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Liquidity premia, interest rates and exchange rate dynamics

Markus Hörmann

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Markus Hörmann
TU Dortmund University and RGS Econ

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Abstract

Empirical failure of uncovered interest rate parity (UIP) has become a stylized fact. VARs by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008) find delayed overshooting of the exchange rate in response to a monetary shock. This result contradicts Dornbusch's (1976) original overshooting, which is based on UIP. This paper presents a model in which assets eligible for central bank's open market operations, such as government bonds, command liquidity premia. Further, I allow for a key currency which is required to participate in international goods trade. Therefore, assets allowing access to key currency liquidity are held by agents around the globe. I show that liquidity premia lead to a modified UIP condition. In response to a monetary policy shock, the model predicts delayed overshooting of the nominal exchange rate, as in Eichenbaum and Evans (1995).

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1 Introduction

Empirical studies reject uncovered interest rate parity (UIP), which states that a currency is expected to depreciate relative to another country's currency when the interest rate difference to that country is positive. One aspect of empirical failure of UIP is that exchange rates do not react to interest rate shocks as predicted by Dornbusch's (1976) overshooting but are characterized by delayed overshooting, as documented by Eichenbaum and Evans (1995). This paper takes the evidence against UIP as a starting point and develops a model in which there is a spread between interest rates paid on assets eligible for central bank's open market operations and those paid on ineligible assets, i.e. a liquidity premium. The model further allows for a key currency which is required to participate in international trade. Therefore, assets allowing access to key currency liquidity are held by agents around the globe. This paper shows that the liquidity premium implied by this setup generates deviations from UIP and offers an explanation for delayed overshooting. Moreover, it analyzes how the international transmission of shocks is affected by modeling key currency liquidity.

Empirical failure of UIP is documented by various types of evidence including forward premium regressions, vector autoregressions (VARs) and model estimations. Testing UIP by applying regression analysis is difficult because expectations cannot be measured. However, as pointed out by Chinn (2006), UIP can be tested jointly with the assumption of rational expectations. In the forward premium regression, empirical studies regress realized exchange rate changes on the interest rate difference (the forward premium) between two countries. Under rational expectations and risk neutrality, UIP predicts this regression to yield a positive coefficient of unity. Froot (1990) finds that the average estimate of this coefficient across 75 published studies is -0.88 with only a few estimates above zero and none greater than unity. The finding of a negative coefficient in the forward premium regression has become known as the forward premium puzzle.[2] It implies that the forward premium predicts exchange rate movements inconsistent with theory not only in magnitude, but also in terms of the direction of the movement.[3] When investors are risk averse, the UIP condition allows for risk premia, which are positive when an asset's domestic currency return is positively correlated to consumption growth. A large literature which follows up on the seminal contribution analyzes the capability of risk premia to reconcile UIP with the data. The seminal contribution of Fama (1984) shows that a negative coefficient in the forward premium regression implies that the risk premium would have to be negatively correlated to, and more volatile than, the expected exchange rate change.

[2] Recent improvements in data availability have spurred a re-evaluation of these results with respect to maturities and countries. Chinn (2006) and Bansal and Dahlquist (2000) confirm the forward premium puzzle for short maturities in developed economies, but find evidence supportive of UIP with respect to long horizons and for emerging economies.

[3] Surveys of this literature include Froot and Thaler (1984), Engel (1996) and Taylor (1995).

There is consensus that the volatility of the risk premium implied by Fama's conditions is too high for any reasonable risk premium (see Froot and Thaler (1984) and Backus, Foresi, and Telmer (2001)), so that empirical UIP failure has become a stylized fact.[4]

A second type of evidence documents the empirical failure of uncovered interest rate parity: Eichenbaum and Evans (1995) estimate a VAR to analyze the impact of monetary policy shocks on exchange rates. Their conclusion is known as the delayed overshooting puzzle: In contrast to Dornbusch's (1976) overshooting, which is based on UIP, they find that a contractionary U.S. monetary policy shock leads the dollar to appreciate continuously until it peaks after around three years. Some studies question the identification assumptions made by Eichenbaum and Evans (1995) and find evidence in line with Dornbusch's overshooting (see Kim and Roubini (2000) and Faust and Rogers (2003)). However, Scholl and Uhlig (2008) reconfirm the delayed overshooting result and find that the exchange rate peaks between 17 and 26 months after a monetary shock.

A third type of evidence stems from estimations of small open economy models, which commonly include a UIP condition. Justiniano and Preston (2010) find that their model cannot account for the observed co-movement of Canadian and U.S. business cycles. Further, volatility in the real exchange rate is virtually entirely caused by shocks to an ad-hoc risk premium, so that the authors find an extreme version of exchange rate disconnect.[5] Justiniano and Preston (2010) suggest that the failure of the model to associate movements of exchange rates with fundamentals is related to its poor performance. Thus, improving the exchange rate predictions of economic models is a promising avenue to enhance the quantitative performance of open economy models.

This paper does not deal with risk premia but combines two features, *liquidity* and *key currency pricing*: First, as is conveyed in anecdotal evidence - for instance about recurring flight to quality and flight to liquidity episodes - and in empirical studies, interest rates on assets vary not only according to their risk but also as a function of their liquidity. For instance, Longstaff (2004) shows that U.S. Treasury bonds pay lower interest rates than Refcorp bonds, which are backed by the Treasury, and finds that the premium is related to indicators of liquidity preferences.[6] In a closed economy, Reynard and Schabert (2009) show that taking into account liquidity premia by modeling open market operations can

[4]Recently, some authors challenge this view: Lustig and Verdelhan (2007) find that high-interest rate currencies depreciate on average when consumption growth is low, so that a consumption based risk premium can explain excess returns if one is willing to assume large coefficients of risk aversion. Alvarez, Atkeson, and Kehoe (2009) build a model where asset markets are segmented, so that the investor's marginal utility varies more than indicated by fluctuations in aggregate consumption. This can increase the fluctuations of the risk premium.

[5]Lubik and Schorfheide (2006) obtain a qualitatively identical result.

[6]Further evidence documenting liquidity premia is given by Longstaff, Mithal, and Neis (2005) and Krishnamurthy and Vissing-Jorgensen (2007) who find that the supply of Treasury debt (relative to GDP) is negatively correlated to the spread between corporate and Treasury bond yields, even when controlling for default risk.

align observed interest rates and their theoretical counterparts. Further, they demonstrate that monetary transmission is fundamentally affected. This suggests that the international transmission of shocks can be improved by a model analyzing the impact of liquidity on interest and exchange rates. The second feature relates to the leading role of the U.S. dollar in the international monetary system. Canzoneri, Cumby, and Diba (2007) coin the term key currency pricing, which states that a large share of international trade is conducted in dollars. Key currency pricing implies that importers and exporters find it convenient to hold dollar assets to facilitate their transactions. Canzoneri, Cumby, and Diba (2007) argue that such liquidity services provided by key currency bonds are the driving force behind relatively low U.S. interest rates, which imply an "exorbitant privilege" for the United States.[7]

This paper combines these two observations and analyzes the impact of key currency pricing and liquidity on exchange rate dynamics. I develop a two-country open economy model with explicit open market operations in the foreign country (the key currency country), which are modeled as in Reynard and Schabert (2009): The foreign central bank supplies cash in exchange for foreign government bonds, so that these pay lower interest rates compared to assets not eligible for open market operations. Liquidity demand is motivated from households' demand for goods purchases, which require cash. Key currency pricing implies that households in the home economy require foreign currency to purchase import goods and hold foreign government bonds despite their low interest rates. I analyze how this setup affects uncovered interest rate parity and exchange rate movements, in particular in response to monetary policy shocks. The goal is to answer the following questions: Can liquidity premia generate deviations from uncovered interest rate parity? Can key currency effects reconcile theory and empirical evidence, for instance with respect to delayed overshooting? Does modeling key currency liquidity affect the international transmission of shocks in a fundamental way?

The main aim of the present paper is thus a positive analysis of monetary transmission, with a particular focus on asset prices and exchange rates. It addresses deficits of current asset pricing conditions, in particular UIP, and aims to advance consumption based asset pricing theory, suggesting that liquidity premia play an important role in determining exchange rates and international interest rate differences. Because asset pricing conditions are an important ingredient to currently used macroeconomic models, this can improve the empirical performance of these models, as suggested by the work of Justiniano and Preston (2010). Further, compared to standard models, the present model implies lower risk free interest rates and can thus contribute to solving the risk free rate puzzle, see Weil (1989).

[7]This quote is attributed to Charles de Gaulle but stems from Valery Giscard d'Estaing, who was French finance minister at the time of the statement. See Canzoneri, Cumby, and Diba (2007).

In the literature, the present work is most closely related to Canzoneri, Cumby, and Diba (2007). Like them, this paper stresses the importance of the U.S. dollar in international trade and models liquidity services provided by government bonds. However, both the setup and goal of this paper are different. The model in this paper builds on Reynard and Schabert (2009), so that liquidity premia in the model analyzed in this paper are microfounded and endogenously derived from households' demand for cash. In contrast, Canzoneri, Cumby, and Diba (2007) employ an ad hoc specification for the liquidity services provided by government bonds. Further, these authors focus on asymmetries in fiscal and monetary policy transmission between countries and do not address delayed overshooting.

The results of my analysis are the following. I show that modeling key currency liquidity generates deviations from UIP and demonstrate that the key currency model predicts delayed overshooting of the nominal exchange rate, as in Eichenbaum and Evans (1995). The reason is that a rising foreign monetary policy rate increases the interest rate on foreign government bonds but reduces liquidity premia overproportionately. This reduces the marginal benefit of investing in foreign government bonds, so that the foreign currency is expected to appreciate. I find an exchange rate peak after seven quarters, in line with the empirical evidence.

This paper is structured as follows. The model is presented in section 2. A modified uncovered interest parity condition, which contains a liquidity premium, is derived in section 3. Building on these results, section 4 analyzes the response of interest and exchange rates to monetary policy shocks. Section 5 concludes.

2 The model

2.1 Setup and timing of events

I model a small open economy (SOE) and its interactions to a large foreign economy, say the United States, which is explicitly modeled so that the impact of shocks to the foreign economy on the small home country can be analyzed. In the domestic and foreign economies, there is a continuum of infinitely lived households. I assume that households in both economies have identical asset endowments and preferences, so that I can consider a representative household in each country. As in Canzoneri, Cumby, and Diba (2007), I assume key currency pricing: International goods trade is carried out in terms of the foreign economy's currency, while domestic goods are purchased with local currency.[8] Moreover, it is assumed that the law of one price holds, so that exchange rate pass-through is perfect. Further, I analyze a foreign economy which is relatively large compared to the home

[8] This is equivalent to assuming producer currency pricing for large economy exports and local currency pricing for small economy exports.

economy, so that the home economy does not influence the foreign economy. However, I take into account the impact of home holdings of foreign assets on asset stocks in the foreign economy.

In the following, the timing of events is described. The representative household in the home economy enters the period with holdings of foreign currency $M_{F,t-1}$, domestic and foreign private debt D_{t-1}, $D_{F,t-1}$ and foreign government bonds $B_{F,t-1}$.[9] Foreign households enter the period with holdings of foreign currency $M^*_{F,t-1}$, foreign government bonds $B^*_{F,t-1}$ and foreign private debt $D^*_{F,t-1}$. For simplicity, I neglect domestic government bonds and assume that domestic households hold domestic currency only within periods. Further, it is assumed that firms in each country are owned by local households.

1. At the beginning of the period, shocks realize, households supply labor n_t and n^*_t and firms produce goods.

2. The foreign money market opens and both domestic and foreign households can exchange foreign government bonds $B_{F,t-1}$ and $B^*_{F,t-1}$ for money at the policy rate R^{m*}_t. The amounts $I_{F,t}$ and $I^*_{F,t}$ of foreign currency which home and foreign households can obtain in open market operations are therefore constrained by

$$I_{F,t} \leq \frac{B_{F,t-1}}{R^{m*}_t}, \tag{1}$$

$$I^*_{F,t} \leq \frac{B^*_{F,t-1}}{R^{m*}_t}. \tag{2}$$

   With respect to the home money market, I assume abundant supply of collateral, so that home households can obtain cash M_t at the opportunity cost $R_t - 1$.[10]

3. Households in both countries enter the goods markets, where goods can be bought with currency only. Key currency pricing requires import goods in both countries to be purchased with foreign currency only. Further, households in both economies purchase domestic goods with their domestic currencies. Thus, households in the

[9] Throughout the paper, the subscripts H and F refer to home and foreign origin of goods and assets. An asterisk denotes variables decided upon by foreign agents. Verbally, I distinguish between both economies by using the terms "foreign" or "large" economy versus "home" or "small" economy. The terms "local" and "domestic" can refer to either economy, depending on the context. Further, upper case letters refer to nominal variables while lower case letters denominate real variables.

