

Comparison of Dynamic Hardening Equations for Metallic Materials with three types of Crystalline Structures

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Outlines

- Introduction
- Review of Dynamic Hardening Equations
- Experiments
- Model Construction
- Conclusions

Introduction

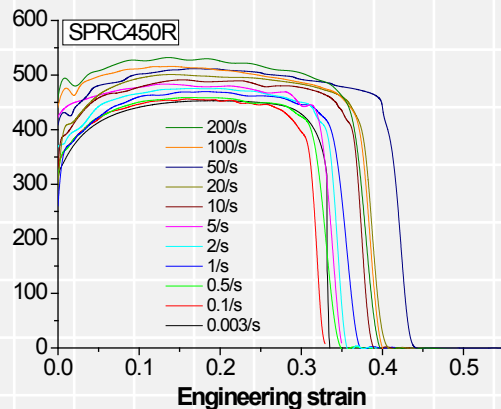
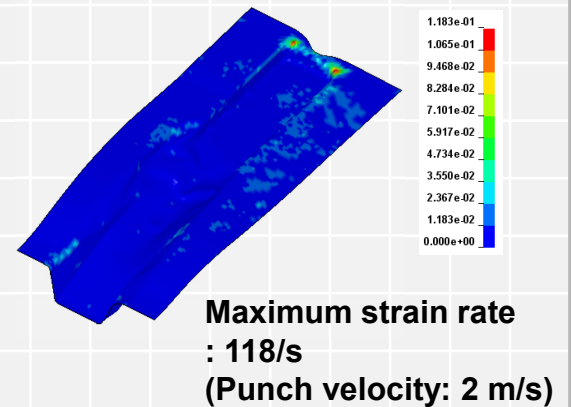
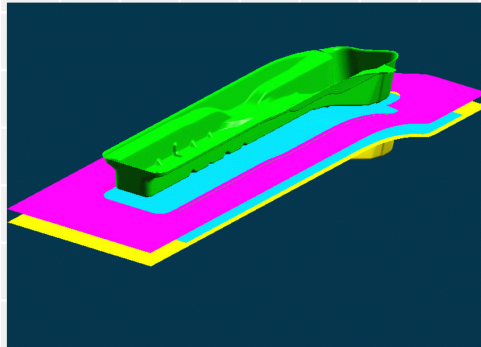


Motivation

- **Most simulations of Electromagnetic Forming / High Speed Forming are carried out prior to a real forming process.**
- **The simulations have to be accurate for the real forming process.**
- **The simulations need an accurate model to predict deformation of a process material.**
- **Difficulties in selecting an appropriate model... Too many models.**

Strain rate effect in high speed forming

- Strain rate effect in high speed forming process is significant because the dynamic response of metals differs considerably from static response

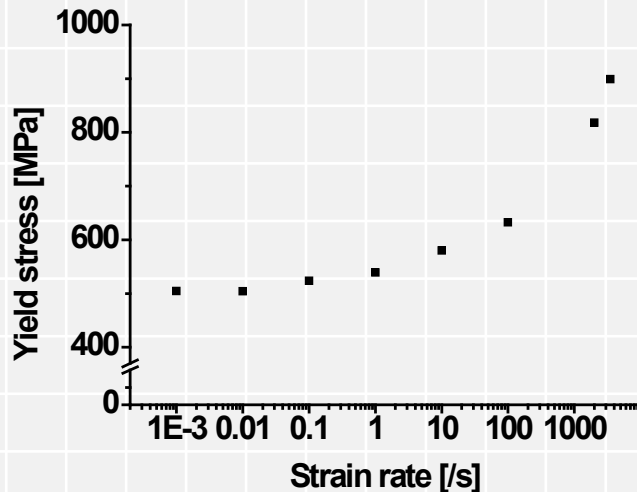


- ✓ Hardening properties of metallic materials are remarkably changed by the level of the strain rate
- ✓ Accurate understanding of material properties at various strain rates is necessary to guarantee the reliability of the high speed forming analysis

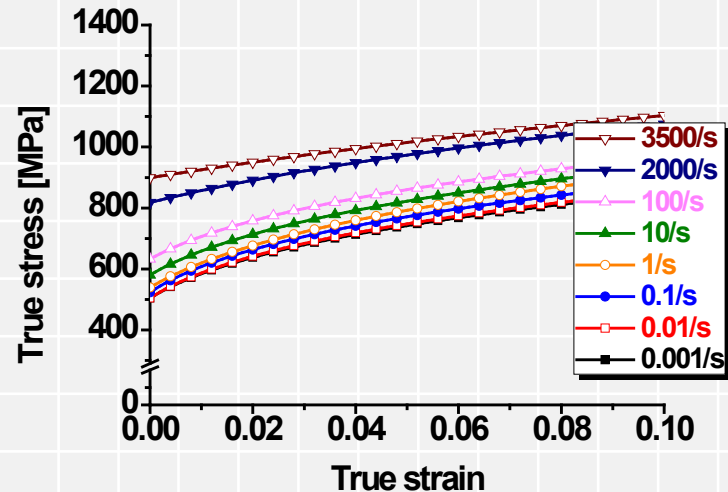
Material properties of metallic materials with the variation of strain rate

- Initial yield stress and strain hardening are changed by the level of strain rate

Change of initial yield stress (4340Steel)



Change of strain hardening (4340Steel)

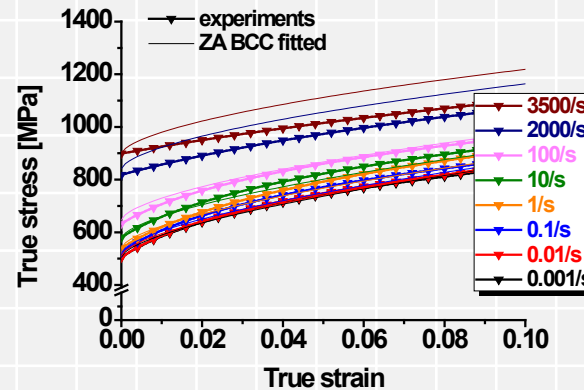
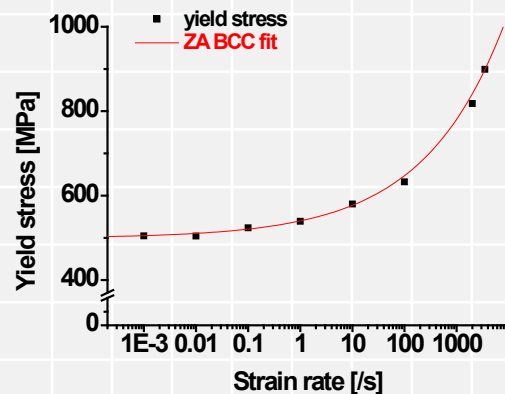


- Initial yield stress and strain hardening with respect to the strain rate are regarded as the inherent characteristics of the material

Representation of dynamic hardening properties using hardening equations

- Dynamic hardening properties of materials can be represented as the one simple equation by using the dynamic hardening equation

Example of Zerilli-Armstrong model for 4340Steel



$$\sigma = 499 + 1279 \times \exp[-0.011T + 9.32 \times 10^{-3} T \ln \dot{\epsilon}] + 1515 \times \epsilon^{0.63}$$

- ✓ There is no unique equation which can represent the dynamic hardening characteristics of all kinds of materials
- ✓ It is important to select and use the most applicable equation which can represent the dynamic hardening characteristics of the material

Research scope

- Understand the characteristics of some of the famous dynamic hardening equations by reviewing of those
- Suggestion of new dynamic hardening equations for more accurate representation of dynamic behavior of materials
- Uniaxial tensile and SHPB tests of three kinds of materials to obtain stress–strain data at various strain rate conditions
 - 4340Steel (BCC)
 - OFHC (FCC)
 - Ti6Al4V (HCP)
- Quantification of test results using six kinds of the dynamic hardening equations
 - Suggestion of the most applicable model for each material

Review of Dynamic Hardening Equations

- Johnson–Cook model
- Zerilli–Armstrong model
- Preston–Tonks–Wallace model
- Modified Johnson–Cook model
- Modified Khan–Huang model

Dynamic hardening equations

- **Representative well-known dynamic hardening equations**
 - Johnson–Cook model (1983)
 - Zerilli–Armstrong model (1987)
 - ✓ for BCC
 - ✓ for FCC
 - Preston–Tonks–Wallace model (2003)

- **Dynamic hardening equations suggested**
 - Modified Johnson–Cook model (1999)
 - Modified Khan–Huang model (2006)

Johnson–Cook model

$$\sigma = \left[A + B\varepsilon^n \right] \left[1 + C \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right] \left[1 - \left(\frac{T - T_r}{T_m - T_r} \right)^m \right]$$

- **The most widely used rate and temperature dependent model due to its simplicity**
- **Purely empirical model**
 - **Model is developed by expressing experimental tendency**
 - **Coefficients can be determined by fitting the experimental results**
- **Cannot represent the strain hardening change as strain rate changes**
- **Strain rate hardening is expressed as a linear function of the logarithm of strain rate**

*G.R. Johnson *et al.*, “A Constitutive Model and Data for Metals Subjected to Large Strains, High Strain Rates and High Temperatures”, in Proceedings of the Seventh International Symposium on Ballistics, 1983.

Zerilli–Armstrong model

$$\sigma = C_0 + [C_1 + C_2\sqrt{\varepsilon}] \exp[-C_3T + C_4T \ln \dot{\varepsilon}] + C_5\varepsilon^n$$

- **The most widely used physically based model**
 - **Based on simplified dislocation dynamics**
- **Different type can be used for BCC and FCC materials**
 - **$C_2=0$ for BCC**

$$\sigma = C_0 + C_1 \exp[-C_3T + C_4T \ln \dot{\varepsilon}] + C_5\varepsilon^n$$

- ✓ **Constant strain hardening as strain rate changes**
- ✓ **Cannot represent hardening change as strain rate changes**
- **$C_1=C_5=0$ for FCC**

$$\sigma = C_0 + C_2\sqrt{\varepsilon} \exp[-C_3T + C_4T \ln \dot{\varepsilon}]$$

- ✓ **Strain hardening increases as strain rate increases**
- ✓ **Constant initial yield stress as strain rate changes**

*F.J. Zerilli *et al.*, “Dislocation-Mechanics-Based Constitutive Relations for Material Dynamics Calculations”, J. Appl. Phys., 1987.

