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Abstract. Interaction effects capture the impact of one explanatory variable x_1 on the marginal effect of another explanatory variable x_2. To explore interaction effects, so-called interaction terms $x_1 x_2$ are typically included in estimation specifications. While in linear models the effect of a marginal change in the interaction term is equal to the interaction effect, this equality generally does not hold in non-linear specifications (AI, NORTON, 2003). This paper provides for a general derivation of marginal and interaction effects in both linear and non-linear models and calculates the formulae of the marginal and interaction effects resulting from the Two-Part Model, a commonly employed censored regression model. Drawing on a survey of automobile use from Germany, we illustrate several subtleties inherent to the substantive interpretation of interaction effects gleaned from non-linear models.

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Key words: Censored regression models, interaction terms, marginal and interaction effects.

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1 Introduction

To explore whether the effect of an explanatory variable x_1 on the expected value $E[y]$ of the dependent variable y depends on the size of another explanatory variable x_2, it is indispensable to estimate the interaction effect, which is formally given by the second derivative $\frac{\partial^2 E[y]}{\partial x_2 \partial x_1}$. To this end, linear estimation specifications typically include so-called interaction terms, consisting of the product $z := x_1 x_2$ of two explanatory variables. In linear contexts, the marginal effect $\frac{\partial E[y]}{\partial (x_1 x_2)}$ of the interaction term $x_1 x_2$ equals the interaction effect $\frac{\partial^2 E[y]}{\partial x_2 \partial x_1}$. This equality, however, generally does not extend to non-linear specifications, as is demonstrated by AI and NORTON (2003) for the example of probit and logit models.

The present paper builds on the work of these authors in two respects. First, we calculate the formulae of both the marginal and interaction effects resulting from the Two-Part model, a commonly employed approach to accommodate censored data. Second, using an empirical example that applies the Two-Part model to travel survey data collected from a sample of motorists in Germany, we illustrate several subtleties inherent to the substantive interpretation of interaction effects gleaned from non-linear models. To this end, we draw a clear distinction between interaction effects, $\frac{\partial^2 E[y]}{\partial x_2 \partial x_1}$, and interaction terms, $x_1 x_2$. This aspect has received short shrift in the analysis of model results.

The following section provides for a general derivation of interaction effects for both linear and non-linear models. Section 3 presents a concise description of the Two-Part model. Section 4 derives the specific formulae of the marginal and interaction effects for this type of model, followed by the presentation of an example in Section 5. The last section summarizes and concludes.

2 Interaction Effects

To provide for a general derivation of interaction effects in both linear and non-linear models, we closely follow NORTON, WANG, and AI (2004).

2.1 Linear Models

We begin by drawing on the following linear specification of the expected value of dependent variable y:

$$E := E[y|x_1, x_2, \mathbf{w}] = \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \mathbf{w}^T \boldsymbol{\beta}, \tag{1}$$

where the parameters $\beta_1, \beta_2, \beta_{12}$, as well as the vector $\boldsymbol{\beta}$ are unknown and vector $\mathbf{w}$ excludes x_1 and x_2, while T denotes transposition of vector $\mathbf{w}$.

Assuming that x_1 and x_2 are continuous variables, the marginal effect of x_1 on the expected value E is dependent on x_2 if $\beta_{12} \neq 0$:

$$\frac{\partial E}{\partial x_1} = \beta_1 + \beta_{12} x_2. \tag{2}$$

The impact of a marginal change in x_2 on the marginal effect of x_1, in other words the interaction effect, is then obtained from taking the derivative of (2) with respect to x_2:

$$\frac{\partial^2 E}{\partial x_2 \partial x_1} = \beta_{12}. \tag{3}$$

In linear specifications, therefore, the interaction effect $\frac{\partial^2 E}{\partial x_2 \partial x_1}$ equals the marginal effect $\frac{\partial E}{\partial (x_1 x_2)}$ of the interaction term $x_1 x_2$. For non-linear models, however, this equality generally does not hold, as is demonstrated in the subsequent section.

