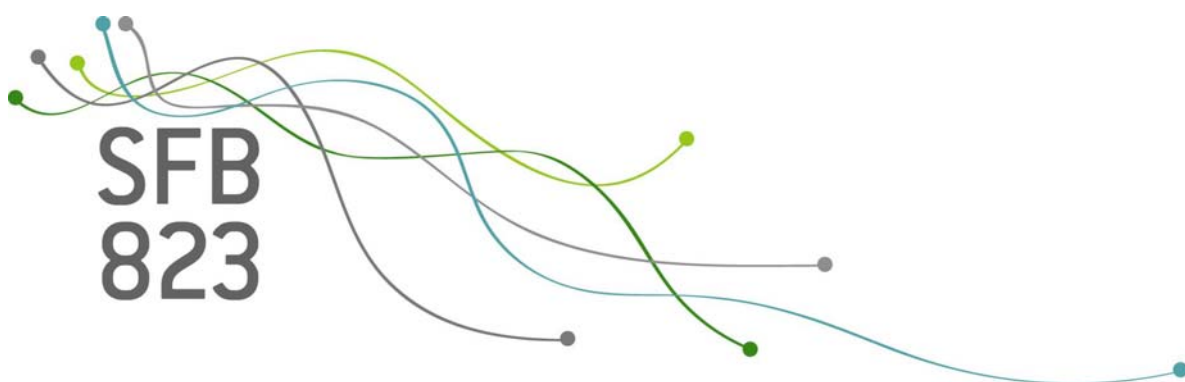# Monetary policy, interest rates, and liquidity premia

Samuel Reynard, Andreas Schabert

Monetary Policy, Interest Rates, and Liquidity Premia[1]

Samuel Reynard[2]

Swiss National Bank

Andreas Schabert

TU Dortmund University

July 23, 2012

Abstract

We augment a standard macroeconomic model by accounting for the fact that central banks supply money only in exchange for eligible assets. Monetary policy implementation then matters for the pass-through of policy rate changes and rationalizes liquidity premia on treasuries. The model explains the observed negative relation between corporate bond yield spreads and the amount of available treasuries. Liquidity premia on eligible assets further provide a structural explanation for the systematic wedge between the policy rate and the consumption Euler rate that standard models equate. Nonetheless, monetary policy effects on real activity and inflation are consistent with broad empirical evidence.

JEL classification: E52; E58; E43; E44; E32.
Keywords: Monetary policy transmission; Open market operations; Treasury debt liquidity premium; Consumption Euler rate.

[2]S. Reynard: Swiss National Bank, Boersenstrasse 15, 8022 Zurich, Switzerland. Phone: +41 44 631 3216. Email: samuel.reynard@snb.ch. A. Schabert: University of Dortmund, Vogelpothsweg 87, 44227 Dortmund, Germany. Phone: +49 231 755 3182. Email: andreas.schabert@udo.edu.

1 Introduction

It is well-established that prices of assets are affected by their liquidity, i.e. the degree to which they facilitate market transactions, such that liquidity premia contribute to interest rate spreads.[3] During the recent financial crises, central banks have proven to be able to affect interest rates at various maturities by introducing lending facilities and direct asset purchases,[4] suggesting that central bank's asset acquisition policies can be relevant for asset pricing in non-crisis times as well. The fact that central banks typically supply reserves only in exchange for a particular set of securities should be internalized by rational investors, leading to a liquidity premium between interest rates on non-eligible and eligible assets, like short-term treasuries. According to this view, monetary policy implementation is causal for the existence of liquidity premia on treasuries, which has not been taken into account in the macroeconomic literature.[5] In this paper, we examine the role of monetary policy for the existence and the dynamics of the liquidity premium on short-term treasuries within a macroeconomic model. We show how interest rate spreads are affected by monetary policy consistent with empirical evidence, and we demonstrate that endogenous liquidity premia can be relevant for monetary transmission.

We present a simple framework which differs from standard macroeconomic models only by specifying money supply as an asset exchange, which is usually neglected in contemporaneous macroeconomic models (see e.g. Smets and Wouters, 2007). The central bank is assumed to supply money against eligible assets, while private agents internalize the eligibility of different assets when they invest. In equilibrium, the interest rate on an eligible asset, i.e. short-term treasuries, closely follows the interest rate the central bank charges when it purchases this asset, and it differs from interest rates on non-eligible assets by a liquidity premium.[6] We show that this model can explain three observations: i.) the negative correlation between the liquidity premium (based on corporate bond yield spreads) and the supply of treasuries, which has been documented by Krishnamurthy and Vissing-Jorgensen (2012), ii.) the negative correlation between the consumption Euler equation residual and the Fed-

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[3]See e.g. Holmstrom and Tirole (2001), Acharya and Pedersen (2005), or Lagos (2010), where liquidity also does not refer to the ease with which assets can be resold.

[4]See Fleming (2012) for an overview of US Federal Reserve liquidity provision policies in 2007-2009.

[5]The liquidity of assets has been considered for various purposes in Bansal and Coleman (1996), Canzoneri et al. (2008), Lagos (2010, 2011), Shi (2012), Kiyotaki and Moore (2012). Liquidity premia emerge in these studies because assets provide transaction services to different degrees, whereas monetary policy matters for asset prices in equilibrium rather than for the private agents' investment decision.

[6]To be more precise, there are two interest rate differentials due to the liquidity of assets: the spread between the rates of return on money and treasuries, and the spread between the rates of return on non-eligible assets and the treasury rate. Throughout the paper, we will focus on the latter.

eral Funds rate, as shown by Canzoneri et al. (2007) and Atkeson and Kehoe (2009), and *iii.*) the fact that this residual, which is also known as a "risk premium shock" (see Smets and Wouters, 2008), is more volatile than the Euler rate itself, which has been stressed by Chari et al. (2009).[7]

The majority of macroeconomic studies on monetary policy focusses on a short-run interest rate as the central bank's operating target, which impacts on the private sector behavior by affecting intertemporal substitution, i.e. by relating the policy rate to inflation and consumption growth via the consumption Euler equation. The well-known failure of the consumption Euler equation to explain the magnitude of risk-free interest rates (see Weil, 1989) has – until now – typically been neglected in standard macroeconomics models, where the policy and the treasury rate are assumed to equal the (consumption) Euler rate.[8] Moreover, the consumption Euler equation residual, i.e. the spread between the Euler rate and the Federal Funds rate, is typically more volatile than the Euler rate (see Chari et al., 2009) and is found to be negatively related to the Federal Funds rate (see Canzoneri et al., 2007, and Atkeson and Kehoe, 2009), which cast severe doubts on the assumed identity between the Euler rate and the Federal Funds rate in standard models. We show analytically and quantitatively that these observations can be reconciled within our macroeconomic framework, where changes in the policy rate are – due to the liquidity premium – not one-for-one passed through to all short-term interest rates. Moreover, we show that the liquidity premium varies endogenously with the availability of treasuries relative to real activity, like in Krishnamurthy and Vissing-Jorgensen (2012).

The model is specified in a stylized way to facilitate the derivation of analytical results. In particular, demand for liquidity/cash is induced by a simple cash-in-advance constraint for the benchmark model, which is replaced by a money-in-the-utility-function specification to assess the robustness of the results (see section 5.4). The model essentially differs from conventional New Keynesian models by two key assumptions: First, open market operations are separated from the asset market, where agents trade which each other and with the government. Before the asset market opens, private agents can acquire cash in open market operations from the central bank in exchange for eligible securities discounted with the policy rate. Eligible assets that are bought today can therefore be cashed in the next period, such that their rate of return closely follows to the expected future policy rate. Second, we

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[7]Chari et al. (2009) suspect that this shock is "hardly likely to be invariant to monetary policy", which accords to the idea presented in this paper.

[8]Aiyagari and Gertler (1991) and Eisfeldt (2007) also argue that the demand for short-term treasury securities (T-bills) cannot solely be explained with consumption smoothing, and suggest considering a transactions demand for liquid assets.

account for common central bank practice (like the Fed's or the BoE's in non-crisis times) and assume that only short-term treasuries are eligible in open market operations ("T-bills only"),[9] while other assets – like equity or corporate bonds – are not accepted by the central bank.[10] Given that access to money relies on holdings of treasuries, private agents demand a (il-)liquidity premium on non-eligible assets. A higher policy rate then raises the price of money in terms of treasuries and therefore leads to a decline in the liquidity premium consistent with Canzoneri et al.'s (2007) and Atkeson and Kehoe's (2009) findings. Moreover, the liquidity premium is shown to be more volatile than the Euler rate, providing a structural explanation of the volatility of "risk premium shock" in Smets and Wouters (2007). The second assumption further implies that changes in the supply of treasuries can alter private agents access to reserves. An increase in treasuries relative to real activity thereby reduces the valuation of liquidity and thus the liquidity premium, which accords to Krishnamurthy and Vissing-Jorgensen's (2012) result regarding the role of liquidity for the treasury yields.[11]

These assumptions imply a monetary transmission mechanism that differs from the way real activity is affected by monetary policy in standard New Keynesian models, where the rate of intertemporal substitution equals the real policy rate. Consider, for example, an unexpected increase in the policy rate. Private agents, who are willing to hold both money and treasuries, then demand a higher treasury rate to be compensated for higher costs of acquiring new money in the next period, such that the treasury rate follows the expected future policy rate, which accords to empirical evidence, e.g., by Simon (1990).[12] Due to the cash constraint, aggregate demand positively depends on available means of payment, while access to the latter is constrained by the amount of privately held treasuries discounted with the current policy rate. A higher policy rate reduces the discounted value of treasuries and has, thereby, a contractionary effect on aggregate demand and inflation. Given that

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[9]Accepting other assets than T-bills was, for example, viewed as a relevant question in 2001 in face of a decline in the amount of outstanding US-treasury debt. See Board of Governors (2001) for a comprehensive discussion on alternative assets that were considered for open market purchases. This issue has regained interest in the current financial crises, where the Fed and other central banks relaxed their asset acquisition policy (see, e.g., Borio and Disyatat, 2009, for an overview)

[10]This assumption accords to the Fed's asset aquisition policy before 2007. In 2006, for example, Treasury bills were the largest position accounting for one-third of the System Open Market Account (SOMA) holdings. Bills and Treasury coupon securities with a maturity below 2 years accounted for about two-third of SOMA holdings, while treasury securities of longer maturities and a relatively small amount of Treasury inflation-indexed securities completed the porfolio.

