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Manuel Frondel, Colin Vance

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Discussion Paper

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Manuel Frondel, Rheinisch-Westfälisches Institut für Wirtschaftsforschung and Ruhr University Bochum

Colin Vance, Rheinisch-Westfälisches Institut für Wirtschaftsforschung and Jacobs University Bremen

Abstract. This note resurrects a largely forgotten technique advanced by TWEETEN and QUANCE (1969) for capturing price asymmetries. Their study employs dummy variables that split up the price variable into two complementing explanatory terms capturing either increasing or decreasing input prices. This approach is criticized by WOLFFRAM (1971) for being mathematically incorrect. He proposes an alternative technique based on price differences, which has since established itself as the primary method for dealing with price asymmetry. We challenge WOLFFRAM's critique and show that the TWEETEN and QUANCE approach provides for a consistent estimation method that is more readily interpretable than WOLFFRAM's alternative.

JEL classification: D13, Q41.

Key words: Irreversibility, decomposition approaches .

Correspondence: Prof. Dr. Manuel Frondel, Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI), Hohenzollernstr. 1-3, D-45128 Essen. frondel@rwi-essen.de.

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1 Introduction

The estimation of irreversible supply and demand functions that allow for asymmetric price responses has been a subject of ongoing research across a range of fields in economics, including agriculture (TRAILL, COLMAN, YOUNG, 1978:528), labor (CAMPBELL, FISHER, 2000) and energy (GRIFFIN, SCHULMAN, 2005). While theoretical arguments in favor of asymmetric responses to rising or falling agricultural input prices were advanced by JOHNSON (1958), empirical work on the topic did not begin until a decade later with an analysis by TWEETEN and QUANCE (1969a, b) of aggregate farm output. Their study, which employs dummy variables that split up the price variable into two complementing explanatory terms capturing either increasing or decreasing input prices, is criticized by WOLFFRAM (1971:356), who claims that the method employed by TWEETEN and QUANCE is "mathematically incorrect [for the] quantification of irreversible supply reactions to increasing and decreasing prices".

WOLFFRAM (1971) proposes an alternative technique based on cumulated price differences that, in their reply to his criticism, TWEETEN and QUANCE (1971:359) concede is superior to their approach, even though the application of the technique to their own data suggests otherwise (TWEETEN, QUANCE, 1971:360). In the aftermath of this exchange, WOLFFRAM's technique, henceforth called the W technique, became the most popular method of partitioning an explanatory variable to allow for the estimation of a non-reversible function (TRAILL, COLMAN, and YOUNG, 1978:528), and has since been expanded upon using more sophisticated approaches, such as error-correction models (MEYER, VON CRAMON-TAUBADEL, 2004). By contrast, the TWEETEN and QUANCE – henceforth T-Q – approach has been largely forgotten and, in the rare instances when cited, is often referenced as an inferior method that WOLFFRAM subsequently improved upon.

Using a slight modification of WOLFFRAM's (1971) misleading example originally conceived to demonstrate the superiority of his method over the T-Q approach, this note challenges WOLFFRAM's (1971) critique. In the subsequent section, we empirically show that the T-Q approach provides for a consistent estimation method if the underlying asymmetric relationship is specified correctly. Beyond this, we argue that because the explanatory variables are measured in levels, rather than in differences, the T-Q approach lends itself to a more intuitive interpretation of the coefficient estimates based on price elasticities. We therefore conclude that the T-Q approach should be considered a viable alternative to WOLFFRAM's (1971) and other methods that today build the fundamental basis of more sophisticated empirical specifications.

2 A Reassessment of WOLFFRAM's Empirical Example

WOLFFRAM (1971:357) criticizes that any irreversible relationship $y = f(x)$ between a dependent variable y and an explanatory variable x cannot be determined exactly with the T-Q approach, which splits x into two complementary variables, x^+ and x^-. Variable x^+ is defined as $x_1^+ = x_1$,

$$x_i^+ := x_i, \quad \text{if} \quad x_i > x_{i-1}, \tag{1}$$

and $x_i^+ := 0$ otherwise, where subscript i is used to denote the observation, while x^- is defined in a similar way: $x_1^- := 0$,

$$x_i^- := x_i, \quad \text{if} \quad x_i \leq x_{i-1}, \tag{2}$$

and $x_i^- := 0$ otherwise. By definition, $x_i^+ + x_i^- = x_i$ for all i.