2.2 Non-Linear Models

Instead of expectation (1), we now depart from

$$E := E[y|x_1, x_2, \mathbf{w}] = F(\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \mathbf{w}^T \boldsymbol{\beta}) = F(u), \tag{4}$$

where $F(u)$ is a non-linear function of its argument $u := \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \mathbf{w}^T \boldsymbol{\beta}$. In the Probit model, for example, $F(u)$ equals the cumulative normal distribution $\Phi(u)$. We now derive general formulae for the interaction effects resulting from non-linear models if (1) x_1 and x_2 are both continuous variables, (2) both are dummy variables, and (3) x_1 is continuous, while x_2 is a dummy variable.

(1) If $F(u)$ is a twice differentiable function, with the first and second derivatives being denoted by $F'(u)$ and $F''(u)$, respectively, the marginal effect with respect to x_1 reads:

$$\frac{\partial E}{\partial x_1} = \frac{\partial F(u)}{\partial x_1} = F'(u)\frac{\partial u}{\partial x_1} = F'(u)(\beta_1 + \beta_{12} x_2), \tag{5}$$

while the interaction effect of two continuous variables x_1 and x_2 is symmetric and given by

$$\frac{\partial^2 E}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_2}(\frac{\partial E}{\partial x_1}) = \frac{\partial}{\partial x_2}(\frac{\partial F(u)}{\partial x_1}) = F'(u)\beta_{12} + (\beta_1 + \beta_{12} x_2)(\beta_2 + \beta_{12} x_1)F''(u). \tag{6}$$

As, in general, $(\beta_1 + \beta_{12} x_2)(\beta_2 + \beta_{12} x_1)F''(u) \neq 0$, the interaction effect $\frac{\partial^2 E}{\partial x_2 \partial x_1}$ generally differs from the marginal effect $\frac{\partial E}{\partial (x_1 x_2)}$ of the interaction term $z = x_1 x_2$:

$$\frac{\partial E}{\partial (x_1 x_2)} = \frac{\partial E}{\partial z} = F'(u)\frac{\partial u}{\partial z} = F'(u)\beta_{12}. \tag{7}$$

(2) If x_1 and x_2 are dummy variables, the discrete interaction effect, which in analogy to $\frac{\partial^2 E}{\partial x_2 \partial x_1}$ shall be designated by $\frac{\Delta^2 E}{\Delta x_2 \Delta x_1}$, is given by the discrete change in E due to a unitary change in both x_1 and x_2, $\Delta x_1 = 1, \Delta x_2 = 1$:

$$\begin{aligned}\frac{\Delta^2 E}{\Delta x_2 \Delta x_1} &:= \frac{\Delta}{\Delta x_2}(\frac{\Delta E}{\Delta x_1}) = \frac{\Delta}{\Delta x_2}(E[y|x_1 = 1, x_2, \mathbf{w}] - E[y|x_1 = 0, x_2, \mathbf{w}]) \\ &= \{E[y|x_1 = 1, x_2 = 1, \mathbf{w}] - E[y|x_1 = 0, x_2 = 1, \mathbf{w}]\} \\ &\quad -\{E[y|x_1 = 1, x_2 = 0, \mathbf{w}] - E[y|x_1 = 0, x_2 = 0, \mathbf{w}]\}.\end{aligned} \tag{8}$$

Note that the discrete interaction effects are symmetric: $\frac{\Delta^2 E}{\Delta x_2 \Delta x_1} = \frac{\Delta^2 E}{\Delta x_1 \Delta x_2}$, as can be seen from (8) by rearranging the terms in the middle of the double difference. Using the non-linear expectation (4), the general expression (8) translates into:

$$\frac{\Delta^2 E}{\Delta x_2 \Delta x_1} = \{F(\beta_1 + \beta_2 + \beta_{12} + \mathbf{w}^T \boldsymbol{\beta})) - F(\beta_2 + \mathbf{w}^T \boldsymbol{\beta})\} - \{F(\beta_1 + \mathbf{w}^T \boldsymbol{\beta}) - F(\mathbf{w}^T \boldsymbol{\beta})\}. \tag{9}$$

