Thomas Sparla

Strategic Real Options
– with the German Electric Power Market in View

Dissertation

Wirtschafts- und Sozialwissenschaftliche Fakultät
der Universität Dortmund

Dortmund, August 2001

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Preface

This dissertation emerged during the past two years when I was a scholarship holder at the Dortmund Graduate School (”Graduiertenkolleg”). The Graduiertenkolleg is a research program on *Allocation Theory, Economic Policy and Collective Decisions* and is financed by the *Deutsche Forschungs-gemeinschaft* (DFG). The present thesis has been strongly influenced by frequent presentations in the Graduiertenkolleg’s workshop and numerous discussions with professors and fellow students of the Graduiertenkolleg. I am grateful to all who supported my research in this way, in particular to Professor Wolfgang Leininger, Ph.D., and Professor Dr. Walter Krämer who both supervised this dissertation. Moreover, I would like to thank Susanne Maidorn and Christian Bayer who helped to markedly improve the final version of this work. My future colleagues from RWE Trading, especially Michael Römmich, who provided a lot of practical knowledge and helpful suggestions should not remain unmentioned. I also thank the *Deutsche Forschungsgemeinschaft* for its financial support. Finally, I am indebted to my wife Nita and my little son Moritz for their all-out support and immense patience.

Thomas Sparla
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Chapter 1

Introduction

Real options theory has revolutionized the neoclassical way of viewing and evaluating investment projects. Pioneers of this approach—such as Bernanke (1983), Brennan and Schwartz (1985) and McDonald and Siegel (1986)—recognized the analogy between financial call options and investment projects and discovered the *option value of waiting* or *value of flexibility* that had been ignored by the orthodox theory of investment. Also, the impact of game theory and the theory of industrial organizations on the economists' understanding of investment decisions has probably not been less. The first explicit analyses of the strategic dimension of investment decisions and their effects on market power are due to Spence (1977) and Dixit (1980). However, the implicit knowledge about it perhaps dates back to von Stackelberg (1934). The main contribution of the latter theories is what we would like to call the *strategic value of investment*[1].

In the thesis at hand it is one of our main concerns to work out the effects of both values on the firms' timing of investment and closure in duopoly. For each model of capacity adjustment we first present the adjustment policy proposed by the standard real options theory. These solutions do not capture the strategic interaction between firms but they do account for the option value of waiting. Its impact on the firms' investment and closure decisions can easily be characterized by contrasting the corresponding adjustment

[1]While the intuition that is associated with the option value of waiting does not depend on whether positive investment or negative investment, i.e. disinvestment, is investigated, this does not apply to the strategic value of investment. Spencer and Brander (1992, p.1601) introduce the notion of the *strategic value of pre-commitment* which only refers to positive investment. Of course, closure and exit opportunities also have an inherent strategic value, but this value does not emerge from pre-commitment. For this reason we use the more general but ambiguous notion of *strategic value of investment* where investment should be understood in a wider sense.

policy with the timing rules of the neoclassical investment theory. Finally, by combining game-theoretic and real options methods we derive the strategically optimal capacity adjustment rules. It should be clear that a comparison of outcomes allows to determine the autonomous effect of the strategic value of investment on the behavior of firms.

1.1 The Real Options Approach

Over the last two decades the real options approach has evolved into a powerful tool for evaluating various kinds of business projects by highlighting three basic features of investment decisions. First, most investment projects are at least partially *irreversible*. Once a firm has invested in new capital equipment, the installation cost are completely sunk. Also the purchase cost cannot be fully retrieved even if the equipment is not firm-specific but industry-specific. The reason is that the equipment's value is almost equally low for all firms in a market that is exposed to an economic downturn. Second, the future flow of revenues that arises after investment is highly *uncertain*. Especially in multi-stage projects this might also apply to the sunk investment expenditure. Third, firms are assumed to have some leeway to choose the *timing* of investment. These three properties give rise to the analogy between a call option and an investment decision and generate the option value of waiting. Let us develop the basic intuition by resorting to a simple example of closure timing.

Example 1 *Suppose a utility has the opportunity or the right to shut down a 150 MW peak load plant that currently makes losses. The retirement cost equals DM 10 million. The utility cannot reinvest or recover its expenditure. Thus, its closure decision is completely irreversible. Uncertainty about future revenues is captured in the simplest possible manner. The flow of net revenues is exposed to a single shock in the next period that either permanently rises the periodwise net revenue* π_t *by DM one million or decreases it by the same amount. Each state occurs with probability* $1/2$. *The stream of net revenues* *is graphically depicted by*

$$
\begin{array}{ccccccc}
t=0 & & t=1 & & t=2 & & ... \\
 & \nearrow_{1/2} & \pi_1 = 0.0 & \longrightarrow & \pi_2 = 0.0 & \longrightarrow & ... \\
\pi_0 = -1.0 & & & & & & \\
 & \searrow^{1/2} & \pi_1 = -2.0 & \longrightarrow & \pi_2 = -2.0 & \longrightarrow & ...
\end{array}
$$

Assuming that plant closure is instantaneous and that the utility discounts future revenues at a rate of 10 per cent, the net present value (NPV) generated by the strategy 'shut down immediately (at $t = 0$*)' is DM* -10 *million which is greater than the expected NPV from maintaining production eternally,*

$$-1 + \sum_{t=1}^{\infty} \frac{\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot (-2)}{1.1^t} = -\sum_{t=0}^{\infty} \frac{1}{1.1^t} = -11.$$

Thus, the orthodox investment theory suggests 'immediate closure' to be the optimal investment policy. This rule ignores the value of owning the right to postpone capacity reduction and, thereby, keeping open the possibility of not closing the plant should revenues rise. Under uncertainty of revenues and with irreversibility of the disinvestment decision, this value of having some leeway to time the plant closure is exactly what has been called the option value of waiting. The question arises how the option value of waiting affects the optimal closure strategy. Suppose the utility plays the strategy 'wait and shut down if price goes down; continue production otherwise' instead of adhering to the orthodox rule. Then its NPV is given by

$$-1 + \frac{1}{2} \cdot \frac{-10}{1.1} + \frac{1}{2} \cdot \sum_{t=0}^{\infty} \frac{0}{1.1^t} \simeq -5.05 > -10.$$

Thus, waiting, resolving the ongoing uncertainty and conditioning the disinvestment decision on new information strictly dominates the orthodox NPV rule.

Though the example is rather rudimentary, it is sufficiently detailed to convey the crucial ideas underlying the theory of real options. It also reveals the analogy of the closure option to an American *put* option in finance. Note that the holder of the disinvestment option owns the right to "sell" the underlying asset—the plant—at an arbitrary exercise date. The option's strike price equals the retirement cost. In our example the option is "deep in the money".

In recent years the number of theoretical and empirical contributions to the real options literature has grown rapidly. Meanwhile, the approach exhibits a sufficient amount of flexibility to cover a wide variety of business problems. However, most researchers in this field restricted their attention to either single-firm settings with an exogenous price process or perfect competition[2] or monopolistic scenarios[3] (called standard real options theory models henceforth). Also, recent research efforts are mainly dedicated to the refinement of fully stochastic and, therefore, exogenous pricing models.[4] Papers dealing with the evaluation of real options in oligopoly are still relatively rare.

1.2 Introducing Strategic Interaction

A unique definition of the strategic value of investment can hardly be given, since it takes different forms with each specific economic context. For example, consider the well-known "Stackelberg leadership game". One exogenously determined firm has the opportunity to *pre-commit* to a certain capacity and, thereby, output level. Or it might wait and enter a simultaneous capacity game with its rival. Suppose we observe that the Stackelberg leader chooses the pre-commitment strategy in equilibrium. In this case the timing of investment is strategically relevant and the strategic value of investment must be strictly positive. Another example is a game of *entry deterrence* (see Spence, 1977, and Dixit, 1980) with an incumbent that has the opportunity to invest excessively in idle capacity to prevent potential rivals from entering the market. If the threat of using the excess

[2] As is shown by Leahy (1993), the investment timing in models of single price-taking firms is identical to that in fully competitive scenarios. The intuition behind this result is that perfect competition reduces not only the option value of waiting to zero, but at the same time the value of installed capital. This two effects exactly offset each other.

[3] For example, see McDonald and Siegel (1986) and Pindyck (1988) for investment options available to a single firm. Dixit (1989, 1991) analyzes chains of product market entry and exit options in, respectively, monopolistic and perfectly competitive settings.

[4] For a collection of recent contributions to the field of real options theory, see Brennan and Trigeorgis (1999).

capacity once the rival has entered is credible[5], then the strategic aspect of investment timing matters again. Finally, consider a game of investment timing with symmetric firms, i.e. both firms have the opportunity to choose their timing of capacity expansion. They may have the incentive to *preempt* their rival to attract additional market shares. In this *preemption game*, if firms do not invest jointly, then the (Stackelberg) leader arises endogenously and, once more, the strategic value of investment is strictly positive. General versions of this game have been proposed, for example, by Fudenberg and Tirole (1985) and, more recently, by Huisman and Kort (1999). The latter contribution is discussed in chapter 3.

As the examples reveal, there often exists a trade-off between the strategically "optimal" timing of investment and the timing that is proposed by the standard real options theory. The latter approach always suggests to delay investment for better information, while the strategic view of investment seems to involve an increased speed of investment. Among others, Spencer and Brander (1992), Sadanand and Sadanand (1996) and Dewit and Leahy (2001) investigate what determines the size of the trade-off in various game settings with endogenous investment timing. Vives (1989), however, makes the important general point that the trade-off does not always emerge in investment timing games. This also applies to the model by Huisman and Kort (1999). Depending on the size of the underlying parameters a delayed joint-investment scenario rather than a preemption-type equilibrium might prevail. In the (Pareto-optimal) joint-investment equilibrium strategic interaction does not increase but slow down the speed of investment. As is shown in chapter 3, this effect can be detected by comparing the equilibrium investment pattern of the competitors with the hypothetic behavior of a firm that fully takes the option-like nature of its investment decision into account. But it completely ignores strategic interaction with its rivals and its own impact on (inverse) demand. Thus, this firm is a *price taker*.

In games of *closure* timing that generally belong to the class of *wars of attrition*[6] one might expect that the option value of waiting and the strategic value of (dis)investment do not offset each other but accumulate. In analogy to the above example of investment

[5] Bulow *et al.* (1985) show that the credibility of the threat to use excess capacity hinges on the shape of the demand curve and on whether Bertrand or Cournot competition is anticipated.

[6] Maynard Smith (1974) proposed this type of game to investigate certain patterns of behavior in animal conflicts.

timing, consider two symmetric firms, each of which holds a single option to reduce capacity. Then firms may have the incentive to postpone disinvestment to prevent the rival from attracting additional market shares. Thus, intuition suggests that strategic interaction and the option-like nature of the disinvestment project *both* tend to delay the timing of closure. In chapter 2 we propose a closure timing game that is an elaborate version of this example. Equivalently to the model by Huisman and Kort (1999), it depends on the initial assumptions whether the basic intuition applies or not.

1.3 Strategic Real Options in Electricity Markets

The real options approach has extensively been used in the electric utility industry. Applications range from plant investment and retirement scenarios (e.g., EPRI, 1996) and the problem of determining optimal plant operating policies (e.g., Gardner and Zhuang, 2000) to environmental investment projects such as the installment of "srubbers"[7] (Herbelot, 1992) and models of transmission network investments (Martzoukos and Teplitz-Sembitzky, 1992).

Experts (e.g., Pilipović, 1998, p.2) agree that spot markets for electric power are highly illiquid as compared, for example, to financial markets, and that agents—especially on the supply side—have significant market power.[8] Nevertheless, research efforts in the field of real asset valuation are mainly dedicated to fully stochastic and, therefore, exogenous price models.[9] The more sophisticated the assumptions about the stochastic nature of the underlying price process, the more likely it is that one has to resort to numerical solutions of the option pricing model. This naturally results in a loss of intuition for the market-specific mechanisms. The research trend prevails, though the limited experience with market pricing in generation markets has confirmed the importance of horizontal

[7] Srubbers are devices that remove the SO_2 emissions of power generation units.

[8] Schuppe and Nolden (1999) provide a detailed but partially out-dated analysis of market power in the European electric power and gas markets. They find that in January 1997 the Herfindahl index shows high concentration in the electric utility industry in Austria, Denmark, England and Wales, France, Finland, the Netherlands, Spain, and Sweden. Only in Germany and Norway the results point to moderate concentration. The remaining countries are not investigated. However, due to a number of mergers the market structure in Germany has dramatically changed since 1997. Certainly, the market power of utilities has significantly increased.

[9] For a collection of recent papers on valuation methods for power generating assets, see Risk Books (publisher, 1999).

market power. For example, by applying the theory of supply function equilibria developed by Klemperer and Meyer (1989) to the U.K. Pool[10], Green (1994) and Green and Newbery (1992) find a substantial markup on marginal cost at peak times.[11] With the Californian electricity spot markets in view, Hogan (1997) makes the point that capacity constraints in electricity transmission networks give rise to strategic interaction and opens up the possibility to exercise horizontal market power.[12] All these contributions, however, are essentially static—they focus on modelling oligopoly pricing in the short run. Capacity is assumed to be fixed and investment is not an issue.

Papers dealing with the strategic dimension of investment in the electric utility industry are extremely rare. The only analyses that are known to us are Wei and Smeers (1996a, 1996b) and Smeers and Wei (1997). The authors use two-stage models à la Kreps and Scheinkmann (1983). On the first stage firms play a game of capacity expansion, while on the second stage they enter in capacity constrained competition in the output market. Though Bertrand competition is assumed to take place on the second stage, Kreps and Scheinkmann obtain an overall Cournot outcome. David and Denecker (1986) find that this result is not universally valid. For different demand rationing rules the overall Cournot scenario does not occur. Smeers and Wei point out that the physical nature of electricity (it cannot be stored), the time structure of demand (peak and base times) and the institutional arrangements in electricity markets (pool mechanism) give rise to a type of demand rationing that does not support Kreps and Scheinkmann's result. Their simulation studies suggest that firms can exert market power only if the Cournot competition paradigm on the second stage is assumed.

There are several drawbacks associated with the two-stage models applied by Smeers and Wei. First, in the capacity game on the first stage the Stackelberg leader is exogenously determined. In a general context Bergman (1998) shows that endogenizing the timing of capacity expansion leads to a variety of possible outcomes, however, not including the extreme overall Cournot scenario. Second, Smeers and Wei establish their

[10] In the U.K. Pool generation is dominated by the duopoly of National Power and PowerGen.

[11] More precisely, they detect a symmetric equilibrium with supply functions that are less steep than in the Cournot model but steeper than in the Bertrand model. In Bunn (eds., 1999) the assumptions required to derive a supply function equilibrium are criticized to be too restrictive. Instead a market simulation study of the U.K. Pool is presented that, however, leads to similar results.

[12] There exists a very recent paper by Mason and Weeds (2001) that models strategic network effects in a real options model. However, it is not related to electricity transmission grids.

results for a radial network and, thereby, ignore the market power that is induced by realistic models of transmission networks (Hogan, 1997). Finally, stochastic fluctuations in demand and the related option value of waiting are not taken into account.

We think that this overview points to a lack of theoretical and empirical evidence concerning the strategic value of investment in the electric utility industry and its impact on the evolution of market prices and quantities. Moreover, several discussions with energy managers and traders[13] lead us to the conclusion that there is not only a lack of explicit knowledge. In practice agents have not even become aware on an intuitive level that the right to choose the timing and the volume of investment opens up the possibility to exercise market power and to influence market pricing. In chapter 4 an empirical model of investment in the German power generation industry is developed. It is based on the assumption that during the sample period from 1977 to 1993 utilities took prices as being exogenously determined when evaluating investment and closure projects. The fact that practical knowledge about the strategic scope of investment and closure decisions is *still* missing justifies this assumption.

Our main motive for investigating "strategic real options" in the thesis at hand has been to shed light on some aspects of the interaction between the strategic dimension of timing capacity expansion and reduction and the option-like nature of these projects. With respect to the electric utility industry, we have particularly been inspired by the following "real world" closure scenario: In the German electricity sector the regional monopolies were disbanded in the course of its deregulation and liberalization in 1998.[14] Responding to the increased competition, each of the two major utilities, RWE and Eon, which are approximately of equal size and together account for about 61.77 per cent of the industry's total capacity, announced the closure of about 5000 MW of excess capacity in October 2000.[15] Moreover, the largest competitors, Veag, EnBW and Bewag, publicly denied similar closure intentions. Table 1.1 summarizes the major utilities' market shares measured in terms of power generation capacity. Note that the category

[13] I am grateful to Michael Römmich, Hoss Hauksson and Christophe Chassard from RWE Trading for spending a lot of their scarce time on answering my questions.

[14] Since the adoption of a European Parliament and European Council directive on the liberalization of the EU electricity markets in 1996, the process of deregulation of the electricity sector was completed in UK, Sweden, Finland and Germany. Many other EU countries will reach this aim soon.

[15] The underlying data stems from the "Handelsblatt", 2000/10/11, and from "Stromzahlen 2001" published by the Association of the Electricity Industry (VDEW).

Table 1.1: Capacity and Market Share of German Utilities

	RWE	Eon	Veag	EnBW	Steag	HEW	Bewag	Other Utilities
I.C.	24412	24553	9838	8420	3795	3758	2702	23702
A.C.	32500	30000	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
M.S., I.C.	24.13	24.27	9.72	8.32	3.75	3.71	2.67	23.43
M.S., A.C.	32.12	29.65	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Sources: Utilities, "Handelsblatt" (2000/10/11), VDEW (2001).
I.C. = installed capacity (in MW), A.C. = available capacity (in MW),
M.S. = market share (in per cent), n.a. = not available.

"installed capacity" does not include those power generation units that utilities have under control either by holding shares or by holding long-term power delivery contracts.

Three facts strongly support the view that the above closure timing problem should be dealt with in a duopoly model. First, RWE and Eon account for more than 3/5 of the industry's total capacity. Second, the largest competitors do not plan capacity reductions. Finally, the structure of the "residual supply side" is nearly atomistic. As we already mentioned, in chapter 2 we propose a general theoretical model of closure timing in duopoly of which the closure scenario "RWE versus Eon" would represent an almost ideal application. In chapter[16] 4 we come back to this example and investigate whether the strategic value of investment is empirically significant and should be taken into account by the rivals.

1.4 Related Literature

Among the small group of researchers who succeeded in building real options pricing models for *duopolistic* settings, the pioneering work of Smets (1991) should be mentioned. The paper examines the timing of irreversible entry in a product market that is subject to aggregate demand uncertainty. Smets derives the optimal entry triggers by fixing exogenously which firm invests first and becomes the Stackelberg leader. Huisman and Kort (1999) consider investment decisions of previously active firms. In contrast to Smets, they allow the leader's role to be assigned endogenously. Weeds (2000) analyzes investment in R&D activity. Though this requires a slightly different model set-up, the classes of equilibria induced by her model are directly comparable to those derived

[16] The content of this chapter is also included in a recent working paper by Sparla (2001c).

from models of market entry and capacity choice. Finally, Grenadier (1996) applies an investment options model to the real-estate market. He provides an explanation for the empirical puzzle of building booms in the face of declining property values. Moreover, Grenadier points out that the problem of finding optimal exercise policies of certain financial assets such as warrants and convertible securities involve equivalent strategic aspects as the corresponding problem in the case of real assets.

In the industrial-organizations literature there exist numerous papers on the optimal timing of entry or investment in duopoly. Fudenberg and Tirole (1985) might be the most prominent example. The timing games with a link to real options theory that has been listed above are basically closely related versions of Fudenberg and Tirole's model. The two-stage game by Kreps and Scheinkmann (1983) that we already discussed in the previous section establishes another approach to assess the strategic value of investment. Bergman (1998) combines the models by Fudenberg and Tirole on the one hand and Kreps and Scheinkmann on the other hand. All these contributions, however, completely neglect the option value of waiting.

While strategic entry and investment decisions have been studied in a real options framework, strategic closure and exit options has not. The closure timing game introduced in chapter[17] 2 should partially fill this gap. Analogously to the contributions on investment options, our model allows the market price (or, equivalently, inverse demand) to be driven by Cournot competition of firms on the one hand and by aggregate uncertainty of a form that is widely used in option pricing theory on the other hand. However, there is a distinguishing feature that limits the analogy. We already mentioned that closure timing games generally belong to the class of wars of attrition, while investment timing games are preemption games. Hendricks *et al.* (1988, p.663) characterize the war of attrition as a game in which "each of the two players must choose a time at which he plans to concede in the event that the other player has not already conceded. The return to conceding decreases with time, but, at any time, a player earns a higher return if the other concedes first."[18]

The existing papers on investment options in duopoly have two features in common.

[17] This chapter summarizes the results of two working papers on closure options. See Sparla (2001a, 2001b).

[18] We will see in chapter 2 that this definition is slightly too restrictive in our case. In particular the return to conceding of the first mover does not uniformly decrease with time.

First, as mentioned above, the underlying timing games have to be characterized as preemption games. The defining property of this class of timing games is that there exist times where moving first yields a higher expected payoff than moving second (Fudenberg and Tirole, 1991, section 4.5.3). Second, the specific pattern of strategic interaction arising in these models is driven by the widely used key assumption of *first-mover advantages*, that is, investing first yields a higher increase in the flow of net revenues *at the time of investment* than investing second. The notions might suggest that preemption games require first-mover advantages, but this conjecture turns out to be wrong. Becoming the leader can still result in a higher payoff than waiting until the opponent has invested if firms are involved in an investment timing game with second-mover advantages[19]. However, this complementary assumption (second-mover advantages) that induces a different kind of strategic interaction has not been analyzed so far.[20]

In the special case of an isoelastic inverse demand function, first-mover advantages can be shown to correspond to a high quantity (low price) elasticity of inverse demand (of demand), while a low quantity (high price) elasticity implies second-mover advantages.[21] One can easily think of a number of cases where demand becomes more elastic in the long run. Consider, for example, the manufacturing sector's demand for electricity. In the short run externally purchased electric power is a complementary input factor and firms' demand is inelastic.[22] But in the long run firms may decide to build up there own power generation capacities. Thus, capital and fuel are long-term substitutes for external electricity. This probably points to second-mover advantages in games that model long term decisions such as investment in or closure of plants. For this reason we analyze both cases, first-mover as well as second-mover advantages, in chapter 2.

In contrast to the literature on real options, the process of firm exit out of a duopolistic

[19]Second-mover advantages require models with firms that are active ex ante to investment. In models of market entry with previously idle firms the revenue functions always exhibit the property of first-mover advantages.

[20]In a recently published paper by Hoppe (2000) with the title "Second-mover advantages in the strategic adoption of new technology under uncertainty" the notion of second-mover advantages is used to describe a special type of equilibrium, not an exogenous assumption. A closer look at p.319 reveals that Hoppe assumes an ordering of revenue flows that implies first-mover advantages in a very similar sense as in Weeds (2000), Huisman and Kort (1999) and the present paper. However, Hoppe's contribution is not related to real options theory.

[21]This property of isoelastic inverse demand functions is shown in chapter 4.

[22]Borenstein *et al.* (1996) indeed find low demand elasticity levels in short-term electricity markets in California.

market has been studied explicitly in the industrial-organizations literature. Ghemawat and Nalebuff (1985) and Fudenberg and Tirole (1986), for example, analyze continuous-time models of asymmetric duopoly. Both articles, however, deal with an evolution of prices that is driven by exogenous *deterministic* "shocks". They exclude aggregate price or demand uncertainty. The model by Fine and Li (1989) is closer to our approach. It considers a fairly general type of demand uncertainty, i.e. demand follows a Markov process. Nevertheless, there are some distinctive features. First, Fine and Li use a discrete-time model that makes it difficult to relate their results to most of the option-pricing literature. Second, in contrast to the real options approach, they need to impose exogenously that demand declines over time in the sense of first-order stochastic dominance to derive their existence results. Finally, they analyze the extreme case of firm exit, while we are concerned with disinvestment decisions.[23]

Last but not least, the remarkable paper by Baldursson (1998) should be mentioned, though it is not directly related to our approach. He is concerned with *incremental investment*, where output and profit flow are available all the time as a function of the installed capital stock. And the capital stock can be altered gradually rather than in large discrete units. The aim is to characterize the firms' optimal investment policies that are rates of capacity expansion in this context. Regarding his solution concept Baldursson draws from Slade (1994) who has shown that the problem of finding oligopoly Nash equilibria is equivalent to a "fictitious social-planner problem". For the special case of a linear demand function, Baldursson succeeds in deriving analytical expressions for the optimal investment and disinvestment triggers of firms in a n-firm oligopoly.

[23] As in entry models with previously idle firms second-mover advantages never occur in models of complete firm exit.

Chapter 2

Closure Options and Wars of Attrition

Strategic closure options in oligopoly has not been analyzed so far, though potential applications are not rare in practice. We already mentioned the "RWE versus Eon" closure scenario in the German electric utility industry. The envisaged termination of long-term supply contracts by automobile manufacturers competing in the same market segment (e.g., "DaimlerChrysler versus BMW") might serve as another example. Finally, one can imagine a wide variety of non-economic applications such as the decision to initiate divorce proceedings (e.g., "Kramer versus Kramer").

2.1 Model Set-Up

We analyze a duopolistic market in continuous time t, $t \in [0, \infty)$. Full capital utilization is assumed, i.e. the two risk neutral[1] firms currently produce at the capacity limit (maximal output flow), $q_i = \overline{q}$, $i \in \{1, 2\}$. The competitors contemplate partial closure of production facilities which irreversibly reduces q_i to the lower capacity (and output flow) level[2] $\underline{q}$. In view of the very long-term horizon of the disinvestment decision, the full-capital-utilization condition does not seem to be too restrictive. Instead of full capital

[1] This seemingly restrictive assumption can easily be relaxed by adjusting the drift term of the underlying stochastic process (see below) to capture a risk premium.

[2] The reduction in capacity $\overline{q} - \underline{q}$ should be considered as being equivalent to shutting down a single discrete production unit of a large system.

utilization one could equivalently assume a constant output-to-capital ratio that might be interpreted as the long-term average utilization rate or as the utilization rate that is most efficient from the side of the production technology.[3]

A firm's only decision (action) in this game is to choose the threshold of current revenue flows, at which the single closure option should optimally be exercised. Giving up capacity is not costless. In order to shut down a factory, firm i incurs the lump-sum cost E. This can be interpreted as severance payments to workers or as cost of demolishing the factory and restoring the site. In some instances E might be negative, e.g., when the scrap value exceeds the cost of closure. Setting the variable cost of production for firm i constant at C_i, we concentrate on output price uncertainty. Suppose that the inverse industry demand function exhibits the specific form

$$P = Y \cdot D\left(Q\right), \tag{2.1}$$

where Q represents the aggregate output, $q_1 + q_2$. The functional specification of the deterministic part $D(Q)$ implies that the firms' products are perfect substitutes.[4] The industry-wide demand shock Y is supposed to follow a geometric Brownian motion[5],

$$dY = \alpha Y dt + \sigma Y dB. \tag{2.2}$$

Additionally, some assumptions about firm i's gross revenue function are required. For ease of notation we define the deterministic part of i's revenues as

$$R\left(q_i, q_j\right) = D\left(q_i + q_j\right) q_i,$$

for $i, j \in \{1, 2\}$, $i \neq j$. Then, let us assume that the market is characterized by strategic

[3]For example, power generation units usually have a maximum degree of efficiency between 50 and 65 per cent in terms of fuel input units per generated MWh.

[4]As long as the conditions on revenues (see below) are satisfied, the assumption of *perfect* substitutibility is not needed.

[5]More precisely, let $(\Omega, \mathfrak{F}, P)$ be a filtered probability space, i.e. a family $\mathbb{F} = \{\mathfrak{F}_t, t \geq 0\}$ of σ-algebras on Ω such that $\mathfrak{F}_t \subseteq \mathfrak{F}$ for all $t \geq 0$ and $\mathfrak{F}_s \subseteq \mathfrak{F}_t$ if $s \leq t$. Further let B denote a standard Brownian motion that is adapted to the space $(\Omega, \mathbb{F}, P)$. Then $Y = \{Y(t, \omega) = Y_0 \cdot \exp[(\alpha - (\sigma^2/2))t + \sigma B(t, \omega)], t \geq 0$ and $\omega \in \Omega\}$ with $Y_0 > 0$ is a geometric Brownian motion.

substitutability of the firms' capacity decisions, i.e.

$$R\left(\overline{q},\underline{q}\right) > R\left(\overline{q},\overline{q}\right) \text{ and } R\left(\underline{q},\underline{q}\right) > R\left(\underline{q},\overline{q}\right).$$

Further, firms cannot raise gross revenues by reducing their capacity ceteris paribus,

$$R\left(\overline{q},\underline{q}\right) > R\left(\underline{q},\underline{q}\right) \text{ and } R\left(\overline{q},\overline{q}\right) > R\left(\underline{q},\overline{q}\right).$$

If the inequalities were reversed, firms would prefer to shut down production facilities immediately, independent of what the rival does. This scenario may occur, but its solution is trivial and uninteresting, since it does not exhibit any scope for strategic interaction. Another rather technically motivated assumption is added,

$$R\left(\overline{q},\overline{q}\right) > R\left(\underline{q},\underline{q}\right).$$

In terms of the inverse demand function, it says that the percentage increase in prices due to simultaneous plant closure is required to be smaller than the percentage change in capacity from $\underline{q}$ to $\overline{q}$, $D(2\underline{q})/D(2\overline{q}) < \overline{q}/\underline{q}$. Intuitively, the price reaction to a jump in quantity should be rather sluggish. Note that this condition is *not* needed to derive most of the results in this chapter.[6] In particular, it is not required to derive the equilibrium behavior of firms. Moreover, the analysis and, especially, the proofs would simplify if the inequality sign were reversed and the complementary case were considered. Nevertheless, we impose the assumption to ensure the comparability of our approach with the investment option models by Huisman and Kort (1999) (see chapter 3) and Weeds (2000). There the condition has a crucial impact on the set of equilibria.[7] Together with the former assumptions one obtains the following ordering of revenues

$$R\left(\overline{q},\underline{q}\right) > R\left(\overline{q},\overline{q}\right) > R\left(\underline{q},\underline{q}\right) > R\left(\underline{q},\overline{q}\right). \tag{2.3}$$

[6] The only exception is the optimal investment timing of a monopolist in the case of second-mover advantages (see section 2.2). Note that the condition just guarantees that the payoff arising from simultaneous closure has a maximum in the time of simultaneous closure. If the inequality sign were reversed, then this payoff function would be uniformly decreasing for all times of joint closure.

[7] In Huisman and Kort's game of investment timing the set of equilibria becomes unbounded and a (well-defined) Pareto-optimal joint-investment equilibrium does no longer exist.

Finally, we have to distinguish the case of "first-mover advantages", i.e.

$$R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)>R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right), \tag{2.4}$$

from the complementary case of "second-mover advantages", i.e.

$$R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)<R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right). \tag{2.5}$$

Both assumptions and their implications for the strategic interaction of firms will be discussed later in this chapter. Nevertheless, let us consider the economic content of these inequalities briefly. If inequality (2.4) is satisfied, then the first mover which is the firm that reduces capacity *before* its rival is exposed to a drop in gross revenues when exercising his closure option that is less severe than the decrease in revenues of the second mover.[8] In the context of a model of complete firm exit we would have $R(\underline{q},\underline{q})=R(\underline{q},\overline{q})=0$. In this case condition (2.5) implies that $R(\overline{q},\underline{q})<R(\overline{q},\overline{q})$ contradicting the assumption about strategic substitutability. Thus, second-mover advantages cannot occur in an exit model.

The only source of uncertainty in our model stems from (future) aggregate demand, so that each firm has common knowledge about the current state of the world and the payoff function of its rival. This setting constitutes a timing game in continuous time under *imperfect*, but complete information. Pitchik (1981), for example, introduces an adequate formalization of strategy spaces and payoffs for timing games in continuous time under *perfect* information. He describes a player's mixed strategy as a function $G_i^t(s)$. It denotes the cumulative probability that player i has moved by time $s\geq t$ conditionally on no player moving before t. Additionally, an intertemporal consistency condition is required to be satisfied for each family of functions G_i^t. We will impose such a condition in section 2.3. Notice that this formalization of strategies allows to test for subgame perfection of any prevailing Nash equilibrium if one interprets t as the root of a subgame.

[8] In the special case of a hyperbolic inverse demand function, $D(q_i+q_j)=a/(q_i+q_j)^b$, inequality (2.4) (inequality (2.5)) is valid, if D is elastic (inelastic) with respect to aggregate production, i.e. $b>1\,(b\in(0,1))$. A linear inverse demand function, $D(q_i+q_j)=a-b(q_i+q_j)$, guarantees that strategic interaction among firms is driven by first-mover advantages $\forall a,b>0$.

In section 2.3 we also show that the closure timing game proposed in this chapter can be characterized as a *war of attrition*. This implies that we do not run into the difficulties that are bound up with the representation of continuous-time analogues of the equilibria of discrete-time *preemption games*. In particular, we do not need to resort to the extended strategy spaces as suggested by Fudenberg and Tirole (1985). In chapter 3 it will be motivated that the corresponding investment timing game is a *preemption game* and, therefore, requires two-dimensional strategy spaces.

In our model players act in an imperfect information world due to the uncertainty about future revenues. This requires some modifications of the strategy concept. In contrast to Pitchik, the simple strategy $G_i^{\mathsf{T}}(t,\omega)$ should not be interpreted as an ordinary function, but as a stochastic process, e.g., if the event $\omega \in \Omega$ has realized and $Y_{\mathsf{T}} > 0$ denotes a fixed threshold, then $G_i^{\mathsf{T}}(t,\omega)$ gives the probability of firm i having moved (having adopted the leader's role) by time t in the (sub)game that starts at $\mathsf{T}(\omega) = \inf(t \geq 0 \,|Y(t,\omega) \leq Y_{\mathsf{T}})$. Notice that $G_i^{\mathsf{T}}(t,\omega) \equiv 0$ for $t < \mathsf{T}(\omega)$. In Figure 2-1 an example of the step function $G_i^{\mathsf{T}}(t,\omega)$ is given. It can be seen that $G_i^{\mathsf{T}}(t,\omega)$ jumps discretely, when $Y(t)$ first hits Y_{T_1} and Y_{T_2}, respectively. For the moment, we have reached a sufficient level of description of strategy spaces. We will come back to this point in section 2.3 that includes more formal definitions. The modifications required with respect to the firms' payoff functions are also discussed in section 2.3.

Remark 1 *Throughout the paper we use $Y(t)$, $t \in [0,\infty)$, to denote the element of the stochastic process Y at t, but Y_{T}, Y_T, or $Y_{\mathcal{T}}$ to be a fixed threshold. In the former case t simply represents a time index, while in the latter case the capital letters T, T, and $\mathcal{T}$ (occasionally used with subindices) should be interpreted as first passage or stopping times, e.g. the stochastic time that is required for the process Y to reach Y_T first given that $Y(0) = Y_0 > Y_T$. Formally, we have $T = \inf(t \geq 0 \,|Y(t) \leq Y_T)$.*

Remark 2 *Throughout the rest of the paper we use either the fixed threshold Y_{T} or the stochastic stopping time T to denote the root of the subgame that starts when the process Y first reaches the threshold Y_{T}.*

Since each firm possesses only a single option, there are in principle four outcomes of strategic interaction that can arise. In the first case firm 1 reduces capacity before firm

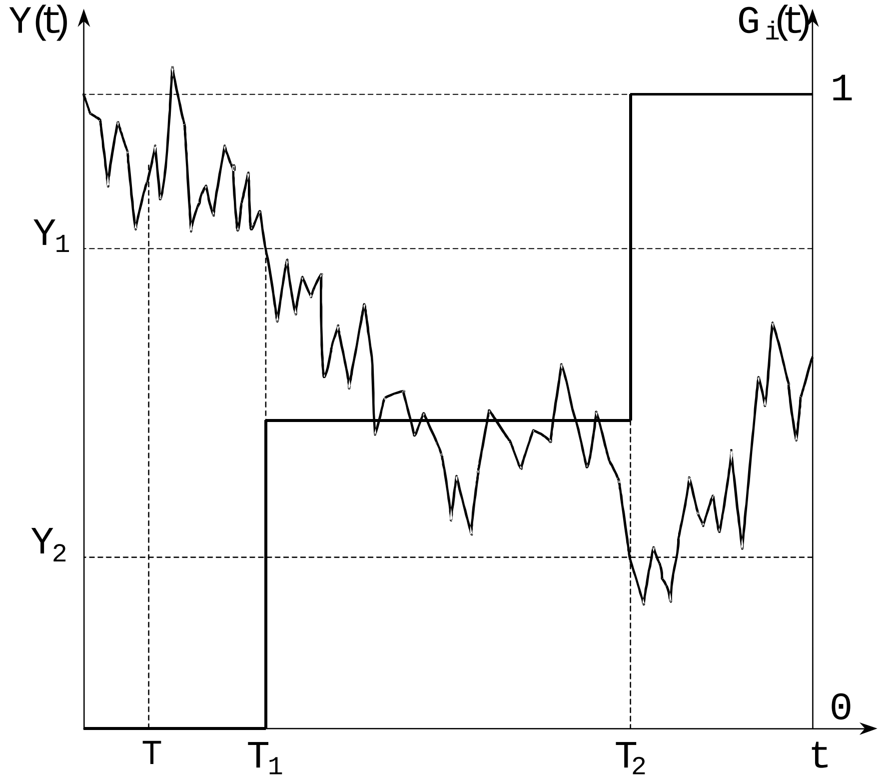

Figure 2-1: Strategies as Stochastic Processes

2 and, thereby, becomes the leader. Then firm 2 follows. The second case is equivalent to the first one, but with interchanged roles—firm 2 becomes the leader and firm 1 the follower. In the third case both firms reduce capacity simultaneously. Finally, in the fourth case neither firm 1 nor firm 2 disinvest. We immediately exclude the last case from our analysis, since both firms will close production facilities, if market conditions become sufficiently bad.[9]

In order to determine which of the remaining outcomes constitutes a (subgame perfect) equilibrium, we analyze the players' potential roles, i.e. leader, follower, and simultaneous mover. Focusing on roles rather than players allows us to calculate the corresponding payoffs and optimal closure thresholds. This is done in section 2.2. In section 2.3 we give a proper definition of players' strategies and combine strategies with the

[9]If market conditions become too unfavorable, it is always a dominant strategy to reduce capacity. Thus a firm will never find it optimal to keep the high capacity level for all $Y \in \mathbb{R}^+$.

roles' payoff functions. The expected payoffs of firms are obtained as functions of their strategy profiles. Then, in section 2.4, the equilibrium assignments of roles to players are derived, i.e. we describe which player prefers to play which role in any subgame perfect equilibrium of the timing game.

2.2 Potential Roles

We start off with the analysis of the follower's role. This is done in a rather extensive way to introduce the basic methodology of real options theory.

2.2.1 Follower

Let us suppose that one firm (the leader) has already reduced its capacity from $\overline{q}$ to $\underline{q}$. In this situation, the follower does not face competition for market shares, since the leader is not allowed to move more than once and to react to the follower's action. Thus, firms do not interact strategically anymore, and the follower can time his disinvestment as if he was the only firm in the market. He remains active as long as Y stays above the optimal closure threshold Y_F which we still have to determine. At Y_F the return on 'reducing capacity immediately' equals the (expected) return on 'sticking to initial production capacity forever'.

If the follower, who discounts future revenues at the constant rate ρ, produced $\overline{q}$ eternally, then the expected net present value of his firm would equal[10]

$$\begin{aligned}\overline{\Phi}\left(Y_{\mathsf{T}}\right) &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)D\left(\overline{q}+\underline{q}\right)-C_F\right]\overline{q}dt\right] \\ &= R\left(\overline{q},\underline{q}\right)\cdot g\left(Y_{\mathsf{T}}\right)-\frac{C_F\overline{q}}{\rho},\end{aligned}$$

where Y_{T} denotes the root of the subgame, i.e. the level of the stochastic process at T. $\mathcal{E}_{\mathsf{T}}$ represents the conditional expectations operator contingent on Y having first reached

[10] Note that $D(Q)$ need not be introduced as a functional argument of $\overline{\Phi}$, since it depends on the actual position of Y whether $D(Q) = D(2\overline{q})$ or $D(Q) = D(\overline{q}+\underline{q})$ or $D(Q) = D(2\underline{q})$.

Y_{T}. The conditional expectation

$$g\left(Y_{\mathsf{T}}\right)=\mathcal{E}_{\mathsf{T}}\left[\int\limits_{\mathsf{T}}^{\infty} e^{-\rho(t-\mathsf{T})} Y\left(t\right) dt\right] \tag{2.6}$$

is written as a function of the root of the subgame. We show in Lemma 6 in Appendix A.1 how to calculate $g(Y_{\mathsf{T}})$. The term $C_F\overline{q}/\rho$ gives the present value of current and future variable cost. The upper bar on Φ indicates that the value of the follower's firm is computed, when he produces at the high capacity limit $\overline{q}$. One yields

$$\overline{\Phi}\left(Y_{\mathsf{T}}\right)=\frac{Y_{\mathsf{T}}R\left(\overline{q},\underline{q}\right)}{\delta}-\frac{C_F\overline{q}}{\rho}.$$

We refer to $\overline{\Phi}$ as the follower's "fundamental value" that arises from eternal production at $\overline{q}$. It is linear in the starting threshold Y_{T}. Note that without any prospect of a positive revenue flow, the value of keeping up production at level $\overline{q}$ becomes negative, but nevertheless remains bounded. Similarly, let us define $\underline{\Phi}$ to be the follower's "fundamental value" *after* capacity reduction, when the follower produces eternally at $\underline{q}$. Obviously it holds that

$$\underline{\Phi}\left(Y_{\mathsf{T}}\right)=\frac{Y_{\mathsf{T}}R\left(\underline{q},\underline{q}\right)}{\delta}-\frac{C_F\underline{q}}{\rho}.$$

We mentioned above that neither firm is forced to produce forever at $\overline{q}$. If market conditions become too unfavorable, the follower may choose to partially abandon capacity and to shut down production facilities, thereby incurring the lump-sum cost E. Since $C_F(\overline{q}-\underline{q})/\rho$ is the maximum loss which the follower can avoid by exercising the closure option, we need $E<C_F(\overline{q}-\underline{q})/\rho$. Otherwise the follower will never choose to reduce capacity. Holding this closure option implies that the value of the follower's firm (with low or high capacity) exhibits a lower bound at $-E$. The question arises, at which point in time the closure option is optimally exercised. The net-present-value rule (NPV) suggests to do so at Y_F^{NPV}, where

$$\overline{\Phi}\left(Y_F^{NPV}\right)=\underline{\Phi}\left(Y_F^{NPV}\right)-E.$$

Among others, Dixit and Pindyck (1994) have shown that the exercise rule Y_F^{NPV} is not an adequate threshold if the follower's decision is made under uncertainty about future market conditions. It ignores the option value of waiting. For Y_{T} not too large, where the closure option is far out of the money, the strategy to wait and refrain from exercising the option has a positive value, since future uncertainty partly resolves as time goes by. Especially, even if demand is unfavorably low, the follower might refrain from partial capacity closure at Y_F^{NPV} if he assigns a high probability to the event that demand will recover again. If the follower shut down at the NPV threshold and, thereby, disregarded the inherent (option) value of the chance to experience future market conditions, he would give up this chance too early.

Taking the additional value from holding the option into account, let us define the value of the follower's firm, equipped with a closure option, as

$$F\left(Y_{\mathsf{T}}\right)=\max\left\{\underline{\Phi}\left(Y_{\mathsf{T}}\right)-E,\widetilde{\Phi}\left(Y_{\mathsf{T}}\right)\right\},$$

where $\widetilde{\Phi}$ is the follower's fundamental value or the expected net revenues from keeping up production at $\overline{q}$ *plus* the value that arises from still holding the option, i.e. $\widetilde{\Phi}\geq\overline{\Phi}$ for all Y_{T}. Notice that $\widetilde{\Phi}$ and F are not yet known. Let us assume for the moment that the optimal exercise threshold Y_F has already been determined. Then an explicit expression for $\widetilde{\Phi}$ and F can be obtained. If $Y_{\mathsf{T}}>Y_F$, then the follower still holds his option and produces at $\overline{q}$. Thus the expected value of his firm equals

$$F\left(Y_{\mathsf{T}}\right)=\widetilde{\Phi}\left(Y_{\mathsf{T}}\right)=\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_F}e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\underline{q}\right)-C_F\overline{q}\right]dt\right]$$
$$+\mathcal{E}_{\mathsf{T}}\left[\int_{T_F}^{\infty}e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\underline{q},\underline{q}\right)-C_F\underline{q}\right]dt\right]-\mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_F-\mathsf{T})}E\right].$$

After rearranging we get

$$F\left(Y_{\mathsf{T}}\right)=\widetilde{\Phi}\left(Y_{\mathsf{T}}\right)=\left(R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)\cdot h\left(Y_{\mathsf{T}}\right)|_{T=T_F}$$
$$+R\left(\underline{q},\underline{q}\right)\cdot g\left(Y_{\mathsf{T}}\right)-\frac{C_F\overline{q}}{\rho}+\left(\frac{C_F\left(\overline{q}-\underline{q}\right)}{\rho}-E\right)\cdot f\left(Y_{\mathsf{T}}\right)|_{T=T_F},\qquad(2.7)$$

where $g(Y_\mathsf{T})$ has already been defined by equation (2.6) and

$$f\left(Y_\mathsf{T}\right) = \mathcal{E}_\mathsf{T}\left[e^{-\rho(T-\mathsf{T})}\right], \tag{2.8}$$

$$h\left(Y_\mathsf{T}\right) = \mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^{T} e^{-\rho(t-\mathsf{T})} Y\left(t\right) dt\right]. \tag{2.9}$$

The expectations $f\left(Y_\mathsf{T}\right)$ and $h\left(Y_\mathsf{T}\right)$ are also calculated in Lemma 6 in Appendix A.1. The resulting expression for the value of the follower's firm is given in Lemma 1 below. If $Y_\mathsf{T} \leq Y_F$, then it is known that the follower already exercised its closure option and eternally produces at the lower capacity limit $\underline{q}$. Hence, we obtain

$$F\left(Y_\mathsf{T}\right) = \underline{\Phi}\left(Y_\mathsf{T}\right) - E \tag{2.10}$$

in this case. Again, see Lemma 1 for the resulting expression. It can easily be verified that F satisfies the so-called "value-matching condition" (Dixit and Pindyck, 1994, p.130ff.),

$$\widetilde{\Phi}\left(Y_F\right) = \underline{\Phi}\left(Y_F\right) - E,$$

i.e. F is a continuous function at Y_F. Intuitively, the condition says that the follower equipped with a closure option can principally choose among two alternatives. On the one hand, he can decide to keep on waiting and producing at the original capacity level $\overline{q}$. On the other hand, the follower can exercise his option, thereby, incurring the lump-sum cost E and producing at the low capacity level $\underline{q}$. At the optimal exercise threshold the follower is just indifferent between these alternatives and, therefore, the payoffs arising from the corresponding choices should be equal at Y_F.

It remains to determine the optimal exercise threshold Y_F. This can be accomplished by maximizing the follower's payoff from still holding the option, $\widetilde{\Phi}$, with respect to Y_F. The first-order condition, $\partial\widetilde{\Phi}/\partial Y_F = 0$, is given by

$$\left(\frac{Y_F}{Y_\mathsf{T}}\right)^{-\beta}\left(-\beta\frac{\left(C_F/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{Y_F} - \left(1-\beta\right)\frac{R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\right) = 0 \tag{2.11}$$

If $Y_F > 0$, then the equation can explicitly be solved for Y_F. The solution is stated in Lemma 1. By using this solution, it can be shown that F satisfies the so-called "smooth-pasting condition" (Dixit and Pindyck, 1994, p.130ff.),

$$\widetilde{\Phi}'(Y_F) = \underline{\Phi}'(Y_F),$$

i.e. F is not only continuous at Y_F, but also smooth. Let us summarize:

Lemma 1 *The expected value of the follower's firm conditionally on Y having first reached Y_{T} is*

$$F(Y_{\mathsf{T}}) = \begin{cases} \frac{Y_{\mathsf{T}} R(\underline{q},\underline{q})}{\delta} - \frac{C_F \underline{q}}{\rho} - E & \text{for } 0 \leq Y_{\mathsf{T}} \leq Y_F, \\ \frac{Y_{\mathsf{T}} R(\overline{q},\underline{q})}{\delta} - \frac{C_F \overline{q}}{\rho} & \\ + \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta} \left(\frac{C_F(\overline{q}-\underline{q})}{\rho} - E - \frac{Y_F\left(R(\overline{q},\underline{q}) - R(\underline{q},\underline{q})\right)}{\delta}\right) & \text{for } Y_{\mathsf{T}} > Y_F, \end{cases} \tag{2.12}$$

where the optimal exercise rule of the follower's closure option is given by the unique fixed threshold

$$Y_F = -\frac{\delta\beta}{1-\beta} \cdot \frac{(C_F/\rho)\left(\overline{q} - \underline{q}\right) - E}{R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)}, \tag{2.13}$$

and

$$\beta = \begin{cases} \beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} < 0 & \text{for } \alpha < \frac{\sigma^2}{2}, \\ \widetilde{\beta}_2 = \beta_2 + 1 - (2\alpha/\sigma^2) < 0 & \text{for } \alpha \geq \frac{\sigma^2}{2}. \end{cases} \tag{2.14}$$

Proof. Equation (2.12) is derived by plugging the formulas for $f(Y_{\mathsf{T}})$, $g(Y_{\mathsf{T}})$, and $h(Y_{\mathsf{T}})$ computed in Lemma 6 in Appendix A.1 into equations (2.7) and (2.10). Y_F as given by equation (2.13) is determined by rearranging equation (2.11). For $\beta < -1$, there exists another solution at the boundary $Y_F = 0$. However, the second-order condition reveals that only equation (2.13) maximizes $\widetilde{\Phi}$, while the function exhibits a saddle point at $Y_F = 0$, $\beta < -1$. ■

Since the "markdown factor" $-\beta/(1-\beta)$ in equation (2.13) is smaller than one, but positive, the closure threshold that is suggested by the net-present-value rule, $Y_F^{NPV}[R(\overline{q},\underline{q}) - R(\underline{q},\underline{q})] = \delta[(C_F(\overline{q}-\underline{q})/\rho) - E]$ is set too high ($Y_F^{NPV} > Y_F$), i.e. following the NPV rule the follower would reduce capacity too early. This result confirms our earlier intuitive considerations. Moreover, it is interesting to see that, if Y_F is smaller than the leader's closure threshold, then the follower's timing decision is not affected by the leader's behavior in the sense that the value of the follower's firm does not depend on the leader's exercising rule.

Another important result is hidden in equation (2.14). It reveals that the expected value of a firm—here: the follower—equipped with a disinvestment option may be overestimated[11] if one directly translates the standard results that have been obtained for investment option models (see chapter 3) into our context. There, it is the positive root

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}}$$

rather than the negative root β_2 of the so-called "fundamental quadratic equation"[12] that affects the roles' payoffs and the corresponding optimal thresholds. One might guess that the formula for the disinvestment case can easily be obtained by substituting β_2 for β_1 in the investment-option formula. However, this assumption is not true in general. The intuition is that the market price and, therefore, the firms' revenues might become infinitely large if the drift α exceeds $\sigma^2/2$. In this case the value of holding a closure option goes to zero, while the value of holding an investment option increases without bound. In the reverse case, where market conditions deteriorate and the price approaches zero, the value of holding an investment option goes to zero. The value of the closure option, however, does not become infinite, but converges to a bounded value. It is this asymmetry that drives our result.

For later purposes we simplify $F(Y_\mathsf{T})$ for $Y_\mathsf{T} > Y_F$ by substituting $-[(1-\beta)/\delta\beta] \cdot Y_F[R(\overline{q},\underline{q}) - R(\underline{q},\underline{q})]$ for $[(C_F(\overline{q}-\underline{q})/\rho) - E]$. One yields

$$\begin{aligned} F\left(Y_\mathsf{T}\right) &= \frac{Y_\mathsf{T} R\left(\overline{q},\underline{q}\right)}{\delta} - \frac{C_F \overline{q}}{\rho} \\ &+ \left(\frac{Y_F}{Y_\mathsf{T}}\right)^{-\beta} \frac{Y_F\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)\right)}{-\delta\beta} \quad \text{for } Y_\mathsf{T} > Y_F. \end{aligned} \tag{2.15}$$

[11] Note that $(Y_F/Y_\mathsf{T})^{-\beta_2} > (Y_F/Y_\mathsf{T})^{-\widetilde{\beta}_2}$, if $Y_\mathsf{T} > Y_F$.

[12] The "fundamental quadratic equation" is introduced briefly in Appendix A.1. See also Dixit and Pindyck (1994, section 5.2).

2.2.2 Leader

By definition of the leader's role, it is implicitly assumed that this "role" has already exercised its closure option at some unknown threshold $\widetilde{Y}_L$. So, in contrast to the follower's threshold Y_F, $\widetilde{Y}_L$ cannot be derived from the payoff function of the corresponding role. Actually, it is the equilibrium strategy profile that determines if and when a particular firm chooses to become the leader. Out of equilibrium $\widetilde{Y}_L$ remains unknown.[13] Nevertheless there is something that should be said about $\widetilde{Y}_L$: If Y_T is the root of the (sub)game and $\widetilde{Y}_F$ denotes the *actual* closure threshold of the follower—to be distinguished from the preferred closure threshold Y_F in equation (2.13)—, then it holds that

$$\widetilde{T}_L = \inf\left(t \geq \mathsf{T} \middle| Y(t) \leq \widetilde{Y}_L\right) \leq \widetilde{T}_F = \inf\left(t \geq \mathsf{T} \middle| Y(t) \leq \widetilde{Y}_F\right),$$

and $\widetilde{Y}_F \leq \widetilde{Y}_L$ again by definition of the leader's role. Since the follower does not reduce capacity before the leader, $\widetilde{Y}_F$ equals the follower's optimal closure rule Y_F, if and only if $Y_F \leq \widetilde{Y}_L$. Otherwise $\widetilde{Y}_F < Y_F$. Thus, the leader's payoff function depends on the (actual) timing of the follower's capacity decision. For $Y_F \geq \widetilde{Y}_L = \widetilde{Y}_F$ both, leader and follower, partially close production facilities at $\widetilde{Y}_L$ and we have an instance of 'simultaneous capacity reduction' which is discussed in the next subsection. So let us concentrate on the scenario $Y_F = \widetilde{Y}_F < \widetilde{Y}_L$. As is proved below, the graph of the leader's value function exhibits a kink at Y_F due to the change in the follower's behavior. Therefore, it is useful to distinguish between the two subcases $Y_\mathsf{T} \leq Y_F < \widetilde{Y}_L$ and $Y_F \leq Y_\mathsf{T} < \widetilde{Y}_L$. We start off with the assumption $Y_\mathsf{T} \leq Y_F < \widetilde{Y}_L$. In this case the follower reduces capacity immediately after the subgame has started—and so does the leader. Hence, the expected value of the leader's firm in this region must equal the "fundamental value" arising from eternal aggregate production at $2\underline{q}$ net of the option's strike price E,

$$\begin{aligned} L(Y_\mathsf{T}) &= \mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^\infty e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R\left(\underline{q},\underline{q}\right) - C_L\underline{q}\right]dt\right] - E \\ &= R\left(\underline{q},\underline{q}\right)\cdot g(Y_\mathsf{T}) - \frac{C_L\underline{q}}{\rho} - E, \end{aligned} \tag{2.16}$$

[13] Of course, if none of the equilibrium strategy profiles implies an assignment of the leader's role to a firm, then $\widetilde{Y}_L$ cannot be determined even *in* any of the corresponding equilibria.

where $g(Y_{\mathsf{T}})$ is defined by equation (2.6). The expression resulting for $L(Y_{\mathsf{T}})$ is given in Lemma 2. Next, let us examine the second case, $Y_F \leq Y_{\mathsf{T}} < \widetilde{Y}_L$, where the follower still refrains from exercising his option, but the leader immediately reduces capacity. The leader's value becomes

$$\begin{aligned} L\left(Y_{\mathsf{T}}\right) &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_F} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\underline{q},\overline{q}\right)-C_L\underline{q}\right]dt\right] \\ &\quad +\mathcal{E}_{\mathsf{T}}\left[\int_{T_F}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\underline{q},\underline{q}\right)-C_L\underline{q}\right]dt\right]-E \\ &= -\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)\cdot h\left(Y_{\mathsf{T}}\right)\big|_{T=T_F} \\ &\quad +R\left(\underline{q},\underline{q}\right)\cdot g\left(Y_{\mathsf{T}}\right)-\frac{C_L\underline{q}}{\rho}-E, \end{aligned} \tag{2.17}$$

where $h(Y_{\mathsf{T}})$ is defined by equation (2.9). In the following lemma we summarize the piecewise functions that give the expected value of the leader's firm for the different regions.