As an alternative to the T-Q decomposition of x, WOLFFRAM (1971) suggests taking cumulated increases and decreases of the explanatory variable x, denoted here by w_i^+ and w_i^-, respectively. In detail, WOLFFRAM (2000:351-352) defines his approach by $w_1^+ = w_1^- := x_1$ and

$$w_i^+ := w_{i-1}^+ + D_i^+(x_i - x_{i-1}), \tag{3}$$

$$w_i^- := w_{i-1}^- + D_i^-(x_i - x_{i-1}) \tag{4}$$

for $i > 1$, where $D_i^+ = 1$ for $x_i > x_{i-1}$ and 0 otherwise, while $D_i^- = 1 - D_i^+$.[1] An

[1] Using the dummy variables D_i^+ and D_i^-, the T-Q decomposition can be concisely described by $x_i^+ = D_i^+ x_i$ and $x_i^- = D_i^- x_i$ for $i > 1$ (MEYER, VON CRAMON-TAUBADEL, 2004:594) and $x_1^+ = x_1, x_i^- := 0$.

alternative definition that is equivalent to (3) is given by $w_i^+ = \sum_{s=2}^{i} (w_s - w_{s-1})$ for $w_s > w_{s-1}$, whereas $w_i^- = \sum_{s=2}^{i} (w_{s-1} - w_s)$ for $w_s \leq w_{s-1}$ (DARGAY, 1992:167). From these definitions, it becomes obvious that w^+ and w^- represent cumulated differences of increasing and decreasing prices, respectively.

To demonstrate the superiority of his approach over the T-Q decomposition, WOLFFRAM (1971) conceives a simple example presented in Table 1. For this purpose, WOLFFRAM (1971:358) assumes the following exact relationship between the predefined values of dependent variable y and those of the explanatory variable x, which is split up into x^+ and x^- according to the T-Q decomposition:

$$y_i = a_i + 5x_i^+ + 3x_i^-. \tag{5}$$

In this equation, potential residual terms u_i are set to zero: $u_i = 0$, thereby attributing the varying differences between the predefined values y_i and the predicted values $\hat{y}_i := 5x_i^+ + 3x_i^-$ to variable a, whose components are shown in Table 1.

This contrasts with the classical Ordinary Least Squares (OLS) framework, in which variable a would adopt the role of a constant: $a = a_0$. It is not surprising, therefore, that when applying OLS methods, WOLFFRAM obtains the following estimation equation for which both coefficient estimates, 6.25 and 6.99, differ greatly from the predefined coefficients in equation (5):[2]

$$y_i = -40.23\ (11.03)\ + 6.25\ (0.74)\ x_i^+ + 6.99\ (0.88)\ x_i^- + \hat{u}_i, \tag{6}$$

with $R^2 = 0.912$, $\hat{u}_i \neq 0$ for all i, and standard errors reported in parentheses. In contrast, WOLFFRAM shows that the correct coefficients 5 and 3 are reproduced – apart from the sign of coefficient 3 – by using the proposed W technique and regressing y on w^+ and w^-:

$$y_i = 0 + 5w_i^+ - 3w_i^-, \tag{7}$$

where $\hat{u}_i = 0$ for all i and, hence, $R^2 = 1$.

[2] WOLFFRAM (1971:358) reported an estimate of -43.16 for the constant, which appears to be wrong.

We now demonstrate that it is inappropriate to blame the T-Q decomposition for a poor performance when using OLS. First, the differences between the coefficient estimates reported in equation (6) and the true coefficients of 5 and 3 is merely the result of the fact that the varying values a_i are approximated by a constant when equation (5) is estimated by OLS. If one estimates equation (5) by employing variable a as an additional regressor, thereby avoiding any omitted-variable bias, one can exactly reproduce the coefficients given in equation (5).

Second, although it appears to be somewhat misleading, one point that immediately emerges from WOLFFRAM's example is that in case of irreversibility, one may expect distinct intercepts a^+ and a^-, $a^+ \neq a^-$, as is shown in the following modification of WOLFFRAM's example:

$$\tilde{y}_i = -30D_i^+ - 20D_i^- + 5x_i^+ + 3x_i^-, \tag{8}$$

with $a^+ = -30D_i^+$ and $a^- = -20D_i^-$ and the modified values $\tilde{y}_i$ for the dependent variable being shown in Table 1. Equation (8) reflects the fact that in case of asymmetry, one would certainly expect two entirely distinct supply functions for the two different regimes of either increasing or decreasing prices.

Table 1: WOLFFRAM's Original Example and its Modification.