(3) If x_1 is a continuous variable and x_2 is a dummy variable, the mixed interaction effect $\frac{\Delta}{\Delta x_2}(\frac{\partial E}{\partial x_1})$ can be computed as follows:

$$\begin{aligned}\frac{\Delta}{\Delta x_2}(\frac{\partial E}{\partial x_1}) &:= \frac{\Delta}{\Delta x_2}(\frac{\partial F(u)}{\partial x_1}) = \frac{\Delta}{\Delta x_2}(F'(u)(\beta_1 + \beta_{12}x_2)) \\ &= F'(\beta_1 x_1 + \beta_2 + \beta_{12}x_1 + \mathbf{w}^T\boldsymbol{\beta})(\beta_1 + \beta_{12}) - F'(\beta_1 x_1 + \mathbf{w}^T\boldsymbol{\beta})\beta_1.\end{aligned} \quad (10)$$

Note that the symmetry observed for the cases when both variables are either continuous or dummies also holds true for the mixed interaction effects: $\frac{\partial E}{\partial x_1}(\frac{\Delta}{\Delta x_2}) = \frac{\Delta}{\Delta x_2}(\frac{\partial E}{\partial x_1})$.

All in all, it bears noting that for a linear function $F(u) = u$, for which $F'(u) = 1$, all three kinds of interaction effects collapse to β_{12}. Furthermore, we shall re-emphasize the point raised by AI and NORTON (2003:124) that, in contrast to linear specifications, the interaction effect gleaned from non-linear models is generally non-vanishing even if $\beta_{12} = 0$, that is, even if no interaction term is included.

Finally, for the special case of the Probit model, the interaction effects are given by (6), (9), and (10) if $F(u)$ is replaced by the cumulative normal distribution $\Phi(u)$, $F'(u)$ is replaced by the density function of the standard normal distribution, $\phi(u) := \exp\{-u^2/2\}/\sqrt{2\pi}$, and $F''(u)$ is replaced by $\phi'(u) = -u\phi(u)$. Similarly, formulae (6), (9), and (10) can be applied to the Logit model if $F(u)$ is replaced by $\Lambda(u) := 1/(1 + \exp\{-u\})$, $F'(u)$ is replaced by $\Lambda'(u) = \Lambda(u)(1 - \Lambda(u))$, and $F''(u)$ is substituted by $\Lambda''(u) = (\Lambda(u)(1 - \Lambda(u)))' = \Lambda(u)(1 - \Lambda(u))(1 - 2\Lambda(u))$.

3 Two-Part Model (2PM)

To accommodate the feature of zero values in observed data, two-stage estimation procedures, such as the Two-Part model (2PM), are frequently employed. The first stage defines a dichotomous variable R indicating the regime into which observations of the dependent variable y falls:

$$R = 1, \text{if} \quad y^* = \mathbf{x_1}^T\boldsymbol{\tau} + \epsilon_1 > 0 \quad \text{and} \quad R = 0, \text{if} \quad y^* \leq 0. \quad (11)$$

y^* is a latent variable, vector $\mathbf{x_1}$ includes its determinants, $\boldsymbol{\tau}$ is a vector of associated parameters, and ϵ_1 is an error term assumed to have a standard normal distribution. $R = 1$ indicates that $y > 0$, whereas $R = 0$ is equivalent to $y = 0$.

After estimating $\boldsymbol{\tau}$ using Probit estimation methods, the second stage involves an OLS regression of the parameters $\boldsymbol{\beta}$ that affect the expected value $E[y|y > 0]$ conditional on $y > 0$, i. e. , $R = 1$:

$$E[y|R = 1, \mathbf{x_2}] = E[y|y > 0, \mathbf{x_2}] = \mathbf{x_2}^T\boldsymbol{\beta} + E(\epsilon_2|y > 0, \mathbf{x_2}), \tag{12}$$

where $\mathbf{x_2}$ includes the determinants of the dependent variable y, and ϵ_2 is another error term.

