]$$

with the standard normal distribution function $\Phi(\cdot)$, the bivariate normal distribution function $\Phi_\alpha(\cdot,\cdot)$ with means zero and variances 1 and the correlation coefficient $|\alpha| < 1$ is an elliptical copula. Its lower tail dependence parameter is $\lambda_L = 0$ und its upper tail dependence parameter is $\lambda_U = 0$. Therefore, it exhibits neither dependence in the negative tail nor in the positive tail. The copula density function $c(u_{t-1}, u_t; \cdot)$ is:

$$(1-\alpha^2)^{-1/2} \exp\left\{-\frac{1}{2}(1-\alpha^2)^{-1}[u_{t-1}^2 + u_t^2 - 2\alpha u_{t-1} u_t]\right\} \exp\left\{\frac{1}{2}[u_{t-1}^2 + u_t^2]\right\}$$

Due to the linearity of the Gauss copula according to Chen and Fan (2006) $\Phi^{-1}(u_t) = \alpha\Phi^{-1}(u_{t-1}) + \varepsilon_t$ with $\varepsilon_t \sim N(0; \sqrt{1-\alpha^2})$ follows. Consequently, $u_t = \Phi(\alpha\Phi^{-1}(u_{t-1}) + \varepsilon_t)$ and $v_t = \Phi(\varepsilon_t/\sqrt{1-\alpha^2})$ follows.

II. The Clayton copula (Clayton (1978))

$$C(u_{t-1}, u_t; \alpha) = \left(u_{t-1}^{-\alpha} + u_t^{-\alpha} - 1\right)^{-\frac{1}{\alpha}} \quad ,$$

$\alpha > 0$, is an asymmetric Archimedean copula. Its lower tail dependence parameter is $\lambda_L = 2^{-\frac{1}{\alpha}}$ und its upper tail dependence parameter is $\lambda_U = 0$. Therefore, it exhibits greater dependence in the negative tail than in the positive tail. The copula density function is:

$$c(u_{t-1}, u_t; \alpha) = (1+\alpha)\,(u_{t-1}u_t)^{-\alpha-1}\left({u_{t-1}}^{-\alpha} + {u_t}^{-\alpha} - 1\right)^{-2-1/\alpha}$$

The inverse of the conditional distribution is:

$$C^{-1}_{t|t-1}(v_t|u_{t-1}; \alpha) = u_t = [(v_t^{-\alpha/(1+\alpha)} - 1)u_{t-1}^{-\alpha} + 1]^{-1/\alpha}$$

III. The Frank copula (Frank (1979))

$$C(u_{t-1}, u_t; \alpha) = -\frac{1}{\alpha}\log\left(1 + \frac{(e^{-\alpha u_{t-1}} - 1)\,(e^{-\alpha u_t} - 1)}{(e^{-\alpha} - 1)}\right) \quad ,$$

$\alpha = (-\infty, +\infty)\backslash\{0\}$, is a symmetric Archimedean copula. Its lower tail dependence parameter is $\lambda_L = 0$ and its upper tail dependence parameter is $\lambda_U = 0$. Therefore, it exhibits neither dependence in the negative tail nor in the positive tail and shows more tail dispersion than the Gauss copula. The copula density function is:

$$c(u_{t-1}, u_t; \alpha) = \alpha\eta e^{-\alpha(u_{t-1}+u_t)}/[\eta - (1 - e^{-\alpha u_{t-1}})(1 - e^{-\alpha u_t})]^2,\ \eta = 1 - e^{-\alpha}$$

The inverse of the conditional distribution is:

$$C^{-1}_{t|t-1}(v_t|u_{t-1}; \alpha) = u_t = -\alpha^{-1}\log\{1 - (1 - e^{-\alpha})/[(v_t^{-1} - 1)e^{-\alpha u_{t-1}} + 1]\}$$

In order to allow for a more flexible copula specification the following two parameter copula will be applied.

IV. The Fang copula (Fang et al. (2000))

$$C(u_{t-1}, u_t; \alpha, \beta) = \frac{u_{t-1}u_t}{\left[1 - \beta\left(1 - {u_{t-1}}^{\frac{1}{\alpha}}\right)\left(1 - {u_t}^{\frac{1}{\alpha}}\right)\right]^{\alpha}} \tag{25}$$