[10] More explicitly, at the beginning of the period, households in the small open economy can exchange their holdings of private debt against cash at a discount identical to the interest rate on private debt, R_t. As private debt can be created by households at no cost, this constraint does not bind in equilibrium. I further assume that home households can engage in repurchase operations only, so that they will not hold domestic money across periods. Seigniorage is transferred back to households via a lump sum transfer.

small open economy are constrained by

$$P^*_{F,t} c_{F,t} \leq I_{F,t} + M_{F,t-1}, \tag{3}$$

$$P_{H,t} c_{H,t} \leq M_t, \tag{4}$$

where $c_{F,t}$ and $c_{H,t}$ denote home consumption of foreign and, respectively, domestic goods and where $P_{H,t}$ is the price of home goods in home currency and $P^*_{F,t}$ is the price of foreign goods in terms of foreign currency. Households in the large economy require foreign currency for their entire goods purchases and are thus constrained by

$$P^*_{F,t} c^*_t \leq I^*_{F,t} + M^*_{F,t-1}. \tag{5}$$

4. Before the asset markets open, households in both countries receive dividends $P_t \delta_t$ and wages $P_t w_t n_t$ as well as government transfers τ_t and τ^*_t. Further, repurchase agreements are settled. I assume that the foreign central bank conducts repo operations amounting to $M^{R*}_{F,t} + M^R_{F,t} = \Omega \left(M^*_{F,t} + M_{F,t} \right)$, where $M^{R*}_{F,t}$ and $M^R_{F,t}$ is the amount of money repurchased from foreign and, respectively, home households.

5. The asset markets open. Home households can carry wealth into the next period by purchasing domestic private debt D_t, foreign government bonds $B_{F,t}$ and foreign currency $M_{F,t}$. Foreign households invest into foreign assets only and acquire government bonds $B^*_{F,t}$, money $M^*_{F,t}$ as well as private debt $D^*_{F,t}$. The interest rates on domestic private debt, foreign government bonds and foreign private debt are given by R_t, R^*_t and R^{D*}_t.

2.2 The home economy

2.2.1 Households

Households maximize the expected sum of the discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u \left(c_t, n_t \right) \right], \tag{6}$$

where u is separable in its arguments, increasing, twice continuously differentiable, strictly concave and satisfies the Inada conditions, β is the households' discount factor and n_t is the share of his time endowment a household spends working. Home households' consumption is a composite good of foreign and domestic goods

$$c_t = \gamma c^{1-\eta}_{H,t} c^{\eta}_{F,t}, \tag{7}$$

where $\gamma^{-1} = \eta^{n}(1-\eta)^{1-\eta}$ and η provides an openness measure of the home country. Households maximize utility subject to the asset market constraint,

$$S_t\left[M_{F,t}-M_{F,t-1}+\frac{B_{F,t}}{R_t^*}-B_{F,t-1}+\frac{D_{F,t}}{R_t^{D*}}-D_{F,t-1}+P_t^*I_{F,t}\left(R_t^{m*}-1\right)\right]$$
$$\leq P_t w_t n_t + P_t\delta_t + P_t\tau_t - M_t\left(R_t-1\right)-\frac{D_t}{R_t}+D_{t-1}-P_{H,t}c_{H,t}-S_tP_{F,t}^*c_{F,t},$$

where S_t refers to the nominal exchange rate, i.e. the price of a unit of foreign currency in terms of domestic currency, the cash in advance constraints for imported and domestic goods, (3)-(4), the open market constraint (1) and the non-negativity constraints $M_{F,t}, M_t, B_{F,t} \geq 0$ as well as the no-Ponzi game condition $\lim_{s\to\infty} E_t \prod_{i=0}^{s} D_{F,t+s}/R_{t+i} \geq 0$. The first order conditions for working time n_t, domestic and foreign consumption $c_{H,t}$ and $c_{F,t}$, open market operations $I_{F,t}$, holdings of domestic and foreign money, and investment into home and foreign private debt as well as foreign government bonds are given by

$$\lambda_t w_t = -u_{n,t}, \tag{8}$$
$$\left(\lambda_t+\psi_{H,t}\right)\frac{P_{H,t}}{P_t} = u_{c,t}\gamma\left(1-\eta\right)\left(\frac{c_{F,t}}{c_{H,t}}\right)^{\eta}, \tag{9}$$
$$\left(\psi_{F,t}+\lambda_t\right)q_t = u_{c,t}\gamma\eta\left(\frac{c_{F,t}}{c_{H,t}}\right)^{\eta-1}, \tag{10}$$
$$\left(\psi_{F,t}+\lambda_t\right)q_t = \left(\lambda_t+\mu_t\right)R_t^m q_t, \tag{11}$$
$$\psi_{H,t} = \lambda_t\left(R_t-1\right), \tag{12}$$
$$\lambda_t q_t = \beta E_t q_{t+1}\frac{\lambda_{t+1}+\psi_{F,t+1}}{\pi_{t+1}^*}, \tag{13}$$
$$\lambda_t = \beta E_t\frac{\lambda_{t+1}R_t}{\pi_{t+1}}, \tag{14}$$
$$\lambda_t q_t = \beta E_t q_{t+1}\frac{\lambda_{t+1}R_t^{D*}}{\pi_{t+1}^*}, \tag{15}$$
$$\lambda_t q_t = \beta E_t q_{t+1}\frac{\lambda_{t+1}+\mu_{t+1}}{\pi_{t+1}^*}R_t^*, \tag{16}$$

where $\psi_{H,t}$, $\psi_{F,t}$, μ_t and λ_t are the respective Lagrange multipliers on the cash, open market, and asset market constraints, $q_t = S_tP_t^*/P_t$ is the real exchange rate and $\pi_t^* = P_t^*/P_{t-1}^*$ and $\pi_t = P_t/P_{t-1}$ are foreign and domestic (CPI) inflation. The budget constraint binds in equilibrium, $\lambda_t > 0$, because the disutility of working is strictly negative, $u_{n,t} < 0$. The complementary slackness conditions are given by

$$\begin{aligned}
&\psi_{H,t}\geq 0, && M_t-P_{H,t}c_{H,t}\geq 0, && \psi_{H,t}\left(M_t-P_{H,t}c_{H,t}\right)=0,\\
&\psi_{F,t}\geq 0, && M_{F,t}+I_{F,t}-P_{F,t}^*c_{F,t}\geq 0, && \psi_{F,t}\left(M_{F,t}+I_{F,t}-P_{F,t}^*c_{F,t}\right)=0,\\
&\mu_t\geq 0, && I_{F,t}-B_{F,t-1}/R_t^{m*}\geq 0, && \mu_t\left(\ I_{F,t}-B_{F,t-1}/R_t^{m*}\right)=0,
\end{aligned}$$

and the transversality condition requires $\lim_{s\to\infty} E_t \prod_{i=0}^{s} D_{F,t+s}/R_{t+i} = 0$. From (9) and (10), observe that both imported and domestically produced goods are subject to a cash credit friction. This implies that households' optimal allocation of consumption good spending depends not only on the relative prices of foreign and domestic goods, but also on foreign and domestic interest rates. Using (9)-(11) and (7), I demonstrate in Appendix A.1.1 that demand for foreign and domestic goods is given by

$$c_{F,t} = \frac{\eta u_{c,t}}{(\lambda_t + \mu_t)\, R_t^{m*}} q_t^{-1} c_t, \tag{17}$$

$$c_{H,t} = \frac{(1-\eta)\, u_{c,t}}{(\lambda_t + \psi_{H,t})} \left(\frac{P_{H,t}}{P_t}\right)^{-1} c_t. \tag{18}$$

Further, (15) and (16) show that households are willing to hold foreign government bonds at an interest rate below that on foreign private debt whenever the open market constraint (1) is binding. Further, as shown in Appendix A.1.1, the consumer price index P_t is given by

$$P_t = \frac{[(\lambda_t + \mu_t)\, R_t^m]^\eta \left(\lambda_t + \psi_{H,t}\right)^{1-\eta}}{c_t^{-\sigma}} P_{F,t}^\eta P_{H,t}^{1-\eta}, \tag{19}$$

where $P_{F,t}$ is the price of foreign goods in terms of the domestic currency. This implies that the cash distortion influences the price index. The reason is that the households take into account the cash credit friction into their optimal choice of consumption goods.

2.2.2 Firms

There is a continuum of monopolistically competitive firms indexed with $j \in [0,1]$. Firms rent labor at the nominal wage $P_t w_t$ and produce a differentiated good using a linear technology,

$$y_{H,t}(j) = n_t(j).$$

Cost minimization implies that marginal cost in real (PPI) terms, mc_t, are constant across firms and given by

$$mc_t = w_t \frac{P_t}{P_{H,t}}. \tag{20}$$

Firms produce varieties which are aggregated to a final good by competitive retailers according to

$$y_{H,t} = \left[\int_0^1 y_{H,t}^{\frac{\varepsilon-1}{\varepsilon}}(j)dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

so that firms face the demand constraint $y_{H,t}(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} y_{H,t}$. Following Calvo (1983), every firm reoptimizes its price in a given period with probability ϕ. Firms who do not reoptimize prices are assumed to increase prices with the steady state PPI inflation rate π_H, as in Ascari (2004). Denoting with Z_t the price of firms which reoptimize their price in period t, optimal forward looking price setting is given by

$$Z_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_s (\phi\beta)^s u_{c,t+s} y_{H,t+s} P^{\varepsilon}_{H,t+s} mc_{t+s}}{\sum_s (\phi\beta)^s u_{c,t+s} y_{H,t+s} P^{\varepsilon-1}_{H,t+s}}. \tag{21}$$

The optimal price setting condition can be rewritten recursively as

$$Z^1_t = \varepsilon / (\varepsilon - 1) u_{c,t} y_{H,t} mc_t + \phi\beta\pi_H^{-\varepsilon} E_t \pi^{\varepsilon}_{H,t+1} Z^1_{t+1}, \tag{22}$$

$$Z^1_t = u_{c,t} y_{H,t} + \phi\beta\pi_H^{1-\varepsilon} E_t \pi^{\varepsilon-1}_{H,t+1} Z^2_{t+1}, \tag{23}$$

where $\tilde{Z}_t = Z_t / P_{H,t} = Z^1_t / Z^2_t$. To determine the PPI inflation rate $\pi_{H,t}$, I use that the price index for home goods satisfies $P_{H,t} y_{H,t} = \int_0^1 P_{H,t}(j) \, y_{H,t}(j) \, dj$. Using the firms' demand constraint, $y_{H,t}(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} y_{H,t}$, this yields

$$1 = (1 - \phi) \left(Z^1_t / Z^2_t \right)^{1-\varepsilon} + \phi \pi_H^{1-\varepsilon} \pi^{\varepsilon-1}_{H,t}. \tag{24}$$

Further, the impact of price dispersion on output is given by

$$y_{H,t} = \frac{n_t}{s_t}, \tag{25}$$

where $s_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj$ captures price dispersion and evolves according to

$$s_t = (1 - \phi) \left(Z^1_t / Z^2_t \right)^{-\varepsilon} + \phi \pi_H^{-\varepsilon} \pi^{\varepsilon}_{H,t} s_{t-1}, \tag{26}$$

as in Schmitt-Grohé and Uribe (2004).

2.2.3 Public sector

The public sector in the home economy has a balanced budget. Thus, seigniorage earnings on domestic cash holdings are redistributed as a lump sum transfer $P_t \tau_t$ to domestic households, so that the public budget constraint reads

$$P_t \tau_t = M_t (R_t - 1).$$

Further, monetary policy is given by the interest rate rule

$$R_t = R^{(1-\rho_R)} R^{\rho_R}_{t-1} (\pi_{H,t}/\pi_H)^{w_\pi (1-\rho_R)} (y_{H,t}/y_H)^{w_y (1-\rho_R)}, \tag{27}$$

where ρ_R governs interest rate inertia and w_π (w_y) describes the central bank's reaction to deviations of producer price inflation (domestic output) from steady state. This rule is a simplified version of Justiniano and Preston (2010).