Preston–Tonks–Wallace model

$$\left[\begin{array}{l} \hat{\tau} = \hat{\tau}_s + \frac{1}{p}(s_0 - \hat{\tau}_y) \ln \left[1 - \left[1 - \exp \left(-p \frac{\hat{\tau}_s - \hat{\tau}_y}{s_0 - \hat{\tau}_y} \right) \right] \times \exp \left\{ -\frac{p\theta\psi}{(s_0 - \hat{\tau}_y) \left[\exp \left(p \frac{\hat{\tau}_s - \hat{\tau}_y}{s_0 - \hat{\tau}_y} \right) - 1 \right]} \right\} \right] \\ \hat{\tau} = \hat{\tau}_s \mu \end{array} \right. \quad \begin{array}{l} \text{At thermal regime } (\dot{\epsilon} < 10^4 / \text{sec}) \\ \text{At shock regime } (\dot{\epsilon} > 10^9 / \text{sec}) \end{array}$$

where

$$\hat{\tau}_s = s_0 - (s_0 - s_\infty) \operatorname{erf} \left[\kappa \hat{T} \ln \left(\gamma \dot{\xi} / \dot{\psi} \right) \right]$$

$$\hat{\tau}_y = y_0 - (y_0 - y_\infty) \operatorname{erf} \left[\kappa \hat{T} \ln \left(\gamma \dot{\xi} / \dot{\psi} \right) \right]$$

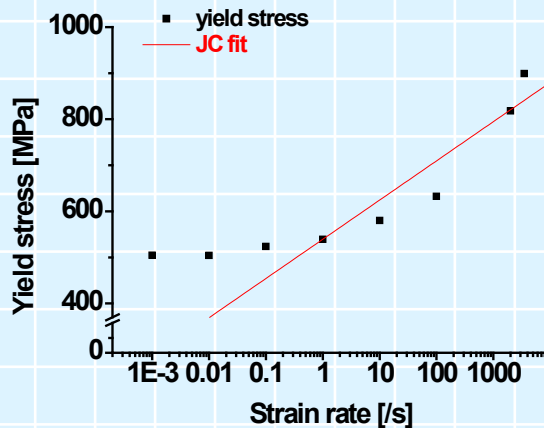
- Physically based model
- Valid for the largest range of strain rates ($\sim 10^{12}$ /sec)
 - Extended to plastic deformation in overdriven shock regime
- Only thermal regime will be considered in this research

*D.L. Preston et al., "Model of Plastic Deformation for Extreme Loading Conditions", J. Appl. Phys., 2003.

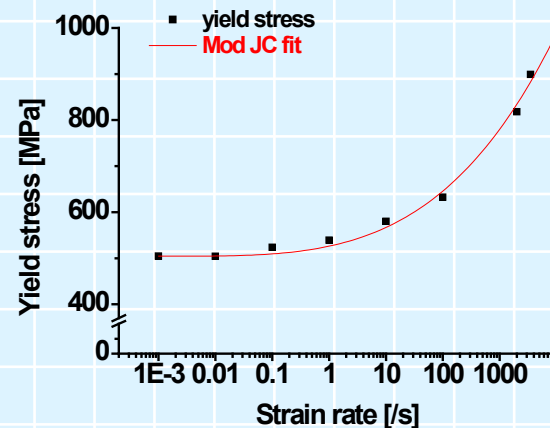
Modified Johnson–Cook model

$$\sigma = \left[A + B\varepsilon^n \right] \left[1 + C \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right] \left[1 - \left(\frac{T - T_r}{T_m - T_r} \right)^m \right] \rightarrow \sigma = \left[A + B\varepsilon^n \right] \left[1 + C \left(\ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^p \right] \left[1 - \left(\frac{T - T_r}{T_m - T_r} \right)^m \right]$$

- Linear expression of strain rate hardening term in Johnson–Cook model is substituted by the exponential expression



Expression of initial yield stress of 4340Steel using Johnson–Cook model



Expression of initial yield stress of 4340Steel using modified Johnson–Cook model

*W.J. Kang et al., "Modified Johnson-Cook Model for Vehicle Body Crashworthiness Simulation", Int. J. Vehicle Design, 1999.

Modified Khan–Huang model

- **Khan–Huang model (1992)**

- **Represent strain hardening change (increase or decrease) as strain rate changes**

✓ **Strain hardening term in first bracket is described by function of strain and strain rate**

$$\bar{\sigma} = \left[A + B \left(1 - \frac{\ln \dot{\varepsilon}^*}{\ln D_0^p} \right)^{n_1} \varepsilon^{n_0} \right] e^{C \ln \dot{\varepsilon}} (1 - T^{*m}) \quad \text{where } D_0^p = 10^6 / s$$

- **Modified Khan–Huang model (2006)**

- **Modify strain rate hardening term in Khan–Huang model as done in modified Johnson–Cook model**

$$\bar{\sigma} = \left[A + B \left(1 - \frac{\ln \dot{\varepsilon}^*}{\ln D_0^p} \right)^{n_1} \varepsilon^{n_0} \right] \left(1 + C (\ln \dot{\varepsilon}^*)^p \right) (1 - T^{*m}) \quad \text{where } D_0^p = 10^9 / s$$

*A.S. Khan *et al.*, “Experimental and Theoretical Study of Mechanical Behavior of 1100 Aluminum in the Strain Rate Change 10^{-5} - 10^4 s $^{-1}$ ”, Int. J. Plast., 1992.

*H.J. Lee *et al.*, “Dynamic Tensile Tests of Auto-body Steel Sheets with the Variation of Temperature”, Solid State Phenomena, 2006.

Summary of dynamic hardening equations

- Dynamic hardening equations used in this research

			Characteristics	
			Yield stress representation	Strain hardening representation
Representative dynamic hardening equations	Johnson-Cook model		Linear increase as strain rate increases	Increase a little as strain rate increases (nearly constant)
	Zerilli-Armstrong model	BCC model	Exponential increase as strain rate increases	Independent on strain rate (constant)
		FCC model	Independent on strain rate (constant)	Increase as strain rate increases
	Preston-Tonks-Wallace model		Error function increase as strain rate increases	Increase or decrease as strain rate increases
Modified model suggested	Modified Johnson-Cook model		Exponential increase as strain rate increases	Increase a little as strain rate increases (nearly constant)
	Modified Khan-Huang model		Exponential increase as strain rate increases	Increase or decrease as strain rate increases

- Material

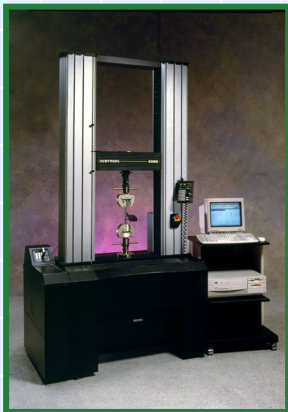
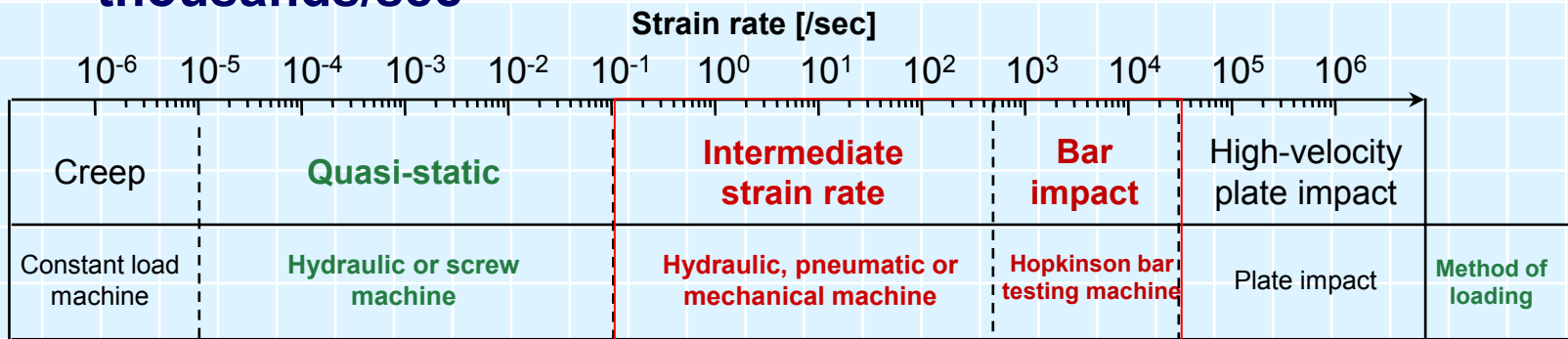
✓ 4340Steel(BCC), OFHC(FCC), Ti6Al4V(HCP)

Experiments

- **Uniaxial tensile tests**
at quasi-static and intermediate strain rates
- **Hopkinson bar tests at high strain rates**

Material test with the variation of strain rate

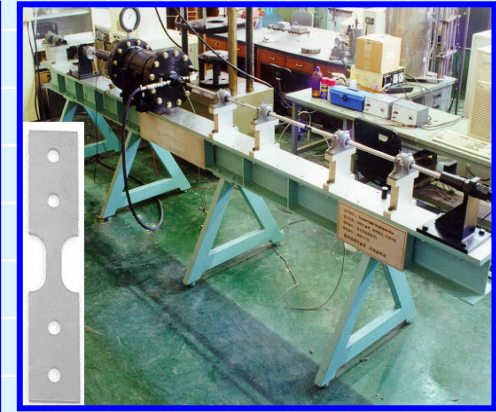
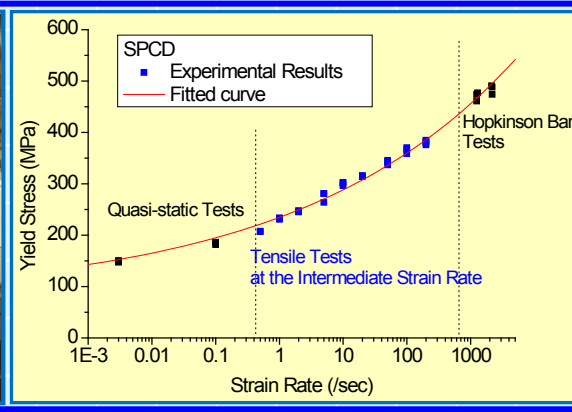
- Strain rate: strain per unit time (unit: /sec)
- Material tests at the strain rate ranged from quasi-static to thousands/sec