[11]This result is also consistent with Friedman and Kuttner's (1998) findings on the effect of relative asset quantities for the spread between commercial papers and treasury bills.

[12]The spread between these rates increases on average with aggregate uncertainty and investors' relative risk aversion, such that the treasury rate and the policy rate differ due to a small risk premium.

the liquidity premium falls, the increase in the real policy rate is not one-for-one passed through to real rates of return on non-eligible assets, like corporate debt or equity. Since monetary policy does not govern the rate of intertemporal substitution, consumption habits are neither necessary nor sufficient for smooth consumption growth (unliek in standard models, see Fuhrer, 2000) and reasonable investment dynamics can already be generated by investment adjustment costs which are much smaller than suggested by estimates based on aggregate data (like in Smets and Wouters, 2007).[13]

The paper is organized as follows. Section 2 presents empirical evidence on interest rates and spreads. In Section 3, the model is developed. In Section 4, we provide analytical results on the behavior of interest rates and spreads. Section 5 presents quantitative results. Section 6 concludes.

2 Empirical evidence

In this Section, we provide an empirical analysis of interest rates and spreads, which we will interpret as liquidity premia. We then show in the subsequent analysis how modeling monetary policy implementation involving quantities of money and treasuries, with a small modification from standard models, can explain the existence and behavior of these premia while leading to conventional monetary policy effects. In the first part of this Section, we analyze liquidity premia and their correlation with the amount of available treasuries, which relates to the analysis of Krishnamurthy and Vissing-Jorgensen (2012). The second part analyzes the spread between the consumption Euler rate and the Federal Funds rate, relating to Canzoneri et al. (2007), Atkeson and Kehoe (2009), and Chari et al. (2009). The results presented in this Section indicate a substantial role for monetary policy in influencing interest rates and spreads, providing the point of departure for our theoretical analysis. Here, we present selected second moments of interest rates and spreads, and we will subsequently show that they can be replicated with our model.

2.1 Interest rate on short-term treasuries

In a recent study, Krishnamurthy and Vissing-Jorgensen (2012) provide empirical evidence on the role of treasuries supply for corporate bond yield spreads. They find a negative relationship in US data between the supply of government debt and spreads between corporate

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[13] Our model suggest investment adjustment costs of a magnitude that is consistent with empirical evidence from disaggregate data (see Groth and Khan, 2011).

and government debt yields.[14] Krishnamurthy and Vissing-Jorgensen (2012) argue that an increase in the supply of treasuries should reduce their "convenience value", representing liquidity and safety attributes, and thereby reduce the corporate bond yield spread. For their baseline specification, they consider the spread between yields on AAA rated corporate bonds and yields on treasury bonds with a long maturity and the ratio of total government debt to GDP. Given that the focus of this paper is on the role of monetary policy on interest rate spreads, we examine correlations of slightly different variables.

When the Federal Reserve implements its interest rate target, it buys or sells assets against reserves in open market operations. In normal times, Treasury bills are the largest asset class held in the Federal Reserve portfolio as a result of open market operations.[15] To assess the particular role of monetary policy, we analyze the behavior of the spread between the 3-month high-grade commercial paper rate and the 3-month Treasury bill rate, which is also examined in Friedman and Kuttner (1998) and in Krishnamurthy and Vissing-Jorgensen's (2012) analysis of short-term interest rates. Using a structural asset pricing equation, the latter estimate the impact of total stock of treasury debt relative to GDP on that spread and argue that it reflects the price of liquidity, as there has never been a default on high grade commercial papers.[16] Given that this spread reflects liquidity (rather than safety) attributes, we will assess the plausibility of our model's predictions by comparing this spread with the model's counterpart.

To account for the argument that the valuation of treasuries should depend on the amount available to the private sector, we remove the Federal Reserve holdings of 3-month T-bills, as we want to isolate the T-bills held by the private sector ($bills_t$). The first line of Table 1 displays the correlation between the short-term spread s_t^{Treas}, identified as the spread between the 3-month high-grade commercial paper rate and the 3-month T-bill rate, and the bills-to-GDP ratio, where we used the total amount of outstanding T-bills minus the amount of T-bills held by the Federal Reserve.[17] The empirical correlation is strongly negative (-0.62), indicating that the supply of eligible assets (T-bills) matters for the

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[14]This result is related to Friedman and Kuttner's (1998) finding that the spread between commercial papers and treasury bills is affected by the relative supply of those assets.

[15]See footnote 10.

[16]The default controls, which Krishnamurthy and Vissing-Jorgensen include in their regressions explaining that spread, are statistically not different from zero.

[17]The sample covers 2003-2007, with quarterly data, due to data availability and to remove the recent financial crisis episode. The sample can be extended back to 1970 if total outstanding T-bills – instead of privately held T-bills – are considered. For the full sample (1970-2007), the correlation is less pronounced and equals -0.36. Data are from the Federal Reserve Bank of St. Louis FRED database, the U.S. Treasury, and the Federal Reserve Board.

Table 1: Selected emprical moments

$corr\left(s_t^{Treas}, bills_t/gdp_t\right)$	-0.62
$corr\left(R_t, R_t^m\right)$	0.99
$corr\left(R_t^{Euler}, R_t^m\right)$	0.53
$corr\left(s_t^{Euler}, R_t^m\right)$	-0.82
$sd(s_t^{Euler})/sd(R_t^m)$	0.85
$sd(s_t^{Euler})/sd(R_t^{Euler})$	1.49

Note: Standard deviations refer to net interest rates.

spread.[18] For the sake of completeness, the second line gives the correlation (0.99) between the T-bills rate R_t and the Federal Funds rate R_t^m, which is well-known to be close to unity. Given that the Federal Reserve controls the latter and influences the private sector holdings of T-bills, these results suggest that monetary policy plays a crucial role for the liquidity premium on short-term treasuries.

2.2 Consumption Euler rate

As demonstrated by Canzoneri et al. (2007), Atkeson and Kehoe (2009), and Chari et al. (2009), the rate implied by the consumption Euler equation hardly mimics the US monetary policy rate, i.e. the Federal Funds rate. Applying different approaches, Canzoneri et al. (2007) and Atkeson and Kehoe (2009) both find that the spread between the Euler rate and the Federal Funds rate is actually negatively related to the Federal Funds rate. Chari et al. (2009) further show that this "Euler equation error" is more than six times larger than the short-term interest rate, which they view as one of several critical properties of standard New Keynesian models.

To assess our model's ability to explain this pattern, we follow Canzoneri et al. (2007) and compute the empirical interest rate implied by standard Euler equations. According to a standard Euler equation, the (gross) Euler rate R_t^{Euler} satisfies $\frac{1}{R_t^{Euler}} = \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \frac{P_t}{P_{t+1}}$, where β is the discount factor, $u_{c,t}$ is marginal utility of consumption, and P_t is the aggregate price level. With a standard CRRA utility function, leading to a marginal utility of consumption

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[18]We find a similar correlation with long-term spreads, as with estimates presented by Krishnamurthy and Vissing-Jorgensen (2012). When supply is measured as the total government debt outstanding relative to GDP, and the spread between yields on AAA corporate bonds and long-term government bonds is considered, the correlation is -0.62. The sample covers 1934-2007, with yearly data. The debt-to-GDP ratio data is from Bohn (2008).

$u_{c,t} = c_t^{-\sigma}$, and under conditional log-normality the Euler equation can be written as

$$\frac{1}{R_t^{Euler}} = \beta \exp \left[\begin{array}{c} -\sigma \left(E_t \log c_{t+1} - \log c_t \right) - E_t \log \pi_{t+1} \\ +\frac{\sigma^2}{2} var_t \log c_{t+1} + \frac{1}{2} var_t \log \pi_{t+1} + \sigma cov_t \left(\log c_{t+1}, \log \pi_{t+1} \right) \end{array} \right], \quad (1)$$

where $\pi_t = P_t/P_{t-1}$. Equation (1) is used to compute the implied standard Euler rate R_t^{Euler}, where the conditional moments are estimated from a six-variable VAR, $Y_t = A_0 + A_1 Y_{t-1} + v_t$, assuming $v \sim i.i.d.N(0, \Sigma)$, $\sigma = 1$ and $\beta = .993$.[19] These parameters are chosen according to our calibration strategy (see Section 5.1). The variables included in Y_t (1966Q1-2007Q4) are log per capita real personal consumption expenditures on nondurable goods and services, log change in the deflator of this consumption measure, log price of industrial commodities, log per capita real disposable personal income, Federal Funds rate, and log per capita real non-consumption GDP.[20] Moreover, a segmented (1974Q1) time trend is included in A_0.