2.3 The foreign economy

In modeling the foreign economy, I closely follow Reynard and Schabert (2009). The only difference is that households import goods from the small economy. However, it is assumed that the foreign economy is large compared to the home economy, so that neither the allocation nor the price system in the small open economy influences the foreign economy. However, the impact of changes in domestic holdings of foreign government bonds on asset stocks in the foreign economy is taken into account.

2.3.1 Households

Foreign households consume an aggregate of consumption goods produced in the foreign and home economies, $c_t^* = \gamma^* \left(c_{F,t}^*\right)^{1-\eta^*} \left(c_{H,t}^*\right)^{\eta^*}$ where $\gamma^* = \left[\eta^{*n^*} \left(1-\eta^*\right)^{1-\eta^*}\right]^{-1}$. As is standard in the literature, the large open economy is treated as approximately closed, i.e. I analyze the case of $\eta^* \to 0$ so that foreign consumption and the price index are approximately given by $c_t^* = c_{F,t}^*$ and $P_t^* = P_{F,t}^*$. However, the demand function for import goods is relevant for the small open economy and given by $c_{H,t}^* = \eta^* \left(\frac{P_{H,t}/S_t}{P_t^*}\right)^{-1} c_t^*$. I assume that foreign households' discount factor is identical to that applied by households in the small economy. Foreign households maximize the expected sum of a discounted stream of instantaneous utilities which are separable in consumption and labor,

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^*, n_t^*\right).$$

subject to the asset market constraint

$$\begin{aligned} & M_{F,t-1}^* + B_{F,t-1}^* + P_t^* w_t^* n_t^* + D_{F,t-1}^* + P_t^* \delta_t^* + P_t^* \tau_t^* \\ & \leq M_{F,t}^* + \frac{B_{F,t}^*}{R_t^*} + \frac{D_{F,t}^*}{R_t^{D*}} + P_t^* c_t^* + \left(R_t^{m*} - 1\right) I_{F,t}^*, \end{aligned} \tag{28}$$

the open market constraint

$$I_{F,t}^* R_t^{m*} \leq B_{F,t-1}^*, \tag{29}$$

the cash in advance constraint

$$P_t^* c_t^* \leq I_{F,t}^* + M_{t-1}^*, \tag{30}$$

and non-negativity conditions $M_{F,t}^* \geq 0$ and $B_{F,t}^* \geq 0$ as well as the no Ponzi game condition $\lim_{s \to \infty} E_t \prod_{i=0}^{s} D_{F,t+s}^* / R_{t+i}^{D*} \geq 0$. The first order conditions with respect to working time, consumption, open market operations and holdings of private as well as government debt and money are given by

$$-\frac{u^*_{n,t}}{w^*_t} = \lambda^*_t, \tag{31}$$

$$u^*_{c,t} = \lambda^*_t + \psi^*_t, \tag{32}$$

$$R^{m*}_t \left(\lambda^*_t + \mu^*_t\right) = \lambda^*_t + \psi^*_t, \tag{33}$$

$$\lambda^*_t = \beta^* E_t \frac{\lambda^*_{t+1}}{\pi^*_{t+1}} R^{D*}_t, \tag{34}$$

$$\lambda^*_t = \beta^* E_t \frac{\lambda^*_{t+1} + \mu^*_{t+1}}{\pi^*_{t+1}} R^*_t, \tag{35}$$

$$\lambda^*_t = \beta^* E_t \frac{\lambda^*_{t+1} + \psi^*_{t+1}}{\pi^*_{t+1}}, \tag{36}$$

where λ^*_t, μ^*_t and ψ^*_t are the Lagrange multipliers on the budget, open market and cash in advance constraints. The complementary slackness conditions are given by

$$\psi^*_t \geq 0, \quad M^*_t + I^*_{F,t} - P^*_t c^*_t \geq 0, \quad \psi^*_t \left(M^*_t + I^*_{F,t} - P^*_t c^*_t\right) = 0,$$
$$\mu^*_t \geq 0, \quad I^*_{F,t} - B^*_{F,t-1}/R^{m*}_t \geq 0, \quad \mu^*_t \left(I^*_{F,t} - B^*_{F,t-1}/R^{m*}_t\right) = 0.$$

Further, the transversality condition, $\lim_{s\to\infty} E_t \prod_{i=0}^{s} D^*_{F,t+s}/R^{D*}_{t+i} = 0$ has to be satisfied.

2.3.2 Firms

The setup of the firm sector is identical to the home economy: A continuum of firms indexed over k rents labor and produces intermediate goods with a linear technology, given exogenous and constant total factor productivity A^*. Intermediate goods are aggregated like in the home economy, $y^*_t = \left[\int_0^1 \left(y^*_t(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}$, where I assume an identical elasticity of substitution, $\varepsilon^* = \varepsilon$. This yields the following equilibrium conditions

$$w^*_t = mc^*_t A^*, \tag{37}$$

$$Z^{1*}_t = \varepsilon/\left(\varepsilon - 1\right) u^*_{c,t} y^*_t mc^*_t + \phi^* \beta \pi^{*-\varepsilon} E_t \pi^{*\varepsilon}_{t+1} Z^{1*}_{t+1}, \tag{38}$$

$$Z^{2*}_t = u^*_{c,t} y^*_t + \phi^* \beta \pi^{*1-\varepsilon} E_t \pi^{*\varepsilon-1}_{t+1} Z^{2*}_{t+1}, \tag{39}$$

$$1 = (1-\phi)\left(Z^{1*}_t/Z^{2*}_t\right)^{1-\varepsilon} + \phi \pi^{*1-\varepsilon} \pi^{*\varepsilon-1}_t. \tag{40}$$

As in the home economy, price dispersion is defined as $s^*_t = \int_0^1 \left(\frac{P^*_t(k)}{P^*_t}\right)^{-\varepsilon} dk$, so that aggregate resources are inefficiently employed whenever $s^*_t > 1$. Aggregate production and price dispersion are given by

$$y^*_t = A^* n^*_t / s^*_t, \tag{41}$$

$$s^*_t = (1-\phi)\left(Z^{1*}_t/Z^{2*}_t\right)^{-\varepsilon} + \phi s^*_{t-1} \pi^{*\varepsilon}_t. \tag{42}$$

2.3.3 Public sector

The public sector is identical to that in Reynard and Schabert (2009) with the exception that I take into account the impact of holdings of foreign government bonds in the home economy on asset stocks in the foreign economy. Given a constant growth rate of the volume of Treasury bonds, which evolve according to

$$B_t^{T*} = \Gamma B_{t-1}^{T*}, \tag{43}$$

the Treasury's budget constraint is given by

$$\frac{B_t^{T*}}{R_t^*} + P_t^* \tau_t^{m*} = B_{t-1}^{T*} + P_t^* \tau_t^*, \tag{44}$$

where $P_t^* \tau_t^{m*}$ are seigniorage revenues and $P_t^* \tau_t^*$ lump-sum transfers to households. The central bank's bond holdings evolve according to

$$\frac{B_t^{CB*}}{R_t^*} + P_t^* \tau_t^{m*} = B_{t-1}^{CB*} + R_t^{m*} I_t^* - M_{F,t}^{R*}, \tag{45}$$

where $I_t^* = I_{F,t}^* + I_{F,t}$ denotes total injections and $M_{F,t}^{R*} + M_{F,t}^R$ are repo operations in both countries. Seigniorage is defined as interest earnings on government bonds held at period end, $P_t^* \tau_t^{m*} = \frac{B_t^{CB*}}{R_t^*} - B_t^{CB*}$. Thus, the central bank's bond holdings evolve according to

$$B_t^{CB*} - B_{t-1}^{CB*} = R_t^m I_t^* - M_{F,t}^{R*} - M_{F,t}^R.$$

Foreign households' bond holdings can now be derived residually from $B_{F,t}^* = B_t^T - B_{F,t} - B_t^{CB}$, which in differences reads

$$B_{F,t}^* - B_{F,t-1}^* = B_t^{T*} - B_{t-1}^{T*} - (B_{F,t} - B_{F,t-1}) - \left(B_t^{CB*} - B_{t-1}^{CB*}\right).$$

Plugging in central bank bond holdings (45) yields

$$B_{F,t}^* - B_{F,t-1}^* = (\Gamma - 1) B_{t-1}^{T*} - (B_{F,t} - B_{F,t-1}) - \left(R_t^m I_t^* - M_{F,t}^{R*} - M_{F,t}^R\right). \tag{46}$$

Monetary policy is assumed to conduct repurchase operations amounting to, $M_{F,t}^{R*} + M_{F,t}^R = \Omega \left(M_{F,t}^* + M_{F,t}\right)$. Further, the foreign policy rate follows an interest rate rule similar to that in the home economy

$$R_t^{m*} = R^{m*(1-\rho)} \left(R_{t-1}^{m*}\right)^\rho \left(\frac{\pi_t^*}{\pi^*}\right)^{w_\pi^*(1-\rho)} \left(\frac{y_t^*}{y^*}\right)^{w_y(1-\rho_R)} \exp(\varepsilon_t^*)^\rho, \tag{47}$$

where ε_t^* is independently identically distributed with $E_{t-1}\varepsilon_t^* = 0$. This closes the description of the foreign economy.

2.4 Equilibrium

In equilibrium markets clear, i.e. $n_t = \int_0^1 n_t(j)dj$, $y_{H,t} = c_{H,t} + c^*_{H,t}$, and for the foreign economy $n^*_t = \int_0^1 n^*_t(k)dk$ and $y^*_t = c^*_{F,t} = c^*_t$ because the home economy's imports $c_{F,t}$ are considered quantitatively negligible for the foreign economy. Further, private debt in both economies is in zero net supply, so that $D_{F,t} = -D^*_{F,t}$ and $D_t = 0$, because foreign households do not invest into home private debt. Throughout, I assume that the central banks in both countries set their instruments so that the cash in advance constraints (3), (4) and (30) bind ($\psi_{H,t}, \psi_{F,t}, \psi^*_t > 0$). I further assume that the share of repurchase agreements in money holdings is identical in both economies, so that the amounts of bonds repurchased by home and foreign households are given by $M^R_{F,t} = \Omega M_{F,t}$ and $M^{R*}_{F,t} = \Omega M^*_{F,t}$. Therefore, home households' holdings of foreign money are given by

$$\begin{aligned} M_{F,t} &= M_{F,t-1} + I_{F,t} - P^*_{F,t} c_{F,t} + P_{H,t} c^*_{H,t}/S_t - M^R_{F,t} \\ &= P_{H,t} c^*_{H,t}/S_t - M^R_{F,t}, \end{aligned} \tag{48}$$

where the second equality uses the binding cash in advance constraint for the home economy's imports.[11] Further, when (4) binds, households in the small economy hold domestic currency amounting to

$$M_t = P_{H,t} c_{H,t}. \tag{49}$$

Foreign households' currency holdings are given by $M^*_{F,t} = M^*_{F,t-1} + I^*_{F,t} - P^*_{F,t} c^*_{F,t} + P^*_t w^*_t n^*_t + P^*_t \delta^*_t - M^{R*}_{F,t}$. Using that foreign firms distribute their revenues entirely to foreign households, this simplifies to

$$M^*_{F,t} = M^*_{F,t-1} + I^*_{F,t} - \Omega M^*_{F,t}. \tag{50}$$

Capital account and the real exchange rate The evolution of net foreign asset holdings is given by[12]

$$\begin{aligned} & S_t \frac{B_{F,t}}{R^*_t} - S_t B_{F,t-1} + S_t \frac{D_{F,t}}{R^{D*}_t} - S_t D_{F,t-1} + S_t M_{F,t} - S_t M_{F,t-1} \\ &= P_{H,t} c^*_{H,t} - S_t P^*_{F,t} c_{F,t} - S_t I_{F,t} \left(R^{m*}_t - 1\right). \end{aligned} \tag{51}$$

Thus, the foreign country receives interest payments from home households' participation in open market operations. Except for this, the capital account is standard: The change in net foreign asset holdings of domestic households equals the current account, which

[11] The reason why exports appear is that they are paid for in foreign currency. Thus, households in the small open economy receive a share of dividends and wages in foreign currency. This share is given by $P_{H,t} c^*_{H,t}/S_t$. The remaining amount $P_t w_t n_t + P_t \delta_t - P_{H,t} c^*_{H,t}$ is received in domestic currency.