considers the parameters $\alpha > 0$ and $0 \leq \beta \leq 1$. When $\beta = 0$, U_{t-1} and U_t are independent. When $\beta = 1$, $C(u_{t-1}, u_t; \alpha, 1)$ in (25) becomes the bivariate Clayton copula. As $\alpha = 1$, $C(u_{t-1}, u_t; 1, \beta)$ is the Ali-Mikhail-Haq copula (Ali et al. (1978)) and the generalized Eyraud-Farlie-Gumbel-Morgenstern copula (Cambanis (1977)). By means of some stochastic transforms, some bivariate distributions can be induced by the Fang copula, such as the generalization of Gumbel's bivariate logistic distribution given by Satterthwaite and Hutchinson (1978). Moreover, it can be shown that if $\beta < 1$, $\lim_{\alpha\to 0} C(u_{t-1}, u_t; \alpha, \beta) = \lim_{\alpha\to\infty} C(u_{t-1}, u_t; \alpha, \beta) = u_{t-1}u_t$. Therefore, U_{t-1} and U_t are independent as $\alpha \to 0$ and $\alpha \to \infty$. To asses the correlation between two random variables, copulas can be used to define Spearman's ρ_s (see Joe (1997)) in general. Analog to the general case the Spearman's correlation coefficient of the Fang copula between U_{t-1} and U_t is representable by a hypergeometric function. A hypergeometric function of x is defined as

$$ {}_pF_q(a_1, \cdots, a_p; b_1, \cdots, b_q; x) = \sum_{k=0}^{\infty} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k} \frac{x^k}{k!} \quad , $$

where $(a)_k = \Gamma(a+k)/\Gamma(a)$ and $a_1, \ldots a_p, b_1, \ldots, b_q$ are parameters. $\Gamma(z)$ stands for the gamma function $\int_0^\infty e^{-t}t^{z-1}dt$. Then, the Spearman's correlation coefficient $\rho_s(\alpha, \beta)$ of the Fang copula in (25) between U_{t-1} and U_t is given by

$$ \rho_s(\alpha, \beta) = 3({}_3F_2(1, 1, \alpha; 1 + 2\alpha, 1 + 2\alpha; \beta) - 1) \; . \tag{26} $$

The copula density function is:

$$ c(u_{t-1}, u_t; \alpha, \beta) = \frac{(\beta^2 + \beta/\alpha)(u_{t-1}u_t)^{1/\alpha} + (\beta - \beta^2)(u_{t-1}^{1/\alpha} + u_t^{1/\alpha}) + (1 - \beta)^2}{[1 - \beta(1 - {u_{t-1}}^{1/\alpha})(1 - {u_t}^{1/\alpha})]^{\alpha+2}} $$

$C_{t|t-1}^{-1}$ does not exist in closed form. $u_t = C_{t|t-1}^{-1}(v_t|u_{t-1}; \alpha, \beta)$ can be obtained from the equation $v_t = C_{t|t-1}(u_t|u_{t-1}; \alpha, \beta)$ using a numerical root-finding routine (here: Newton's procedure).

Copula-specific forecasting errors:

Consider the nonparametric estimated conditional quantiles $\widehat{u}_t$, which contain no information about a parametric copula. On the other hand if a parametric copula is selected, it is possible to calculate copula implied conditional quantiles which are used to construct a copula-based confidence interval of the conditional quantiles. Regarding the level of significance ϵ it follows for the upper interval bound

$$ \widehat{u}_{t,\overline{\epsilon}} = C_{t|t-1}^{-1}(1 - \epsilon/2|\widehat{u}_{t-1}; \widehat{\alpha}) \tag{27} $$

and for the lower interval bound

$$ \widehat{u}_{t,\underline{\epsilon}} = C_{t|t-1}^{-1}(\epsilon/2|\widehat{u}_{t-1}; \widehat{\alpha}) \; . \tag{28} $$

The ,,overall region" of Table 3 reports the estimated error rates for all conditional quantiles $\widehat{u}_t$, $t = 2, \ldots, n$. Therefore, given $\widehat{u}_t$, $\widehat{u}_{t,\overline{\epsilon}}$ and $\widehat{u}_{t,\underline{\epsilon}}$ copula-based error rates are:

$$ \widehat{\epsilon}_{overall} = 1 - \left(\frac{1}{n-1} \sum_{t=2}^{n} 1\{\widehat{u}_{t,\underline{\epsilon}} \le \widehat{u}_t \le \widehat{u}_{t,\overline{\epsilon}}\} \right) \tag{29} $$