Lemma 2 *The expected value of the leader's firm conditionally on Y having first reached Y_{T} is*

$$L\left(Y_{\mathsf{T}}\right)=\begin{cases} \frac{Y_{\mathsf{T}}R\left(\underline{q},\underline{q}\right)}{\delta}-\frac{C_L\underline{q}}{\rho}-E & \text{for } 0\leq Y_{\mathsf{T}}\leq Y_F, \\ \frac{Y_{\mathsf{T}}R\left(\underline{q},\overline{q}\right)}{\delta}-\frac{C_L\underline{q}}{\rho}-E & \\ +\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_F\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta} & \text{for } Y_F<Y_{\mathsf{T}}<\widetilde{Y}_L, \end{cases} \tag{2.18}$$

where Y_F and β are given by equations (2.13) and (2.14), respectively. Further, L is not smooth at Y_F.

Proof. Equation (2.18) is derived by plugging the formulas for $g(Y_{\mathsf{T}})$ and $h(Y_{\mathsf{T}})$ in Lemma 6 in Appendix A.1 into equations (2.16) and (2.17). In Appendix A.2 we verify that $L'(Y_{\mathsf{T}})$ exhibits a point of discontinuity at Y_F. ■

As mentioned above, the actual closure threshold of the leader, $\widetilde{Y}_L$, results as an outcome of strategic interaction and can only be determined in equilibrium. The equilibria of the closure timing game will be discussed in section 2.4. Nevertheless, by *pre*assigning roles to players, we can compute an exercise rule Y_L that turns out to be payoff maximizing for the firm that gets *pre*assigned to the leader's role. This threshold represents an important benchmark in the case of *first-mover advantages* (see condition (2.4)) ,when comparing our equilibrium exercising rules with the optimal policies as suggested by the standard real options theory. To be more precise, consider a myopic firm currently producing at its capacity limit $\overline{q}$ and contemplating capacity reduction by $\overline{q}-\underline{q}$ to $\underline{q}$. The firm is myopic in the sense that it ignores strategic interaction, i.e. it has static expectations with respect to its rival's actions. However, in contrast to a price-taking firm it takes the impact of its own disinvestment decision on the price process into account. As the following proposition says, if the leader's variable cost is not too different from the follower's variable cost, then not only the myopic firm finds it optimal to exercise its closure option, when $Y(t)$ first hits the threshold Y_M but also the leader in the duopolistic timing game. Thus, the leader's optimal timing is identical to that of a myopic firm if the strategic interaction is driven by first-mover advantages. However, the following proposition also suggests that, under the assumption of *second-mover advantages*, a strikingly different outcome arises, if the difference in variable cost is not too extreme.

Proposition 1 *(i) Let us preassign roles to players such that one firm is the leader and the other firm becomes the follower. Further, let*

$$\mathcal{C}=\frac{(C_L/\rho)\left(\overline{q}-\underline{q}\right)-E}{(C_F/\rho)\left(\overline{q}-\underline{q}\right)-E},\mathcal{R}_1=\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)}{R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)},\mathcal{R}_2=\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)}{R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)}.$$

Under the assumption of first-mover (second-mover) advantages, $\mathcal{R}_1$ and $\mathcal{R}_2$ satisfy $\mathcal{R}_1<\mathcal{R}_2<1$ ($\mathcal{R}_1<1<\mathcal{R}_2$). If the (sub)game starts at $Y_{\mathsf{T}}>Y_L$, then the leading firm maximizes its payoff by reducing capacity from $\overline{q}$ to $\underline{q}$, when $Y(t)$ first reaches the fixed threshold

$$Y_L=\begin{cases} Y_{S_i}\big|_{C_i=C_L} & \text{if } \mathcal{C}<\mathcal{R}_1 \\ Y_F & \text{if } \mathcal{R}_1\leq\mathcal{C}\leq\mathcal{R}_2 \\ Y_M\big|_{C_M=C_L} & \text{if } \mathcal{C}>\mathcal{R}_2 \end{cases}, \tag{2.19}$$

where Y_F, Y_M and Y_{S_i} are defined by equations (2.13), (2.20) and (2.27), respectively, $i\in\{1,2\}$. If $Y_{\mathsf{T}}\leq Y_L$ holds, then the leader maximizes its payoff by reducing capacity immediately at Y_{T}.

(ii) Suppose that a myopic firm with constant marginal cost of production $C_M\in\{C_L,C_F\}$ is equipped with the option to reduce capacity from $\overline{q}$ to $\underline{q}$. Then the myopic firm maxi-

mizes its payoff by exercising the option at

$$Y_M = -\frac{\delta\beta}{1-\beta} \cdot \frac{(C_M/\rho)\left(\overline{q} - \underline{q}\right) - E}{R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right)} \tag{2.20}$$

if $Y_{\mathsf{T}} > Y_M$*, and at* Y_{T} *if* $Y_{\mathsf{T}} \leq Y_M$*.*

(iii) Suppose that a price-taking firm with constant marginal cost of production $C_P \in \{C_L, C_F\}$ *facing an exogenous price process* $P = YD(2\overline{q})$ *is equipped with the option to reduce capacity from* $\overline{q}$ *to* $\underline{q}$*. Then this firm maximizes its payoff by exercising the option at*

$$Y_P = -\frac{\delta\beta}{1-\beta} \cdot \frac{(C_P/\rho)\left(\overline{q} - \underline{q}\right) - E}{D\left(2\overline{q}\right)\left(\overline{q} - \underline{q}\right)} \tag{2.21}$$

if $Y_{\mathsf{T}} > Y_P$*, and at* Y_{T} *if* $Y_{\mathsf{T}} \leq Y_P$*. The same disinvestment threshold is optimal for a single price-taking firm that owns the option to reduce capacity from* $2\overline{q}$ *to* $\overline{q} + \underline{q}$*.*

(iv) Suppose that $C_M = C_P = C_L = C_F \Leftrightarrow \mathcal{C} = 1$*. Then the disinvestment thresholds can be ordered as*

$$\min\{Y_P, Y_F\} < \max\{Y_P, Y_F\} < Y_M = Y_L \tag{2.22}$$

if condition (2.4) holds. The corresponding ordering is given by

$$Y_P < Y_M < Y_F = Y_L \tag{2.23}$$

if condition (2.5) holds.

Proof. See Appendix A.3. ■

Note that Y_{S_i} denotes the optimal exercise threshold of firm i conditionally on both firms moving simultaneously. The corresponding formula is derived in the next subsection. The importance of Proposition 1 will become obvious in section 2.4, where the equilibria of the timing game are derived. However, let us anticipate later derivations and point out that e.g. with identical firms, $C_1 = C_2 \Leftrightarrow C_F = C_L$, and with second-mover advantages the unique equilibrium prediction will be that both firms should exercise their closure options at $Y_L = Y_F$. If the second-mover advantage (2.5) is of significant size, then the ordering (2.23) suggests that this policy might be very different from the myopic firm's exercising rule Y_M. Moreover, Y_F lies even farther away from the price taker's threshold Y_P than from the myopic firm's disinvestment trigger. Thus, Proposition 1 implies that fully rational competitors in a duopoly with second-mover advantages disinvest earlier than price-taking firms would do.

In the case of first-mover advantages (2.4) the equilibrium prediction will be that one firm exercises its closure option at $Y_L = Y_M$, while the other firm follows at Y_F. As the ordering (2.22) reveals, the leader certainly shuts down production facilities before the price taker would do. However, the picture is not that clear regarding the follower's disinvestment timing. To realize the full profits from having attracted additional market shares, the follower might even reduce capacity much later than a price-taking firm.

Under fairly general conditions—including infinitely divisible investment projects—Leahy (1993) shows that the optimal investment threshold of a single firm facing an exogenous price process is identical to the optimal exercise threshold of a firm in *perfect competition*. Unfortunately, there does not exist a general analogous result that applies to the price taker's optimal closure threshold Y_P in our model due the limited divisibility of the underlying disinvestment project. However, in the special case of a linear adjustment cost function, i.e. $E = k \cdot (\overline{q} - \underline{q})$ with k denoting the closure cost per retired unit, Y_P equals the perfect competition benchmark as derived by Leahy. The next section contains a result on the optimal disinvestment timing of a monopolist.

2.2.3 Joint Capacity Closure

The third role that firms can play is that of simultaneous movers. In this case firms reduce capacity simultaneously at $T_S = \inf(t \geq \mathsf{T} \,|Y(t) \leq Y_S)$, where Y_S denotes the threshold of simultaneous capacity closure. The expected value of firm i, $i \in \{1, 2\}$, conditionally on Y having first reached the threshold $Y_\mathsf{T} > Y_S$ equals

$$\begin{aligned} S_i(Y_\mathsf{T}, Y_S) &= \mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^{T_S} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q}, \overline{q}) - C_i\overline{q}\right] dt \right. \\ &\quad \left. + \int_{T_S}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R\left(\underline{q}, \underline{q}\right) - C_i\underline{q}\right] dt - e^{-\rho(T_t-\mathsf{T})}E\right]. \end{aligned}$$

After rearranging one yields

$$S_i(Y_\mathsf{T}, Y_S) = \left(R(\overline{q}, \overline{q}) - R\left(\underline{q}, \underline{q}\right)\right) \cdot h(Y_\mathsf{T})|_{T=T_S} + R\left(\underline{q}, \underline{q}\right) \cdot g(Y_\mathsf{T})$$

$$-\frac{C_i\overline{q}}{\rho}+\left(\frac{C_i\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\cdot f\left(Y_{\mathsf{T}}\right)|_{T=T_S}, \tag{2.24}$$

where $f(Y_{\mathsf{T}})$, $g(Y_{\mathsf{T}})$, and $h(Y_{\mathsf{T}})$ are defined by equations (2.8), (2.6), and (2.9), respectively, and are calculated in Lemma 6 in Appendix A.1. If $Y_{\mathsf{T}} \leq Y_S$ both firms reduce capacity immediately and the expected value of firm i becomes

$$\begin{aligned} S_i\left(Y_{\mathsf{T}}, Y_S\right) &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\underline{q},\underline{q}\right)-C_i\underline{q}\right]dt-E\right] \\ &= R\left(\underline{q},\underline{q}\right)\cdot g\left(Y_{\mathsf{T}}\right)-\frac{C_i\underline{q}}{\rho}-E. \end{aligned} \tag{2.25}$$

The resulting expressions for firm i's payoff are summarized in the following lemma.

Lemma 3 *The expected value of firm i that arises from simultaneous closure conditionally on Y having first reached Y_{T} is*

$$S_i\left(Y_{\mathsf{T}}, Y_S\right)=\begin{cases} \frac{Y_{\mathsf{T}}R\left(\underline{q},\underline{q}\right)}{\delta}-\frac{C_i\underline{q}}{\rho}-E & \text{for } 0\leq Y_{\mathsf{T}}\leq Y_S, \\ \frac{Y_{\mathsf{T}}R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C_i\overline{q}}{\rho} & \\ -\left(\frac{Y_S}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_S\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C_i\left(\overline{q}-\underline{q}\right)}{\rho}+E\right) & \text{for } Y_{\mathsf{T}}>Y_S, \end{cases} \tag{2.26}$$

where β is given by equation (2.14).

Proof. Equation (2.26) is derived by plugging the formulas for $f(Y_{\mathsf{T}})$, $g(Y_{\mathsf{T}})$, and $h(Y_{\mathsf{T}})$ in Lemma 6 in Appendix A.1 into equations (2.24) and (2.25). ■

So far, it is left undetermined what should be the optimal exercise rule if both firms coordinate on reducing capacity simultaneously. We expect that, as long as $C_1 \neq C_2$, there does not exist any joint closure time $T_{S^*} = \inf(t\,|Y(t) \leq Y_{S^*})$, where both players find it optimal to reduce capacity. Let us confirm this assertion. From the point of view of player i the optimal joint reduction time is $T_{S_i} = \inf(t\,|Y(t) \leq Y_{S_i})$ with $Y_{S_i} = \arg\max_{Y_{\mathsf{T}}>Y_s>0} S_i\left(Y_{\mathsf{T}}, Y_S\right)$. We compute the first-order condition $\partial S_i/\partial Y_S = 0$ and obtain

$$Y_{S_i}=\frac{-\delta\beta}{1-\beta}\cdot\frac{\left(C_i/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)}, \tag{2.27}$$

which turns out to be an interior local maximum, since the second partial derivative evaluated at Y_{S_i} is negative, i.e.

$$\left.\frac{\partial^2 S_i}{(\partial Y_S)^2}\right|_{Y_S=Y_{S_i}} = -(1-\beta)\left(\frac{Y_{S_i}}{Y_\mathsf{T}}\right)^{-\beta} \frac{R(\overline{q},\overline{q}) - R(\underline{q},\underline{q})}{\delta Y_{S_i}} < 0.$$

The optimal joint closure thresholds for firm one and two indeed differ for $C_1 \neq C_2$. We have $Y_{S_i} > Y_{S_j}$ for $C_i > C_j$, $i \neq j$, $i,j \in \{1,2\}$. Moreover, a comparison with the follower's optimal exercise threshold in equation (2.13) reveals that $Y_{F_i} < Y_{S_i}$.

Further, note that the payoff from *immediate and simultaneous* capacity reduction (at Y_T) is given by

$$I_i(Y_\mathsf{T}) = S_i(Y_\mathsf{T}, Y_\mathsf{T}) = \frac{Y_\mathsf{T} R(\underline{q},\underline{q})}{\delta} - \frac{C_i \underline{q}}{\rho} - E. \tag{2.28}$$

The threshold Y_{S_i} is not only the optimal joint reduction trigger of firm i but also of a monopolist in the model with second-mover advantages as the following proposition suggests.

Proposition 2 *Suppose that a monopolist with constant marginal cost of production C_i, $i \in \{1,2\}$, is equipped with a chain of options—an option to reduce capacity from $2\overline{q}$ to $\overline{q}+\underline{q}$ and a subsequent option to reduce capacity from $\overline{q}+\underline{q}$ to $2\underline{q}$.*
(i) Suppose that condition (2.4) is satisfied. Then the monopolist maximizes his payoff by exercising his options sequentially at

$$Y_O' = \frac{-\delta\beta}{1-\beta} \cdot \frac{(C_i/\rho)(\overline{q}-\underline{q}) - E}{2R(\overline{q},\overline{q}) - R(\overline{q},\underline{q}) - R(\underline{q},\overline{q})} \tag{2.29}$$

and

$$Y_O'' = \frac{-\delta\beta}{1-\beta} \cdot \frac{(C_i/\rho)(\overline{q}-\underline{q}) - E}{R(\overline{q},\underline{q}) + R(\underline{q},\overline{q}) - 2R(\underline{q},\underline{q})} < Y_O' \tag{2.30}$$

if $Y_\mathsf{T} > Y_O'$, at Y_T and Y_O'' if $Y_O'' < Y_\mathsf{T} \leq Y_O'$, and simultaneously at Y_T if $Y_\mathsf{T} \leq Y_O''$.
(ii) Suppose that condition (2.5) is satisfied. Then the monopolist maximizes his payoff by exercising both options simultaneously at Y_{S_i} as given by equation (2.27) if $Y_\mathsf{T} > Y_{S_i}$, and at Y_T if $Y_\mathsf{T} \leq Y_{S_i}$.
(iii) Suppose that $C = C_1 = C_2$. If condition (2.4) is satisfied, then the disinvestment

thresholds can be ordered as

$$Y_F < \min\{Y_M, Y_O''\} < \max\{Y_M, Y_O''\} < Y_{S^*} < Y_O', \tag{2.31}$$

where $Y_{S^*} = Y_{S_i}|_{C_i=C}$ *, and* Y_F*,* Y_M*,* Y_P *and* Y_{S_i} *are defined by equations (2.13), (2.20), (2.21) and (2.27), respectively. If condition (2.5) is satisfied, then*

$$Y_M < Y_F < Y_{S^*}. \tag{2.32}$$

Proof. See Appendix A.4. ■

Since the monopolist internalizes the strategic externalities that arise in duopoly, his disinvestment timing represents the *cooperative optimum.* Suppose that firms are identical, $C = C_1 = C_2$. If strategic interaction among firms is driven by second-mover advantages (2.3), then the optimal cooperative disinvestment trigger, Y_{S^*}, equals the optimal closure threshold of two fully rational competitors *contingent* on these firms moving simultaneously. Our equilibrium prediction in this case will be that both firms should exercise their closure options at $Y_L = Y_F$. Since $Y_{S^*} > Y_F$, Proposition 2 suggests that the equilibrium timing of disinvestment in a duopoly with second-mover advantages is delayed as compared to the optimal cooperative disinvestment pattern.

In the complementary case of first-mover advantages (2.4) the equilibrium prediction will be that one firm exercises its closure option at $Y_L = Y_M$, while its rival follows at Y_F. As the ordering (2.31) reveals, the equilibrium follower certainly exercises his closure option after the monopolist has finished his disinvestment sequence. However, the results are more ambiguous with respect to the equilibrium leader's disinvestment timing. To realize the full profits from having attracted the first-mover advantage, the leader might have completed capacity reduction before the monopolist exercises his second option. Nevertheless, at least on average firms still reduce capacity earlier than a monopolist.

Let us provide some closer investigation of the relative magnitude of Y_M as compared to Y_O''. Assume that D is a downward slopping C^1 function. Suppose further that consumers have a finite reservation price, i.e. $D(0) < +\infty$. To order Y_O'' and the myopic firm's threshold Y_M by size, we compare the denominators,

$$\begin{aligned} & Y_M \gtreqless Y_O'' \\ \Leftrightarrow\ & R\left(\overline{q},\underline{q}\right) - 2R\left(\underline{q},\underline{q}\right) + R\left(\underline{q},\overline{q}\right) \gtreqless R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right) \\ \Leftrightarrow\ & D\left(\overline{q}+\underline{q}\right)\overline{q} - D\left(2\underline{q}\right)2\underline{q} + D\left(\overline{q}+\underline{q}\right)\underline{q} \gtreqless D\left(2\overline{q}\right)\overline{q} - D\left(\overline{q}+\underline{q}\right)\underline{q} \\ \Leftrightarrow\ & D\left(\left(1+c\right)\overline{q}\right)\left(1+c\right) - D\left(2c\overline{q}\right)2c \gtreqless D\left(2\overline{q}\right) - D\left(\left(1+c\right)\overline{q}\right)c \end{aligned}$$

where $c = \underline{q}/\overline{q} \in (0,1)$. For $c \to 1$, both sides of the latter inequality converge to zero, i.e. $\lim_{c\to 1} Y_M = \lim_{c\to 1} Y_O'' = +\infty$. However, for $c \to 0$, D being strictly decreasing and $D(0) < +\infty$, the left-hand side's expression converges to $D(\overline{q})$ which is strictly greater than the right-hand side's limit value, $D(2\overline{q})$. Moreover, evaluating the derivative of the left-hand side term with respect to c,

$$D'\left(\left(1+c\right)\overline{q}\right)\left(1+c\right)\overline{q} + D\left(\left(1+c\right)\overline{q}\right) - D'\left(2c\overline{q}\right)4c\overline{q} - 2D\left(2c\overline{q}\right),$$

at $c = 1$ yields $-D'(2\overline{q})2\overline{q} - D(2\overline{q})$. The corresponding derivative of the right-hand side,

$$-D'\left(\left(1+c\right)\overline{q}\right)c\overline{q} - D\left(\left(1+c\right)\overline{q}\right),$$

attains the value $-D'(2\overline{q})\overline{q} - D(2\overline{q})$ at $c = 1$ which is strictly smaller than $-D'(2\overline{q})2\overline{q} - D(2\overline{q})$. This implies that there exists $\varepsilon > 0$ such that $Y_M > Y_O''$ for $c \in (1-\varepsilon, 1)$. However, if c becomes sufficiently small, then the reverse relation, $Y_M < Y_O''$, holds. Additional regularity conditions on the second derivative of the inverse demand function ensure that there exists a unique c^* such that $Y_M > Y_O''$ for $c > c^*$ and $Y_M < Y_O''$ for $c < c^*$.

We conclude that the myopic firm in the model with first-mover advantages reduces capacity after the monopolist has finished his disinvestment sequence if its closure option resembles an exit option (regarding the ratio of ex-post to ex-ante capacity).

2.3 Strategies and Payoffs

It has already been mentioned in section 2.1 that the choice of adequate strategy spaces depends on whether the underlying timing game is a war of attrition or a preemption game.[14] So, it seems reasonable to proceed with showing that the closure timing game proposed in this chapter is a war of attrition. It should be stressed that there does not exist a uniform definition of this type of game. For example, Hendricks *et al.* (1988) present a fairly general characterization of the war of attrition in continuous time with *complete* information. However, it is straightforward to show that the payoff functions of our closure timing game are too complex to satisfy their rather restrictive assumptions. Fudenberg and Tirole (1991) provide more general classifying criteria, but focus on timing games with *perfect information.* Thus, as a prerequisite we have to modify their definition to fit the *imperfect*-information case. Moreover, notice that the roles' payoffs in the standard timing-game literature are defined as functions of the leader's stopping time, T_L, which generally does not coincide with the root of the subgame, T. In Huisman and Kort (1999) and this paper the roles' payoffs, however, are written as functions of the root of the subgame, T, conditionally on the leader moving at T. To ensure compatibility, we adopt the standard notation in the following two paragraphs and switch back to our original notation afterwards.[15]

In accordance with the modified definition of the roles' payoffs, if player i, $i \in \{1, 2\}$, gets assigned to the leader's role and the leader stops before the follower's optimal stopping time, i.e. $Y_L > Y_{F_j}$, $j \in \{1, 2\}$, $i \neq j$, then player i's payoff function is given by,

$$\begin{aligned} L_i\left(Y_{\mathsf{T}}, Y_L\right) &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_L} e^{-\rho(t-\mathsf{T})}\left(Y\left(t\right)R\left(\overline{q},\overline{q}\right) - C_i\overline{q}\right)dt - e^{-\rho(T_L-\mathsf{T})}E\right. \\ &\quad + \int_{T_L}^{T_{F_j}} e^{-\rho(t-\mathsf{T})}\left(Y\left(t\right)R\left(\underline{q},\overline{q}\right) - C_i\underline{q}\right)dt \\ &\quad \left. + \int_{T_{F_j}}^{\infty} e^{-\rho(t-\mathsf{T})}\left(Y\left(t\right)R\left(\underline{q},\underline{q}\right) - C_i\underline{q}\right)dt\right], \end{aligned}$$

where $T_L = \inf(t \geq \mathsf{T}\,|Y\left(t\right) \leq Y_L)$, $Y_{F_j} = Y_F|_{C_F=C_j}$, and Y_F is defined in equation (2.13).

[14]See Fudenberg and Tirole (1991) for a general classification of timing games.

[15]Alternatively, one could use the roles' payoff functions as defined in the previous section and adapt the classifying criteria. However, in this case the analogy to the criteria as suggested by Fudenberg and Tirole (1991) would become less clear.

Equivalently, if player i becomes the follower at $Y_L > Y_{F_i}$, then i's payoff can be written as

$$\begin{aligned} F_i(Y_\mathsf{T}, Y_L) &= \mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^{T_L} e^{-\rho(t-\mathsf{T})}\left(Y(t)R(\overline{q},\overline{q}) - C_i\overline{q}\right)dt\right. \\ &+ \int_{T_L}^{T_{F_i}} e^{-\rho(t-\mathsf{T})}\left(Y(t)R\left(\overline{q},\underline{q}\right) - C_i\overline{q}\right)dt - e^{-\rho\left(T_{F_i}-\mathsf{T}\right)}E \\ &+ \left.\int_{T_{F_i}}^{\infty} e^{-\rho(t-\mathsf{T})}\left(Y(t)R\left(\underline{q},\underline{q}\right) - C_i\underline{q}\right)dt\right]. \end{aligned}$$

Now, with this modified definitions of the roles' payoffs at hand, let us adapt the classifying criteria for war-of-attrition games suggested by Fudenberg and Tirole (1991) to cover the imperfect information case. That is, continuous-time timing games with aggregate price uncertainty as defined by equation (2.1) can be viewed as wars of attrition if they satisfy the following conditions: For all players i, $i \in \{1, 2\}$, and all thresholds $Y_L, Y_L' \in (0, Y_\mathsf{T}]$, *(i)* $F_i(Y_\mathsf{T}, Y_L) \geq F_i(Y_\mathsf{T}, Y_L')$ for $Y_L > Y_L'$, *(ii)* $F_i(Y_\mathsf{T}, Y_L) \geq L_i(Y_\mathsf{T}, Y_L')$ for $Y_L > Y_L'$, *(iii)* $L_i(Y_\mathsf{T}, Y_\mathsf{T}) > \lim_{Y_L\to 0} L_i(Y_\mathsf{T}, Y_L)$, and *(iv)* $\lim_{Y_L\to 0} L_i(Y_\mathsf{T}, Y_L) = \lim_{Y_L\to 0} F_i(Y_\mathsf{T}, Y_L)$.

To show the validity of conditions *(i)* to *(iv)*, let us assume without loss of generality that $C_2 > C_1 \Rightarrow Y_{F_2} \geq Y_{F_1}$. Then one obtains for $Y_L > Y_{F_2}$

$$\begin{aligned} &F_1(Y_\mathsf{T}, Y_L) - L_1(Y_\mathsf{T}, Y_L) \\ &= \mathcal{E}_\mathsf{T}\left[\int_{T_L}^{T_{F_2}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)\right) - C_1\left(\overline{q}-\underline{q}\right)\right]dt\right. \\ &+ \int_{T_{F_2}}^{T_{F_1}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)\right) - C_1\left(\overline{q}-\underline{q}\right)\right]dt \\ &+ \left.\left(e^{-\rho(T_L-\mathsf{T})} - e^{-\rho\left(T_{F_1}-\mathsf{T}\right)}\right)E\right] \\ &> \mathcal{E}_\mathsf{T}\left[\int_{T_L}^{T_{F_1}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)\right) - C_1\left(\overline{q}-\underline{q}\right)\right]dt\right] \\ &+ \mathcal{E}_\mathsf{T}\left[e^{-\rho(T_L-\mathsf{T})} - e^{-\rho\left(T_{F_1}-\mathsf{T}\right)}\right]\cdot E \end{aligned}$$

$$= \frac{R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \underline{q}\right)}{\delta} \times$$
$$\times \left[\left(\frac{Y_L}{Y_{\mathsf{T}}} \right)^{-\beta} \left(Y_L - \frac{1-\beta}{-\beta} Y_{F_1} \right) + \left(\frac{Y_{F_1}}{Y_{\mathsf{T}}} \right)^{-\beta} \frac{Y_{F_1}}{-\beta} \right] > 0 \, ,$$

and

$$F_2\left(Y_{\mathsf{T}}, Y_L\right) - L_2\left(Y_{\mathsf{T}}, Y_L\right)$$
$$= \mathcal{E}_{\mathsf{T}} \left[\int_{T_L}^{T_{F_2}} e^{-\rho(t-\mathsf{T})} \left[Y(t) \left(R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right) \right) - C_2 \left(\overline{q} - \underline{q} \right) \right] dt \right.$$
$$\left. + \int_{T_{F_2}}^{T_{F_1}} e^{-\rho(t-\mathsf{T})} Y(t) \left(R\left(\underline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right) \right) dt + \left(e^{-\rho(T_L - \mathsf{T})} - e^{-\rho\left(T_{F_2} - \mathsf{T}\right)} \right) E \right]$$
$$> \mathcal{E}_{\mathsf{T}} \left[\int_{T_L}^{T_{F_2}} e^{-\rho(t-\mathsf{T})} \left[Y(t) \left(R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \underline{q}\right) \right) - C_2 \left(\overline{q} - \underline{q} \right) \right] dt \right]$$
$$+ \mathcal{E}_{\mathsf{T}} \left[e^{-\rho(T_L - \mathsf{T})} - e^{-\rho\left(T_{F_2} - \mathsf{T}\right)} \right] \cdot E$$
$$= \frac{R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \underline{q}\right)}{\delta} \times$$
$$\times \left[\left(\frac{Y_L}{Y_{\mathsf{T}}} \right)^{-\beta} \left(Y_L - \frac{1-\beta}{-\beta} Y_{F_2} \right) + \left(\frac{Y_{F_2}}{Y_{\mathsf{T}}} \right)^{-\beta} \frac{Y_{F_2}}{-\beta} \right] > 0 \, .$$

If $Y_L \leq Y_{F_1}$, then the follower exercises his option immediately after the leader has done so. Thus, the roles payoffs are equal,

$$F_2\left(Y_{\mathsf{T}}, Y_L\right) = L_2\left(Y_{\mathsf{T}}, Y_L\right)$$
$$= \mathcal{E}_{\mathsf{T}} \left[\int_{\mathsf{T}}^{T_L} e^{-\rho(t-\mathsf{T})} \left[Y(t) \, R\left(\overline{q}, \overline{q}\right) - C_i \overline{q} \right] dt - e^{-\rho(T_L - \mathsf{T})} E \right.$$
$$\left. + \int_{T_L}^{\infty} e^{-\rho(t-\mathsf{T})} \left[Y(t) \, R\left(\underline{q}, \underline{q}\right) - C_i \underline{q} \right] dt \right] .$$

If $Y_L \in (Y_{F_1}, Y_{F_2}]$, then only the high-cost firm (firm 2) finds it optimal to move immediately after the leader conditionally on the low-cost firm getting assigned to the leader's role. With interchanged roles, however, the low-cost firm does not follow the high-cost firm at Y_L, but waits until Y_{F_1} is reached. We get

$$\begin{aligned}
& F_1\left(Y_{\mathsf{T}}, Y_L\right) - L_1\left(Y_{\mathsf{T}}, Y_L\right) \\
& = \mathcal{E}_{\mathsf{T}}\left[\int_{T_L}^{T_{F_1}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\left(R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right) - C_1\left(\overline{q} - \underline{q}\right)\right] dt\right] \\
& + \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_L-\mathsf{T})} - e^{-\rho\left(T_{F_1}-\mathsf{T}\right)}\right] \cdot E \\
& > \mathcal{E}_{\mathsf{T}}\left[\int_{T_L}^{T_{F_1}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\left(R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \underline{q}\right)\right) - C_1\left(\overline{q} - \underline{q}\right)\right] dt\right] \\
& + \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_L-\mathsf{T})} - e^{-\rho\left(T_{F_1}-\mathsf{T}\right)}\right] \cdot E \\
& = \frac{R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \underline{q}\right)}{\delta} \times \\
& \times \left[\left(\frac{Y_L}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_L - \frac{1-\beta}{-\beta} Y_{F_1}\right) + \left(\frac{Y_{F_1}}{Y_{\mathsf{T}}}\right)^{-\beta} \frac{Y_{F_1}}{-\beta}\right] > 0\ ,
\end{aligned}$$

and

$$F_2\left(Y_{\mathsf{T}}, Y_L\right) - L_2\left(Y_{\mathsf{T}}, Y_L\right) = \mathcal{E}_{\mathsf{T}}\left[\int_{T_L}^{T_{F_1}} e^{-\rho(t-\mathsf{T})} Y(t)\left(R\left(\underline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right) dt\right] > 0.$$

Finally, notice that $\partial F_i\left(Y_{\mathsf{T}}, Y_L\right) / \partial Y_L > 0\ \forall Y_L \in (0, Y_{\mathsf{T}}]$. From these derivations it follows immediately that the closure timing game proposed in this chapter satisfies the classifying conditions *(i)* to *(iv)*. Therefore, we do not need to resort to the extended strategy spaces suggested by Fudenberg and Tirole (1985).

In what follows we give a full description of the firms' payoff functions. This can be managed by combining the roles' payoff functions and the firms' strategies in an adequate way. Recall that each mixed strategy profile gives the probability of a certain assignment of roles to players for every subgame and every state of the world at each instant of time. By modifying the formalism of Pitchik (1982) let us state what we understand by a strategy of a subgame of the timing game more precisely.

Definition 1 *A simple strategy for firm i in the subgame starting at $\mathsf{T} = \inf(t \geq 0\,|Y(t) \leq Y_{\mathsf{T}})$, $Y_{\mathsf{T}} > 0$, is a stochastic process $G_i^{\mathsf{T}} : [0, \infty) \times (\Omega, \mathfrak{F}) \rightarrow [0, 1]$ satisfying for all ω that $G_i^{\mathsf{T}}(\cdot, \omega)$ is non-decreasing and right-continuous.*

This condition represents the natural extension of the definition by Pitchik (1981, p.208) to the imperfect information world. In section 2.1 we provided some intuition for the above definition. From hereafter we suppress the upper index, T, and the second functional argument, ω, of G_i for the ease of notation, whenever there is no risk of confusion. Let $G_i^-(t) = \lim_{s \uparrow t} G_i(s)$ and impose $G_i^-(\mathsf{T}) \equiv 0$, $i \in \{1, 2\}$, i.e. the probability that firm i moves before the (sub)game has started equals zero. Then firm i's expected payoff conditionally on Y having first reached Y_T is given by

$$\begin{aligned} & V_i\left(Y_\mathsf{T}, G_i, G_j\right) \\ & = \mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^\infty e^{-\rho(t-\mathsf{T})}\left(Y\left(t\right) R\left(\overline{q}, \overline{q}\right) - C_i \overline{q}\right)\left(1 - G_i\left(t\right)\right)\left(1 - G_j\left(t\right)\right) dt\right. \\ & + \int_\mathsf{T}^\infty e^{-\rho(t-\mathsf{T})} L_i\left(Y\left(t\right)\right)\left(1 - G_j\left(t\right)\right) dG_i\left(t\right) \\ & + \int_\mathsf{T}^\infty e^{-\rho(t-\mathsf{T})} F_i\left(Y\left(t\right)\right)\left(1 - G_i\left(t\right)\right) dG_j\left(t\right) \\ & \left. + \sum_{t \geq \mathsf{T}} e^{-\rho(t-\mathsf{T})}\left(G_i\left(t\right) - G_i^-\left(t\right)\right)\left(G_j\left(t\right) - G_j^-\left(t\right)\right) I_i\left(Y\left(t\right)\right)\right], \end{aligned} \tag{2.33}$$

where $F_i\left(\cdot\right) = F\left(\cdot\right)|_{C_F = C_i}$ and $L_i\left(\cdot\right) = L\left(\cdot\right)|_{C_L = C_i, C_F = C_j}$, $i \neq j$, $i, j \in \{1, 2\}$. Further, $F\left(\cdot\right)$, $L\left(\cdot\right)$, and $I_i\left(\cdot\right)$ are defined by equations (2.12), (2.18), and (2.28), respectively. By taking expectations contingent on the information that is available at the root of the subgame T in equation (2.33) we generalize the corresponding payoff functions given in Pitchik to account for the inherent uncertainty regarding future revenues. Recall that the stochastic process Y first hits Y_T at T. The second line of equation (2.33) captures the expected profits ex ante to capacity-reduction. The third and the fourth line of the formula give firm i's expected payoff from becoming the leader and the follower, respectively, when i shuts down production facilities with probability G_i, while firm j moves with probability G_j. Finally, the fifth line of equation (2.33) determines firm i's payoff if both firms move simultaneously with positive probability. Some more definitions are required. First, let us define what is understood by a Nash equilibrium in simple strategies.

Definition 2 *A pair of simple strategies $G_1(\cdot, \cdot)$ and $G_2(\cdot, \cdot)$ is a Nash equilibrium of the* *timing game starting at Y_T (with neither firm having yet reduced capacity) if each firm's strategy maximizes its expected payoff $V_i(Y_\mathsf{T}, G_i, G_j)$, $i, j \in \{1, 2\}, i \neq j$, holding the other firm's strategy fixed.*

Second, closed-loop strategies must satisfy the following intertemporal consistency condition.

Definition 3 *A closed-loop strategy for firm i is a collection of simple strategies $\{G_i^\mathsf{T}(\cdot,\cdot)\}_{Y_\mathsf{T}>0}$ satisfying for all $\omega \in \Omega$ and $0 \leq u \leq v$ that $G_i^\mathsf{T}(\mathsf{T}+v) = G_i^\mathsf{T}(\mathsf{T}+u) + (1 - G_i^\mathsf{T}(\mathsf{T}+u)) \cdot G_i^{\mathsf{T}+u}(\mathsf{T}+v)$.*

Thus, a closed-loop strategy determines a simple strategy for every subgame $Y_\mathsf{T} > 0$. The condition ensures that firms are aware of the intertemporal nature of the game, i.e. firms take into account that they have to reconsider tomorrow what is currently supposed to be the optimal action at some future point in time if they do not move with probability one today. In our case this requires $G_i^\mathsf{T}(\mathsf{T}+v)$ to be greater or equal than $G_i^{\mathsf{T}+u}(\mathsf{T}+v)$ for $0 < u \leq v$. Finally, let us define what is meant by a subgame perfect equilibrium of the timing game.

Definition 4 *A pair of closed-loop strategies $\{G_1^\mathsf{T}(\cdot,\cdot)\}_{Y_\mathsf{T}>0}$ and $\{G_2^\mathsf{T}(\cdot,\cdot)\}_{Y_\mathsf{T}>0}$ is a subgame perfect equilibrium if for every $Y_\mathsf{T} > 0$ the simple strategies $G_1^\mathsf{T}(\cdot,\cdot)$ and $G_2^\mathsf{T}(\cdot,\cdot)$ are a Nash equilibrium.*

2.4 Equilibria

We start off by examining the special case where firms are identical, i.e. $C_1 = C_2$. Then our analysis is extended to the general case, $C_1, C_2 \in \mathbb{R}^+$.

2.4.1 Identical Firms

The problem of determining the equilibria of the closure timing game simplifies significantly, when identical variable costs, i.e. $C_1 = C_2 = C$, are assumed. In this special case each role's payoff function does not differ across firms,

$$L_1(\cdot) = L(\cdot)|_{C_L=C_1, C_F=C_2} = L_2(\cdot) = L(\cdot)|_{C_L=C_2, C_F=C_1},$$

$$F_1(\cdot) = F(\cdot)|_{C_L=C_1} = F_2(\cdot) = F(\cdot)|_{C_L=C_2},$$

$$S_1(\cdot,\cdot) = S_2(\cdot,\cdot) = S(\cdot,\cdot) \quad \text{and} \quad I_1(\cdot) = I_2(\cdot) = I(\cdot),$$

and the reduction thresholds can be ordered as

$$Y_F < Y_M = Y_L < Y_{S_1} = Y_{S_2} = Y_{S^*}$$

in the case of first-mover advantages (2.4), and

$$Y_M < Y_F = Y_L < Y_{S_1} = Y_{S_2} = Y_{S^*}$$

in the case of second-mover advantages (2.5). The following Lemma 4 suggests an ordering of the roles' payoff functions that will turn out to be crucial in the derivation of the equilibria of the timing game.

Lemma 4 *Let $C = C_1 = C_2$. (i) Then it holds that*

$$\begin{aligned} L(Y_\mathsf{T}) &< I(Y_\mathsf{T}) \leq S(Y_\mathsf{T}, Y_{S^*}) < F(Y_\mathsf{T}) \quad \text{for } Y_\mathsf{T} > Y_F, \\ L(Y_\mathsf{T}) &= I(Y_\mathsf{T}) = S(Y_\mathsf{T}, Y_{S^*}) = F(Y_\mathsf{T}) \quad \text{for } 0 \leq Y_\mathsf{T} \leq Y_F. \end{aligned}$$

(ii) $L(\cdot)$ and $F(\cdot)$ are strictly convex for $Y_\mathsf{T} > Y_F$. $S(\cdot, Y_{S^*})$ is strictly convex for $Y_\mathsf{T} > Y_{S^*}$.

Proof. See Appendix A.5. ■

The graphs of F, L, S and I are depicted in Figure 2-2. Though L might be more convex than F and/or S, as $Y_\mathsf{T} \to \infty$, F and S converge to higher "fundamental values" than L. The fundamental values are defined by

$$\lim_{Y_\mathsf{T}\to\infty} \left(F(Y_\mathsf{T}) - \frac{Y_\mathsf{T} R(\overline{q}, \underline{q})}{\delta} + \frac{C\overline{q}}{\rho} \right) = 0,$$

$$\lim_{Y_\mathsf{T}\to\infty} \left(L(Y_\mathsf{T}) - \frac{Y_\mathsf{T} R(\underline{q}, \overline{q})}{\delta} + \frac{C\underline{q}}{\rho} + E \right) = 0,$$

and

$$\lim_{Y_\mathsf{T}\to\infty} \left(S(Y_\mathsf{T}) - \frac{Y_\mathsf{T} R(\overline{q}, \overline{q})}{\delta} + \frac{C\overline{q}}{\rho} \right) = 0.$$

Recall that $R(\overline{q}, \underline{q}) > R(\overline{q}, \overline{q}) > R(\underline{q}, \overline{q})$. The fundamental values are depicted by dashed lines in Figure 2-2.

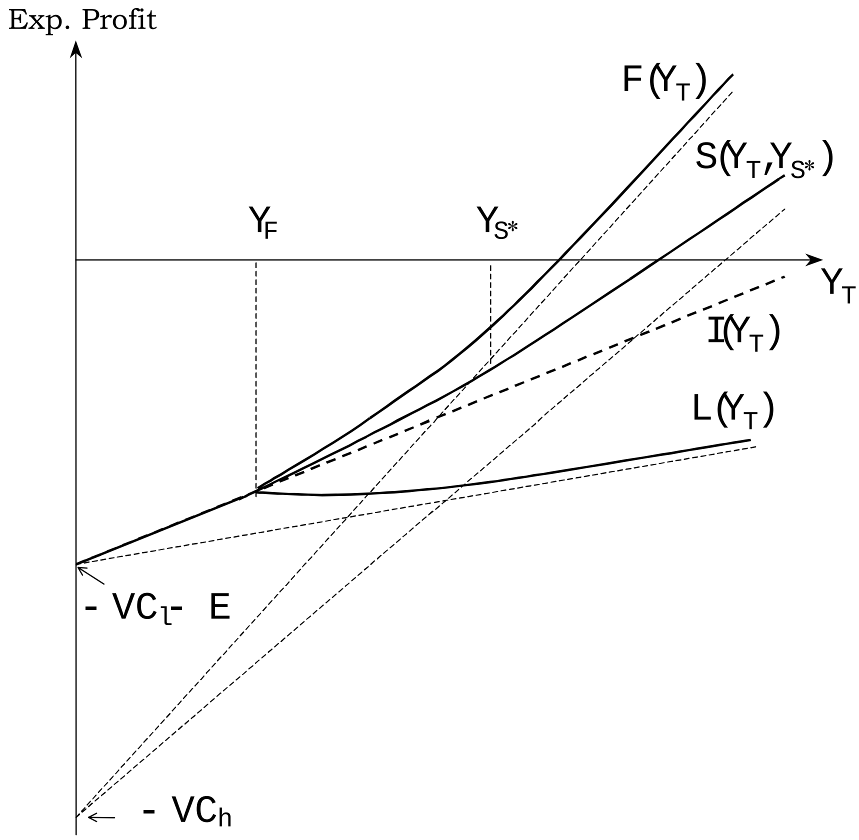

Figure 2-2: Graphs of F, L, S, and I for Identical Firms; $VC_l = \frac{C}{\rho}\underline{q}$ and $VC_h = \frac{C}{\rho}\overline{q}$.

It seems as if the subgame perfect equilibria of this game could be derived just by having a closer look at Figure 2-2. Suppose that for any subgame starting at $\mathsf{T} < T_F$ or, equivalently, for any current position $Y_\mathsf{T} > Y_F$ of the stochastic process Y, there exists a firm choosing the leader's role with positive probability. The ordering of payoff functions depicted in Figure 2-2 suggests that such a strategy will never be played in an equilibrium of the timing game, since $F(Y_\mathsf{T}) > L(Y_\mathsf{T})$ implies that the potential leader has an incentive to deviate, i.e. to postpone capacity reduction and to refrain from giving up market shares. Because $F(Y_\mathsf{T}) > S(Y_\mathsf{T}, Y_{S^*})$ holds for $\mathsf{T} < T_F$, the same line of arguments seems to apply to any strategy profile that induces simultaneous capacity reduction. Each firm has an individual incentive to deviate from simultaneous closure, thereby attracting some of the rival's market shares. Thus, our graphical approach points to the following equilibrium outcome: Both firms decide to wait, until either the rival chooses to play the leader's role or market conditions become unfavorable enough, so that firms are forced to partially leave the market irrespective of what the rival does. We guess that the latter scenario arises, when $Y(t)$ first hits Y_F, since $F(Y_F) = L(Y_F) = I(Y_F)$, i.e. the firms are indifferent between leading, following and moving simultaneously at Y_F. So, simultaneous closure at Y_F seems to be a natural equilibrium candidate.

However, the graphs do not capture the costs of *not* assigning roles to players that are captured by the ex-ante-to-closure revenues in the firms' payoff functions (see line two of equation (2.33). If $Y(t)$ is close to Y_F, market conditions are sufficiently bad, so that the revenues ex ante to capacity reduction become negative. Thus, at each instant of time, at which firms do not "agree upon" the equilibrium assignment of roles, they accumulate losses from currently producing at the high capacity level. The question arises whether these "costs of coordination failure" are high enough to make at least one firm prefer the leader's role and to offset the opportunity cost of the foregone market shares. The following proposition indicates that the answer is closely linked to the presence of first-mover versus second-mover advantages.

Proposition 3 *(i) The asymmetric strategy profiles* $\left(G_1^{\top}(t), G_2^{\top}(t)\right) = \left(G_1(t), G_2(t)\right)$ *such that*

$$G_i(t) = \begin{cases} 0 & \text{for } t < T_M, \\ 1 & \text{for } t \geq T_M, \end{cases} \quad \text{and} \quad G_j(t) = \begin{cases} 0 & \text{for } t < T_F, \\ 1 & \text{for } t \geq T_F, \end{cases}$$

for $i, j \in \{1, 2\}$, $i \neq j$, *are subgame perfect equilibria of the capacity-reduction game with identical firms and first-mover advantages.*

(ii) The symmetric strategy profile $\left(G_1^{\top}(t), G_2^{\top}(t)\right) = \left(G_1(t), G_2(t)\right)$ *such that*

$$G_i(t) = \begin{cases} 0 & \text{for } t < T_F, \\ 1 & \text{for } t \geq T_F, \end{cases}$$

for $i \in \{1, 2\}$, *is a subgame perfect equilibrium of the capacity-reduction game with identical firms and second-mover advantages.*

Proof. See Appendix A.6. ■

Thus, in the case of second-mover advantages the costs of coordination failure do not destroy the intuition that we gained from interpreting Figure 2-2. The reason is that with second-mover advantages the threat of losing market shares carries more weight. First-mover advantages that work into the same direction as the costs of coordination failure, however, induce a strikingly different set of equilibria. Firms choose a kind of maximum differentiation outcome[16], i.e. one obtains two asymmetric pure strategy equilibria, where firm i chooses the leader's role at the myopic firm's threshold Y_M, while firm j follows and reduces capacity at Y_F. Recall that Y_F and $Y_L = Y_M$ were shown to represent the optimal closure rules of firms that get preassigned to the corresponding roles in Lemma 1 and Proposition 1, respectively.

The following nonexistence results considerably enforce the predictive power of the pure strategy subgame perfect equilibria presented in Proposition 3.

Proposition 4 *(i) In the capacity-reduction game with identical firms and first-mover advantages there does not exist any symmetric pure strategy Nash equilibrium.*
(ii) In the capacity-reduction game with identical firms and second-mover advantages there does not exist any other symmetric pure strategy Nash equilibrium beside the one given in Proposition 3.

Proof. See Appendix A.7. ■

In both cases there are additional asymmetric equilibria. Under the assumption of first-mover advantages we get a continuum of pure strategy subgame perfect equilibria $(G_1(t), G_2(t))$, where

$$G_i(t) = \begin{cases} 0 & \text{for } t < T_M, \\ 1 & \text{for } t \geq T_M, \end{cases} \quad \text{and} \quad G_j(t) = \begin{cases} 0 & \text{for } t < T', \\ 1 & \text{for } t \geq T', \end{cases},$$

$i, j \in \{1, 2\}$, $i \neq j$, $T' > T_F$. Under the assumption of second-mover advantages the equivalent set of equilibria is given by $(G_1(t), G_2(t))$ such that

$$G_i(t) = \begin{cases} 0 & \text{for } t < T_F, \\ 1 & \text{for } t \geq T_F, \end{cases} \quad \text{and} \quad G_j(t) = \begin{cases} 0 & \text{for } t < T', \\ 1 & \text{for } t \geq T', \end{cases},$$

[16] This notion is borrowed from the literature about Hotelling's spatial location game.

$i, j \in \{1, 2\}$, $i \neq j$, $T' > T_F$. However, note that each of these equilibria generates the same pattern of disinvestment timing than the corresponding equilibrium in Proposition 3. Their existence should be considered as an artefact of the specific construction of the payoff functions. Recall that players do not choose closure times but roles. And roles themselves are assumed to time the reduction of capacity optimally. Particularly, the follower finds it optimal to move at Y_F conditionally on the leader having moved before or at Y_F. Since the assignment of the follower's role is determined by the leading player's strategy and since moving first yields the same payoff than moving second or moving simultaneously at all closure thresholds $Y_T \leq Y_F$, one can find a continuum of equilibrium strategies of the follower, though we have a unique equilibrium outcome. We stress that there does not exist any other asymmetric pure strategy equilibrium than these "artificial" ones.

The subsequent proposition excludes mixed-strategy equilibria.

Proposition 5 *(i) In the capacity-reduction game with identical firms and first-mover advantages there does not exist any symmetric non-degenerate mixed strategy equilibrium where strategies are restricted to be discrete lotteries over the space of thresholds.*
(ii) In the capacity-reduction game with identical firms and second-mover advantages there does not exist any symmetric non-degenerate mixed strategy equilibrium.

Proof. See Appendix A.8. ■

Another result is closely related to Proposition 3.

Proposition 6 *Suppose that strategic interaction of firms is driven by second-mover advantages. Suppose further that firms play the unique symmetric equilibrium strategy profile suggested in part (ii) of Proposition 3. Let $V_i^*(Y_\mathsf{T})$ denote the equilibrium payoff of firm i, $i \in \{1, 2\}$. Then*

$$
\begin{aligned}
L(Y_\mathsf{T}) &< V_i^*(Y_\mathsf{T}) < S(Y_\mathsf{T}, Y_{S^*}) \quad \text{for } Y_\mathsf{T} > Y_F, \\
L(Y_\mathsf{T}) &= V_i^*(Y_\mathsf{T}) = S(Y_\mathsf{T}, Y_{S^*}) \quad \text{for } 0 \leq Y_\mathsf{T} \leq Y_F,
\end{aligned}
$$

where $Y_{S^*} = Y_{S_i}|_{C_i = C}$ and Y_{S_i} is defined by equation (2.27).

Proof. See Appendix A.9. ■

In the case of second-mover advantages Proposition 6 implies that firms would Pareto-improve their position if they were able to coordinate on reducing capacity simultaneously at some threshold $Y_S = \min\{Y_\mathsf{T}, Y_{S^*}\} > Y_F$. However, joint disinvestment at Y_S never prevails in equilibrium, since the net payoffs from deviating and delaying disinvestment are positive. Especially, firms cannot commit to the cooperative optimum Y_{S^*}. For that reason one may characterize the capacity-reduction timing game with identical firms and second-mover advantages as a prisoners' dilemma. Interestingly, we were not able to establish an equivalent result in the case of first-mover advantages. Though the sum of the duopolists' values would be maximized if they could coordinate on playing the monopolist's optimal *sequential* disinvestment program, this strategy is not necessarily Pareto-optimal. Of course, Pareto-optimality can be achieved by allowing for side payments.

2.4.2 Heterogeneous Firms

In this subsection we return to the original model by relaxing the assumption of identical costs, i.e. C_1 might be different from C_2.

Lemma 5 *Let $C_1, C_2 \in \mathbb{R}^+$, $L_i(\cdot) = L(\cdot)|_{C_L=C_i, Y_F=Y_{F_j}}$, $F_i(\cdot) = F(\cdot)|_{C_F=C_i, Y_F=Y_{F_i}}$, and $Y_{F_i} = Y_F|_{C_F=C_i}$, $i \neq j$, $i, j \in \{1, 2\}$. Further, let $T_F^{\min} = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_F^{\min})$ and $Y_F^{\min} = Y_F|_{C_F=\min\{C_1,C_2\}}$.*
(i) Then it holds that

$$\begin{aligned}
L_i(Y_\mathsf{T}) &= I_i(Y_\mathsf{T}) = S_i(Y_\mathsf{T}, Y_{S_i}) = F_i(Y_\mathsf{T}) \quad \text{for } 0 \leq Y_\mathsf{T} \leq Y_F^{\min}, \\
L_i(Y_\mathsf{T}) &\leq I_i(Y_\mathsf{T}) \leq S_i(Y_\mathsf{T}, Y_{S_i}) < F_i(Y_\mathsf{T}) \quad \text{for } Y_\mathsf{T} > Y_{F_i},\ C_i < C_j, \\
L_i(Y_\mathsf{T}) &< I_i(Y_\mathsf{T}) \leq S_i(Y_\mathsf{T}, Y_{S_i}) < F_i(Y_\mathsf{T}) \quad \text{for } Y_\mathsf{T} > Y_{F_i},\ C_i > C_j.
\end{aligned}$$

(ii) $L_i(Y_\mathsf{T})$ is strictly convex for $Y_\mathsf{T} > Y_{F_j}$ and $F_i(Y_\mathsf{T})$ is strictly convex for $Y_\mathsf{T} > Y_{F_i}$. $S_i(Y_\mathsf{T}, Y_{S_i})$ is strictly convex for $Y_\mathsf{T} > Y_{S_i}$

Proof. See Appendix A.10. ■

Suppose without loss of generality that firm 2 exhibits higher variable cost than firm 1, i.e. $C_1 < C_2 \Leftrightarrow Y_{F_1} < Y_{F_2}$. The corresponding graphs of F_i, L_i, and S_i, $i \in \{1, 2\}$, are depicted in Figure 2-3.

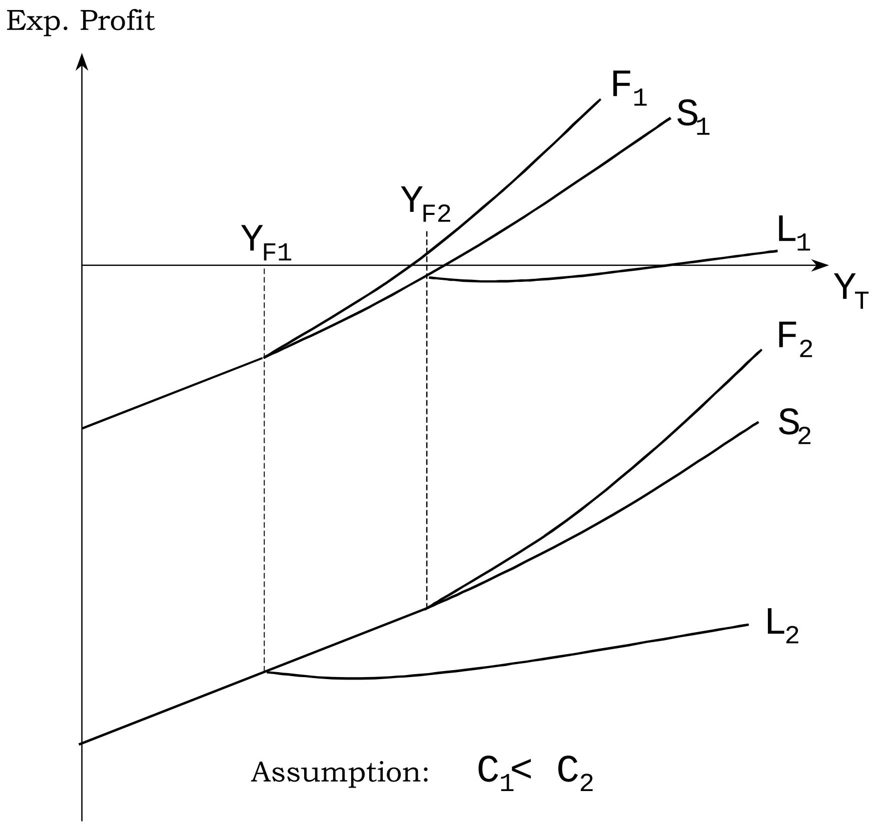

Figure 2-3: Graphs of F_i, L_i, and S_i, $i \in \{1, 2\}$, in the General Case.