[12] This is derived from the households' budget constraint, using that home firms distribute all revenues as dividends and wages to home households, and applying the public sector's budget constraint (44).

consists of interest rate payments and the trade balance. Further, (19) can be rewritten by using the law of one price and the assumption of a large foreign economy, which implies that $P_{F,t} = S_t P^*_{F,t} = S_t P^*_t$. The real exchange rate is defined as

$$q_t = \frac{S_t P^*_t}{P_t} = \frac{P_{F,t}}{P_t}. \tag{52}$$

Using this, (19) can be rewritten as $\frac{P_{H,t}}{P_t} = \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}}$, which in differences reads

$$\pi_t = \pi_{H,t} \left(\frac{\Phi_t}{\Phi_{t-1}}\right)^{\frac{1}{1-\eta}} \left(\frac{q_t}{q_{t-1}}\right)^{\frac{\eta}{1-\eta}}, \tag{53}$$

where $\Phi_t = \frac{[(\lambda_t+\mu_t)R^m_t]^\eta (\lambda_t+\psi_{H,t})^{1-\eta}}{c_t^{-\sigma}}$.

Binding cash and open market constraints With the exception of section 3.1, I only consider equilibria where the open market constraints in both economies bind. In steady state, this is guaranteed by $R^{m*} < \frac{\pi^*}{\beta^*}$.[13] This implies that money injections are given by households' holdings of foreign government bonds,

$$I_{F,t} = \frac{B_{F,t-1}}{R^{m*}_t}, \tag{54}$$

$$I^*_{F,t} = \frac{B^*_{F,t-1}}{R^{m*}_t}. \tag{55}$$

Further, binding open market constraints in both economies imply that total injections are given by $I^*_t = \frac{B^*_{F,t-1}+B_{F,t-1}}{R^{m*}_t}$, so that foreign households' bond holdings evolve according to

$$B^*_{F,t} = (\Gamma - 1)\, B^{T*}_{t-1} - B_{F,t} + \Omega M^*_{F,t} + \Omega M_{F,t}. \tag{56}$$

A rational expectations equilibrium is a set of sequences $\{c_t, c_{F,t}, c_{H,t}, n_t, P_{H,t}, P_t, M_t, S_t, q_t, M_{F,t}, I_{F,t}, D_{F,t}, B_{F,t}, w_t, \lambda_t, \psi_{H,t}, \psi_{F,t}, \mu_t, y_{H,t}, mc_t, Z^1_t, Z^2_t, s_t, R_t, c^*_t, c^*_{H,t}, n^*_t, P^*_t, \lambda^*_t, \psi^*_t, \mu^*_t, M^*_{F,t}, I^*_{F,t}, B^*_{F,t}, B^{T*}_t, R^{m*}_t, R^{D*}_t, R^*_t, w^*_t, mc^*_t, y^*_t, Z^{1*}_t, Z^{2*}_t, s^*_t\}_{t=0}^{\infty}$ satisfying the households' and firms' first order conditions including the transversality conditions, the open market constraints (1) and (2), binding cash in advance constraints (3), (4) and (5), the households' holdings of foreign and home currency and foreign bonds, (48), (50) and (46), the capital account (51), the definition of the real exchange rate (52) and the home CPI (53) and PPI (24), aggregate production $y_{H,t} = c_{H,t} + c^*_{H,t} = n_t/s_t$ and $y^*_t = c^*_t = A^* n^*_t / s^*_t$ with price dispersion (26) and (42), export demand $c^*_{H,t} = \eta^* P^*_t S_t / P_{H,t} c^*_t$ and monetary policy rules (27) and (47) as well as the supply of foreign government bonds (43) for given A^* and initial values $M_{F,-1}, M^*_{F,-1} \geq 0$, $B_{F,-1}, B^*_{F,-1}, B^{T*}_{-1} > 0$ and $D_{F,-1} = -D^*_{F,-1}$, and $P_{-1}, P_{H,-1}, P^*_{-1}, S_{-1} > 0$. A summary of equilibrium conditions for the case of binding

[13] For a derivation of this property, see Appendix A.3.

open market constraints is given in Appendix A.2.

3 Uncovered interest rate parity

In this section, I derive the uncovered interest rate parity conditions implied by the model economy. When open market constraints bind, the model gives rise to a modified UIP condition, which contains a liquidity premium. This condition collapses to the standard UIP condition when open market constraints do not bind.

3.1 A standard UIP condition

Assume that $\mu_t = \mu_t^* = 0$ so that the open market constraints in both economies, (1) and (2), do not bind. In steady state, this is the case if foreign monetary policy sets the long-run policy rate to $R^{m*} = \pi^*/\beta^*$. The foreign households' first order conditions (34)-(35) imply that in this case, there is no spread between interest rates on private and government debt, which must then equal the policy rate, $R_t^{D*} = R_t^* = R_t^{m*}$. Thus, there are no liquidity premia when open market constraints do not bind. Consider the home households' first order conditions for investment in domestic private debt and foreign government bonds, (14) and (16). Using the definition of the real exchange rate (52) and combining the two equations yields

$$E_t \frac{S_{t+1}}{S_t} = \frac{R_t}{R_t^*} + \Upsilon_t, \tag{57}$$

using that the Inada conditions imply $\lambda_t > 0 \ \forall t$ and where terms of order higher than one are summarized in $\Upsilon_t = \frac{1}{R_t^* E_t \lambda_{t+1} E_t \pi_{t+1}^{-1}} \Big\{ R_t Cov\left(\lambda_{t+1}, \pi_{t+1}^{-1}\right) - \frac{R_t^*}{S_t} Cov\left(\lambda_{t+1} S_{t+1}, \pi_{t+1}^{-1}\right)$ $- \frac{R_t^*}{S_t} E_t \lambda_{t+1} Cov\left(\lambda_{t+1}, S_{t+1}\right) \Big\}$. I am not interested in effects of order two and above and thus ignore covariance terms in the analysis in this and the following sections. Equation (57) is a standard uncovered interest rate parity condition, which can be found in many small open economy models, such as Galí and Monacelli (2005). It requires the expected nominal depreciation to be equal to the interest rate difference between the home and foreign economies.

3.2 A modified UIP condition

When open market constraints bind, $\mu_t, \mu_t^* > 0$, foreign government bonds will pay a lower interest rate compared to foreign private debt. The reason is that foreign government bonds can be exchanged into cash, which households in the home economy need to purchase internationally traded goods. Combining the domestic households' optimality conditions for investment into domestic and foreign private debt (14) and (15) and using that $\lambda_t > 0$ yields

$$E_t \frac{S_{t+1}}{S_t} = \frac{R_t}{R_t^{D*}} + \Upsilon_t', \tag{58}$$

where $\Upsilon_t' = \frac{1}{R_t^{D*} E_t \lambda_{t+1} E_t \pi_{t+1}^{-1}} \Big\{ R_t Cov\left(\lambda_{t+1}, \pi_{t+1}^{-1}\right) - \frac{R_t^{D*}}{S_t} \big[Cov\left(\lambda_{t+1} S_{t+1}, \pi_{t+1}^{-1}\right)$ $+ E_t \pi_{t+1}^{-1} Cov\left(\lambda_{t+1}, S_{t+1}\right)\big]\Big\}$ summarizes terms of order two and higher. Thus, a standard UIP condition holds with respect to the interest rate difference in terms of the foreign debt rate R_t^{D*}. This rate is usually not observable. To obtain a UIP condition in the observable interest rate difference of home to foreign government bonds, I use the domestic households' optimality condition for investment into foreign government bonds, (16). Combining this with (15) gives

$$R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{R_t^*}{S_t} E_t S_{t+1} \frac{\theta_{t+1}}{\pi_{t+1}},$$

which can be written in the form of a modified UIP condition

$$E_t \frac{S_{t+1}}{S_t} = \frac{R_t}{R_t^* \theta_t} + \Upsilon_t'', \tag{59}$$

where $\theta_t = \left(1 + E_t \frac{\mu_{t+1}}{\lambda_{t+1}}\right)$ and with higher order terms summarized in $\Upsilon_t'' = \frac{1}{R_t^* \theta_t E_t \lambda_{t+1} E_t \pi_{t+1}^{-1}}$ $\Big[R_t Cov\left(\lambda_{t+1}, \pi_{t+1}^{-1}\right) - \frac{R_t^*}{S_t} Cov\left(\pi_{t+1}^{-1}, S_{t+1}\theta_{t+1}\right) - \frac{R_t^*}{S_t} E_t \pi_{t+1}^{-1} Cov\left(S_{t+1}, \theta_{t+1}\right)\Big]$. Thus, the interest rate difference between home and foreign government bonds is not the only determinant of exchange rate behavior. When the open market constraint in the home economy binds, $\mu_t > 0$, the term θ_t exceeds unity, reflecting the liquidity value of foreign government bonds. (58) and (59) imply that

$$\theta_t = \frac{R_t^{D*}}{R_t^*} \left(1 + \Upsilon_t'''\right), \tag{60}$$

where $\Upsilon_t''' = (\Upsilon_t' - \Upsilon_t'')$ summarizes higher order terms. The interest rate spread R_t^{D*}/R_t^* represents the opportunity cost of holding foreign government bonds, which in equilibrium, up to first order, will be equal to the premium θ_t. This premium captures the marginal liquidity value of holding foreign government bonds and will thus be called a liquidity premium.

4 Monetary policy and exchange rates

The goal of this section is to analyze the response of the exchange rate to a foreign monetary policy shock when open market constraints bind, so that a non-standard UIP condition holds. Further, I analyze a log-linear approximation to the equilibrium conditions around the model's steady state, which is derived in Appendix A.3. Let $\hat{x}_t = 100 \log(x_t/x)$ denote the percentage deviation of x_t from its steady state x. The linearized version of (59) then reads

$$E_t \hat{S}_{t+1} - \hat{S}_t = \hat{R}_t - \hat{R}_t^* - \hat{\theta}_t, \tag{61}$$

where $\hat{\theta}_t = \hat{R}_t^{D*} - \hat{R}_t^*$. The liquidity premium can be reexpressed as a function of the policy rate using that (35) and (36) imply $\hat{R}_t^* = E_t \hat{R}_{t+1}^{m*}$, so that

$$\hat{\theta}_t = \hat{R}_t^{D*} - E_t \hat{R}_{t+1}^{m*}. \tag{62}$$

Because a closed form solution for the general model version cannot be derived, I analyze a simplified model version.

4.1 Flexible prices

Assume flexible prices in the foreign economy, so that (37) becomes $w_t^* = A^*$ and (38)-(40) are redundant. Further, assume a utility function of the form $u(c_t^*, n_t^*) = \log c_t^* - \chi^* n_t^*$ and an exogenous instrument rule for the foreign policy rate, $R_t^{m*} = (R^{m*})^{1-\rho^*} \left(R_{t-1}^{m*}\right)^{\rho^*} \exp \varepsilon_t^*$.[14] Moreover, nominal growth of foreign government debt is given by $\Gamma^* = 1$, and the central bank targets zero steady state inflation, $\pi^* = 1$.[15] Further, I assume that the impact of home households' holdings of foreign government bonds on foreign households' holdings $B_{F,t}^*$ is negligible, so that (56) collapses to $B_{F,t}^* = \Omega M_{F,t}^*$. This implies that the foreign allocation and price system are independent from the home economy.

It can be shown that a shock to the foreign policy rate R_t^{m*} leads to an increase in the interest rate on foreign government debt which is more than compensated by a decline in the liquidity premium. Intuitively, the rising foreign policy rate makes it more costly to exchange government bonds for cash, so that the marginal liquidity value of holding foreign government bonds declines. This result is summarized in the following proposition.

Proposition 1 *Consider the simplified model version. A foreign monetary policy shock then leads to a decline in the liquidity premium which is larger than the rise in the interest rate on foreign government bonds, $\hat{\theta}_t > \hat{R}_t^*$.*

Proof. See Appendix A.4. ■

I now turn to exchange rate dynamics. Proposition 1 shows that in response to a contractionary foreign policy shock, the liquidity premium declines and overcompensates the rise in the government bond interest rate. Thus, at a constant home interest rate, the expected rate of depreciation $E_t \hat{S}_{t+1} - \hat{S}_t$ increases in order to compensate for the lower marginal benefit of investing into foreign government bonds. This result is in stark contrast to

[14] Note that the model does not imply equilibrium indeterminacy under an interest rate peg, which would be the case in a standard small open economy model. The reason is that the supply of collateral determines the price level path in the long run and thus prevents indeterminacy.