The expected value of the dependent variable y then consists of two parts, with the first part resulting from the first stage (11), $P(y > 0) = \Phi(\mathbf{x_1}^T\boldsymbol{\tau})$, and the second part being the conditional expectation $E[y|y > 0]$ from the second stage (12):

$$E[y] = P(y > 0) \cdot E[y|y > 0] + P(y = 0) \cdot \underbrace{E[y|y = 0]}_{=0} = P(y > 0) \cdot E[y|y > 0].$$

In contrast to HECKMAN's (1979) sample selection model, the 2PM assumes that $E(\epsilon_2|y > 0, \mathbf{x_2}) = \mathbf{0}$ and, hence, $E[y|y > 0] = \mathbf{x_2}^T\boldsymbol{\beta}$, so that the unconditional expectation $E[y]$ is given by

$$E[y] = \Phi(\mathbf{x_1}^T\boldsymbol{\tau}) \cdot \mathbf{x_2}^T\boldsymbol{\beta}. \tag{13}$$

The relative merits of the 2PM have been the subject of a vigorous debate in the literature (DOW and NORTON, 2003), with much of the discussion focusing on the simplifying assumption $E(\epsilon_2|y > 0, \mathbf{x_2}) = \mathbf{0}$.

4 Marginal and Interaction Effects in Two-Part Models

Using a slightly more detailed version of prediction (13),

$$E := E[y|x_1, x_2, \mathbf{w_1}, \mathbf{w_2}] = \Phi(u_1)u_2,$$

where $u_1 := \tau_1 x_1 + \tau_2 x_2 + \tau_{12} x_1 x_2 + \mathbf{w_1}^T \boldsymbol{\tau}$, $u_2 := \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \mathbf{w_2}^T \boldsymbol{\beta}$, and $\mathbf{w_1}$ and $\mathbf{w_2}$ neither include x_1 nor x_2, we now derive formulae for the interaction effects if (1) x_1 and x_2 are both continuous variables, (2) x_1 is continuous, while x_2 is a dummy variable, and (3) both are dummy variables.

(1) To calculate the interaction effect $\frac{\partial^2 E}{\partial x_2 \partial x_1}$, we first need to calculate the marginal effect:

$$\frac{\partial E}{\partial x_1} = (\tau_1 + \tau_{12} x_2) \cdot \phi(u_1) \cdot u_2 + \Phi(u_1) \cdot (\beta_1 + \beta_{12} x_2). \tag{14}$$

Apparently, marginal effects resulting from non-linear models generally depend on all other variables. As elaborated in the empirical example below, accurate interpretation necessitates that in calculating the marginal effect $\frac{\partial E}{\partial x_1}$ the derivatives $\tau_{12} x_2$ and $\beta_{12} x_2$ of the interaction terms must be taken into account.

By now taking the derivative with respect to x_2 and employing $\phi'(u_1) = -u_1 \phi(u_1)$, we get the interaction effect:

$$\begin{aligned} \frac{\partial^2 E}{\partial x_2 \partial x_1} \;=\; & \tau_{12} \cdot \phi(u_1) \cdot u_2 - (\tau_1 + \tau_{12} x_2) \cdot (\tau_2 + \tau_{12} x_1) \cdot \phi(u_1) \cdot u_1 \cdot u_2 \\ & + (\tau_1 + \tau_{12} x_2) \cdot \phi(u_1) \cdot (\beta_2 + \beta_{12} x_1) \\ & + (\tau_2 + \tau_{12} x_1) \cdot \phi(u_1) \cdot (\beta_1 + \beta_{12} x_2) + \Phi(u_1) \cdot \beta_{12}. \end{aligned} \tag{15}$$

Note that, in general, it would be incorrect to calculate the interaction effect by taking the marginal effect of the interaction term $z = x_1 x_2$:

$$\frac{\partial E}{\partial z} = \tau_{12} \cdot \phi(u_1) \cdot u_2 + \Phi(u_1) \cdot \beta_{12}. \tag{16}$$

(2) The mixed interaction effect $\frac{\Delta}{\Delta x_2}(\frac{\partial E}{\partial x_1})$ follows immediately from the marginal effect (14):

$$\begin{aligned} \frac{\Delta}{\Delta x_2}\left(\frac{\partial E}{\partial x_1}\right) \;=\; & \frac{\partial E}{\partial x_1}\Big|_{x_2=1} - \frac{\partial E}{\partial x_1}\Big|_{x_2=0} \\ =\; & (\tau_1 + \tau_{12}) \cdot \phi(\tau_1 x_1 + \tau_2 + \tau_{12} x_1 + \mathbf{w_1}^T \boldsymbol{\tau}) \cdot \{\beta_1 x_1 + \beta_2 + \beta_{12} x_1 + \mathbf{w_2}^T \boldsymbol{\beta}\} \\ & + \Phi(\tau_1 x_1 + \tau_2 + \tau_{12} x_1 + \mathbf{w_1}^T \boldsymbol{\tau}) \cdot (\beta_1 + \beta_{12}) \\ & - \tau_1 \cdot \phi(\tau_1 x_1 + \mathbf{w_1}^T \boldsymbol{\tau}) \cdot \{\beta_1 x_1 + \mathbf{w_2}^T \boldsymbol{\beta}\} - \Phi(\tau_1 x_1 + \mathbf{w_1}^T \boldsymbol{\tau}) \cdot \beta_1. \end{aligned} \tag{17}$$