Let us begin with the closure scenario in the case of second-mover advantages. If the "prisoners' dilemma effect", i.e. the payoff constellation that creates an incentive for firms to reduce capacity as late as possible, were still effective in the general model, we would expect that the competition for market shares "forces" the high-cost firm (firm 2) not to adopt the leader's role before $Y(t)$ first hits Y_{F_1} where the low-cost firm is known to follow immediately (notice that $L_2(Y_\mathsf{T}) < F_2(Y_\mathsf{T}) = S(Y_\mathsf{T}, Y_{S_2})$ for $Y_\mathsf{T} > Y_{F_1}$, but $L_2(Y_\mathsf{T}) = F_2(Y_\mathsf{T}) = S(Y_\mathsf{T}, Y_{S_2})$ for $Y_\mathsf{T} \leq Y_{F_1}$). Proposition 7 shows that this intuition is indeed applicable if the difference in variable cost is not too extreme. Nevertheless, the equilibrium qualitatively differs from the model with identical firms, even if $C_2 - C_1$ becomes arbitrarily small: The low-cost firm (firm 1) adopting the leader's role at any $Y_\mathcal{T}$, $Y_{F_1} < Y_\mathcal{T} \leq Y_{F_2}$, knows that its rival will follow immediately. Thus, firm 1 yields $S_1(Y_\mathsf{T}, Y_\mathcal{T})$ in this case. In section 2.2 we proved $S_1(Y_\mathsf{T}, Y_S)$ to be increasing in Y_S for all $Y_S < Y_{S_1}$. This implies that firm 1 always prefers to adopt the leader's role at $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$ rather than moving late at Y_{F_1} and, thereby, obtaining $S_1(Y_\mathsf{T}, Y_{F_1})$. Intuitively, its higher efficiency or more advanced production technology allows firm 1 to avoid the extreme prisoners' dilemma outcome and to realize a higher joint-reduction payoff than in the identical firm case.

If the difference in variable cost exceeds a certain upper bound, the prisoners' dilemma effect is predominated by a "cost-pressure effect" that forces the high-cost firm to avoid the ruinous run for market shares. In the proof of Proposition 1 we showed that the high-cost firm being preassigned to the leader's role finds it optimal to move at $Y_{L_2} = Y_{M_2} > Y_{F_1}$, if $C_L - C_F = C_2 - C_1$ is sufficiently large. Proposition 7 indicates that the cost-pressure effect indeed gives rise to an additional equilibrium, where firm 2 moves first at $Y_{L_2} = Y_{M_2}$ and firm 1 follows later at Y_{F_1}. However, the former equilibrium, where firm 1 reduces capacity at $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$ and firm 2 follows immediately, may still occur for a wide range of parameter values implying that we obtain a multiplicity of equilibrium outcomes. Proposition 7 says that the joint-reduction equilibrium vanishes if the degree of heterogeneity becomes even more extreme.

Proposition 7 *Let F_i, S_i, L_i, and Y_{F_i} be defined by Lemma 5, and let $Y_{M_i} = Y_M|_{C_M = C_i}$, Y_M and Y_{S_i} be defined by equations (2.20) and (2.27), respectively, $i \in \{1, 2\}$. Let $C_1 < C_2$ w.l.o.g. and suppose that condition (2.5) is satisfied. Then $\mathcal{R}_1 < 1 < \mathcal{R}_2$, $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$ and $Y_{L_2} = \max\{Y_{M_2}, Y_{F_1}\}$ according to Proposition 1. There exists a unique threshold $\widehat{Y}_1 \in (Y_{F_1}, Y_{L_1})$, $Y_\mathsf{T} \geq Y_{L_1}$, such that*

$$\mathcal{E}_\mathsf{T}\left[e^{-\rho\left(T_{L_1} - \mathsf{T}\right)} I_1(Y_{L_1})\right] =$$
$$\mathcal{E}_\mathsf{T}\left[\int_{T_{L_1}}^{\widehat{T}_1} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q}, \overline{q}) - C_1 \overline{q}\right] dt + e^{-\rho\left(\widehat{T}_1 - \mathsf{T}\right)} F_1\left(\widehat{Y}_1\right)\right] \tag{2.34}$$

with $T_{L_1} = \inf(t \geq \mathsf{T}\, |Y(t) \leq Y_{L_1})$ and $\widehat{T}_1 = \inf(t \geq \mathsf{T} \left| Y(t) \leq \widehat{Y}_1\right.)$. If $Y_{L_2} = Y_{M_2}$, then there exists a unique threshold $\widehat{Y}_2 \in (Y_{F_1}, Y_{L_2})$, $Y_\mathsf{T} \geq Y_{L_2}$, such that

$$\mathcal{E}_\mathsf{T}\left[e^{-\rho\left(T_{L_2} - \mathsf{T}\right)} L_2(Y_{L_2})\right] =$$

$$\mathcal{E}_{\mathsf{T}}\left[\int_{T_{L_2}}^{\widehat{T}_2} e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q})-C_2\overline{q}\right]dt+e^{-\rho\left(\widehat{T}_2-\mathsf{T}\right)}I_2\left(\widehat{Y}_2\right)\right] \tag{2.35}$$

with $T_{L_2}=\inf(t\geq\mathsf{T}\,|Y(t)\leq Y_{L_2})$ *and* $\widehat{T}_2=\inf(t\geq\mathsf{T}\,\middle|Y(t)\leq\widehat{Y}_2)$.

(i) If $Y_{L_1}\geq\widehat{Y}_2\vee Y_{L_2}=Y_{F_1}$*, then all strategy profiles* $(G_1^{\mathsf{T}}(t),G_2^{\mathsf{T}}(t))=(G_1(t),G_2(t))$ *such that*

$$G_1(t)=\begin{cases}0 & \text{for } t<T_{L_1},\\ 1 & \text{for } t\geq T_{L_1},\end{cases}\quad \text{and}\quad G_2(t)=\begin{cases}0 & \text{for } t<T_2',\\ 1 & \text{for } t\geq T_2',\end{cases}$$

with $T_2'=\inf(t\geq\mathsf{T}\,\middle|Y(t)\leq Y_2')\geq T_{F_1}=\inf(t\geq\mathsf{T}\,|Y(t)\leq Y_{F_1})$ *are pure strategy subgame perfect equilibria of the general capacity-reduction game.*

(ii) If $Y_{L_2}=Y_{M_2}\geq\widehat{Y}_1$*, then all strategy profiles* $(G_1^{\mathsf{T}}(t),G_2^{\mathsf{T}}(t))=(G_1(t),G_2(t))$ *such that*

$$G_1(t)=\begin{cases}0 & \text{for } t<T_1',\\ 1 & \text{for } t\geq T_1',\end{cases}\quad \text{and}\quad G_2(t)=\begin{cases}0 & \text{for } t<T_{L_2},\\ 1 & \text{for } t\geq T_{L_2},\end{cases}$$

with $T_1'=\inf(t\geq\mathsf{T}\,|Y(t)\leq Y_1')\geq T_{F_1}$ *are pure strategy subgame perfect equilibria of the general capacity-reduction game.*

(iii) If $Y_{L_2}<\widehat{Y}_1$*, then there does not exist any other pure strategy subgame perfect equilibrium of the general capacity-reduction game than those proposed in part (i).*

If $Y_{L_1}=Y_{S_1}<\widehat{Y}_2$*, then there does not exist any other pure strategy subgame perfect equilibrium of the general capacity-reduction game than those proposed in part (ii).*

Otherwise all elements of the union of sets of equilibria proposed in part (i) and (ii) are pure strategy subgame perfect equilibria of the general capacity-reduction game.

Proof. See Appendix A.11. ■

Note that all strategy profiles with $G_2^{\mathsf{T}}(t)=\mathbb{I}_{\{t\leq T\}}$ for $T\in[\widehat{T}_1,T_{F_1})$, $\mathsf{T}\leq\widehat{T}_1$, $\mathbb{I}$ denoting the indicator function, in part *(i)* of the above proposition constitute Nash equilibria of the general timing game. But they can be excluded by the requirement of subgame perfection. The same argument applies with respect to all strategy profiles such that $G_1^{\mathsf{T}}(t)=\mathbb{I}_{\{t\leq T\}}$ for $T\in[\widehat{T}_2,T_{F_1})$, $\mathsf{T}\leq\widehat{T}_2$, in part *(ii)*. Moreover, the multiplicity of equilibria in part *(i)* and *(ii)*, respectively, can be regarded as an artefact of the specific construction of the payoff functions. We already justified that view with respect to the continuum of asymmetric equilibria in the identical firm case. Again, one can find a continuum of equilibrium strategies of the follower, though we have a unique equilibrium outcome in part *(i)* and *(ii)*, respectively.

In the case of first-mover advantages the set of equilibrium strategy profiles does not vary, but the number of possible equilibrium outcomes increases. One might guess that both asymmetric equilibria from the identical firm model carry over to the general case when just a small difference in variable cost is introduced. Note that with a small cost difference Proposition 1 suggests that the low-cost (high-cost) firm being preassigned to the leader's role finds it optimal to move at $Y_{L_1} = Y_{M_1} > Y_{F_1}$ ($Y_{L_2} = Y_{M_2} > Y_{L_1}$). In the following proposition the threshold $\widetilde{Y}_2 \in (Y_{F_1}, Y_{L_2})$ is defined such that the high-cost firm is indifferent between adopting the leader's role at Y_{L_2} or following at $\widetilde{Y}_2$. If $Y_{L_1} < \widetilde{Y}_2$, then the high-cost firm prefers to attract the first-mover advantage rather than additional market shares and *always* takes over the leader's role. However, since $Y_{L_2} \downarrow Y_{L_1} > Y_{F_1}$ and $\widetilde{Y}_2 \downarrow Y_{F_1}$ for $C_2 \downarrow C_1$, the equilibrium that assigns the leader's role to the low-cost firm survives the introduction of heterogeneity if the difference in variable cost stays sufficiently small. Thus, by slightly departing from the identical firm model one obtains both asymmetric equilibria.

When the difference in variable cost gets more significant, one might expect that the "cost-pressure effect" makes the high-cost firm less patient in the sense that an equilibrium assignment of roles with the high-cost firm becoming the follower is observed less likely. Proposition 8 reveals that this intuition indeed applies. With a high difference in variable cost Proposition 1 suggests that the low-cost firm being preassigned to the leader's role finds it optimal to move at $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\} \leq Y_{F_2}$. If $Y_{L_1} < \widetilde{Y}_2$, then the sequential disinvestment equilibrium, where the high-cost firm disinvests before its rival, is unique. If $Y_{L_1} \geq \widetilde{Y}_2$, then there exists an additional joint-reduction equilibrium where the low-cost firm adopts the leader's role at Y_{L_1} but the high-cost firm follows immediately. Thus, the other sequential disinvestment equilibrium in which the high-cost firm delays capacity reduction compared to the low-cost firm has vanished.

Proposition 8 *Let $C_1 < C_2$ w.l.o.g. and suppose that condition (2.4) is satisfied. Then $\mathcal{R}_1 < \mathcal{R}_2 < 1$, $Y_{L_1} = \min\{\max\{Y_{M_1}, Y_{F_2}\}, Y_{S_1}\}$ and $Y_{L_2} = Y_{M_2}$ according to Proposition 1. Let $\widetilde{T}_2 = \inf(t \geq \mathsf{T} \,|\, Y(t) \leq \widetilde{Y}_2)$ and let T_1' and T_2' be defined as in Proposition 7. There* *exists a unique threshold* $\widetilde{Y}_2$, $Y_{\mathsf{T}} \geq Y_{L_2}$, *such that*

$$\mathcal{E}_{\mathsf{T}}\left[e^{-\rho\left(T_{L_2}-\mathsf{T}\right)}L_2(Y_{L_2})\right] = \mathcal{E}_{\mathsf{T}}\left[\int_{T_{L_2}}^{\widetilde{T}_2} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C_2\overline{q}\right]dt + e^{-\rho\left(\widetilde{T}_2-\mathsf{T}\right)}F_2\left(\widetilde{Y}_2\right)\right] \tag{2.36}$$

if $\widetilde{Y}_2 \in (Y_{F_2}, Y_{L_2})$, *and*

$$\mathcal{E}_{\mathsf{T}}\left[e^{-\rho\left(T_{L_2}-\mathsf{T}\right)}L_2(Y_{L_2})\right] = \mathcal{E}_{\mathsf{T}}\left[\int_{T_{L_2}}^{\widetilde{T}_2} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C_2\overline{q}\right]dt + e^{-\rho\left(\widetilde{T}_2-\mathsf{T}\right)}I_2\left(\widetilde{Y}_2\right)\right] \tag{2.37}$$

if $\widetilde{Y}_2 \in (Y_{F_1}, Y_{F_2}]$.
(i) If $Y_{L_1} \geq \widetilde{Y}_2$, *then all strategy profiles* $\left(G_1^{\mathsf{T}}(t), G_2^{\mathsf{T}}(t)\right) = \left(G_1(t), G_2(t)\right)$ *such that*

$$G_i(t) = \begin{cases} 0 & \text{for } t < T_{L_i}, \\ 1 & \text{for } t \geq T_{L_i}, \end{cases} \quad \text{and} \quad G_j(t) = \begin{cases} 0 & \text{for } t < T_j', \\ 1 & \text{for } t \geq T_j', \end{cases}$$

with $T_j' \geq T_{F_1}$, $i, j \in \{1, 2\}$, $i \neq j$, *are pure strategy subgame perfect equilibria of the general capacity-reduction game.*
(ii) If $Y_{L_1} < \widetilde{Y}_2$, *then there does not exist any other pure strategy subgame perfect equilibrium of the general capacity-reduction game than the equilibria proposed in part (i) that satisfy* $i = 2$ *and* $j = 1$.

Proof. See Appendix A.12. ■

There exist parameterizations of the closure timing model with first-mover advantages where the joint-reduction outcome does not occur for any pair of cost parameters C_1, C_2, $C_1 < C_2$, such that $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$. The inequality

$$\frac{\mathcal{R}_1}{\mathcal{R}_2} - 1 - \mathcal{R}_2^{\beta-2}\left(\mathcal{R}_1 + \frac{\mathcal{R}_2}{-\beta} - \frac{1-\beta}{-\beta}\right) < 0 \tag{2.38}$$

that is derived from equation (2.36) represents a sufficient condition for the non-occurrence of the joint-reduction equilibrium. Let us briefly verify this assertion. Suppose that the model is parameterized such that Y_{L_1} attains a maximum with respect to C_1 conditionally on $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$. According to Proposition 1, this is fulfilled for $\mathcal{C} = \mathcal{R}_2 \Rightarrow Y_{L_1} = Y_{F_2}$. Given the parameterization $\mathcal{C} = \mathcal{R}_2$ the condition says that, if the right-hand side of equation (2.36) is still greater than its left-hand side at the greatest lower bound of $\widetilde{Y}_2$, $\widetilde{Y}_2 = Y_{F_2}$, then $\widetilde{Y}_2 > Y_{L_1} = Y_{F_2}$ for all C_1, C_2 such that $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$. Inequality (2.38) is certainly not satisfied if the first-mover advantage vanishes ($\mathcal{R}_2 = 1$). However, it is certainly valid if the first-mover advantage increases without bound ($\mathcal{R}_2 \to 0$). This is the result that we have expected. Intuition suggests that a higher first-mover advantage makes the occurrence of a "diffusion" equilibrium outcome—one firm adopts the leaders role, the other firm follows "later"—more likely.

As in the model with second-mover advantages there are some Nash equilibria of the supergame that do not satisfy subgame perfection. Moreover, the same multiplicity regarding the follower's equilibrium strategies arises due to the specific construction of the players' payoff functions.

A reasonable extension of the general capacity-reduction model is to allow firms not only to be heterogeneous with respect to variable cost but also with respect to initial size. We will not develop a formal model but illustrate the main issues by discussing some special cases. Let us assume that firm 1 is initially bigger than firm 2, i.e. $\overline{q}_1 > \overline{q}_2$. The crucial relation that determines whether the same qualitative results—especially, the same disinvestment sequence in equilibrium—as in Proposition 7 or Proposition 8 can be obtained is given by the ordering $Y_{F_1} < Y_{F_2}$. If $Y_{F_1} > Y_{F_2}$ holds instead, then the qualitative equilibrium outcomes still applies but with interchanged roles. So, let us compare the thresholds,

$$Y_{F_1} < Y_{F_2} \Leftrightarrow \frac{(C_2/\rho)\left(\overline{q}_2 - \underline{q}_2\right) - E_2}{(C_1/\rho)\left(\overline{q}_1 - \underline{q}_1\right) - E_1} > \frac{R_2\left(\overline{q}_2, \underline{q}_1\right) - R_2\left(\underline{q}_2, \underline{q}_1\right)}{R_1\left(\overline{q}_1, \underline{q}_2\right) - R_1\left(\underline{q}_1, \underline{q}_2\right)}, \tag{2.39}$$

where $R_i(q_i, q_j) = D(q_i + q_j) \cdot q_i$, $i, j \in \{1, 2\}$, $i \neq j$. Let us assume that the adjustment cost depend linearly on the amount of reduced capacity, $E_i = k \cdot (\overline{q}_i - \underline{q}_i)$ with k denoting the closure cost per retired unit. Further, suppose that analogous conditions on gross revenues are satisfied as in the identical firm case, i.e. $R_i(\overline{q}_i, \underline{q}_j) > R_i(\overline{q}_i, \overline{q}_j) > R_i(\underline{q}_i, \underline{q}_j)$

$> R_i(\underline{q}_i, \overline{q}_j)$.

Example 2 *Write* $\underline{q}_i = c_i\overline{q}_i$*,* $c_i \in (0,1)$*,* $i \in \{1,2\}$*. Let us consider the special case, where both firms reduce capacity by the same percentage, i.e.* $c = c_1 = c_2$*. Then, inequality (2.39) can be rewritten as*

$$\frac{(C_2/\rho) - k}{(C_1/\rho) - k} > \frac{D\left(\underline{q}_1 + \overline{q}_2\right) - cD\left(\underline{q}_1 + \underline{q}_2\right)}{D\left(\overline{q}_1 + \underline{q}_2\right) - cD\left(\underline{q}_1 + \underline{q}_2\right)} > 1$$

due to $\underline{q}_1 + \overline{q}_2 < \overline{q}_1 + \underline{q}_2$*. Thus, if firm 1's production technology is not sufficiently better than firm 2's technology, then* $Y_{F_1} > Y_{F_2}$*. According to Proposition 7, in the case of second-mover advantages, we either obtain a sequential equilibrium with the bigger firm closing first or a joint-reduction equilibrium. With first-mover advantages (see Proposition 8) an additional equilibrium with the smaller firm disinvesting first may prevail, though its occurrence gets less likely if the difference between firms becomes more significant. Though the set-up is different from the market exit model by Ghemawat and Nalebuff (1985), the result turns out to be similar. Their model suggests that the bigger firm quits first in the absence of economics of scale. The authors point out that considerable economics of scale may soften or even reverse this equilibrium outcome.*

Example 3 *Next let us assume that* $\underline{q}_1 = \underline{q}_2 = \underline{q} > 0$*. This setting is much closer to the model by Ghemawat and Nalebuff than the one above. Notice that* $\underline{q}_1 = \underline{q}_2 = \underline{q} = 0$ *would represent an exit scenario. Nevertheless, it becomes significantly less likely that the bigger firm moves first under the new assumption. This can be seen by applying the assumption to inequality (2.39). We obtain*

$$\frac{(C_2/\rho) - k}{(C_1/\rho) - k} > \frac{D\left(\overline{q}_2 + \underline{q}\right)\overline{q}_2 - D\left(2\underline{q}\right)\underline{q}}{D\left(\overline{q}_1 + \underline{q}\right)\overline{q}_1 - D\left(2\underline{q}\right)\underline{q}} \cdot \frac{\overline{q}_1 - \underline{q}}{\overline{q}_2 - \underline{q}}.$$

The right-hand side of this inequality is smaller than or equal to one if and only if

$$D\left(\overline{q}_2 + \underline{q}\right)\overline{q}_2\left(\overline{q}_1 - \underline{q}\right) - D\left(\overline{q}_1 + \underline{q}\right)\overline{q}_1\left(\overline{q}_2 - \underline{q}\right) - D\left(2\underline{q}\right)\underline{q}\left(\overline{q}_1 - \overline{q}_2\right) \leq 0.$$

Note that the first term is greater than the second one if $D(\cdot)$ *is downward slopping. Nevertheless, due to the third term the inequality might well be satisfied given that* $\underline{q} > 0$*.*

In this case the difference in variable cost does not have any effect on the qualitative equilibrium outcome that always excludes the bigger firm from moving first in the case of second-mover advantages. According to Proposition 7, either the smaller firm moves first or both firms reduce capacity simultaneously even if the difference in variable cost $C_2 - C_1 > 0$ *becomes arbitrarily small. The same equilibrium outcomes prevail in the closure timing model with first-mover advantages if the difference in initial size* $\overline{q}_1 - \overline{q}_2 > 0$ *is not too small. These results contrast with the findings by Ghemawat and Nalebuff.*

2.5 Welfare Analysis

In this section we briefly address some welfare issues for the special case of identical firms (or production units). The social planner equipped with a chain of two closure options—one option to reduce capacity from $2\overline{q}$ to $\overline{q}+\underline{q}$ and a subsequent option to reduce capacity from $\overline{q}+\underline{q}$ to $2\underline{q}$—maximizes consumers' surplus net of production cost by timing the exercise of these options optimally.[17] Principally, the social planner may decide to reduce capacity sequentially or to shut down both production units at the same time. That is, either he chooses two distinct closure thresholds Y'_W and Y''_W, $Y'_W > Y''_W$, or he exercises both options simultaneously at Y_W. Suppose that a sequential disinvestment pattern is socially optimal. Then Y'_W and Y''_W are the solutions to the welfare maximization problem $\max_{Y'_W, Y''_W} W^{seq}(Y_{\mathsf{T}})$ with

$$\begin{aligned} W^{seq}\left(Y_{\mathsf{T}}\right) = \mathcal{E}_{\mathsf{T}} \Bigg[& \int_{\mathsf{T}}^{T'_W} \int_0^{2\overline{q}} e^{-\rho(t-\mathsf{T})} \left(Y\left(t\right) D\left(q\right) - C\right) dq dt \\ & + \int_{T'_W}^{T''_W} \int_0^{\overline{q}+\underline{q}} e^{-\rho(t-\mathsf{T})} \left(Y\left(t\right) D\left(q\right) - C\right) dq dt \\ & + \int_{T''_W}^{\infty} \int_0^{2\underline{q}} e^{-\rho(t-\mathsf{T})} \left(Y\left(t\right) D\left(q\right) - C\right) dq dt \\ & - \left(e^{-\rho\left(T'_W - \mathsf{T}\right)} + e^{-\rho\left(T''_W - \mathsf{T}\right)} \right) E \Bigg], \end{aligned}$$

[17] Since we assumed constant marginal cost of production, maximizing consumers' surplus is equivalent to maximizing total welfare.

$T_W' = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_W')$ and $T_W'' = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_W'')$ and ρ denoting the social discount rate. According to earlier findings we can explicitly express the welfare function in terms of the closure thresholds,

$$\begin{aligned} W^{seq}(Y_\mathsf{T}) &= \frac{Y_\mathsf{T}}{\delta}\int_0^{2\overline{q}} D(q)\,dq - \frac{C\cdot 2\overline{q}}{\rho} \\ &- \left(\frac{Y_W'}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{Y_W'}{\delta}\int_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)\,dq - \frac{C\left(\overline{q}-\underline{q}\right)}{\rho} + E\right) \\ &- \left(\frac{Y_W''}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{Y_W''}{\delta}\int_{2\underline{q}}^{\overline{q}+\underline{q}} D(q)\,dq - \frac{C\left(\overline{q}-\underline{q}\right)}{\rho} + E\right). \end{aligned}$$

The first-order condition with respect to Y_W',

$$\frac{1-\beta}{\delta}\left(\frac{Y_W'}{Y_\mathsf{T}}\right)^{-\beta}\left(\int_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)\,dq - \frac{-\beta\delta}{1-\beta}\frac{(C/\rho)\left(\overline{q}-\underline{q}\right) - E}{Y_W'}\right) = 0,$$

is satisfied for $Y_W' = 0$, $\beta < -1$, and

$$Y_W' = \frac{-\beta\delta}{1-\beta}\frac{(C/\rho)\left(\overline{q}-\underline{q}\right) - E}{\int_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)\,dq}. \tag{2.40}$$

The second-order condition reveals that only the second value is a maximizer of social welfare. With respect to Y_W'' a similar result is obtained. The maximizing argument turns out to be

$$Y_W'' = \frac{-\beta\delta}{1-\beta}\frac{(C/\rho)\left(\overline{q}-\underline{q}\right) - E}{\int_{2\underline{q}}^{\overline{q}+\underline{q}} D(q)\,dq}. \tag{2.41}$$

Now, suppose that the social planner finds it optimal to exercise both options simultaneously at some threshold Y_W that is the solution to the maximization problem $\max_{Y_W}$ $W^{sim}(Y_\mathsf{T})$ with

$$W^{sim}(Y_\mathsf{T}) = \mathcal{E}_\mathsf{T}[\int_\mathsf{T}^{T_W}\int_0^{2\overline{q}} e^{-\rho(t-\mathsf{T})}\left(Y(t)\,D(q) - C\right) dq dt$$

$$+\int_{T_W}^{\infty}\int_{0}^{2\underline{q}} e^{-\rho(t-\mathsf{T})}\left(Y\left(t\right)D\left(q\right)-C\right)dqdt-2e^{-\rho(T_W-\mathsf{T})}E].$$

The welfare function can be rewritten as

$$\begin{aligned} W^{sim}\left(Y_{\mathsf{T}}\right) &= \frac{Y_{\mathsf{T}}}{\delta}\int_{0}^{2\overline{q}} D\left(q\right)dq-\frac{C\cdot 2\overline{q}}{\rho} \\ &-\left(\frac{Y_W}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_W}{\delta}\int_{2\underline{q}}^{2\overline{q}} D\left(q\right)dq-\frac{2C\left(\overline{q}-\underline{q}\right)}{\rho}+2E\right). \end{aligned}$$

The first-order condition with respect to Y_W,

$$\frac{(1-\beta)}{\delta}\left(\frac{Y_W}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\int_{2\underline{q}}^{2\overline{q}} D\left(q\right)dq-\frac{-2\beta\delta}{1-\beta}\frac{\left(C/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{Y_W}\right)=0,$$

is satisfied for $Y_W=0$, $\beta<-1$, and

$$Y_W=\frac{-2\beta\delta}{1-\beta}\frac{\left(C/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{\int_{2\underline{q}}^{2\overline{q}} D\left(q\right)dq}. \tag{2.42}$$

The second-order condition reveals that only the second value is a maximizer of social welfare. As one would expect, we have $Y_W=Y_W'=Y_W''$, if and only if $\int_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)dq = \int_{2\underline{q}}^{\overline{q}+\underline{q}} D(q)dq$. A first conclusion about the socially optimal timing of capacity reduction can be drawn:

Proposition 9 *The social planner exercises the closure options sequentially at Y_W' and Y_W'' as given by equations (2.40) and (2.41), respectively, if*

$$\int_{\overline{q}+\underline{q}}^{2\overline{q}} D\left(q\right)dq<\int_{2\underline{q}}^{\overline{q}+\underline{q}} D\left(q\right)dq.$$

He exercises both options simultaneously at Y_W as given by equation (2.42), if

$$\int_{\overline{q}+\underline{q}}^{2\overline{q}} D\left(q\right)dq\geq\int_{2\underline{q}}^{\overline{q}+\underline{q}} D\left(q\right)dq.$$

As the intervals $[2\underline{q}, \overline{q}+\underline{q}]$ and $[\overline{q}+\underline{q}, 2\overline{q}]$ are of equal length, a sufficient condition for the first inequality to be satisfied is a downward slopping inverse demand function on $[2\underline{q}, 2\overline{q}]$. Since we expect $D(q)$ to be well-behaved in this sense, we conclude that a sequential disinvestment pattern is likely to be observed under the social planner's regime.

Next, let us compare the socially optimal closure thresholds with the disinvestment thresholds Y_P, Y_F, Y_M, Y_{S^*}, Y_O' and Y_O'' which are known from equations (2.21), (2.13), (2.20), (2.27), (2.29) and (2.30), respectively. Note that the denominators of Y_W' and Y_W'', $\int_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)dq$ and $\int_{2\underline{q}}^{\overline{q}+\underline{q}} D(q)dq$, can be rewritten as $\int_c^1 D((1+\zeta)\overline{q})\overline{q}d\zeta$ and $\int_c^1 D((\zeta+c)\overline{q})\overline{q}d\zeta$ with $c = \underline{q}/\overline{q} \in (0,1)$. Further assume that D is a downward slopping C^1 function. To order Y_W' and the price taker's threshold Y_P by size, we compare the denominators,

$$Y_P \gtreqless Y_W' \quad \Leftrightarrow \quad \int\limits_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)\, dq \gtreqless D(2\overline{q})\left(\overline{q}-\underline{q}\right)$$
$$\Leftrightarrow \quad \int\limits_{c}^{1} D\left((1+\zeta)\,\overline{q}\right)\overline{q}d\zeta \gtreqless D(2\overline{q})\,(1-c)\,\overline{q}.$$

For $c \to 1$, both sides of the latter inequality converge to zero, i.e. $\lim_{c\to 1} Y_P = \lim_{c\to 1} Y_W' = +\infty$. However, for $c \to 0$ and D being strictly decreasing, the area under the curve of $D(q)$ given by the resulting left-hand side expression, $\int_{\overline{q}}^{2\overline{q}} D(q)dq$, becomes strictly greater than the area of the rectangle $D(2\overline{q})\overline{q}$ on the inequality's right-hand side. Moreover, by Leibniz' rule,

$$\frac{d\left[\int\limits_c^1 D\left((1+\zeta)\,\overline{q}\right)\overline{q}d\zeta\right]}{dc} = -D\left((1+c)\,\overline{q}\right)\overline{q}$$
$$< \frac{d\left[D(2\overline{q})\,(1-c)\,\overline{q}\right]}{dc} = -D(2\overline{q})\,\overline{q} = const.$$

for all $c \in (0,1)$. The derivatives are equal for $c \to 1$. This implies that $Y_W' > Y_P$ holds given a minimal set of assumptions on the inverse demand function D. The relevant thresholds in the case of second-mover advantages can be ordered as follows,

$$0 < Y_W'' < Y_W' < Y_P < Y_M < Y_F < Y_{S^*}. \tag{2.43}$$

In the case of first-mover advantages it should be analyzed whether the social planner delays his disinvestment decision as compared to the equilibrium follower, since we detected some ambiguity in the ordering of thresholds. We impose the additional assumption that $\lim_{m\to+\infty} D(m)\cdot m = 0$. Further, note that the denominator of Y_W'', $\int_{2\underline{q}}^{\overline{q}+\underline{q}} D(q)dq$, can be rewritten as $\int_1^{\widetilde{c}} D((1+\zeta)\underline{q})\underline{q}d\zeta$ with $\widetilde{c}=\overline{q}/\underline{q}\in(1,+\infty)$. To compare Y_W'' with the follower's threshold Y_F, we write

$$\begin{aligned} Y_F \gtreqless Y_W'' &\Leftrightarrow \int_{2\underline{q}}^{\overline{q}+\underline{q}} D(q)\,dq \gtreqless D\left(\overline{q}+\underline{q}\right)\overline{q} - D\left(2\underline{q}\right)\underline{q} \\ &\Leftrightarrow \int_1^{\widetilde{c}} D\left((1+\zeta)\,\underline{q}\right)\underline{q}d\zeta \gtreqless D\left((1+\widetilde{c})\,\underline{q}\right)\widetilde{c}\underline{q} - D\left(2\underline{q}\right)\underline{q}. \end{aligned}$$

For $\widetilde{c}\to 1$, both sides of the latter inequality converge to zero, i.e. $\lim_{\widetilde{c}\to 1} Y_F = \lim_{\widetilde{c}\to 1} Y_W'' = +\infty$. However, for $\widetilde{c}\to+\infty$, D being strictly decreasing and $m = o(1/D(m))$, the resulting left-hand side integral, $\int_1^{+\infty} D((1+\zeta)\underline{q})\underline{q}d\zeta$, exists and is strictly positive, while the expression on the right-hand side converges to $-D(2\underline{q})\underline{q}<0$. By Leibniz' rule,

$$\begin{aligned} &\frac{d\left[\int_1^{\widetilde{c}} D\left((1+\zeta)\,\underline{q}\right)\underline{q}d\zeta\right]}{d\widetilde{c}} = D\left((1+\widetilde{c})\,\underline{q}\right)\underline{q} \\ > &\frac{d\left[D\left((1+\widetilde{c})\,\underline{q}\right)\widetilde{c}\underline{q} - D\left(2\underline{q}\right)\underline{q}\right]}{d\widetilde{c}} = D'\left((1+\widetilde{c})\,\underline{q}\right)\widetilde{c}\underline{q}^2 + D\left((1+\widetilde{c})\,\underline{q}\right)\underline{q} \end{aligned}$$

for all $\widetilde{c}\in(1,+\infty)$. This implies that $Y_W''>Y_F$ holds if the inverse demand function D satisfies the imposed conditions.

Next, let us compare Y_W' and Y_F. We introduce the additional assumption that consumers have a finite reservation price, i.e. $D(0)<+\infty$. One obtains

$$\begin{aligned} Y_F \gtreqless Y_W' &\Leftrightarrow \int_{\overline{q}+\underline{q}}^{2\overline{q}} D(q)\,dq \gtreqless D\left(\overline{q}+\underline{q}\right)\overline{q} - D\left(2\underline{q}\right)\underline{q} \\ &\Leftrightarrow \int_c^1 D\left((1+\zeta)\,\overline{q}\right)\overline{q}d\zeta \gtreqless D\left((1+c)\,\overline{q}\right)\overline{q} - D\left(2c\overline{q}\right)c\overline{q}. \end{aligned}$$

For $c \to 1$, both sides of the latter inequality converge to zero, i.e. $\lim_{c\to 1} Y_F = \lim_{c\to 1} Y'_W = +\infty$. However, for $c \to 0$, D being strictly decreasing and $D(0) < +\infty$, the left-hand side integral, $\int_{\overline{q}}^{2\overline{q}} D(q)dq$, becomes strictly smaller than the resulting expression on the right-hand side, $D(\overline{q})\overline{q}$. By Leibniz' rule,

$$\frac{d\left[\int\limits_c^1 D\left((1+\zeta)\overline{q}\right)\overline{q}d\zeta\right]}{dc} = -D\left((1+c)\overline{q}\right)\overline{q},$$

while the right-hand side's derivative with respect to c is given by

$$D'\left((1+c)\overline{q}\right)\overline{q}^2 - D'\left(2c\overline{q}\right)2c\overline{q}^2 - D\left(2c\overline{q}\right)\overline{q}.$$

Note that

$$\begin{aligned} & \lim_{c\to 1}\left[D'\left((1+c)\overline{q}\right)\overline{q}^2 - D'\left(2c\overline{q}\right)2c\overline{q}^2 - D\left(2c\overline{q}\right)\overline{q}\right] \\ > \; & \lim_{c\to 1}\left[-D\left((1+c)\overline{q}\right)\overline{q}\right] \Leftrightarrow -D'\left(2\overline{q}\right)\overline{q}^2 > 0, \end{aligned}$$

implying that there exists $\varepsilon > 0$ such that $Y_F > Y'_W$ for $c \in (1-\varepsilon, 1)$. However, if c gets small enough, then the reverse relation, $Y_F < Y'_W$, holds. The second derivative of the expression on the right hand-side with respect to c,

$$\overline{q}^3\left[D''\left((1+c)\overline{q}\right) - 4cD''\left(2c\overline{q}\right) - 4D'\left(2c\overline{q}\right)\right],$$

reveals that the right-hand side is strictly convex in c, if D'' is not too large in absolute value. In this case there exists a unique c^* such that $Y_F > Y'_W$ for $c > c^*$ and $Y_F < Y'_W$ for $c < c^*$. We conclude that the relevant thresholds in a closure timing model with first-mover advantages can be ordered as follows,

$$Y''_W < Y'_W < Y_P < \min\left\{Y_M, Y''_O\right\} < \max\left\{Y_M, Y''_O\right\} < Y_{S^*} < Y'_O \tag{2.44}$$

and

$$Y_F \in \left(Y''_W, \min\left\{Y_M, Y''_O\right\}\right).$$

Moreover, we point out that the more the follower's closure option resembles an exit option the more likely it gets that the follower reduces capacity after the social planner has exercised his first option. A similar result was obtained in section 2.2 with respect to the ordering of Y_M and Y_O'', Thus, if c becomes sufficiently small, we obtain the following ordering of thresholds in the model with first-mover advantages,

$$Y_W'' < Y_F < Y_W' < Y_P < Y_M < Y_O'' < Y_{S^*} < Y_O'. \tag{2.45}$$

Some conclusions can be drawn. First, by choosing a sequential rather than simultaneous disinvestment pattern, the social planner realizes the full value of owning a chain of options rather than one single option. Notice that holding a chain of options—one option to reduce capacity from $2\overline{q}$ to $\overline{q} + \underline{q}$ and a subsequent option to reduce capacity from $\overline{q} + \underline{q}$ to $2\underline{q}$—adds at least as much value to the owner's position as just holding a single option to reduce capacity from $2\overline{q}$ to $2\underline{q}$. By exercising the chain of options simultaneously the holder may always realize the same disinvestment pattern as the holder of the single option. The former option holder, however, can choose among an infinite number of additional disinvestment patterns. Thus, he cannot be worse off. In contrast to the monopolist's value function, social welfare is not exposed to forces that work via the deterministic part of the demand function and may reduce the additional option value to zero. So the natural outcome is $Y_W'' < Y_W'$.

Second, the ordering (2.45) reveals that the standard real options theory rules are not only inadequate when deriving the precise closure timing of duopolists. These rules might not even serve as benchmarks. An intuitive approach might suggest that duopolists reduce capacity earlier than price takers but later than a monopolist. According to the ordering (2.45), this intuition does not have general validity.

Third, due to the fact that $W^{seq} > W^{sim}$ and W^{sim} strictly decreases, if one raises Y_W above its optimal level (2.41), the ordering (2.43) together with $Y_W'' < Y_W < Y_W'$ implies that the closure timing of price taking and myopic firms is socially preferable to the equilibrium timing of two fully rational competitors in a duopoly with second-mover advantages. Moreover, we conclude that the cooperative outcome Y_{S^*} must be even worse in terms of social welfare. The picture appears to be more ambiguous in the case of first-mover advantages (see the ordering (2.44) and (2.45)). There might exist extreme scenarios—especially, if firms' closure decisions are almost equivalent to exit scenarios—where (sequential) disinvestment in duopoly occurs even later on average than disinvestment in a market with price-taking firms. In these cases duopolistic competition leads to a disinvestment pattern that might well be socially preferable to the closure behavior of price-taking firms. Of course, the closure timing of price takers compared to the monopolist's closure timing should be ranked with respect to social welfare as in the case of second-mover advantage.

These considerations have some immediate implications for policy issues. The unique symmetric equilibrium outcome Y_F in the case of second-mover advantages and the sequential disinvestment pattern (Y_F, Y_M) in the case of first-mover advantages can be seen as the best achievable cartels. Since the cooperative optima has been shown to be worse than the cartel outcomes from the point of view of the social planner, regulation authorities should adopt a restrictive approach to the assessment of mergers and cooperation in declining industries or markets with excess capacities. Our results contrast with the findings of Weeds (2000) who recommends a loose regulation policy with respect to joint ventures in R&D investment projects. We conclude that it depends on the industry's potential for future growth whether a restrictive or a loose regulation policy is socially optimal.

Chapter 3

Investment Options and Preemption Games

In contrast to the optimal exercise policies for closure options, the related timing problem of investment options in duopoly has been studied in the literature. In chapter 1 several papers that derive and investigate the equilibrium investment thresholds in duopoly were mentioned. We think that the recent working paper by Huisman and Kort (1999) represents the most advanced contribution. The authors do not restrict their analysis to pure strategies and, thereby, obtain additional equilibrium outcomes that are likely to occur in an investment timing game with two identical firms. For this reason we prefer to discuss Huisman and Kort's model rather than an alternative approach in this chapter. Note, however, that we do not confine ourselves to review their findings, but supplement some own results. First, we give formal definitions of strategy spaces and payoff functions that are missing in Huisman and Kort's paper. Second, to assess the strategic value of investment and to compare the duopoly outcome with the predictions of the standard real options theory, we investigate the behavior of a price-taking firm and of a monopolist. Third, with respect to the electric utility industry in Germany we provide evidence that a collusion-type equilibrium would prevail if the two largest utilities, RWE and Eon, entered into an investment timing game.

3.1 Model Set-Up

The model set-up resembles the one presented in section 2.1. However, some minor changes have to be introduced—in particular, with respect to strategies and payoff functions. Again, let us consider a duopolistic market in continuous time. As in section 2.1 we assume a constant output-to-capital ratio, i.e. the two risk neutral firms currently produce at the constant capacity level, $q_i = \underline{q}$, $i \in \{1, 2\}$. The competitors contemplate investment in new production facilities which irreversibly raises q_i to the higher capacity and output flow level $\overline{q}$. Firms are assumed to hold just a single investment option rather than a chain of options. By extending its capacity by $\overline{q} - \underline{q}$, firm i incurs the lump-sum cost[1] E. For ease of exposition we confine ourselves to the case of identical firms. Moreover, without loss of generality, we set the variable cost of production of firm i to zero, $C_i = C = 0$.

Firms face an inverse industry demand that is identical to the price model defined by equation (2.1). The assumptions about firm i's gross revenue function also remain the same as in section 2.1 with a single exception. Huisman and Kort (1999) stick to the standard setting used in the literature and focus on the case of first-mover advantages. So, in our notation we have

$$R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \underline{q}\right) > R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right).$$

By analogy with chapter 2, this set-up also establishes a timing game in continuous time under imperfect information. However, there is a crucial difference. While the closure timing game of chapter 2 constitutes a *war of attrition*, the investment timing game turns out to be a *preemption game* as will be explained in the next section. Fudenberg and Tirole (1985) suggest a formalization of strategy spaces and payoffs for preemption games in continuous time under *perfect* information. They describe a player's mixed strategy as a tupel $(G_i^t(s), \alpha_i^t(s))$. The first entry, $G_i^t(s)$, denotes the same cumulative probability as in chapter 2. The tupel's second entry, $\alpha_i^t(s)$, represents the "intensity" with which players move at times "just after" $G_i^t(s)$ jumps to one. That is, $\alpha_i^t(s)$ will

[1] For variables that have an analogous meaning to the quantities in the closure timing game, we use the same notation as in chapter 2. Hopefully, there is no risk of confusion.

represent the "intensity" of an "interval of consecutive atoms" according to Fudenberg and Tirole (p.391).

Extending formerly common strategy spaces for timing games by a second dimension enables us to determine properly the continuous-time analogues of the equilibria of discrete-time games of timing. Fudenberg and Tirole motivate this point by introducing the "grab-the-dollar-game" example. In what follows we present a modified version of this game.

Example 4 *Suppose that there are two firms with common discount rate $\rho = 0$ playing a game of investment timing in discrete time $s = 0, 1, 2, \ldots$. The game ends in period s if at least one firm invests in s and no firm has invested previously. Further, suppose that firm i's investment probability, $G_i^t(s)$, does not depend on s and t, i.e. $G_i^t(s) = G_i = const.$, $i \in \{1, 2\}$. Then G_1 (G_2) denotes the probability that firm 1 (firm 2) plays the first row (the first column) in the matrix depicted in Table 3.1. Firm i yields a payoff of L if it becomes the leading firm in period s, i.e. the only firm that invests in the current period. In this case firm j's payoff is F, $j \in \{1, 2\}$, $j \neq i$. If both firms invest simultaneously, they are paid I. Otherwise the game is repeated. The payoffs can be ordered as $I < F < L$. Since the leader's payoff exceeds the follower's payoff, the timing game is of the preemption type. Firm i's expected payoff is given by*

$$\Lambda \cdot \sum_{t=0}^{\infty} \left[\frac{(1 - G_i)(1 - G_j)}{1 + \rho} \right]^t = \frac{\Lambda}{G_i + G_j - G_i G_j},$$

where

$$\Lambda = G_i G_j I + G_i (1 - G_j) L + (1 - G_i) G_j F.$$

The unique symmetric mixed strategy equilibrium is $(G_i, G_j) = (G^*, G^*)$ with $G^* = (L - F)/(L - I) > 0$. Consequently, a "mistake"—an equilibrium outcome where both firms invest simultaneously—occurs with probability $(G^*)^2 > 0$. Note that the probability that by any positive time a firm has tried to invest equals $\sum_{t=0}^{\infty}(1 - G^*)^{2t}[G^*(2 - G^*)] = 1$. Thus, as the period length goes to zero, $\Delta s \to 0$, the probability that investment takes place in period $s = 0$ converges to one implying that firms' strategies should be represented by a unit mass at time zero, i.e. $G_i^0(s) = 1$ for all $s \geq 0$, $i \in \{1, 2\}$. It follows that a mistake occurs almost surely at $s = 0$, though the limiting value of the probability of mistake is

Table 3.1: Payoff Matrix of Investment Game

		firm 2	
		G_2	$1-G_2$
firm 1	G_1	(I, I)	(L, F)
	$1-G_1$	(F, L)	repeat game

$\sum_{t=0}^{\infty}(1-G^*)^{2t}(G^*)^2 = G^*(2-G^*)^{-1} < 1$. Further, $G_i^0(s) = 1$ implies that the probability of firm i being the only firm that invests at $s = 0$ equals zero. However, the limiting value of that probability is represented by $\sum_{t=0}^{\infty}(1-G^*)^{2t}G^*(1-G^*) = (1-G^*)(2-G^*)^{-1} > 0$.

The example makes evident that the specification of strategy spaces and payoff functions as used in wars of attrition—such as the closure timing game of chapter 2—should not be applied in preemption games, since it does not allow the symmetric discrete-time equilibrium, where coordination failure occurs with probability less than one, to "survive" the transition into continuous time. According to Fudenberg and Tirole (p.390), passing to the continuous-time limit involves a "loss of information".

For this reason it was suggested to extend the strategy space known from chapter 2 by the additional function $\alpha_i^t(s)$. Let $\tau_i(t) \geq t$ denote the minimum time at which G_i^t jumps to one, that is, $G_i^t(\tau_i(t)) = 1$ and $G_i^t(s) < 1$ for $s < \tau_i(t)$. Further, let $\tau(t) = \min\{\tau_1(t), \tau_2(t)\}$. Now, consider the game that is played just after $\tau(t)$ has been reached. The payoff functions à la Fudenberg and Tirole are constructed such that the intensities $\alpha_1^t(\tau) = \alpha_2^t(\tau) = \alpha^* = [L(\tau) - F(\tau)]/[L(\tau) - I(\tau)]$ replicate the symmetric discrete-time equilibrium described in the above example in a continuous-time framework. More generally, these payoff functions ensure that firm i gets assigned to the leader's role at τ with probability $\alpha_i^t(\tau)[1-\alpha_j^t(\tau)]/\,[\alpha_1^t(\tau) + \alpha_2^t(\tau) - \alpha_1^t(\tau)\alpha_2^t(\tau)]$ and that a mistake occurs at τ with probability $\alpha_1^t(\tau)\alpha_2^t(\tau)/\,[\alpha_1^t(\tau) + \alpha_2^t(\tau) - \alpha_1^t(\tau)\alpha_2^t(\tau)]$ which is less than one if $\alpha_1^t(\tau) + \alpha_2^t(\tau) < 2$. If strategies are symmetric, one immediately obtains the corresponding limiting values as computed in Example 4. Note that it is this probability of mistake and the corresponding equilibrium outcome of coordination failure that is explicitly taken into account by Huisman and Kort (1999) but ignored by Weeds (2000) and Grenadier (1996).

Due to the fact that firms in the investment timing game introduced in this chapter have *im*perfect information about future revenues, Fudenberg and Tirole's concept has to be modified. Similar to chapter 2, the entries of the simple-strategy tupel $(G_i^{\mathsf{T}}(t,\omega), \alpha_i^{\mathsf{T}}(t,\omega))$ with $\mathsf{T}(\omega) = \inf(t \geq 0\,|Y(t,\omega) \leq Y_{\mathsf{T}})$ and $\omega \in \Omega$ do not denote ordinary functions but stochastic processes. Precise definitions that are missing in the paper by Huisman and Kort are provided in section 3.3.

3.2 Potential Roles

As in the closure timing game the firms' payoff functions are combinations of the roles' payoff functions and the firms' strategies. Thus, as a prerequisite for describing the firms' payoffs, the roles' payoff functions have to be defined. According to Huisman and Kort (1999) the expected value of the follower's firm conditionally on the process Y having first reached the root of the (sub)game Y_{T} is given by[2]

$$F\left(Y_{\mathsf{T}}\right) = \begin{cases} \frac{Y_{\mathsf{T}} R\left(\underline{q},\overline{q}\right)}{\delta} + \left(\frac{Y_{\mathsf{T}}}{Y_F}\right)^{\beta}\left(\frac{Y_F\left(R(\overline{q},\overline{q})-R\left(\underline{q},\overline{q}\right)\right)}{\delta} - E\right) & \text{for } 0 \leq Y_{\mathsf{T}} \leq Y_F, \\ \frac{Y_{\mathsf{T}} R(\overline{q},\overline{q})}{\delta} - E & \text{for } Y_{\mathsf{T}} > Y_F, \end{cases} \tag{3.1}$$

where the optimal exercise rule for the follower's investment option is given by the unique fixed threshold

$$Y_F = \frac{\beta}{\beta - 1} \cdot \frac{\delta E}{R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right)}, \tag{3.2}$$

and

$$\beta = \beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{3.3}$$

The "markup" factor $\beta/(\beta - 1)$ that makes the difference between the follower's optimal investment trigger, Y_F, and the corresponding threshold suggested by the net-present-value (NPV) rule, $Y_F^{NPV} = \delta E/[R(\overline{q},\overline{q}) - R(\underline{q},\overline{q})]$, is strictly greater than one. Thus, by adhering to the NPV rule, a firm ignores the inherent uncertainty about future revenues and the option value of waiting, and invests to early. In contrast to the corresponding parameter in the closure timing model, the functional form of β does not depend on the

[2]We do not use the original notation but adopt the notation introduced in chapter 2.

sign of $2\alpha - \sigma^2$ according to Dixit and Pindyck (1994, section 5.2). We already provided some intuition for this distinctive feature of the investment option in section 2.2.

The expected value of the leader's firm conditionally on Y having first reached Y_{T} is

$$L\left(Y_{\mathsf{T}}\right) = \begin{cases} \frac{Y_{\mathsf{T}} R\left(\overline{q}, \underline{q}\right)}{\delta} - E - \left(\frac{Y_{\mathsf{T}}}{Y_F}\right)^{\beta} \frac{Y_F\left(R\left(\overline{q}, \underline{q}\right) - R\left(\overline{q}, \overline{q}\right)\right)}{\delta} & \text{for } 0 \leq Y_{\mathsf{T}} \leq Y_F, \\ \frac{Y_{\mathsf{T}} R\left(\overline{q}, \overline{q}\right)}{\delta} - E & \text{for } Y_{\mathsf{T}} > Y_F, \end{cases} \tag{3.4}$$

where Y_F and β are given by equations (3.2) and (3.3), respectively. Since the follower's optimal action on the interval $[Y_F, \infty)$ is to invest immediately at Y_{T}, one obtains $L\left(Y_{\mathsf{T}}\right) = F\left(Y_{\mathsf{T}}\right)$ for $Y_{\mathsf{T}} \geq Y_F$. As the leader's value function in the closure timing game, $L(Y_{\mathsf{T}})$ exhibits a kink at the follower's investment threshold, Y_F.

According to Fudenberg and Tirole (1991), the defining property of a preemption game is that there are times where the leader's payoff exceeds the follower's payoff. The criterion applies to perfect-information games of timing. To show that the property is also satisfied in the investment timing game suggested by Huisman and Kort, it has to be modified to fit the imperfect-information case. This can easily be managed by switching from the (stopping) time domain to the threshold domain and computing expectations over payoffs. In contrast to section 2.3, there is no need to rewrite the roles' payoffs as functions of the leader's future stopping time, T_L. One can resort directly to the functions as defined by equations (3.1) and (3.4). Then it is sufficient to show that $L\left(Y_{\mathsf{T}}\right) > F\left(Y_{\mathsf{T}}\right)$ for some $Y_{\mathsf{T}} \in (0, Y_F)$. Huisman and Kort (1999) proved that there exists a unique threshold[3] Y_{RE} such that $L\left(Y_{\mathsf{T}}\right) < F\left(Y_{\mathsf{T}}\right)$ for $Y_{\mathsf{T}} < Y_{RE}$, $L\left(Y_{\mathsf{T}}\right) = F\left(Y_{\mathsf{T}}\right)$ for $Y_{\mathsf{T}} = Y_{RE}$, and $L\left(Y_{\mathsf{T}}\right) > F\left(Y_{\mathsf{T}}\right)$ for $Y_{\mathsf{T}} \in (Y_{RE}, Y_F)$. Thus, the game of investment timing indeed belongs to the class of preemption games.

The expected value of firm i, $i \in \{1, 2\}$, that arises from simultaneous investment at Y_S conditionally on Y having first reached Y_{T} is

$$S\left(Y_{\mathsf{T}}, Y_S\right) = \begin{cases} \frac{Y_{\mathsf{T}} R\left(\underline{q}, \underline{q}\right)}{\delta} + \left(\frac{Y_{\mathsf{T}}}{Y_S}\right)^{\beta} \left(\frac{Y_S\left(R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \underline{q}\right)\right)}{\delta} - E\right) & \text{for } 0 \leq Y_{\mathsf{T}} \leq Y_S, \\ \frac{Y_{\mathsf{T}} R\left(\overline{q}, \overline{q}\right)}{\delta} - E & \text{for } Y_{\mathsf{T}} > Y_S, \end{cases} \tag{3.5}$$

where β is given by equation (3.3). From equation (3.5) one can immediately derive the

[3]Note that RE abbreviates 'rent equalization'.

optimal joint investment trigger

$$Y_{S^*} = \frac{\beta}{\beta - 1} \cdot \frac{\delta E}{R(\overline{q}, \overline{q}) - R(\underline{q}, \underline{q})}. \tag{3.6}$$

Note that $Y_{S^*} > Y_F$ according to condition (2.3). Finally, the payoff from immediate and simultaneous investment (at Y_{T}) is given by

$$I(Y_{\mathsf{T}}) = S(Y_{\mathsf{T}}, Y_{\mathsf{T}}) = \frac{Y_{\mathsf{T}} R(\overline{q}, \overline{q})}{\delta} - E. \tag{3.7}$$

Equation (3.7) reveals that $L(Y_{\mathsf{T}}) = F(Y_{\mathsf{T}}) = I(Y_{\mathsf{T}})$ for all $Y_{\mathsf{T}} \geq Y_F$. In section 3.4 below we will explain why the relative size of $L(Y_{\mathsf{T}})$ and $S(Y_{\mathsf{T}}, Y_S)$ determines which type of equilibrium prevails in the investment timing game.