[15] Existence of a steady state then requires a long-run policy rate of $R^{m*} = 1$ because a positive policy rate in the steady state would imply that the central bank in every period acquires a share of households' bond holdings. With a constant supply of bonds, this would imply that foreign households' holdings of foreign government bonds, and thus foreign consumption, would converge to zero. Note that in principle, the central bank could also target an inflation rate different from zero, as long as $\pi > \beta^*$ so that the cash constraints in both economies continue to bind. For a steady state to exist, the policy rate then must satisfy $R^{m*} = (\Omega/(\Omega\pi^* + \pi^* - 1))$. For details, see Appendix A.3.2.

standard UIP conditions, which predict that a rise in the foreign interest rate (which in a standard model is identical to the foreign policy rate) leads to a decline in the expected rate of depreciation. This result is summarized in the following:

Corollary 2 *Consider the effect of a rise in the foreign policy rate on exchange rates given a constant home interest rate in the simplified model version. When the open market constraints do not bind, a rise in R_t^* leads to a decline in the expected rate of depreciation of the home currency, $E_t\hat{S}_{t+1} - \hat{S}_t < 0$. Under binding open market constraints, a positive shock to the foreign policy rate implies that the expected rate of depreciation is positive, $E_t\hat{S}_{t+1} - \hat{S}_t > 0$.*

Thus, endogenous movements in the liquidity premium can alter exchange rate dynamics to an extent that the sign of the exchange rate change can switch. This is in line with the empirical evidence by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008), who find that a foreign monetary shock lets the home currency depreciate for several quarters. Because it is difficult to derive analytical results for the full version of the model, I analyze a calibrated version in the next section.

4.2 Sticky prices

This section analyzes a calibrated version of the model economy with sticky prices in both economies, using a first-order approximation to the model's equilibrium conditions around the steady state.[16] Foreign monetary policy is assumed to set the long-run policy rate according to $R^{m*} < \frac{\pi^*}{\beta}$ and targets long-run inflation $\pi^* > \beta^*$, so that the the open market and cash constraints in the home and the foreign economy bind in steady state (see Appendix A.3). I analyze the model in a local neighborhood of the steady state where shocks are sufficiently small so that open market and cash constraints continue to bind. Households in both economies are assumed to maximize utility functions of the form

$$u(c_t, n_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \chi \frac{n_t^{1+\omega}}{1+\omega}, \tag{63}$$

$$u(c_t^*, n_t^*) = \frac{c_t^{*1-\sigma^*} - 1}{1-\sigma^*} - \chi^* \frac{n_t^{*1+\omega^*}}{1+\omega^*}. \tag{64}$$

4.2.1 Calibration

Table 1 summarizes the calibration. With respect to the intertemporal substitution elasticity of consumption goods and the Frisch elasticity of labor supply, I choose $\sigma = \sigma^* = 1.5$ and $\omega = \omega^* = 1$, which I consider a reasonable trade-off between diverging estimates resulting from microeconomic and macroeconomic data: Card (1994) suggests a range of 0.2 to 0.5 for the Frisch elasticity while Smets and Wouters (2007) estimate $\omega = 1.92$. With respect to the intertemporal substitutability of consumption, Barsky, Kimball, Juster,

[16] The full set of (non-linearized) equilibrium conditions can be found in Appendix A.2.

Discount Factor	$\beta = \beta^* = 0.9889$
Inverse of intertemporal substitution elasticity	$\sigma = \sigma^* = 1.5$
Inverse of Frisch elasticity of labor supply	$\omega = \omega^* = 1$
Openness home economy	$\eta = 0.27$
Openness foreign economy	$\eta^* = 0.01$
Subst. elasticity home and foreign varieties	$\varepsilon = \varepsilon^* = 10$
Calvo price stickiness	$\phi = 0.85$; $\phi^* = 0.75$
Taylor rule coefficients - Inflation	$w_\pi = w_\pi^* = 2$
Taylor rule coefficients - Output	$w_y = 0.2$, $w_y^* = 0.1$
Interest rate inertia	$\rho = 0.88$; $\rho^* = 0.80$
Share of repos to outright purchases	$\Omega = 1.5$
Steady state inflation	$\Gamma = 1.00575 = \pi^* = \pi$
Steady state foreign policy rate	$R^{m*} = 1.0105$
Steady state labor supply	$n = n^* = 0.33$
Foreign labor productivity	$A^* = 10$
Home net foreign asset position relative to imports (steady steady)	$\frac{b_F + d_F + m_F}{c_F} = -1$

Table 1: Paramater calibration

and Shapiro (1997) estimate an elasticity of 0.18 using micro data, implying a value of around 5 for σ. Macroeconomic data generally implies lower estimates, e.g. Smets and Wouters (2007) estimate $\sigma = 1.39$. I further choose χ and χ^* to calibrate working time in both economies to $n = n^* = 0.33$. Foreign labor productivity is set to $A^* = 10$, so that the relative size of the economies matches the ratio of Canadian to U.S. gross domestic product. I follow Justiniano and Preston's (2010) estimate of openness and price stickiness for Canada, $\eta = 0.27$ and $\phi = 0.85$. With respect to the foreign economy, I choose $\phi^* = 0.75$ as a compromise between the estimates of Smets and Wouters (2007), Justiniano and Primiceri (2008) and Justiniano and Preston (2010) for the United States, which range between 0.65 and 0.90. Monetary policy in both countries sets the interest rate according to a Taylor rule, where home policy is calibrated to $w_\pi = 2$, $w_y = 0.2$ and $\rho = 0.88$, as estimated by Justiniano and Preston (2010) for the Canadian economy. In the foreign economy, monetary policy is characterized by $w_\pi^* = 2$, $w_y^* = 0.1$ and $\rho^* = 0.80$, which is in line with Smets and Wouters (2007) and Justiniano and Primiceri (2008), who estimate models with Bayesian techniques using U.S. data. The parameter Ω is chosen to match the observed share of reserves supplied in repurchase operations to total reserves, as in Reynard and Schabert (2009). The long-run inflation rate and the policy rate in the foreign economy are set to the 20-year averages of U.S. consumer price inflation and, respectively, the Federal Funds rate, $\pi^* = 1.00575$ and $R^{m*} = 1.0105$. The home central bank is assumed to adopt an identical long-run inflation target, $\pi = \pi^*$. The discount factor is assumed to be equal across both countries and calibrated to the liquidity premium, i.e. the spread between the debt rate R^{D*} and the rate on foreign government bonds R^*. The debt rate is the interest rate on a safe but illiquid bond. I follow Canzoneri, Cumby, and Diba (2007) and calibrate the spread to 65 basis points, which equals the difference between the interest rate faced by high-quality (AAA) borrowers and the interest rate on 3 months Treasury bills. Because there is no asset without any liquidity value, it is likely that this figure underestimates the true liquidity premium. Thus, the discount factor is set to $\beta = \frac{\pi}{R^m + 65 \cdot 10^{-4}} = 0.9889$. Further, the home economy is assumed to be a net debtor in steady state, with debt equivalent to 100% of the home country's quarterly imports, $\frac{b_F + d_F + m_F}{c_F} = -1$. This is in line with the ratio of Canadian foreign debt to average imports over the past 20 years and leads to a ratio of debt to domestic absorption of 9%, as in Bouakez and Rebei (2008).[17]

4.2.2 Responses to a shock to the foreign policy rate

This section analyzes the impact of a foreign monetary policy shock. Figure 1 shows the impact of a 12.5 basis point innovation to R_t^{m*} on the foreign economy. All variables are in per cent deviations from steady state, $\widehat{z}_t = 100 \left[\log(z_t) - \log(z)\right]$, except for interest rates and inflation, which are given in absolute deviations, $\hat{R}_t^* = 100 * (R_t^* - R^*)$. The increase

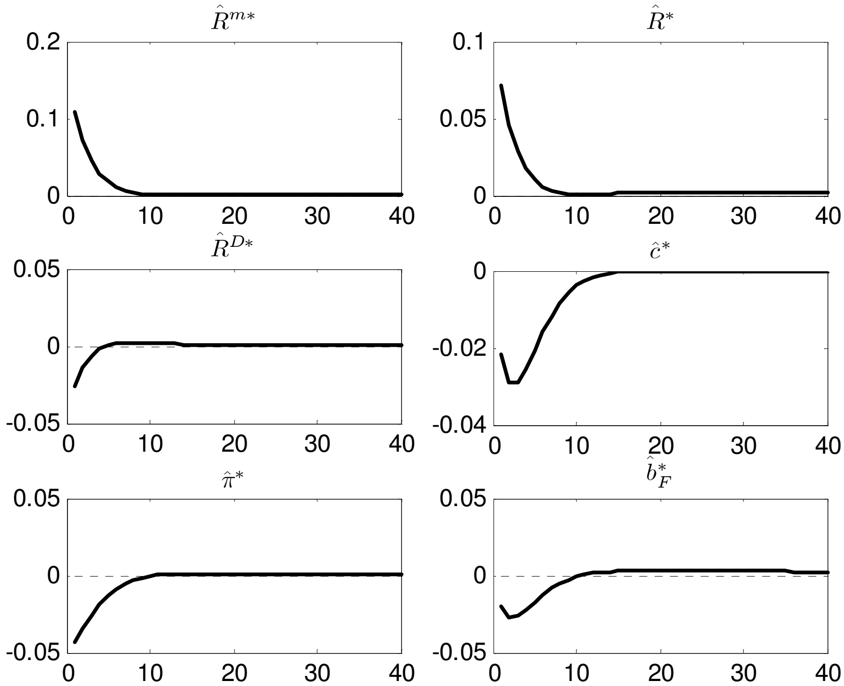

Figure 1: Responses to a foreign monetary policy shock in the foreign economy

[17]Data on imports and net foreign debt were taken from Statistics Canada, Publications 67-202-X and 13-019-X.

in the foreign policy rate induces a decline in foreign consumption and a reduction in inflation in the foreign economy. Consumption responds in a hump-shaped way because a rising policy rate increases seigniorage and thus reduces households' bond holdings, which implies that consumption declines with a lag. Further, the increase in the policy rate reduces the liquidity value of government bonds, so that the interest rate on these rises. The nominal interest rate on private debt declines because inflation falls.

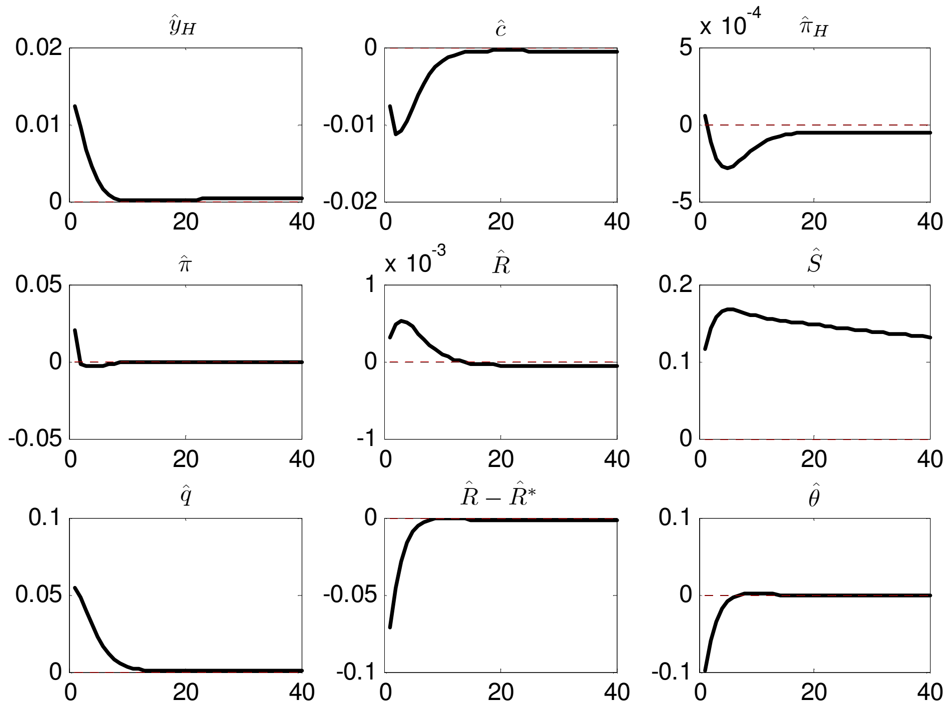

Figure 2: Responses to a foreign monetary policy shock in the home economy

Figure 2 shows the responses of the home economy. The foreign interest rate shock affects the home economy through different channels. First, it renders imports more expensive because foreign currency becomes more costly. Further, the decline in foreign consumption reduces export demand and implies that the home currency devalues both in nominal and real terms. This makes imports even more expensive for domestic households, who reduce consumption and increase worked hours, so that production rises. Turning attention to the exchange rate, a pattern different from that implied by standard models is observed: The nominal exchange rate depreciates on impact, and continues to depreciate until it peaks in the seventh quarter, consistent with Corollary 2. Thus, the model predicts delayed overshooting in line with the analysis by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008). The driving force behind delayed overshooting is the liquidity premium. A rising foreign policy rate implies that government bonds become less liquid, so that the liquidity premium declines. As in Proposition 1, the decline in the liquidity premium exceeds the increase in the foreign government bond interest rate.