(3) Applying formula (8) to $E[y|x_1, x_2, \mathbf{w_1}, \mathbf{w_2}]$, the discrete interaction effect $\frac{\Delta^2 E}{\Delta x_2 \Delta x_1}$ is obtained as follows:

$$
\begin{aligned}
\frac{\Delta^2 E}{\Delta x_2 \Delta x_1} &= \{[E[y|x_1 = 1, x_2 = 1, \mathbf{w_1}, \mathbf{w_2}] - E[y|x_1 = 0, x_2 = 1, \mathbf{w_1}, \mathbf{w_2}]\} \\
&\quad -\{[E[y|x_1 = 1, x_2 = 0, \mathbf{w_1}, \mathbf{w_2}] - E[y|x_1 = 0, x_2 = 0, \mathbf{w_1}, \mathbf{w_2}]\} \\
&= \Phi(\tau_1 + \tau_2 + \tau_{12} + \mathbf{w_1}^T \tau) \cdot \{\beta_1 + \beta_2 + \beta_{12} + \mathbf{w_2}^T \beta\} \qquad (18) \\
&\quad -\Phi(\tau_2 + \mathbf{w_1}^T \tau) \cdot \{\beta_2 + \mathbf{w_2}^T \beta\} - \Phi(\tau_1 + \mathbf{w_1}^T \tau) \cdot \{\beta_1 + \mathbf{w_2}^T \beta\} \\
&\quad +\Phi(\mathbf{w_1}^T \tau) \cdot \{\mathbf{w_2}^T \beta\}.
\end{aligned}
$$

5 Empirical Example

To illustrate illustrate several subtleties inherent to the substantive interpretation of interaction effects gleaned from a 2PM, we employ household data drawn from the German Mobility Panel (MOP 2011) using the following specification:

$$E[s] = \Phi(\mathbf{x}^T \boldsymbol{\tau}) \cdot \{\mathbf{x}^T \boldsymbol{\beta}\}, \qquad (19)$$

where the dependent variable s is the daily distance driven for non-work travel and the set of explanatory variables **x** includes the individual and household attributes that are hypothesized to influence the extent of this travel. A detailed data description can be found in FRONDEL, PETERS, VANCE (2008) or FRONDEL, VANCE (2009, 2010).

The key attributes of interest in the following example are the individual's *age*, the number (#) of *children*, and the dummy variable *enoughcars* indicating whether the individual lives in a household in which the number of cars is at least equal to the number of licensed drivers. Each of these variables is interacted with a *female* dummy variable, which is intended to capture the role played by household responsibilities, social status, and competition among household members in dictating access to the car. In addition, we interact the variable measuring the number of children with the age of the individual. The specification thus yields all combinations of interactions: between (1) two continuous variables, (2) two dummies, and (3) a dummy and continuous variable.

Table 1 reports the results of two model specifications, one in which several interaction terms are included and another in which these are omitted entirely. To focus on the salient results, we refrain here from reporting the estimation results of the first-stage Probit models, and instead present both the coefficient estimates of the (second-stage) OLS regression, as well as the marginal and interaction *effects* of the explanatory variables on distance driven resulting from the 2PM. Given that the marginal and interaction effects are comprised of multiple parameters that makes analytical computation of the variance impossible, the standard errors are calculated by applying the Delta method, which uses a first-order Taylor expansion to create a linear approximation of a non-linear function.

Table 1: Estimation Results of the Two-Part model (2PM) on Distance driven.