Huisman and Kort do not discuss how the standard real options theory is related to the implications of their model. Though the value of a "monopolistic" firm and its optimal investment threshold are derived, we would like to point out that this is not the monopolistic scenario corresponding to the size of the market in their model. What they label as "monopolist" actually turns out to be the myopic firm that should be well-known from chapter 2.[4] Recall that this firm is myopic in the sense that it has static expectations with respect to its rival's actions. However, in contrast to a price-taking firm it takes the impact of its own investment decision on the price process into account. Its optimal investment trigger is given by

$$Y_M = \frac{\beta}{\beta - 1} \cdot \frac{\delta E}{R(\overline{q}, \underline{q}) - R(\underline{q}, \underline{q})}, \tag{3.8}$$

for $Y_{\mathsf{T}} \leq Y_M$. Moreover, Huisman and Kort show that, if one firm is given the leader's role beforehand, then it also finds it optimal to invest at the myopic firm's threshold, i.e. $Y_L = Y_M$. Note that $Y_M < Y_F$ due to the assumption of first-mover advantages. Intuitively, for $Y_{\mathsf{T}} \in (0, Y_F)$ the leader knows that its investment decision does not affect the optimal response of the follower. Thus, he ignores the rival and acts as a myopic firm.

[4]Huisman and Kort compute the optimal exercise threshold of a firm equipped with a single option and half of the total capacity that is available in the duopoly. So, this threshold cannot be compared with the optimal exercise rules of a monopolist in that market.

In the following proposition we describe the optimal investment pattern of a price-taking firm and the monopolist.

Proposition 10 *(i) Suppose that a price-taking firm facing an exogenous price process* $P = YD(2\underline{q})$ *is equipped with the option to extend capacity from* $\underline{q}$ *to* $\overline{q}$*. Then this firm maximizes its payoff by exercising the option at*

$$Y_P = \frac{\beta}{\beta - 1} \cdot \frac{\delta E}{D\left(2\underline{q}\right)\left(\overline{q} - \underline{q}\right)} \tag{3.9}$$

if $Y_{\mathsf{T}} \leq Y_P$*, and at* Y_{T} *if* $Y_{\mathsf{T}} > Y_P$*.*

(ii) Suppose that a monopolist is equipped with a chain of options—an option to extend capacity from $2\underline{q}$ *to* $\overline{q} + \underline{q}$ *and a subsequent option to extend capacity from* $\overline{q} + \underline{q}$ *to* $2\overline{q}$*. Then the monopolist maximizes his payoff by exercising his options sequentially at*

$$Y_O' = \frac{\beta}{\beta - 1} \cdot \frac{\delta E}{R\left(\overline{q}, \underline{q}\right) + R\left(\underline{q}, \overline{q}\right) - 2R\left(\underline{q}, \underline{q}\right)} \tag{3.10}$$

and

$$Y_O'' = \frac{\beta}{\beta - 1} \cdot \frac{\delta E}{2R\left(\overline{q}, \overline{q}\right) - R\left(\overline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right)} > Y_O' \tag{3.11}$$

if $Y_{\mathsf{T}} < Y_O'$*, at* Y_{T} *and* Y_O'' *if* $Y_O' < Y_{\mathsf{T}} \leq Y_O''$*, and simultaneously at* Y_{T} *if* $Y_{\mathsf{T}} \geq Y_O''$*.*

(iii) The investment thresholds can be ordered as follows,

$$Y_P < Y_M < \min\left\{Y_F, Y_O'\right\} < \max\left\{Y_F, Y_O'\right\} < Y_{S^*} < Y_O'', \tag{3.12}$$

where Y_F*,* Y_{S^*}*, and* Y_M *are defined by equations (3.2), (3.6), and (3.8), respectively.*

Proof. See Appendix B.1. ■

As will become clear below, we are also interested in the relation between the price taker's threshold, Y_P, and the "rent equalization" threshold, Y_{RE}. The following proposition states the required ordering.

Proposition 11 *The price taker's optimal investment threshold,* Y_P*, and the threshold* Y_{RE} *that is implicitly defined by* $L(Y_{RE}) = F(Y_{RE})$*,* $Y_{RE} \in (0, Y_F)$*, satisfy*

$$Y_{RE} < Y_P.$$

Proof. See Appendix B.2. ■

The importance of Propositions 10 and 11 will become obvious in section 3.4, where the equilibria of the investment timing game are introduced. However, let us anticipate later considerations and point out that either a preemption-type equilibrium or a collusion-type equilibrium will prevail depending on the relative size of the payoffs L and S. Suppose that $Y_{\mathsf{T}} < Y_{RE}$. Then in the preemption-type equilibrium the equilibrium prediction will be that one firm becomes the leader at Y_{RE} while the other firm follows at Y_F, while the unique Pareto-dominant equilibrium of the collusion type suggests that both firms invest simultaneously at Y_{S^*}. For $Y_{\mathsf{T}} \in (Y_{RE}, Y_F)$ there exists an additional outcome in the preemption equilibrium, where firms make a mistake, i.e. both players move at Y_{T} with positive probability.

The ordering (3.12) suggest that the standard real options theory might provide an adequate approximation to the actual investment pattern observed in duopoly if the Pareto-dominant joint-investment equilibrium, Y_{S^*}, prevails. Notice that Y_O' and Y_O'' both converge to Y_{S^*} if the first-mover advantage becomes small. Thus, in this case the monopolist's sequential capacity expansion and the joint investment of the duopolists are likely to occur almost simultaneously. Of course, the approximation gets worse if the first-mover advantage becomes more significant.

In the case of preemption, where $Y_{RE} < Y_P < Y_M < Y_F$ represents the relevant ordering, an equivalent result cannot be obtained. Even if the first-mover advantage turns out to be of marginal size and approaches zero (resulting in $Y_F - Y_M \rightarrow 0$), the price taker's threshold, Y_P, may lie far away from any of the duopolistic equilibrium investment triggers, Y_{RE} and Y_F for $Y_{\mathsf{T}} < Y_{RE}$. For $Y_{\mathsf{T}} \in (Y_{RE}, Y_P)$ we additionally get the outcome with positive probability that both firms make a mistake and invest simultaneously at Y_{T}. Thus, *both* duopolists might have finished investment before the price-taking firms would prefer to invest. We conclude that, similar to the findings in the closure timing game, the rules suggested by the standard real options theory cannot serve as proper benchmarks in the sense that the speed of investment in duopoly is expected to be increased compared to the monopoly case and to slow down when switching from price-taking to fully rational firms.

3.3 Strategies and Payoffs

Huisman and Kort (1999) neither present a precise definition of strategies nor a sketch of the underlying payoff functions of firms. By modifying the formalism of Fudenberg and Tirole (1985) we fill this gap. First, let us define the notion of a "simple strategy" of the preemption-type game of investment timing.

Definition 5 *A simple strategy for firm i in the subgame starting at* $\mathsf{T} = \inf(t \geq 0\,|Y(t) \leq Y_{\mathsf{T}})$, $Y_{\mathsf{T}} > 0$, *is a pair of stochastic processes* $(G_i^{\mathsf{T}}, \alpha_i^{\mathsf{T}}) : ([0,\infty) \times (\Omega, \mathfrak{F}))^2 \rightarrow [0,1]^2$ *satisfying for all* ω*:*
(i) $G_i^{\mathsf{T}}(\cdot, \omega)$ *is non-decreasing and right-continuous,*
(ii) $\alpha_i^{\mathsf{T}}(t, \omega) > 0$ *implies* $G_i^{\mathsf{T}}(t, \omega) = 1$,
(iii) $\alpha_i^{\mathsf{T}}(\cdot, \omega)$ *is right-differentiable,*
(iv) if $\alpha_i^{\mathsf{T}}(t, \omega) = 0$ *and* $t = \inf(s \geq \mathsf{T}(\omega)\,|\alpha_i^{\mathsf{T}}(s, \omega) > 0)$, *then* $\alpha_i^{\mathsf{T}}(\cdot, \omega)$ *has positive right derivative.*

Condition *(i)* to *(iv)* represent the natural extension of the definition by Fudenberg and Tirole (1985, p.392) to the imperfect information world. In section 3.1 we provided some intuition for the above definition. Let us suppress the upper index, T, and the second functional argument, ω, of G_i and α_i for the ease of notation. Let the time of the first "interval of atoms" in firm i's strategy be defined as

$$\tau_i(\mathsf{T}) = \begin{cases} \infty & \text{if } \alpha_i(t) = 0 \ \forall t \geq \mathsf{T} \\ \inf(t \geq \mathsf{T}\,|\alpha_i(t) > 0) & \text{else} \end{cases},$$

and let $\tau(\mathsf{T}) = \min\{\tau_1(\mathsf{T}), \tau_2(\mathsf{T})\}$ with T denoting the root of the subgame. Due to condition *(ii)* in the above definition, at least one firm invests in capacity by time $\tau(\mathsf{T})$ in the subgame starting at T almost surely. Finally, let $G_i^-(t) = \lim_{s\uparrow t} G_i(s)$ and impose $G_i^-(\mathsf{T}) \equiv 0$, $i \in \{1, 2\}$, i.e. the probability that firm i moves before the (sub)game has started equals zero. Then firm i's expected payoff conditionally on Y having first reached Y_{T} is given by

$$V\left(Y_{\mathsf{T}}, (G_i, \alpha_i), (G_j, \alpha_j)\right)$$

$$
\begin{aligned}
&= \mathcal{E}_{\mathsf{T}} \left[\int_{\mathsf{T}}^{\tau(\mathsf{T})} e^{-\rho(t-\mathsf{T})} Y(t)\, R\left(\underline{q}, \underline{q}\right) (1 - G_i(t))\, (1 - G_j(t))\, dt \right. \\
&+ \int_{\mathsf{T}}^{\tau(\mathsf{T})} e^{-\rho(t-\mathsf{T})} L(Y(t))\, (1 - G_j(t))\, dG_i(t) \\
&+ \int_{\mathsf{T}}^{\tau(\mathsf{T})} e^{-\rho(t-\mathsf{T})} F(Y(t))\, (1 - G_i(t))\, dG_j(t) \qquad (3.13) \\
&+ \sum_{t<\tau(\mathsf{T})} e^{-\rho(t-\mathsf{T})} \left(G_i(t) - G_i^-(t)\right) \left(G_j(t) - G_j^-(t)\right) I(Y(t)) \\
&+ e^{-\rho(\tau(\mathsf{T})-\mathsf{T})} \left(1 - G_i^-(\tau(\mathsf{T}))\right) \left(1 - G_j^-(\tau(\mathsf{T}))\right) \times \\
&\left. \times W(\tau(\mathsf{T}), (G_i, \alpha_i), (G_j, \alpha_j)) \right],
\end{aligned}
$$

where $j \in \{1, 2\}$, $i \neq j$, and $F(\cdot)$, $L(\cdot)$, and $I(\cdot)$ are defined by equations (3.1), (3.4), and (3.7), respectively. Comparing equation (3.13) with the corresponding payoff function in Fudenberg and Tirole reveals that we have extended their concept. We account for the inherent uncertainty regarding future revenues by taking expectations contingent on the information that is available at the root of the subgame T (or, equivalently, Y_{T}).

One will not find an expression in the payoff functions à la Fudenberg and Tirole that is equivalent to the second line of equation (3.13) representing the expected revenues ex ante to investment. The reason is that in the standard timing-game literature these revenues are already captured by the roles' payoff functions, while they are treated explicitly in the model by Huisman and Kort and our approach. The third and the fourth line of this formula give firm i's expected payoff from becoming the leader and the follower, respectively, when i extends its production facilities with probability G_i, while firm j moves with probability G_j. The fifth line of (3.13) determines firm i's payoff if both firms move simultaneously with positive probability before $\tau(\mathsf{T})$ is reached. Finally, the last two lines and the related formulas for W define what firm i yields, after the first player has moved with probability one and the first interval of atoms has begun. Note that, if $\tau_i(\mathsf{T}) \neq \tau_j(\mathsf{T})$, then

$$
W(\tau, (G_i, \alpha_i), (G_j, \alpha_j))
$$

$$
= \begin{cases} \frac{G_j(\tau)-G_j^-(\tau)}{1-G_j^-(\tau)} \left[\left(1-\alpha_i(\tau)\right) F\left(Y(\tau)\right) + \alpha_i(\tau) I\left(Y(\tau)\right) \right] \\ +\frac{1-G_j(\tau)}{1-G_j^-(\tau)} L\left(Y(\tau)\right) \quad \text{for } \tau_i(\mathsf{T}) < \tau_j(\mathsf{T}), \\ \frac{G_i(\tau)-G_i^-(\tau)}{1-G_i^-(\tau)} \left[\left(1-\alpha_j(\tau)\right) L\left(Y(\tau)\right) + \alpha_j(\tau) I\left(Y(\tau)\right) \right] \\ +\frac{1-G_i(\tau)}{1-G_i^-(\tau)} F\left(Y(\tau)\right) \quad \text{for } \tau_i(\mathsf{T}) > \tau_j(\mathsf{T}), \end{cases}
$$

and, if $\tau_i(\mathsf{T}) = \tau_j(\mathsf{T})$, then

$$
\begin{aligned} & W\left(\tau, \left(G_i, \alpha_i\right), \left(G_j, \alpha_j\right)\right) \\ & = \begin{cases} I\left(Y(\tau)\right) \\ \quad \text{for } \alpha_i(\tau) = \alpha_j(\tau) = 1, \\ \frac{\alpha_i(\tau)(1-\alpha_j(\tau))L(Y(\tau))+\alpha_j(\tau)(1-\alpha_i(\tau))F(Y(\tau))+\alpha_i(\tau)\alpha_j(\tau)I(Y(\tau))}{\alpha_i(\tau)+\alpha_j(\tau)-\alpha_i(\tau)\alpha_j(\tau)} \\ \quad \text{for } 0 < \alpha_i(\tau) + \alpha_j(\tau) < 2, \\ \frac{\alpha_i'(\tau)L(Y(\tau))+\alpha_j'(\tau)F(Y(\tau))}{\alpha_i'(\tau)+\alpha_j'(\tau)} \\ \quad \text{for } \alpha_i(\tau) = \alpha_j(\tau) = 0. \end{cases} \end{aligned}
$$

In Fudenberg and Tirole W is chosen such that the payoffs from playing with intensities α_i and α_j during the first interval of atoms equal the payoffs that would arise if firms played the discrete-time game of investment timing described in Example 4 with probabilities of investment α_i and α_j. In the case that $\alpha_i(\tau) = \alpha_j(\tau) = 0$, Fudenberg and Tirole suggest a first-order (or, alternatively, higher-order) Taylor expansion. It remains to be defined what is understood by a Nash equilibrium, by a closed-loop strategy and by a subgame perfect equilibrium of the investment-timing game. However, we refrain from stating explicit definitions, since they are just straightforward modifications of the corresponding definitions in section 2.3 (see also Fudenberg and Tirole, 1985, p.392).

3.4 Equilibria

As we already mentioned, the relative size of the leader's payoff, L, compared to the payoff from joint investment, S, determines which set of equilibria arises. Huisman and Kort distinguish between the following two cases—the preemption and the collusion case:

1. There are incentives to become the leader, i.e.

$$\exists Y_{\mathsf{T}} \in (0, Y_F), \text{ such that } L(Y_{\mathsf{T}}) \geq S(Y_{\mathsf{T}}, Y_{S^*}). \tag{3.14}$$

2. There are no incentives to become the leader, i.e.

$$L(Y_{\mathsf{T}}) \leq S(Y_{\mathsf{T}}, Y_{S^*}) \quad \forall Y_{\mathsf{T}} \in (0, Y_F). \tag{3.15}$$

The shape of the roles' payoff functions in the preemption and the collusion case are depicted in Figures 3-1 and 3-2, respectively.In the preemption case each firm would

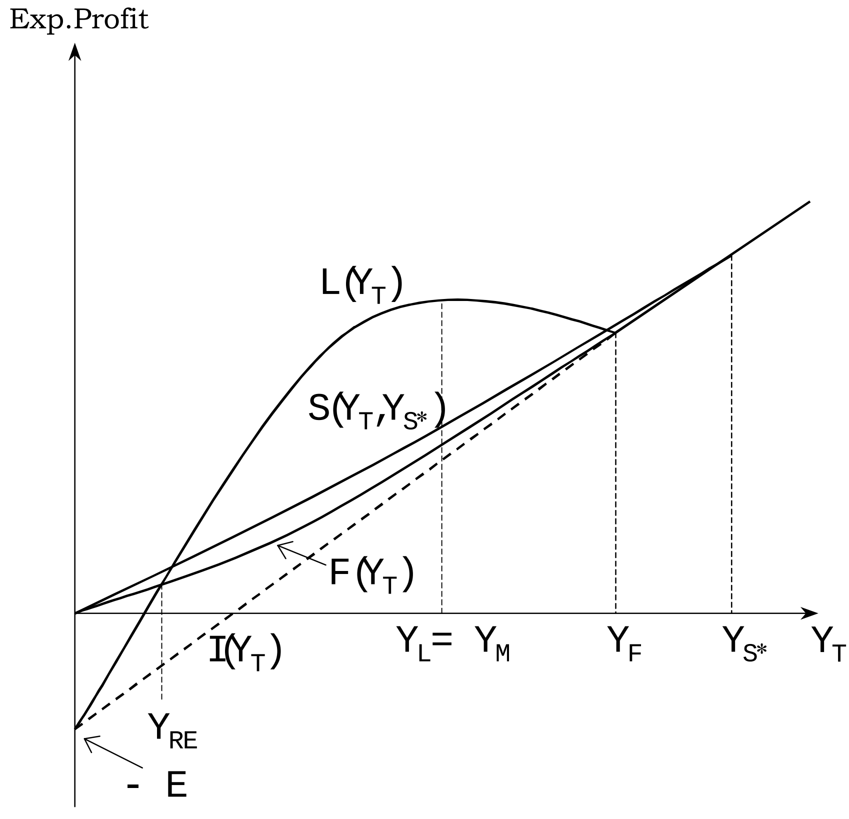

Figure 3-1: Graphs of F, L, S, and I in the Preemption Case.

like to become the leader at $Y_L = Y_M$, where the payoff from taking over the leader's role attains its maximum. Suppose firm i, $i \in \{1, 2\}$, plans to invest at Y_L. The best response of firm j, $j \in \{1, 2\}$, $j \neq i$, is to preempt firm i by investing just before firm

i at $Y_L - \varepsilon$. Certainly, firm i knows firm j's best response and, therefore, will try to invest at $Y_L - 2\varepsilon$. By backward induction, if $Y_\mathsf{T} < Y_{RE}$, the first investment option will be exercised no later than Y_{RE}, where firms are indifferent between taking over the leadership and getting assigned to the follower's role. Moreover, investment will not take place earlier than Y_{RE} due to $F(Y_T) < L(Y_T)$ for $Y_T < Y_{RE}$. This is the principle of *rent equalization* introduced by Fudenberg and Tirole (1985) that applies in the preemption-type equilibrium $(G_1^\mathsf{T}(t), \alpha_1^\mathsf{T}(t)) \times (G_2^\mathsf{T}(t), \alpha_2^\mathsf{T}(t)) = (G(t), \alpha(t))^2$ with

$$G(t) = \begin{cases} 0 & \text{for } t < T_{RE}, \\ 1 & \text{for } t \geq T_{RE}, \end{cases} \quad \text{and} \quad \alpha(t) = \begin{cases} 0 & \text{for } t < T_{RE}, \\ \frac{L(Y(t)) - F(Y(t))}{L(Y(t)) - I(Y(t))} & \text{for } T_{RE} \leq t < T_F, \\ 1 & \text{for } t \geq T_F, \end{cases} \tag{3.16}$$

and $T_{RE} = \inf(t \geq \mathsf{T} \,|Y(t) \leq Y_{RE})$. In Example 4 the intensity $\alpha = (L - F)/(L - I)$ has been shown to maximize the firms' payoffs in the discrete-time game of investment timing as given by Table 3.1. Further, note that $\alpha(T_{RE}) = 0$ implying that the probability of mistake as derived in Example 4 for symmetric strategies equals $[\alpha(T_{RE})]^2/[2-\alpha(T_{RE})] = 0$ at Y_{RE}. However, the probability that firm i is the only firm that invests at Y_{RE} equals $[1 - \alpha(T_{RE})]/[2 - \alpha(T_{RE})] = 1/2$. Thus, for $Y_\mathsf{T} \leq Y_{RE}$ there exist two equilibrium outcomes each occurring with probability 1/2. In the first scenario firm 1 takes over the leadership by investing at Y_{RE}, while firm 2 gets assigned to the follower's role and invests at Y_F. The second scenario is identical but with interchanged roles. For $Y_\mathsf{T} \in (Y_{RE}, Y_P)$ the probability of coordination failure attains positive values. Thus, either firm 1 becomes the leader at Y_T and firm 2 follows at Y_F, or roles are interchanged, or both firms erroneously invest at Y_T and, thereby, yield $I(Y_\mathsf{T}) < F(Y_\mathsf{T}) < L(Y_\mathsf{T})$. Finally, if the subgame starts beyond the optimal follower's threshold, $Y_\mathsf{T} \geq Y_F$, then joint investment is the unique outcome and firms again are paid off $I(Y_\mathsf{T})$.

In the collusion case the strategy profile given by equation (3.16) still represents a subgame perfect equilibrium of the investment timing game, but it is no longer unique. There additionally exists a continuum of equilibria involving that firms invest late and jointly. Recall that $Y_S = Y_{S^*}$ denotes the unique maximizer of the payoff from simultaneous investment, $S(Y_\mathsf{T}, Y_S)$, implying that $S(Y_\mathsf{T}, Y_S) < S(Y_\mathsf{T}, Y_{S^*})$ for all $Y_S \neq Y_{S^*}$. Now, let $\widetilde{Y} < Y_{S^*}$ be defined as the minimal threshold such that the payoff from becoming

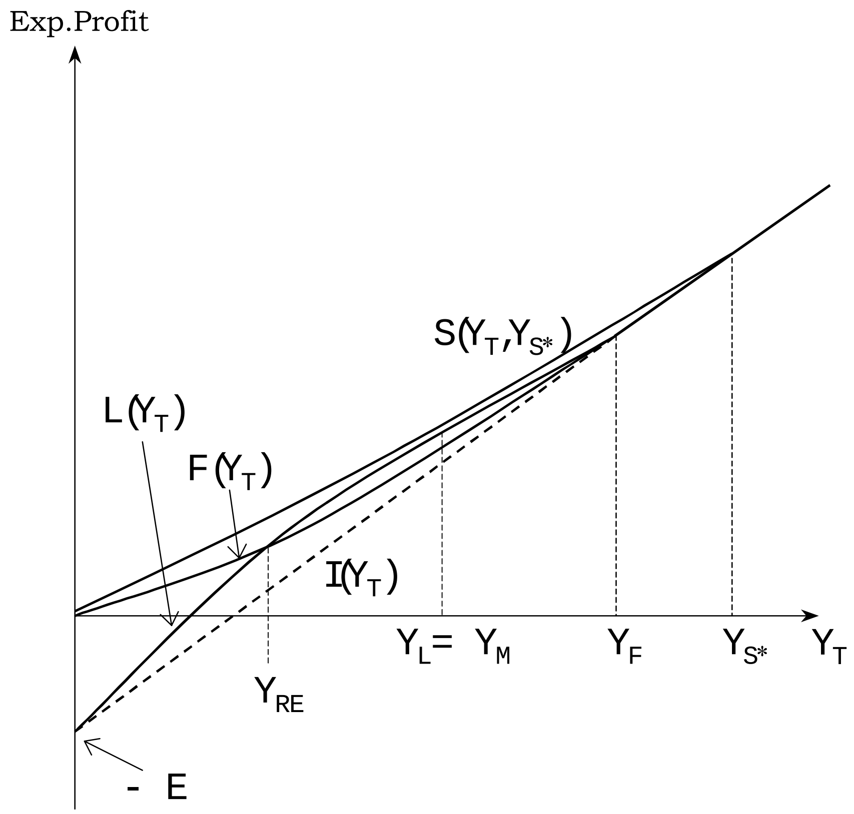

Figure 3-2: Graphs of F, L, S, and I in the Collusion Case.

the leader at Y_{T} never exceeds the payoff that arises from joint investment at $\widetilde{Y}$, i.e. $L(Y_{\mathsf{T}}) \leq S(Y_{\mathsf{T}}, \widetilde{Y}) \leq S(Y_{\mathsf{T}}, Y_{S^*})$ for all Y_{T}. As sketched in Figure 3-2, the functional forms of L and S imply that $Y_F < \widetilde{Y} \leq Y_{S^*}$. Then the continuum of collusion-type equilibria is given by the strategy profiles $(G_1^{\mathsf{T}}(t), \alpha_1^{\mathsf{T}}(t)) \times (G_2^{\mathsf{T}}(t), \alpha_2^{\mathsf{T}}(t)) = (G(t), \alpha(t))^2$ with

$$G(t) = \begin{cases} 0 & \text{for } t < T, \\ 1 & \text{for } t \geq T, \end{cases} \quad \text{and} \quad \alpha(t) = \begin{cases} 0 & \text{for } t < T, \\ 1 & \text{for } t \geq T, \end{cases} \tag{3.17}$$

$T = \inf(t \geq \mathsf{T} \,|Y(t) \leq Y_T)$ and $Y_T \in [\widetilde{Y}, Y_{S^*}]$. From the definition of Y_{S^*} it follows that $Y_T = Y_{S^*}$ Pareto-dominates all "earlier" simultaneous investment equilibria. This result prompts Huisman and Kort (1999, p.17) to call $Y_T = Y_{S^*}$ the "most reasonable outcome" in the collusion case.

Huisman and Kort determine whether the preemption or the collusion case applies depending on the size of the drift and the volatility parameter of the price function.

They find that the preemption-type (collusion-type) equilibrium occurs more likely if volatility decreases (increases) and the drift grows (diminishes). Increasing volatility and a decreasing drift raise the option value of waiting which in turn has a negative impact on the profitability of preemption, and, thereby, on the speed of investment.

Moreover, they point out that the option value of waiting is also inversely related to the firms' discount rate, ρ. That is, the profitability of waiting increases, if firms become more patient. Since the number of (sustainable) equilibrium outcomes increases when switching from the preemption case to the collusion case, this feature is consistent with the general findings by Dutta (1995). There it is shown that for a large class of dynamic games in *discrete* time, any equilibrium outcome that is sustainable by less patient players is also an equilibrium outcome when players are more patient.

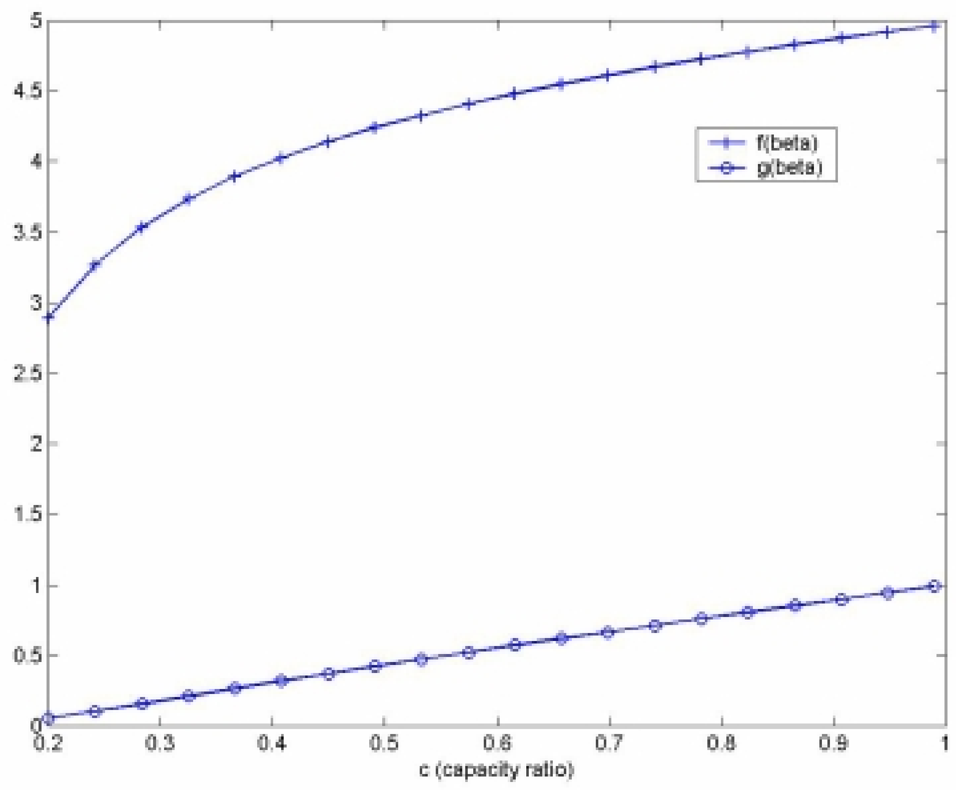

Figure 3-3: Collusion vs. Preemption—Comparison of $f(\beta_1)$ with $g(\beta_1)$.

Huisman and Kort's comparative static analysis is based on the comparison of two quantities, f and g, that are written as functions of the positive root, β_1, of the fundamental quadratic equation (see section 3.2). According to equation (3.3), β_1 depends on the variables of interest, i.e. on the drift and volatility parameters, α and σ. Huisman and Kort show that a preemption equilibrium prevails whenever $f(\beta_1) < g(\beta_1)$. Otherwise the collusion case applies. In the next chapter, in section 4.2, we provide estimates of the drift and the volatility of the price process in the German electric power market. Then, with our knowledge about the size of α and σ and by using a reasonable benchmark value for the firm's discount rate, ρ, we can compute β_1. However, the information is not sufficient to determine the sign of $f(\beta_1) - g(\beta_1)$, since f and g are also affected by the inverse demand elasticity, η, and the ratio of capacity ex ante to investment to capacity ex post to investment, $c = \underline{q}/\overline{q}$. In Figure 3-3 both quantities, f and g, are plotted against the ratio c. We implicitly use the estimate of η given in section 4.2 to approximate the elasticity's "true" value. The figure reveals that there is a significant and stable difference between f and g, i.e. $f(\beta_1) > g(\beta_1)$. We would like to consider this result as strong evidence in favor of the collusion case as the relevant scenario that applies in the German electricity sector.

Chapter 4

Empirical Issues—The Electric Power Market in Germany

The task of this chapter is to investigate whether the strategic dimension of option-timing problems as analyzed in the previous chapters bears any practical relevance for commodity traders and managers in electricity markets. One may argue that the costs of building and implementing richer real options models that properly depict the competition among utilities are likely to be higher than the gains from explicitly taking strategic interaction in the electricity sector into account. In particular, if the "true" values of the model parameters suggest that there is little scope for strategic behavior and that the strategic value of investment is small, then it might be a more recommendable strategy for utilities to act as price takers rather than fully rational players. As will be shown, this "pessimistic" view has to be rejected with respect to the German electric power market.

4.1 Best Response

Let us start off with some remarks. First, in chapter 1 we already motivated why a duopoly model with identical firms represents an adequate approximation of the German electric utility industry. Second, note that the assumption of perfect strategic substitutability of the firms' products imposed in chapters 2 and 3 appears to be highly natural in electricity markets, since almost homogeneous products are traded. Third, in this section and in section 4.4 we are not interested in equilibrium predictions but in the gain in profits that a utility yields by playing a best response to its rival rather than following the standard real options theory, i.e. the price taker's rule. Thereby, we implicitly assume that utilities usually behave as price takers. As we justified in chapter 1, this assumption about today's electricity markets does not seem to be too unrealistic. Finally, due to the fact that the electricity sector in Germany is currently equipped with a considerable amount of excess capacity (see chapter 1), we consider the optimal timing of capacity reduction to be the more urgent problem compared to the optimal timing of investment projects. Therefore, this chapter is dedicated to closure options.

To "test" the adequacy and significance of our approach, we begin with determining the best response of a duopolist (called utility 1 henceforth) who faces a price-taking rival with identical variable cost. The best response Y^* of utility 1 (that can be written more precisely as $G_1^{\mathsf{T}}(t) = \mathbb{I}_{\{t \leq T*\}}$ for $T^* = \inf(t \geq \mathsf{T} \,|Y(t) \leq Y^*))$ is the solution to the payoff maximization problem $\max_{\{Y^*\}} V_1(Y_{\mathsf{T}}, Y^*, Y_P)$ with $Y_{\mathsf{T}} > \max\{Y^*, Y_P\}$ and Y_P denoting the optimal closure threshold of a price taker (see equation (2.21)). Suppose that $Y^* > Y_P$. Clearly, this assertion has to verified below. Then

$$
\begin{aligned}
V_1\left(Y_{\mathsf{T}}, Y^*, Y_P\right) \;=\; \mathcal{E}_{\mathsf{T}} \Bigg[& \int_{\mathsf{T}}^{T^*} e^{-\rho(t-\mathsf{T})} \left[Y(t)\, R\left(\overline{q}, \overline{q}\right) - C\overline{q}\right] dt \\
& + \int_{T^*}^{T_P} e^{-\rho(t-\mathsf{T})} \left[Y(t)\, R\left(\underline{q}, \overline{q}\right) - C\underline{q}\right] dt \\
& + \int_{T_P}^{\infty} e^{-\rho(t-\mathsf{T})} \left[Y(t)\, R\left(\underline{q}, \underline{q}\right) - C\underline{q}\right] dt - e^{-\rho(T^*-\mathsf{T})} E \Bigg]
\end{aligned}
$$

according to chapter 2. From earlier derivations it is known that this conditional expectation can be rewritten as

$$
\begin{aligned}
& \frac{Y_{\mathsf{T}} R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho} \\
& - \left(\frac{Y^*}{Y_{\mathsf{T}}}\right)^{-\beta} \left(\frac{Y^*\left(R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right)}{\delta} - \frac{C\left(\overline{q} - \underline{q}\right)}{\rho} + E\right) \\
& + \left(\frac{Y_P}{Y_{\mathsf{T}}}\right)^{-\beta} \frac{Y_P\left(R\left(\underline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right)}{\delta}.
\end{aligned}
$$

By substituting $(Y_M/\delta)(R(\overline{q}, \overline{q}) - R(\underline{q}, \overline{q}))$ for $(C/\rho)(\overline{q} - \underline{q}) - E$, the derivative $\partial V_1(Y_{\mathsf{T}},$

Y^*, Y_P) $/\, \partial Y^*$ becomes

$$-\frac{R(\overline{q},\overline{q})-R(\underline{q},\overline{q})}{\delta}\cdot\left(\frac{Y^*}{Y_{\mathsf{T}}}\right)^{-\beta}\left(1-\frac{Y_M}{Y^*}\right).$$

The expression has one root at $Y^*=0$, $\beta<-1$, and another root at the myopic firm's threshold $Y^*=Y_M$. However, the second-order condition reveals that only the second root is a maximizer of $V_1(Y_{\mathsf{T}},Y^*,Y_P)$. Moreover, in chapter 2 we showed that $Y_M>Y_P$ holds in both cases—first-mover and second-mover advantages. This verifies our initial assertion.

Since we are interested in the improvement in utility 1's payoff position that it achieves by switching from the price taker's disinvestment trigger to the optimal closure threshold of the myopic firm, we compute the corresponding difference in payoffs,

$$\begin{aligned}
&V_1(Y_{\mathsf{T}},Y_M,Y_P)-V_1(Y_{\mathsf{T}},Y_P,Y_P)=\\
&-\mathcal{E}_{\mathsf{T}}\left[\int_{T_M}^{T_P}e^{-\rho(t-\mathsf{T})}\left[Y(t)\left(R(\overline{q},\overline{q})-R(\underline{q},\overline{q})\right)-C(\overline{q}-\underline{q})\right]dt\right]\\
&-\mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_M-\mathsf{T})}-e^{-\rho(T_P-\mathsf{T})}\right]\cdot E\\
&=\left(\frac{Y_M}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_M\left(R(\overline{q},\overline{q})-R(\underline{q},\overline{q})\right)}{\delta}\left[\frac{1}{-\beta}-\left(\frac{Y_P}{Y_M}\right)^{-\beta}\left(\frac{1-\beta}{-\beta}-\left(\frac{Y_P}{Y_M}\right)\right)\right]
\end{aligned}$$

Of course, this expression is strictly positive for all $Y_{\mathsf{T}}\geq Y_M$. Let us define $c=\underline{q}/\overline{q}\in(0,1)$. Then the payoff difference can be simplified to

$$\begin{aligned}
&V_1(Y_{\mathsf{T}},Y_M,Y_P)-V_1(Y_{\mathsf{T}},Y_P,Y_P)=\\
&\left(\frac{Y_M}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{-\beta}{1-\beta}\left(\frac{(1-c)\,C\overline{q}}{\rho}-E\right)\times\\
&\times\left[\frac{1}{-\beta}-\left(\frac{1-\frac{D((1+c)\overline{q})}{D(2\overline{q})}c}{1-c}\right)^{-\beta}\left(\frac{1-\beta}{-\beta}-\frac{1-\frac{D((1+c)\overline{q})}{D(2\overline{q})}c}{1-c}\right)\right].
\end{aligned}$$

The latter transformation reveals that the magnitude of the gain in profits at its maximum $Y_{\mathsf{T}}=Y_M$ depends on the marginal production cost C, on the total adjustment cost E, on the firm's discount factor ρ, on the drift and volatility parameters α and σ (via β), on the current capacity level $\overline{q}$, on the ratio of ex-post to ex-ante capacity c and on the functional form of the inverse demand function D. A numerical assessment of the impact of the proposed best response on profits requires that we find realistic values for the parameters and an adequate specification of D. A nonparametric estimation of D based on the demand equation, $P = Y \cdot D(Q)$, is out of question due to the nonstationarity of the involved variables, measurement errors in variables (see below) and the limited number of observations. Similar problems occur when trying to estimate general nonlinear parameterizations of this function. These difficulties resolve if one assumes D to be isoelastic, i.e. $D(Q) = Q^{-\eta}$ with $\eta = -D'(Q)Q/D(Q) = const. > 0$. In this case the gain in profits can be written as

$$\begin{aligned} & V_1(Y_\mathsf{T}, Y_M, Y_P) - V_1(Y_\mathsf{T}, Y_P, Y_P) = \\ & \left(\frac{Y_M}{Y_\mathsf{T}}\right)^{-\beta} \frac{-\beta}{1-\beta} \left(\frac{(1-c)\,C\overline{q}}{\rho} - E\right) \times \\ & \times \left[\frac{1}{-\beta} - \left(\frac{1-\left(\frac{2}{1+c}\right)^{\eta} c}{1-c}\right)^{-\beta} \left(\frac{1-\beta}{-\beta} - \frac{1-\left(\frac{2}{1+c}\right)^{\eta} c}{1-c}\right)\right]. \end{aligned} \tag{4.1}$$

An isoelastic inverse demand function has another appealing feature. One can easily distinguish between first-mover and second-mover advantages as the following calculations indicate. According to condition (2.4) (condition (2.5)), if

$$(\overline{q})^{1-\eta} + (\underline{q})^{1-\eta} - 2^{\eta} (\overline{q} + \underline{q})^{1-\eta}$$

is smaller (greater) than zero, then firms face first-mover advantages (second-mover advantages). Again, by expressing the ratio $\underline{q}/\overline{q}$ by the constant c, these terms can be written as

$$\left(1 + c^{1-\eta}\right) - 2^{\eta} (1+c)^{1-\eta}.$$

The difference would be zero if c was equal to one. However, $c \in (0,1)$ and

$$\frac{\partial\left[\left(1 + c^{1-\eta}\right) - 2^{\eta} (1+c)^{1-\eta}\right]}{\partial c} = (1-\eta)\, c^{-\eta} \left[1 - \left(\frac{2c}{1+c}\right)^{\eta}\right] \gtreqless 0 \text{ for } \eta \gtreqless 1.$$

Thus, with an inverse demand function given by $D(Q) = Q^{-\eta}$, the assumption of first-mover advantages (second-mover advantages) is satisfied if and only if $\eta > 1$ ($\eta \in (0,1)$).

4.2 The Empirical Model

The must crucial parameters in standard option pricing models—such as the Black and Scholes model—are the drift α that captures the rate of certain demand growth in our model and the volatility σ that represents a measure for the uncertainty in demand. In the context of Cournot competition in duopoly the quantity elasticity η is at least equally important. In order to roughly assess the "correct" size of these parameters and to generate a "real-world" benchmark, we would like to estimate the price or inverse demand model (2.1), $P = Y \cdot D(Q)$.

However, several difficulties arise. First, the stochastic trend in this level equation, Y, is not observable. Nevertheless, an estimable equation can be obtained by deriving the discrete-time analogue of a model of growth rates rather than levels and assuming that the inverse demand function is isoelastic and time-invariant. Second, electricity demand itself is unobservable. All that is available are data about market equilibria. This implies that a simultaneous equation system with an inverse demand *and* a supply equation has to be formulated. Third, if the system should be estimated simultaneously, not only the inverse demand model but also the supply equation must be set up in growth rates. This differencing naturally results in a loss of information. Alternatively, one can search for a supply model that just depends on one of the endogenous variables, price, P, and quantity, Q. The system of equations would become triangular and could be estimated recursively implying that the supply equation need not to be written in growth rates. Fortunately, there exists a reasonable supply model that only depends on price, P. Finally, to use the fitted values of the price series resulting from the estimated supply equation, a model of demand rather than inverse demand is required. "Inverting" the inverse demand equation inevitably results in a demand model with measurement errors in variables. In what follows we elaborate on each of these difficulties but in a different order.

Let us come back to our starting point and consider the price model (2.1). Recall that Y intuitively represents the unstable long-term evolution of consumers' preferences. We modelled this component of electricity demand by a geometric Brownian motion with drift. As mentioned above, the stochastic trend in Y is not observable. However, Y and, thereby, P are known to be lognormally distributed. Then $\ln P$ is normally distributed and, due to $dY = \alpha Y dt + \sigma Y dB$,

$$d \ln P = \frac{dD}{D} + \frac{dY}{Y} - \frac{dY}{2Y^2} = d \ln D + \left(\alpha - \frac{\sigma^2}{2}\right) dt + \sigma dB \tag{4.2}$$

by the Itô lemma. Moreover, by assuming that D is isoelastic and time-invariant, one yields

$$d \ln D(Q) = \frac{D'(Q)\,Q}{D(Q)} \frac{dQ}{Q} = -\eta d \ln Q.$$

This allows us to write the discrete-time analogue of equation (4.2) as

$$\Delta \ln P_t = \widetilde{\alpha} - \eta \Delta \ln Q_t + \varepsilon_t,$$

where $\widetilde{\alpha} = \alpha - \sigma^2/2$ and ε_t denotes the demand shock that is normally distributed with mean zero and variance σ^2. For reasons that we explained above, we do not want to estimate inverse demand but the demand function itself. Thus, we obtain the following empirical model of electricity demand,

$$\Delta \ln Q_t = \phi_0 + \phi_1 (\varepsilon_t - \Delta \ln P_t) + u_t, \tag{4.3}$$

with $\phi_0 = \widetilde{\alpha}/\eta$ and $\phi_1 = 1/\eta$. The error term u_t, $\mathcal{E}(u_t) = 0$ and $\text{var}(u_t) = \sigma_u^2$, should capture any unexpected changes of exogenous variables that are not explicitly modelled but influence electricity demand. Unexpected non-permanent deviations of weather from its regular seasonal pattern may serve as an example.[1]

Equation (4.3) establishes a linear regression model with (stochastic) additive measurement error, ε_t, in the variables. The effect of measurement error in a simple univariate linear regression is to bias the standard slope estimate in the direction of zero. An (asymptotic) bias of this kind is commonly referred to as *attenuation* (see, for example, Fuller, 1987). Note that an ordinary least-squares (OLS) estimate of $\Delta \ln Q_t$ on $\Delta \ln P_t$ is a consistent estimate not of ϕ_1, but instead of $\phi_1^* = \lambda \phi_1$, where

$$\lambda = \frac{\sigma_X^2}{\sigma_X^2 + \sigma^2} < 1 \text{ and } X_t = \varepsilon_t - \Delta \ln P_t.$$

[1]Note that we seasonally adjusted our data before running the estimation procedures. See section 4.3 below.

The factor λ is called the *reliability ratio* (Fuller, 1987, p.3). The model (4.3) cannot be consistently estimated without additional information (there are more parameters—ϕ_0, ϕ_1, σ^2, σ_X^2, and σ_u^2—than pieces of information). Of course, if either σ_u^2 or the reliability ratio were known, we could easily construct a consistent (but not efficient) method-of-moments estimator for ϕ_1.

Another well publicized estimation method for linear regression in the presence of measurement error is *orthogonal regression* (Fuller, 1987, section 1.3.3). The estimator minimizes not the vertical but the orthogonal distance of $(\Delta \ln Q_t, \Delta \ln P_t)$ to $\phi_0 + \phi_1 X_t$, weighted by the ratio $\kappa = \sigma_u^2/\sigma^2$ which is assumed to be known. However, we neither have any information about the reliability ratio and σ_u^2 nor about κ. As we already mentioned, the OLS estimate of ϕ_1, $\widehat{\phi}_1^{OLS}$, is biased towards zero. Thus, $\left|\widehat{\phi}_1^{OLS}\right| \leq \left|\widehat{\phi}_1\right|$, where $\widehat{\phi}_1$ denotes the (consistent) orthogonal least-squares estimator of ϕ_1. Moreover, by ignoring the measurement error, we get a OLS slope estimate in the inverse regression that is biased upwards, i.e. the so-called *inverse* (ordinary) least-squares estimate, $\widehat{\phi}_1^{INV}$, satisfies $\left|\widehat{\phi}_1\right| \leq \left|\widehat{\phi}_1^{INV}\right|$ (Schneeweiß and Mittag, 1986, section 2.2.2). Therefore, by calculating $\widehat{\phi}_1^{OLS}$ and $\widehat{\phi}_1^{INV}$, we may obtain some useful benchmarks if the interval $\left[\left|\widehat{\phi}_1^{OLS}\right|, \left|\widehat{\phi}_1^{INV}\right|\right]$ is not too large. The results of this computational exercise will be given in the next section.

Consistent estimates of ϕ_1 and the other parameters can also be obtained by using an *instrumental variable* (IV) estimator (Fuller, 1987, section 2.2.4). A variable Z that is correlated with $\Delta \ln P$ but not with the measurement error ε, may serve as an instrumental variable. Then the ratio $\widehat{\phi}_1^{IV} = \sum(Z_t \Delta \ln Q_t)/\sum(Z_t \Delta \ln P_t)$ provides a consistent estimator of the slope parameter ϕ_1. In the next subsection we will present such IV estimates of the relevant parameters.

Errors in the model variables are not the only difficulty that occurs when estimating our demand function. As discussed above, another problem is that electricity demand itself cannot be observed. The only data sets that are available include ex post market equilibria, i.e. points of balanced supply and demand. Thus, without introducing further assumptions a two-dimensional simultaneous equation system has to be estimated. This raises the question which kind of aggregate supply function is consistent with our idea about the (dis)investment behavior of firms. Suppose that utilities use three input factors in the production of electricity, capital K, labor L and fuel F. Further, let the production function be of the Cobb-Douglas type, $Q = AK^aL^bF^c$ with the coefficients $A, a, b, c > 0$. Then the least-cost factor combinations are given by

$$\frac{rK^*}{a} = \frac{wL^*}{b} = \frac{fF^*}{c} \quad \text{or} \quad L^*(K) = \frac{br}{aw}K \quad \text{and} \quad F^*(K) = \frac{cr}{af}K,$$

where r denotes the real rental cost of capital, w is the real wage of workers in the electric power generation industry and f symbolizes the deflated market price of fuel. Plugging these expressions into the utility's profit function yields

$$\Pi(K) = P(K) \cdot A\left(\frac{br}{aw}\right)^b \left(\frac{cr}{af}\right)^c K^{a+b+c} - \left(1 + \frac{b}{a} + \frac{c}{a}\right) rK.$$

We already stressed that utilities in the German electricity market can be considered as being price takers. In this case the first-order condition for a profit-maximizing level of capital is given by

$$P \cdot A(a+b+c)\left(\frac{br}{aw}\right)^b \left(\frac{cr}{af}\right)^c K^{a+b+c-1} = \left(1 + \frac{b}{a} + \frac{c}{a}\right) r.$$

Further, the models proposed in chapters 2 and 3 require that the output-to-capital ratio is constant in the long run. So we should observe a constant-returns-to-scale technology on the firm level as well as the aggregate (sector) level. One obtains the following long-run supply function of the firm written in logs and with time indices

$$\ln P_t = \ln \frac{b^{-b}c^{-c}}{A(1-b-c)^{1-b-c}} + (1-b-c)\ln r_t + b \ln w_t + c \ln f_t. \tag{4.4}$$

In the short run, however, the individual firm's output-to-capital ratio should be allowed to fluctuate around its constant long-run level due to the impact of supply shocks. Thus, an error term $v = v_i + v^A$ has to be introduced to the long-run relation in order to capture the impact of the ideosynchratic shock, v_i, and the aggregate shock, v^A, on the deviations of firm i's production from its long-run output level. The investment and closure timing models proposed in the previous chapters suggest that a firm invests (disinvests) if the firm's cash flow has reached an individually optimal upper (lower) barrier. This conclusion holds irrespective of the sector's supply side structure. Thus, the individual firm adjusts its capacity if the deviation from the optimal long-run output level, $|v|$, is large enough such that the firm's cash flow hits either the upper or the lower barrier. This reveals that our understanding of equilibrium capacity adjustment on the firm level amounts to a (S, s) model of aggregate investment dynamics with heterogeneous firms as proposed, for example, by Caballero et al. (1995) and Caballero and Engel (1999). In these papers firms are heterogenous with respect to their adjustment cost (sunk investment or closure cost). To estimate the sector's distribution of adjustment cost, detailed information about the behavior of the individual firm, i.e. panel data sets, are required. Such data are not available with respect to the German electric power market. We have to content ourselves with time series on the aggregate level.

A simplifying condition has to be imposed to get a rough impression about the utilities' supply even though there is a lack of information. The utilities that had had the legal status of regional monopolists before the deregulation of the German electric power sector are assumed to face the same output price level during the sample period (1977 to 1989). This does not seem to be too much of an abstraction from reality, since the political sector in Germany has always reacted very sensitive to price changes in energy markets. Thus, due to political pressure diverging price paths in different regional markets are very unlikely. With this simplification the aggregate supply function resembles the individual firm's supply function (4.4) and we obtain the following regression model,

$$\ln P_t = \gamma_0 + \gamma_1 \ln r_t + \gamma_2 \ln w_t + \gamma_3 \ln f_t + v_t. \tag{4.5}$$

The coefficients are defined as $\gamma_0 = \ln b^{-b} c^{-c} - \ln A(1-b-c)^{1-b-c}$, $\gamma_1 = 1-b-c$, $\gamma_2 = b$ and $\gamma_3 = c$. Of course, the error term, v_t, in equation (4.5) now captures the sector's investment dynamics. Since investment in or retirement of power generation units is likely to be lumpy in nature even on an aggregate level, the deviations from the long-run supply function cannot be normally distributed.

The inverse demand function, $\ln P = \ln Y + \ln D(Q)$, as well as the supply function suggest stationary linear combinations between nonstationary variables (see section 4.3). So, the model of market equilibrium is a system of cointegrating relations. However, as we already argued above, inverse demand cannot be estimated in levels, since the stochastic trend in Y is not observable. For that reason we derived a regression model of electricity demand in first differences as given by equation (4.3). To estimate the system simultaneously, the supply equation (4.5) must also be written in first differences. If the variables in equation (4.5) are indeed cointegrated, then differencing results in a loss of information. Fortunately, the supply equation includes only one endogenous variable—the price. Thus, the equations can be estimated recursively. We first estimate the supply model (in levels) by ordinary least squares (OLS) which guarantees superconsistent coefficient estimators.[2] Then the fitted values of the price series, $\widehat{\Delta \ln P_t}$, derived from the supply regression are plugged into the differenced demand model (4.3) with measurement errors. This clarifies why we have to estimate the demand function rather than inverse demand. Finally, the desired parameter estimates are derived as discussed above. Note that, by substituting $\widehat{\Delta \ln P_t}$ for $\Delta \ln P_t$, the interpretation of the measurement error, ε, changes. Now, it does not any longer capture demand shocks exclusively but also supply shocks.

Irrespective of the drawback that simultaneous estimation requires differencing of the supply equation, the standard system estimation methods, i.e. full-information maximum-likelihood (FIML), are not efficient. The reason is that at least the errors in the supply equation are not normally distributed (see, for example, Greene, 2000). Though these methods may still provide adequate quasi-maximum-likelihood estimates, one might prefer OLS anyway because of its simplicity and robustness.

4.3 Data and Estimation Results

Our sample consists of six time series that contain data of monthly frequency ranging from January 1977 to December 1989. We chose the producers' price index of electric power sold to the manufacturing sector and the corresponding volume series to represent the electricity price, P, and the traded quantity, Q, respectively. Unit capital costs, r, unit labor costs, w, and unit fuel costs, f, should be measured by the market interest rate of industrial bonds with a minimum maturity of ten years, the standard wage per hour of workers in public utilities and the manufacturer's price index of hard coal, respectively.

[2] The superconsistency of OLS estimators of cointegrating relations was shown by Stock (1987).

Finally, we use the consumers' price index to deflate the other price series.[3] All series are drawn from the database of the German "Statistisches Bundesamt" and correspond to the former territory of West Germany.

Though more recent data samples are available, we decide to cut off the series at December 1989. Especially, the evolution of the electricity price series dropping down in 1990 points to the existence of a structural break that is induced by the German unification in that year. The price crash might be explained by the considerable increase in excess capacity that resulted after the huge East German brown-coal based power plants had been integrated into the national electricity grid. Since we are not interested in the specific behavior of the supply side but in consumers' preferences, it seems to be an adequate measure to cut off the original sample before 1990. A total number of 156 observations per series results.

A test for seasonal integration as proposed by Beaulieu and Miron (1993) for monthly data reveals that all logarithmized series exhibit unit roots at zero frequency.[4] Only in the quantity series and the wage series, however, we find evidence on the presence of seasonal unit roots. The results are reported in Table 4.1.[5] Note that we test for the presence of complex conjugated seasonal unit roots by applying an F test to the two corresponding coefficient estimates in the underlying regression equation. Our findings suggest that the nonstationary stochastic seasonal component in the wage series can be removed by applying the filter $1 + B + B^2$ with B denoting the backshift operator. With respect to the quantity series there is some ambiguity whether it exhibits two, four or six seasonal unit roots. We decide in favor of six roots, because the resulting estimates of the supply and demand models are the most plausible ones. Then the filter that has to be applied to the quantity series is given by $1+B^2+B^3+B^5$. The deflated and seasonally adjusted logs of all series are depicted in Figure 4-1.It should be stressed that our estimation results respond highly sensitive to a change in the seasonal adjustment procedure. Suppose, for

[3] To obtain the real interest rate, we compute a (moving-window) proxy for the inflation rate that is expected to prevail at the end of the following ten-years period. This expectation is the relevant benchmark for holders of bonds with a maturity of ten years. To derive the proxy, the series of the consumers' price index is required to range up to December 1999.

[4] This test is based on the well-known HEGY procedure developed by Hylleberg et al. (1990) for quarterly data.

[5] The results are derived by including a constant in the corresponding regression equation. All deterministic components like constant, trend or dummies influence the asymptotic as well as the finite sample distributions.

Table 4.1: Results from Test for Unit Roots at Zero and Seasonal Frequencies

Frequency	$\ln P$	$\ln Q$	$\ln r$	$\ln w$	$\ln f$
0	o	o	o	o	o
π	+	o*	+	+	+
$\pm\pi/2$	+	o**	+	+	+
$\mp 2\pi/3$	+	+	+	o	+
$\pm\pi/3$	+	o	+	+	+
$\mp 5\pi/6$	+	+	+	+	+
$\pm\pi/3$	+	+	+	+	+

o = H0 (unit root) accepted at all conventional significance levels,
+ = H0 rejected at all conventional significance levels,
o* = H0 accepted at 5 per cent level, rejected at 10 per cent level ,
o** = H0 accepted at 2.5 per cent level, rejected at 5 per cent level.

example, one chooses the standard filter $(1-B^{12})/(1-B)$ for $\ln Q$ and $\ln w$. It removes all seasonal unit roots if they exist, but leads to overdifferencing and loss of information if they do not exist. In this case our estimation model suggests strong and significant second-mover advantages, while in the present case we found first-mover advantages as will be shown below.