With respect to the real exchange rate, the model does not predict delayed overshooting: In real terms, the domestic currency depreciates on impact, peaks in the shock period and then appreciates gradually back toward its steady state. The reason for the divergence between nominal and real exchange rates is the persistent decline in foreign inflation, which implies that the real rate of appreciation is negative while the rate of nominal depreciation is positive in the shock period. In line with the high observed correlation between real and nominal exchange rates, the VAR evidence quoted above predicts delayed overshooting for both the nominal and the real exchange rate. Although the key currency model does not predict delayed overshooting for the real exchange rate, the liquidity premium increases the rate of real appreciation, so that real exchange rate movements are closer to the pattern observed by Eichenbaum and Evans (1995) and Scholl and Uhlig (2008), as predicted by standard UIP.

4.2.3 Comparing exchange rate dynamics to standard UIP

This section compares exchange rate dynamics to those predicted by a standard UIP condition. In principle, the model without binding open market constraints is characterized by such a standard UIP. However, analyzing the impact of a shock to the foreign policy rate within the model without binding open market constraints would imply that, apart from the different UIP condition, general equilibrium effects would affect exchange rate movements. For instance, the reaction of inflation in the foreign economy would be different due to differences in monetary transmission. Thus, I construct a counterfactual scenario which shows how exchange rates would behave under a standard UIP condition, all other things equal.[18] Denoting ex ante real interest rates as $\hat{r}_t = \hat{R}_t - E_t\hat{\pi}_{t+1}$ and $\hat{r}_t^* = \hat{R}_t^* - E_t\hat{\pi}_{t+1}^*$, time series for the expected nominal and real exchange rates are constructed from standard UIP conditions

$$E_t\hat{S}_{t+1} - \hat{S}_t = \hat{R}_t - \hat{R}_t^*,$$
$$E_t\hat{q}_{t+1} - \hat{q}_t = \hat{r}_t - \hat{r}_t^*,$$

where the series for $\hat{R}_t - \hat{R}_t^*$ and $\hat{r}_t - \hat{r}_t^*$ are given by the responses to a foreign policy rate shock in the model with liquidity premia. These are compared to the exchange rate movements which result when taking into account the liquidity premium, which are identical to those presented in Figure 2. Figure 3 shows the results of this analysis.

[18] "All other things" also refers to the long-run equilibrium values for the nominal and real exchange rates. In other words, I assume that in the counterfactual scenario, the nominal and real exchange rates converge to long-run equilibrium values identical to those in the model with liquidity premia. This assumption is required to compute the impact response of the exchange rates in the counterfactual scenario.

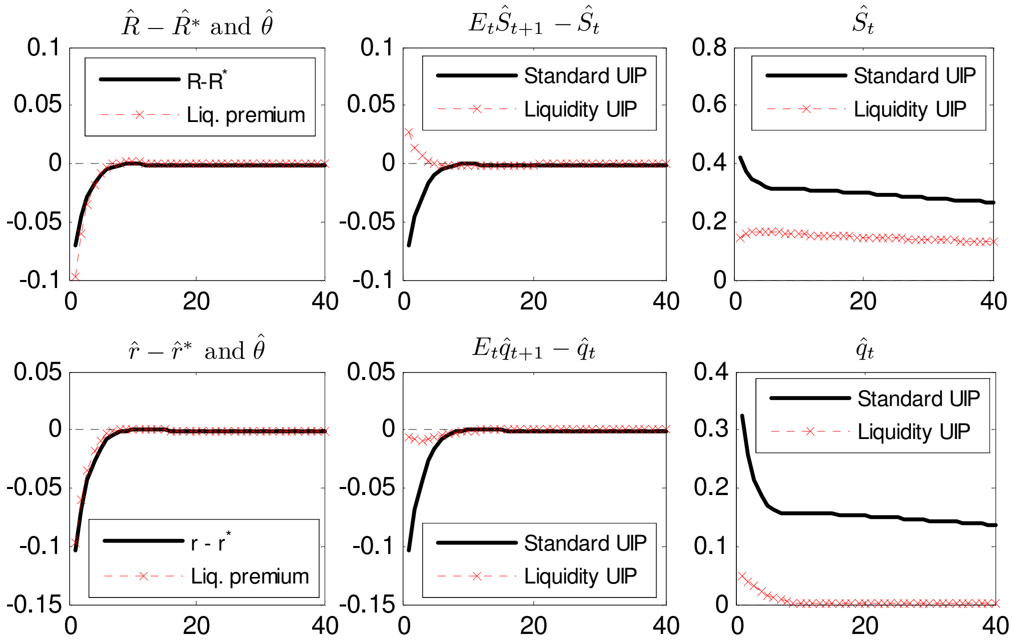

Figure 3: Comparison of exchange rate dynamics under standard and modified UIP

Under a conventional UIP, a rise in the foreign interest rate leads to an impact nominal depreciation, followed by a persistent appreciation. This is Dornbusch's (1976) famous "overshooting" result: The nominal exchange rate jumps on impact after a monetary shock and overshoots its new long-run equilibrium value. Given that the decline in the nominal interest rate on foreign government bonds under sticky prices implies a decline in the real interest rate, the standard UIP condition predicts overshooting for the real exchange rate as well.

Taking into account movements of the liquidity premium fundamentally affects exchange rate dynamics: An increase in the foreign policy rate reduces the liquidity premium and leads to an impact depreciation of the domestic currency, as before. However, because the liquidity premium falls more strongly than the interest rate difference for the first seven quarters, in nominal terms the domestic currency continues to depreciate (for seven quarters). Thus, the liquidity premium reverses the sign of the expected rate of nominal depreciation, compared to a standard UIP. Apart from the pattern of the response, also the timing of the peak, which occurs in the seventh quarter is in line with the estimates by Scholl and Uhlig (2008), who find that the median of the peak in the exchange rates of the U.S. dollar to the currencies of Germany, the U.K., and Japan occurs after 17-26 months.

The response of the real exchange rate under the modified UIP condition depends on real interest rates in both countries and the liquidity premium. The foreign monetary policy shock leads to a persistent decline in foreign inflation, which implies that the foreign real interest rate (on government bonds) increases more strongly than its nominal counterpart. Figure 4 shows that this leads to a decline in the real interest rate difference which slightly exceeds the decline in the liquidity premium, so that the real exchange rate will appreciate and return toward its steady state after its peak in the first period. Thus, the pattern of the real exchange rate's response to a foreign monetary policy shock under the modified UIP condition is similar to standard UIP. However, the decline in the liquidity premium moderates the appreciation after the peak, so that the predictions of the modified UIP condition become closer to the empirical evidence, which finds delayed overshooting for nominal and real exchange rates.

5 Conclusion

This paper asks if the leading role of the U.S. dollar in international trade can explain observed deviations from uncovered interest rate parity, focusing on the impact of monetary policy shocks on exchange rates. It derives a macroeconomic model in which U.S. government bonds trade at a liquidity premium because they facilitate access to key currency liquidity. This liquidity premium enters the UIP condition and can explain delayed overshooting of the nominal (but not the real) exchange rate: In response to a contractionary U.S. monetary policy shock, the premium falls (reflecting the higher cost of obtaining liquidity) and overcompensates the rise in the interest rate on government bonds.

Thus, the paper contributes to consumption based asset pricing theory by demonstrating that liquidity premia can improve exchange rate predictions. In a similar vein, Reynard and Schabert (2009) show that they can align model-implied and observed interest rates. Because asset pricing conditions are an important determinant of the equilibrium allocation in macroeconomic models, this can crucially affect the transmission of shocks. Further, Justiniano and Preston (2010) argue that the empirical failure of UIP is at the root of the deficits of estimated New Keynesian models in explaining the international transmission of shocks. Therefore, it would be interesting to analyze if a full-fledged model incorporating the effects of liquidity premia on asset prices and the macroeconomic allocation can perform better in this respect. Further, the model contains a channel through which contagion, i.e. financial crises spreading across seemingly unrelated countries, can be explained: When investors' liquidity demand changes, this affects liquidity premia and has an effect on exchange rates and import demand in all countries that use the key currency in international transactions.

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A Appendix

The appendix contains the derivation of equilibrium conditions of the home economy as well as summaries of home and foreign equilibrium conditions for the case of binding open market constraints, a derivation of the steady states and a proof of proposition 1.

A.1 Home economy equilibrium conditions

A.1.1 Price index and households' goods demand

Households' goods demand First, I rewrite (10) by using (11) and $\left(\frac{c_{F,t}}{c_{H,t}}\right)^{1-\eta} = \frac{c_t}{c_{F,t}\gamma}$ to obtain

$$c_{F,t} = \frac{\eta u_{c,t}}{\left(\lambda_t + \mu_t\right) R_t^m q_t} c_t. \tag{65}$$

Similarly, rewriting (9) by using $\left(\frac{c_{F,t}}{c_{H,t}}\right)^{\eta} = \frac{c_t}{c_{H,t}\gamma}$ implies

$$c_{H,t} = \frac{\left(1-\eta\right) u_{c,t}}{\left(\lambda_t + \psi_{H,t}\right) \frac{P_{H,t}}{P_t}} c_t. \tag{66}$$

Using (65) and (66) in the definition of the price index yields

$$\begin{aligned} P_t c_t &= P_{H,t} c_{H,t} + P_{F,t} c_{F,t} \\ &\Longleftrightarrow \\ 1 &= \left(1-\eta\right) \frac{u_{c,t}}{\lambda_t + \psi_{H,t}} + \eta \frac{u_{c,t}}{\left(\lambda_t + \mu_t\right) R_t^m}, \end{aligned} \tag{67}$$

which characterizes the optimal labor leisure trade-off given that domestic and imported goods are subject to cash credit frictions.