On the one hand, the Federal Funds rate and the Euler rate,[21] which should be identical according to standard macroeconomic models are positively correlated by 0.53 (see Table 1). On the other hand, the spread between the computed standard Euler interest rate and Federal Funds rate, $s_t^{Euler} = R_t^{Euler} - R_t^m$ is strongly negatively correlated with the Federal Funds rate (-0.82).[22] The standard deviation of the spread between the Euler and the policy rate is 2.78, compared to 1.87 for the Euler rate and 3.26 for the Federal Funds rate. Hence, the spread is less volatile than the Federal Funds rate and is much more volatile than the Euler rate; the latter relation being less pronounced than for Smets and Wouters' (2007) model, according to Chari et al. (2009).

To summarize, the spread between the Euler rate and the observed policy rate, which are equated in standard models, exhibits two properties, i.e. a systematic co-movement with monetary policy and a high relative volatility, which casts doubts on the validity and the precision of the transmission mechanism of standard New Keynesian models even when consumption habits are considered (see Atkeson and Kehoe, 2009, and Chari et al., 2009). The following analysis will show that these observations can be explained by an endogenous liquidity premium induced by monetary policy implementation.

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[19]Our results differ slightly from Canzoneri et al. (2007), who use a CRRA utility function with $\sigma = 2$.

[20]Data are from the Federal Reserve Bank of St. Louis FRED database and are released by the Federal Reserve Board, the Bureau of Economic Analysis (U.S. Department of Commerce), the Bureau of Labor Statistics (U.S. Department of Labor), and the Census Bureau (U.S. Department of Commerce).

[21]The computed Euler rate, the Federal Funds rate, and the spread between these two rates, $s_t^{Euler} = R_t^{Euler} - R_t^m$, are displayed in Appendix B.2.

[22]This accords to Atkeson and Kehoe's (2008) result of the analysis of the Euler equation error (i..e. of $-s_t = R_t^m - R_t^{Euler}$) using Smets and Wouters' (2007) model.

3 The model

In this section, we develop a macroeconomic framework which can explain the results presented in the previous section. The benchmark version of the model is held deliberately simple to facilitate comparisons with the textbook New Keynesian model (see Woodford, 2003) and to be able to derive analytical results.[23] We therefore abstract from financial intermediation and assume that households directly trade with the central bank in open market operations. One can interpret the set-up as a stylized specification of an economy with financial intermediaries which receive deposits from households, lend funds to firms (see Section 5), and hold cash to satisfy reserve requirements and/or to meet random withdrawals of deposits. This structure would lead to a model which is isomorphic to the one presented below, as long as financial frictions and costs of financial intermediation are neglected.

Households hold short-term treasuries (i.e. one-period government bonds), equity, and non-interest bearing money. Their demand for money is induced by assuming that goods market transactions cannot be conducted by using credit. To allows for a transparent specification of markets and the timing of events, we apply a basic cash-in-advance constraint (which is replaced by a money-in-the-utility-function specification in Section 5.4). We consider that the central bank supplies money only in exchange for eligible securities. Specifically, we restrict our attention to the case where only short-term treasuries are eligible in open market operations (like in Lacker, 1997), which accords to the "T-bills only" regime of the US Federal Reserve.

Throughout the paper, upper case letters denote nominal variables and lower case letters real variables. Though, agents are not heterogenous, we introduce indices for individual agents (i, j, and k) to describe individual choices in a transparent way.

3.1 Timing of events

There are infinitely many households, firms, and retailer indexed with $i \in [0,1]$, $j \in [0,1]$, and $k \in [0,1]$. A household i enters a period t with assets carried over from the previous period $t-1$: $M_{i,t-1}^H + B_{i,t-1} + V_t z_{i,t-1}$, where $M_{i,t}^H \geq 0$ denotes holdings of money, $B_{i,t} \geq 0$ one-period nominally risk-free government bonds, and $z_{i,t-1} \in [0,1]$ shares of firms valued at the price V_t. The timing of events in each period is as follows:

1. Aggregate shocks materialize, labor is supplied by households, intermediate goods are produced by firms and sold to retailers.

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[23]In Section 5, the model is augmented (e.g. by habit formation and investments in physical capital) to allow for a quantitative analysis.

2. Households and the central bank participate in open market operations. Here, money is exchanged against eligible securities via outright sales/purchases and via repurchase agreements, which are essentially "collateralized loans".[24] The relative price of money R_t^m (for both types of trades) is controlled by the central bank and will be called policy rate. Assuming that only government bonds are eligible, household i faces the following constraint:

$$I_{i,t} \leq B_{i,t-1}/R_t^m, \tag{2}$$

which will be referred to as the collateral constraint. It limits the amount of new money $I_{i,t}$ that can be acquired by household i in period t. Its bond holdings then equals $B_{i,t-1} - \Delta B_{i,t}^c$, where $\Delta B_{i,t}^c$ denotes treasuries received by the central bank.

3. The final goods market opens, where money is assumed to be the only accepted means of payment. Thus, goods market expenditures are restricted by money carried over from the previous period plus money acquired from the central bank in current period open market operations:

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H, \tag{3}$$

where $c_{i,t}$ denotes consumption purchases of the final good and P_t the price level.

4. Before the asset market opens, current labor income is paid back in cash to households, such that household i's money holdings equals $\widetilde{M}_{i,t} = I_{i,t} + M_{i,t-1}^H + P_t w_t n_{i,t} - P_t c_{i,t}$, where w_t denotes the real wage rate and $n_{i,t}$ working time. Further, it can repurchase treasuries, $B_{i,t}^R = R_t^m M_{i,t}^R$, such that its bond holdings equals $\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B_{i,t}^c + B_{i,t}^R$. Then, the asset market opens, where households trade money, equity, and treasuries at the price $1/R_t$ subject to

$$(B_{i,t}/R_t) + M_{i,t}^H + V_t z_{i,t} \leq \widetilde{B}_{i,t} + \widetilde{M}_{i,t} - R_t^m M_{i,t}^R + (V_t + P_t \varrho_t) z_{i,t-1} - P_t \tau_t + P_t \varphi_t, \tag{4}$$

where τ_t denotes lump-sum tax, ϱ_t dividends from intermediate goods producing firms, and φ_t profits from retailer. The central bank reinvests its payoffs from maturing bonds in new bonds and does not change money supply, $\int_0^1 M_{i,t}^H di = \int_0^1 (M_{i,t-1}^H + I_{i,t} - M_{i,t}^R) di$.

Since private agents are homogenous, we can abstract from a market for federal funds and identify the policy rate with the price of money in open market operations. A federal funds market would allow exchanging federal funds among heterogenous participants within the

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[24]See Fedpoint "Repurchase and Reverse Repurchase Transactions" (http://www.newyorkfed.org/ about-thefed/ fedpoint/fed04.html)

maintenance period, for example, to flexibly respond to idiosyncratic liquidity demands. When there are no further frictions, the price of federal funds charged between different participants would then be equal to the price charged by the central bank, R_t^m.

3.2 Private sector

Households Households are infinitely lived and have identical endowments and identical preferences. Household i maximizes the expected sum of a discounted stream of instantaneous utilities u :

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, n_{i,t}\right), \tag{5}$$

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility function is assumed to satisfy $u\left(c_{i,t}, n_{i,t}\right) = [(c_{i,t}^{1-\sigma} - 1)/(1 - \sigma)] - \theta n_{i,t}^{1+\sigma_n}/(1 + \sigma_n)$, where $\sigma \geq 1$, $\sigma_n \geq 0$, and $\theta > 0$.

As described above, household i faces three constraints in each period. In open market operations, it can acquire additional money $I_{i,t}$ up to the amount of government bonds carried over from the previous period $B_{i,t-1}$ discounted by R_t^m (see 2). Hence, other assets (e.g. equity) are not eligible in open market operations, which accords to the common practice of central banks, like the BoE or the US-Fed, of restricting the set of eligible securities mainly to short-term government debt. In principle, the central bank can also withdraw money from the private sector ($I_{i,t} < 0$). Here, however, we focus on the empirically relevant case where the central bank creates a "structural deficiency" when it supplies money,[25] by choosing a suited relation between money supplied under repurchase agreements and under outright sales/purchases. This implies to a sufficiently large fraction of money supplied under repurchase agreements to guarantee $I_{i,t} \geq 0$ in equilibrium (see Section 3.3).

Households are further assumed to rely on cash for transactions in the goods market (see 3). Given that they can first trade with the central bank in open market operations, the cash-in-advance constraint (3) differs from Svensson's (1985) cash-in-advance constraint by $I_{i,t}$. In the asset market, household i receives payoffs from maturing bonds and dividends. It can buy bonds from the government and invest in shares of intermediate goods producing firms $j \in [0,1]$, $z_{i,t} = \int z_{i,t}^j dj$. Substituting out the stock of bonds and money held before

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[25]This strategy has for example been applied by the US-Federal Reserve: "*To most effectively influence the level of reserve balances, the Federal Reserve has created what is called a 'structural deficiency'. That is, it has created permanent additions to the supply of reserve balances that are somewhat less than the total need. Then on a seasonal and daily basis, the Desk is in a position to add balances temporarily to get to the desired level.*" (see "Fedpoint: Open Market Operations", http://www.newyorkfed.org/aboutthefed/fedpoint/fed32.html).

the asset market opens, $\widetilde{B}_{i,t}$ and $\widetilde{M}_{i,t}$, in (4), household $i's$ asset market constraint reads

$$(B_{i,t}/R_t) + M_{i,t}^H + (R_t^m - 1) I_{i,t} + V_t z_{i,t} \tag{6}$$
$$\leq B_{i,t-1} + M_{i,t-1}^H + (V_t + P_t \varrho_t) z_{i,t-1} + P_t w_t n_{i,t} - P_t c_{i,t} - P_t \tau_t + P_t \varphi_t.$$

while its $i's$ borrowing is restricted by $z_{i,t} \geq 0$, by $M_{i,t}^H \geq 0$, and $B_{i,t} \geq 0 \ \forall t \geq 0$. The term $(R_t^m - 1) I_{i,t}$ measures the costs of money acquired in open market operations, because it receives money $I_{i,t}$ in exchange for $R_t^m I_{i,t}$ treasuries.