Table 4.2 summarizes the estimation results. It does not contain the estimates of the production elasticicities a, b and c. We have $\widehat{a} = 0.0884$, $\widehat{b} = 0.5284$ and $\widehat{c} = 0.0688$. The augmented Dickey-Fuller (ADF) test clearly rejects the null hypothesis that the cointegration errors still contain a unit root indicating that the suggested supply function indeed is a cointegrating vector. The estimated coefficients $\widehat{a}$, $\widehat{b}$ and $\widehat{c}$ sum up to 0.6856. This value might indicate that our constant-returns-to-scale production technology is a misspecification. However, we think that the production elasticities of capital and fuel are underestimated. The chosen series seem to be bad proxies for the underlying variables in these two cases. The real interest rate, for example, appears to be too volatile to represent the return-on-capital of long-term bonds. One reason might be that our deflation procedure does not remove a sufficient amount of nominal volatility. Regarding the hard coal price series we stress that a regression based on a slightly modified seasonal filter leads to significantly higher coefficient estimate of c.

The method-of-moments estimates, $\widehat{\eta}^{OLS} = -(\widehat{\phi_1}^{OLS})^{-1}$ and $\widehat{\eta}^{INV} = -(\widehat{\phi_1}^{INV})^{-1}$ (see section 4.2), that should establish an upper and a lower bound for the consistent elasticity estimator $\widehat{\eta}$, respectively, lie fairly close to each other. However, the interval $[\widehat{\eta}^{INV}, \widehat{\eta}^{OLS}]$

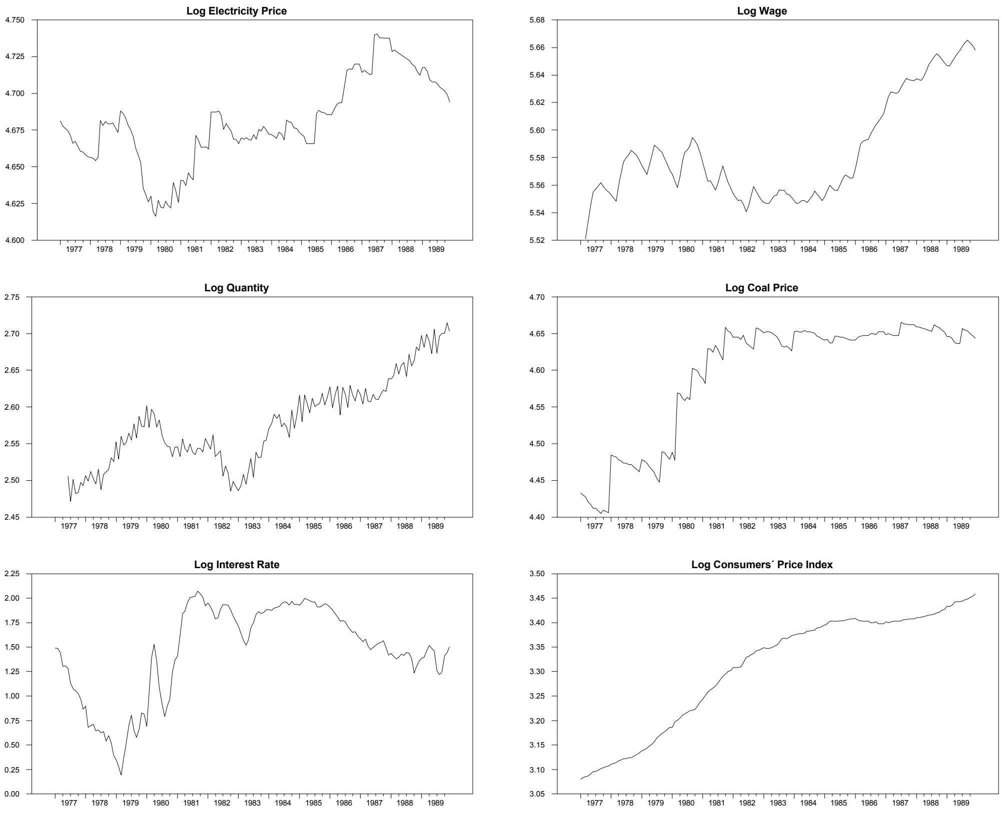

Figure 4-1: The Deflated and Seasonally Adjusted Time Series in Logarithms

is not sufficiently narrow to exclude either first-mover or second-mover advantages as a reasonable model assumption from the analysis. A naive IV estimate with the constant and the first lag of $\widehat{\ln P}$ as instruments yields $\widehat{\eta} < 1$ which might be interpreted as evidence in favor of second-mover advantages. However, any inference based on the $\widehat{\sigma}_u^2$ about the significance of $\widehat{\phi}_0$ and $\widehat{\phi}_1$ is invalidated by the presence of strong first-order serial correlation in the residuals, $\widehat{u}$ (see the DW statistic). Moreover, least squares estimation is at least inefficient in this case. Whether it is also inconsistent depends on the amount of residual autocorrelation across observations (Greene, 2000, section 13.2.2). Recall that we have introduced instrumental variables that are independent of the measurement error, ε, to obtain consistent OLS estimates. Though we used lagged regressors as instrumental variables, this approach is not invalidated by the serial correlation of the disturbances, u, as long as the measurement errors are not autocorrelated.

Table 4.2: Coeffcient Estimates - Different Estimation Methods

Parameter	MM	IV	(p-value)	IV with FGLS	(p-value)
$\widehat{\phi}_0$	-	0.00211	(0.1803)	0.00187	(0.0128)
$\widehat{\phi}_1^{OLS}$	-0.54575	-	-	-	-
$\widehat{\phi}_1^{INV}$	-116.74219	-	-	-	-
$\widehat{\phi}_1$	-	-1.14310	(0.2264)	-0.73856	(0.0254)
$\widehat{\sigma}_u$	-	0.01703	-	0.01715	-
$\widehat{\eta}^{OLS}$	1.83234	-	-	-	-
$\widehat{\eta}^{INV}$	0.00857	-	-	-	-
$\widehat{\eta}$	-	0.87481	-	1.35399	-
$\widehat{\widehat{\alpha}}$	-	0.00185	-	0.00253	-
$\widehat{\sigma}$	-	0.01562	-	0.01095	-
$\widehat{\alpha}$	-	0.00197	-	0.00259	-
DW	-	3.21552	-	2.34364	-
R^2	-	0.01160	-	0.38477	-

p-value = significance of H0: "coefficient=0",
MM = Method of Moments, IV = Instrumental Variables,
FGLS = Feasible Generalized Least Squares,
DW = Durbin-Watson test statistic for first-oder serial autocorrelation.

If the shocks, u, can be approximated by an AR(1) model, $u_t = \rho u_{t-1} + \zeta_t$ with $|\rho| < 1$ and ζ_t being i.i.d. with mean zero, then the first-round ordinary least squares estimates of ϕ_0 and ϕ_1 are consistent and we can correct for autocorrelated disturbances by applying the Cochrane-Orcutt procedure of *feasible generalized least squares* (FGLS) (Greene, 2000, section 15.7). The estimates resulting from the FGLS method are reported in the last two columns of Table 4.2. The null hypothesis $\widehat{\phi}_1 = 0$ is rejected—at least at the 5 per cent significance level. Thus, we obtain $\widehat{\eta} > 1$ which clearly supports the assumption of first-mover advantages. The coefficient, $\widehat{\phi}_0$, and, therefore, the drift estimate $\widehat{\widehat{\alpha}}$ are significantly greater than zero suggesting that $\alpha > \sigma^2/2$ and $\beta = \beta_2$ according to Lemma 1 in chapter 2.

Since utilities do not decide about the retirement of power generation units on a month-by-month basis, we are interested in the long-run levels of the coefficients. Thus, it remains to calculate the estimated *annual* drift and volatility. We get $\widehat{\alpha} = 0.03109$ and $\widehat{\sigma} = 0.03795$ for the parameters of the geometric Brownian motion Y. Note that σ is overestimated by $\widehat{\sigma}$, since the measurement error, ε, captures demand *and* supply shocks in our model. So, $\widehat{\sigma}$ should be understood as an upper bound of the "true" estimate of the demand shock's volatility. Further, it is likely that the price elasticity of electricity

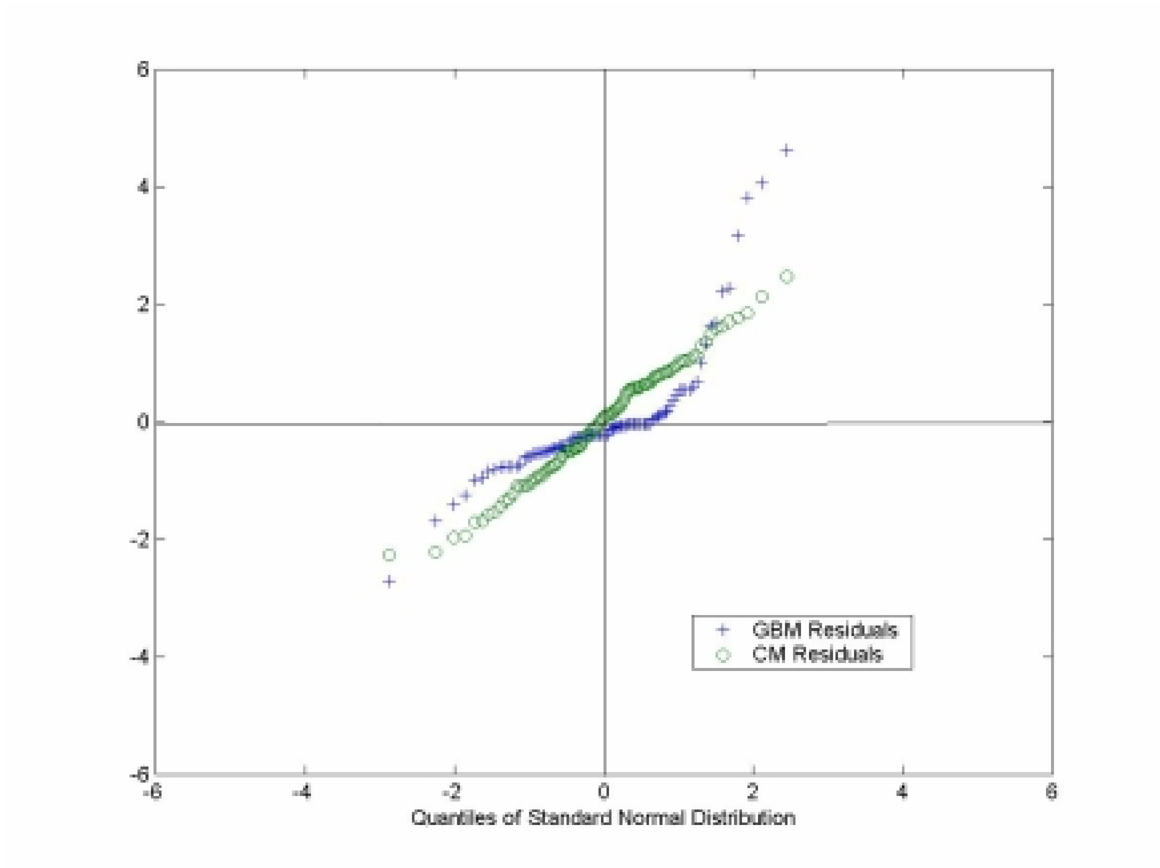

Figure 4-2: Q-Q Plots of Residuals—Geom. Browian Motion vs. Cournot Model

demand, η^{-1}, increases also with the time horizon, since firms in the manufacturing sector may substitute "externally acquired electricity" by "insourcing" electric power production in the long run, but not in the short run. Hence, $\widehat{\eta}$ may be interpreted as an upper bound for the annual inverse demand elasticity. This implies that even a scenario with second-mover advantages may occur when switching from months to years as basic units of time. The next section that investigates the profitability of strategic closure timing rules, includes a sensitivity analysis with respect to the parameter η.

Finally, we raise the question whether our model fits "reality" also in a statistical sense. In Figure 4-2 and Table 4.3 some distributional characteristics of our price model (called Cournot model) are compared to the characteristics of a standard price model—a geometric Brownian motion. Both models require that the residuals from the logarithmized discrete-time analogue are normally distributed. Obviously, the residuals from our model satisfy this requirement much better than the disturbances of the standard model.

Table 4.3: Charateristics of Empirical Distribution of Model Residuals

Empirical Moment	$\Delta \ln P$	$\Delta \ln P + \widehat{\eta}\Delta \ln Q$
standard deviation	0.00658	0.02508
skewness	1.56774	-0.14062
(p-value)	(0.0000)	(0.4528)
excess kurtosis	8.33692	-0.52708
(p-value)	(0.0000)	(0.1643)

Especially the quantile-to-quantile (Q-Q) plots in Figure 4-2 reveal that our residual distribution does not exhibit *fat tails*, while the distribution of the geometric Brownian motion does. We conclude that combining a geometric Brownian motion as a model for exogenous demand shocks with a deterministic model of supply variations seems to generate a price model that fits the "real world" much better than a single geometric Brownian motion does.

4.4 Superiority of Strategic Timing—An Example

Let us return to the initial question whether the gains in profits of utility 1 are of significant size when it switches from the price taker's closure rule suggested by the standard real options theory, Y_P, to the best-response threshold, Y_M. Resorting to numerical solutions illustrates that the answer must be a clear 'yes'—at least with respect to the closure scenario "RWE versus Eon" that we introduced in chapter 1.

Whether to shut down an entire plant or not is certainly a long-run decision. For that reason, we express all parameters in annual rather than monthly or quarterly terms henceforth. Further, the numerical specifications of the model parameters as detailed below were either given by experienced energy managers[6] or derived from the estimation model of the German electricity market presented in the last sections. Thus, the figures should be considered as plausible approximations of the "true" values. Nevertheless, we will provide a sensitivity analysis for most of the parameters to ensure that our conclusions are drawn on a firm base. Unless we explicitly refer to a different set of parameter values, the computations in this section use the following specification. Each utility controls

[6] I am grateful to Stefan Florek and Thomas Niedrig from RWE Trading for providing the details.

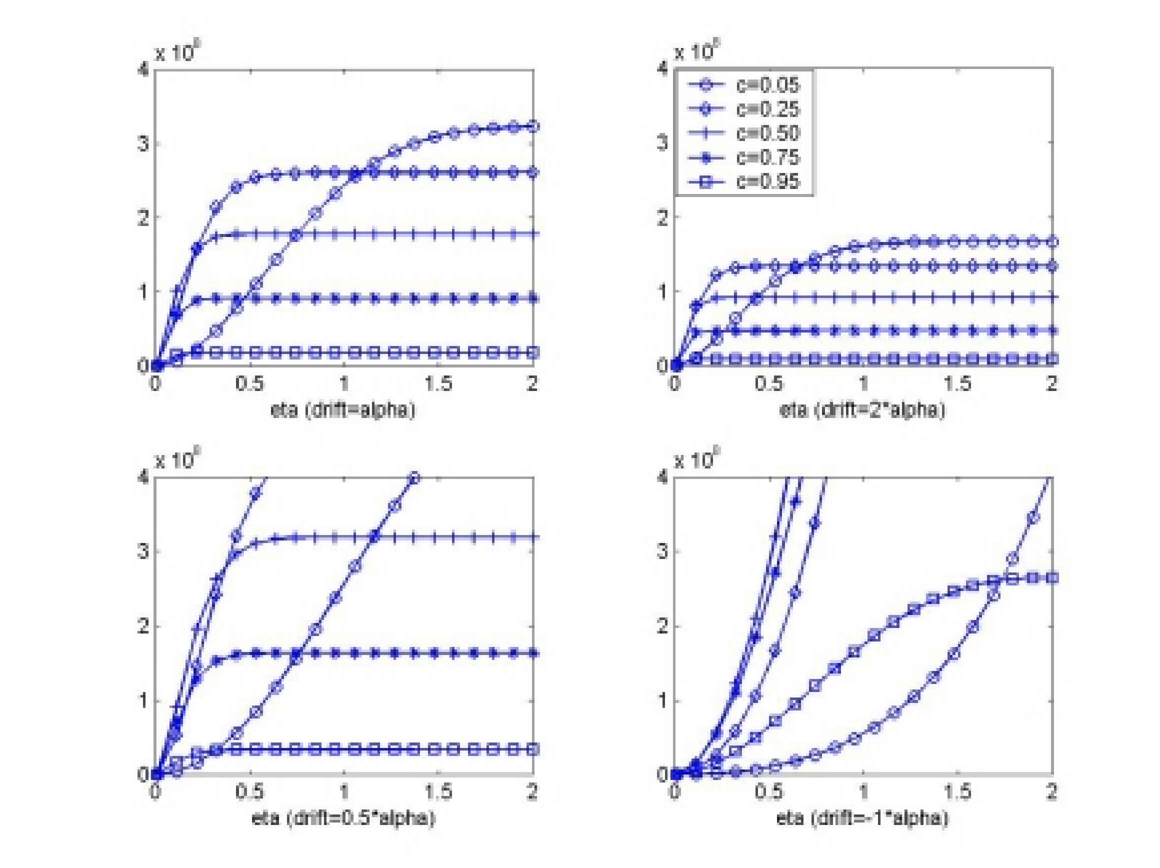

Figure 4-3: Net Gain $\overline{\Delta}$ for Various Values of c, α and η.

for $30,000$ MW of power generation capacity.[7] Assuming an average annual utilization rate of 10 per cent amounts to an average annual power production of $\overline{q} = 26.28$ million MWh.[8] Both duopolists plan to close 15 per cent of their total capacity. Thus, we have $c = \underline{q}/\overline{q} = 0.85$. Let us assume that the sunk cost of capacity reduction, E, are linear in the reduced quantity, i.e. $E = k(\overline{q} - \underline{q})$, and that the marginal adjustment cost, k, equal DM $66,667$ per retired MW.[9] The variable cost (capital, fuel and labor cost) per generated MWh are given by $C =$ DM 80.00 and the annual discount rate by $\rho = 0.10$. Together with the estimates of α, σ and η, this set of parameter values establishes a complete numeric assessment of the closure scenario as brought up by RWE and Eon in the German electric power market. Moreover, it numerically specifies the function $\overline{\Delta}$

[7] See Table 1.1 in chapter 1 for the available capacity of RWE and Eon.

[8] An average utilization rate of 10 per cent is set relatively high if the power generation units that should be closed belong to the peak load segment. It is set comparatively low if the units are base load plants.

[9] This amounts to adjustment cost of DM 10 million per 150 MW of retired capacity.

which is defined as the net payoff that utility 1 gains by switching from Y_P to Y_M (see equation (4.1)) contingent on $Y_T = Y_M$. We will show that $\overline{\Delta}$ attains very significant values not only for this specification.

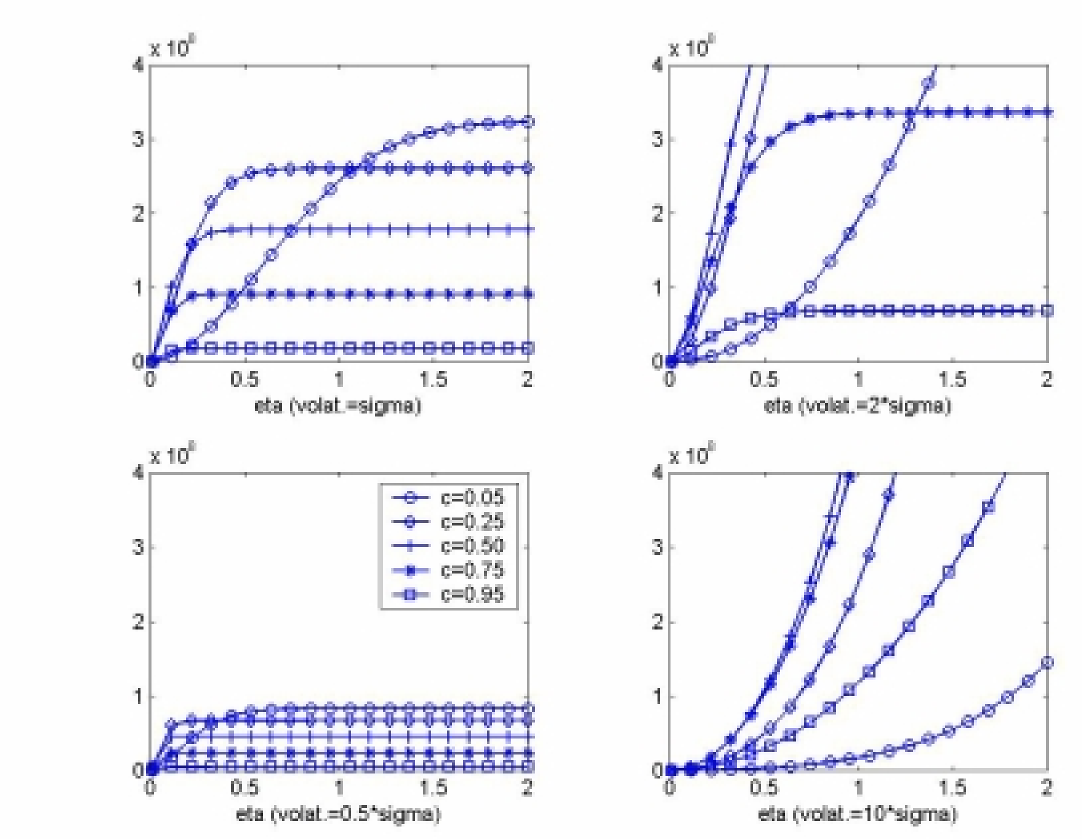

Figure 4-4: Net Gain $\overline{\Delta}$ for Various Values of c, σ and η.

Before investigating utility 1's gain in profits, it is useful to develop some intuition about the impact of the different parameters on the closure thresholds, Y_P and Y_M, rather than on $\overline{\Delta}$. Both thresholds are decreasing functions of the drift, α. The higher the certain demand growth the more likely the event occurs that demand will recover soon. Consequently, firms become more reluctant to exercise their closure option. If α approaches the discount rate, ρ, from below, then $Y_P, Y_M \rightarrow 0$. The reason is that for $\alpha \geq \rho$ the "fundamental value" of each utility equals infinity implying that no firm would ever exercise its closure option independent of what the rival does. Y_M and Y_F are not only decreasing in α, but also in the volatility, σ. This is a well-known feature of exercising rules in standard option pricing theory (see, for example, Dixit and Pindyck, 1994, section

6.2). In the limit, if σ becomes large, both thresholds approach zero and so does their difference. The intuition is clear: the value of waiting and keeping the option to resolve the ongoing uncertainty increases with the volatility in demand. Thus, firms prefer to delay capacity reduction and the corresponding thresholds shrink. Due to $E = k(\overline{q} - \underline{q})$ the thresholds are linear in the variable cost, C, the ratio c and the marginal adjustment cost, k. Moreover, Y_M and Y_F are proportional to $(C/\rho) - k$ and the total capacity ex ante to capacity reduction, $\overline{q}$. Of course, if the adjustment cost, $k(\overline{q} - \underline{q})$, converge to the present value of the operating cost arising from infinite production, $(C/\rho)(\overline{q} - \underline{q})$, then the thresholds go to zero, $Y_P, Y_M \rightarrow 0$, since the value of the closure option becomes worthless.

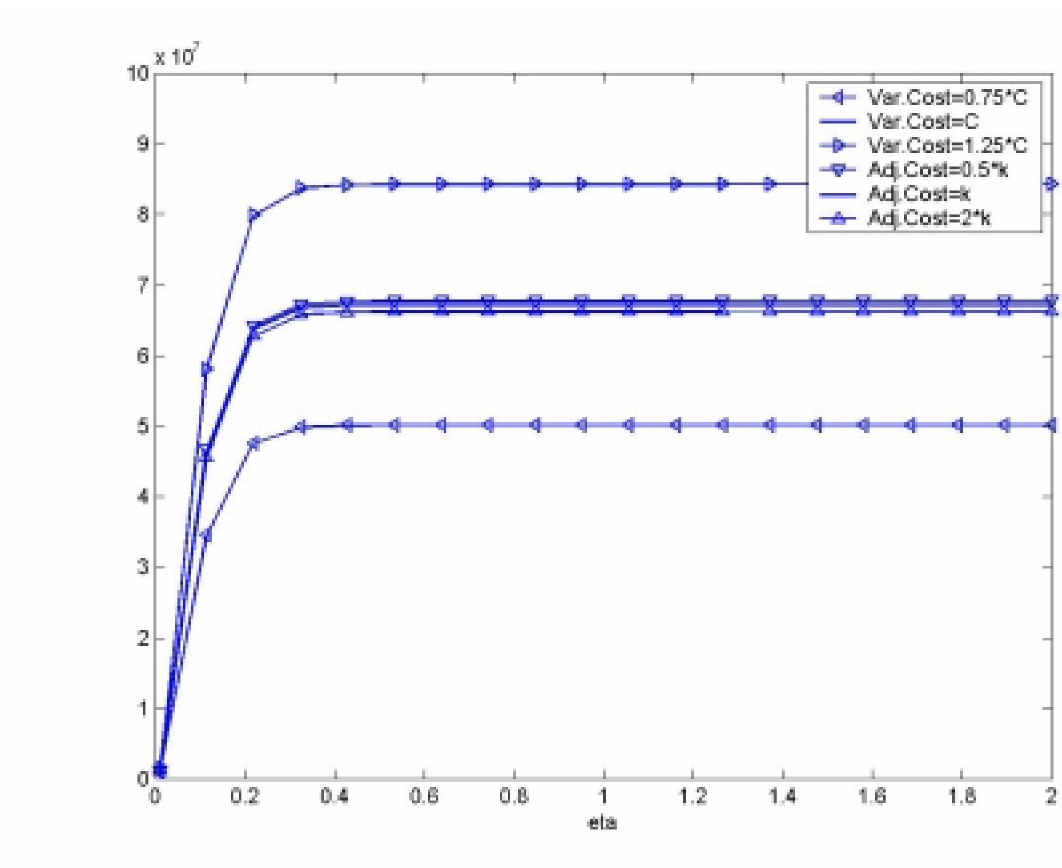

Figure 4-5: Net Gain $\overline{\Delta}$ for Various Values of C, k and η.

Of course, our main concern is about profits rather than thresholds. Figure 4-3 depicts utility 1's net gain in payoffs, $\overline{\Delta}$, as a function of the inverse demand elasticity, η, for various values of the ratio c and the drift α ($c \in \{0.05, 0.25, 0.50, 0.75, 0.95\}$ and

$\alpha \in \{\widehat{\alpha} = 0.03109, 2\widehat{\alpha}, 0.5\widehat{\alpha}. - \widehat{\alpha}\})$. In contrast to the thresholds Y_M and Y_P, $\overline{\Delta}$ is not uniformly decreasing in c and α. However, Figure 4-3 suggests that there exist functions $\eta_1(\alpha)$ and $\eta_2(c)$ such that $\overline{\Delta}(c = c^l, \alpha, \eta) > \overline{\Delta}(c = c^h, \alpha, \eta)$ for all $c^l < c^h$, $\eta > \eta_1(\alpha)$, $\alpha \in \mathbb{R}$, and $\overline{\Delta}(c, \alpha = \alpha^l, \eta) > \overline{\Delta}(c, \alpha = \alpha^h, \eta)$ for all $\alpha^l < \alpha^h$, $\eta > \eta_2(c)$, $c \in (0, 1)$. Thus, for η large enough, $\overline{\Delta}$ is decreasing in c and α. It should be clear that the price taker's closure option as well as the closure option of utility 1 get worthless if the retired quantity becomes marginal, $c \to 1$. So, $\overline{\Delta}$ approaches zero in this case. If α gets large, than firms become more and more reluctant to exercise their closure option, since the probability that demand will recover soon grows rapidly. Again, the options lose value and $\overline{\Delta} \to \dot{0}$. Figure 4-3 also reveals that $\overline{\Delta}$ seems to converge to an upper bound if η increases. $\overline{\Delta}$ converges faster for small values of c and α ceteris paribus.

In Figure 4-4 $\overline{\Delta}$ is again plotted as a function of η, but we use varying values for c and the volatility σ. For small values of η the impact of σ on $\overline{\Delta}$ is ambiguous too. However, if η becomes sufficiently large, then it seems that $\overline{\Delta}$ always increases with σ. The more volatile the underlying price the higher the value of the option that allows to hedge the price risk. Further, the rate of convergence of $\overline{\Delta}$ in η declines with σ.

Finally, Figure 4-5 reveals that the net gain in profits that utility 1 yields by playing a best response against its price-taking rival, utility 2, reacts very insensitive to changes in the inverse demand elasticity, η, for $\eta > 0.6$. The thick line depicts $\overline{\Delta}$ as a function of η given that the other parameters attain the "real-world" benchmark values as detailed above. It shows that utility 1 can gain about $\overline{\Delta} =$ DM 68 million by choosing the best-response trigger, Y_M, rather than adhering to the price taker's disinvestment threshold, Y_P. Further, $\overline{\Delta}$ does not change significantly even if the "true" η lies far below its estimate $\widehat{\eta} = 1.35399$. As compared to the total disinvestment cost, E, that equal DM 300 million we consider $\overline{\Delta}$ as being of remarkable size. These findings strongly support the relevance of our approach. They make evident that accounting for strategic interaction in closure-option timing problems is not just an exercise in game theory being merely of academic interest. This view obviously pays off and, therefore, should not be ignored by practitioners in the electric power generation sector.

The other curves in Figure 4-5 also represent plots of $\overline{\Delta}$ as a function of η, but for distinct values of the marginal operating cost, C, and the marginal adjustment cost, k. The plots indicate that $\overline{\Delta}$ is influenced rather strongly by fluctuations in C, while the impact of k appears to be much less significant. The reason is clear; C/ρ is about ten times as large as k. Thus, doubling C should have a much stronger effect on $\overline{\Delta}$ than doubling k.

Chapter 5

Summary

In chapter 2 we analyzed the set of subgame perfect equilibria of a disinvestment timing game in a market that is subject to aggregate shocks. We found that the timing game with *identical* firms and *second-mover* advantages exhibits a unique symmetric subgame perfect equilibrium and, moreover, a unique equilibrium outcome with firms reducing capacity jointly. In equilibrium firms are involved in a prisoners' dilemma. They reduce capacity late and simultaneously to avoid that their rival attracts additional market shares and yields the second-mover advantage. A different set of equilibria is obtained for the disinvestment timing game with identical firms and *first-mover* advantages. In that case there exist two asymmetric pure strategy equilibria, where one firm becomes the leader at the myopic firm's threshold and, thereby, yields the first-mover advantage, while the other firm follows later and attracts additional market shares.

The result obtained in the game with first-mover advantages is somewhat unsatisfactory as it leaves undetermined which firm actually chooses the "first-mover strategy" and, thereby, takes over the leadership. With identical firms we would like to obtain a symmetric equilibrium, since identical equilibrium strategies solve this coordination problem. As we discussed in chapter 3, Fudenberg and Tirole (1985) show the existence of a symmetric mixed strategy equilibrium of the preemption type in a game of investment timing. Though equilibrium strategies are identical, the equilibrium outcome of "diffusion"—one firm invests early and the other follows later—occurs with probability one.[1] We showed, however, that an analogous equilibrium where strategies are discrete

[1] As we explained in chapter 3, this statement only holds for those subgames that start before the

lotteries over thresholds, does not exist in the game of closure timing. The reason is that the leading and the following firm's payoff do not equalize in equilibrium, i.e. in contrast to the preemption equilibrium of the investment timing game we do not observe "rent equalization".[2]

By assuming a linear inverse demand function that implies first-mover advantages, Baldursson (1998) succeeded in deriving analytical expressions for the optimal investment and disinvestment triggers of firms in a n-firm oligopoly. He obtains identical thresholds for identical firms. This is not in line with our main proposition that identical firms are likely to behave very asymmetric if they face first-mover advantages. The reason for this discrepancy is that in Baldursson's incremental investment model the process of adjusting the capital stock is shaded off in adding or removing small units, while we are concerned with discrete large changes in the stock of capital. In the (dis)investment timing games of the previous chapters, however, the leader's optimal adjustment trigger would also converge to the follower's optimal threshold if the difference between capacity ex post and capacity ex ante to (dis)investment approached zero and if the inverse demand function was well-behaved. Note that firms prefer to adjust their capital stock in small units if firms face decreasing returns to capital, while with increasing returns to capital the capital stock is optimally altered in large units. Thus, the model by Baldursson is addressed to a different kind of industry.

In both cases, with first-mover as well as second-mover advantages, the equilibrium strategy profiles imply that firms generally should not move as suggested by the standard real options theory. In a duopoly, where firms face second-mover advantages, they bring forward their disinvestment as compared to a price-taking firm. Thus, the effect of the strategic value of investment is to increase the speed of investment. Moreover, we found

point of rent equalization is reached.

[2] Our analysis of the case of first-mover advantages does not exclude the existence of a symmetric equilibrium where strategies are continuous or mixed distributions over thresholds. In chapter 2 we mentioned that the diffusion outcome reminds us of the principle of "maximum differentiation" introduced by d'Aspremont *et al.* (1979) in Hotelling's model of spacial competition. Bester *et al.* (1996) show that due to coordination failure the location game exhibits an infinity of asymmetric mixed strategy Nash equilibria that do not involve maximum differentiation as a possible equilibrium outcome. Additionally, they detect a symmetric mixed strategy equilibrium where strategies are mixed distributions over the space of locations. The distributions possess discrete mass at the endpoints. Unfortunately, the approach by Bester *et al.* cannot be applied to our closure timing game. It requires the validity of two symmetry conditions that are not satisfied by our payoff functions. Of course, the fact that we do not observe rent equalization in the diffusion equilibrium of the closure timing is just a consequence of this more fundamental asymmetry of payoff functions.

that the rivals delay capacity reduction as compared to a monopolist. In a duopoly with first-mover advantages we detected closure scenarios where the equilibrium follower delays his disinvestment as compared to a price-taking firm. In particular, such an outcome is likely to occur if the rivals plan to shut down large parts of their production facilities. As a consequence, the standard rules cannot even serve as benchmarks in the sense that the speed of disinvestment in duopoly is expected to be generally higher than in a market with price-taking firms and lower than in monopoly. If the follower exercises his closure option before a price taker would do, then the strategic value of investment speeds up disinvestment. Otherwise its effect is ambiguous.

In chapter 2 we also investigated the investment timing game with *heterogeneous* firms. In the case of *second-mover advantages* the unique equilibrium outcome of the identical-firm model "survives" the transition to the general model if the degree of heterogeneity remains relatively small. The intuition that underlies the simultaneous closure equilibrium, however, is not quite the same as before. Due to its higher efficiency the low-cost firm has the scope to avoid the extreme prisoners' dilemma outcome arising in the identical-firm case. It disinvests a little bit earlier and, thereby, adopts the leader's role. A simultaneous closure scenario nevertheless occurs, since the high-cost firm is known to follow immediately. The payoff from joint disinvestment is an increasing function of the disinvestment trigger in that region. Thus, both firms can raise their payoffs above the level that would result from the extreme prisoners' dilemma outcome.

If the difference in variable cost grows, one obtains an additional "diffusion" equilibrium in which the high-cost firm chooses the leader's role and the low-cost firm follows later. This equilibrium with successive disinvestment dates becomes unique when the cost difference rises even more. The intuition is that the high-cost firm being exposed to a strong "cost-pressure effect" can only avoid this pressure by reducing capacity early.

We also showed that in the case of *first-mover advantages* it is not sufficient to introduce just a small cost difference between firms to solve the coordination problem of assigning the leader's role. Again, one obtains two asymmetric equilibria that correspond to the equilibria of the model with identical firms. Of course, if the degree of heterogeneity is raised sufficiently high, the same cost pressure as in the case of second-mover advantages guarantees a unique equilibrium with the high-cost firm adopting the leader's role. If the difference in variable cost attains intermediate values, it depends on the parameters of the model whether a joint-disinvestment equilibrium additionally exists. As we expected, a sufficiently small first-mover advantage ensures its existence. We conclude that the set of equilibria might be richer than in the game with second-mover advantages.

Fine and Li (1989) stress that the multiplicity of equilibria arising from their discrete-time model of exit timing is an immediate consequence of a discretely jumping demand process.[3] The intuition behind this result is appealing. In an asymmetric duopoly in a continuously declining industry there will be an initial period of time where both firms are viable as duopolists. In the subsequent period the stronger firm is still viable as a duopolist, but the weaker firm is not. It could, however, survive as monopolist. In the following period both firms are only viable as monopolist. In the fourth period even monopoly profits are not sufficient to prevent the weaker firm from exiting and in the fifth period the same applies to the stronger firm. Intuition suggests that there exists a unique (subgame perfect) equilibrium where the weaker firm exits the market in the second period and the stronger firm follows in the fifth period. On a heuristic level this approach characterizes the continuous-time games of exit timing by Ghemawat and Nalebuff (1985) and Fudenberg and Tirole (1986).

Now, suppose that demand does not decline continuously but jumps discretely. Then a scenario might occur where both firms has been viable as duopolists before some time t. At t firms' revenues drop sufficiently sharp such that none of the firms is still viable as a duopolist afterwards. In this case intuition suggests that there exist two asymmetric equilibria. In the first equilibrium the weak firm exits first and the strong firm follows later. In the other equilibrium roles are interchanged. So, by allowing the demand process to be discontinuous, one obtains a multiplicity of equilibria as in the model by Fine and Li. They conclude that the equilibrium uniqueness in the models by Ghemawat and Nalebuff and Fudenberg and Tirole could be destroyed if the assumption of a continuously declining demand were relaxed and discrete (stochastic or deterministic) jumps were allowed. In our closure timing game with heterogeneous firms, however, we also obtain multiple asymmetric equilibria of the above kind, though our inverse demand process is perfectly continuous.[4]

[3] "Jumps", i.e. discontinuities in demand naturally occurs in such a discrete-time framework.

[4] Of course, if the degree of heterogeneity becomes sufficiently large, we obtain a unique equilibrium. But an equivalent result also holds in the model by Fine and Li. There the intuition is that if the weak firm becomes sufficiently weak, the length of the third period converges to zero, i.e. the strong firm is

One might guess that the multiplicity of equilibria in our model is due to fact that we consider closure decisions rather than the extreme case of firm exit. But this conjecture turns out to be wrong. We modified our model and analyzed the case of firm exit. However, multiple equilibria were still obtained. We were not able to resolve the apparent inconsistency between Fine and Li's findings and our result. The puzzle has to be dedicated to future research.

The non-stochastic model of firm exit by Ghemawat and Nalebuff (1985) provides some evidence that the bigger firm quits earlier than the smaller rival. Moreover, if economics of scale are sufficiently large, this equilibrium pattern of disinvestment might be reversed. Our results contrast with these findings. Using a model set-up that is almost analogous to the one by Ghemawat and Nalebuff, we showed that for a wide range of parameter values either the smaller firm reduces capacity before the big firm or both firms shut down production facilities simultaneously. Moreover, the outcome is independent of the magnitude of the difference in variable cost. As we discussed in chapter 2, our result differs from that of Ghemawat and Nalebuff because we assumed firms to be still active after capacity reduction, while they are concerned with the extreme case of firm exit. This does not necessarily imply that in the limit, when the "activity level" of firms ex post to capacity reduction converges to zero, our timing game generates the same outcome as Ghemawat and Nalebuff's model. Results do qualitatively coincide if consumers have a finite reservation price.

Turning to policy issues, our welfare analysis provides some evidence that regulation authorities should adopt a restrictive approach to the assessment of mergers, joint ventures and cooperation in declining industries or markets with excess capacities. Our results contrast with the findings of Weeds (2000) who recommends a loose regulation policy with respect to joint ventures in R&D investment projects. This implies that the socially optimal regulation policy should depend on the industry's potential for future growth.

In chapter 3 we discussed the corresponding *investment* timing game *with identical firms* as proposed by Huisman and Kort (1999). There, either an equilibrium of the preemption type or of the collusion type prevails depending on the parameter values.

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still viable as a duopolist when only monopoly profits can guarantee the survival of the weak firm.

While trying to preempt each other, firms might invest simultaneously, leaving them off with the worst possible payoff. Weeds (2000) and Grenadier (1996) restrict on pure strategies and, thereby, avoid the outcome of erroneous joint investment. Huisman and Kort deal with this kind of coordination failure by using the appropriate strategy spaces as suggested by Fudenberg and Tirole (1985). They prove that erroneous joint investment indeed occurs with positive probability in the preemption equilibrium of any subgame that starts after the point of rent equalization has been passed.

We demonstrated that a monopolist equipped with two investment options—one from each firm in duopoly—exercises the first option earlier and the second later than duopolists do in the Pareto-optimal equilibrium of the *collusion type*. We argued that, if first-mover advantages are small, the actual timing of investment in duopoly might be very similar to the optimal investment sequence in monopoly. Empirical evidence was provided that points to the collusion case rather than the preemption case in the German electric utility industry. Moreover, in chapter 4 we presented estimates of the inverse demand elasticity that suggests rather moderate first-mover advantages. These facts probably imply that in the German electric power market the practical relevance of the game-theoretic approach to investment options is limited as compared to the standard real options theory—the discrepancy between the monopolist's investment pattern and the equilibrium investment timing in duopoly might be negligible; the strategic value of investment seems to be small.

In chapter 4 we investigated the closure scenario "RWE versus Eon" in the German electric power market. Our main concern was the empirical relevance of the "strategic real options approach" propagated in the previous chapters. We analytically derived the gain in profits that a utility yields when switching from a standard real-options-theory rule to the best response against its price-taking rival. For the purpose of computing realistic values for this gain, we estimated the main parameters of our model with respect to the German electricity sector. By using the estimates and some other reasonable benchmark values, it was shown that a utility can increase its profits by a very remarkable amount if it follows our suggestions rather than the standard real options theory. Further, we provided some evidence that, with respect to its distributional characteristics, our price model fits the theoretical requirements much better than the conventional price model. That is, the residuals from our model follow a normal distribution, while the residuals from the discrete-time analogue of the geometric Brownian motion do not. Finally, a primitive model of aggregate investment (and disinvestment) in the German electricity sector was derived.

We did not compare numerically the utilities' payoffs that result if both competitors play the equilibrium strategy profile with their payoffs from jointly sticking to the price-taker's rule. From an analytical point of view, however, the equilibrium payoffs Pareto-dominate the price takers' payoffs in the case of identical firms facing second-mover advantages. The reason is that price takers (jointly) disinvest later than firms do in the unique joint-closure equilibrium outcome and that the payoff from simultaneous closure decreases with the time of disinvestment in that region. In the game with first-mover advantages our findings are more ambiguous. In the case that the equilibrium follower disinvests earlier than a price taker, the equilibrium payoffs of the follower and the leader are greater than a price taker's payoff. This can be verified as follows. Suppose the equilibrium leader deviates to the equilibrium follower's threshold. Then both firms yield a strictly smaller return on closure than in equilibrium. By moving simultaneously at the equilibrium follower's threshold, however, they are still better off than the price takers. Again, the reason is that the payoff from simultaneous closure decreases with the time of disinvestment in that region. In the case that the equilibrium follower moves later than a price taker, this line of arguments does not apply any longer. It cannot be excluded that the equilibrium leader is worse off than a price taker with a rival that shows the same irrational behavior.

Appendix A

Proofs of Chapter 2

A.1 Conditional Expectations

In the text we make extensive use of the conditional expectations

$$f\left(Y_{\mathsf{T}}\right)=\mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T-\mathsf{T})}\right],$$

$$g\left(Y_{\mathsf{T}}\right)=\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\infty}e^{-\rho(t-\mathsf{T})}Y\left(t\right)dt\right] \text{ and } h\left(Y_{\mathsf{T}}\right)=\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T}e^{-\rho(t-\mathsf{T})}Y\left(t\right)dt\right],$$

where $\mathcal{E}_{\mathsf{T}}[\cdot]=\mathcal{E}[\cdot\,|\mathfrak{F}_{\mathsf{T}},\mathsf{T}<\infty]$ and Y_{T} and Y_T are some fixed thresholds. T and T denote the corresponding first-passage or stopping times, i.e. $\mathsf{T}=\inf(t\geq 0\,|Y(t)\leq Y_{\mathsf{T}})$ and $T=\inf(t\geq\mathsf{T}\,|Y(t)\leq Y_T)$. Further, note that $\mathfrak{F}_{\mathsf{T}}=\{A\in\mathfrak{F}:A\cap\{\mathsf{T}(\omega)\leq t\}\in\mathfrak{F}_t\}$. Intuitively, $\mathfrak{F}_{\mathsf{T}}$ is the set of all events whose occurrence or non-occurrence is known at the time of stopping. First, let us formally state the results in the following lemma.

Lemma 6 *For Y_{T}, Y_T, T, T and $\mathcal{E}_{\mathsf{T}}\left[\cdot\right]$ defined as above it holds that*

$$f\left(Y_{\mathsf{T}}\right)=\left(\frac{Y_T}{Y_{\mathsf{T}}}\right)^{-\beta},\ g\left(Y_{\mathsf{T}}\right)=\frac{Y_{\mathsf{T}}}{\delta} \text{ and } h\left(Y_{\mathsf{T}}\right)=\frac{Y_{\mathsf{T}}}{\delta}-\left(\frac{Y_T}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_T}{\delta},$$

where $\delta=\rho-\alpha$ and

$$\beta=\begin{cases}\beta_2=\frac{1}{2}-\frac{\alpha}{\sigma^2}-\sqrt{\left[\frac{\alpha}{\sigma^2}-\frac{1}{2}\right]^2+\frac{2\rho}{\sigma^2}}<0 & \text{for } \alpha<\frac{\sigma^2}{2},\\ \widetilde{\beta}_2=\beta_2+1-(2\alpha/\sigma^2)<0 & \text{for } \alpha\geq\frac{\sigma^2}{2}.\end{cases}$$

The remaining part of this appendix contains the proof of Lemma 6. To keep the exposition simple we focus on deriving explicit expressions for $f(Y_0)$, $g(Y_0)$, and $h(Y_0)$, where Y_0 is assumed to be the initial value of the underlying geometric Brownian motion $\{Y(t), t \geq 0\}$ and $\mathcal{E}_0[\cdot] = \mathcal{E}[\cdot\,|Y(0) = Y_0]$. This simplification requires to clarify in which sense the latter expectations are equivalent to the former ones. For this purpose we need a version of the strong Markov property (e.g., see Harrison, 1985, p.5).

Theorem 1 *Let $T < \infty$ be a stopping time, and define $Z^*(t) = Z(T+t)$ for $t \geq 0$. Then Z^* is a (μ, σ^2) Brownian motion with starting state Z_T and Z^* is independent of $\mathfrak{F}_T = \{A \in \mathfrak{F} : A \cap \{T(\omega) \leq t\} \in \mathfrak{F}_t\}$.*

Let $X(t) = \ln Y(t) = \ln Y_0 + (\alpha - (\sigma^2/2))t + \sigma B(t)$ with $\{B(t), t \geq 0\}$ denoting standard Brownian motion. Notice that $\{X(t), t \geq 0\}$ is a $((\alpha - (\sigma^2/2), \sigma^2)$ Brownian motion with starting state $\ln Y_0$. Now, define $X^*(t) = X(\mathsf{T} + t)$ and $Y^*(t) = Y(\mathsf{T} + t)$. Then not only $X^*(t)$, but also $Y^*(t)$ is independent of $\mathfrak{F}_\mathsf{T}$.

Regarding the simplest expectation f define $T^* = \inf(t \geq 0\,|Y^*(t) \leq Y_T)$ and $Y_0^* = Y_\mathsf{T}$. Then it follows immediately that

$$f\left(Y_\mathsf{T}\right) = \mathcal{E}_\mathsf{T}\left[e^{-\rho(T-\mathsf{T})}\right] = \mathcal{E}\left[e^{-\rho T^*}\,|Y^*\left(0\right) = Y_0^*\right] = f\left(Y_0^*\right).$$

Next, we analyze the expectation h. We have

$$h\left(Y_\mathsf{T}\right) = \mathcal{E}_\mathsf{T}\left[\int\limits_0^{T-\mathsf{T}} e^{-\rho t} Y^*\left(t\right) dt\right] = \mathcal{E}_\mathsf{T}\left[\int\limits_0^{T^*} e^{-\rho t} Y^*\left(t\right) dt\right].$$

$>$From the strong Markov property of Y^* it follows that

$$\mathcal{E}_\mathsf{T}\left[\int\limits_0^{T^*} e^{-\rho t} Y^*\left(t\right) dt\right] = \mathcal{E}\left[\int\limits_0^{T^*} e^{-\rho t} Y^*\left(t\right) dt \,\middle|\, Y^*\left(0\right) = Y_0^*\right] = h\left(Y_0^*\right).$$

Finally, the corresponding equivalence with respect to g can be shown by setting $T = \infty$ in the formula for h. To conclude, w.l.o.g. the expectations can be calculated contingent on the event that Y_0 rather than Y_T is the root of the (sub)game.

Let us start off with the computation of $f(Y_0)$ by drawing partially from Karlin and Taylor (1975, p.357f.).[1] The key result that is needed in order to solve $f(Y_0)$ is the Martingale Stopping Theorem (e.g., see Harrison, 1985, p.130),

Theorem 2 *Let $(\Omega, \mathbb{F}, P)$ be a filtered probability space and T a stopping time on this space. If (i) $Z(t)$ is a martingale with right-continuous sample paths, (ii) $T < \infty$ almost surely, and (iii) $\{Z(T \wedge t), t \geq 0\}$ is uniformly bounded, then the stopped process $\{Z(T \wedge t), t \geq 0\}$ is also a martingale and it holds that $\mathcal{E}[Z(T)] = \mathcal{E}[Z(0)]$.*

For notational convenience we write $T \wedge t$ instead of $\min\{T, t\}$. Unfortunately, condition *(ii),* i.e. $T < \infty$ almost surely, is not necessarily satisfied here. By Theorem 5.3 in Karlin and Taylor (1975, p.361) the probability that $X(t) = \ln Y(t) = \ln Y_0 + (\alpha - (\sigma^2/2))t + \sigma B(t)$ ever reaches $X_T = \ln Y_T < X_0 = \ln Y_0$ (or, equivalently, that $T < \infty$) is given by

$$\lim_{a \to \infty} \left(1 - \frac{\exp\left(-\left(2\alpha - \sigma^2\right) X_0/\sigma^2\right) - \exp\left(-\left(2\alpha - \sigma^2\right) X_T/\sigma^2\right)}{\exp\left(-\left(2\alpha - \sigma^2\right) a/\sigma^2\right) - \exp\left(-\left(2\alpha - \sigma^2\right) X_T/\sigma^2\right)} \right).$$

For $\alpha < \sigma^2/2$ this probability converges to one, i.e. $T < \infty$ almost surely. When $\alpha \geq \sigma^2/2$, however, T is infinite with positive probability (Karlin and Taylor, 1975, p.361). Let us summarize,

$$\Pr\{T < \infty\} = \begin{cases} 1 & \text{for } \alpha < \frac{\sigma^2}{2}, \\ \exp\left(-\left(2\alpha - \sigma^2\right)\left(X_0 - X_T\right)/\sigma^2\right) \\ = \left(Y_T/Y_0\right)^{\left(2\alpha/\sigma^2\right)-1} & \text{for } \alpha \geq \frac{\sigma^2}{2}. \end{cases} \tag{A.1}$$

Hence, we have well defined what is understood by the following transformation of $f(Y_0)$,

$$\mathcal{E}_0\left[e^{-\rho T}\right] = \Pr\{T < \infty\} \cdot \mathcal{E}_0\left[e^{-\rho T}; T < \infty\right] + \Pr\{T = \infty\} \cdot \mathcal{E}_0\left[e^{-\rho T}; T = \infty\right].$$

[1] Note that Karlin and Taylor calculate this expectation under the assumptions that $0 < X(0) = x < \ln Y_T$ rather than $x > \ln Y_T$ (see below) and that $Y_t = Y_0 \exp[\alpha t - \sigma B(t)]$ rather than $Y_t = Y_0 \exp[(\alpha - \sigma^2)t - \sigma B(t)]$. Both assumptions are of major importance for the solution. In this appendix we put emphasis on the formal aspects of the computation. For a more intuitive, but heuristic derivation, see Dixit and Pindyck (1994, appendix of chap.9).

Since the discount rate ρ is positive by assumption, $\mathcal{E}_0[e^{-\rho T}; T = \infty] = 0$ and

$$f\left(Y_0\right) = \Pr\left\{T < \infty\right\} \cdot \mathcal{E}_0\left[e^{-\rho T}; T < \infty\right].$$

Now, we can focus on the computation of $\mathcal{E}_0[e^{-\rho T}; T < \infty]$. In what follows the contingency on the event $\{T < \infty\}$ is suppressed for ease of notation. Let us define $V(t) = \exp(\beta X(t) - \gamma t)$, where $\gamma = \beta(\alpha - (\sigma^2/2)) + \beta^2(\sigma^2/2)$. The so-called *Wald martingale* $V(t)$ is chosen such that it satisfies the martingale property. We have

$$\begin{aligned}
&\mathcal{E}\left[V\left(t+s\right)\left|Y\left(r\right), r \leq t\right.\right] \\
&= e^{-\gamma(t+s)} \cdot \mathcal{E}\left[e^{\beta(X(t+s)-X(t))} \cdot e^{\beta X(t)}\left|Y\left(r\right), r \leq t\right.\right] \\
&= V\left(t\right) \cdot e^{-\gamma s} \cdot \mathcal{E}\left[e^{\beta(X(t+s)-X(t))}\left|Y\left(r\right), r \leq t\right.\right] \\
&= V\left(t\right) \cdot e^{-\beta^2(\sigma^2/2)s} \cdot \mathcal{E}\left[e^{\beta\sigma(B(t+s)-B(t))}\left|Y\left(r\right), r \leq t\right.\right],
\end{aligned}$$

with $t, s > 0$. Due to the independence of $B(t+s) - B(t)$ with respect to the σ-algebra $\sigma(Y(r), r \leq t)$, the self-similarity property of Brownian motion, and the fact that $B(1) \sim N(0,1)$, one yields (e.g. see Mikosch, 1998, p.42)

$$\begin{aligned}
&\mathcal{E}_t\left[e^{\beta\sigma(B(t+s)-B(t))}\left|Y\left(r\right), r \leq t\right.\right] \\
&= \mathcal{E}\left[e^{\beta\sigma(B(t+s)-B(t))}\right] = \mathcal{E}\left[e^{\beta\sigma B(s)}\right] = \mathcal{E}\left[e^{\beta\sigma\sqrt{s}B(1)}\right] = e^{\beta^2(\sigma^2/2)s}.
\end{aligned}$$

Thus $\mathcal{E}[V\left(t+s\right)\left|Y\left(r\right), r \leq t\right] = V\left(t\right)$. From the martingale property it follows

$$\mathcal{E}\left[V\left(t\right)\right] = \mathcal{E}\left[V\left(0\right)\right] \quad \text{and} \quad \mathcal{E}\left[V\left(T \wedge t\right)\right] = \mathcal{E}\left[V\left(0\right)\right].$$

However, we are interested in $\mathcal{E}[V(T)]$ rather than $\mathcal{E}[V(T \wedge t)]$. Since $T < \infty$ is implicitly assumed, $\lim_{t\to\infty} V(T \wedge t) = V(T)$ holds generally. Further, in order to ensure that

$$\mathcal{E}\left[\lim_{t\to\infty} V\left(T \wedge t\right)\right] = \lim_{t\to\infty} \mathcal{E}\left[V\left(T \wedge t\right)\right] \Leftrightarrow \mathcal{E}\left[V\left(T\right)\right] = \mathcal{E}\left[V\left(0\right)\right]$$

is satisfied, we additionally need $V(T \wedge t) = \exp(\beta X(T \wedge t) - \gamma(T \wedge t))$ to be uniformly bounded. Fortunately, the restrictions $\beta \leq 0$ *and* $\gamma \geq 0 \Leftrightarrow \beta \leq 1 - 2\alpha/\sigma^2$ guarantee uniform boundedness of $V(T \wedge t)$, because in that case $0 < V(T \wedge t) \leq e^{\beta \ln Y_T}$ almost surely (again contingent on $T < \infty$). One yields

$$\begin{aligned} & \mathcal{E}\left[V\left(T\right)\right] = \mathcal{E}\left[V\left(0\right)\right] \\ \Leftrightarrow \quad & \mathcal{E}\left[\exp\left(\beta \ln Y_T - \gamma T\right)\right] = \exp\left(\beta \ln Y_0\right) \\ \Leftrightarrow \quad & \mathcal{E}\left[\exp\left(-\gamma T\right)\right] = \left(\frac{Y_T}{Y_0}\right)^{-\beta}. \end{aligned}$$

Recall that we still have to relate $f(Y_0) = \mathcal{E}_0[V(T)]|_{\gamma=\rho}$ with $\mathcal{E}[V(T)]$. Since $V(t)$ is a martingale, the conditional expectation $\mathcal{E}_0[V(T)]$ satisfies

$$\begin{aligned} & \mathcal{E}_0\left[V\left(T\right)\right] = V\left(0\right) = \mathcal{E}\left[V\left(T\right)\right] \\ \Leftrightarrow \quad & \mathcal{E}_0\left[V\left(T\right)\right] = \left(Y_T\right)^{-\beta} \mathcal{E}_0\left[e^{-\gamma T}\right] = \left(Y_T\right)^{-\beta} \mathcal{E}\left[e^{-\gamma T}\right] \end{aligned}$$

This leaves us with the desired result $\mathcal{E}_0\left[e^{-\gamma T}\right] = \mathcal{E}\left[e^{-\gamma T}\right]$. Setting γ equal to the discount rate ρ gives $f(Y_0) = (Y_T/Y_0)^{-\beta}$. It remains to relate β and $\gamma = \rho$ by the so-called "fundamental quadratic equation" (e.g., see Dixit and Pindyck, 1994), $0 = \beta^2(\sigma^2/2) + \beta(\alpha - (\sigma^2/2)) - \rho$, the two roots of which are given by

$$\beta_{1,2} = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}}.$$

While β_1 turns out to be greater than one, $\beta = \beta_2$ satisfies $\beta \leq 0$ *and* $\gamma \geq 0$. So we finally obtain

$$f\left(Y_0\right) = \Pr\left\{T < \infty\right\} \cdot \left(\frac{Y_T}{Y_0}\right)^{-\beta_2} = \begin{cases} \left(\frac{Y_T}{Y_0}\right)^{-\beta_2} & \text{for } \alpha < \frac{\sigma^2}{2}, \\ \left(\frac{Y_T}{Y_0}\right)^{-\beta_2 + \left(2\alpha/\sigma^2\right) - 1} & \text{for } \alpha \geq \frac{\sigma^2}{2}. \end{cases}$$

To guarantee the general validity of the equilibria derived in section 2.4, it is of essential importance to see that not only β_2 turns out to be negative, but also $\widetilde{\beta}_2 = \beta_2 - (2\alpha/\sigma^2) + 1 < 0$ given that $\alpha \geq \sigma^2/2$.