	Original Variables		T-Q technique			W technique		Modified y
	y	x	x^+	x^-	a	w^+	w^-	$\tilde{y}$
1	20	10	10	0	-30	10	10	20
2	35	13	13	0	-30	13	10	35
3	29	11	0	11	-4	13	12	13
4	44	14	14	0	-26	16	12	40
5	59	17	17	0	-26	19	12	55
6	44	12	0	12	8	19	17	16
7	35	9	0	9	8	19	20	7
8	70	16	16	0	-10	26	20	50
9	90	20	20	0	-10	30	20	70
10	84	18	0	18	30	30	22	34

If one falsely estimates the modified example (8) by using a common intercept, the following OLS results are obtained:

$$\tilde{y}_i = -24.87\ (1.89) + 4.67\ (0.13)\ x_i^+ + 3.36\ (0.15)\ x_i^-. \tag{9}$$

In statistical terms, the coefficient estimates of x_i^+ and x_i^- are significantly different from the true vales 5 and 3, respectively. Clearly, these estimation results, which seem to support WOLFFRAM' criticism with respect to the T-Q decomposition, are due to omitted-variable bias. This bias could be readily avoided by including two dummy variables that capture the different intercepts, rather than employing a common constant, thereby perfectly reproducing the modified example (8).

In short, if an empirical model correctly specifies the underlying asymmetric relationship, nothing should go wrong when estimating it using the T-Q decomposition. Indeed, modeling price asymmetry by using two complementary price variables x^+ and x^- appears to be a highly natural approach that can be flexibly adapted in other contexts. In a study of car use, for example, ROUWENDAL (1996) splits the fuel price variable x in two complementary price variables x^d and x^p to distinguish between diesel and petrol fuel types. In contrast, splitting up the price variable x a la WOLFFRAM (1971) into two complementary variables w^+ and w^-, which reflect either cumulated price increases or decreases, respectively, is far from being intuitive, most notably, because the W technique implies that the *level* of dependent variable y is then supposed to be explained by cumulated *changes* of an explanatory variable x. The classical estimation of demand or supply functions, however, relates the *levels* of supply or demand to the *levels* of prices.

Despite WOLFFRAM's (1971) and TWEETEN and QUANCE's (1971) common belief of the superiority of the W technique, a number of articles have pointed to several weaknesses in its application, including the high dependence on the starting point of the data (GRIFFIN, SCHULMAN, 2005:7). [3] Moreover, we argue that for estimating demand functions, e. g. to gauge the responsiveness of motorists to fuel price changes

[3] TRAILL, COLMAN, and YOUNG (1978:528), for example, criticize the W technique for its implication that for given starting and finishing prices, the greater the price changes in the intermediate period, the larger is the output at the end of the period. Yet, most economists would probably argue that highly variable prices would lead to a reduction in output due to risk considerations. According to these authors,

(see e. g. FRONDEL, PETERS, and VANCE, 2008; FRONDEL, RITTER, and VANCE, 2012), the T-Q decomposition is more appropriate than the W technique, because it yields estimates in terms of price elasticities (see FRONDEL, VANCE, 2012).

3 Conclusion

This paper has demonstrated that WOLFRAM's (1971) critique of the technique advanced by TWEETEN and QUANCE (1969) for capturing price asymmetries is misplaced, being based on a contrived example that omits a key explanatory variable. Nevertheless, the critique has had a major impact on the subsequent trajectory of the literature, as evidenced by the frequency with which WOLFRAM's paper is cited and the corresponding neglect of the T-Q approach. To wit, based on an extensive review of articles on the topic, including 40 articles covered in a survey paper by MEYER and VON CRAMON-TAUBADEL (2004), we were hard pressed to find a single case in which the T-Q approach was employed to empirically test for asymmetry. We regard this neglect as a missed opportunity for three reasons.

First, as we have demonstrated above, the T-Q approach is both a highly flexible and natural method to estimate asymmetric supply and demand functions. Second, it is not subject to various shortcomings of the W approach, such as the dependence of the start date. Lastly, the measurement of the explanatory variable in levels, rather than in differences, affords a more intuitive interpretation of the coefficient estimates in terms of elasticites. These features lend themselves to further development of the T-Q approach using more sophisticated estimation techniques, including error correction models.

the generally poor empirical results obtained when using the W technique may thus be attributed to the implied unrealistic pattern of supply responses, rather than to multi-collinearity between the prices series, as is suggested by HOUK (1977) and SAYLOR (1974).

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