Derivation of the price index Using $c_{H,t} = \left(\frac{c_t}{\gamma c_{F,t}^{\eta}}\right)^{\frac{1}{1-\eta}}$ to cancel out $c_{H,t}$ in (66), solving for $c_{F,t}$ and combining this with (65) yields

$$\left[\frac{\left(1-\eta\right) u_{c,t}}{\left(\lambda_t + \psi_{H,t}\right) P_{H,t}/P_t}\right]^{\frac{\eta-1}{\eta}} c_t \gamma^{\frac{-1}{\eta}} = \frac{\eta u_{c,t}}{\left(\lambda_t + \mu_t\right) R_t^m P_{F,t}/P_t} c_t,$$

where $\gamma^{1/\eta} = \eta^{-\eta/\eta} \left(1-\eta\right)^{\frac{\eta-1}{\eta}}$. Solving for the price level yields

$$P_t = \frac{\left[\left(\lambda_t + \mu_t\right) R_t^m\right]^{\eta} \left(\lambda_t + \psi_{H,t}\right)^{1-\eta}}{u_{c,t}} P_{F,t}^{\eta} P_{H,t}^{1-\eta}. \tag{68}$$

Thus, the price index takes into account that households' consumption choice is influenced by the cash credit friction. For simplicity, define $\Phi_t = \frac{\left[\left(\lambda_t + \mu_t\right) R_t^{m*}\right]^{\eta} \left(\lambda_t + \psi_{H,t}\right)^{1-\eta}}{u_{c,t}}$ which measures the extent of the cash-credit friction. Introducing the real exchange rate $q_t = \frac{S_t P_t^*}{P_t} = \frac{P_{F,t}}{P_t}$ and using $z_t = P_{H,t}/P_{F,t} = \frac{P_{H,t}}{S_t P_t^*} = q_t^{-1} \frac{P_{H,t}}{P_t}$, which implies $\Phi_t \left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} =$

$\frac{P_t}{P_{F,t}}$, I can rewrite (68) as $\frac{P_{H,t}}{P_t} = \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}}$, which in differences reads

$$\pi_t = \pi_{H,t} \left(\frac{\Phi_t}{\Phi_{t-1}}\right)^{\frac{1}{1-\eta}} \left(\frac{q_t}{q_{t-1}}\right)^{\frac{\eta}{1-\eta}}. \tag{69}$$

A.2 Equilibrium conditions when open market constraints bind

A.2.1 Home economy

The representative household's first order conditions can be summarized by

$$\lambda_t w_t = -u_{n,t}, \tag{70}$$

$$1 = (1-\eta)\frac{u_{c,t}}{\lambda_t + \psi_{H,t}} + \eta \frac{u_{c,t}}{(\lambda_t + \mu_t) R_t^m}, \tag{71}$$

$$c_{F,t} = \frac{\eta u_{c,t}}{(\lambda_t + \mu_t) R_t^{m*} q_t} c_t, \tag{72}$$

$$c_{H,t} = \frac{(1-\eta) u_{c,t}}{\left(\lambda_t + \psi_{H,t}\right) \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}}} c_t, \tag{73}$$

$$\lambda_t = \beta E_t \frac{\lambda_{t+1} R_t}{\pi_{t+1}}, \tag{74}$$

$$\lambda_t q_t = \beta E_t q_{t+1} \frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}^*} R_t^*, \tag{75}$$

$$\frac{R_t}{E_t \pi_{t+1}} = E_t \frac{q_{t+1}}{q_t} \frac{R_t^{D*}}{\pi_{t+1}^*}, \tag{76}$$

$$\psi_{H,t} = \lambda_t (R_t - 1), \tag{77}$$

$$\pi_{H,t} = \pi_t \left(\frac{\Phi_t}{\Phi_{t-1}}\right)^{\frac{1}{\eta-1}} \left(\frac{q_t}{q_{t-1}}\right)^{\frac{\eta}{\eta-1}}, \tag{78}$$

where $\Phi_t = \frac{[(\lambda_t+\mu_t)R_t^{m*}]^\eta \left(\lambda_t + \psi_{H,t}\right)^{1-\eta}}{u_{c,t}}$. The binding cash and open market constraints read

$$c_{F,t} = \frac{b_{F,t-1}}{R_t^{m*}\pi_t^*} + \frac{m_{F,t-1}}{\pi_t^*}, \tag{79}$$

$$m_{F,t} = \frac{1}{1+\Omega}\eta^* c_t^*, \tag{80}$$

$$m_t = \Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{\eta}{\eta-1}} c_{H,t}, \tag{81}$$

where $m_{F,t} = M_{F,t}/P_t^*$, $b_{F,t} = B_{F,t}/P_t^*$ and $m_t = M_t/P_t$ denote real money and bond holdings. The firms' block of first order conditions is given by

$$mc_t = w_t \Phi_t^{\frac{1}{1-\eta}} q_t^{\frac{\eta}{1-\eta}}, \tag{82}$$

$$Z_t^1 = u_{c,t} y_{H,t} mc_t + \phi\beta\pi_H^{-\varepsilon} E_t \pi_{H,t+1}^{\varepsilon} Z_{t+1}^1, \tag{83}$$

$$Z_t^2 = u_{c,t} y_{H,t} + \phi\beta\pi_H^{1-\varepsilon} E_t \pi_{H,t+1}^{\varepsilon-1} Z_{t+1}^2, \tag{84}$$

$$1 = (1-\phi)\left(Z_t^1/Z_t^2\right)^{1-\varepsilon} + \phi\pi_H^{1-\varepsilon}\pi_{H,t}^{\varepsilon-1}. \tag{85}$$

The final block of equilibrium conditions contains, among others, the resource constraint, the production function including price dispersion and the evolution of foreign debt,

$$y_{H,t} = c_{H,t} + c_{H,t}^*, \tag{86}$$

$$y_{H,t} = n_t^{\alpha}/s_t, \tag{87}$$

$$s_t = (1-\phi)\left(Z_t^1/Z_t^2\right)^{-\varepsilon} + \phi\pi_H^{-\varepsilon}\pi_{H,t}^{\varepsilon} s_{t-1}, \tag{88}$$

$$\Phi_t^{\frac{1}{\eta-1}} q_t^{\frac{1}{\eta-1}} c_{H,t}^* - c_{F,t} = \frac{b_{F,t}}{R_t^*} - \frac{b_{F,t-1}}{R_t^{m*}\pi_t^*} + \frac{d_{F,t}}{R_t^{D*}} - \frac{d_{F,t-1}}{\pi_t^*} + m_{F,t} - \frac{m_{F,t-1}}{\pi_t^*},$$

$$\frac{q_t}{q_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\pi_t^*}{\pi_t}, \tag{89}$$

$$c_{H,t}^* = q_t^{\frac{1}{1-\eta}} \Phi_t^{\frac{1}{1-\eta}} \eta^* c_t^*, \tag{90}$$

where $d_{F,t} = D_{F,t}/P_t^*$ denotes real holdings of foreign private debt. Monetary policy follows a Taylor rule.

$$R_t = R^{(1-\rho_R)} R_{t-1}^{\rho_R} \left(\pi_{H,t}/\pi_H\right)^{w_\pi(1-\rho_R)} \left(y_{H,t}/y_H\right)^{w_y(1-\rho_R)}, \tag{91}$$

where $R = \pi/\beta$ is the steady state interest rate in the home economy.

A.2.2 Foreign economy

When cash and open market constraints bind, the foreign economy can be described by the behavior of households,

$$\frac{-u_{n,t}^*}{w_t^*} = \beta^* E_t \frac{u_{c,t+1}^*}{\pi_{t+1^*}}, \tag{92}$$

$$E_t \frac{u_{c,t+1}^*}{\pi_{t+1}^*} = R_t^* E_t \frac{u_{c,t+1}^*}{\pi_{t+1}^* R_{t+1}^{m*}}, \tag{93}$$

$$\frac{u_{n,t}^*}{w_t^*} = \beta R_t^{D*} \frac{u_{n,t+1^*}}{w_{t+1}^* \pi_{t+1}^*}, \tag{94}$$

$$c_t^* = (1+\Omega) m_{F,t}^*, \tag{95}$$

$$m_{F,t}^*(1+\Omega) = \frac{m_{F,t-1}^*}{\pi_t^*} + \frac{b_{F,t-1}^*/\pi_t^*}{R_t^{m*}}, \tag{96}$$

firms,

$$w_t^* = mc_t^* A^*, \tag{97}$$

$$Z_t^{1*} = \varepsilon / (\varepsilon - 1) u_{c,t}^* y_t^* mc_t^* + \phi^* \beta \pi^{*-\varepsilon} E_t \pi_{t+1}^{*\varepsilon} Z_{t+1}^{1*}, \tag{98}$$

$$Z_t^{2*} = u_{c,t}^* y_t^* + \phi^* \beta \pi^{*1-\varepsilon} E_t \pi_{t+1}^{*\varepsilon-1} Z_{t+1}^{2*}, \tag{99}$$

$$1 = (1-\phi) \left(Z_t^{1*} / Z_t^{2*} \right)^{1-\varepsilon} + \phi \pi^{*1-\varepsilon} \pi_t^{*\varepsilon-1}, \tag{100}$$

the public sector,

$$b_{F,t}^* = (\Gamma - 1) b_{t-1}^{T*} / \pi_t^* - b_{F,t} + m_{F,t}^{R*}, \tag{101}$$

$$b_t^{T*} = \Gamma b_{t-1}^{T*} / \pi_t^*, \tag{102}$$

$$R_t^{m*} = R^{m*(1-\rho)} \left(R_{t-1}^{m*} \right)^{\rho} (\pi_t^* / \pi^*)^{w_\pi^*(1-\rho)} (y_t^* / y^*)^{w_y(1-\rho_R)} \exp(\varepsilon_t^*)^{\rho}, \tag{103}$$

and aggregate resources,

$$y_t^* = c_t^*, \tag{104}$$

$$y_t^* = A^* n_t^* / s_t^*, \tag{105}$$

$$s_t^* = (1-\phi^*) \left(Z_t^{1*} / Z_t^{2*} \right)^{-\varepsilon} + \phi^* s_{t-1}^* \pi_t^{*\varepsilon}, \tag{106}$$

where $m_{F,t}^* = M_{F,t}^* / P_t^*$ and $b_{F,t}^* = B_{F,t}^* / P_t^*$ denote real money and bond holdings, A^* is exogenous labor productivity and $b_t^{T*} = B_t^{T*} / P_t^*$ denotes the real stock of foreign bonds in circulation.

A.3 Steady States under binding open market constraints

This section derives the steady state of the model given binding open market constraints. This is required for the log-linear approximation used in section 4.2.

A.3.1 Home economy

I use that the utility function is given by (63), which is repeated here for convenience

$$u(c_t, n_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta}.$$

The first order conditions for price setting imply

$$\begin{array}{ll} Z^1 = \frac{\varepsilon}{\varepsilon-1} \frac{u_c y_H}{1-\phi\beta} mc, & Z^2 = \frac{u_c y_H}{1-\phi\beta}, \\ mc = \frac{\varepsilon-1}{\varepsilon}, & s = \left(\frac{Z^1}{Z^2} \right)^{-\varepsilon} = 1. \end{array}$$

The steady state inflation rate of home goods, π_H, can be set by the central bank through the interest rate rule. There is no price dispersion in steady state due to indexation of non-optimized prices to steady state inflation. The domestic Euler rate is given by $R = \pi/\beta$, and the UIP condition implies identical real interest rates $R/\pi = R^{D*}/\pi^*$ and thus identical discount factors, $\beta = \beta^*$. Further, in steady state CPI inflation equals PPI inflation, $\pi = \pi_H$. Moreover, I assume that the home central bank targets an inflation rate identical to foreign inflation, $\pi = \pi^*$, so that $R = R^{D*}$ and the nominal exchange rate is constant, $S_t/S_{t-1} = 1$ but in its level not determined. Consider the remaining system of equilibrium conditions,

$$\chi = \lambda w n^{-\omega} \tag{107}$$
$$\lambda = c^{-\sigma}\left[\frac{1-\eta}{R^D} + \frac{\eta}{R^{D*}}\right] = c^{-\sigma}/R^D \tag{108}$$
$$\lambda = \eta q^{-1} c^{-\sigma}\left(\frac{c}{c_F}\right)\frac{1}{R^{D*}} \tag{109}$$
$$\mu = \lambda\left(\frac{R^{D*}}{R^*} - 1\right) \tag{110}$$
$$\psi_F = \lambda\left(R^{D*} - 1\right) \tag{111}$$
$$\psi_H = \lambda\left(R^D - 1\right) \tag{112}$$
$$\Phi = \frac{\lambda}{c^{-\sigma}}\left(R^{D*}\right)^{\eta}\left(R^D\right)^{1-\eta} = 1 \tag{113}$$
$$c_F = \frac{b_F}{R^{m*}\pi^*} + \frac{m_F}{\pi^*} \tag{114}$$
$$m_F = \frac{1}{1+\Omega}\eta^* c^* \tag{115}$$
$$w = \Phi^{\frac{1}{\eta-1}} q^{\frac{\eta}{\eta-1}} \frac{\varepsilon - 1}{\varepsilon} \tag{116}$$
$$n^{\alpha} = \left(c/c_F^{\eta}\gamma\right)^{\frac{1}{1-\eta}} + \eta^* q^{\frac{1}{1-\eta}} \Phi^{\frac{1}{1-\eta}} c^* \tag{117}$$
$$\frac{b_F}{R^*} = \eta^* c^* - d_F\left(\frac{1}{R^{D*}} - \frac{1}{\pi^*}\right) - m_F, \tag{118}$$

where the last equation uses $R^m = R^*$ as well as the binding open market constraint. Observe from the multipliers on the cash in advance constraint (ψ_F, ψ_H) and the open market constraint (μ) that a foreign interest rate policy satisfying $R^{m*} < \frac{\pi^*}{\beta^*}$ and $\pi^* > \beta^*$ as well as a positive domestic interest rate in the long run ($\pi > \beta$) implies that all cash and open market constraints bind in the long run. Using (115), I can rewrite (118) as

$$b_F + d_F\left(\frac{R^*}{R^{D*}} - \frac{R^*}{\pi^*}\right) = \eta^* c^* R^* \frac{\Omega}{1+\Omega}$$

and can solve for b_F given a level of total foreign asset holdings relative to imports $\bar{d} = \frac{b_F + d_F + m_F}{c_F}$, which yields

$$b_F = \frac{\eta^* c^* R^* \frac{\Omega}{1+\Omega} + \left(\bar{d} c_F - \frac{\eta^* c^*}{1+\Omega}\right)\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right)}{1 + \frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}}.$$