Next let us compute $g(Y_0)$. A basic condition for the existence of $g(Y_0)$ is that $\int_0^\infty e^{-\rho t}$ $\mathcal{E}_0[|Y(t)|]dt < \infty$. Due to the fact that the geometric Brownian motion Y is a positive process as $Y_0 > 0$, we have $|Y(t)| = Y(t)$. Then the conditional expectation can be written as

$$\mathcal{E}_0\left[|Y(t)|\right] = \mathcal{E}_0\left[Y(t)\right] = Y_0 \cdot e^{\left(\alpha - \left(\sigma^2/2\right)\right)t} \cdot \mathcal{E}_0\left[e^{\sigma B(t)}\right]$$

Above we have already shown that $\mathcal{E}_t[\exp(\sigma B(t))] = \exp((\sigma^2/2)t)$ holds. One obtains

$$\int_0^\infty e^{-\rho t}\mathcal{E}_0\left[|Y(t)|\right] dt = Y_0 \cdot \int_0^\infty e^{-(\rho-\alpha)t} dt.$$

Obviously, the integral is well-defined if $\delta = \rho - \alpha > 0$, which is assumed hereafter. By Tonelli's Theorem—a version of Fubini's Theorem (e.g., see Harrison, 1985, p.131) that requires the integrand to be a positive process (which is satisfied by $\widetilde{Y}(t) = e^{-\rho t}Y(t)$)—, the original function $g(Y_0)$ can be transformed as follows,

$$\begin{aligned} g(Y_0) &= \mathcal{E}_0\left[\int_0^\infty e^{-\rho t} Y(t)\, dt\right] = \int_0^\infty e^{-\rho t}\mathcal{E}_0\left[Y(t)\right] dt \\ &= Y_0 \cdot \int_0^\infty e^{-(\rho-\alpha)t} dt = \frac{Y_0}{\delta}. \end{aligned}$$

Finally, we compute $h(Y_0)$. Again, one has to take into account that $Y(t)$ might never reach Y_T such that $T = \infty$ with positive probability. Therefore

$$\begin{aligned} h(Y_0) &= \Pr\{T < \infty\} \cdot \left(g(Y_0) - l(Y_0)\right) + \Pr\{T = \infty\} \cdot g(Y_0) \\ &= g(Y_0) - \Pr\{T < \infty\} \cdot l(Y_0), \end{aligned}$$

where

$$l(Y_0) = \mathcal{E}_0\left[\int_T^\infty e^{-\rho t} Y(t)\, dt; T < \infty\right].$$

The probability that $T < \infty$ (or equivalently that $X(t)$ ever reaches X_T) has already been computed in equation (A.1). Further note that $Y(t) = \exp(X(t))$. Then $l(Y_0)$ can be transformed as follows,

$$l(Y_0) = \mathcal{E}_0\left[\int_T^\infty e^{-\rho t} e^{X(t)} dt; T < \infty\right] = \mathcal{E}_0\left[e^{-\rho T}\int_0^\infty e^{-\rho t} e^{X^*(t)} dt; T < \infty\right].$$

Since $\mathcal{E}_0[\cdot] = \mathcal{E}_0[\mathcal{E}[\cdot\,|\mathfrak{F}_T]]$ and $\exp(-\rho T)$ is measurable with respect to $\mathfrak{F}_T$, one obtains

$$l\left(Y_0\right) = \mathcal{E}_0\left[e^{-\rho T}\cdot \mathcal{E}_0\left[\left.\int\limits_0^{\infty} e^{-\rho t} e^{X^*(t)} dt\right| \mathfrak{F}_T\right]; T < \infty\right].$$

Then by the strong Markov property we get

$$\mathcal{E}_0\left[\left.\int\limits_0^{\infty} e^{-\rho t} e^{X^*(t)} dt\right| \mathfrak{F}_T\right] = \mathcal{E}_0\left[\int\limits_0^{\infty} e^{-\rho t} e^{X^*(t)} dt\right] = g\left(Y_T\right),$$

and $l(Y_0)$ becomes

$$l\left(Y_0\right) = \mathcal{E}_0\left[e^{-\rho T} g\left(Y_T\right); T < \infty\right] = g\left(Y_T\right)\cdot f\left(Y_0\right) = \frac{Y_T}{\delta}\cdot\left(\frac{Y_T}{Y_0}\right)^{-\beta_2}.$$

We can conclude that

$$h\left(Y_0\right) = \begin{cases} \frac{Y_0}{\delta} - \left(\frac{Y_T}{Y_0}\right)^{-\beta_2}\frac{Y_T}{\delta} & \text{for } \alpha < \frac{\sigma^2}{2}, \\ \frac{Y_0}{\delta} - \left(\frac{Y_T}{Y_0}\right)^{-\widetilde{\beta}_2}\frac{Y_T}{\delta} & \text{for } \alpha \geq \frac{\sigma^2}{2}, \end{cases}$$

with $\beta_2 = 0.5 - (\alpha/\sigma^2) - [(0.5 - \alpha/\sigma^2)^2 + 2\rho/\sigma^2]^2$ and $\widetilde{\beta}_2 = \beta_2 - (2\alpha/\sigma^2) + 1 < 0$.

A.2 Leader's Value Function

Let us prove the assertion that there exists a discontinuity of $L^{'}(Y_{\mathsf{T}})$ at Y_F. The r-limit $Y_{\mathsf{T}} \downarrow Y_F$ of $L^{'}\left(Y_{\mathsf{T}}\right)$ is given by

$$\begin{aligned} \lim_{Y_{\mathsf{T}}\downarrow Y_F} L^{'}\left(Y_{\mathsf{T}}\right) &= \lim_{Y_{\mathsf{T}}\downarrow Y_F}\left[\frac{R\left(\underline{q},\overline{q}\right)}{\delta} + \beta\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\underline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta}\right] \\ &= (1-\beta)\frac{R\left(\underline{q},\overline{q}\right)}{\delta} + \beta\frac{R\left(\underline{q},\underline{q}\right)}{\delta} \gtreqless 0 \quad \text{for } \frac{R\left(\underline{q},\overline{q}\right)}{R\left(\underline{q},\underline{q}\right)} \gtreqless \frac{-\beta}{1-\beta}. \end{aligned}$$

The l-limit $Y_{\mathsf{T}} \uparrow Y_F$ turns out to be $\lim_{Y_{\mathsf{T}}\uparrow Y_F} L^{'}(Y_{\mathsf{T}}) = R(\underline{q},\underline{q})/\delta > 0$. It follows that there indeed exists a discontinuity of $L^{'}(Y_{\mathsf{T}})$ at Y_F, since $\lim_{Y_{\mathsf{T}}\downarrow Y_F} L^{'}(Y_{\mathsf{T}}) < R(\underline{q},\underline{q})/\delta = \lim_{Y_{\mathsf{T}}\uparrow Y_F} L^{'}(Y_{\mathsf{T}})$. One can quickly verify the general validity of this inequality by recalling that $R(\underline{q},\underline{q}) > R(\underline{q},\overline{q})$.

A.3 Proof of Proposition 1

Throughout this proof we assume that roles are exogenously preassigned to firms.

Proof of part *(i)*: We have to distinguish between the three cases *(a)* $\mathcal{R}_1 \leq \mathcal{C} \leq \mathcal{R}_2$, *(b)* $\mathcal{C} < \mathcal{R}_1$, and *(c)* $\mathcal{C} > \mathcal{R}_2$.

Case *(a)*: Let us start with the assertion $Y_L \geq Y_F$. Clearly we have to prove below that this assertion is correct. Since we exogenously assign roles to players, the leader's threshold Y_L can be derived equivalently to the follower's threshold Y_F. According to equation (2.17) the firm that decides to become the leader at Y_{T}, $Y_L \geq Y_{\mathsf{T}} > Y_F$, yields the expected revenues $\underline{\Lambda}(Y_{\mathsf{T}})$ net of the option's strike price E,

$$\begin{aligned} L\left(Y_{\mathsf{T}}\right) = \underline{\Lambda}\left(Y_{\mathsf{T}}\right) - E = \quad & \frac{Y_{\mathsf{T}} R\left(\underline{q}, \overline{q}\right)}{\delta} - \frac{C_L \underline{q}}{\rho} - E \\ & + \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta} \frac{Y_F \left(R\left(\underline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right)}{\delta}. \end{aligned}$$

$>$From our considerations about the follower's value we know that the value of the leader's firm in the region $Y_{\mathsf{T}} > Y_L$ can be determined by assuming implicitly that Y_L is known. For $Y_{\mathsf{T}} > Y_L$ one obtains

$$\begin{aligned} L\left(Y_{\mathsf{T}}\right) = \widetilde{\Lambda}\left(Y_{\mathsf{T}}\right) = \quad & \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_L} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right) R\left(\overline{q}, \overline{q}\right) - C_L \overline{q}\right] dt\right] \\ & + \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_L - \mathsf{T})}\left(\underline{\Lambda}\left(Y_L\right) - E\right)\right]. \end{aligned}$$

Note that, since Y_L represents a predetermined threshold, $\underline{\Lambda}(Y_L)$ is constant and measurable with respect to the σ-algebra $\mathfrak{F}_{\mathsf{T}}$. Thus, after rearranging, we can write

$$\widetilde{\Lambda}\left(Y_{\mathsf{T}}\right) = R\left(\overline{q}, \overline{q}\right) \cdot h\left(Y_{\mathsf{T}}\right)\big|_{T=T_L} - \frac{C_L \overline{q}}{\rho} + \left(\frac{C_L \overline{q}}{\rho} + \underline{\Lambda}\left(Y_L\right) - E\right) \cdot f\left(Y_{\mathsf{T}}\right)\big|_{T=T_L}.$$

By using the expectation formulas from Lemma 6 in Appendix A.1, we get

$$\widetilde{\Lambda}\left(Y_{\mathsf{T}}\right) = \frac{Y_{\mathsf{T}} R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C_L \overline{q}}{\rho} - \left(\frac{Y_L}{Y_{\mathsf{T}}}\right)^{-\beta} \left(\frac{Y_L R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C_L \overline{q}}{\rho} - \underline{\Lambda}\left(Y_L\right) + E\right).$$

As for the follower, the leader's optimal closure threshold can be determined by maximizing $\widetilde{\Lambda}$ with respect to Y_L. The first derivative is given by

$$\frac{\partial\widetilde{\Lambda}}{\partial Y_L}=(1-\beta)\left(\frac{Y_L}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{R(\overline{q},\overline{q})-R\left(\underline{q},\overline{q}\right)}{\delta}\left(\frac{Y_F}{Y_L}\cdot\frac{\mathcal{C}}{\mathcal{R}_2}-1\right) \tag{A.2}$$

Since we assumed that $\mathcal{C}\leq\mathcal{R}_2$, $\partial\widetilde{\Lambda}/\partial Y_L$ is negative for all $Y_L>Y_F$. In that case it follows that either $Y_L=Y_F$ or our initial assertion, $Y_L\geq Y_F$, turns out to be wrong. To decide about this we consider the complementary assumption $Y_L<Y_F$. Under this scenario the follower will reduce his capacity immediately after the leader has done so (at Y_L). Consequently, for $Y_{\mathsf{T}}>Y_L$, the leader's value function is given by

$$\begin{aligned}L(Y_{\mathsf{T}}) &= \Lambda(Y_{\mathsf{T}})=\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_L}e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q})-C_L\overline{q}\right]dt\right]\\ &\quad+\mathcal{E}_{\mathsf{T}}\left[\int_{T_L}^{\infty}e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R\left(\underline{q},\underline{q}\right)-C_L\underline{q}dt\right]-e^{-\rho(T_L-\mathsf{T})}E\right].\end{aligned}$$

Again, by using the expectation formulas from Lemma 6 in Appendix A.1, we get

$$\begin{aligned}\Lambda(Y_{\mathsf{T}}) &= \frac{Y_{\mathsf{T}}R(\overline{q},\overline{q})}{\delta}-\frac{C_L\overline{q}}{\rho}-\left(\frac{Y_L}{Y_{\mathsf{T}}}\right)^{-\beta}\times\\ &\quad\times\left(\frac{Y_L\left(R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C_L\left(\overline{q}-\underline{q}\right)}{\rho}+E\right).\end{aligned}$$

Since C_L-C_F is assumed to be not too negative, i.e. $\mathcal{C}\geq\mathcal{R}_1$ is satisfied, the first derivative with respect to Y_L,

$$\frac{\partial\Lambda}{\partial Y_L}=(1-\beta)\left(\frac{Y_L}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)}{\delta}\left(\frac{Y_F}{Y_L}\cdot\frac{\mathcal{C}}{\mathcal{R}_1}-1\right), \tag{A.3}$$

is greater than zero for all $Y_L<Y_F$. So, if $\mathcal{R}_1\leq\mathcal{C}\leq\mathcal{R}_2$, then $Y_L=Y_F$ is indeed the optimal exercise threshold of the firm that gets preassigned to the leader's role.

Case *(b)*: If the leader's cost is low compared to the follower's cost, i.e. $\mathcal{C}<\mathcal{R}_1$ is satisfied, then the derivative in equation (A.3) has roots at $Y_L=0$, $\beta<-1$, and

$$Y_L=\frac{-\beta\delta}{1-\beta}\cdot\frac{(C_L/\rho)\left(\overline{q}-\underline{q}\right)-E}{R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)}=Y_{S_i}\big|_{C_i=C_L}<Y_F.$$

The second-order condition shows that Λ exhibits a saddle point at $Y_L = 0$, while the other solution indeed maximizes Λ. Moreover, the derivative in equation (A.2) is negative for all $Y_L \geq Y_F$. So, if $\mathcal{C} < \mathcal{R}_1$, then $Y_L = Y_{S_i}|_{C_i = C_L} < Y_F$ is the optimal exercise threshold of the firm that gets preassigned to the leader's role.

Case *(c)*: If the leader's and the follower's variable cost satisfy $\mathcal{C} > \mathcal{R}_2$, then the derivative in equation (A.2) has roots at $Y_L = 0$, $\beta < -1$, and

$$Y_L = \frac{-\beta\delta}{1-\beta} \cdot \frac{(C_L/\rho)\left(\overline{q} - \underline{q}\right) - E}{R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right)} = Y_M|_{C_M = C_L} > Y_F.$$

The second-order condition shows that $\widetilde{\Lambda}$ exhibits a saddle point at $Y_L = 0$, while the other solution maximizes $\widetilde{\Lambda}$. Moreover, the derivative in equation (A.3) is positive for all $Y_L < Y_F$. So, if $\mathcal{C} > \mathcal{R}_2$, then $Y_L = Y_M|_{C_M = C_L} > Y_F$ is the optimal exercise threshold of the firm that gets preassigned to the leader's role.

Proof of part *(ii)*: Imagine a myopic firm that faces constant marginal cost of production C_M and that is equipped with the option to reduce capacity from $\overline{q}$ to $\underline{q}$. If Y_T is greater than its optimal closure threshold Y_M, the myopic firm has the following value,

$$\begin{aligned} M\left(Y_\mathsf{T}\right) &= \mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right) - C_M\overline{q}\right]dt\right] \\ &\quad + \mathcal{E}_\mathsf{T}\left[\int_{T_M}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\underline{q},\overline{q}\right) - C_M\underline{q}\right]dt\right] - \mathcal{E}_\mathsf{T}\left[e^{-\rho(T_M-\mathsf{T})}E\right] \\ &= \frac{Y_\mathsf{T}R\left(\overline{q},\overline{q}\right)}{\delta} - \frac{C_M\overline{q}}{\rho} - \left(\frac{Y_M}{Y_\mathsf{T}}\right)^{-\beta} \times \\ &\quad \times \left(\frac{Y_M\left(R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta} - \frac{C_M\left(\overline{q}-\underline{q}\right)}{\rho} + E\right), \end{aligned}$$

where $T_M = \inf(t > \mathsf{T}\,|Y\left(t\right) \leq Y_M)$. The partial derivative of $M\left(Y_\mathsf{T}\right)$ with respect to Y_M,

$$\frac{\partial M}{\partial Y_M} = (1-\beta)\left(\frac{Y_M}{Y_\mathsf{T}}\right)^{-\beta}\frac{R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta}\left(\frac{-\beta\delta}{1-\beta}\frac{(C_M/\rho)\left(\overline{q}-\underline{q}\right) - E}{Y_M\left(R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)} - 1\right),$$

exhibits roots at

$$Y_M = \frac{-\beta\delta}{1-\beta} \cdot \frac{(C_M/\rho)\left(\overline{q}-\underline{q}\right)-E}{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)}$$

and $Y_M = 0$, $\beta < -1$. Again, from the second-order condition, $Y_M > 0$ is known to be the unique local maximizer, i.e. the high-cost firm that gets preassigned to the leader's role reduces capacity when the myopic firm's reduction threshold is hit.

The result in part (*iii*) follows from a simple modification of the proof of part (*ii*), i.e. substitute $R(\underline{q},\overline{q})$ for $D(2\overline{q})\underline{q}$. Finally, let us prove part (*iv*). The inequality $Y_M > Y_F$ ($Y_M < Y_F$) immediately follows from condition (2.4) (condition (2.5)). Moreover, $Y_M > Y_P$ is an implication of the fact that $D(2\overline{q})(\overline{q}-\underline{q}) > R(\overline{q},\overline{q})-R(\underline{q},\overline{q}) \Leftrightarrow D(2\overline{q}) < D(\overline{q}+\underline{q})$. This completes the proof of Proposition 1.

A.4 Proof of Proposition 2

Proof of part *(i)* and *(ii)*: Principally, the monopolist may decide to reduce capacity sequentially or to shut down both production units at the same time. I.e. either he chooses two distinct closure thresholds Y_O' and Y_O'', $Y_O' > Y_O''$, or he exercises both options simultaneously at Y_O. Suppose that a sequential disinvestment pattern is optimal. Then Y_O' and Y_O'', $Y_O' > Y_O''$, are the solutions to the maximization problem $\max_{Y_O',Y_O''} O^{seq}(Y_{\mathsf{T}})$ with

$$\begin{aligned} O^{seq}\left(Y_{\mathsf{T}}\right) &= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T_O'} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)D\left(2\overline{q}\right)-C\right]2\overline{q}dt \\ &\quad + \int_{T_O'}^{T_O''} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)D\left(\overline{q}+\underline{q}\right)-C\right]\left(\overline{q}+\underline{q}\right)dt \\ &\quad + \int_{T_O''}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)D\left(2\underline{q}\right)-C\right]2\underline{q}dt \\ &\quad - \left(e^{-\rho\left(T_O'-\mathsf{T}\right)}+e^{-\rho\left(T_O''-\mathsf{T}\right)}\right)E], \end{aligned}$$

$T_O' = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_O')$ and $T_O'' = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_O'')$. According to earlier findings we can express the monopolist's value function explicitly in terms of the closure thresholds,

$$
\begin{aligned}
&O^{seq}\left(Y_{\mathsf{T}}\right)=\frac{2Y_{\mathsf{T}}R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\cdot 2\overline{q}}{\rho}\\
&-\left(\frac{Y_O'}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_O'\left(2R\left(\overline{q},\overline{q}\right)-R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&-\left(\frac{Y_O''}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_O''\left(R\left(\overline{q},\underline{q}\right)+R\left(\underline{q},\overline{q}\right)-2R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right).
\end{aligned}
$$

The first-order condition with respect to Y_O',

$$
\begin{aligned}
0=&\frac{1-\beta}{\delta}\left(\frac{Y_O'}{Y_{\mathsf{T}}}\right)^{-\beta}\times\\
&\times\left(2R\left(\overline{q},\overline{q}\right)-R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)-\frac{-\beta\delta}{1-\beta}\frac{\left(C/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{Y_O'}\right),
\end{aligned}
$$

is satisfied for $Y_O'=0$, $\beta<-1$, and

$$
Y_O'=\frac{-\beta\delta}{1-\beta}\frac{\left(C/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{2R\left(\overline{q},\overline{q}\right)-R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)}.
$$

The second-order condition reveals that only the second value is a maximizer of O^{seq}. With respect to Y_O'' a similar result is obtained. The maximizing argument turns out to be

$$
Y_O''=\frac{-\beta\delta}{1-\beta}\frac{\left(C/\rho\right)\left(\overline{q}-\underline{q}\right)-E}{R\left(\overline{q},\underline{q}\right)+R\left(\underline{q},\overline{q}\right)-2R\left(\underline{q},\underline{q}\right)}.
$$

Substracting the denominator in the expression for Y_O'' from the denominator of Y_O' yields

$$
2\left[R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)-R\left(\overline{q},\underline{q}\right)+R\left(\underline{q},\underline{q}\right)\right]
$$

which is smaller (greater) than zero if condition (2.4) (condition (2.5)) holds. Thus, in the case of first-mover advantages we have $Y_O'>Y_O''$ verifying our initial assumption. However, with second-mover advantages one obtains $Y_O'<Y_O''$ contradicting our initial assumption. In this case a simultaneous rather than a sequential disinvestment pattern must be optimal for the monopolist. The corresponding simultaneous exercising threshold

Y_O is the solution to the maximization problem $\max_{Y_O} O^{sim}(Y_\mathsf{T})$ with

$$O^{sim}(Y_\mathsf{T}) = \mathcal{E}_\mathsf{T}[\int_\mathsf{T}^{T_O} e^{-\rho(t-\mathsf{T})}[Y(t)D(2\overline{q})-C]2\overline{q}dt + \int_{T_O}^{\infty} e^{-\rho(t-\mathsf{T})}\left[Y(t)D\left(2\underline{q}\right)-C\right]2\underline{q}dt - 2e^{-\rho(T_O-\mathsf{T})}E].$$

The monopolist's value function can be rewritten as

$$O^{sim}(Y_\mathsf{T}) = \frac{2Y_\mathsf{T}R(\overline{q},\overline{q})}{\delta} - \frac{C\cdot 2\overline{q}}{\rho} - \left(\frac{Y_O}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{2Y_O\left(R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)\right)}{\delta} - \frac{2C\left(\overline{q}-\underline{q}\right)}{\rho} - 2E\right).$$

The first-order condition with respect to Y_O,

$$\frac{2(1-\beta)}{\delta}\left(\frac{Y_O}{Y_\mathsf{T}}\right)^{-\beta}\left(R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right) - \frac{-\beta\delta}{1-\beta}\frac{(C/\rho)\left(\overline{q}-\underline{q}\right)-E}{Y_O}\right) = 0,$$

is satisfied for $Y_O = Y_{S^*} = Y_{S_i}|_{C_i=C}$.

Proof of part (*iii*): Note that $Y_F < Y_{S^*}$ as well as $Y_M < Y_{S^*}$ immediately follow from condition (2.3). These inequalities hold irrespective of the occurrence of first-mover or second-mover advantages. Moreover, the validity of $Y_O'' < Y_{S^*} < Y_O'$ was already proved above. It remains to be shown that $Y_F < Y_O''$. This requires that

$$R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right) > R\left(\overline{q},\underline{q}\right) + R\left(\underline{q},\overline{q}\right) - 2R\left(\underline{q},\underline{q}\right) \Leftrightarrow R\left(\underline{q},\underline{q}\right) > R\left(\underline{q},\overline{q}\right),$$

which is satisfied according to condition (2.3). This completes the proof of Proposition 2.

A.5 Proof of Lemma 4

Proof of part *(i)*: First, $L(Y_\mathsf{T}) = S(Y_\mathsf{T}, Y_{S^*}) = F(Y_\mathsf{T})$ for $Y_\mathsf{T} \le Y_F$ and $C = C_1 = C_2$ is an obvious implication of equations (2.12), (2.18), (2.26), and (2.28). Second, $L(Y_\mathsf{T})$, $F(Y_\mathsf{T})$, and $S(Y_\mathsf{T}, Y_{S^*})$ are (at least) C^2-functions for $Y_\mathsf{T} > Y_F$ and continuous for all Y_T.

Third, $L^{'}(Y_{\mathsf{T}}) < \partial S(Y_{\mathsf{T}}, Y_{S^*})/\partial Y_{\mathsf{T}} < F^{'}(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} > Y_F$ will be shown immediately. As a prerequisite the derivatives have to be computed. The slope of $F(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} > Y_F$ turns out to be

$$F^{'}(Y_{\mathsf{T}}) = \frac{R(\overline{q}, \underline{q})}{\delta} - \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta} \frac{R(\overline{q}, \underline{q}) - R(\underline{q}, \underline{q})}{\delta},$$

which is smaller than $R(\overline{q}, \underline{q})/\delta$, but greater than $R(\underline{q}, \underline{q})/\delta$. The partial derivative of $S(Y_{\mathsf{T}}, Y_{S^*})$ with respect to Y_{T} for $Y_{\mathsf{T}} > Y_{S^*}$ can be written as

$$\frac{\partial S(Y_{\mathsf{T}}, Y_{S^*})}{\partial Y_{\mathsf{T}}} = \frac{R(\overline{q}, \overline{q})}{\delta} - \left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{1-\beta} \frac{R(\overline{q}, \overline{q}) - R(\underline{q}, \underline{q})}{\delta},$$

which is smaller than $R(\overline{q}, \overline{q})/\delta$, but greater than $R(\underline{q}, \underline{q})/\delta$. The slope of $L(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} > Y_{S^*}$ is given by

$$L^{'}(Y_{\mathsf{T}}) = \frac{R(\underline{q}, \overline{q})}{\delta} + \beta \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta} \frac{R(\underline{q}, \underline{q}) - R(\underline{q}, \overline{q})}{\delta}.$$

If β is small enough, then $L^{'}(Y_{\mathsf{T}})$ might become negative. However, for Y_{T} becoming large it approaches $R(\underline{q}, \overline{q})/\delta$.

Now we can show that $L^{'}(Y_{\mathsf{T}}) < \partial S(Y_{\mathsf{T}}, Y_{S^*})/\partial Y_{\mathsf{T}} < F^{'}(Y_{\mathsf{T}})$ generally holds for $Y_{\mathsf{T}} > Y_F$ under the assumption of identical variable cost, i.e. $C_1 = C_2 = C$. Let us begin with the first inequality. For $Y_{\mathsf{T}} > Y_{S^*}$ we get

$$\begin{aligned}
& L^{'}(Y_{\mathsf{T}}) < \frac{\partial S(Y_{\mathsf{T}}, Y_{S^*})}{\partial Y_{\mathsf{T}}} \\
\Leftrightarrow\quad & \frac{R(\underline{q}, \overline{q})}{\delta} + \beta \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta} \frac{R(\underline{q}, \underline{q}) - R(\underline{q}, \overline{q})}{\delta} \\
& < \frac{R(\overline{q}, \overline{q})}{\delta} - \left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{1-\beta} \frac{R(\overline{q}, \overline{q}) - R(\underline{q}, \underline{q})}{\delta} \\
\Leftarrow\quad & R(\overline{q}, \overline{q}) - R(\underline{q}, \overline{q}) > \left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{1-\beta} \left(R(\overline{q}, \overline{q}) - R(\underline{q}, \underline{q})\right) \\
\Leftarrow\quad & R(\overline{q}, \overline{q}) - R(\underline{q}, \overline{q}) > R(\overline{q}, \overline{q}) - R(\underline{q}, \underline{q}),
\end{aligned}$$

which is satisfied, since $R(\underline{q}, \overline{q}) < R(\underline{q}, \underline{q})$. For $Y_F < Y_{\mathsf{T}} \leq Y_{S^*}$ we have

$$L^{'}(Y_{\mathsf{T}}) < \frac{\partial S(Y_{\mathsf{T}}, Y_{S^*})}{\partial Y_{\mathsf{T}}}$$

$$\Leftrightarrow \quad \frac{R\left(\underline{q},\overline{q}\right)}{\delta} + \beta\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\underline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta} < \frac{R\left(\underline{q},\underline{q}\right)}{\delta} \Leftrightarrow \beta\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta} < 0.$$

The inequality is valid due to $\beta < 0$.

Next, we examine the second assertion for $Y_{\mathsf{T}} > Y_{S^*}$,

$$\begin{aligned} & \frac{\partial S\left(Y_{\mathsf{T}}, Y_{S^*}\right)}{\partial Y_{\mathsf{T}}} < F^{'}\left(Y_{\mathsf{T}}\right) \\ \Leftrightarrow \quad & \frac{R\left(\overline{q},\overline{q}\right)}{\delta} - \left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\underline{q}\right)}{\delta} \\ & < \frac{R\left(\overline{q},\underline{q}\right)}{\delta} - \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)}{\delta} \\ \Leftarrow \quad & \left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{1-\beta}\left(R\left(\overline{q},\underline{q}\right) - R\left(\overline{q},\overline{q}\right)\right) < R\left(\overline{q},\underline{q}\right) - R\left(\overline{q},\overline{q}\right), \end{aligned}$$

which is satisfied by assumption. For $Y_F < Y_{\mathsf{T}} \leq Y_{S^*}$ we get

$$\begin{aligned} & \frac{\partial S\left(Y_{\mathsf{T}}, Y_{S^*}\right)}{\partial Y_{\mathsf{T}}} < F^{'}\left(Y_{\mathsf{T}}\right) \\ \Leftrightarrow \quad & \frac{R\left(\underline{q},\underline{q}\right)}{\delta} < \frac{R\left(\overline{q},\underline{q}\right)}{\delta} - \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)}{\delta} \\ \Leftrightarrow \quad & \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)\right) < R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right), \end{aligned}$$

which is obviously true, since $Y_{\mathsf{T}} > Y_F$ by assumption. It follows that $L\left(Y_{\mathsf{T}}\right) < S\left(Y_{\mathsf{T}}, Y_{S^*}\right) < F\left(Y_{\mathsf{T}}\right)$ for $Y_{\mathsf{T}} > Y_F$.

Proof of part *(ii)*: The shape of $F\left(Y_{\mathsf{T}}\right)$ for $Y_{\mathsf{T}} > Y_F$ is given by

$$F^{''}\left(Y_{\mathsf{T}}\right) = \left(1-\beta\right)\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right)}{Y_{\mathsf{T}}\delta} > 0.$$

Thus, $F\left(Y_{\mathsf{T}}\right)$ is strictly convex for $Y_{\mathsf{T}} > Y_F$. The shape of $S\left(Y_{\mathsf{T}}, Y_{S^*}\right)$ with respect to Y_{T} for $Y_{\mathsf{T}} > Y_{S^*}$ is strictly convex,

$$\frac{\partial^2 S\left(Y_{\mathsf{T}}, Y_{S^*}\right)}{\left(\partial Y_{\mathsf{T}}\right)^2} = \left(1-\beta\right)\left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\underline{q}\right)}{\delta Y_{\mathsf{T}}} > 0.$$

The shape of $L(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} > Y_F$ is strictly convex,

$$L''(Y_{\mathsf{T}}) = -\beta(1-\beta)\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{1-\beta}\frac{R(\underline{q},\underline{q}) - R(\underline{q},\overline{q})}{\delta Y_{\mathsf{T}}} > 0.$$

This completes the proof of Lemma 4.

A.6 Proof of Proposition 3

For ease of notation let Y_T denote player i's pure strategy $G_i^{\mathsf{T}}(t) = \mathbb{I}_{\{t\leq T\}}$ with $T = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_T)$ and $\mathbb{I}$ denoting the indicator function.

Proof of part (i): We start by pointing out that the equilibrium strategies satisfy the intertemporal consistency condition in Definition 3. Let us compute the expected equilibrium payoffs of the firm that chooses the leader's role in equilibrium (called the "equilibrium leader" henceforth) and of the firm that gets assigned to the follower's role in equilibrium (called the "equilibrium follower" henceforth) for later references. First, let $Y_{\mathsf{T}} > Y_L = Y_M$. Then one gets

$$\begin{aligned} V_L^* &= V_L(Y_{\mathsf{T}}, Y_L, Y_F) \\ &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)R(\overline{q},\overline{q}) - C\overline{q}\right]dt + e^{-\rho(T_M-\mathsf{T})}L(Y_M)\right] \end{aligned} \tag{A.4}$$

and

$$\begin{aligned} V_F^* &= V_F(Y_{\mathsf{T}}, Y_F, Y_L) \\ &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)R(\overline{q},\overline{q}) - C\overline{q}\right]dt + e^{-\rho(T_M-\mathsf{T})}F(Y_M)\right] \end{aligned}$$

Second, for $0 < Y_{\mathsf{T}} \leq Y_L$ the corresponding equilibrium payoffs turn out to be

$$V_L^* = L(Y_{\mathsf{T}}) \text{ and } V_F^* = F(Y_{\mathsf{T}}).$$

Note that F and L are defined in equations (2.12) and (2.18), respectively.

Next, we prove the proposition by showing that neither the equilibrium leader nor the equilibrium follower have an incentive to deviate. Let us start off with the leader. Proposition 1 implies that, if a firm gets assigned to the leader's role, then its payoff attains a unique global maximum at $Y_L = Y_M$. Thus, the equilibrium leader does not have any incentive to deviate to some exercise threshold $Y_{\mathcal{T}} > Y_M$ or $Y_{\mathcal{T}} \in (Y_F, Y_M)$. Moreover, by deviating to the some $Y_{\mathcal{T}} \leq Y_F$, the equilibrium leader yields $S(Y_{\mathsf{T}}, Y_F)$. So, for $Y_L = Y_M$ to be an equilibrium threshold, $V_L^* - S(Y_{\mathsf{T}}, Y_F) \geq 0$ should be satisfied. The difference on the left-hand side can be written as

$$
\begin{aligned}
& \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q}) - C\overline{q}\right]dt + e^{-\rho(T_M-\mathsf{T})}L(Y_M)\right] \\
& -\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_F} e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q}) - C\overline{q}\right]dt + e^{-\rho(T_F-\mathsf{T})}I(Y_F)\right] \\
= \; & \mathcal{E}_{\mathsf{T}}\left[-\int_{T_M}^{T_F} e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q}) - C\overline{q}\right]dt\right] \\
& +\mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_M-\mathsf{T})}L(Y_M) - e^{-\rho(T_F-\mathsf{T})}I(Y_F)\right] \\
= \; & -R(\overline{q},\overline{q})\,\mathcal{E}_{\mathsf{T}}\left[\int_{T_M}^{T_F} e^{-\rho(t-\mathsf{T})}Y(t)\,dt\right] + \frac{C\overline{q}}{\rho}\left(f(Y_{\mathsf{T}})|_{T=T_M} - f(Y_{\mathsf{T}})|_{T=T_F}\right) \\
& +L(Y_M)\,f(Y_{\mathsf{T}})|_{T=T_M} - I(Y_F)\,f(Y_{\mathsf{T}})|_{T=T_F}\,. \qquad \text{(A.5)}
\end{aligned}
$$

The functions L and I are defined by equations (2.18) and (2.28), respectively, and the Laplace transform $f(Y_{\mathsf{T}})$ has been calculated in Lemma 6 in Appendix A.1. Due to the linearity of the conditional expectations operator the expectation of the Riemann-Stieltjes integral can be rewritten as the difference of two standard expressions,

$$
\mathcal{E}_{\mathsf{T}}\left[\int_{T_M}^{T_F} e^{-\rho(t-\mathsf{T})}Y(t)\,dt\right] = \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_F} e^{-\rho(t-\mathsf{T})}Y(t)\,dt\right] - \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}Y(t)\,dt\right].
$$

The conditional expectations on the right-hand side are known from Lemma 6 in Appendix A.1, i.e. they equal $g(Y_{\mathsf{T}})|_{T=T_F}$ and $g(Y_{\mathsf{T}})|_{T=T_M}$, respectively. We obtain for the difference in payoffs

$$
V_L^* - S(Y_{\mathsf{T}}, Y_F) =
$$

$$
\begin{aligned}
&-\left(\frac{Y_M}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_M R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\overline{q}}{\rho}\right)+\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\overline{q}}{\rho}\right)\\
&+\left(\frac{Y_M}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_M R\left(\underline{q},\overline{q}\right)}{\delta}-\frac{C\underline{q}}{\rho}-E\right)+\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_F\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}\\
&+\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F R\left(\underline{q},\underline{q}\right)}{\delta}-\frac{C\underline{q}}{\rho}-E\right)
\end{aligned}
$$

or after rearranging

$$
\begin{aligned}
&V_L^*-S(Y_{\mathsf{T}},Y_F)=\\
&-\left(\frac{Y_M}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_M\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&+\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right),
\end{aligned}
$$

which is greater than zero. The validity of the inequality becomes more apparent if we replace $(C(\overline{q}-\underline{q})/\rho)-E$ by $((1-\beta)/Y_M(R(\overline{q},\overline{q})-R(\underline{q},\overline{q}))/(-\beta\delta)$ and divide through by $-\delta(Y_{\mathsf{T}})^{\beta}/(R(\overline{q},\overline{q})-R(\underline{q},\overline{q}))$. This yields

$$
(Y_M)^{-\beta}\left(Y_M-\frac{1-\beta}{-\beta}Y_M\right)<(Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_M\right).
$$

If $Y_F = Y_M$, both sides would be equal. However, $Y_F < Y_M$ by the assumption of first-mover advantages (2.4) and

$$
\frac{\partial\left[(Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_M\right)\right]}{\partial Y_F}=(1-\beta)\left(Y_F\right)^{-\beta}\left(1-\frac{Y_M}{Y_F}\right)<0.
$$

Hence, the inequality holds and the suggested deviation from the equilibrium strategy decreases the equilibrium leader's payoff.

So far we have shown that there does not exist any profitable deviation strategy for the equilibrium leader. Let us analyze the equilibrium follower. First, suppose that the equilibrium follower deviates to some threshold $Y_T \in (0,Y_F)\cup(Y_F,Y_M)$. Since a player conditionally on being assigned to the follower's role at some $Y_T \geq Y_F$ reduces capacity at Y_F anyway (see Lemma 1), the equilibrium follower's payoff is not affected by this modified choice of the adoption date of the leader's role. Second, if the equilibrium follower deviates to the leader's threshold $Y_{\mathcal{T}} = Y_M$, then firms move simultaneously and the equilibrium follower yields $S(Y_{\mathsf{T}}, Y_M)$. For Y_F to be an equilibrium threshold $V_F^* - S(Y_{\mathsf{T}}, Y_M) \geq 0$, should be satisfied. The difference on the left-hand side is given by

$$\begin{aligned}
& \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C\overline{q}\right] dt + e^{-\rho(T_M-\mathsf{T})} F(Y_M)\right] \\
& -\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C\overline{q}\right] dt + e^{-\rho(T_M-\mathsf{T})} I(Y_M)\right] \\
= \; & \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_M-\mathsf{T})}\left[F(Y_M) - I(Y_M)\right]\right].
\end{aligned}$$

Since Lemma 4 implies that $F(Y_{\mathcal{T}}) > I(Y_{\mathcal{T}})$ for all $Y_{\mathcal{T}} > Y_F$, the inequality holds and the suggested deviation from the equilibrium strategy decreases the equilibrium follower's payoff. Finally, if the equilibrium follower deviates to even "earlier" exercise thresholds $Y_{\mathcal{T}} > Y_M$, then it becomes the leader at $Y_{\mathcal{T}}$ implying that the other firm will follow at Y_F. Again, conditionally on getting assigned to the leader's role at $Y_{\mathcal{T}}$, from the above derivations, the equilibrium follower cannot do better than reducing capacity at Y_M. Thus, the equilibrium leader's payoff, V_L^*, is a least upper bound to the equilibrium follower's payoff from deviating to some threshold $Y_{\mathcal{T}} > Y_M$. It follows that $V_F^* - V_L^* \geq 0$ must be satisfied in order to guarantee that the proposed strategy profiles are indeed Nash equilibria. The difference on the inequality's left-hand side can be written as

$$\begin{aligned}
& \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C\overline{q}\right] dt + e^{-\rho(T_M-\mathsf{T})} F(Y_M)\right] \\
& -\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_M} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C\overline{q}\right] dt + e^{-\rho(T_M-\mathsf{T})} L(Y_M)\right] \\
= \; & \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_M-\mathsf{T})}\left[F(Y_M) - L(Y_M)\right]\right].
\end{aligned}$$

Since Lemma 4 implies that $F(Y_{\mathcal{T}}) > L(Y_{\mathcal{T}})$ for all $Y_{\mathcal{T}} > Y_F$, the inequality holds and the suggested deviation from the equilibrium strategy decreases the equilibrium follower's payoff.

The requirement of subgame perfection remains to be discussed. It is sufficient to show that

(1.)

$$L\left(Y_{\mathsf{T}}\right) \geq \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C\overline{q}\right]dt+e^{-\rho(\mathcal{T}-\mathsf{T})}L\left(Y_{\mathcal{T}}\right)\right]$$

for $Y_M > Y_{\mathsf{T}} > Y_{\mathcal{T}} > Y_F$,

(2.)

$$L(Y_{\mathsf{T}}) \geq S(Y_{\mathsf{T}}, Y_F)$$

for $Y_M > Y_{\mathsf{T}} > Y_F$, and

(3.)

$$S\left(Y_{\mathsf{T}},Y_{\mathsf{T}}\right)=I\left(Y_{\mathsf{T}}\right) \geq \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C\overline{q}\right]dt+e^{-\rho(\mathcal{T}-\mathsf{T})}F\left(Y_{\mathcal{T}}\right)\right]$$

for $Y_F \geq Y_{\mathsf{T}} > Y_{\mathcal{T}} > 0$. Proposition 1 says that the equilibrium leader's payoff attains a unique global maximum at $Y_L = Y_M$ and strictly increases in the leader's adoption threshold Y_L if $Y_L < Y_M$. Hence, the first inequality holds. The second inequality can be shown to hold by replacing Y_M by Y_{T} and T_M by T in equation (A.5). Lemma 4 indicates that $F(Y_{\mathcal{T}}) = I(Y_{\mathcal{T}})$ for all $Y_{\mathcal{T}} \leq Y_F$. Thus, the third inequality can be written as

$$\begin{aligned}
& I\left(Y_{\mathsf{T}}\right) \geq \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C\overline{q}\right]dt+e^{-\rho(\mathcal{T}-\mathsf{T})}I\left(Y_{\mathcal{T}}\right)\right] \\
\Leftrightarrow\quad & -\frac{Y_{\mathsf{T}}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}+\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}-E \\
& +\left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{\mathcal{T}}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right) \geq 0 \\
\Leftrightarrow\quad & -\left(Y_{\mathsf{T}}\right)^{-\beta}\left(Y_{\mathsf{T}}-\frac{1-\beta}{-\beta}Y_{S^*}\right)+\left(Y_{\mathcal{T}}\right)^{-\beta}\left(Y_{\mathcal{T}}-\frac{1-\beta}{-\beta}Y_{S^*}\right) \geq 0,
\end{aligned}$$

where we replaced $(C(\overline{q}-\underline{q})/\rho)-E$ by $((1-\beta)/Y_{S^*}(R(\overline{q},\overline{q})-R(\underline{q},\underline{q}))/(-\beta\delta)$. Note that $Y_{S^*} = Y_{S_i}|_{C_i=C}$ with Y_{S_i} defined in equation (2.27). If $Y_{\mathcal{T}} = Y_{\mathsf{T}}$, both sides would be equal. However, $Y_{\mathcal{T}} < Y_{\mathsf{T}}$ by assumption and

$$\frac{\partial\left[\left(Y_{\mathcal{T}}\right)^{-\beta}\left(Y_{\mathcal{T}}-\frac{1-\beta}{-\beta}Y_{S^*}\right)\right]}{\partial Y_{\mathcal{T}}}=\left(1-\beta\right)\left(Y_{\mathcal{T}}\right)^{-\beta}\left(1-\frac{Y_{S^*}}{Y_{\mathcal{T}}}\right)<0,$$

since $Y_{\mathcal{T}} < Y_F < Y_{S^*}$ according to Lemma 4. Hence, the third inequality is satisfied too. This completes the proof of part (i).

Proof of part (ii): Again, we start by stressing that the suggested equilibrium strategies satisfy the intertemporal consistency condition in Definition 3. For later references let us state the (expected) equilibrium payoff of firm i, $i \in \{1, 2\}$. We have

$$V_i^* = V_i\left(Y_{\mathsf{T}}, Y_F, Y_F\right) = \begin{cases} S\left(Y_{\mathsf{T}}, Y_F\right) & \text{for } Y_{\mathsf{T}} > Y_F \\ I\left(Y_{\mathsf{T}}\right) & \text{for } Y_{\mathsf{T}} \leq Y_F \end{cases},$$

where S and I are defined by equations (2.26) and (2.28), respectively. We prove this part by showing that firm i has no incentive to deviate. First, if firm i deviates to some "earlier" threshold $Y_{\mathcal{T}} > Y_F$, then it becomes the leader at $Y_{\mathcal{T}}$ and its payoff from deviation can be written as

$$\begin{aligned} \widetilde{V}_i &= V_i\left(Y_{\mathsf{T}}, Y_{\mathcal{T}}, Y_F\right) \\ &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R\left(\overline{q}, \overline{q}\right) - C\overline{q}\right] dt + e^{-\rho(\mathcal{T}-\mathsf{T})} L\left(Y_{\mathcal{T}}\right)\right]. \end{aligned}$$

Substracting $\widetilde{V}_i$ from V_i^* should give a non-negative valued function. We have

$$\begin{aligned} V_i^* - \widetilde{V}_i &= \mathcal{E}_{\mathsf{T}}[\int_{\mathcal{T}}^{T_F} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R\left(\overline{q}, \overline{q}\right) - C\overline{q}\right] dt \\ &\quad + e^{-\rho(T_F-\mathsf{T})} I\left(Y_F\right) - e^{-\rho(\mathcal{T}-\mathsf{T})} L\left(Y_{\mathcal{T}}\right)], \end{aligned}$$

where L is defined by equation (2.18). Following the same procedure as applied to equation (A.5) in part (i) gives

$$\begin{aligned} & V_i^* - \widetilde{V}_i = \\ & \left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{\mathcal{T}} R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho}\right) - \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho}\right) \\ & + \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F R\left(\underline{q}, \underline{q}\right)}{\delta} - \frac{C\underline{q}}{\rho} - E\right) \\ & - \left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{\mathcal{T}} R\left(\underline{q}, \overline{q}\right)}{\delta} - \frac{C\underline{q}}{\rho} - E\right) - \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta} \frac{Y_F\left(R\left(\underline{q}, \underline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right)}{\delta} \end{aligned}$$

or after rearranging

$$
\begin{aligned}
&V_i^* - \widetilde{V}_i = \\
&\left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{\mathcal{T}}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right) \\
&-\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right),
\end{aligned}
$$

which is strictly greater than zero. The validity of the inequality becomes more apparent, if we replace $(C(\overline{q}-\underline{q})/\rho)-E$ by $((1-\beta)/Y_M(R(\overline{q},\overline{q})-R(\underline{q},\overline{q}))/(-\beta\delta)$ and divide through by $\delta(Y_{\mathsf{T}})^{\beta}/(R(\overline{q},\overline{q})-R(\underline{q},\overline{q}))$. This yields

$$
(Y_{\mathcal{T}})^{-\beta}\left(Y_{\mathcal{T}}-\frac{1-\beta}{-\beta}Y_M\right) > (Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_M\right).
$$

If $Y_{\mathcal{T}} = Y_F$, both sides would be equal. However, $Y_{\mathcal{T}} > Y_F$ and

$$
\frac{\partial\left[(Y_{\mathcal{T}})^{-\beta}\left(Y_{\mathcal{T}}-\frac{1-\beta}{-\beta}Y_M\right)\right]}{\partial Y_{\mathcal{T}}} = (1-\beta)\,(Y_{\mathcal{T}})^{-\beta}\left(1-\frac{Y_M}{Y_{\mathcal{T}}}\right) > 0,
$$

since $Y_F > Y_M$ by the assumption of second-mover advantages (2.5). Hence, the inequality holds and the suggested deviation from the equilibrium strategy decreases firm i's payoff.

Second, suppose firm i deviates to some "later" threshold $Y_{\mathcal{T}} < Y_F$. Then it gets assigned to the follower's role at Y_F. However, this strategy does not change firm i's payoff, since $F(Y_{\mathcal{T}}) = I(Y_{\mathcal{T}})$ for all $Y_{\mathcal{T}} \leq Y_F$ according to Lemma 4. Thus, firm i has no incentive to deviate.

Subgame perfection follows from

$$
\begin{aligned}
&S\left(Y_{\mathsf{T}},Y_{\mathsf{T}}\right) = I\left(Y_{\mathsf{T}}\right) \\
&\geq \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C\overline{q}\right]dt + e^{-\rho(\mathcal{T}-\mathsf{T})}F\left(Y_{\mathcal{T}}\right)\right] \\
&= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}\left[Y\left(t\right)R\left(\overline{q},\overline{q}\right)-C\overline{q}\right]dt + e^{-\rho(\mathcal{T}-\mathsf{T})}I\left(Y_{\mathcal{T}}\right)\right]
\end{aligned}
$$

for $Y_F \geq Y_\mathsf{T} > Y_T > 0$ what we already proved to be satisfied in part (i). This completes the proof of Proposition 3.

A.7 Proof of Proposition 4

Proof of part (i): We show that there does not exist any symmetric pure strategy equilibrium in the timing game with identical firms and first-mover advantages. Suppose the contrary, i.e. suppose that the strategy profile (Y^*, Y^*) with $Y^* \neq Y_F$ is a Nash equilibrium of the capacity-reduction game starting at $Y_\mathsf{T} > Y^*$. Three cases have to be distinguished, $Y^* > Y_F$, $Y^* < Y_F$, and $Y^* = Y_F$. First, if $Y^* > Y_F$, then for (Y^*, Y^*) to be a Nash equilibrium the deviation strategy Y_F should not increase firm i's payoff, i.e.

$$V_i^* = V_i\left(Y_\mathsf{T}, Y^*, Y^*\right) \geq \widetilde{V}_i = V_i\left(Y_\mathsf{T}, Y_F, Y^*\right)$$

must hold for $i \in \{1, 2\}$. We have

$$\begin{aligned}
&V_i^* - \widetilde{V}_i = \\
&\frac{Y_\mathsf{T} R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho} - \left(\frac{Y^*}{Y_\mathsf{T}}\right)^{-\beta} \left(\frac{Y^* R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho} - I\left(Y^*\right)\right) \\
&-\frac{Y_\mathsf{T} R\left(\overline{q}, \overline{q}\right)}{\delta} + \frac{C\overline{q}}{\rho} + \left(\frac{Y^*}{Y_\mathsf{T}}\right)^{-\beta} \left(\frac{Y^* R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho} - F\left(Y^*\right)\right).
\end{aligned}$$

Since $I(Y_{T^*}) < F(Y_{T^*})$ for $Y_{T^*} > Y_F$, the inequality $V_i^* \geq \widetilde{V}_i$ does not hold proving that (Y^*, Y^*), $Y^* > Y_F$, is not a Nash equilibrium.

Second, if $Y^* < Y_F$, then for (Y^*, Y^*) to be a Nash equilibrium the deviation strategy Y_F should not increase firm i's payoff, i.e.

$$V_i^* = V_i\left(Y_\mathsf{T}, Y^*, Y^*\right) \geq \widehat{V}_i = V_i\left(Y_\mathsf{T}, Y_F, Y^*\right)$$

must hold for $i \in \{1, 2\}$. We have

$$\begin{aligned}
&V_i^* - \widehat{V}_i = \\
&\frac{Y_\mathsf{T} R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho} - \left(\frac{Y^*}{Y_\mathsf{T}}\right)^{-\beta} \left(\frac{Y^* R\left(\overline{q}, \overline{q}\right)}{\delta} - \frac{C\overline{q}}{\rho} - I\left(Y^*\right)\right)
\end{aligned}$$

$$-\frac{Y_{\mathsf{T}} R\left(\overline{q}, \overline{q}\right)}{\delta}+\frac{C \overline{q}}{\rho}+\left(\frac{Y_{F}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{F} R\left(\overline{q}, \overline{q}\right)}{\delta}-\frac{C \overline{q}}{\rho}-L\left(Y_{F}\right)\right).$$

Since $Y_{T^*} < Y_F$, it holds that $I(Y_{T^*}) = F(Y_{T^*}) = L(Y_{T^*})$ and $I(Y_F) = F(Y_F) = L(Y_F)$. Thus,

$$\begin{aligned}
V_i^* - \widehat{V}_i = &-\left(\frac{Y^*}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y^* R\left(\overline{q}, \overline{q}\right)}{\delta}-\frac{C \overline{q}}{\rho}-I\left(Y_{T^*}\right)\right) \\
&+\left(\frac{Y_{F}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{F} R\left(\overline{q}, \overline{q}\right)}{\delta}-\frac{C \overline{q}}{\rho}-I\left(Y_{F}\right)\right) \\
&=\left(\frac{Y_{F}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{F}\left(R\left(\overline{q}, \overline{q}\right)-R\left(\underline{q}, \underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right) \\
&-\left(\frac{Y^*}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y^*\left(R\left(\overline{q}, \overline{q}\right)-R\left(\underline{q}, \underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right) < 0\,,
\end{aligned}$$

which is strictly smaller than zero. The validity of the inequality becomes more obvious if one replaces $(C(\overline{q}-\underline{q})/\rho) - E$ by $((1-\beta)/Y_{S^*}(R(\overline{q},\overline{q}) - R(\underline{q},\underline{q}))/(-\beta\delta)$ such that

$$V_i^* < \widehat{V}_i \Leftrightarrow \left(Y_F\right)^{-\beta}\left(Y_F - \frac{1-\beta}{-\beta} Y_{S^*}\right) - \left(Y^*\right)^{-\beta}\left(Y^* - \frac{1-\beta}{-\beta} Y_{S^*}\right) < 0$$

If Y^* was equal to Y_F, then $V^* = \widehat{V}$. However, $Y^* < Y_F$ by assumption and

$$\frac{\partial\left[-\left(Y^*\right)^{-\beta}\left(Y^* - \frac{1-\beta}{-\beta} Y_{S^*}\right)\right]}{\partial Y^*} = -\left(1-\beta\right)\left(Y^*\right)^{-\beta}\left(1 - \frac{Y_{S^*}}{Y^*}\right) > 0,$$

since $Y_F < Y_{S^*}$. Hence, $V_i^* < \widehat{V}_i$ holds indeed, and the suggested deviation from the equilibrium candidate (Y^*, Y^*), $Y^* < Y_F$, is profitable.