Instron(static)



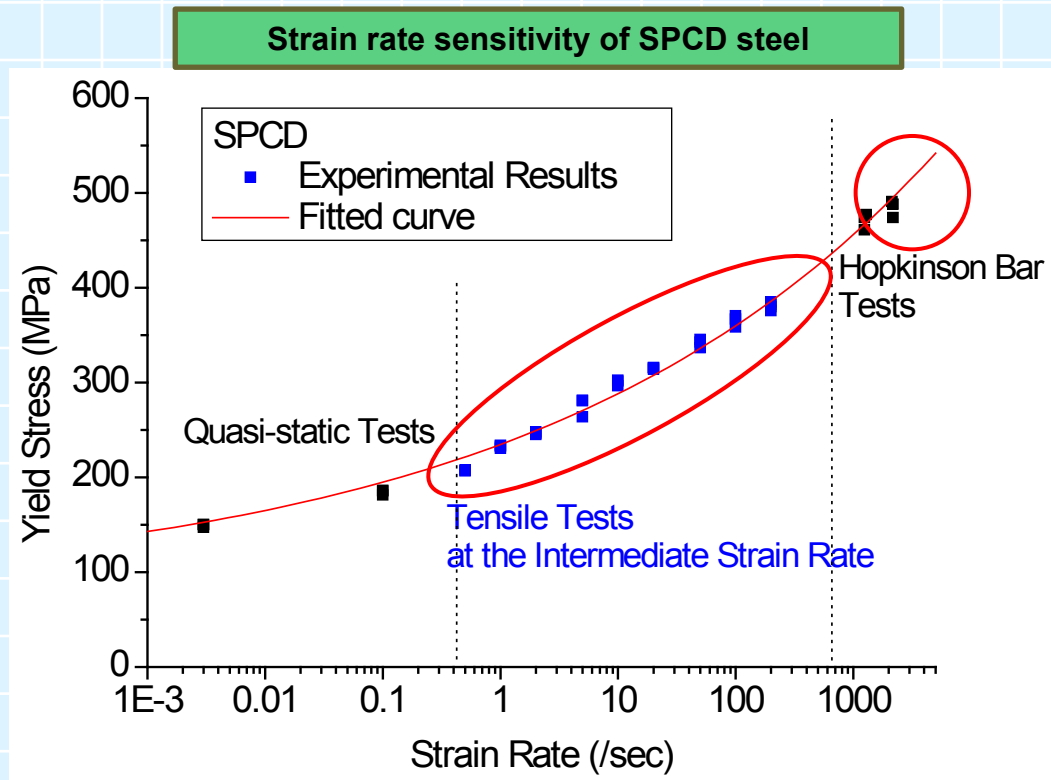
High speed material testing machine



Hopkinson bar

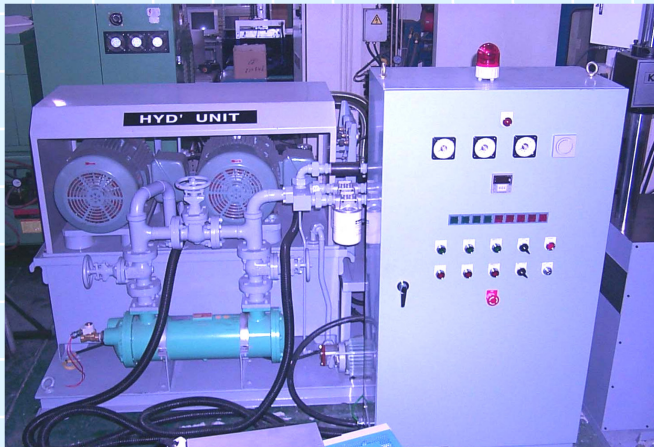
Necessity of material properties at intermediate strain rates

- Accurate understanding of material properties at wide range of strain rates



High speed material testing machine

- Dynamic material properties at intermediate strain rates
- Range of strain rate: 0.1 ~ 500/sec
- Servo-hydraulic system
- Max. speed: 7,800 mm/sec
- Max. load : 30 kN



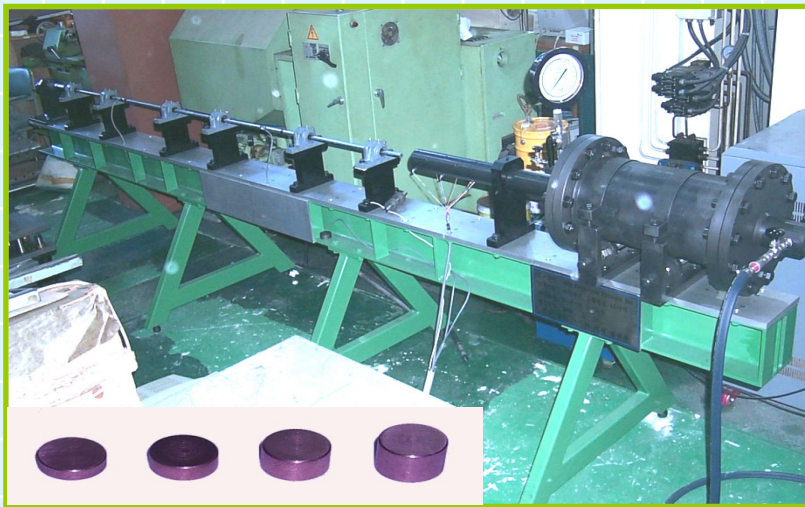
Hydraulic Unit



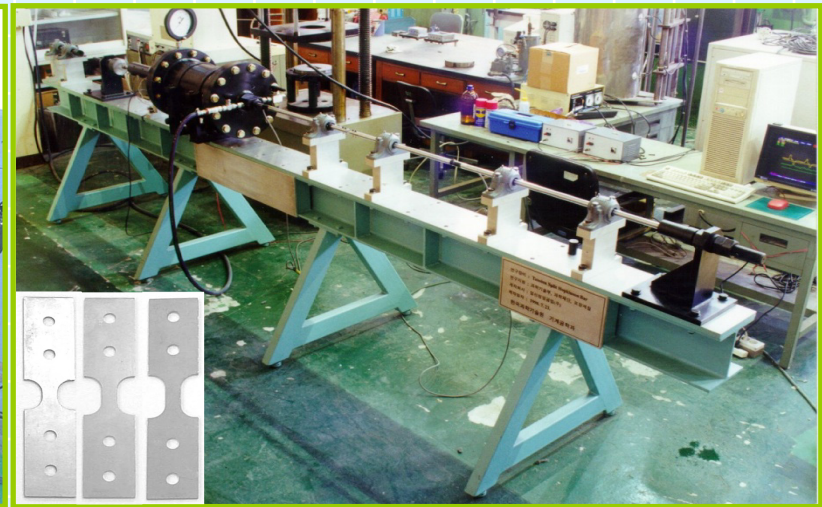
Testing Machine

Split Hopkinson pressure bar

- Dynamic material properties at high strain rates
- Range of strain rate: 1,000 ~ 10,000/sec
- Pneumatic system
- Max. speed: 35,000 mm/sec
- Striker bar(tube), incident bar, transmitted bar ($\Phi 20$)



Split Hopkinson Pressure Bar



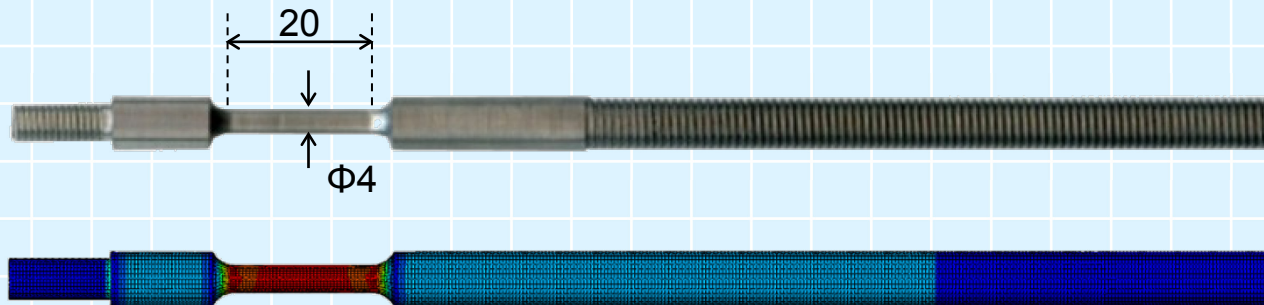
Tension Split Hopkinson Bar

Dimensions of specimens for tests

- Specimen for tensile tests

- Cylindrical type tensile specimen

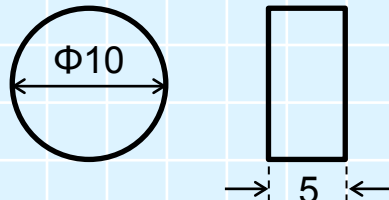
- ✓ Determined from finite element analysis for the gauge section to be uniformly elongated at intermediate strain rate



- Specimen for SHPB tests

- Cylindrical type specimen

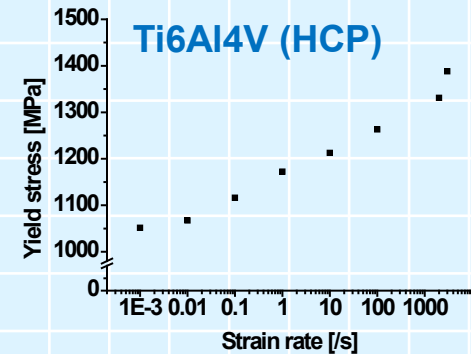
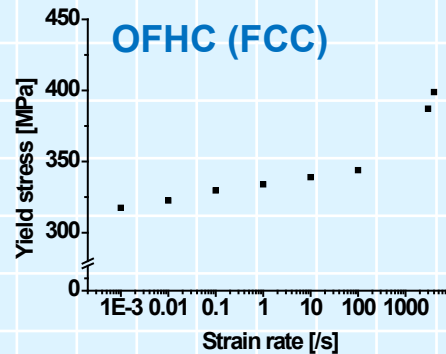
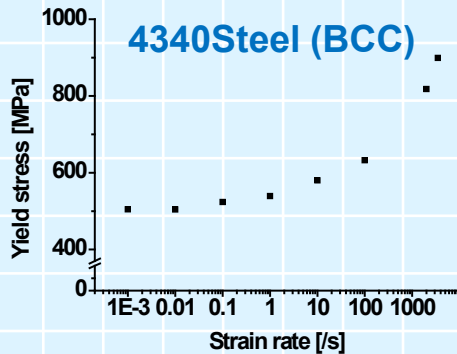
- ✓ Determined to induce force equilibrium during the tests