	Interaction Terms Included:				No Interaction Terms:			
	$\tau_{12} \neq 0, \beta_{12} \neq 0$				$\tau_{12} = \beta_{12} = 0$			
	OLS		2PM		OLS		2PM	
	Coeff.s	Errors	*Effects*	Errors	Coeff.s	Errors	*Effects*	Errors
female	-1.624	(1.292)	-0.285	(0.305)	** -1.491	(0.299)	** -0.581	(0.164)
employed	** -1.181	(0.427)	** -1.087	(0.237)	** -1.212	(0.413)	** -1.302	(0.229)
commute distance	** 0.030	(0.007)	0.006	(0.003)	** 0.029	(0.007)	0.003	(0.003)
age	-0.028	(0.019)	0.008	(0.013)	* -0.033	(0.013)	0.005	(0.007)
female × *age*	-0.020	(0.023)	** -0.058	(0.012)	–	–	** -0.003	(0.001)
age × *# children*	0.012	(0.020)	0.009	(0.011)	–	–	-0.002	(0.001)
high-school diploma	** 0.947	(0.320)	* 0.346	(0.173)	** 1.024	(0.316)	** 0.502	(0.173)
# children	-0.996	(0.873)	** 0.711	(0.211)	-0.277	(0.157)	** 0.722	(0.094)
female × *# children*	0.426	(0.331)	** 1.113	(0.222)	–	–	** -0.110	(0.024)
# employed	-0.033	(0.240)	-0.080	(0.129)	-0.022	(0.240)	0.003	(0.128)
enoughcars	0.741	(0.422)	** 1.854	(0.223)	** 1.368	(0.292)	** 1.966	(0.165)
female × *enoughcars*	* 1.308	(0.552)	** 1.855	(0.298)	–	–	** -0.180	(0.046)
city region	* -0.714	(0.288)	-0.090	(0.162)	* -0.712	(0.289)	-0.079	(0.161)
# observations used for estimation: 17,798								

Note: * denotes significance at the 5 %-level and ** at the 1 %-level, respectively. In the 2PM, interaction terms, such as *female* × *enoughcars*, stand for the interaction effect, here $\frac{\Delta^2 E}{\Delta x_2 \Delta x_1}$.

Turning first to the model that includes the interaction terms, the OLS estimates and associated marginal effects of the 2PM are seen to differ markedly, both with respect to their magnitude and statistical significance. For some of the variables, such as *commute distance* and *city region*, significant coefficient estimates correspond to insignificant estimates of the marginal effects, while for others, such as # *children* and *enoughcars*, the converse is true. Furthermore, it appears to be particularly important to distinguish between interaction terms and interaction effects[1]: For example, while the OLS estimates of the coefficients of the interaction terms *female* $\times$ *age* and *female* $\times$ # *children* do not statistically differ from zero, the associated interaction effects are significantly negative and positive, respectively.

Although no interaction terms are included in the specification presented on the right-hand panel, the corresponding interaction effects, which are calculated using the formulae (15), (17), and (18) and setting $\tau_{12} = \beta_{12} = 0$, are still significantly different from zero in three out of four cases. This serves to highlight the fact that in non-linear models such as 2PM, the marginal effect of a variable x_1 depends on variable x_2, even when no interaction term $x_1 x_2$ is included in the model.

Finally, it bears noting that the marginal effects of variables that are interacted with others are distinct to those when no interaction terms are employed in a specification. For example, the marginal effect of the gender dummy *female* is statistically significant and negative in the specification without interaction terms, but insignificant in the more general model specification including interaction terms.

6 Summary and Conclusion

By providing for a general derivation of marginal and interaction effects in both linear and non-linear models and the specific formulae of marginal and interaction effects gleaned from the Two-Part Model (2PM), this paper has analyzed the significance of

[1] Note that the coefficient estimate of 0.009 of the interaction effect pertaining to age and the number of children, for example, which appears on the left-hand panel of Table 1, is calculated on the basis of (10), rather than (16), and, hence, is not simply the marginal effect of the interaction term *age* $\times$ # *children*.

these effects. Drawing on a survey of automobile use from Germany, we have illustrated that in non-linear models such as the 2PM, a clear distinction between interaction effects, given by the second derivative $\frac{\partial^2 E[y]}{\partial x_2 \partial x_1}$, and interaction terms $x_1 x_2$ must be drawn. This aspect has received short shrift in the analysis of model results.

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