Focusing the tails of the bivariate copula leads to further information about the copula adequacy. The calculation of the estimated error rates of the „lower region" of Table 3 is analog to (29), but only valid for lower $\widehat{u}_t$. We define the region for lower quantiles by $\widehat{u}_t < \pi$ with $\pi = 1/3$.[13] According to

$$\widehat{\epsilon}_{lower} = 1 - \left(\frac{1}{\underline{n}} \sum_{t=2}^{\underline{n}} 1\{\widehat{u}_{t,\underline{\epsilon}} \leq \widehat{u}_t \leq \widehat{u}_{t,\overline{\epsilon}} \text{ and } \widehat{u}_t < \pi\} \right) \tag{30}$$

the estimated error rate for the lower region are computed. Consequently, for the „upper region"

$$\widehat{\epsilon}_{upper} = 1 - \left(\frac{1}{\overline{n}} \sum_{t=2}^{\overline{n}} 1\{\widehat{u}_{t,\underline{\epsilon}} \leq \widehat{u}_t \leq \widehat{u}_{t,\overline{\epsilon}} \text{ and } \widehat{u}_t > 1 - \pi\} \right) \tag{31}$$

holds. $\underline{n}$ stands for the cases with $\widehat{u}_t < \pi$ and $\overline{n}$ for the cases with $\widehat{u}_t > 1 - \pi$. Table 3 shows additionally the root mean squared error of the true and estimated error rates separated according to different regions.

[13] Also for varying π similar error rates are observed.

Table 3: Estimated conditional quantile error rates of the $VDAX$

Copula	Lower region ϵ		Upper region ϵ		Overall region ϵ	
	0.10	0.05	0.10	0.05	0.10	0.05
Gauss	0.12	0.04	0.14	0.12	0.10	0.03
	(0.0907)		(0.1322)		(0.0796)	
	{0.1134}					
Clayton	0.23	0.19	0.07	0.03	0.07	0.07
	(0.2104)		(0.0501)		(0.0873)	
	{0.1530}					
Frank	0.03	0.00	0.11	0.07	0.10	0.03
	(0.0191)		(0.0878)		(0.0562)	
	{0.0635}					
Fang	0.08	0.03	0.08	0.05	0.07	0.04
	(0.0604)		(0.0671)		(0.0583)	
	{0.0639}					

Sample: 1992:1-2010:12 • The estimated conditional quantiles $\widehat{u}_t$ are computed by the empirical distribution. By assuming a certain parametric copula a level of significance ϵ determines a $(1-\epsilon)$ confidence interval of the nonparametric estimated conditional quantiles $\widehat{u}_t$. With respect to the inverse conditional distributions for the upper interval bound $v_t = 1 - \epsilon/2$ and for the lower bound $v_t = \epsilon/2$ holds. The unknown copula parameters are substituted by appropriate ML-estimates according to Table 4. • The copula specific numbers are the relative frequencies for the nonparametric estimated conditional quantiles outside the parametric confidence interval. The lower quantile region is defined by quantiles in a range of $(0; 1/3)$. For the upper quantile region $(2/3; 1)$ holds. • Root mean squared errors of the regions in parenthesis and root mean squared errors of the extreme regions in curly brackets (root of the arithmetic mean of the squared errors).

Table 4: VIX results

Copula	Estimate	Estimated autocorrelation						G-o-f
		1	2	3	4	5	6	
Gauss	0.849	-0.16*	-0.04	-0.05	0.03	0.04	-0.01	
	(0.035)	(0.02)	(0.60)	(0.50)	(0.68)	(0.59)	(0.92)	(0.39)
Clayton	1.920	0.07	0.24*	0.10	0.12	0.14*	0.09	
	(0.120)	(0.30)	(0.00)	(0.15)	(0.07)	(0.04)	(0.19)	(0.08)
Frank	10.995	-0.13*	-0.01	-0.03	0.02	0.03	-0.07	
	(0.835)	(0.05)	(0.91)	(0.70)	(0.76)	(0.67)	(0.33)	(0.95)
Fang, α	0.157	-0.08	0.00	0.00	0.03	0.00	-0.08	
	(0.014)	(0.22)	(0.99)	(0.99)	(0.64)	(0.97)	(0.22)	(0.91)
Fang, β	0.9995							
	(0.0004)							

Sample: 1992:1-2010:12 • Included observations $T = 227$ • Initial value of the one parameter copulas is 1 and of the Fang copula are $\widehat{\alpha}_1 = 0.3$ and $\widehat{\beta}_1 = 1$ • ML-estimates are different from zero at any level of significance (standard errors in brackets) • Spearman's correlation coefficients and p-values of the hypothesis $\rho_s(V_t, V_{t-l}) = 0$, $l = 1, \ldots, 6$, in brackets • * indicates a significant autocorrelation on the 10% overall error rate using Bonferroni's adjustment (see e.g. (Sokal & Rohlf, 1995)) • 2 is the number of tests performed (correlation test up to a specific lag and goodness-of-fit (G-o-f) test) • Finite sample adjustment of the Kolmogorov statistic and corresponding p-values of the hypothesis $V_t \sim U[0,1]$ in brackets (see e.g. (D'Agostino & Stephens, 1986))