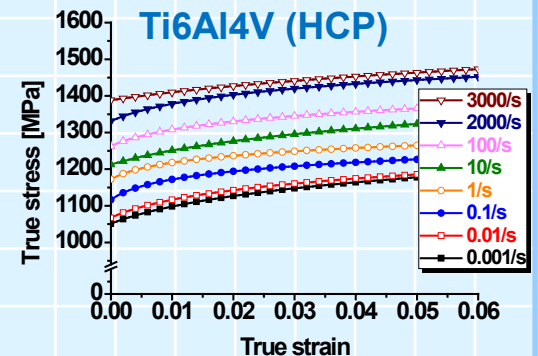
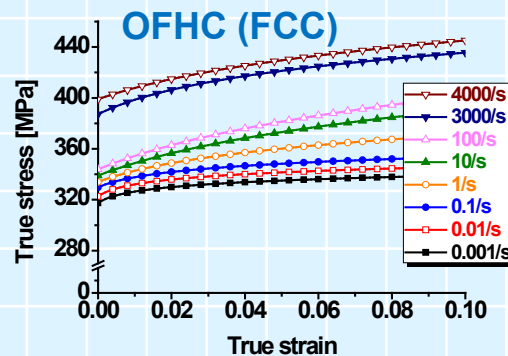
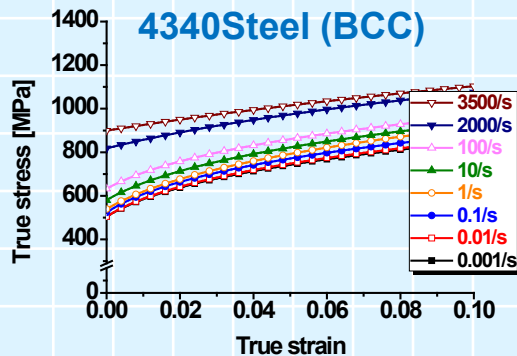
Experimental results

- Dynamic material properties for 3 kinds of materials
 - Initial yield stress and flow stress of 3 kinds of materials increase as strain rate increases

Initial yield stress



Flow stress

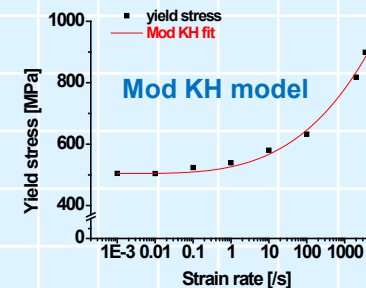
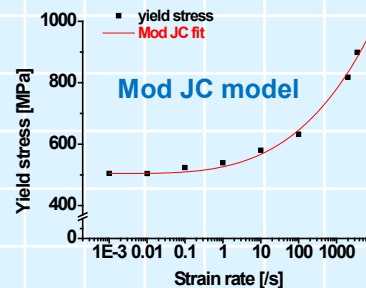
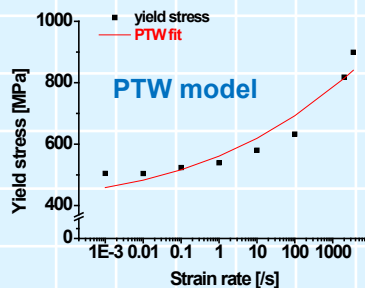
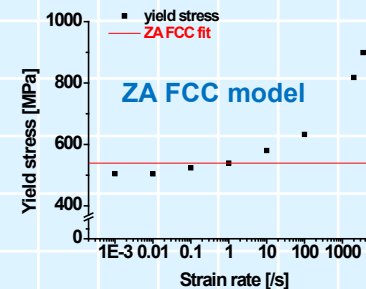
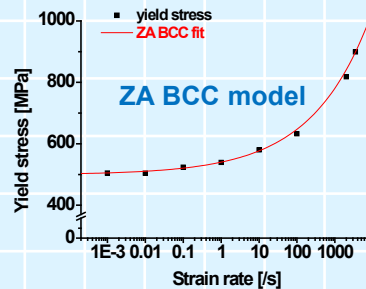
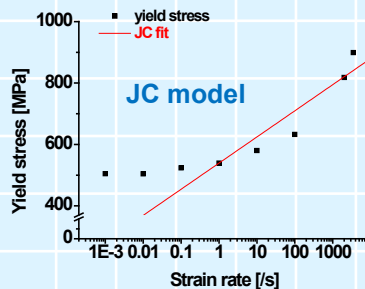


Evaluation and Comparison of Selected Models Constructed

Model construction procedure

1. Determine initial yield stress related parameters

- Initial yield stress is the most important to represent plastic deformation of materials since the initial yield stress indicates the onset of plastic deformation
- Example: Representation of initial yield stress change of 4340steel with respect to the strain rate using six kinds of hardening models

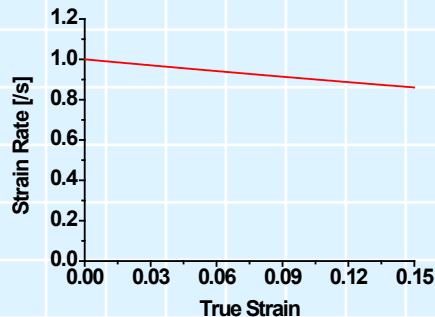


Model construction procedure (Cont'd)

2. Determine strain hardening related parameters

- Consider strain rate and temperature change during the tests for more accurate fitting

- ✓ The strain rate changes during the tests continuously since the gauge length of the specimen changes



- ✓ The strain rate changes although the tensile speed is constant

$$\dot{\epsilon} = \dot{\epsilon}(\epsilon) = \frac{V_0}{L_0} \exp(-\epsilon) = \dot{\epsilon}_0 \exp(-\epsilon)$$

- ✓ The strain rate can be expressed as a function of strain

- ✓ Temperature of the specimen changes during the tests
 - At high strain rate conditions ($\dot{\epsilon} \geq 0.01/\text{sec}$), 90% of the plastic deformation energy is converted to heat energy

$$\Delta T = \frac{0.9}{\rho c} \int_0^\epsilon \sigma(\epsilon) d\epsilon$$

Model construction procedure (Cont'd)

- Experimental data when strain rate and temperature change are considered
- Example: experimental data of 4340steel at 10/s and 300K

Strain	Strain rate [1/sec]	Temperature [K]	Stress [MPa]
0.000	10	300	599.56
0.001	10	300	605.08
0.002	10	300	614.24
0.003	10	300	614.86
0.004	10	300	625.31
0.005	10	300	628.36
0.006	10	300	633.90
0.007	10	300	643.16
0.008	10	300	647.49
0.009	10	300	655.53
0.010	10	300	661.17
⋮	⋮	⋮	⋮



$$\dot{\epsilon} = \dot{\epsilon}_0 \exp(-\epsilon)$$

$$\Delta T = \frac{0.9}{\rho c} \int_0^\epsilon \sigma(\epsilon) d\epsilon$$

Strain	Strain rate [1/sec]	Temperature [K]	Stress [MPa]
0.000	10.000	300.00	599.56
0.001	9.989	300.16	605.08
0.002	9.980	300.31	614.24
0.003	9.970	300.47	614.86
0.004	9.960	300.62	625.31
0.005	9.950	300.78	628.36
0.006	9.940	300.94	633.90
0.007	9.930	301.10	643.16
0.008	9.921	301.27	647.49
0.009	9.911	301.43	655.53
0.010	9.900	301.61	661.17
⋮	⋮	⋮	⋮

Constant strain rate and temperature

Using true strain rate and thermal softening condition

$$\sigma_{JC} = [A + B\epsilon^n] [1 + C \ln(10)] \left[1 - \left(\frac{0}{T_m - T_r} \right)^m \right]$$

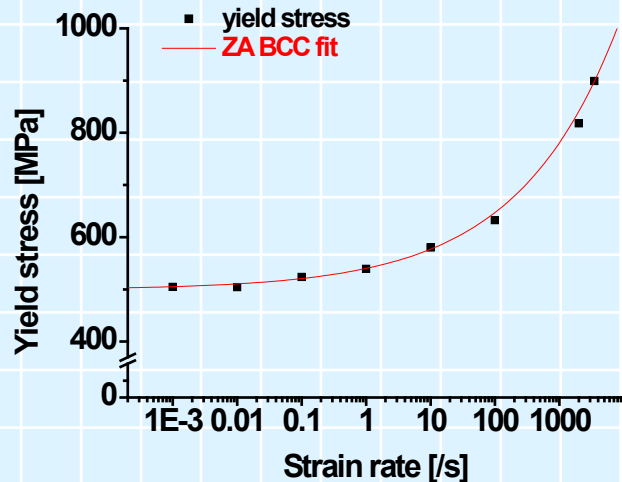
$$\sigma_{JC} = [A + B\epsilon^n] [1 + C \ln(10 \exp(-\epsilon))] \left[1 - \left(\frac{\Delta T}{T_m - T_r} \right)^m \right]$$

Model construction procedure (Cont'd)

- Example of model construction procedure
 - Construction of Zerilli-Armstrong BCC model for 4340steel

$$\sigma = C_0 + C_1 \exp[-C_3 T + C_4 T \ln \dot{\epsilon}] + C_5 \epsilon^n$$

- ✓ 1. Determine the initial yield stress related parameters using the initial yield stress at the various strain rates