Maximizing (5) subject to the collateral constraint (2), the goods market constraint (3), the asset market constraints (6) and the borrowing constraints, for given initial values $M_{i,-1}^H$, $B_{i,-1}$, and $z_{i,-1}$ leads to the following first order conditions for working time, consumption, additional money, as well as for holdings of government bonds, money, and equity:

$$-u_{i,nt}/w_t = \lambda_{i,t}, \tag{7}$$
$$u_{i,ct} = \lambda_{i,t} + \psi_{i,t}, \tag{8}$$
$$R_t^m \left(\lambda_{i,t} + \eta_{i,t}\right) = \lambda_{i,t} + \psi_{i,t}, \tag{9}$$
$$\beta E_t \left[\left(\lambda_{i,t+1} + \eta_{i,t+1}\right) \pi_{t+1}^{-1}\right] = \lambda_{i,t}/R_t, \tag{10}$$
$$\beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right) \pi_{t+1}^{-1}\right] = \lambda_{i,t}, \tag{11}$$
$$\beta E_t \left[\lambda_{i,t+1} R_{t+1}^q \pi_{t+1}^{-1}\right] = \lambda_{i,t}, \tag{12}$$

where $R_t^q = (V_t + P_t \varrho_t)/V_{t-1}$ is the nominal rate of return on equity, and $\eta_{i,t}$, $\lambda_{i,t}$ and $\psi_{i,t}$ denote the multiplier on the collateral, asset market, and goods market constraint. Finally, the following complementary slackness conditions hold in the household's optimum $i.)$ $0 \leq b_{i,t-1}\pi_t^{-1} - R_t^m i_{i,t}$, $\eta_{i,t} \geq 0$, $\eta_{i,t} \left(b_{i,t-1}\pi_t^{-1} - R_t^m i_{i,t}\right) = 0$, and $ii.)$ $0 \leq i_{i,t} + m_{i,t-1}^H \pi_t^{-1} - c_{i,t}$, $\psi_{i,t} \geq 0$, $\psi_{i,t} \left(i_{i,t} + m_{i,t-1}^H \pi_t^{-1} - c_{i,t}\right) = 0$, where $m_{i,t}^H = M_{i,t}^H/P_t$, $b_{i,t} = B_{i,t}/P_t$, and $i_{i,t} = I_{i,t}/P_t$, as well as (6) with equality and associated transversality conditions.

The conditions (7) and (8) show that $\lambda_{i,t} > 0$ and that a binding goods market constraint, $\psi_{i,t} > 0$, distorts the intratemporal consumption-leisure decision in a conventional way.[26] Condition (10) shows that a rise in the multiplier $\eta_{i,t}$, which measures the liquidity value of treasuries, tends to lower the interest rate on treasuries R_t. Hence, a positive liquidity value of treasuries $\eta_{i,t} > 0$ gives rise to a liquidity premium, between the treasury interest and the rate of return on equity as can be seen from combining (10) and (12)

$$R_t E_t \left[\left(\lambda_{i,t+1} + \eta_{i,t+1}\right) \pi_{t+1}^{-1}\right] = E_t \left[R_{t+1}^{eq} \cdot \lambda_{i,t+1} \pi_{t+1}^{-1}\right]. \tag{13}$$

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[26]Combining (7) and (8) with (11), discloses the standard inflation tax: $\beta E_t \left[u_{i,ct+1}/\pi_{t+1}\right] = -u_{i,nt}/w_t$.

The household's investment decisions further relate the treasury rate to the policy rate. It is willing to hold both assets, money and treasuries, if the rate of return on treasuries compensates for the costs of acquiring new money in the next period. This can be seen by combining (7), (9), (10), and (11) to

$$1/R_t = \frac{E_t\left[\left(1/R_{t+1}^m\right)\left(u_{i,ct+1}/\pi_{t+1}\right)\right]}{E_t\left[\left(u_{i,ct+1}/\pi_{t+1}\right)\right]}, \tag{14}$$

implying that the interest rate on treasuries equals the expected future policy rate up to first order in accordance with Simon's (1990) evidence. Households are then willing to hold both types of money, i.e. money held under repos $M_{i,t}^R$ and outright $M_{i,t}^H$.

Firms There are perfectly competitive intermediate goods producing firms, which sell their goods to monopolistically competitive retailers. The latter sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology.

There is a continuum of intermediate goods producing firms indexed with $j \in [0,1]$. They are perfectly competitive, owned by the households, and produce an identical intermediate good with labor. Production depends on a stochastic productivity levels, which materialize after the labor market closes. Firm j produces according to the production function

$$IO_{j,t} = a_t n_{j,t}^{\alpha}, \ \ \alpha \in (0,1), \tag{15}$$

and sells the intermediate good to retailers at the price P_t^m. The productivity level a_t is generated by a stochastic process satisfying $a_t = a_{t-1}^{\rho_a} \exp \varepsilon_{a,t}$, where $\rho_a \geq 0$, and $\varepsilon_{a,t}'s$ are normally and i.i.d. distributed with $E_{t-1}\varepsilon_{a,t} = 0$ and a constant standard deviation $st.dev.(\varepsilon_a) \geq 0$. Labor demand then satisfies

$$w_t = mc_t a_t \alpha n_{j,t}^{\alpha-1}, \tag{16}$$

where $mc_t = P_t^m/P_t$ denotes real marginal costs, and dividends are given by $P_t \varrho_{j,t} = P_t^m(1-\alpha)a_t n_{j,t}^{\alpha}$. Given that firms face the same prices, they behave in an identical way.

Monopolistically competitive retailers buy intermediate goods $IO_t = \int_0^1 IO_{j,t}dj$ at the common price P_t^m. A retailer $k \in [0,1]$ relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive bundlers, who bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk$, where $\varepsilon > 1$. The cost minimizing demand for $y_{k,t}$ is therefore given by $y_{k,t} = (P_{k,t}/P_t)^{-\varepsilon} y_t$. Retailers set their prices to maximize profits, where we consider a nominal rigidity in form of staggered price setting as in Yun (1995). Each period a measure $1-\phi$ of randomly selected retailers may reset

their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0, 1)$ of retailers do not adjust their prices. The fraction $1-\phi$ of retailers set their price to maximize the expected sum of discounted future profits, $\max_{\widetilde{P}_{k,t}} E_t \sum_{s=0}^{\infty} \phi^s \varphi_{t,t+s}(\widetilde{P}_{k,t}y_{k,t+s} - P_{t+s}mc_{t+s}y_{k,t+s})$, s.t. $y_{k,t+s} = \widetilde{P}_{k,t}^{-\epsilon}P_{t+s}^{\epsilon}y_{t+s}$. The first order condition for their price $\widetilde{P}_{k,t}$ is given by (where we use that P_t^m/P_t are real marginal cost, mc_t) $\tilde{Z}_t = \frac{\varepsilon}{\varepsilon-1}Z_t^1/Z_t^2$, where $\tilde{Z}_t = \widetilde{P}_{k,t}/P_t$, $Z_t^1 = c_t^{-\sigma}y_tmc_t + \phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{t+1}^1$ and $Z_t^2 = c_t^{-\sigma}y_t + \phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{t+1}^2$. With perfectly competitive bundlers and the homogenous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon}dk$. Using the demand constraint, we obtain $1 = (1-\phi)\tilde{Z}_t^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$.

Aggregate intermediate output is then given by $IO_t = n_t^{\alpha}$, where $n_t = \int_0^1 n_{j,t}dj$, while price dispersion across retailers affects aggregate output. Specifically, the market clearing condition in the intermediate goods market, $IO_t = \int_0^1 y_{k,t}dk$, gives $n_t^{\alpha} = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} y_tdk \Leftrightarrow y_t = n_t^{\alpha}/s_t$, where $s_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ and $s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}$ given s_{-1}.

3.3 Public sector

Fiscal authority The government issues one-period nominally risk-free bonds B_t^T, which are either held by households or the central bank. Throughout, we also refer to these bonds as *T-bills* to emphasize that B_t^T consists of short-term treasuries that typically serve as collateral in open market operations. To minimize interactions between fiscal policy and monetary policies, which are beyond the scope of the analysis, we assume that the supply of government bonds is exogenously determined. Specifically, we consider a simple bond supply regime that keeps the growth rate of T-bills constant,

$$B_t^T = \Gamma B_{t-1}^T, \tag{17}$$

where $\Gamma > \beta$. As mentioned above, (17) describes the supply of treasury securities that are declared as eligible by the central bank, and is not aimed at modelling the evolution of total public debt. The latter usually also contains debt with longer maturity that might grow with a rate different from Γ, which will not be modelled here to keep the exposition simple. In order to avoid any further effects of fiscal policy, we assume that the government can raise tax revenues in a non-distortionary way, $P_t\tau_t$. Accounting for transfers $P_t\tau_t^m$ from the central bank, the government budget constraint is given by

$$\left(B_t^T/R_t\right) + P_t\tau_t^m + P_t\tau_t = B_{t-1}^T.$$

As long as bonds with longer maturities are not eligible, they can be neglected without any consequences for the analysis of monetary policy effects. In fact, all qualitative results derived in this paper will not be affected either if we add non-eligible government bonds with longer maturity or if we assume that only few of them are eligible.