Using (114) to solve for b_F yields

$$
\begin{aligned}
c_F = & B^{-1}\eta^* c^* R^* \frac{\Omega}{1+\Omega} - \frac{\eta^* c^*}{1+\Omega}\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) \\
& + B^{-1}\left(1 + \frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) R^{m*}\pi^* \frac{m_F}{\pi^*},
\end{aligned}
$$

where $B = \left[\left(R^{m*}\pi^* - \bar{d}\right)\left(\frac{R^*}{\pi^*} - \frac{R^*}{R^{D*}}\right) + R^{m*}\pi^*\right]$. Then, back out d_F by using holdings of foreign private debt by using $d_F = \bar{d}c_F - b_F - m_F$. Further, with (108) and (109), λ can be eliminated, so that consumption is given by

$$
c = q\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right) c_F.
$$

To obtain q, I use this in (117) to replace c, yielding

$$
n^\alpha = q^{\frac{1}{1-\eta}} \left\{ \left[\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right)\gamma^{-1}\right]^{\frac{1}{1-\eta}} c_F + \eta^* \Phi^{\frac{1}{1-\eta}} c^* \right\}. \tag{119}
$$

Further, I set $n = 0.33$ and use (107) to back out χ after the other steady state variables are determined. Thus, (119) can be used to solve for the real exchange rate,

$$
q = n^{\alpha(1-\eta)} \left\{ \left[\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right)\gamma^{-1}\right]^{\frac{1}{1-\eta}} c_F + \eta^* \Phi^{\frac{1}{1-\eta}} c^* \right\}^{\eta-1}.
$$

Thus, home consumption is given by

$$
c = q\left(1 + \frac{R^{D*}}{R^D}\frac{1-\eta}{\eta}\right) c_F.
$$

With this result at hand, the remaining variables can be backed out, yielding

$$
\begin{aligned}
& w = \tfrac{\varepsilon-1}{\varepsilon}\Phi^{\frac{1}{\eta-1}} q^{\frac{\eta}{\eta-1}}, && m_F = \tfrac{\eta^* c^*}{1+\Omega}, \\
& b_F = c_F R^{m*}\pi^* - m_F R^{m*}, && \lambda = \tfrac{c^{-\sigma}}{R^D}, \\
& \chi = \lambda w n^{-\omega}, && \psi_H = \lambda\left(R^D - 1\right), \\
& \psi_F = \lambda\left(R^{D*} - 1\right), && \mu = \lambda\left(\tfrac{R^{D*}}{R^*} - 1\right), \\
& tb = q\left(\eta^* c^* - c_F\right), && c_H = \left(\tfrac{c}{c_F^\eta \gamma}\right)^{\frac{1}{1-\eta}}, \\
& c_H^* = q^{\frac{1}{1-\eta}}\Phi^{\frac{1}{1-\eta}}\eta^* c^*.
\end{aligned}
$$

A.3.2 Foreign economy

I use that the utility function is given by (64). As shown in Reynard and Schabert (2009), steady state inflation is determined by the growth rate of short-term government bonds, Γ^*. The central bank is assumed to adjust its long-run inflation target to this value, $\pi^* = \Gamma^*$. The households' first order conditions imply that the steady state interest rate on private debt is given by $R^{D*} = \frac{\pi^*}{\beta^*}$. Further, using the first order conditions for money holdings and consumption yields

$$\mu^* = c^{*-\sigma}\left(\frac{1}{R^{m*}} - \frac{1}{R^{D*}}\right).$$

Thus, the open market constraint binds in steady state when policy sets $R^{m*} < R^{D*} = \frac{\pi^*}{\beta}$. Further, the multiplier on the cash in advance constraint is given by $\psi^* = R^{m*}\eta^* + \lambda^*(R^{m*} - 1)$ where $\lambda^* = \beta^* \frac{c^{*-\sigma}}{\pi^*}$ implies that $\psi^* = c^{*-\sigma}\left[1 - \frac{\beta^*}{\pi^*}\right]$. Thus, the cash in advance constraint binds whenever $\pi^* > \beta^*$, which is assumed to be fulfilled throughout the paper. Further, attention is restricted to a small neighborhood of the steady state, where the open market and cash in advance constraints bind. The steady state can be derived analytically from the remaining equilibrium conditions. Using the households' and firms' first order conditions (as well as the aggregate resource constraint $c^* = \frac{n^*}{s^*}$) gives

$$\begin{aligned}
&n^* = \left[\frac{\varepsilon-1}{\varepsilon}\frac{A^{*1-\sigma}\beta^*}{\chi^*\pi^*}\right]^{\frac{1}{\omega^*+\sigma^*}}, && w^* = A^* mc^*, \\
&c^* = \frac{n^*}{s^*}, && Z^{1*} = \frac{\varepsilon}{\varepsilon-1}\frac{c^{*-\sigma}y^*}{1-\phi\beta}mc^*, \\
&R^{D*} = \frac{\pi^*}{\beta^*}, && Z^{2*} = \frac{c^{*-\sigma}y_H^*}{1-\phi\beta}, \\
&R^* = R^{m*}, && Z^{1*}/Z^{2*} = 1 \Longrightarrow mc^* = \frac{\varepsilon-1}{\varepsilon}, \\
&s^* = \left(Z^{1*}/Z^{2*}\right)^{-\varepsilon} = 1.
\end{aligned}$$

Further, the cash-in-advance constraint and the households' holdings of money and bonds can be used to obtain the steady state values for m, b and b^T,

$$\begin{aligned}
m_F^* &= \frac{c^*}{1+\Omega}, \\
b_F^* &= R^{m*} m_F^* \pi^* \left(1 + \Omega - \pi^{*-1}\right), \\
b^{T*} &= \frac{\pi^*}{\Gamma^* - 1}\left[b_F^* + b_F - \Omega\left(m_F^* + m_F\right)\right].
\end{aligned}$$

Steady state under $\Gamma^* = 1$ Consider the case analyzed in section 4.1 where nominal bond growth is zero, $\Gamma^* = 1$. In this case, the foreign economy's equilibrium conditions are fundamentally affected. (101) changes to $b_{F,t}^* = m_{F,t}^{R*}$ and thus, the real stock of government bond holdings becomes irrelevant for the equilibrium allocation. Ignoring the influence of foreign asset holdings (as in section 4.1), I obtain identical conditions as above, except for the steady state holdings of government bonds. Household money holdings (96) require

$$b_F = R^{m*} m_F^* \pi^* \left(1 + \Omega - \pi^{*-1}\right),$$

while the evolution of households' bond holdings (101) requires $b_F^* = \Omega m_F^*$. A steady state exists only if both equations are satisfied, i.e. if

$$R^{m*} = \frac{\Omega}{\Omega\pi^* + \pi^* - 1}.$$

Thus, if the central bank targets zero inflation, $\pi^* = 1$, the long-run policy rate has to be zero as well. For $\pi^* = 1$ and $R^{m*} > 1$, the economy has no steady state. The reason is that the central bank acquires bonds every period in its open market operations when $R^m > 1$. Given a nominally constant amount of bonds, and no steady state inflation, households' real bond holdings then must decline.

A.4 Proof of Proposition 1

(60) implies that the decline in the liquidity premium is larger than the increase in the interest rate on foreign government bonds if the foreign debt rate falls below its steady state, $\hat{R}_t^{D*} < 0$. Consider the foreign economy under the assumptions in section 4.1, i.e. $u(c_t^*, n_t^*) = \log c_t^* - \chi n_t^*$, binding cash and open market constraints, flexible prices, constant nominal foreign government debt, $\Gamma^* = 1$, zero steady state inflation $\pi^* = 1$ as well as a policy rate governed by $R^{m*} = 1$ and $R_t^{m*} = R_{t-1}^{m*\rho^*} \exp\left(\varepsilon_t^R\right)$ and a negligible impact of home households' holdings of foreign government bonds on foreign households' holdings, $b_{F,t}^* = \Omega m_{F,t}^*$. The set of equilibrium conditions describing the foreign economy is then given by the linearized versions of (31) - (36), (41) with zero price dispersion $s_t^* = 1$, (43), binding open market and cash constraints (29) and (30), households' money holdings (50), labor demand $w_t^* = A^*$, the resource constraint $y_t^* = c_t^*$ and the policy rule $R_t^{m*} = R_{t-1}^{m*\rho^*} \exp\left(\varepsilon_t^R\right)$. Substituting out Lagrange multipliers in (31) - (36) yields the following system of linear equilibrium conditions

$$\hat{R}_t^* = E_t \hat{R}_{t+1}^{m*}, \tag{120}$$

$$\hat{R}_t^{D*} = E_t \hat{\pi}_{t+1}^*, \tag{121}$$

$$-E_t \hat{c}_{t+1}^* = E_t \hat{\pi}_{t+1}^*, \tag{122}$$

$$\hat{c}_t^* = \frac{m_F^*}{c^* \pi^*} \hat{m}_{F,t-1}^* + \frac{b_F^*}{c^* \pi^* R^{m*}} \left(\hat{b}_{F,t-1}^* - \hat{R}_t^{m*}\right) - \hat{\pi}_t^*, \tag{123}$$

$$\hat{m}_{F,t}^* = \hat{c}_t^*, \tag{124}$$

$$\hat{R}_t^{m*} = \rho^* R_{t-1}^{m*} + \varepsilon_t^R, \tag{125}$$

$$\hat{b}_{F,t}^* = \Omega \frac{m^*}{b_F^*} \hat{c}_t^*, \tag{126}$$

and conditions for the wage, production, injections and real government debt. Applying the expectations operator to (123), and using (124) yields,

$$E_t \hat{c}_{t+1}^* = \frac{m_F^*}{c^* \pi^*} \hat{c}_t^* + \frac{b_F^*}{c^* \pi^* R^{m*}} \left(\hat{b}_{F,t}^* - E_t \hat{R}_{t+1}^{m*}\right) - E_t \hat{\pi}_{t+1}^*.$$

Thus, (122) can be rewritten as

$$\hat{c}_t^* = -\pi^* \left(1 + \Omega - \pi^{*-1}\right) \left(\hat{b}_{F,t}^* - E_t \hat{R}_{t+1}^{m*}\right),$$

where I use the steady state relation $b_F^*/m_F^* = R^{m*}\pi^*\left(1+\Omega-\pi^{*-1}\right)$ derived in Appendix A.3.2. Replacing bond holdings by (126) yields

$$\begin{aligned}\hat{c}_t^* &= -\pi^*\left(1+\Omega-\pi^{*-1}\right)\left(\Omega\frac{m^*}{b_F^*}\hat{c}_t^* - E_t\hat{R}_{t+1}^{m*}\right)\\ &= \frac{\Omega}{1+\Omega}E_t\hat{R}_{t+1}^{m*}.\end{aligned}$$

The debt rate is given by $\hat{R}_t^{D*} = E_t\hat{\pi}_{t+1}^* = -E_t\hat{c}_{t+1}^*$, so that its solution reads

$$\hat{R}_t^{D*} = -\rho a_1\hat{R}_{t-1}^{m*} - a_1\varepsilon_t^R,$$

where $a_1 = \rho^2\frac{\Omega}{1+\Omega} > 0$. Thus, a positive foreign policy shock leads to a decrease in the private debt rate, which persists until the shock fades out. □