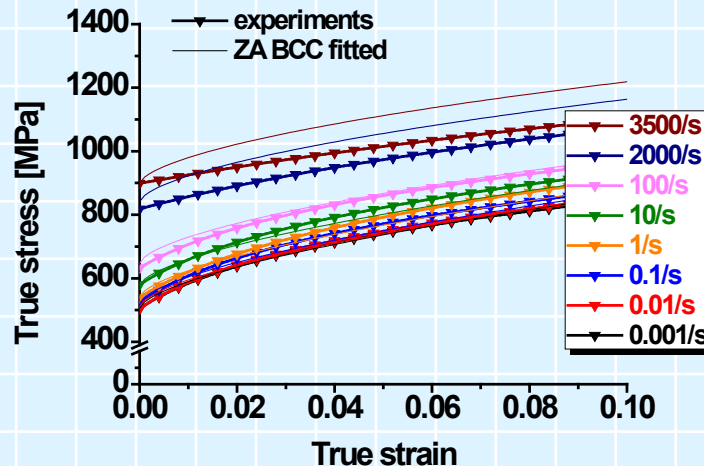


$C_0, C_1, C_3,$ and C_4 are determined

$$\sigma = C_0 + C_1 \exp[-C_3 T + C_4 T \ln \dot{\epsilon}] + C_5 \epsilon^n$$

Model construction procedure (Cont'd)

- ✓ 2. Determine the strain hardening related parameters considering the strain rate and the temperature change



C_5 and n
are determined

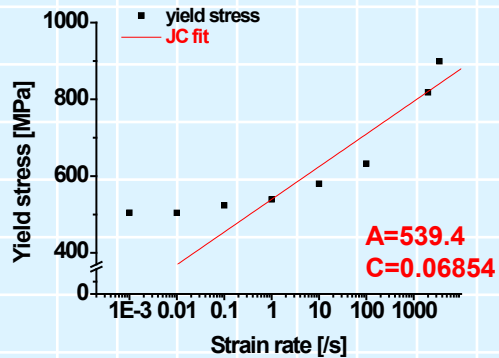
$$\sigma = C_0 + C_1 \exp[-C_3 T + C_4 T \ln \dot{\epsilon}] + C_5 \epsilon^n$$

- ✓ Strain hardening related parameters are determined by the least square method using all strain, strain rate, temperature, and stress data
- ✓ The other models can be constructed using the same procedure

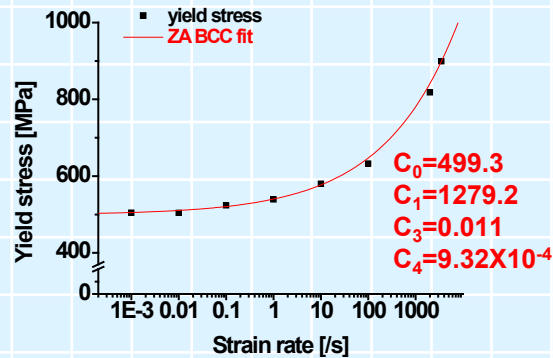
Evaluation of Selected Models (BCC)

- Initial yield stress of 4340steel (BCC)

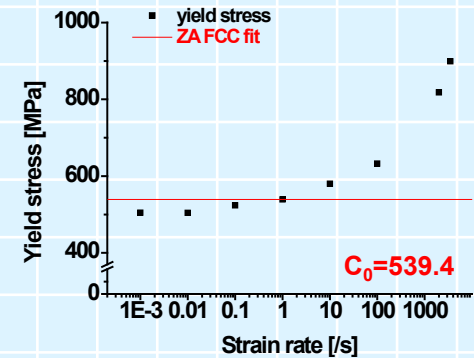
Johnson-Cook



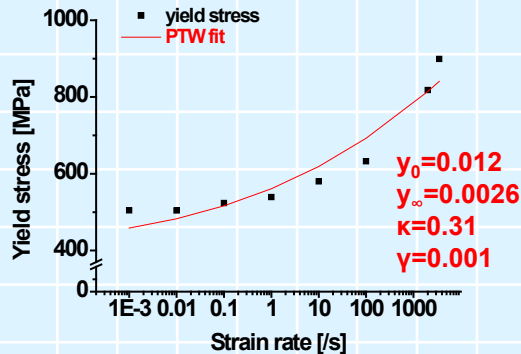
Zerilli-Armstrong (BCC)



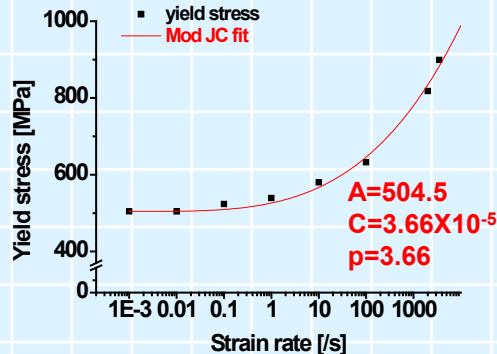
Zerilli-Armstrong (FCC)



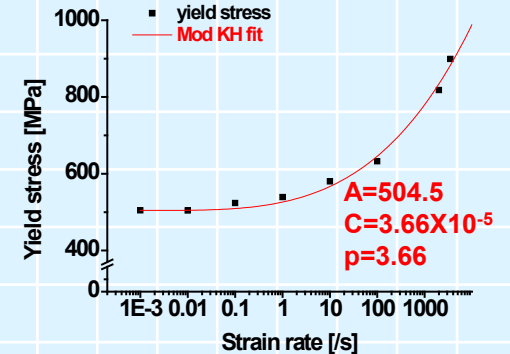
Preston-Tonks-Wallace



Modified Johnson-Cook



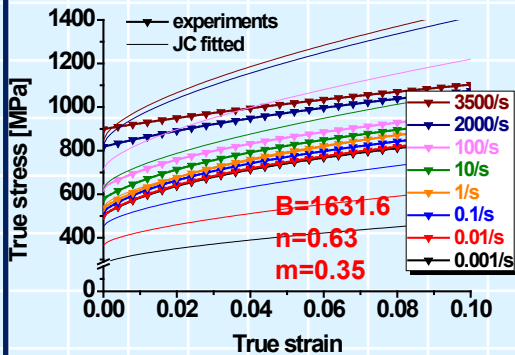
Modified Khan-Huang



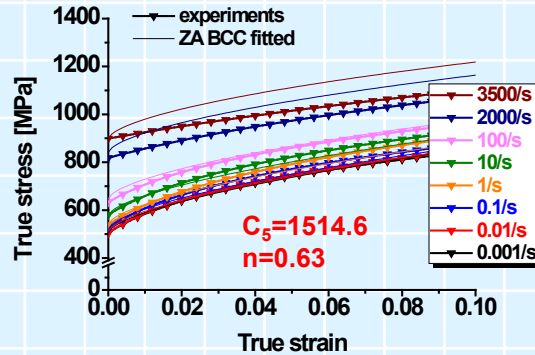
Evaluation of Selected Models (BCC)

● Strain hardening of 4340steel (BCC)

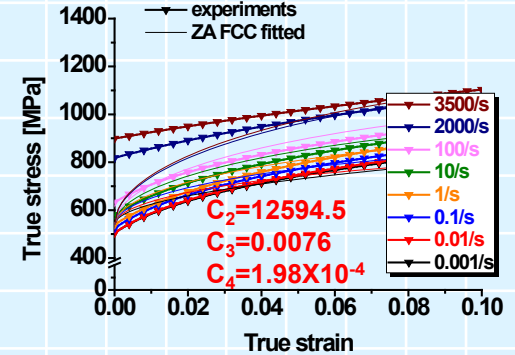
Johnson-Cook



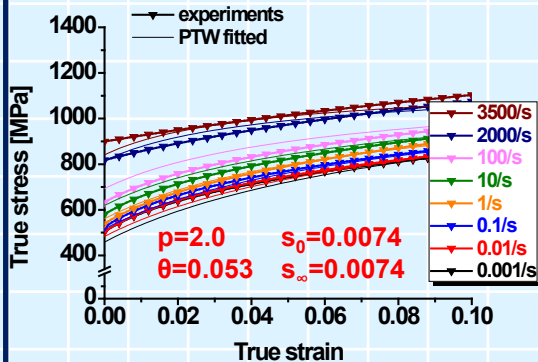
Zerilli-Armstrong (BCC)



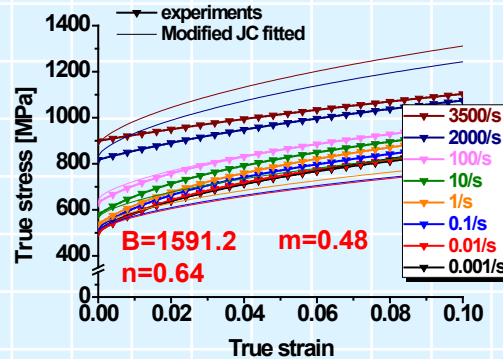
Zerilli-Armstrong (FCC)



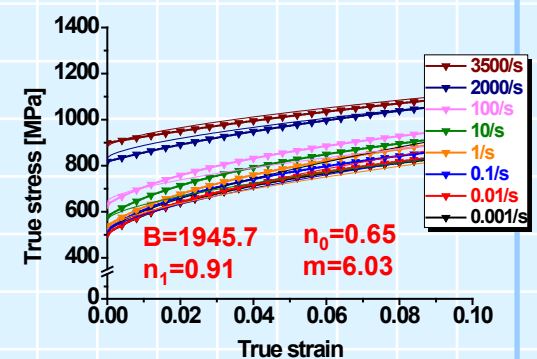
Preston-Tonks-Wallace



Modified Johnson-Cook



Modified Khan-Huang



Evaluation of Selected Models (BCC)

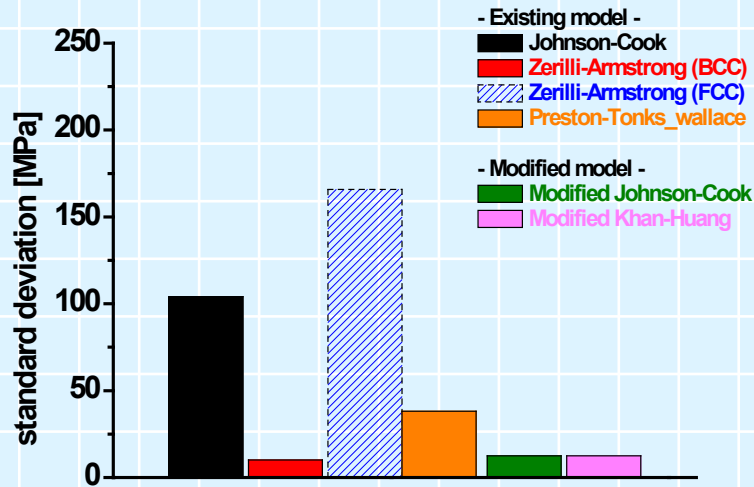
- Comparison of strain hardening characteristics from experiments and models