Central bank The central bank supplies money in exchange for treasuries in open market operations in form of outright sales/purchases M_t^H and repurchase agreements M_t^R. At the beginning of each period, the central bank's stock of treasuries equals B_{t-1}^c and the stock of outstanding money equals M_{t-1}^H. It then receives an amount ΔB_t^c of treasuries in exchange for money I_t, and after repurchase agreements are settled its holdings of treasuries reduces by B_t^R and the amount of outstanding money by $M_t^R = B_t^R$. Before the asset market opens, where the central bank can invest in new T-bills B_t^c, it holds an amount equal to $\widetilde{B}_t^c = \Delta B_t^c + B_{t-1}^c - B_t^R$. Its budget constraint is given by

$$(B_t^c/R_t) + P_t \tau_t^m = \Delta B_t^c + B_{t-1}^c - B_t^R + M_t^H - M_{t-1}^H - \left(I_t - M_t^R\right).$$

In accordance with the operational practice of central banks, we assume that it rolls over its maturing assets (see e.g. Meulendyke, 1998, Ch.7). For this, the central bank enters the asset market at the end of each period and reinvests in treasuries to the amount that equals its current stock of maturing assets $B_t^c = \widetilde{B}_t^c$. Further using $B_t^R = M_t^R$ and $\Delta B_t^c = R_t^m I_t$, the budget constraint can be simplified to $(B_t^c/R_t) - B_{t-1}^c = R_t^m \left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1) M_t^R - P_t \tau_t^m$. Following common central bank practice, we assume that the central bank transfers interest earnings to the government:

$$P_t \tau_t^m = B_t^c \left(1 - 1/R_t\right) + (R_t^m - 1) M_t^R. \tag{18}$$

Note that these transfers will not be negative in equilibrium, such that the central bank will never rely on funds from the government.[27] Accordingly, its holdings of treasuries will evolve according to

$$B_t^c - B_{t-1}^c = R_t^m \left(M_t^H - M_{t-1}^H\right). \tag{19}$$

implying that it tends to hold more treasuries, when money supply or the policy rate are high. Regarding the implementation of monetary policy, we assume that the central bank conducts monetary policy by using a standard feedback rule for the current policy rate R_t^m

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[27]This is different in standard models, where central bank transfers seigniorage (defined as the change in the monetary base) to the government in each period. A discussion of government transfers and central bank independence can be found in Sims (2003).

and by choosing an average policy rate $R^m > 1$:

$$R_t^m = \left(R_{t-1}^m\right)^{\rho_R} (R^m)^{1-\rho_R} (\pi_t/\pi)^{\rho_\pi (1-\rho_R)} [(y_t/y)/(\widetilde{y}_t/\widetilde{y})]^{\rho_y(1-\rho_R)} \exp \varepsilon_{r,t}, \qquad (20)$$

where $R^m > 1$, $\rho_R \geq 0$, $\rho_\pi \geq 0$, and $\rho_y \geq 0$, $\widetilde{y}_t$ denotes first-best output, which is given by $\widetilde{y}_t = a_t^{\frac{1+\sigma_n}{1+\sigma_n+\alpha(\sigma-1)}} (\alpha/\theta)^{\frac{\alpha}{1+\sigma_n+\alpha(\sigma-1)}}$, and the $\varepsilon'_{r,t}s$ are normally and i.i.d. with $E_{t-1}\varepsilon_{r,t} = 0$. The central bank further sets an inflation target, which is consistent with the long-run inflation rate and satisfies $\pi > \beta$. To avoid further complexities, we will assume that the growth rate of T-bills Γ equals the central bank's inflation target, $\Gamma = \pi$, which actually accords to the estimated growth rate of T-bills (corrected by GDP growth) for the sample period 1966-2007 (see Section 5.1). The model can, however, easily be extended to allow for $\Gamma \neq \pi$ (see Schabert, 2012).[28]

The central bank can further decide on whether money is traded in form of outright sales/purchases or in form of repurchase agreements. For simplicity, we assume that it exogenously sets the ratio of money supply under both types of open market operations $\Omega : M_t^R = \Omega \cdot M_t^H$. We assume that $\Omega > 0$ in accordance with the practice of the US Fed. In fact, the Trading Desk of the New York Fed "structures its outright holdings to maintain a need to routinely add to balances by arranging repurchase agreements" (see Federal Reserve Bank of New York, 2006). The choice of Ω does not affect the pattern but only the size of monetary policy effects, i.e. the size of the responses to an innovation to the policy rate.[29]

3.4 Rational expectations equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{jt}dj = \int_0^1 n_{it}di$ and $y_t = \int_0^1 y_{jt}dj = \int_0^1 c_{it}di = c_t$. Aggregate asset holdings satisfy $\forall t \geq 0 : \int z_{i,t}di = 1$, $B_t = \int B_{i,t}di$, $M_t^R = \int_0^1 M_{i,t}^R di$, $M_t^H = \int_0^1 M_{i,t}^H di$, $\int I_{i,t}di = I_t = M_t^H - M_{t-1}^H + M_t^R$, and $B_t^T = B_t + B_t^c$. Given that households (firms) behave in an identical way, we will omit indices in the subsequent analysis. In a rational expectations (REE) all plans and constraints of households and firms are satisfied and consistent with monetary and fiscal policy, for given initial endowments (see definition 1 in Appendix A.1).

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[28]If the central bank would adjust the amount of eligible treasuries, it can chose an inflation target that differs from Γ. When, for example, the central bank chooses for a smaller inflation target $\pi < \Gamma$, it might accept smaller fractions of treasuries in open market operations. Otherwise, for $\pi > \Gamma$, it might also declare other assets (or a fraction of them) as eligible, which grow with a rate that exceeds Γ.

[29]In fact, a higher ratio of money supplied under repurchase agreements relative to money supplied outright increases the effectiveness of changes in the policy rate, which provides a rationale why central banks create a "structural deficiency" with respect to the outright supply of money, like the Fed (see "Fedpoint: Open Market Operations", http://www.newyorkfed.org/aboutthefed/fedpoint/fed32.html).

The main difference to a standard New Keynesian model is the existence of the collateral constraint in open market operations (2), which restricts households' access to money. The model reduces to a conventional sticky price model if the collateral constraint,

$$M_t^H - M_{t-1}^H + M_t^R \leq B_{t-1}/R_t^m, \tag{21}$$

is slack, i.e. if the multiplier η_t equals zero (see definition 2 in Appendix A.1). In this case, there is no liquidity premium on eligible securities, such that the expected equity return equals the treasury rate up to first order (see 13). Throughout the subsequent analysis, we are interested in the case where the collateral constraint (21) is binding. To see when this is the case, eliminate $\lambda_{i,t}$ and $\psi_{i,t}$ in (9) by (8) and (11), which leads to $\frac{\eta_{i,t}}{u_{i,ct}} = \frac{1}{R_t^m} - \beta E_t \frac{u_{i,ct+1}}{u_{i,ct}\pi_{t+1}}$ that can – in equilibrium – be written as

$$\eta_t/u_{c,t} = (1/R_t^m) - (1/R_t^{Euler}) \geq 0, \tag{22}$$

where R_t^{Euler} is the standard Euler rate defined as $1/R_t^{Euler} = \beta E_t \frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}$ (as in Section 2). As it is well-known, the nominal Euler rate measures the marginal valuation of money. Agents are willing to spend a price $R_t^{Euler} - 1$ to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today. Hence, if the central bank supplies money at a lower price $R_t^m < R_t^{Euler}$, households earn a positive rent and are willing to get the maximum amount of money they can get. Given that this amount is restricted by holdings of eligible assets, the collateral constraint (21) will then be binding, indicating a positive liquidity value of treasuries, $\eta_t > 0$. A binding collateral constraint (21), which relies on a positive valuation of liquidity, implies the cash constraint (3) to be binding as well, $\psi_t > 0$.[30]

If the central bank would supply money at the rate R_t^m in an unrestricted way (e.g. by accepting securities that can be issued by private sector agents in an unbounded way), then households will adjust their consumption pattern until their marginal valuation of money equals the price, i.e. $R_t^{Euler} = R_t^m$. This accords to the case typically considered in standard macroeconomic models, where the policy rate affects aggregate demand via the consumption Euler equation and there is no liquidity premium on treasuries (see Appendix A.1).

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[30]To see this, combine (8) and (11), which leads to $c_{i,t}^{-\sigma} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{\pi_{t+1}} + \psi_{i,t}$. Hence, the multiplier ψ_t on the goods market constraint (3) satisfies $\psi_t/u_{c,t} = 1 - (1/R_t^{Euler}) \geq 0$ in equilibrium, which obviously implies $\psi_t > 0$ if $1 \leq R_t^m < R_t^{Euler}$.

4   Analytical results

In this Section, we analytically examine some main properties of the model. In particular, we show how the liquidity premium is affected by the bills-to-gdp ratio and by monetary policy. For this, we apply several parameter values that simplify the model. We assume that utility is logarithmic in consumption ($\sigma = 1$) and that money is only supplied via repos ($\Omega \rightarrow \infty$). The latter assumption implies that T-bills are only held by the private sector, $B_t = B_t^T$. We further assume that the central bank sets the policy rate exogenously, i.e. $\rho_y = \rho_\pi = 0$ (see 20), and the inflation target at $\pi = 1$. It should be noted that the equilibrium is uniquely determined under an exogenous policy rate.[31] Appendix A.2 provides an equilibrium determinacy analysis, which shows that the model exhibits equilibrium determinacy under a large set of reasonable feedback coefficients (see lemma 1). For the quantitative analysis in Section 5, we consider a more realistic monetary policy and apply an endogenous policy rate rule, which satisfies the Taylor-principle.