Third, if $Y^* = Y_F$, then for (Y^*, Y^*) to be a Nash equilibrium the deviation strategy $Y_L = Y_M$ should not increase firm i's payoff, i.e.

$$V_i^* = V_i\left(Y_{\mathsf{T}}, Y_F, Y_F\right) \geq \overline{V}_i = V_i\left(Y_{\mathsf{T}}, Y_M, Y_F\right)$$

must hold for $i \in \{1, 2\}$. We have

$$V_i^* - \overline{V}_i = S\left(Y_{\mathsf{T}}, Y_F\right) - V_L^*,$$

where V_L^* denotes the equilibrium leader's payoff in the proof of Proposition 3 (see equation (A.4)). There, $V_L^* - S(Y_\mathsf{T}, Y_F)$ was shown to be strictly positive if the condition of first-mover advantages is satisfied. Thus, Y_L represents a profitable deviation strategy proving that $(Y^*, Y^*) = (Y_F, Y_F)$ cannot be a Nash equilibrium.

Proof of part (ii): We show that there does not exist any other symmetric pure strategy equilibrium than the one that is suggested in Proposition 3 if strategic interaction is driven by second-mover advantages. In contrast to part (i) only two cases have to be distinguished, $Y^* > Y_F$ and $Y^* < Y_F$. In both cases Y_F turns out to be a profitable deviation strategy from the equilibrium candidate (Y^*, Y^*). The proofs are identical to the corresponding proofs of part (i). This completes the proof of Proposition 4.

A.8 Proof of Proposition 5

We show that there does not exist any symmetric non-degenerate mixed strategy equilibrium in the timing game with identical firms. This is done by making extensive use of the following well-known fundamental lemma,

Lemma 7 *Let Y_T denote player i's pure strategy $G_i^\mathsf{T}(t) = \mathbb{I}_{\{t \leq T\}}$ with $T = \inf(t \geq \mathsf{T} \,|Y(t) \leq Y_T)$ and $\mathbb{I}$ denoting the indicator function. Further, let* $\text{supp}(G_i - G_i^-)$ *denote the set of closure thresholds that player i plays with positive probability in the mixed strategy profile $G = (G_1, G_2)$. Strategy profile G is a Nash equilibrium in the closure timing game if and only if $\forall i, j \in \{1, 2\}$, $i \neq j$,*

$$V_i(Y_\mathsf{T}, Y_T, G_j) = V_i(Y_\mathsf{T}, Y'_T, G_j)$$

$\forall Y_T, Y'_T \in \text{supp}(G_i - G_i^-)$,

$$V_i(Y_\mathsf{T}, Y_T, G_j) \geq V_i(Y_\mathsf{T}, Y'_T, G_j)$$

$\forall Y_T \in \text{supp}(G_i - G_i^-)$, $Y'_T \notin \text{supp}(G_i - G_i^-)$.

As our concern is exclusively dedicated to *symmetric* mixed strategy equilibria, we can set $i = 1$ and $j = 2$ without loss of generality. Of course, symmetry of equilibrium strategies means that $G_1 = G_2$.

It seems to be a major difficulty to derive explicit expressions for the firms' payoffs as player i's strategy, G_i, can take on any general functional form that represents a cumulative distribution function. Consider, however, the following partition

$$\Theta_K : Y_{T(K)} < Y_{T(K-1)} < ... < Y_{T(1)} < Y_{T(0)}$$

of the deterministic interval $[Y_{T(K)}, Y_{T(0)}]$ and the corresponding partition

$$\tau_K : T(0) < T(1) < ... < T(K-1) < T(K)$$

of the stochastic interval $[T(0), T(K)]$, where $Y_{T(0)}, ..., Y_{T(K)}$, is a sequence of fixed thresholds with mesh$(\Theta_K) \to 0$ for $K \to \infty$ and $T(k) = \inf(t > T(k-1) \,|Y(t) \leq Y_{T(k)})$, $k = 1, ..., K$, $T(0) = T$ is a sequence of stopping times with mesh$(\tau_K) \overset{a.s.}{\to} 0$ for $K \to \infty$. Then there exists a step function

$$\Gamma_i(t) = \begin{cases} 0 & \text{for } t < T(0) \\ G_i(T(k)) - G_i(T(k-1)) & \text{for } T(k) \leq t < T(k+1), \\ & \text{and } k \in \{0, ..., K-1\} \\ 1 & \text{for } t \geq T(K) \end{cases}$$

with $G_i(T(-1)) \equiv 0$ such that $\lim_{K\to\infty} \Gamma_i(t) = G_i(t) - G_i^-(t)\ \forall t \in [0, \infty)$ almost surely. Thus, we can express the cumulative distribution function $G_i(\cdot)$ as a limit of a sequence of values, $G_i(T(0)), ..., G_i(T(K))$. These functions of stopping times are measurable with respect to the information set that is available at the root of the timing game. In what follows it is assumed w.l.o.g. that $Y_{T(0)} = \max\{Y_T \in \text{supp}(G_i - G_i^-)\}$ and $Y_{T(K)} = \min\{Y_T \in \text{supp}(G_i - G_i^-)\}$.

Proof of part *(i)*: The proof works as follows. In subpart (a) we show that there does not exist any symmetric non-degenerate mixed strategy profile that has at least partial support on $[\mathsf{T}, T_M)$ or $(Y_M, Y_\mathsf{T}]$, respectively, and constitutes an equilibrium. Then, in subpart (b), it is proved that there does not exist any symmetric non-degenerate mixed strategy profile that has at least partial support on the interval (T_F, ∞) or $(0, Y_F)$, and that is an equilibrium. Finally, in subpart (c) we show that there does not exist any symmetric non-degenerate mixed strategy profile with *exclusive* support on the remaining interval $[T_M, T_F]$ or $[Y_F, Y_M]$, and is an equilibrium. In the last subpart, however, we have to confine ourselves to mixed strategies that are *discrete* lotteries over the space of closure thresholds.

Proof of subpart (a): Suppose that there exists a symmetric non-degenerate mixed strategy equilibrium G with $\text{supp}(G_1 - G_1^-) \cap (Y_M, Y_{\mathsf{T}}]$ nonempty. Two cases have to be distinguished. First, suppose that $T(K) < T_M$. Then

$$V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) = V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \tag{A.6}$$

should be satisfied $\forall Y_{T(k)} \in \{Y_{T(0)}, ..., Y_{T(K-1)}\} \subset \text{supp}(G_2 - G_2^-)$ in equilibrium. For ease of notation let us define

$$\Pi(t) \equiv e^{-\rho(t-\mathsf{T})}\left[Y(t) R(\overline{q}, \overline{q}) - C\overline{q}\right].$$

Then, we can write

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(K)} \Pi(t)(1 - G_2(t))\, dt + \int_{\mathsf{T}}^{T(K)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) \\
&+ e^{-\rho(T(K)-\mathsf{T})}\left(1 - G_2^-(T(K))\right) I\left(Y_{T(K)}\right)] \\
&- \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(k)} \Pi(t)(1 - G_2(t))\, dt + \int_{\mathsf{T}}^{T(k)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) + e^{-\rho(T(k)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(k)) - G_2^-(T(k))\right) I\left(Y_{T(k)}\right) + (1 - G_2(T(k))) L\left(Y_{T(k)}\right)\right\}].
\end{aligned}$$

After rearrangement one obtains

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{T(k)}^{T(K)} \Pi(t)(1 - G_2(t))\, dt + \int_{T(k)}^{T(K)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) \\
&+ e^{-\rho(T(K)-\mathsf{T})}\left(1 - G_2^-(T(K))\right) I\left(Y_{T(K)}\right) - e^{-\rho(T(k)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(k)) - G_2^-(T(k))\right) I\left(Y_{T(k)}\right) + (1 - G_2(T(k))) L\left(Y_{T(k)}\right)\right\}].
\end{aligned}$$

According to the above considerations about limit representations of the cumulative distribution function G_2, the difference of payoffs can be rewritten as

$$
\begin{aligned}
& V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{T(k)}^{T(K)} \Pi(t)\, dt - \lim_{K\to\infty} \sum_{m=k}^{K-1} G_2\left(T\left(m\right)\right) \int_{T(m)}^{T(m+1)} \Pi(t)\, dt \\
&+ \lim_{K\to\infty} \sum_{m=k}^{K-1} \Gamma_2\left(T\left(m\right)\right) e^{-\rho(T(m)-\mathsf{T})} F\left(Y_{T(m)}\right) \\
&+ \lim_{K\to\infty} \Gamma_2\left(T\left(K\right)\right) e^{-\rho(T(K)-\mathsf{T})} I\left(Y_{T(K)}\right) \\
&- e^{-\rho(T(k)-\mathsf{T})} \left\{ \lim_{K\to\infty} \Gamma_2\left(T\left(k\right)\right) I\left(Y_{T(k)}\right) + \left(1 - G_2\left(T\left(k\right)\right)\right) L\left(Y_{T(k)}\right) \right\}].
\end{aligned}
$$

Note that

$$
\begin{aligned}
& \sum_{m=k}^{K-1} G_2\left(T\left(m\right)\right) \int_{T(m)}^{T(m+1)} \Pi(t)\, dt \\
= \; & \sum_{m=k}^{K-1} G_2\left(T\left(m\right)\right) \int_{T(m)}^{T(m+1)} \Pi(t)\, dt + \sum_{m=k-1}^{K-1} G_2\left(T\left(m\right)\right) \int_{T(m+1)}^{T(K)} \Pi(t)\, dt \\
& - \sum_{m=k}^{K-1} G_2\left(T\left(m-1\right)\right) \int_{T(m)}^{T(K)} \Pi(t)\, dt \\
= \; & \sum_{m=k}^{K-1} \Gamma_2\left(T\left(m\right)\right) \int_{T(m)}^{T(K)} \Pi(t)\, dt + G_2\left(T\left(k-1\right)\right) \int_{T(k)}^{T(K)} \Pi(t)\, dt
\end{aligned}
$$

and

$$
1 = G_2\left(T\left(K\right)\right) = G_2\left(T\left(k-1\right)\right) + \Gamma_2\left(T\left(k\right)\right) + \sum_{m=k+1}^{K-1} \Gamma_2\left(T\left(m\right)\right) + \Gamma_2\left(T\left(K\right)\right).
$$

By inserting the former transformation and applying the latter one to the terms $\int_{T(k)}^{T(K)} \Pi(t)dt$ and $\exp(-\rho(T(k)-\mathsf{T}))L(Y_{T(k)})$, we can summarize

$$
\begin{aligned}
& V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}\left[\lim_{K\to\infty} \Gamma_2\left(T\left(k\right)\right) \cdot e^{-\rho(T(k)-\mathsf{T})} \left\{F\left(Y_{T(k)}\right) - I\left(Y_{T(k)}\right)\right\}\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\mathcal{E}_{\mathsf{T}}[\lim_{K\to\infty}\sum_{m=k+1}^{K-1}\Gamma_2\left(T\left(m\right)\right)\times \\
&\times\left\{\int_{T(k)}^{T(m)}\Pi\left(t\right)dt+e^{-\rho(T(m)-\mathsf{T})}F\left(Y_{T(m)}\right)-e^{-\rho(T(k)-\mathsf{T})}L\left(Y_{T(k)}\right)\right\}] \\
&+\mathcal{E}_{\mathsf{T}}[\lim_{K\to\infty}\Gamma_2\left(T\left(K\right)\right)\times \\
&\times\left\{\int_{T(k)}^{T(K)}\Pi\left(t\right)dt+e^{-\rho(T(K)-\mathsf{T})}I\left(Y_{T(K)}\right)-e^{-\rho(T(k)-\mathsf{T})}L\left(Y_{T(k)}\right)\right\}].
\end{aligned}
$$

Since the expressions to the right of each limes operator—especially the sums—turn out to be bounded, the Fubini Theorem applies and one is allowed to put the limes in front of the conditional expectation. Moreover, the functions of stopping times are measurable with respect to the information set at T such that we end up with

$$
\begin{aligned}
&V_1\left(Y_{\mathsf{T}},Y_{T(K)},G_2\right)-V_1\left(Y_{\mathsf{T}},Y_{T(k)},G_2\right) \\
&=\lim_{K\to\infty}\Gamma_2\left(T\left(k\right)\right)\cdot\mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T(k)-\mathsf{T})}\left\{F\left(Y_{T(k)}\right)-I\left(Y_{T(k)}\right)\right\}\right] \\
&+\lim_{K\to\infty}\sum_{m=k+1}^{K-1}\Gamma_2\left(T\left(m\right)\right)\times \\
&\times\mathcal{E}_{\mathsf{T}}\left[\int_{T(k)}^{T(m)}\Pi\left(t\right)dt+e^{-\rho(T(m)-\mathsf{T})}F\left(Y_{T(m)}\right)-e^{-\rho(T(k)-\mathsf{T})}L\left(Y_{T(k)}\right)\right] \\
&+\lim_{K\to\infty}\Gamma_2\left(T\left(K\right)\right)\times \\
&\times\mathcal{E}_{\mathsf{T}}\left[\int_{T(k)}^{T(K)}\Pi\left(t\right)dt+e^{-\rho(T(K)-\mathsf{T})}I\left(Y_{T(K)}\right)-e^{-\rho(T(k)-\mathsf{T})}L\left(Y_{T(k)}\right)\right].
\end{aligned}
\tag{A.7}
$$

It remains to be shown that the conditional expectations are non-negative and at least one is strictly positive. In this case we can conclude that the difference in payoffs is greater than zero, since the limes of weighted sums of exclusively non-negative terms and at least one positive term also attains a strictly positive value. From $F(Y_T) > I(Y_T)\ \forall Y_T > Y_F$ and $Y_{T(k)} > Y_{T(K)} > Y_M > Y_F$, one can immediately conclude that the expectation in the second line is strictly positive. Moreover, due to $I(Y_T) \geq L\left(Y_T\right)\ \forall Y_T$, a lower bound of the conditional expectation in line six is given by

$$\mathcal{E}_{\mathsf{T}}\left[\int\limits_{T(k)}^{T(K)}\Pi\left(t\right)dt+e^{-\rho(T(K)-\mathsf{T})}L\left(Y_{T(K)}\right)-e^{-\rho(T(k)-\mathsf{T})}L\left(Y_{T(k)}\right)\right]. \tag{A.8}$$

>From previous considerations it is known that this expression can be written as

$$\begin{aligned}
&\left(\frac{Y_{T(K)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(K)}R\left(\underline{q},\overline{q}\right)}{\delta}-\frac{C\underline{q}}{\rho}-E\right)+\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_F\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}\\
&-\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(k)}R\left(\underline{q},\overline{q}\right)}{\delta}-\frac{C\underline{q}}{\rho}-E\right)-\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_F\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}\\
&+\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(k)}R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\overline{q}}{\rho}\right)-\left(\frac{Y_{T(K)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(K)}R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\overline{q}}{\rho}\right).
\end{aligned}$$

Rearrangement yields

$$\begin{aligned}
&\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)}{\delta}\times\\
&\times\left[\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(k)}-\frac{1-\beta}{-\beta}Y_M\right)-\left(\frac{Y_{T(K)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(K)}-\frac{1-\beta}{-\beta}Y_M\right)\right].
\end{aligned}$$

Note that this expression would equal zero if $Y_{T(k)}=Y_{T(K)}$. However, $Y_{T(k)}>Y_{T(K)}>Y_M$ and

$$\frac{\partial\left(\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(k)}-\frac{1-\beta}{-\beta}Y_M\right)\right)}{\partial Y_{T(k)}}=\left(1-\beta\right)\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(1-\frac{Y_M}{Y_{T(k)}}\right)>0.$$

Thus, the lower bound (A.8) and the conditional expectation in line six of equation (A.7) are strictly positive. The same outcome results if we replace $Y_{T(K)}$ by $Y_{T(m)}$ with $m\in\{k{+}1,...,K{-}1\}$. Moreover, since $F(Y_T)>I(Y_T)\ \forall Y_T>Y_F$, not only the conditional expectation in line six, but also the one in line four of equation (A.7),

$$\mathcal{E}_{\mathsf{T}}\left[\int\limits_{T(k)}^{T(m)}\Pi\left(t\right)dt+e^{-\rho(T(m)-\mathsf{T})}F\left(Y_{T(m)}\right)-e^{-\rho(T(k)-\mathsf{T})}L\left(Y_{T(k)}\right)\right],$$

turns out to be strictly positive. So far we have shown that equation (A.6) cannot be satisfied for any $Y_{T(k)} \in \{Y_{T(0)}, ..., Y_{T(K-1)}\} \subset \text{supp}(G_2 - G_2^-)$, $T(K) < T_M$.

Second, in the complementary case we have $T(K) \geq T_M$. Then, in equilibrium

$$V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) = V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right) \tag{A.9}$$

should be satisfied $\forall Y_{T(k)}, Y_{T(l)} \in \{Y_{T(0)}, ..., Y_{T(K-1)}\} \subset \text{supp}(G_2 - G_2^-)$, $Y_{T(k+1)} < Y_M \leq Y_{T(k)} < Y_{T(l)}$. We have

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(k)} \Pi(t)(1 - G_2(t))\, dt + \int_{\mathsf{T}}^{T(k)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) + e^{-\rho(T(k)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(k)) - G_2^-(T(k))\right) I\left(Y_{T(k)}\right) + (1 - G_2(T(k)))\, L\left(Y_{T(k)}\right)\right\}] \\
&-\mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(l)} \Pi(t)(1 - G_2(t))\, dt + \int_{\mathsf{T}}^{T(l)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) + e^{-\rho(T(l)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(l)) - G_2^-(T(l))\right) I\left(Y_{T(l)}\right) + (1 - G_2(T(l)))\, L\left(Y_{T(l)}\right)\right\}].
\end{aligned}$$

After rearrangement one obtains

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{T(l)}^{T(k)} \Pi(t)(1 - G_2(t))\, dt + \int_{T(l)}^{T(k)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) + e^{-\rho(T(k)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(k)) - G_2^-(T(k))\right) I\left(Y_{T(k)}\right) + (1 - G_2(T(k)))\, L\left(Y_{T(k)}\right)\right\} \\
&-e^{-\rho(T(l)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(l)) - G_2^-(T(l))\right) I\left(Y_{T(l)}\right) + (1 - G_2(T(l)))\, L\left(Y_{T(l)}\right)\right\}].
\end{aligned}$$

By proceeding equivalently as above we get

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}\left[\int_{T(l)}^{T(k)} \Pi(t)\, dt - \lim_{K\to\infty} \sum_{m=l}^{k-1} G_2(T(m)) \int_{T(m)}^{T(m+1)} \Pi(t)\, dt\right.
\end{aligned}$$

$$
\begin{aligned}
&+\lim_{K\to\infty}\sum_{m=l}^{k-1}\Gamma_2\left(T\left(m\right)\right)e^{-\rho\left(T\left(m\right)-\mathsf{T}\right)}F\left(Y_{T(m)}\right)\\
&+e^{-\rho\left(T\left(k\right)-\mathsf{T}\right)}\left\{\lim_{K\to\infty}\Gamma_2\left(T\left(k\right)\right)I\left(Y_{T(k)}\right)+\left(1-G_2\left(T\left(k\right)\right)\right)L\left(Y_{T(k)}\right)\right\}\\
&-e^{-\rho\left(T\left(l\right)-\mathsf{T}\right)}\left\{\lim_{K\to\infty}\Gamma_2\left(T\left(l\right)\right)I\left(Y_{T(l)}\right)+\left(1-G_2\left(T\left(l\right)\right)\right)L\left(Y_{T(l)}\right)\right\}\Bigg].
\end{aligned}
$$

Analogous to the procedure that we followed above the term

$$
\lim_{K\to\infty}\sum_{m=l}^{k-1}G_2\left(T\left(m\right)\right)\int\limits_{T(m)}^{T(m+1)}\Pi\left(t\right)dt
$$

can be written as

$$
\lim_{K\to\infty}\sum_{m=l}^{k-1}\Gamma_2\left(T\left(m\right)\right)\int\limits_{T(m)}^{T(k)}\Pi\left(t\right)dt+G_2\left(T\left(l-1\right)\right)\int\limits_{T(l)}^{T(k)}\Pi\left(t\right)dt.
$$

Moreover, if the probability-weight transformation

$$
1=G_2\left(T\left(l-1\right)\right)+\Gamma_2\left(T\left(l\right)\right)+\sum_{m=l+1}^{k}\Gamma_2\left(T\left(m\right)\right)+1-G_2\left(T\left(k\right)\right)
$$

is applied to the terms $\int_{T(l)}^{T(k)}\Pi(t)dt$ and $\exp(-\rho(T(l)-\mathsf{T}))L(Y_{T(l)})$, then the payoff difference can be written as

$$
\begin{aligned}
&V_1\left(Y_{\mathsf{T}},Y_{T(k)},G_2\right)-V_1\left(Y_{\mathsf{T}},Y_{T(l)},G_2\right)\\
&=\lim_{K\to\infty}\Gamma_2\left(T\left(l\right)\right)\mathcal{E}_{\mathsf{T}}\left[e^{-\rho\left(T\left(l\right)-\mathsf{T}\right)}\left\{F\left(Y_{T(l)}\right)-I\left(Y_{T(l)}\right)\right\}\right]\\
&+\lim_{K\to\infty}\sum_{m=l+1}^{k-1}\Gamma_2\left(T\left(m\right)\right)\times\\
&\times\mathcal{E}_{\mathsf{T}}\left[\int\limits_{T(l)}^{T(m)}\Pi\left(t\right)dt+e^{-\rho\left(T\left(m\right)-\mathsf{T}\right)}F\left(Y_{T(m)}\right)-e^{-\rho\left(T\left(l\right)-\mathsf{T}\right)}L\left(Y_{T(l)}\right)\right]\\
&+\lim_{K\to\infty}\Gamma_2\left(T\left(k\right)\right)\times\\
&\times\mathcal{E}_{\mathsf{T}}\left[\int\limits_{T(l)}^{T(k)}\Pi\left(t\right)dt+e^{-\rho\left(T\left(k\right)-\mathsf{T}\right)}I\left(Y_{T(m)}\right)-e^{-\rho\left(T\left(l\right)-\mathsf{T}\right)}L\left(Y_{T(l)}\right)\right]\\
&+\lim_{K\to\infty}\left(1-G_2\left(T\left(k\right)\right)\right)\times
\end{aligned}
$$

$$\times \mathcal{E}_{\mathsf{T}} \left[\int\limits_{T(l)}^{T(k)} \Pi(t)\, dt + e^{-\rho(T(k)-\mathsf{T})} L\left(Y_{T(k)}\right) - e^{-\rho(T(l)-\mathsf{T})} L\left(Y_{T(l)}\right) \right].$$

Obviously, the conditional expectation in line two is strictly positive. Comparing the conditional expectation in line eight to the expectation in equation (A.8) reveals that the expressions are identical if we substitute $Y_{T(k)}$ for $Y_{T(K)}$ and $Y_{T(l)}$ for $Y_{T(k)}$. It immediately follows that the conditional expectation in line eight is strictly positive. The same outcome results if we replace $Y_{T(k)}$ by $Y_{T(m)}$ with $m \in \{l+1, ..., k-1\}$. Moreover, since $F(Y_T) > L(Y_T)\ \forall Y_T > Y_F$ and $I(Y_T) \geq L(Y_T)\ \forall Y_T \geq Y_F$, not only the conditional expectation in line eight, but also those in line four and line six turn out to be strictly positive. Thus, equation (A.9) cannot be satisfied for any $Y_{T(k)}, Y_{T(l)} \in \{Y_{T(0)}, ..., Y_{T(K-1)}\} \subset \text{supp}(G_2 - G_2^-)$, $Y_{T(k+1)} < Y_M \leq Y_{T(k)} < Y_{T(l)}$. This completes the proof of subpart (a).

Proof of subpart (b): In subpart (a) we have shown that a symmetric non-degenerate mixed strategy equilibrium cannot have support on $Y_T > Y_M$. In the following part of the proof we concentrate on mixed strategies that are lotteries with at least partial support on the interval $(0, Y_F]$. Contrary, to subpart (a) the structure of the payoff functions does not require to distinguish between the analogous cases $Y_{T(0)} \leq Y_F$ and $Y_{T(0)} > T_F$. Again, we start off by assuming that there exists a symmetric mixed strategy profile with support on $\{Y_{T(0)}, ..., Y_{T(K)}\}$, $Y_{T(K)} < Y_F$, and is an equilibrium. Then

$$V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) = V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right) \tag{A.10}$$

should be satisfied $\forall Y_{T(l)}$, $l \in \{k+1, ..., K\}$, in any equilibrium with

$$Y_{T(k)} = \begin{cases} Y_{T(0)} & \text{for } Y_{T(0)} \leq Y_F, \\ \max\left\{Y_T \in \{Y_{T(0)}, ..., Y_{T(K)}\} : Y_T \leq Y_F\right\} & \text{for } Y_{T(0)} > Y_F. \end{cases}$$

Again, we focus on the difference of payoffs[2],

$$V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right)$$

[2]If $T(k) = T(0)$, then $G_2^-(T(k)) = G_2(T(k-1)) = G_2(T(-1)) = 0$.

$$= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(k)} \Pi(t)(1-G_2(t))\,dt + \int_{\mathsf{T}}^{T(k)} e^{-\rho(t-\mathsf{T})} F(Y(t))\,dG_2(t) + e^{-\rho(T(k)-\mathsf{T})} \times$$
$$\times \left\{ \left(G_2(T(k)) - G_2^-(T(k))\right) I\left(Y_{T(k)}\right) + (1-G_2(T(k)))\,L\left(Y_{T(k)}\right)\right\}]$$
$$-\mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(l)} \Pi(t)(1-G_2(t))\,dt + \int_{\mathsf{T}}^{T(l)} e^{-\rho(t-\mathsf{T})} F(Y(t))\,dG_2(t) + e^{-\rho(T(l)-\mathsf{T})} \times$$
$$\times \left\{ \left(G_2(T(l)) - G_2^-(T(l))\right) I\left(Y_{T(l)}\right) + (1-G_2(T(l)))\,L\left(Y_{T(l)}\right)\right\}].$$

Recall that $F(Y_T) = L(Y_T) = I(Y_T)\ \forall Y_T \leq Y_F$. Thus, one obtains

$$V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right)$$
$$= \mathcal{E}_{\mathsf{T}}[-\int_{T(k)}^{T(l)} \Pi(t)(1-G_2(t))\,dt - \int_{T(k)}^{T(l)} e^{-\rho(t-\mathsf{T})} I(Y(t))\,dG_2(t)$$
$$+e^{-\rho(T(k)-\mathsf{T})}\left(1-G_2^-(T(k))\right) I\left(Y_{T(k)}\right) - e^{-\rho(T(l)-\mathsf{T})}\left(1-G_2^-(T(l))\right) I\left(Y_{T(l)}\right)]$$

after rearrangement. By using the limit representation of the cumulative distribution function G_2, the difference of payoffs can be rewritten as

$$V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(l)}, G_2\right)$$
$$= \mathcal{E}_{\mathsf{T}}[-\int_{T(k)}^{T(l)} \Pi(t)\,dt + \lim_{K\to\infty} \sum_{m=k}^{l-1} \Gamma_2(T(m)) \int_{T(m)}^{T(l)} \Pi(t)\,dt$$
$$+ \lim_{K\to\infty} G_2(T(k-1)) \int_{T(k)}^{T(l)} \Pi(t)\,dt - \lim_{K\to\infty} \sum_{m=k}^{l-1} \Gamma_2(T(m))\, e^{-\rho(T(m)-\mathsf{T})} I\left(Y_{T(m)}\right)$$
$$+e^{-\rho(T(k)-\mathsf{T})}\left(1-\lim_{K\to\infty} G_2(T(k-1))\right) I\left(Y_{T(k)}\right)$$
$$-e^{-\rho(T(l)-\mathsf{T})}\left(1-\lim_{K\to\infty} G_2(T(l-1))\right) I\left(Y_{T(l)}\right)].$$

Consider the following transformations of probability weights,

$$1 = \sum_{m=l}^{K} \Gamma_2(T(m)) + \sum_{m=k}^{l-1} \Gamma_2(T(m)) + G_2(T(k-1))$$

and

$$G_2\left(T\left(l-1\right)\right)=\sum_{m=k}^{l-1}\Gamma_2\left(T\left(m\right)\right)+G_2\left(T\left(k-1\right)\right).$$

If one applies the former probability-weight transformation to $-\int_{T(k)}^{T(l)}\Pi(t)$ and $\exp(-\rho(T(k)-\mathsf{T}))I(Y_{T(k)})$, and both transformations to $(1-\lim_{K\to\infty}G_2(T(l-1)))\cdot\exp(-\rho(T(l)-\mathsf{T}))$ $I(Y_{T(l)})$, it results that

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}},Y_{T(k)},G_2\right)-V_1\left(Y_{\mathsf{T}},Y_{T(l)},G_2\right)\\
&=\lim_{K\to\infty}\sum_{m=l}^{K}\Gamma_2\left(T\left(m\right)\right)\times\\
&\times\mathcal{E}_{\mathsf{T}}\left[-\int\limits_{T(k)}^{T(l)}\Pi\left(t\right)dt+e^{-\rho(T(k)-\mathsf{T})}I\left(Y_{T(k)}\right)-e^{-\rho(T(l)-\mathsf{T})}I\left(Y_{T(l)}\right)\right]\\
&+\lim_{K\to\infty}\sum_{m=k}^{l-1}\Gamma_2\left(T\left(m\right)\right)\times\\
&\times\mathcal{E}_{\mathsf{T}}\left[-\int\limits_{T(k)}^{T(m)}\Pi\left(t\right)dt+e^{-\rho(T(k)-\mathsf{T})}I\left(Y_{T(k)}\right)-e^{-\rho(T(m)-\mathsf{T})}I\left(Y_{T(m)}\right)\right],
\end{aligned}\tag{A.11}$$

Evaluating the first conditional expectation in line three gives

$$\begin{aligned}
&\left(\frac{Y_{T(l)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(l)}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&-\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(k)}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right).
\end{aligned}$$

After rearrangement one obtains

$$\begin{aligned}
&\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\times\\
&\times\left[\left(\frac{Y_{T(l)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(l)}-\frac{1-\beta}{-\beta}Y_{S^*}\right)-\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(k)}-\frac{1-\beta}{-\beta}Y_{S^*}\right)\right].
\end{aligned}$$

This expression would be zero if $Y_{T(l)}$ was equal to $Y_{T(k)}$. However, $Y_{T(l)} < Y_{T(k)} \leq Y_F < Y_{S^*}$ and

$$\frac{\partial \left(\frac{Y_{T(l)}}{Y_\mathsf{T}}\right)^{-\beta} \left(Y_{T(l)} - \frac{1-\beta}{-\beta} Y_{S^*}\right)}{\partial Y_{T(l)}} = (1-\beta)\left(\frac{Y_{T(l)}}{Y_\mathsf{T}}\right)^{-\beta}\left(1 - \frac{Y_{S^*}}{Y_{T(l)}}\right) < 0.$$

Thus, the expectation in line three of equation (A.11) is greater than zero. The same outcome results if we replace $Y_{T(l)}$ by $Y_{T(m)}$ with $m \in \{1, ..., l-1\}$. Hence, not only the conditional expectation in line three, but also the one in line five of equation (A.11) turns out to be strictly positive. We conclude that equation (A.10) cannot be satisfied for any $Y_{T(k)}, Y_{T(l)} \in \{Y_{T(0)}, ..., Y_{T(K)}\} \subset \text{supp}(G_2 - G_2^-)$, $Y_{T(l)} < Y_{T(k)} \leq Y_F < Y_{T(k-1)}$ unless the equilibrium strategies become degenerate. This completes the proof of subpart (b).

Proof of subpart (c): We have shown so far that there does not exist any symmetric mixed strategy profile that has support on some $Y_T \in (0, Y_F) \cup (Y_M, \infty)$ and that constitutes an equilibrium. The non-existence of any symmetric equilibrium mixed strategy profile that has exclusive support on an arbitrary subset of the interval $[Y_F, Y_M]$ is proved differently. We start off by assuming that there exists a mixed strategy equilibrium (G_1, G_2) with support on $Y_{T(0)}, ..., Y_{T(K)}$, $Y_M \geq Y_{T(0)} > ... > Y_{T(K)} \geq Y_F$.

First, we prove that $Y_{T(K)} = Y_F$ must be satisfied in any mixed strategy equilibrium. Suppose the contrary, i.e. suppose that there exists a mixed strategy equilibrium with $Y_{T(K)} > Y_F$. Then the inequality

$$V_1\left(Y_\mathsf{T}, Y_{T(K)}, G_2\right) \geq V_1\left(Y_\mathsf{T}, Y_F, G_2\right)$$

should be satisfied. Let us investigate the difference of payoffs,

$$\begin{aligned}
&V_1\left(Y_\mathsf{T}, Y_{T(K)}, G_2\right) - V_1\left(Y_\mathsf{T}, Y_F, G_2\right)\\
&\mathcal{E}_\mathsf{T}[\int_\mathsf{T}^{T(K)} \Pi(t)\left(1 - G_2(t)\right) dt + \int_\mathsf{T}^{T(K)} e^{-\rho(t-\mathsf{T})} F\left(Y(t)\right) dG_2(t)\\
&+ e^{-\rho(T(K)-\mathsf{T})}\left(1 - G_2^-\left(T(K)\right)\right) I\left(Y_{T(K)}\right)]\\
&- \mathcal{E}_\mathsf{T}[\int_\mathsf{T}^{T(K)} \Pi(t)\left(1 - G_2(t)\right) dt + \int_\mathsf{T}^{T(K)} e^{-\rho(t-\mathsf{T})} F\left(Y(t)\right) dG_2(t)\\
&+ e^{-\rho(T(K)-\mathsf{T})}\left(1 - G_2^-\left(T(K)\right)\right) F\left(Y_{T(K)}\right)]
\end{aligned}$$

$$= \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T(K)-\mathsf{T})}\left(1-G_2^-\left(T\left(K\right)\right)\right)\left\{I\left(Y_{T(K)}\right)-F\left(Y_{T(K)}\right)\right\}\right].$$

Since $I(Y_T) < F(Y_T)\ \forall Y_T > Y_F$, the above inequality is not satisfied unless $Y_{T(K)} = Y_F$.

Second, in any mixed strategy equilibrium, where strategies are *discrete* lotteries over some support $Y_{T(0)}, ..., Y_{T(K)}$, it should be satisfied that

$$V_1\left(Y_{\mathsf{T}}, Y_{T(K-1)}, G_2\right) = V_1\left(Y_{\mathsf{T}}, Y_{T(K)} = Y_F, G_2\right) \geq V_1\left(Y_{\mathsf{T}}, Y_{\mathcal{T}}, G_2\right)$$

with $Y_{\mathcal{T}} \in (Y_{T(K)}, Y_{T(K-1)})$. Again, let us consider the difference of payoffs,

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{\mathcal{T}}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(K)} \Pi\left(t\right)\left(1-G_2\left(t\right)\right)dt + \int_{\mathsf{T}}^{T(K)} e^{-\rho(t-\mathsf{T})}F\left(Y\left(t\right)\right)dG_2\left(t\right) \\
&+ e^{-\rho(T(K)-\mathsf{T})}\left\{\left(1-G_2^-\left(T\left(K\right)\right)\right)I\left(Y_{T(K)}\right)\right\}] \\
&- \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{\mathcal{T}} \Pi\left(t\right)\left(1-G_2\left(t\right)\right)dt + \int_{\mathsf{T}}^{\mathcal{T}} e^{-\rho(t-\mathsf{T})}F\left(Y\left(t\right)\right)dG_2\left(t\right) \\
&+ e^{-\rho(\mathcal{T}-\mathsf{T})}\left\{\left(G_2\left(\mathcal{T}\right)-G_2^-\left(\mathcal{T}\right)\right)I\left(Y_{\mathcal{T}}\right) + \left(1-G_2\left(\mathcal{T}\right)\right)L\left(Y_{\mathcal{T}}\right)\right\}].
\end{aligned}$$

After rearrangement one obtains

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{\mathcal{T}}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{\mathcal{T}}^{T(K)} \Pi\left(t\right)\left(1-G_2\left(t\right)\right)dt + \int_{\mathcal{T}}^{T(K)} e^{-\rho(t-\mathsf{T})}F\left(Y\left(t\right)\right)dG_2\left(t\right) \\
&+ e^{-\rho(T(K)-\mathsf{T})}\left\{\left(1-G_2^-\left(T\left(K\right)\right)\right)I\left(Y_{T(K)}\right)\right\} \\
&- e^{-\rho(\mathcal{T}-\mathsf{T})}\left\{\left(G_2\left(\mathcal{T}\right)-G_2^-\left(\mathcal{T}\right)\right)I\left(Y_{\mathcal{T}}\right) + \left(1-G_2\left(\mathcal{T}\right)\right)L\left(Y_{\mathcal{T}}\right)\right\}].
\end{aligned}$$

Since the equilibrium strategies are discrete lotteries with positive mass on $Y_{T(0)}, ..., Y_{T(K)}$, we have $dG_2\left(t\right) = 0$, $G_2(\mathcal{T}) - G_2^-(\mathcal{T}) = 0$, and

$$\gamma\left(T\left(K\right)\right) := 1 - G_2(t) = 1 - G_2^-\left(T\left(K\right)\right) = 1 - G_2\left(\mathcal{T}\right) > 0$$

for $t \in [\mathcal{T}, T(K))$. Thus, the difference of payoffs simplifies to

$$V_1\left(Y_{\mathsf{T}}, Y_{T(K)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{\mathcal{T}}, G_2\right) = \gamma\left(T\left(K\right)\right) \times$$
$$\times \mathcal{E}_{\mathsf{T}}\left[\left\{\int_{\mathcal{T}}^{T(K)} \Pi\left(t\right) dt + e^{-\rho(T(K)-\mathsf{T})} I\left(Y_{T(K)}\right) - e^{-\rho(\mathcal{T}-\mathsf{T})} L\left(Y_{\mathcal{T}}\right)\right\}\right]$$
$$= \gamma\left(T\left(K\right)\right) \cdot \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(\mathcal{T}-\mathsf{T})}\left\{S\left(Y_{\mathcal{T}}, Y_{T(K)} = Y_F\right) - L\left(Y_{\mathcal{T}}\right)\right\}\right].$$

By plugging in the formula for S and L one obtains for the conditional expectation

$$\left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{\mathcal{T}}\left(R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right)}{\delta} - \frac{C\left(\overline{q} - \underline{q}\right)}{\rho} + E\right)$$
$$-\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_F\left(R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right)\right)}{\delta} - \frac{C\left(\overline{q} - \underline{q}\right)}{\rho} + E\right).$$

After rearrangement we get

$$\frac{R\left(\overline{q}, \overline{q}\right) - R\left(\underline{q}, \overline{q}\right)}{\delta} \times$$
$$\times\left[\left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{\mathcal{T}} - \frac{1-\beta}{-\beta} Y_M\right) - \left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_F - \frac{1-\beta}{-\beta} Y_M\right)\right].$$

This expression would be zero if $Y_{\mathcal{T}}$ was equal to Y_F. However, $Y_{T(K)} = Y_F < Y_{\mathcal{T}} < Y_M$ and

$$\frac{\partial\left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{\mathcal{T}} - \frac{1-\beta}{-\beta} Y_M\right)}{\partial Y_{\mathcal{T}}} = (1-\beta)\left(\frac{Y_{\mathcal{T}}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(1 - \frac{Y_M}{Y_{\mathcal{T}}}\right) < 0.$$

Consequently, $Y_{\mathcal{T}}$ is a profitable deviation strategy for player 1 revealing that our initial assertion was wrong. Moreover, this proves that there does not exist any symmetric mixed strategy equilibrium with $\text{supp}(G_2 - G_2^-) \subset [Y_F, Y_M]$ and $\text{supp}(G_2 - G_2^-)$ discrete. This completes the proof of part (i).

Proof of part *(ii)*: The proof works as follows. In subpart (a) we show that there does not exist any non-degenerate mixed strategy profile that has at least partial support on $[\mathsf{T}, T_F)$ or $(Y_F, Y_{\mathsf{T}}]$, respectively, and constitutes an equilibrium. Then, in subpart (b), it is proved that there does not exist any non-degenerate mixed strategy profile that has at least partial support on the complementary interval (T_F, ∞) or $(0, Y_F)$, respectively, and that is an equilibrium.

Proof of subpart (a): Immediately follows from the proof of subpart (a) in part (i) after replacing Y_M by Y_F.

Proof of subpart (b): We have shown that any symmetric non-degenerate mixed strategy equilibrium cannot have support on $Y_T > Y_F$. So in the following part of the proof we can concentrate on mixed strategies that are lotteries over the interval $(0, Y_F]$. Again, we start off by assuming that there exists a symmetric mixed strategy profile G with $\text{supp}(G_1 - G_1^-) \subset (0, Y_F]$. In equilibrium

$$V_1\left(Y_{\mathsf{T}}, Y_{T(0)}, G_2\right) = V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \tag{A.12}$$

should be satisfied $\forall Y_{T(k)} \in \{Y_{T(1)}, ..., Y_{T(K)}\} \subset \text{supp}(G_2 - G_2^-)$, $T(0) \geq T_F$. We can write

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(0)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(0)} \Pi(t)\left(1 - G_2(t)\right) dt \\
&+e^{-\rho(T(0)-\mathsf{T})}\left\{G_2(T(0)) F\left(Y_{T(0)}\right) + \left(1 - G_2(T(0))\right) L\left(Y_{T(0)}\right)\right\}] \\
&-\mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T(k)} \Pi(t)\left(1 - G_2(t)\right) dt + \int_{\mathsf{T}}^{T(k)} e^{-\rho(t-\mathsf{T})} F(Y(t))\, dG_2(t) + e^{-\rho(T(k)-\mathsf{T})} \times \\
&\times \left\{\left(G_2(T(k)) - G_2^-(T(k))\right) I\left(Y_{T(k)}\right) + \left(1 - G_2(T(k))\right) L\left(Y_{T(k)}\right)\right\}].
\end{aligned}$$

It is important to note that $F(Y_T) = L(Y_T) = I(Y_T)\ \forall Y_T \leq Y_F$. Thus, one obtains

$$\begin{aligned}
&V_1\left(Y_{\mathsf{T}}, Y_{T(0)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right) \\
&= \mathcal{E}_{\mathsf{T}}[-\int_{T(0)}^{T(k)} \Pi(t)\left(1 - G_2(t)\right) dt - \int_{T(0)}^{T(k)} e^{-\rho(t-\mathsf{T})} I(Y(t))\, dG_2(t) \\
&+e^{-\rho(T(0)-\mathsf{T})} I\left(Y_{T(0)}\right) - e^{-\rho(T(k)-\mathsf{T})}\left(1 - G_2^-(T(k))\right) I\left(Y_{T(k)}\right)]
\end{aligned}$$

after rearrangement. By using the limit representation of the cumulative distribution function G_2, the difference of payoffs can be rewritten as

$$V_1\left(Y_{\mathsf{T}}, Y_{T(0)}, G_2\right) - V_1\left(Y_{\mathsf{T}}, Y_{T(k)}, G_2\right)$$

$$= \mathcal{E}_{\mathsf{T}}[-\int_{T(0)}^{T(k)} \Pi(t)\,dt + \lim_{K\to\infty}\sum_{m=0}^{k-1}\Gamma_2(T(m))\int_{T(m)}^{T(k)}\Pi(t)\,dt$$
$$-\lim_{K\to\infty}\sum_{m=0}^{k-1}\Gamma_2(T(m))\,e^{-\rho(T(m)-\mathsf{T})}I\left(Y_{T(m)}\right)$$
$$+e^{-\rho(T(0)-\mathsf{T})}I\left(Y_{T(0)}\right)-e^{-\rho(T(k)-\mathsf{T})}\left(1-\lim_{K\to\infty}G_2(T(k-1))\right)I\left(Y_{T(k)}\right)],$$

since $G_2(T(-1))\equiv 0$. This identity also implies that

$$1=\sum_{m=0}^{K}\Gamma_2(T(m))=\sum_{m=k}^{K}\Gamma_2(T(m))+\sum_{m=0}^{k-1}\Gamma_2(T(m))$$

and

$$G_2(T(k-1))=\sum_{m=0}^{k-1}\Gamma_2(T(m)).$$

If one applies the former probability-weight transformation to $-\int_{T(0)}^{T(k)}\Pi(t)$ and $\exp(-\rho(T(0)-\mathsf{T}))I(Y_{T(0)})$, and both transformations to $(1-\lim_{K\to\infty}G_2(T(k-1)))\cdot\exp(-\rho(T(k)-\mathsf{T}))\,I(Y_{T(k)})$, it results that

$$V_1\left(Y_{\mathsf{T}},Y_{T(0)},G_2\right)-V_1\left(Y_{\mathsf{T}},Y_{T(k)},G_2\right)$$
$$=\lim_{K\to\infty}\sum_{m=k}^{K}\Gamma_2(T(m))\times$$
$$\times\mathcal{E}_{\mathsf{T}}\left[-\int_{T(0)}^{T(k)}\Pi(t)\,dt+e^{-\rho(T(0)-\mathsf{T})}I\left(Y_{T(0)}\right)-e^{-\rho(T(k)-\mathsf{T})}I\left(Y_{T(k)}\right)\right]$$
$$+\lim_{K\to\infty}\sum_{m=0}^{k-1}\Gamma_2(T(m))\times$$
$$\times\mathcal{E}_{\mathsf{T}}\left[-\int_{T(0)}^{T(m)}\Pi(t)\,dt+e^{-\rho(T(0)-\mathsf{T})}I\left(Y_{T(0)}\right)-e^{-\rho(T(m)-\mathsf{T})}I\left(Y_{T(m)}\right)\right].$$

Evaluating the first conditional expectation in line three gives

$$\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(k)}\left(R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)$$

$$-\left(\frac{Y_{T(0)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{T(0)}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right).$$

After rearrangement one obtains

$$\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\times$$
$$\times\left[\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(k)}-\frac{1-\beta}{-\beta}Y_{S^*}\right)-\left(\frac{Y_{T(0)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(0)}-\frac{1-\beta}{-\beta}Y_{S^*}\right)\right].$$

This expression would be zero if $Y_{T(k)}$ was equal to $Y_{T(0)}$. However, $Y_{T(k)} < Y_{T(0)} \leq Y_F$ and

$$\frac{\partial\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(Y_{T(k)}-\frac{1-\beta}{-\beta}Y_{S^*}\right)}{\partial Y_{T(k)}}=(1-\beta)\left(\frac{Y_{T(k)}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(1-\frac{Y_{S^*}}{Y_{T(k)}}\right)<0.$$

Thus, the expectation in line three is greater than zero. The same outcome results if we replace $Y_{T(k)}$ by $Y_{T(m)}$ with $m \in \{1, ..., k-1\}$. Hence, not only the conditional expectation in line three, but also the one in line five turns out to be strictly positive. Thus, equation (A.12) cannot be satisfied for any $\forall Y_{T(k)} \in \{Y_{T(1)}, ..., Y_{T(K)}\} \subset \text{supp}(G_2 - G_2^-)$, $T(0) \geq T_F$. This completes the proof of subpart (b) and of Proposition 5.

A.9 Proof of Proposition 6

From the proof of Proposition 3 in Appendix A.6 the equilibrium payoff $V^*(Y_{\mathsf{T}}) = V_i^*(Y_{\mathsf{T}}) = V_j^*(Y_{\mathsf{T}})$, $i \neq j$, $i, j \in \{1, 2\}$ is already known. Then $L(Y_{\mathsf{T}}) = V^*(Y_{\mathsf{T}}) = S(Y_{\mathsf{T}}, Y_{S^*})$ for $Y_{\mathsf{T}} \leq Y_F$ is an obvious implication of equations (2.18), (2.28), and Appendix A.6. Next, let us compare L and V^* for $Y_{\mathsf{T}} > Y_F$. We have

$$L\left(Y_{\mathsf{T}}\right)<V^*\left(Y_{\mathsf{T}}\right)$$
$$\Leftrightarrow\quad\frac{Y_{\mathsf{T}}R\left(\underline{q},\overline{q}\right)}{\delta}-\frac{C\underline{q}}{\rho}-E+\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_F\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}$$
$$<\frac{Y_{\mathsf{T}}R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\overline{q}}{\rho}-\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\times$$
$$\times\left(\frac{Y_F\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right).$$

After some rearrangement one gets

$$
\begin{aligned}
&(Y_{\mathsf{T}})^{-\beta}\left(\frac{Y_{\mathsf{T}}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&>(Y_F)^{-\beta}\left(\frac{Y_F\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)
\end{aligned}
$$

or

$$
(Y_{\mathsf{T}})^{-\beta}\left(Y_{\mathsf{T}}-\frac{1-\beta}{-\beta}Y_M\right)>(Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_M\right).
$$

If $Y_{\mathsf{T}}=Y_F$, the left-hand side would equal the right-hand side. However, $Y_{\mathsf{T}}>Y_F$ and

$$
\frac{\partial\left[(Y_{\mathsf{T}})^{-\beta}\left(Y_{\mathsf{T}}-\frac{1-\beta}{-\beta}Y_M\right)\right]}{\partial Y_{\mathsf{T}}}=(1-\beta)\,(Y_{\mathsf{T}})^{-\beta}\left(1-\frac{Y_M}{Y_{\mathsf{T}}}\right)>0,
$$

since $Y_F>Y_M$ due to the assumption of second-mover advantages. Thus, the inequality holds. Finally, we compare V^* and S for $Y_{\mathsf{T}}>Y_F$. Consider the subcase $Y_F<Y_{\mathsf{T}}\leq Y_{S^*}$. Then

$$
\begin{aligned}
&V^*\left(Y_{\mathsf{T}}\right)<S\left(Y_{\mathsf{T}},Y_{S^*}\right)\\
\Leftrightarrow\;&\frac{Y_{\mathsf{T}}R\left(\overline{q},\overline{q}\right)}{\delta}-\frac{C\overline{q}}{\rho}-\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\times\\
&\times\left(\frac{Y_F\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&<\frac{Y_{\mathsf{T}}R\left(\underline{q},\underline{q}\right)}{\delta}-\frac{C\underline{q}}{\rho}-E.
\end{aligned}
$$

Rearranging gives

$$
\begin{aligned}
&V^*\left(Y_{\mathsf{T}}\right)<S\left(Y_{\mathsf{T}},Y_{S^*}\right)\\
\Leftrightarrow\;&(Y_{\mathsf{T}})^{-\beta}\left(\frac{Y_{\mathsf{T}}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&<(Y_F)^{-\beta}\left(\frac{Y_F\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)
\end{aligned}
$$

or

$$
(Y_{\mathsf{T}})^{-\beta}\left(Y_{\mathsf{T}}-\frac{1-\beta}{-\beta}Y_{S^*}\right)<(Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_{S^*}\right).
$$

Again, both sides would be equal if $Y_{\mathsf{T}} = Y_F$. However, $Y_{\mathsf{T}} > Y_F$. It follows that

$$\frac{\partial\left[(Y_{\mathsf{T}})^{-\beta}\left(Y_{\mathsf{T}}-\frac{1-\beta}{-\beta}Y_{S^*}\right)\right]}{\partial Y_{\mathsf{T}}}=(1-\beta)\,(Y_{\mathsf{T}})^{-\beta}\left(1-\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)\leq 0,$$

since $Y_{\mathsf{T}} \leq Y_{S^*}$ by assumption. Moreover, the inequality is strictly satisfied for $Y_{\mathsf{T}} = Y_F$. Thus, $V^*(Y_{\mathsf{T}}) < S(Y_{\mathsf{T}}, Y_{S^*})$. In the remaining subcase, $Y_{\mathsf{T}} > Y_{S^*}$, the comparison of V^* and S yields

$$\begin{aligned}
&V^*(Y_{\mathsf{T}}) < S(Y_{\mathsf{T}}, Y_{S^*})\\
\Leftrightarrow\quad &\frac{Y_{\mathsf{T}}R(\overline{q},\overline{q})}{\delta}-\frac{C\overline{q}}{\rho}-\left(\frac{Y_F}{Y_{\mathsf{T}}}\right)^{-\beta}\times\\
&\times\left(\frac{Y_F\left(R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&<\frac{Y_{\mathsf{T}}R(\overline{q},\overline{q})}{\delta}-\frac{C\overline{q}}{\rho}-\left(\frac{Y_{S^*}}{Y_{\mathsf{T}}}\right)^{-\beta}\times\\
&\times\left(\frac{Y_{S^*}\left(R(\overline{q},\overline{q})-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C\left(\overline{q}-\underline{q}\right)}{\rho}+E\right).
\end{aligned}$$

Summarizing terms gives

$$(Y_{S^*})^{-\beta}\left(Y_{S^*}-\frac{1-\beta}{-\beta}Y_{S^*}\right)<(Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_{S^*}\right).$$

Suppose that $Y_F = Y_{S^*}$. Then both sides would be equal. However, we have

$$\frac{\partial\left[(Y_F)^{-\beta}\left(Y_F-\frac{1-\beta}{-\beta}Y_{S^*}\right)\right]}{\partial Y_F}=(1-\beta)\,(Y_F)^{-\beta}\left(1-\frac{Y_{S^*}}{Y_F}\right)<0,$$

since $Y_F < Y_{S^*}$ by assumption (2.3). Hence the inequality is generally true. This completes the proof of Proposition 6.

A.10 Proof of Lemma 5

Proof of part *(i)*: First, $L_i(Y_{\mathsf{T}}) = I_i(Y_{\mathsf{T}}) = S_i(Y_{\mathsf{T}}, Y_{S_i}) = F_i(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} \leq Y_F^{\min}$ is an obvious implication of equations (2.12), (2.18), (2.26), and (2.28). Second, due to the fact that $I_i(Y_{\mathsf{T}})$, $S_i(Y_{\mathsf{T}}, Y_{S_i})$, and $F_i(Y_{\mathsf{T}})$ are not influenced by the rival's behavior and, especially, do not depend on C_j, $i \neq j$, the results obtained in the symmetric case immediately translates to the general case, i.e.

$$\begin{aligned} I_i(Y_\mathsf{T}) &< S_i(Y_\mathsf{T}, Y_{S_i}) < F_i(Y_\mathsf{T}) \quad \text{for } Y_\mathsf{T} > Y_{S_i}, \\ I_i(Y_\mathsf{T}) &= S_i(Y_\mathsf{T}, Y_{S_i}) < F_i(Y_\mathsf{T}) \quad \text{for } Y_{F_i} < Y_\mathsf{T} \leq Y_{S_i}. \end{aligned}$$

It remains to be shown that

$$\begin{aligned} L_i(Y_\mathsf{T}) &\leq I_i(Y_\mathsf{T}) \quad \text{for } Y_\mathsf{T} > Y_{F,i},\, C_i < C_j, \\ L_i(Y_\mathsf{T}) &< I_i(Y_\mathsf{T}) \quad \text{for } Y_\mathsf{T} > Y_{F_i},\, C_i > C_j. \end{aligned}$$

As we extensively discussed above, the follower's behavior has an impact on the leader's value L_i via Y_{F_j}. And Y_{F_j} itself is determined by C_j. Thus the results from the symmetric case do not apply here. In order to compare L_i to I_i we have to analyze some subcases: *(a)* $Y_\mathsf{T} > Y_F^{\max}$, *(b)* $Y_{F_j} < Y_\mathsf{T} \leq Y_{F_i}$, $Y_{F_i} < Y_{F_j}$, *(c)* $Y_{F_i} < Y_\mathsf{T} \leq Y_{F_j}$, $Y_{F_i} < Y_{F_j}$, and *(d)* $Y_\mathsf{T} \leq Y_F^{\min}$. In case *(a)*

$$\begin{aligned} & L_i(Y_\mathsf{T}) < I_i(Y_\mathsf{T}) \\ \Leftrightarrow\ & \frac{Y_\mathsf{T} R(\underline{q}, \overline{q})}{\delta} - \frac{C_i \underline{q}}{\rho} - E + \left(\frac{Y_{F_j}}{Y_\mathsf{T}}\right)^{-\beta} \frac{Y_{F_j}\left(R(\underline{q}, \underline{q}) - R(\underline{q}, \overline{q})\right)}{\delta} \\ & < \frac{Y_\mathsf{T} R(\underline{q}, \underline{q})}{\delta} - \frac{C_i \underline{q}}{\rho} - E \\ \Leftrightarrow\ & \left(\frac{Y_{F_j}}{Y_\mathsf{T}}\right)^{1-\beta} \frac{R(\underline{q}, \underline{q}) - R(\underline{q}, \overline{q})}{\delta} < \frac{R(\underline{q}, \underline{q}) - R(\underline{q}, \overline{q})}{\delta} \end{aligned}$$

generally holds as $Y_\mathsf{T} > Y_F^{\max} \geq Y_{F_j}$. Case *(b)* gives the same result as case *(a)*, since the follower still refrains from exercising the option. In case *(c)* it is assumed that the follower has already exercised his option. Thus, $L_i(Y_\mathsf{T}) = I_i(Y_\mathsf{T})$. The same argument applies in case *(d)* such that $L_i(Y_\mathsf{T}) = I_i(Y_\mathsf{T})$ also holds if $Y_\mathsf{T} \leq Y_F^{\min}$.