			Characteristics of model		Characteristics of 4340Steel	
			Yield stress representation	Strain hardening representation	Yield stress	Strain hardening
Representative dynamic hardening equations	JC model		Linear increase as strain rate increases	Increase a little as strain rate increases (nearly constant)	Exponential increase as strain rate increases	Decrease as strain rate increases
	ZA model	BCC	Exponential increase as strain rate increases	Independent on strain rate (constant)		
		FCC	Independent on strain rate (constant)	Increase as strain rate increases		
	PTW model		Error function increase as strain rate increases	Increase or decrease as strain rate increases		
Modified model suggested	Mod. JC model		Exponential increase as strain rate increases	Increase a little as strain rate increases (nearly constant)		
	Mod. KH model		Exponential increase as strain rate increases	Increase or decrease as strain rate increases		

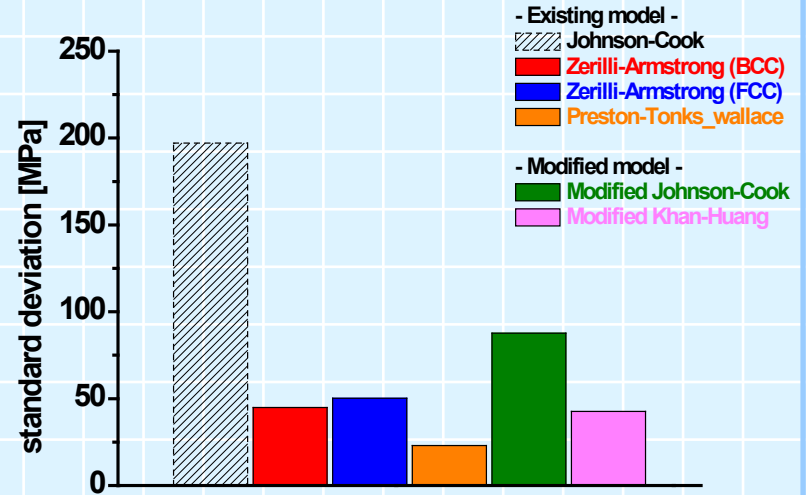
Evaluation of Selected Models (BCC)

- Quantitative evaluation of construction results of 4340steel

Deviation for yield stress characteristics



Deviation for strain hardening characteristics

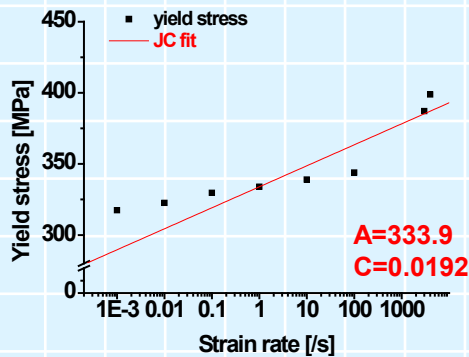


- ✓ JC and ZA FCC models show very poor results for the representation of yield stress
- ✓ JC and Mod JC models show poor results for hardening characteristics
- ✓ ZA BCC and Mod KH model are the best models for 4340steel

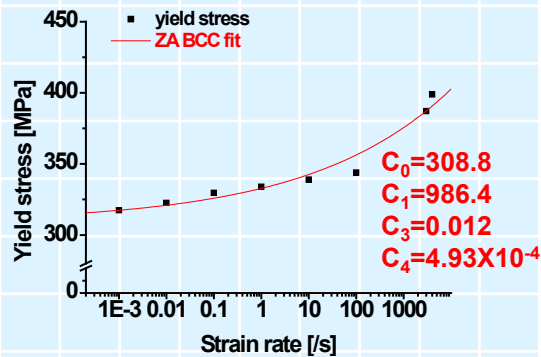
Evaluation of Selected Models (FCC)

- Initial yield stress of OFHC (FCC)

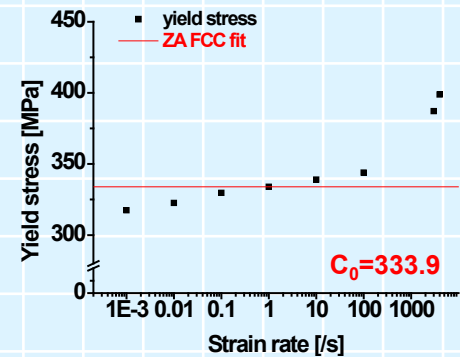
Johnson-Cook



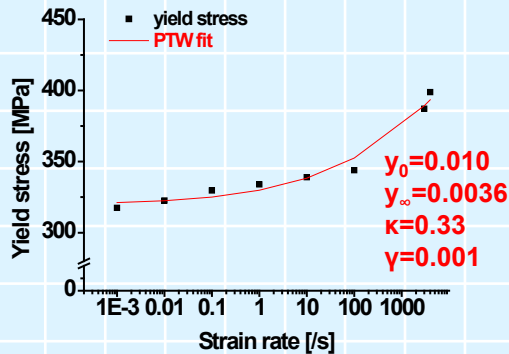
Zerilli-Armstrong (BCC)



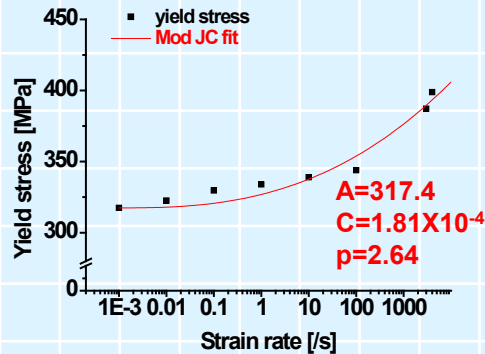
Zerilli-Armstrong (FCC)



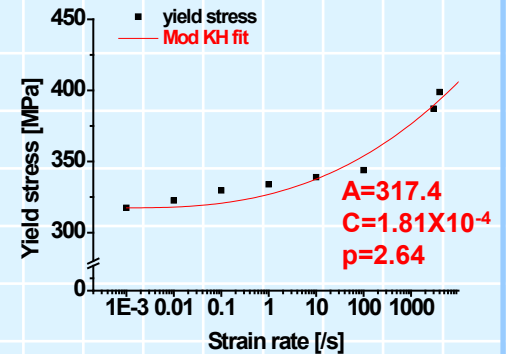
Preston-Tonks-Wallace



Modified Johnson-Cook



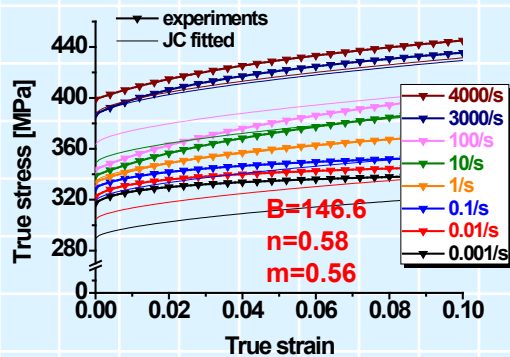
Modified Khan-Huang



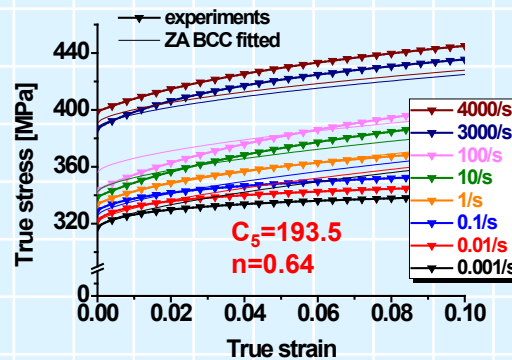
Evaluation of Selected Models (FCC)

● Strain hardening of OFHC (FCC)

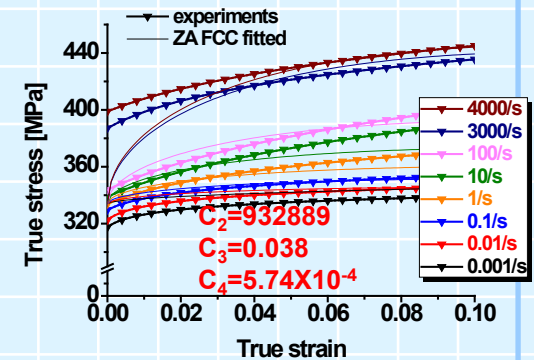
Johnson-Cook



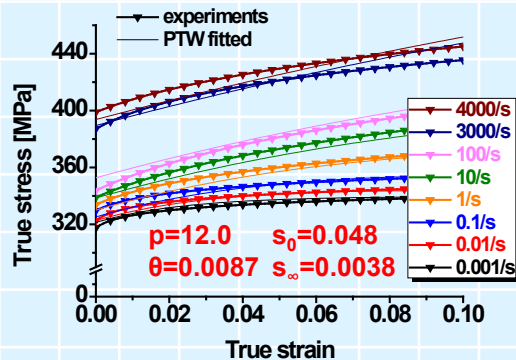
Zerilli-Armstrong (BCC)



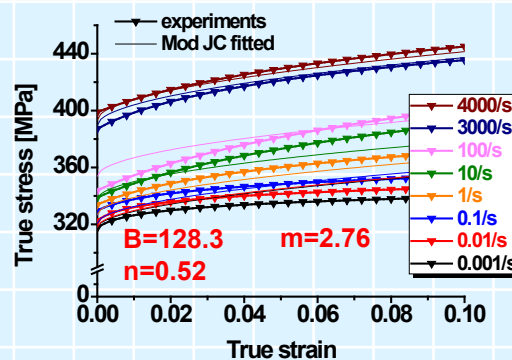
Zerilli-Armstrong (FCC)