Euler rate versus policy rate   We first examine how the spread between the Euler rate R_t^{Euler} and the policy rate R_t^m is related to monetary policy and the bills-to-gdp ratio. Throughout the following analysis, we restrict our attention to the case where the central bank sets the policy rate below the equilibrium Euler rate. As implied by (22), the collateral constraint (21) is binding in this case. For perfectly flexible prices, $\phi = 0$, it can be shown that this can be guaranteed by the central bank if it sets the policy rate in a way that its expected value $E_t R_{t+1}^m$ is below the long-run Euler rate, which equals π/β as usual. The spread Euler $s_t^{Euler} = R_t^{Euler} - R_t^m$, which proxies the liquidity premium on treasuries as the treasury rate, closely follows the expected policy rate (see 14), can further be shown to be negatively related the expected policy rate and to the expected bills-to-gdp ratio. These properties are summarized in the following proposition.

Proposition 1 *Consider the case of flexible prices, $\phi = 0$, with $\Omega \rightarrow \infty$, $\sigma = 1$, and $\Gamma = 1$. If the central bank sets the policy rate and the inflation target according to $E_t R_{t+1}^m < \pi/\beta$ and $\pi > \beta$, the collateral constraint (21) is binding in equilibrium. The Euler spread s_t^{Euler} is then negatively related to the expected policy rate and to the expected bills-to-gdp ratio.*

Proof. See Appendix A.2.   ∎

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[31]The reason why local equilibrium determinacy does not depend on the Taylor-principle is that the stock of eligible securities serves as a nominal anchor (like under a money growth policy). This closely relates to the determinacy property of Adao et al.'s (2003) cash-in-advance model with sticky prices, where both, the nominal interest rate and the supply of money, are controlled by the central bank at the same time.

By setting the policy rate, the central bank decides on whether the collateral constraint is binding or not. Under a binding collateral constraint, the supply of money is bounded and the households' consumption choice is restricted by the available amount of collateral. The associated optimal consumption growth rate as well as the expected inflation rate determine the willingness to pay for an extra unit of money, i.e. the Euler rate. The central bank can set the price of money in open market operations below the Euler rate to support this equilibrium. If, in contrast, it sets the policy rate equal to the Euler rate $R_t^m = R_t^{Euler}$, there is no liquidity premium, since the price of money equals the households' marginal willingness to pay for cash, implying that the collateral constraint is slack.

Proposition 1 further shows that the ratio of the Euler rate to the policy rate is negatively related to the expected policy rate and the expected bills-to-gdp ratio in the simplified model. Here, a binding collateral constraint, $m_t^R = b_t/R_t^m$ and a binding cash constraint, $y_t = m_t^R$, directly equate the policy rate and the bills-to-gdp rate. Hence, both negative correlations are due to the property that the Euler rate moves less than one for one with the policy rate. In the subsequent analysis, we will show that the model generates negative correlations of the liquidity premium with the bills-to-gdp ratio and the policy rate which are consistent with the empirical evidence provided in Section 2.

Monetary policy effects   Under a binding collateral constraint, the model's monetary transmission mechanism differs from the well known logic of New Keynesian models, since the (real) policy rate does not directly govern consumption growth according to the standard Euler equation. For a given beginning-of-period nominal stock of eligible assets, a higher policy rate (i.e. an increase in the price of money in terms of eligible assets) tends to reduce the amount of money that can be acquired via repos. Hence, an increase in the policy rate exerts a contractionary effect on nominal expenditures by making money more expensive.

To examine the monetary policy effects, we consider both cases of flexible and sticky prices separately. For the flexible price case, we extend the analysis on which Proposition 1 is based upon. For the sticky price case, we apply a local approximation to the model at a steady state with a binding collateral constraint, which requires the central bank to set its policy rate target R^m at a value smaller than π/β (see also Proposition 1). We then solve the set of equilibrium conditions, which are log-linearized at this steady state. To derive the unique solution under sticky prices, we apply the local determinacy conditions of the model. This analysis, which is summarized in Lemma 1 in Appendix A.2, shows that an exogenous policy rate, $\rho_\pi = \rho_y = 0$, is associated with equilibrium determinacy. For both cases, we restrict changes in the policy rate to be sufficiently small such that $E_t R_{t+1}^m < \pi/\beta$. The

following proposition summarizes the effects of monetary policy for flexible prices, $\phi = 0$, and sticky prices, $\phi > 0$.

Proposition 2 *Consider a simplified version of the model with $\phi \geq 0$, $\Omega \to \infty$, $\sigma = 1$, and a monetary policy satisfying (20) with $\rho_\pi = \rho_y = 0$, $\rho_R > 1/2$, $E_t R_{t+1}^m < \pi/\beta$, and $\pi = 1$.*

1. *A rise in the policy rate leads on impact to a fall in output and inflation and to a rise in the Euler rate.*

2. *The spread s_t^{Euler} decreases with the policy rate, is negatively related to the bills-to-gdp ratio, and is more volatile than the (net) Euler rate.*

Proof. See Appendix A.2. ∎

As summarized by Proposition 2, the qualitative properties of monetary policy effects accord to conventional expectations (e.g. a higher policy rate lowers output and inflation) and do not depend on the degree of price flexibility. In both cases, the Euler rate increases with the policy rate by less than one for one such that the spread s_t^{Euler} decreases in response to a policy rate increase. The simple reason is that the liquidity value of treasuries is immediately reduced when the policy rate rises, since the price of money in terms of treasuries in open market operations increases (which requires more treasuries for a particular amount of money). At the same time, the willingness to transfer means of payment to the future falls, implying a rise in the Euler rate. Given that the increase in the latter is less pronounced than the rise in the policy rate, the spread between both rates falls.

Hence, the liquidity premium on treasuries, which originates in their convertibility into means of payments in open market operations, falls if the price of money in terms of treasuries R_t^m increases. Likewise, the liquidity premium falls when more eligible assets are available, i.e. when the bills-to-gdp ratio b_t/y_t increases. Given that money is only supplied via repos, all T-bills are held the private sector, such that $b_t/y_t = b_t^T/y_t$. This will not be the case in the calibrated version, which will be examined in the subsequent section. To summarize, the spread between the Euler rate and the policy rate is negatively related to the policy rate itself and to the bills-to-gdp ratio, which is consistent with the empirical evidence provided in Section 2. Moreover, the observation that the Euler spread s_t^{Euler} is more volatile than the (net) Euler rate $R_t^{Euler} - 1$ (see Section 2.2 or Chari et al., 2009) is also implied by the model (see part 2 of Proposition 2).[32]

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[32]For the case of flexible prices it can further be shown that a higher variance of the policy rate reduces the liquidity premium as well (see proof of proposition 2), since the liquidity value of bonds for open market operations becomes more uncertain. Put differently, when the costs associated with the liquidation of bonds get more uncertain, the compensating interest rate increases. This effect accords to the idea of a liquidity *risk* premium (see also Acharya and Pedersen, 2005).

5 Quantitative analysis

In this Section, we present a quantitative analysis of a calibrated version of the model. In the first part of this Section, we describe how the model is extended and calibrated. The moments of simulated series are compared with the corresponding empirical moments of Section 2, which do not serve as targets for the model calibration. We then demonstrate that the model is able to generate macroeconomic effects of monetary policy shocks, which are consistent with broad empirical (VAR) evidence. In the last part of this Section, we assess the robustness of the results by applying an alternative money demand specification.

5.1 Model extension and calibration

For the quantitative analysis of the model, we follow the literature on quantitative New Keynesian models (see, e.g., Smets and Wouters, 2007) and allow for habit formation as well as for accumulation of productive physical capital with investment adjustment costs. Specifically, we allow for external habits by assuming that household utility is given by: $u(c_{i,t}, n_{i,t}) = [(c_{i,t} - h \cdot c_{t-1})^{1-\sigma}/(1-\sigma)^{-1}] - \theta n_{i,t}^{1+\sigma_n}/(1+\sigma_n)$, where $h \geq 0$. We further assume that the stock of capital k_t contributes to the production of intermediate goods according to a neoclassical production function, $IO_{j,t} = a_t n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha}$, where $\alpha \in (0,1)$, which replaces (15). The accumulation of physical capital is associated with adjustment costs: $k_{j,t} = (1-\delta) k_{j,t-1} + x_{j,t} G(x_{j,t}/x_{j,t-1})$, where $x_{j,t}$ denotes investment expenditures and investment adjustment costs are $G(x_{j,t}/x_{j,t-1}) = 1 - \xi \frac{1}{2} (x_{j,t}/x_{j,t-1} - 1)^2$ with $G(1) = G'(1) = 0$ and $G''(1) = \xi > 0$. Intermediate goods producing firms are – like households – assumed to rely on cash $L_{j,t}$ for goods market purchases. They borrow cash from households at the price $1/R_t^L$ and pay back the loan after goods are sold, such that firm j faces the constraint $L_{j,t}/R_t^L \geq P_t x_{j,t}$. Household i's goods market constraint (3) then changes to $P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H - L_{i,t}/R_t^L$, implying that the demanded loan rate equals the Euler rate, $R_t^L = R_t^{Euler}$.[33] The problem of a firm j can then be summarized as $\max E_t \sum_{k=0}^{\infty} \beta^k \frac{u_{c,t+k}}{u_{c,t}} \varrho_{j,t+k}$, where real dividends are now given by $\varrho_{j,t} = (Z_t/P_t)a_t n_{j,t}^{\alpha} k_{t-1}^{1-\alpha} - w_t n_{j,t} - x_{j,t} - (1-1/R_t^L)L_{j,t}/P_t$, subject to capital accumulation and the firm's cash constraint. The former gives rise to the price q_t of physical capital in terms of the final good and the latter distorts the investment decision if $R_t^L > 1$. The full set of equilibrium conditions can be found in Appendix A.3.