Proof of part *(ii)*: The strict convexity of L_i, F_i, and S_i, immediately follows from the strict convexity of their "homogeneous counterparts". See Appendix A.5. This completes the proof of Lemma 5.

A.11 Proof of Proposition 7

W.l.o.g. let us assume that $C_1 < C_2$ holds henceforth. Before proving the various parts of Proposition 7, we show the existence and uniqueness of the thresholds $\widehat{Y}_1 \in (Y_{F_1}, Y_{L_1})$, $Y_{L_1} = \min\{Y_{S_1}, Y_{F_2}\}$, and $\widehat{Y}_2 \in (Y_{F_1}, Y_{L_2})$, $Y_{L_2} = Y_{M_2}$. First, in Proposition 7, $\widehat{Y}_1$ is implicitly defined by equation (2.34) for $Y_\mathsf{T} \geq Y_{L_1}$. The condition can be rewritten as

$$\begin{aligned}
&\left(\frac{Y_{L_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{Y_{L_1}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C_1\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)\\
&+\left(\frac{\widehat{Y}_1}{Y_\mathsf{T}}\right)^{-\beta}\frac{\widehat{Y}_1\left(R\left(\overline{q},\underline{q}\right)-R\left(\overline{q},\overline{q}\right)\right)}{\delta}\\
&-\left(\frac{Y_{F_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{Y_{F_1}\left(R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C_1\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)=0\,.
\end{aligned}$$

At the least upper bound of $\widehat{Y}_1$, $\widehat{Y}_1 = Y_{L_1}$, the left-hand side of the above equation becomes

$$\begin{aligned}
&\frac{R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\times\\
&\times\left[\left(\frac{Y_{L_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(Y_{L_1}-\frac{1-\beta}{-\beta}Y_{F_1}\right)-\left(\frac{Y_{F_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(Y_{F_1}-\frac{1-\beta}{-\beta}Y_{F_1}\right)\right],
\end{aligned}$$

which can be shown to be strictly positive due to $Y_{L_1} > Y_{F_1}$. At the greatest lower bound of $\widehat{Y}_1$, $\widehat{Y}_1 = Y_{F_1}$, one obtains

$$\begin{aligned}
&\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\times\\
&\times\left[\left(\frac{Y_{L_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(Y_{L_1}-\frac{1-\beta}{-\beta}Y_{S_1}\right)-\left(\frac{Y_{F_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(Y_{F_1}-\frac{1-\beta}{-\beta}Y_{S_1}\right)\right]
\end{aligned}$$

for the left-hand side. However, due to $Y_{L_1} \leq Y_{S_1}$ this expression can be shown to be strictly negative. Since the partial derivative of the condition's left-hand side terms with respect to $\widehat{Y}_1$ equals

$$(1-\beta)\,\frac{R\left(\overline{q},\underline{q}\right)-R\left(\overline{q},\overline{q}\right)}{\delta}\left(\frac{\widehat{Y}_1}{Y_\mathsf{T}}\right)^{-\beta}>0$$

for all $\widehat{Y}_1 > 0$, we have established the existence and uniqueness of $\widehat{Y}_1$. Second, suppose that $Y_{L_2} = Y_{M_2}$. Then $\widehat{Y}_2$ is implicitly defined by equation (2.35) for $Y_\mathsf{T} \geq Y_{L_2}$. We rewrite the condition as

$$\left(\frac{Y_{L_2}}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{Y_{L_2}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C_2\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)$$
$$-\left(\frac{\widehat{Y}_2}{Y_\mathsf{T}}\right)^{-\beta}\left(\frac{\widehat{Y}_2\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C_2\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)$$
$$-\left(\frac{Y_{F_1}}{Y_\mathsf{T}}\right)^{-\beta}\frac{Y_{F_1}\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}=0\,.$$

At the least upper bound of $\widehat{Y}_2$, $\widehat{Y}_2 = Y_{L_2}$, the left-hand side of the above equation becomes

$$\frac{R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)}{\delta}\cdot\left[\left(\frac{Y_{L_2}}{Y_\mathsf{T}}\right)^{-\beta}-\left(\frac{Y_{F_1}}{Y_\mathsf{T}}\right)^{-\beta}\right],$$

which is obviously greater than zero. At the greatest lower bound of $\widehat{Y}_2$, $\widehat{Y}_2 = Y_{F_1}$, one yields

$$\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)}{\delta}\times$$
$$\times\left[\left(\frac{Y_{L_2}}{Y_\mathsf{T}}\right)^{-\beta}\left(Y_{L_2}-\frac{1-\beta}{-\beta}Y_{L_2}\right)-\left(\frac{Y_{F_1}}{Y_\mathsf{T}}\right)^{-\beta}\left(Y_{F_1}-\frac{1-\beta}{-\beta}Y_{L_2}\right)\right]$$

for the left-hand side. However, due to $Y_{F_1} < Y_{L_2} = Y_{M_2}$ this expression can be shown to be strictly smaller than zero. Again, since the partial derivative of the condition's left-hand side with respect to $\widehat{Y}_2$,

$$-\left(1-\beta\right)\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\left(\frac{\widehat{Y}_2}{Y_\mathsf{T}}\right)^{-\beta}\left(1-\frac{Y_{S_2}}{\widehat{Y}_2}\right),$$

turns out to be strictly positive due to $\widehat{Y}_2 < Y_{L_2} < Y_{S_2}$, we have established the existence and uniqueness of $\widehat{Y}_2$ conditionally on $Y_{L_2} = Y_{M_2}$.

Proof of part *(i)*: As in the identical firm case the equilibrium strategies satisfy the intertemporal consistency condition in Definition 3. We have $Y_{L_1} \geq \widehat{Y}_2 \vee Y_{L_2} = Y_{F_1}$ by assumption. Though the high-cost firm (firm 2), is known to follow immediately, it is the low-cost firm (firm 1) that adopts the leader's role at Y_{L_1} in the equilibrium proposed in part *(i)* and, thereby, yields $S_1(Y_\mathsf{T}, Y_{L_1})$. Proposition 1 implies that, if the low-cost firm gets assigned to the leader's role, then its payoff attains a unique global maximum at $\min\{Y_{F_2}, Y_{S_1}\} = Y_{L_1}$ (note that $\mathcal{R}_1 \leq \mathcal{C}|_{C_L=C_1} \Leftrightarrow Y_{F_2} \leq Y_{S_1}$). Thus, firm 1 does not have any incentive to deviate to $Y_\mathcal{T} > Y_{L_1}$ or $Y_\mathcal{T} \in (Y_{F_1}, Y_{L_1})$. Moreover, by deviating to some $Y_\mathcal{T} \leq Y_{F_1}$, firm 1 yields $S_1(Y_\mathsf{T}, \max\{Y_\mathcal{T}, Y_2'\})$ with Y_2' denoting the equilibrium adoption threshold of firm 2, $Y_2' \leq Y_{F_1}$. In section 2.2 it was shown that $S_1(Y_\mathsf{T}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_1}$. Therefore, $\max\{Y_\mathcal{T}, Y_2'\} \leq Y_{F_1} < Y_{L_1}$ implies that $S_1(Y_\mathsf{T}, \max\{Y_\mathcal{T}, Y_2'\}) < S_1(Y_\mathsf{T}, Y_{L_1})$. We conclude that reducing capacity later than Y_{F_1} cannot be a profitable deviation strategy for firm 1.

Now suppose that the high-cost firm (firm 2) deviates to some threshold $Y_\mathcal{T} \in (Y_{F_1}, Y_{L_1}]$. Since firm 2 immediately follows firm 1 at Y_{L_1} anyway, firm 2's payoff is not affected by this modified choice of the adoption date of the leader's role. If firm 2 deviates to even earlier exercise thresholds $Y_\mathcal{T} > Y_{L_1}$, then it becomes the leader at $Y_\mathcal{T}$. Conditionally on the high-cost firm getting assigned to the leader's role, from Proposition 1 its payoff is known to be strictly decreasing in the adoption threshold $Y_\mathcal{T}$ for all $Y_\mathcal{T} > \max\{Y_{M_2}, Y_{F_1}\} = Y_{L_2}$ (note that $\mathcal{C}|_{C_L=C_2} \leq \mathcal{R}_2 \Leftrightarrow Y_{M_2} \leq Y_{F_1}$). Now suppose that $Y_{L_2} = Y_{F_1} \Rightarrow Y_{L_1} > Y_{L_2}$ implying that firm 2's payoff from getting assigned to the leader's role at Y_{L_2},

$$\mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^{T_{L_2}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C_2\overline{q}\right] dt + e^{-\rho\left(T_{L_2}-\mathsf{T}\right)} L_2(Y_{L_2})\right] = S_2\left(Y_\mathsf{T}, Y_{L_2} = Y_{F_1}\right),$$

is strictly greater than the payoff from deviating to $Y_\mathcal{T} > Y_{L_1}$. However, in equilibrium firm 2 gets $S_2(Y_\mathsf{T}, Y_{L_1})$. From the derivations in section 2.2, we known that $S_2(Y_\mathsf{T}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_2}$. Therefore, $Y_{L_2} = Y_{F_1} < Y_{L_1} < Y_{S_2}$ implies that $S_2(Y_\mathsf{T}, Y_{L_2}) < S_2(Y_\mathsf{T}, Y_{L_1})$ in this case. Thus, early reduction at $Y_\mathcal{T} > Y_{L_1}$ is not a profitable deviation strategy for firm 2 if $Y_{L_2} = Y_{F_1}$. Next suppose that $Y_{L_1} \geq \widehat{Y}_2$ and note that the existence of $\widehat{Y}_2$ requires $Y_{L_2} = Y_{M_2}$. Further, note that equation (2.35) that implicitly defines $\widehat{Y}_2$ can be rewritten as

$$\mathcal{E}_\mathsf{T}\left[\int_\mathsf{T}^{T_{L_2}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C_2\overline{q}\right] dt + e^{-\rho\left(T_{L_2}-\mathsf{T}\right)} L_2(Y_{L_2})\right] = S_2\left(Y_\mathsf{T}, \widehat{Y}_2\right).$$

The left-hand side denotes firm 2's maximum payoff from getting assigned to the leader's role. Again, since $S_2(Y_{\mathsf{T}}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_2}$, the ordering $\widehat{Y}_2 < Y_{L_1} < Y_{S_2}$ implies that $S_2(Y_{\mathsf{T}}, \widehat{Y}_2) < S_2(Y_{\mathsf{T}}, Y_{L_1})$. We conclude that adopting the leader's role at $Y_{\mathcal{T}} > Y_{L_1}$ is not profitable for firm 2. Subgame perfection is not shown explicitly but can easily be proved by following a similar line of arguments as in the identical-firm case. This completes the proof of part *(i)*.

Proof of part *(ii)*: Again, note that the intertemporal consistency condition in Definition 3 holds with respect to the equilibrium strategies. We have $Y_{L_2} = Y_{M_2} \geq \widehat{Y}_1$. Firm 2 adopts the leader's role at $Y_{L_2} = Y_{M_2}$ in the equilibrium proposed in part *(ii)* and, thereby, yields $S_2(Y_{\mathsf{T}}, \widehat{Y}_2)$ according to the derivations in the proof of part *(i)*. Proposition 1 implies that, if the high-cost firm gets assigned to the leader's role and if the difference in variable cost is sufficiently large (i.e. $\mathcal{C}|_{C_L = C_2} > \mathcal{R}_2 \Leftrightarrow Y_{L_2} = Y_{M_2} > Y_{F_1}$), then its payoff attains a unique global maximum at $Y_{L_2} = Y_{M_2}$. Thus, firm 2 does not have any incentive to deviate to $Y_{\mathcal{T}} > Y_{L_2}$ or $Y_{\mathcal{T}} \in (Y_{F_1}, Y_{L_2})$. Moreover, by deviating to some $Y_{\mathcal{T}} \leq Y_{F_1}$, firm 2 yields $S_2(Y_{\mathsf{T}}, \max\{Y_{\mathcal{T}}, Y_1'\})$ with Y_1' denoting the equilibrium adoption threshold of firm 1, $Y_1' \leq Y_{F_1}$. Since $S_2(Y_{\mathsf{T}}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_2}$, the ordering $\max\{Y_{\mathcal{T}}, Y_1'\} \leq Y_{F_1} < \widehat{Y}_2 < Y_{L_2} = Y_{M_2} < Y_{S_2}$ implies that $S_2(Y_{\mathsf{T}}, \max\{Y_{\mathcal{T}}, Y_1'\}) < S_2(Y_{\mathsf{T}}, \widehat{Y}_2)$. We conclude that reducing capacity later than Y_{F_1} cannot be a profitable deviation strategy for firm 2.

Now suppose that firm 1 deviates to some threshold $Y_{\mathcal{T}} \in (Y_{F_1}, Y_{L_2})$. Since firm 1 conditionally on being assigned to the follower's role does not reduce capacity before Y_{F_1} anyway, firm 1's payoff is not affected by this modified choice of the adoption date of the leader's role. If firm 1 deviates to even earlier exercise thresholds $Y_{\mathcal{T}} \geq Y_{L_2}$, then it either becomes the leader at $Y_{\mathcal{T}} > Y_{L_2}$ or firms move simultaneously at $Y_{\mathcal{T}} = Y_{L_2}$. Conditionally on getting assigned to the leader's role or moving jointly at $Y_{\mathcal{T}}$, from the above derivations, firm 1's payoff is known not to be greater than $S_1(Y_{\mathsf{T}}, L_1)$. Further, note that equation (2.34) that implicitly defines $\widehat{Y}_1$ can be rewritten as

$$S_1(Y_{\mathsf{T}}, L_1) = \mathcal{E}_{\mathsf{T}} \left[\int_{\mathsf{T}}^{\widehat{T}_1} e^{-\rho(t-\mathsf{T})} \left[Y(t) \, R(\overline{q}, \overline{q}) - C_1 \overline{q} \right] dt + e^{-\rho\left(\widehat{T}_1 - \mathsf{T}\right)} F_1 \left(\widehat{Y}_1 \right) \right].$$

In equilibrium firm 1 gets

$$\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_{L_2}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q})-C_1\overline{q}\right]dt+e^{-\rho\left(T_{L_2}-\mathsf{T}\right)}F_1(Y_{L_2})\right].$$

By substracting the former expression from the equilibrium payoff, we obtain

$$\begin{aligned}
&\mathcal{E}_{\mathsf{T}}[e^{-\rho\left(T_{L_2}-\mathsf{T}\right)}F_1(Y_{L_2})-e^{-\rho\left(\widehat{T}_1-\mathsf{T}\right)}F_1\left(\widehat{Y}_1\right)\\
&-\int_{T_{L_2}}^{\widehat{T}_1} e^{-\rho(t-\mathsf{T})}\left[Y(t)\,R(\overline{q},\overline{q})-C_1\overline{q}\right]dt]\\
&=\frac{R\left(\overline{q},\underline{q}\right)-R(\overline{q},\overline{q})}{\delta}\left[\left(\frac{Y_{L_2}}{Y_{\mathsf{T}}}\right)^{-\beta}Y_{L_2}-\left(\frac{\widehat{Y}_1}{Y_{\mathsf{T}}}\right)^{-\beta}\widehat{Y}_1\right]\geq 0
\end{aligned}$$

due to $Y_{L_2} \geq \widehat{Y}_1$. Thus, firm 1's payoff from deviating to an earlier exercise thresholds $Y_{\mathcal{T}} > Y_{L_2}$ is strictly smaller than its equilibrium payoff. Again, we refrain from showing subgame perfection explicitly but refer the reader to the identical-firm case. This completes the proof of part *(ii)*.

Proof of part *(iii)*: To prove that the strategy profiles suggested in part *(i)* of the proposition are the only subgame perfect equilibria of the general timing game if $Y_{L_2} < \widehat{Y}_1$, we can resort to the above findings. In the proof of part *(i)* it was shown that any deviation from the equilibrium exercise threshold Y_{L_1} leaves firm 1 with a strictly smaller payoff than playing its equilibrium strategy. This implies that firm 1 always has an incentive to deviate to Y_{L_1} from any equilibrium candidate where firm 1 adopts the leader's role at $Y_{\mathcal{T}} \neq Y_{L_1}$, while firm 2 sticks to the equilibrium strategy proposed in part *(i)*. So none of these candidates is a Nash equilibrium.

Now suppose that firm 2 does not stick to its equilibrium exercise rule but starts adopting the leader's role with probability one when $Y(t)$ first hits $Y_{\mathcal{T}} \in (Y_{F_1}, \widehat{Y}_1]$. Note that firm 1 is indifferent between adopting the leader's role at Y_{L_1} or getting assigned to the follower's role at $\widehat{Y}_1$. Hence, firm 1 cannot do better than moving first at Y_{L_1} in all subgames that start at $Y_{\mathsf{T}} \geq Y_{L_1}$. However, in any (off-equilibrium-path) subgame that starts at $Y_{\mathsf{T}} < Y_{\mathcal{T}}$, conditionally on no player having moved before Y_{T}, firm 1 has an incentive to refrain from adopting the leader's role and to become the follower at Y_{T}, since

$F_1(Y_\mathsf{T}) > L_1(Y_\mathsf{T})$ for $Y_\mathsf{T} > Y_{F_1}$ according to Lemma 5. So all strategy profiles such that firm 1 reduces capacity at Y_{L_1} and firm 2 plays $Y_\mathcal{T} \in (Y_{F_1}, \widehat{Y}_1]$ are Nash equilibria of the supergame that starts at $Y_\mathsf{T} \geq Y_{L_1}$, but they do not satisfy subgame perfection. Finally, suppose that there exists an equilibrium such that firm 2 plays $Y_\mathcal{T} > \widehat{Y}_1$. Following our derivations in the proof of part *(ii)*, it can be shown that firm 1 prefers to become the follower at $Y_\mathcal{T} \geq \widehat{Y}_1$ rather than trying to exercise first at Y_{L_1}. Since roles are assumed to behave optimally, firm 1 reduces capacity at Y_{F_1} in this case. However, the proposed strategy profile cannot be a Nash equilibrium: Firm 2 conditionally on getting assigned to the leader's role strictly maximizes its payoff by exercising its option at Y_{L_2}. Since $Y_{L_2} < \widehat{Y}_1$ by assumption, firm 2 has an incentive to deviate from any equilibrium candidate involving $Y_\mathcal{T} > \widehat{Y}_1$.

Next we prove that the strategy profiles suggested in part *(ii)* of the proposition are the only subgame perfect equilibria of the general timing game if $Y_{L_1} = Y_{S_1} < \widehat{Y}_2$ (recall that the existence of $\widehat{Y}_2$ requires $Y_{L_2} = Y_{M_2}$). In the proof of part *(ii)* it was shown that any deviation from the equilibrium exercise threshold $Y_{L_2} = Y_{M_2}$ leaves firm 2 with a strictly smaller payoff than playing its equilibrium strategy. This implies that firm 2 always has an incentive to deviate to Y_{L_2} from any equilibrium candidate where firm 2 adopts the leader's role at $Y_\mathcal{T} \neq Y_{L_2}$, while firm 1 sticks to the equilibrium strategy proposed in part *(ii)*. Hence, none of these candidates is a Nash equilibrium.

Now suppose that firm 1 does not stick to its equilibrium exercise rule but starts adopting the leader's role with probability one at $Y_\mathcal{T} \in (Y_{F_1}, \widehat{Y}_2]$. Note that firm 2 is indifferent between adopting the leader's role at Y_{L_2} or getting assigned to the follower's role at $\widehat{Y}_2$. Hence, firm 2 cannot do better than moving first at Y_{L_2} in all subgames that start at $Y_\mathsf{T} \geq Y_{L_2}$. However, in any (off-equilibrium-path) subgame that starts at $Y_\mathsf{T} < Y_\mathcal{T}$, conditionally on no player having moved before Y_T, firm 2 has an incentive to refrain from adopting the leader's role and to become the follower at Y_T, since $F_2(Y_\mathsf{T}) > L_2(Y_\mathsf{T})$ for $Y_\mathsf{T} > Y_{F_1}$ according to Lemma 5. So all strategy profiles such that firm 2 reduces capacity at Y_{L_2} and firm 1 plays $Y_\mathcal{T} \in (Y_{F_1}, \widehat{Y}_2]$ are Nash equilibria of the supergame that starts at $Y_\mathsf{T} \geq Y_{L_2}$, but they do not satisfy subgame perfection. Finally, suppose that there exists an equilibrium such that firm 1 plays $Y_\mathcal{T} > \widehat{Y}_2$ (implying that firm 2 follows at $\min\{Y_{F_2}, Y_\mathcal{T}\}$. However, our derivations in the proof of part (i) implies that, conditionally on getting assigned to the leader's role, firm 1's payoff attains a unique global maximum at Y_{L_1}. Since $Y_{L_1} = Y_{S_1} < \widehat{Y}_2$ by assumption, firm 1 has an incentive to deviate from any equilibrium candidate involving $Y_{\mathcal{T}} > \widehat{Y}_2$.

A.12 Proof of Proposition 8

W.l.o.g. let us assume that $C_1 < C_2$ holds henceforth. Before proving Proposition 8, we show the existence and uniqueness of the threshold $\widetilde{Y}_2$. In Proposition 8, $\widetilde{Y}_2$ has implicitly been defined by equations (2.36) and (2.37) for $Y_{\mathsf{T}} \geq Y_{L_2}$. The first condition can be rewritten as

$$\left(\frac{Y_{L_2}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{L_2}\left(R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}-\frac{C_2\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)$$
$$-\left(\frac{Y_{F_1}}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{Y_{F_1}\left(R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)\right)}{\delta}$$
$$+\left(\frac{\widetilde{Y}_2}{Y_{\mathsf{T}}}\right)^{-\beta}\frac{\widetilde{Y}_2\left(R\left(\overline{q},\underline{q}\right)-R\left(\overline{q},\overline{q}\right)\right)}{\delta}$$
$$-\left(\frac{Y_{F_2}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\frac{Y_{F_2}\left(R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)\right)}{\delta}-\frac{C_2\left(\overline{q}-\underline{q}\right)}{\rho}+E\right)=0\,.$$

At the least upper bound of $\widetilde{Y}_2$, $\widetilde{Y}_2 = Y_{L_2}$, the left-hand side of the above equation becomes

$$\frac{R\left(\overline{q},\underline{q}\right)-R\left(\underline{q},\underline{q}\right)}{\delta}\times$$
$$\times\left[\left(\frac{Y_{L_2}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\Gamma Y_{L_2}-\frac{1-\beta}{-\beta}Y_{F_2}\right)-\left(\frac{Y_{F_2}}{Y_{\mathsf{T}}}\right)^{-\beta}\left(\Gamma Y_{F_2}-\frac{1-\beta}{-\beta}Y_{F_2}\right)\right]$$
$$+\frac{R\left(\underline{q},\underline{q}\right)-R\left(\underline{q},\overline{q}\right)}{\delta}\cdot\left[\left(\frac{Y_{F_2}}{Y_{\mathsf{T}}}\right)^{-\beta}Y_{F_2}-\left(\frac{Y_{F_1}}{Y_{\mathsf{T}}}\right)^{-\beta}Y_{F_1}\right],$$

where $\Gamma = (R(\overline{q},\underline{q}) - R(\underline{q},\overline{q}))/(R(\overline{q},\underline{q}) - R(\underline{q},\underline{q})) > 1$. The resulting expression can be shown to be strictly positive due to $Y_{L_2} > Y_{F_2} > Y_{F_1}$. At the greatest lower bound of $\widetilde{Y}_2$, $\widetilde{Y}_2 = Y_{F_2}$, one obtains

$$\frac{R\left(\overline{q},\overline{q}\right)-R\left(\underline{q},\overline{q}\right)}{\delta}\times$$

$$\times \left[\left(\frac{Y_{L_2}}{Y_\mathsf{T}} \right)^{-\beta} \left(Y_{L_2} - \frac{1-\beta}{-\beta} Y_{L_2} \right) - \left(\frac{Y_{F_2}}{Y_\mathsf{T}} \right)^{-\beta} \left(Y_{F_2} - \frac{1-\beta}{-\beta} Y_{L_2} \right) \right]$$
$$+ \frac{R\left(\underline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta} \cdot \left[\left(\frac{Y_{F_2}}{Y_\mathsf{T}} \right)^{-\beta} Y_{F_2} - \left(\frac{Y_{F_1}}{Y_\mathsf{T}} \right)^{-\beta} Y_{F_1} \right]$$

for the left-hand side. Due to $Y_{F_2} < Y_{L_2} = Y_{M_2}$ the former expression in square brackets turns out to be strictly negative. However, the latter expression in square brackets is obviously strictly positive. Whether the positive or the negative terms dominate cannot be determined generally. It depends on the specific parameterization of the model. If the positive terms dominate, then equation (2.36) cannot be satisfied for any $\widetilde{Y}_2 \in (Y_{F_2}, Y_{L_2})$, since the partial derivative of the condition's left-hand side terms with respect to $\widetilde{Y}_2$ is strictly greater than zero for all $\widetilde{Y}_2 > 0$. However, in this case the complementary equation (2.37) must hold for some $\widetilde{Y}_2 \in (Y_{F_1}, Y_{F_2}]$ as we will show immediately. Equation (2.37) can be written as

$$\left(\frac{Y_{L_2}}{Y_\mathsf{T}} \right)^{-\beta} \left(\frac{Y_{L_2}\left(R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta} - \frac{C_2\left(\overline{q}-\underline{q}\right)}{\rho} + E \right)$$
$$- \left(\frac{\widetilde{Y}_2}{Y_\mathsf{T}} \right)^{-\beta} \left(\frac{\widehat{Y}_2\left(R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\underline{q}\right)\right)}{\delta} - \frac{C_2\left(\overline{q}-\underline{q}\right)}{\rho} + E \right)$$
$$- \left(\frac{Y_{F_1}}{Y_\mathsf{T}} \right)^{-\beta} \frac{Y_{F_1}\left(R\left(\underline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta} = 0 \,.$$

At the greatest lower bound of $\widetilde{Y}_2$, $\widetilde{Y}_2 = Y_{F_1}$, one yields

$$\frac{R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta} \times$$
$$\times \left[\left(\frac{Y_{L_2}}{Y_\mathsf{T}} \right)^{-\beta} \left(Y_{L_2} - \frac{1-\beta}{-\beta} Y_{L_2} \right) - \left(\frac{Y_{F_1}}{Y_\mathsf{T}} \right)^{-\beta} \left(Y_{F_1} - \frac{1-\beta}{-\beta} Y_{L_2} \right) \right]$$

for the left-hand side. However, due to $Y_{F_1} < Y_{L_2} = Y_{M_2}$ this expression can be shown to be strictly smaller than zero. At the least upper bound of $\widetilde{Y}_2$, $\widetilde{Y}_2 = Y_{F_2}$, the left-hand side of the above equation is identical to equation (2.36) evaluated at its greatest lower bound, $\widetilde{Y}_2 = Y_{F_2}$. Since the partial derivative of the left-hand side of equation (2.37) with respect to $\widetilde{Y}_2$,

$$-\left(1-\beta\right) \frac{R\left(\overline{q},\overline{q}\right) - R\left(\underline{q},\underline{q}\right)}{\delta} \left(\frac{\widetilde{Y}_2}{Y_\mathsf{T}} \right)^{-\beta} \left(1 - \frac{Y_{S_2}}{\widetilde{Y}_2} \right),$$

turns out to be strictly positive due to $\widetilde{Y}_2 < Y_{L_2} < Y_{S_2}$, there must exist a unique $\widetilde{Y}_2$ that satisfies either equation (2.36) or equation (2.37).

Proof of part *(i)*: Note that all equilibrium strategies satisfy the intertemporal consistency condition in Definition 3. We have $Y_{L_1} \geq \widetilde{Y}_2$ by assumption. Let us begin to analyze the equilibrium strategy profile where the low-cost-firm (firm 1) adopts the leader's role at Y_{L_1} and firm 2's probability of adopting the leader's role jumps to one at or after Y_{F_1}.

First, suppose that $\mathcal{C}|_{C_L=C_1} \leq \mathcal{R}_2 < 1 \Leftrightarrow \max\{Y_{M_1}, Y_{F_2}\} = Y_{F_2}$ and, therefore, $Y_{L_1} = \min\{Y_{F_2}, Y_{S_1}\}$. Due to $Y_{L_1} \leq Y_{F_2}$ firm 2 is known to exercise its closure option immediately after firm 1 has done so. Thus, firm 1 yields $S_1(Y_\mathsf{T}, Y_{L_1})$. Proposition 1 implies that, if the low-cost firm gets assigned to the leader's role, then its payoff attains a unique global maximum at $\min\{Y_{F_2}, Y_{S_1}\} = Y_{L_1}$ (note that $\mathcal{R}_1 \leq \mathcal{C}|_{C_L=C_1} \Leftrightarrow Y_{F_2} \leq Y_{S_1}$). Thus, firm 1 does not have any incentive to deviate to $Y_\mathcal{T} > Y_{L_1}$ or $Y_\mathcal{T} \in (Y_{F_1}, Y_{L_1})$. Moreover, by deviating to some $Y_\mathcal{T} \leq Y_{F_1}$, firm 1 yields $S_1(Y_\mathsf{T}, \max\{Y_\mathcal{T}, Y_2'\})$, $Y_2' \leq Y_{F_1}$. As $S_1(Y_\mathsf{T}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_1}$, the ordering $\max\{Y_\mathcal{T}, Y_2'\} \leq Y_{F_1} < Y_{L_1} \leq Y_{S_1}$ implies that $S_1(Y_\mathsf{T}, \max\{Y_\mathcal{T}, Y_2'\}) < S_1(Y_\mathsf{T}, Y_{L_1})$. We conclude that reducing capacity later than Y_{F_1} cannot be a profitable deviation strategy for firm 1.

Now suppose that firm 2 deviates to some threshold $Y_\mathcal{T} \in (Y_{F_1}, Y_{L_1}]$. Since firm 2 immediately follows firm 1 at Y_{L_1} anyway, firm 2's payoff is not affected by this modified choice of the adoption date of the leader's role. If firm 2 deviates to even earlier exercise thresholds $Y_\mathcal{T} > Y_{L_1}$, then it becomes the leader at $Y_\mathcal{T}$. Conditionally on the high-cost firm getting assigned to the leader's role, we known from Proposition 1 that firm 2 maximizes its deviation payoff by adopting the leader's role at $Y_{L_2} = Y_{M_2}$ (note that $\mathcal{C}|_{C_L=C_2} > 1 > \mathcal{R}_2 \Leftrightarrow Y_{M_2} > Y_{F_1}$). From the assumption $Y_{L_1} = \min\{Y_{F_2}, Y_{S_1}\} \geq \widetilde{Y}_2$ it follows that $\widetilde{Y}_2 \in (Y_{F_1}, Y_{F_2}]$. Then equation (2.37) says that firm 2 is indifferent between deviating to $Y_{L_2} = Y_{M_2}$ and moving simultaneously at $\widetilde{Y}_2$. Both alternatives generate a payoff that equals $S_2(Y_\mathsf{T}, \widetilde{Y}_2)$. Since $S_2(Y_\mathsf{T}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_2}$, $\widetilde{Y}_2 \leq Y_{L_1} = \min\{Y_{F_2}, Y_{S_1}\} < Y_{S_2}$ implies that the payoff from deviation is not greater than firm 2's equilibrium payoff $S_2(Y_\mathsf{T}, Y_{L_1})$. Thus, neither firm 1 nor firm 2 has an incentive to deviate from the proposed equilibrium. The reader should notice that $Y_{L_1} \geq \widetilde{Y}_2$ is a critical assumption. As we stressed above, one may find a parameterization of the model such that $\widetilde{Y}_2 \in (Y_{F_2}, Y_{L_2})$ for all $\mathcal{C}|_{C_L=C_1}$, $\mathcal{R}_1$ and $\mathcal{R}_2$ with $\mathcal{C}|_{C_L=C_1} \leq \mathcal{R}_2$. In this case

$Y_{L_1} = \min\{Y_{F_2}, Y_{S_1}\} \geq \widetilde{Y}_2$ can never occur and firm 2 has always an incentive to deviate from the suggested joint-reduction strategy profile.

Second, suppose that $\mathcal{R}_2 < \mathcal{C}|_{C_L=C_1} < 1 \Leftrightarrow \max\{Y_{M_1}, Y_{F_2}\} = Y_{M_1}$ and, therefore, $Y_{L_1} = Y_{M_1}$. Following an analogous line of arguments as in the previous case reveals that firm 1 neither prefers to deviate to earlier nor to later exercise thresholds. Similar, firm 2 is indifferent between trying to adopt the leader's role at some $Y_{\mathcal{T}} \leq (Y_{F_1}, Y_{L_1})$ or sticking to one of the equilibrium strategies, since its payoff does not change. Playing $Y_{\mathcal{T}} = Y_{L_1}$ as a deviation strategy leaves firm 2 with a payoff of $S_2(Y_{\mathsf{T}}, Y_{L_1})$, while its equilibrium payoff amounts to

$$\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_{L_1}} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C_2\overline{q}\right] dt + e^{-\rho\left(T_{L_1}-\mathsf{T}\right)} F_2(Y_{L_1})\right].$$

Since $I_2(Y_T) < F_2(Y_T)$ for all $Y_T \geq Y_{F_2}$ according to Lemma 5, $Y_{L_1} > Y_{F_2}$ implies that firm 2's equilibrium payoff exceeds $S_2(Y_{\mathsf{T}}, Y_{L_1})$. Finally, if firm 2 deviates to $Y_{\mathcal{T}} > Y_{L_1}$, it becomes the leader. As we pointed out above, firm 2 maximizes its payoff from deviation by adopting the leader's at $Y_{L_2} = Y_{M_2}$. According to equations (2.36) and (2.37), this deviation payoff either equals $S_2(Y_{\mathsf{T}}, \widetilde{Y}_2)$ for $\widetilde{Y}_2 \in (Y_{F_1}, Y_{F_2}]$ or

$$\mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{\widetilde{T}_2} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C_2\overline{q}\right] dt + e^{-\rho\left(\widetilde{T}_2-\mathsf{T}\right)} F_2\left(\widetilde{Y}_2\right)\right]$$

for $\widetilde{Y}_2 \in (Y_{F_2}, Y_{L_2})$. Because $I_2(Y_T) < F_2(Y_T)$ for all $Y_T \geq Y_{F_2}$, $S_2(Y_{\mathsf{T}}, Y_S)$ increases for all $Y_S < Y_{S_2}$ and $Y_{L_2} < Y_{S_2}$, the latter payoff exceeds the former payoff, $S_2(Y_{\mathsf{T}}, \widetilde{Y}_2)$. Let us compare this maximum payoff from deviation with the equilibrium payoff. If the strategy profile proposed above is indeed an equilibrium, we should have

$$\mathcal{E}_{\mathsf{T}}\left[-\int_{T_{L_1}}^{\widetilde{T}_2} e^{-\rho(t-\mathsf{T})}\left[Y(t)\, R(\overline{q},\overline{q}) - C_2\overline{q}\right] dt \right.$$
$$\left. + e^{-\rho\left(T_{L_1}-\mathsf{T}\right)} F_2(Y_{L_1}) - e^{-\rho\left(\widetilde{T}_2-\mathsf{T}\right)} F_2\left(\widetilde{Y}_2\right)\right] \geq 0\,.$$

In the proof of part (*ii*) of Proposition 7 an almost identical expression was already shown to be strictly positive. We conclude that the proposed strategy profile that involves sequential disinvestment with firm 1 being the leader is indeed an equilibrium if $Y_{L_1} = Y_{M_1} \geq \widetilde{Y}_2$. Again, we stress that $Y_{L_1} \geq \widetilde{Y}_2$ might be a critical assumption.

Let us investigate the equilibrium strategy profile where firm 2 adopts the leader's role at $Y_{L_2} = Y_{M_2}$ and firm 1 follows and reduces capacity at Y_{F_1}. A similar line of arguments as we applied above suggests that firm 2 cannot do better than reducing capacity at Y_{L_2} conditionally on being assigned to the leader's role. Thus, firm 2 does not have any incentive to deviate to some $Y_{\mathcal{T}} \in (Y_{F_1}, Y_{L_2})$ or $Y_{\mathcal{T}} > Y_{L_2}$. If firm 2 deviates to even later thresholds, $Y_{\mathcal{T}} \leq Y_{F_1}$, it yields $S_2(Y_{\mathsf{T}}, \max\{Y_{\mathcal{T}}, Y_1'\})$, $Y_1' \leq Y_{F_1}$. As $S_2(Y_{\mathsf{T}}, Y_S)$ is strictly increasing in Y_S for all $Y_S < Y_{S_2}$, the ordering $\max\{Y_{\mathcal{T}}, Y_1'\} \leq Y_{F_1} < \widetilde{Y}_2 < Y_{L_2} < Y_{S_2}$ implies that the deviation payoff, $S_2(Y_{\mathsf{T}}, \max\{Y_{\mathcal{T}}, Y_1'\})$, is strictly smaller than the lower bound to firm 2's equilibrium payoff, $S_2(Y_{\mathsf{T}}, \widetilde{Y}_2)$. Firm 1's payoff does not vary if it deviates to an earlier threshold $Y_{\mathcal{T}} \in (Y_{F_1}, Y_{L_2})$. By playing $Y_{\mathcal{T}} = Y_{L_2}$, firm 1 obtains $S_1(Y_{\mathsf{T}}, Y_{L_2})$ which is known to be strictly smaller than its equilibrium payoff

$$\mathcal{E}_{\mathsf{T}} \left[\int_{\mathsf{T}}^{T_{L_2}} e^{-\rho(t-\mathsf{T})} \left[Y(t) R(\overline{q}, \overline{q}) - C_2 \overline{q} \right] dt + e^{-\rho\left(T_{L_2} - \mathsf{T}\right)} F_1(Y_{L_2}) \right]$$

(see previous derivations of almost identical expressions). If firm 1 brings forward its adoption date to some $Y_{\mathcal{T}} > Y_{L_2}$, then it becomes the leader at $Y_{\mathcal{T}}$. From Proposition 1 we know that the payoff of the leader who exhibits variable cost of C_1 is decreasing in $Y_T > Y_{L_1} = \min\{\max\{Y_{M_1}, Y_{F_2}\}, Y_{S_1}\}$. Since $Y_{L_2} > Y_{L_1}$,

$$\mathcal{E}_{\mathsf{T}} \left[\int_{\mathsf{T}}^{T_{L_2}} e^{-\rho(t-\mathsf{T})} \left[Y(t) R(\overline{q}, \overline{q}) - C_2 \overline{q} \right] dt + e^{-\rho\left(T_{L_2} - \mathsf{T}\right)} L_1(Y_{L_2}) \right]$$

represents an upper bound to the payoff from deviating to $Y_{\mathcal{T}} > Y_{L_2}$. Because $L_1(Y_T) < F_1(Y_T)$ for all $Y_T > Y_{F_1}$ according to Lemma 5, we can conclude that the maximum payoff from deviation is strictly smaller than the equilibrium payoff. Thus, the equilibrium strategy profile generating a sequential disinvestment pattern with the high-cost firm as the leader is always an equilibrium irrespective of the concrete parameter constellation. We refrain from proving subgame perfection explicitly. However, this can easily be accomplished by following a similar line of arguments as in the identical-firm case. This completes the proof of part (i).

Proof of part (ii): Our considerations in part (i) immediately imply that firm 2 always has an incentive to take over the leader's role if $Y_{L_1} < \widetilde{Y}_2$. Thus, the joint-reduction equilibrium outcome and the sequential disinvestment scenario with firm 1 becoming the leader vanishes in this case. It remains to be shown that there does not exist any other subgame perfect equilibrium of the general closure timing game. Again, we can resort to earlier findings. First, consider those equilibria where firm 1 adopts the leader's role at Y_{L_1} and firm 2's probability of adoption does not raise before Y_{F_1}. It was shown that any deviation from the equilibrium exercise threshold Y_{L_1} leaves firm 1 with a strictly smaller payoff than playing its equilibrium strategy. This implies that firm 1 always has an incentive to deviate to Y_{L_1} from any equilibrium candidate where firm 1 adopts the leader's role at $Y_{\mathcal{T}} \neq Y_{L_1}$, while firm 2 sticks to its equilibrium strategy proposed in part *(i)*. So none of these candidates is a Nash equilibrium.

Now suppose that firm 2 does not stick to its equilibrium exercise rule but starts adopting the leader's role with probability one at $Y_{\mathcal{T}} \in (Y_{F_1}, \widetilde{Y}_1]$. Let $\widetilde{Y}_1$ be defined such that firm 1 is indifferent between getting assigned to the follower's role at $\widetilde{Y}_1$ and adopting the leader's role at Y_{L_1}. Existence and uniqueness can be shown equivalently to the considerations on $\widehat{Y}_1$ in the proof of Proposition 7. Hence, firm 1 cannot do better than moving first at Y_{L_1} in all subgames that start at $Y_{\mathsf{T}} \geq Y_{L_1}$. However, in any (off-equilibrium-path) subgame that starts at $Y_{\mathsf{T}} < Y_{\mathcal{T}}$, conditionally on no player having moved before Y_{T}, firm 1 has an incentive to refrain from adopting the leader's role and to become the follower at Y_{T}, since $F_1(Y_{\mathsf{T}}) > L_1(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} > Y_{F_1}$ according to Lemma 5. So all strategy profiles such that firm 1 reduces capacity at Y_{L_1} and firm 2 plays $Y_{\mathcal{T}} \in (Y_{F_1}, \widetilde{Y}_1]$ are Nash equilibria of the supergame that starts at $Y_{\mathsf{T}} \geq Y_{L_1}$, but they do not satisfy subgame perfection. Finally, suppose that there exists an equilibrium such that firm 2 plays $Y_{\mathcal{T}} > \widetilde{Y}_1$. Following our derivations in the proof of part *(i)*, it can be shown that firm 1 prefers to become the follower at $Y_{\mathcal{T}} > \widetilde{Y}_1$ rather than trying to exercise first at Y_{L_1}. Since roles are assumed to behave optimally, firm 1 reduces capacity at Y_{F_1} in this case. However, the proposed strategy profile cannot be a Nash equilibrium: Firm 2 conditionally on getting assigned to the leader's role strictly maximizes its payoff by exercising its option at Y_{L_2}. Thus, firm 2 has an incentive to deviate from any equilibrium candidate that involves $Y_{\mathcal{T}} \neq Y_{L_2}$.

Next we analyze the equilibrium strategy profiles where firm 2 adopts the leader's role at Y_{L_2} and firm 1 does not raise its adoption probability before Y_{F_1}. It was shown that any deviation from the equilibrium exercise threshold $Y_{L_2} = Y_{M_2}$ leaves firm 2 with a strictly smaller payoff than playing its equilibrium strategy. This implies that firm 2 always has an incentive to deviate to Y_{L_2} from any equilibrium candidate where it adopts the leader's role at $Y_{\mathcal{T}} \neq Y_{L_2}$, while firm 1 sticks to the equilibrium strategy proposed in part *(i)*. Hence, none of these candidates is a Nash equilibrium.

Now suppose that firm 1 does not stick to its equilibrium exercise rule but starts adopting the leader's role with probability one at $Y_{\mathcal{T}} \in (Y_{F_1}, \widetilde{Y}_2]$. Recall that firm 2 is indifferent between adopting the leader's role at Y_{L_2} or getting assigned to the follower's role at $\widetilde{Y}_2$. Hence, firm 2 cannot do better than moving first at Y_{L_2} in all subgames that start at $Y_{\mathsf{T}} \geq Y_{L_2}$. However, in any (off-equilibrium-path) subgame that starts at $Y_{\mathsf{T}} < Y_{\mathcal{T}}$, conditionally on no player having moved before Y_{T}, firm 2 has an incentive to refrain from adopting the leader's role and to become the follower at Y_{T}, since $F_2(Y_{\mathsf{T}}) > L_2(Y_{\mathsf{T}})$ for $Y_{\mathsf{T}} > Y_{F_1}$. So all strategy profiles such that firm 2 reduces capacity at Y_{L_2} and firm 1 plays $Y_{\mathcal{T}} \in (Y_{F_1}, \widetilde{Y}_2]$ are Nash equilibria of the supergame that starts at $Y_{\mathsf{T}} \geq Y_{L_2}$, but they do not satisfy subgame perfection. Finally, suppose that there exists an equilibrium such that firm 1 plays $Y_{\mathcal{T}} > \widetilde{Y}_2$ (implying that firm 2 reduces capacity at $\min\{Y_{F_2}, Y_{\mathcal{T}}\}$). Our derivations in the proof of part (i) implies that, conditionally on getting assigned to the leader's role, firm 1's payoff attains a unique global maximum at Y_{L_1}. Thus, firm 1 has an incentive to deviate from any equilibrium candidate involving $Y_{\mathcal{T}} \neq Y_{L_1}$. This completes the proof of Proposition 7.

Appendix B

Proofs of Chapter 3

B.1 Proof of Proposition 9

Proof of part *(i)*: Imagine a price-taking firm that is equipped with the option to extend capacity from $\underline{q}$ to $\overline{q}$. If Y_{T} is smaller than its optimal closure threshold Y_P, the price taker has the following value,

$$\begin{aligned}
\Upsilon\left(Y_{\mathsf{T}}\right) &= \mathcal{E}_{\mathsf{T}}\left[\int_{\mathsf{T}}^{T_P} e^{-\rho(t-\mathsf{T})} Y\left(t\right) D\left(2\underline{q}\right) \underline{q} dt\right] \\
&\quad + \int_{T_P}^{\infty} e^{-\rho(t-\mathsf{T})} Y\left(t\right) D\left(2\underline{q}\right) \overline{q} dt - \mathcal{E}_{\mathsf{T}}\left[e^{-\rho(T_P-\mathsf{T})} E\right] \\
&= \frac{Y_{\mathsf{T}} D\left(2\underline{q}\right) \underline{q}}{\delta} + \left(\frac{Y_{\mathsf{T}}}{Y_P}\right)^{\beta} \left(\frac{Y_P D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)}{\delta} - E\right),
\end{aligned}$$

where $T_P = \inf(t > \mathsf{T}\,|Y\left(t\right) \leq Y_P)$. The partial derivative of $\Upsilon\left(Y_{\mathsf{T}}\right)$ with respect to Y_P,

$$\frac{\partial \Upsilon}{\partial Y_P} = \left(\beta - 1\right)\left(\frac{Y_{\mathsf{T}}}{Y_P}\right)^{\beta} \frac{D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)}{\delta}\left(\frac{\beta}{\beta-1}\frac{\delta E}{Y_P D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)} - 1\right),$$

has its unique root at

$$Y_P = \frac{\beta}{\beta-1} \cdot \frac{\delta E}{D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)}.$$

$>$From the second-order condition, $Y_P > 0$ is known to be the unique global maximizer.

Proof of part *(ii)*: Principally, the monopolist may decide to invest in additional capacity sequentially or to extend capacity in a single step from $2\underline{q}$ to $2\overline{q}$. That is, either he chooses two distinct investment thresholds Y_O' and Y_O'', $Y_O' < Y_O''$, or he exercises both options simultaneously at Y_O. Suppose that a sequential disinvestment pattern is optimal. Then Y_O' and Y_O'', $Y_O' < Y_O''$, are the solutions to the maximization problem $\max_{Y_O', Y_O''} O^{seq}(Y_{\mathsf{T}})$ with

$$\begin{aligned} O^{seq}\left(Y_{\mathsf{T}}\right) &= \mathcal{E}_{\mathsf{T}}[\int_{\mathsf{T}}^{T_O'} e^{-\rho(t-\mathsf{T})} Y\left(t\right) D\left(2\underline{q}\right) 2\underline{q} dt \\ &+ \int_{T_O'}^{T_O''} e^{-\rho(t-\mathsf{T})} Y\left(t\right) D\left(\overline{q}+\underline{q}\right)\left(\overline{q}+\underline{q}\right) dt \\ &+ \int_{T_O''}^{\infty} e^{-\rho(t-\mathsf{T})} Y\left(t\right) D\left(2\overline{q}\right) 2\overline{q} dt \\ &- \left(e^{-\rho\left(T_O'-\mathsf{T}\right)} + e^{-\rho\left(T_O''-\mathsf{T}\right)}\right) E], \end{aligned}$$

$T_O' = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_O')$ and $T_O'' = \inf(t \geq \mathsf{T}\,|Y(t) \leq Y_O'')$. The monopolist's value function can be expressed in terms of the closure thresholds,

$$\begin{aligned} &O^{seq}\left(Y_{\mathsf{T}}\right) = \frac{2Y_{\mathsf{T}} R\left(\underline{q},\underline{q}\right)}{\delta} \\ &+ \left(\frac{Y_{\mathsf{T}}}{Y_O'}\right)^{\beta} \left(\frac{Y_O'\left(R\left(\overline{q},\underline{q}\right) + R\left(\underline{q},\overline{q}\right) - 2R\left(\underline{q},\underline{q}\right)\right)}{\delta} - E\right) \\ &+ \left(\frac{Y_{\mathsf{T}}}{Y_O''}\right)^{\beta} \left(\frac{Y_O''\left(2R\left(\overline{q},\overline{q}\right) - R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta} - E\right). \end{aligned}$$

The first-order condition with respect to Y_O',

$$\frac{1-\beta}{\delta}\left(\frac{Y_{\mathsf{T}}}{Y_O'}\right)^{\beta}\left(R\left(\overline{q},\underline{q}\right) + R\left(\underline{q},\overline{q}\right) - 2R\left(\underline{q},\underline{q}\right) - \frac{\beta}{\beta-1}\frac{\delta E}{Y_O'}\right) = 0,$$

is satisfied for

$$Y_O' = \frac{\beta}{\beta-1}\frac{\delta E}{R\left(\overline{q},\underline{q}\right) + R\left(\underline{q},\overline{q}\right) - 2R\left(\underline{q},\underline{q}\right)}.$$

The second-order condition reveals that Y_O' is a global maximizer of O^{seq}. With respect to Y_O'' a similar result is obtained. The maximizing argument turns out to be

$$Y_O'' = \frac{\beta}{\beta-1}\frac{\delta E}{2R\left(\overline{q},\overline{q}\right) - R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)}.$$

Substracting the denominator in the expression for Y_O'' from the denominator of Y_O' yields

$$2\left[R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\underline{q}\right) - R\left(\overline{q},\overline{q}\right) + R\left(\underline{q},\overline{q}\right)\right]$$

which is greater than zero due to the assumption of first-mover advantages. Thus, we have $Y_O' < Y_O''$ verifying our initial assertion.

Proof of part (*iii*): Recall condition (2.3) in section 2.1. Then $Y_M > Y_P$ immediately follows from $D(2\underline{q})(\overline{q}-\underline{q}) > R(\overline{q},\underline{q}) - R(\underline{q},\underline{q}) \Leftrightarrow D(2\underline{q}) > D(\overline{q}+\underline{q})$. $Y_M < Y_O'$ is an implication of the fact that $R(\overline{q},\underline{q}) + R(\underline{q},\overline{q}) - 2R(\underline{q},\underline{q}) < R(\overline{q},\underline{q}) - R(\underline{q},\underline{q}) \Leftrightarrow R(\underline{q},\overline{q}) < R(\underline{q},\underline{q})$. Finally, $Y_O' < Y_{S^*}$ and $Y_{S^*} < Y_O''$ is satisfied due to $R(\overline{q},\overline{q}) - R(\underline{q},\underline{q}) < R(\overline{q},\underline{q}) + R(\underline{q},\overline{q}) - 2R(\underline{q},\underline{q}) \Leftrightarrow R(\overline{q},\overline{q}) - R(\underline{q},\overline{q}) < R(\overline{q},\underline{q}) - R(\underline{q},\underline{q})$ and $2R(\overline{q},\overline{q}) - R(\overline{q},\underline{q}) - R(\underline{q},\overline{q}) < R(\overline{q},\overline{q}) - R(\underline{q},\underline{q}) \Leftrightarrow R(\overline{q},\overline{q}) - R(\underline{q},\overline{q}) < R(\overline{q},\underline{q}) - R(\underline{q},\underline{q})$, respectively. This completes the proof of Proposition 10.

B.2 Proof of Proposition 10

The ordering $Y_{RE} < Y_P$ immediately follows from the definition of Y_{RE} and from $L(Y_P) > F(Y_P)$. The validity of the latter inequality can be verified by calculating the difference $L(Y_P) - F(Y_P)$. According to equations (3.1) and (3.4) one obtains

$$L\left(Y_P\right) - F\left(Y_P\right) = \frac{Y_P\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta} - E$$
$$-\left(\frac{Y_P}{Y_F}\right)^{\beta}\left(\frac{Y_F\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta} - E\right).$$

According to equation (3.9) E can be replaced by

$$\frac{\beta-1}{\beta}\cdot\frac{Y_P D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)}{\delta}.$$

Due to $R(\overline{q},\underline{q}) - R(\underline{q},\overline{q}) = D(\overline{q}+\underline{q})(\overline{q}-\underline{q})$ we can write

$$L\left(Y_P\right) - F\left(Y_P\right) = \frac{Y_P\left(R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)\right)}{\delta}\times$$
$$\times\left[1 - \frac{\beta-1}{\beta}\frac{D\left(2\underline{q}\right)}{D\left(\overline{q}+\underline{q}\right)} - \left(\frac{Y_P}{Y_F}\right)^{\beta}\left(\frac{Y_F}{Y_P} - \frac{\beta-1}{\beta}\frac{D\left(2\underline{q}\right)}{D\left(\overline{q}+\underline{q}\right)}\right)\right].$$

If $Y_F = Y_P$, then $L\left(Y_P\right) - F\left(Y_P\right)$ would equal zero. However, $Y_F > Y_P$ and

$$\frac{\partial\left[L\left(Y_P\right) - F\left(Y_P\right)\right]}{\partial Y_F} = \frac{R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta}\times$$
$$\times\left(\beta-1\right)\left(\frac{Y_P}{Y_F}\right)^{\beta+1}\left(\frac{Y_F}{Y_P} - \frac{D\left(2\underline{q}\right)}{D\left(\overline{q}+\underline{q}\right)}\right)$$
$$= \frac{R\left(\overline{q},\underline{q}\right) - R\left(\underline{q},\overline{q}\right)}{\delta}\times$$
$$\times\left(\beta-1\right)\left(\frac{Y_P}{Y_F}\right)^{\beta+1}\left(\frac{D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)}{D\left(2\overline{q}\right)\overline{q} - D\left(\overline{q}+\underline{q}\right)\underline{q}} - \frac{D\left(2\underline{q}\right)}{D\left(\overline{q}+\underline{q}\right)}\right),$$

by using the definitions of Y_F annd Y_P in equations (3.2) and (3.9), respectively. Since

$$\frac{D\left(2\underline{q}\right)\left(\overline{q}-\underline{q}\right)}{D\left(2\overline{q}\right)\overline{q} - D\left(\overline{q}+\underline{q}\right)\underline{q}} > \frac{D\left(2\underline{q}\right)}{D\left(\overline{q}+\underline{q}\right)} \Leftrightarrow D\left(\overline{q}+\underline{q}\right) > D\left(2\overline{q}\right),$$

one yields $\partial\left[L\left(Y_P\right) - F\left(Y_P\right)\right]/\partial Y_F > 0$ and, therefore, $L\left(Y_P\right) > F\left(Y_P\right)$. This completes the proof of Proposition 11.

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