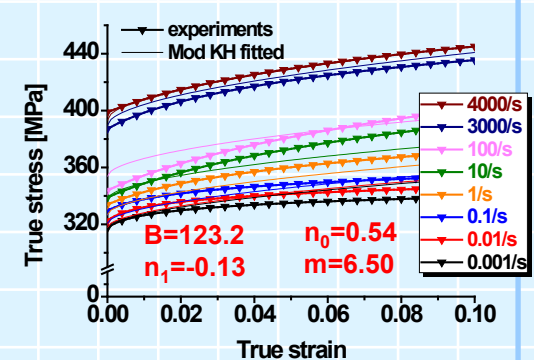
Preston-Tonks-Wallace



Modified Johnson-Cook



Modified Khan-Huang



Evaluation of Selected Models (FCC)

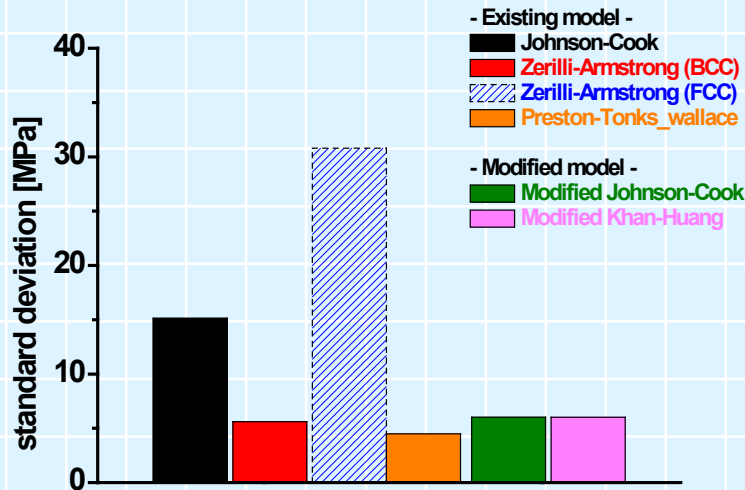
- Comparison of strain hardening characteristics from experiments and models

			Characteristics of model		Characteristics of OFHC	
			Yield stress representation	Strain hardening representation	Yield stress	Strain hardening
Representative dynamic hardening equations	JC model		Linear increase as strain rate increases	Increase a little as strain rate increases (nearly constant)	Exponential increase as strain rate increases	Increase as strain rate increases
	ZA model	BCC	Exponential increase as strain rate increases	Independent on strain rate (constant)		
		FCC	Independent on strain rate (constant)	Increase as strain rate increases		
	PTW model		Error function increase as strain rate increases	Increase or decrease as strain rate increases		
Modified model suggested	Mod. JC model		Exponential increase as strain rate increases	Increase a little as strain rate increases (nearly constant)		
	Mod. KH model		Exponential increase as strain rate increases	Increase or decrease as strain rate increases		

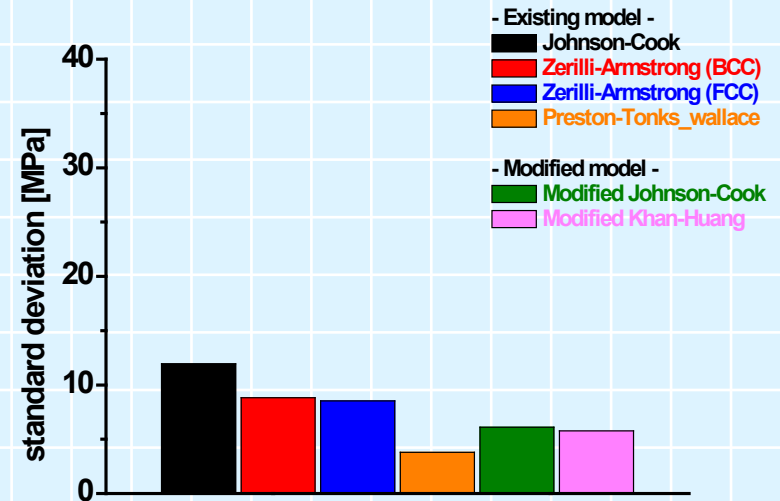
Evaluation of Selected Models (FCC)

- Quantitative evaluation of construction results of OFHC

Deviation for yield stress characteristics



Deviation for strain hardening characteristics

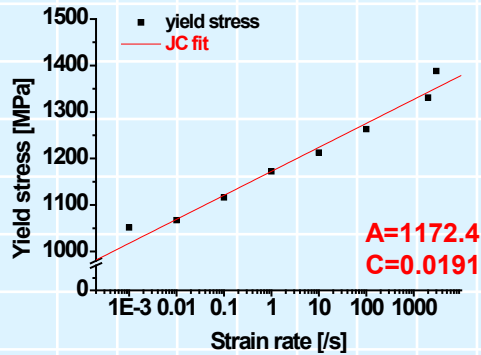


- ✓ JC and ZA FCC models show very poor results for the representation of yield stress
- ✓ PTW model is the best models for OFHC

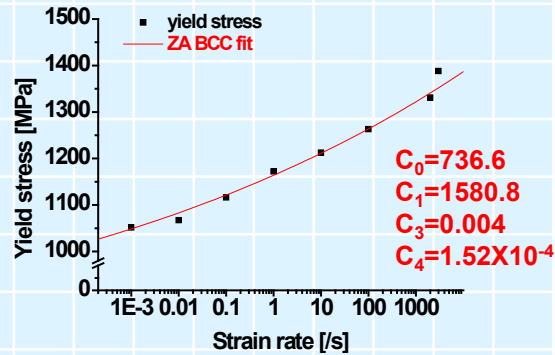
Evaluation of Selected Models (HCP)

Initial yield stress of Ti6Al4V (HCP)

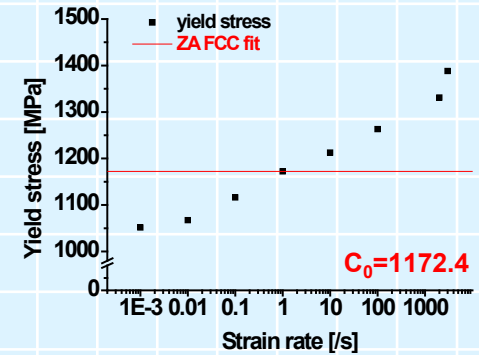
Johnson-Cook



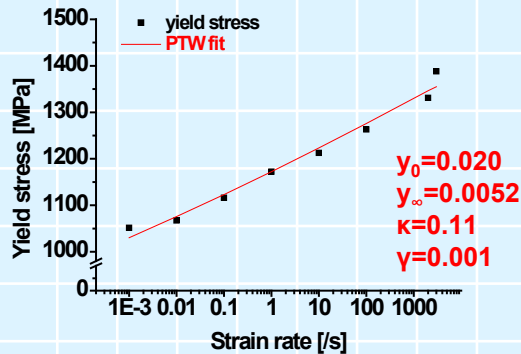
Zerilli-Armstrong (BCC)



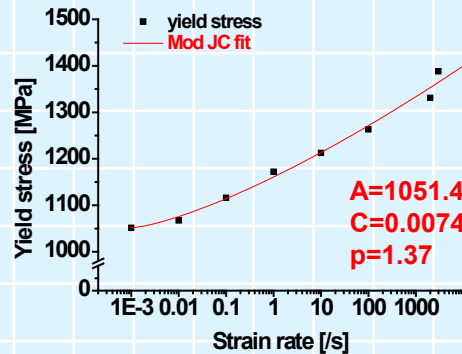
Zerilli-Armstrong (FCC)



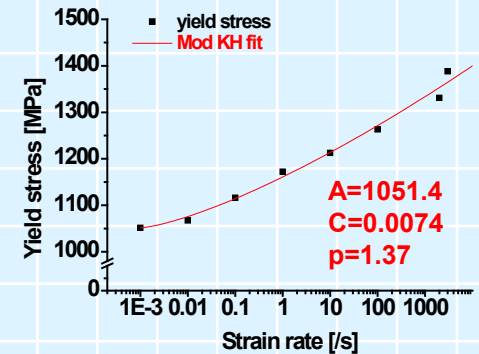
Preston-Tonks-Wallace



Modified Johnson-Cook



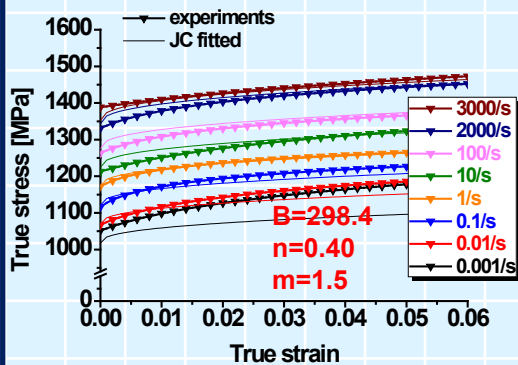
Modified Khan-Huang



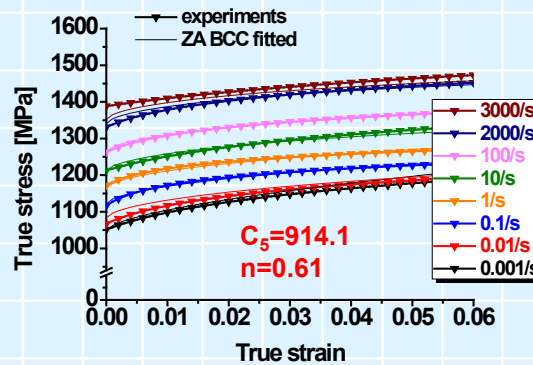
Evaluation of Selected Models (HCP)

● Strain hardening of Ti6Al4V (HCP)