For most of the parameters we apply standard values, which accord to an interpretation

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[33]Using that its budget constraint (6) further contains interest earnings from intraperiod loans $(1 - 1/R_t^L)L_{i,t}$, the first order condition for the supply of loans is $\lambda_{i,t} = (\lambda_{i,t} + \psi_{i,t})/R_t^L$, which can be combined with (8) and (11) to get $1/R_t^L = \beta E_t \frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}$.

of a period as a quarter (see Appendix B.1 for an overview). We adopt Christiano et al.'s (2005) choice of non-estimated parameters and set the inverses of the elasticities of intertemporal substitution at $\sigma = 1$ and $\sigma_n = 1$, the labor income share at $\alpha = 2/3$, and the depreciation rate at $\delta = 0.025$. For the fraction of non-optimally price adjusting firms ϕ, and the elasticity of substitution ϵ we chose the values $\phi = 0.8$ and $\epsilon = 13$, and the utility parameter θ is chosen to lead to a steady state working time of $n = 1/3$. While the investment adjustment cost parameter ξ is typically identified by model estimates based on aggregate data (see e.g. Christiano et al., 2005, or Smets and Wouters, 2007), we apply a benchmark value of $\xi = 0.065$ that accords to Groth and Khan's (2010) estimates based on disaggregate data, which is much smaller than typical estimates based on aggregate data (e.g. $\xi = 2.48$, see Christiano et al., 2005). For a sensitivity analysis, we vary the values of the cost parameter ξ and of the habit parameter h, for which we apply a benchmark value of 0.7 (see Smets and Wouters, 2007).

For the policy rate, which is identified with the Federal Funds rate, we set the average value R^m equal to the sample mean of the Federal Funds rate for the sample 1966-2007: $R^m = 1.065^{1/4}$. The inflation target is set equal to the mean inflation rate of the same sample period, $\pi = 1.046^{1/4}$.[34] The central bank sets the policy rate according to the feedback rule (20), where the output gap is measured by deviations from the first best value $\widetilde{y}_t$ (see Appendix A.3). For the policy rule coefficients we apply Mehra and Minton's (2007) estimates, which accord to standard values: $\rho_R = 0.73$, $\rho_\pi = 1.5$, $\rho_y = 0.78^{1/4}$, and $sd(\varepsilon_R) = 0.003$. For choice of the discount factor β, we can further use that the model's predictions can be compared to observable spreads, like the corporate bond yield spread $R_t^L - R_t$ or the spread between the rate of return on equity and the treasury rate $R_t^q - R_t$. We decided to set β at an intermediate value, $\beta = 0.993$, which implies that the steady state spread $R^L - R$ equals 0.0025 (where we used $R^L = R^{Euler} = \pi/\beta$ and $R = R^m$), or 100 basis points for annualized rates, which accords to the corporate bond yield spread in Krishnamurthy and Vissing-Jorgensen (2012).[35] It further implies an equity premium $E_0(R_t^q - R_t)$ of 2.32% per annum (based on a second-order approximation of the model), which is 2-3 times smaller than typically estimated for US data. We estimate the growth rate Γ of T-bills (see 17) using data for the total stock of T-bills for 1966-2007 from the U.S. Treasury. The estimated value equals $\Gamma = 7.2\%$, which almost exactly equals the growth rate of nominal GDP, implying that we can set Γ equal to the inflation target, $\Gamma = \pi$. We

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[34]Data for the Federal Funds rate and the inflation rate are taken from FRED database.

[35]The paper-bill spread applied in Section 2, which has been choosen for the empirical analysis to isolate a pure liquidity premium, is in fact smaller.

further set the policy parameter Ω equal to 12 to match the maximum output response to monetary policy shocks (i.e. an increase in output by 0.25% in deviations from its steady state value in response to an increase of the policy rate by 30 $b.p.$) as identified in the VAR of Section 2.2. The autocorrelation coefficient of the AR1-process for total factor productivity (TFP) is set equal to 0.8, while the standard deviation of its innovations ε_a is calibrated to match the observed standard deviation of the hp-filtered GDP series for 1966-2007, i.e. $st.dev.((y_t - y)/y) = 1.53$, leading to $sd(\varepsilon_a) = 0.0173$.

5.2 Selected moments

In this Section, we examine selected moments of interest rates and spreads, which correspond to the moments presented in Table 1 in Section 2. Specifically, we consider the liquidity premium $s_t = R_t^{Euler} - R_t^m$, which has been estimated in Section 2.1, and the spread between corporate and government bonds $s_t^{Treas} = R_t^L - R_t$, whose empirical counterpart has been discussed in Section 2.2. Note that the latter spread s_t^{Treas} closely relates to s_t^{Euler}, since the borrowing rate of firms R_t^L equals the Euler rate, $R_t^L = R_t^{Euler}$, and the treasury rate R_t equals the expected policy rate up to first order (see 14): $R_t \approx E_t R_{t+1}^m$, which accords to empirical evidence provided by Simon (1990). Table 2 presents correlations between interest rates and the spreads with the bills-to-gdp ratio and with the policy rate, and relative standard deviations. To facilitate comparisons with the empirical results in section 2, we focus on the model version without habits ($h = 0$). The first column presents the empirical moments (see Table 1), for convenience. The following three columns refer to a model specification without habits ($h = 0$), the next column presents moments of simulated series with external habits ($h = 0.7$), and the last column refers to an alternative model version (see Section 5.4).

The correlation between the treasury spread s_t^{Treas} and the ratio of T-bills (held by private agents) to gdp b_{t-1}/y_t that is computed from simulated series exhibits the same sign as its empirical counterpart. The correlation for the version with habits comes closest but is still substantially larger than found empirically, which is mainly due to the simple cash-in-advance specification of money demand (see section 5.4).[36] The correlation between the treasury rate and the policy rate almost equals unity, as in the data. The correlations between the Euler rate and the policy rate $corr\left(R_t^{Euler}, R_t^m\right)$ as well as the correlation between the Euler spread and the policy rate $corr(s_t^{Euler}, R_t^m)$ exhibit the same signs and similar magnitudes as their empirical counterparts. The former, $corr\left(R_t^{Euler}, R_t^m\right)$,

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[36]These correlations are slighly smaller when private sector holdings of treasuries B_t instead of their total stock B_t^T are taken into account.

Table 2: Unconditional moments of selected series

| | Data | W/o Habits h=0 | | | Habits h=0.7 | MIU |
		PR & TFP	PR	TFP	PR & TFP	PR & TFP
$corr\left(s_t^{Treas}, b_{t-1}/y_t\right)$	-0.62	-0.92	-0.90	-0.99	-0.86	-0.36
$corr\left(R_t, R_t^m\right)$	0.99	0.98	0.99	0.98	0.98	0.99
$corr\left(R_t^{Euler}, R_t^m\right)$	0.53	0.71	0.58	0.79	0.66	0.68
$corr(s_t^{Euler}, R_t^m)$	-0.82	-0.95	-0.99	-0.52	-0.90	-0.99
$sd(s_t^{Euler})/sd(R_t^m)$	0.85	0.79	0.95	0.71	0.78	0.93
$sd(s_t^{Euler})/sd(R_t^{Euler})$	1.49	2.19	7.6	1.68	1.75	8.6

Note: Standard deviations refer to net interest rates and the abbreviations PR, TFP, and MIU denote policy rate shocks, technology shocks, and a money-in-the-utility-function versions with habits.

is overestimated by the model, which is mainly driven by technology shocks. The latter, $corr(s_t^{Euler}, R_t^m)$, is highly negative when only interest rate shocks are considered, and it is less pronounced when only technology shocks are considered. Under both types of shocks, it is still somewhat larger than its empirical counterpart. Regarding the relative volatilities, Table 2 shows that the Euler spread s_t^{Euler} exhibits a smaller standard deviation than the policy rate, whereas it is more volatile than the Euler rate. The magnitudes of the relative standard deviations are similar to their empirical counterparts when both shocks are present, while the spread is much more volatile than the Euler rate when only policy shocks are considered. Overall, the model is able to reasonably replicate the empirical moments of interest rates and spreads. Notably, the moments of simulated series are not substantially altered when habits are taken into account.

5.3 Monetary transmission

In this Section, we show that responses to policy rate shocks accord to broad empirical evidence,[37] even though the transmission mechanism differs from monetary transmission in standard New Keynesian models, where the real policy rate equals the rate of intertemporal substitution. Figure 1 presents the impulse responses of interest rates and macroeconomic aggregates to a one standard deviation innovation to the policy rate, $\varepsilon_{r,t} > 0$ (see 20). Note that interest rates, the inflation rate, and the bills-to-gdp ratio are presented as absolute deviations from their steady state values, while output, consumption, and investments are presented in percentage deviations from their steady state values, e.g. $\widehat{y}_t = 100 * (y_t - y)/y$. The black solid line shows the impulse responses of the version of the model with habits,