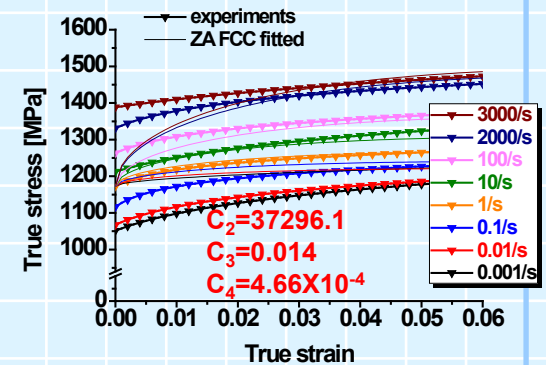
Johnson-Cook



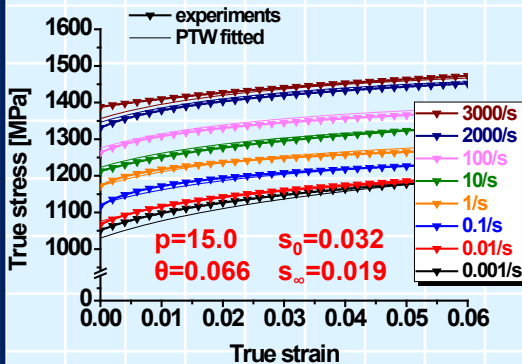
Zerilli-Armstrong (BCC)



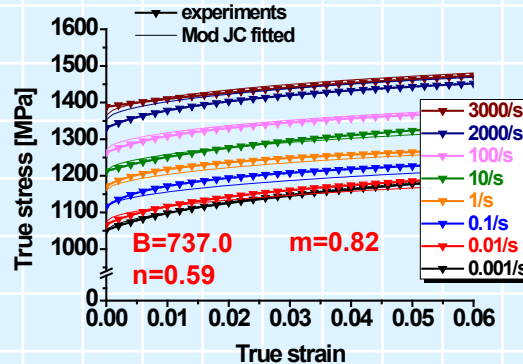
Zerilli-Armstrong (FCC)



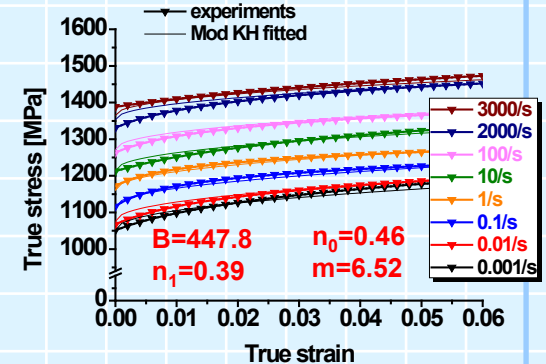
Preston-Tonks-Wallace



Modified Johnson-Cook



Modified Khan-Huang



Evaluation of Selected Models (HCP)

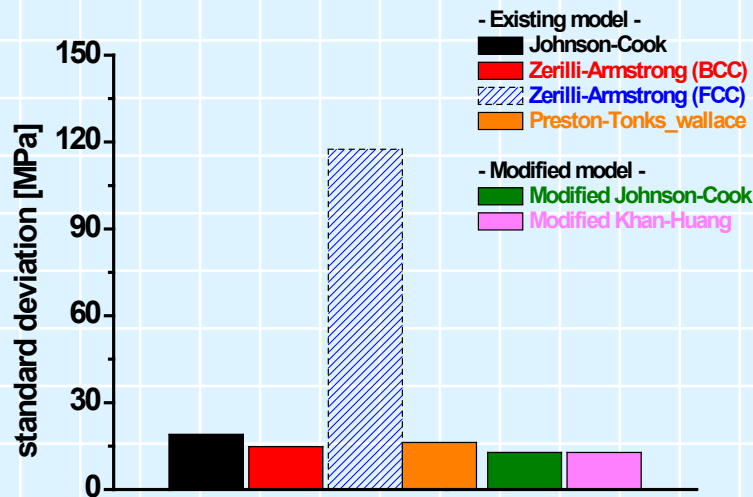
- Comparison of strain hardening characteristics from experiments and models

			Characteristics of model		Characteristics of Ti6Al4V	
			Yield stress representation	Strain hardening representation	Yield stress	Strain hardening
Representative dynamic hardening equations	JC model		Linear increase as strain rate increases	Increase a little as strain rate increases (nearly constant)	Exponential increase as strain rate increases	Nearly constant as strain rate increases
	ZA model	BCC	Exponential increase as strain rate increases	Independent on strain rate (constant)		
		FCC	Independent on strain rate (constant)	Increase as strain rate increases		
	PTW model		Error function increase as strain rate increases	Increase or decrease as strain rate increases		
Modified model suggested	Mod. JC model		Exponential increase as strain rate increases	Increase a little as strain rate increases (nearly constant)		
	Mod. KH model		Exponential increase as strain rate increases	Increase or decrease as strain rate increases		

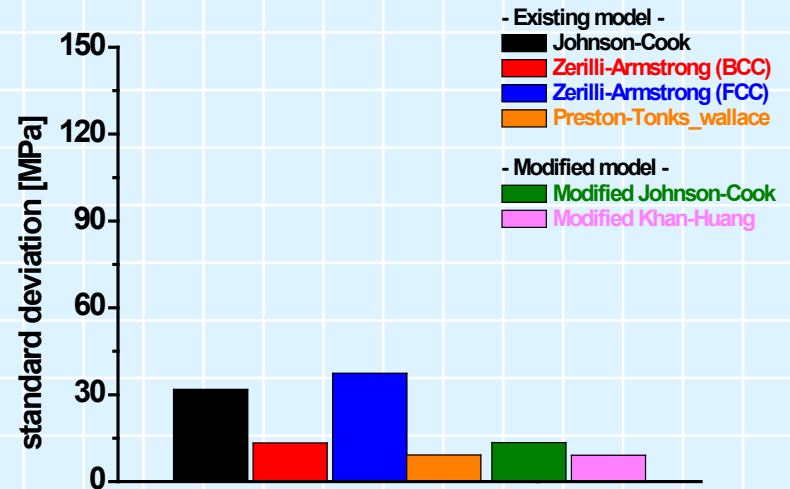
Evaluation of Selected Models (HCP)

- Quantitative evaluation of construction results of Ti6Al4V

Deviation for yield stress characteristics



Deviation for strain hardening characteristics



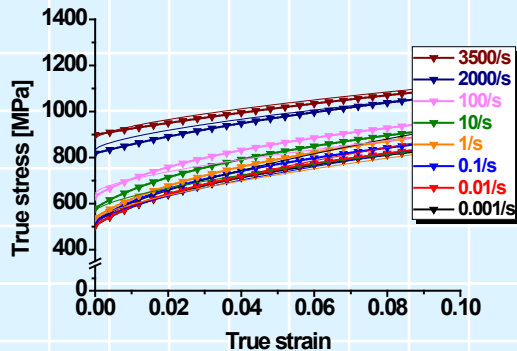
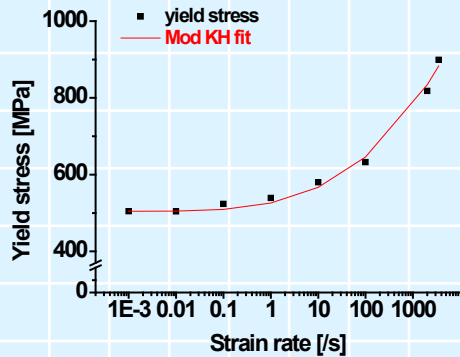
- ✓ ZA FCC model shows very poor results for the representation of yield stress
- ✓ ZA BCC, PTW, Mod JC, and Mod KH models are the best models for Ti6Al4V

Suggestion of Adequate Models

- Suggestion of the best model for each material

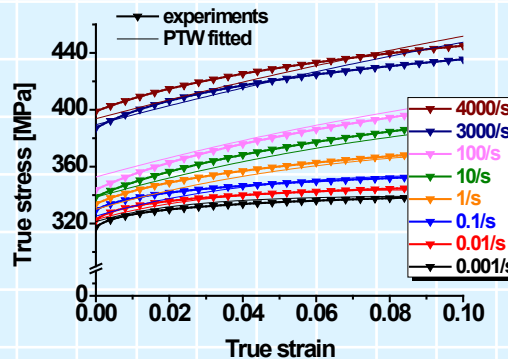
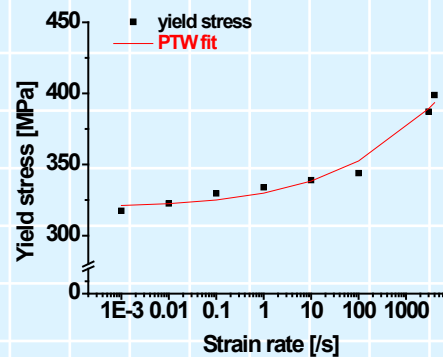
4340Steel (BCC)

Modified Khan-Huang model



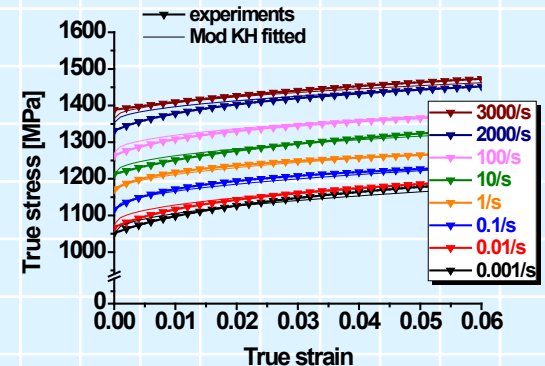
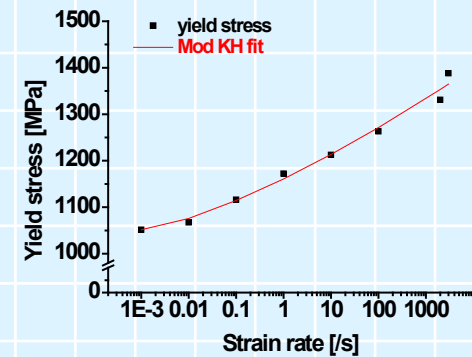
OFHC (FCC)

Preston-Tonks-Wallace model



Ti6Al4V (HCP)

Modified Khan-Huang model



Conclusions

Conclusions

- **Dynamic properties of the materials are the inherent characteristics of the material**
- **There is no unique equation which can represent the dynamic properties of all kinds of materials**
- **It is important to select and use the most applicable equation which can represent the dynamic hardening characteristics of material**
 - **Accurate understanding of the dynamic material properties by using reliable testing procedure**
 - **Accurate understanding of the characteristics of dynamic hardening models**
- **Modified Khan–Huang model, Preston–Tonks–Wallace model, and Modified Khan–Huang model show the best fit for 4340steel, OFHC, and Ti5Al4V, respectively.**

Thank you for attention