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[37]Impulse responses a technology shocks can be found in Appendix B.2.
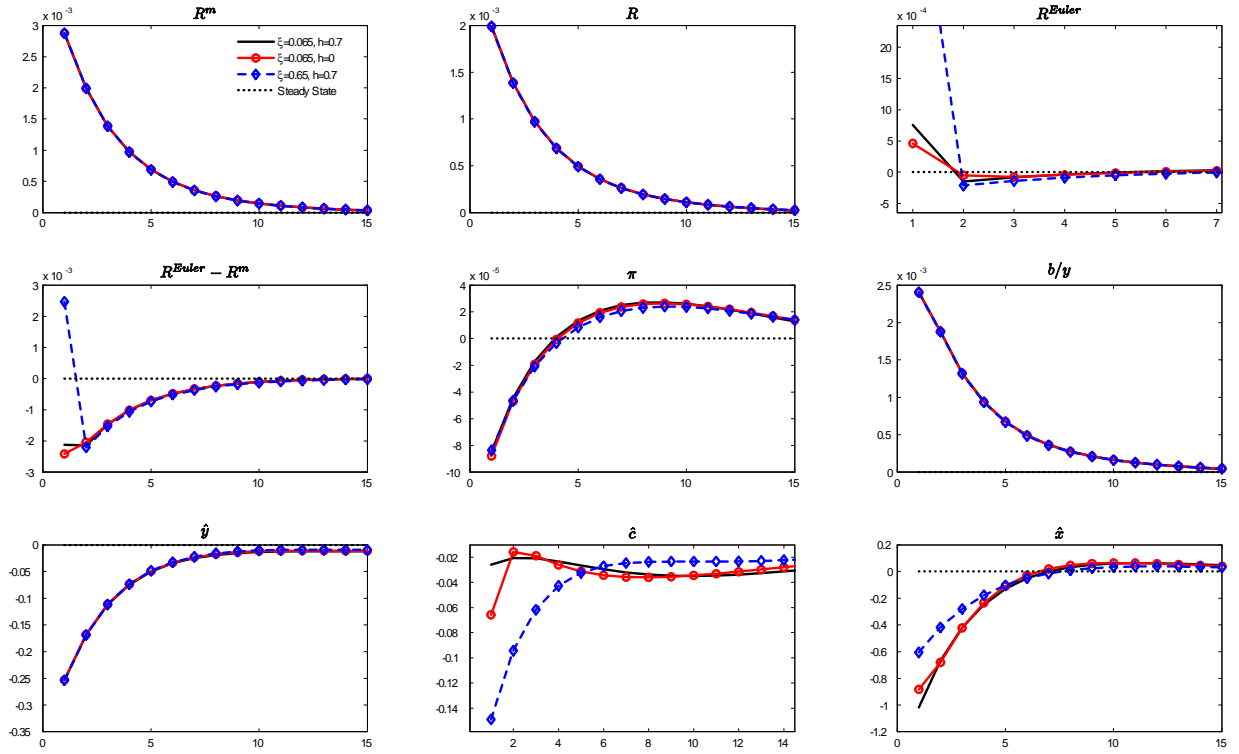
Figure 1: Impulse responses to policy rate shocks

$h = 0.7$, and investment adjustment costs with $\xi = 0.065$. To illustrate that monetary transmission is not based on controlling the rate of intertemporal substitution, we consider two additional versions: The red solid circled line presents impulse responses without habits ($h = 0$) and the blue dashed line with diamonds presents impulse responses with investment adjustment costs that are 10 times larger than in the benchmark case ($\xi = 0.65$).

An increase of the policy rate from its steady state value leads to a smaller rise in the treasury rate R_t, since it follows the future expected policy rate (see 14). Output and inflation decrease, which imply – together with the supply of T-bills (17) – that the bills-to-gdp ratio b_{t-1}/y_t increases with the policy rate. Notably, these responses are virtually not affected by habit formation or investment adjustment costs. The Euler rate rises on impact and returns back to its steady state value from below. This behavior is reflected by the response of consumption, which grows after it falls on impact. Omitting habits (see red solid circled line), leads to a more pronounced impact effect on consumption and slightly reduced Euler rate response. When investment adjustment costs are raised by the factor 10 (see blue dashed line with diamonds), the maximum investment response is reduced by 40%, which is compensated by a more pronounced consumption response. Accordingly, the

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initial increase in the Euler rate is 7 times larger (not displayed) than in the benchmark case. These experiments show that habits and investment adjustment costs mainly alter the composition of aggregate demand, implying that the output and inflation effects of monetary policy are not primarily governed by intertemporal substitution effects. It should be noted that the insensitivity of the output response is mainly due to the money demand specification, which implies that aggregate demand is restricted by $c_t + x_t \geq m_t^H + m_t^R$ in equilibrium.

In this model, a higher policy rate predominantly impacts on the level of consumption and investments due to the increased price of money in open market operations and the binding cash and collateral constraints. In contrast, in a standard model, where the policy rate equals the Euler rate, a higher (real) policy rate immediately increases the growth rate of consumption via the consumption Euler equation. Here, part of an increase in the policy rate is reflected by a decrease in liquidity premium (see 10) such that consumption growth is not one-for-one affected by the real policy rate. Likewise, the latter does not directly impact on the rate of return on investments, such that the investment response is less pronounced – for a given magnitude of investment adjustment costs – than in a standard model.[38]

5.4 Alternative money demand specification

In this Section, we consider an alternative money demand specification to assess the robustness of our main results. To provide a transparent motivation for the demand for liquid assets, we imposed a cash constraint for consumption expenditures (3) in the previous analysis, while we added a cash constraint for firms to treat investment and consumption expenditures in a symmetric way. Given that these cash constraints are rather rigid and imply an unrealistic velocity, we apply a widely used money-in-the-utility function specification. Instead of considering cash constraints, we follow Christiano et al. (2005) and assume that real balances enter household $i's$ utility function in a separable way:

$$u\left(c_{i,t}, M_{i,t}/P_t, n_{i,t}\right) = \frac{(c_{i,t} - hc_{i,t-1})^{1-\sigma}}{1-\sigma} - \theta \frac{n_{i,t}^{1+\sigma_n}}{1+\sigma_n} + \phi \frac{(M_{i,t}/P_t)^{1-\sigma_m}}{1-\sigma_m}, \qquad (23)$$

where $\sigma_m \geq 1$, $\phi > 0$, and $M_{i,t} = I_{i,t} + M_{i,t-1}^H$, while access to money is still constrained by (2). Household $i's$ demand for additional money and holdings of money then satisfies $(R_t^m - 1)\lambda_{i,t} + R_t^m \eta_{i,t} = u_{i,mt}$ and $\beta E_t (u_{i,m,t+1}/\pi_{t+1}) = \left[\left(R_t^{Euler} - 1\right)/R_t^{Euler}\right] u_{i,ct}$ instead of

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[38]Note that a magnitude of investment adjustment costs that is obtained from estimates based on aggregate data, e.g. $\xi = 5.88$ in Smets and Wouters (2007), is much larger and would lead to an investment response that is smaller than the output response, which is clearly at odds with VAR evidence (see, e.g. Christiano et al., 2005).
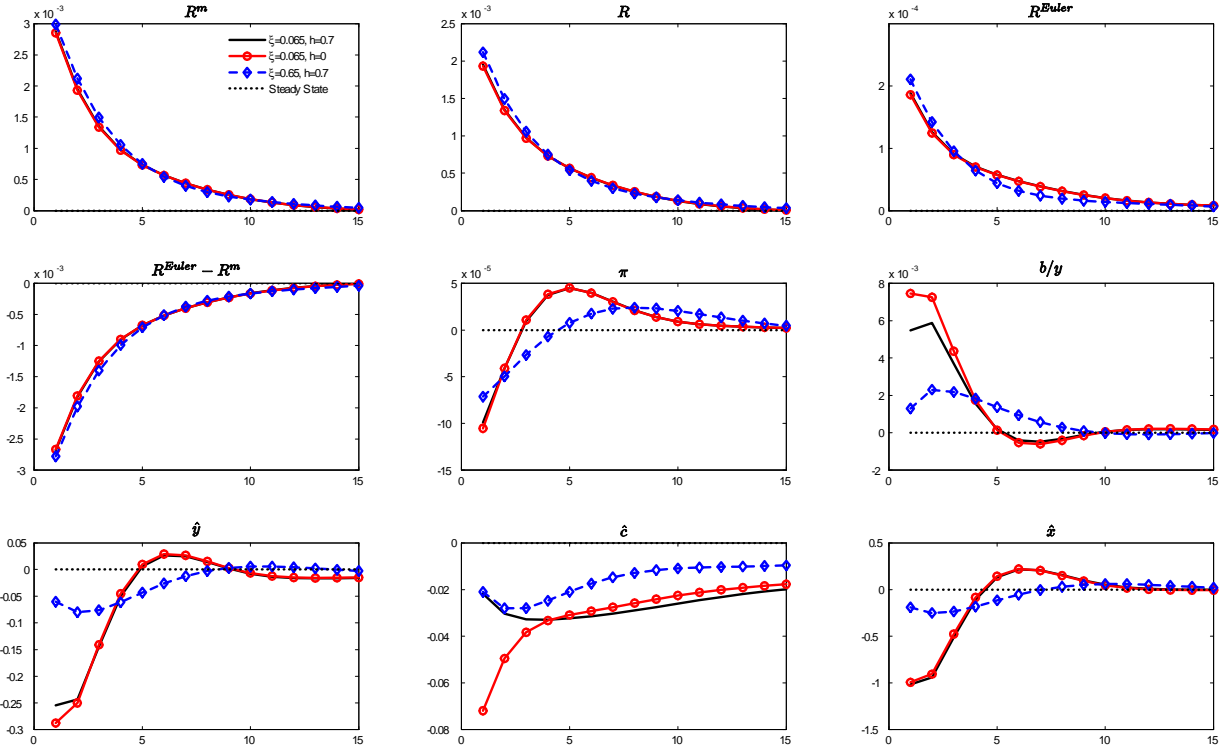
Figure 2: Impulse responses to policy rate shocks for an alternative money demand

(9) and (10),[39] where the Euler rate measures the opportunity costs of holding money from one period to the other (see discussion in Section 3.4). Like households, firms are assumed not to rely on cash for goods purchases, such that the set of equilibrium conditions (given in definition 5 in Appendix A.3) changes by $\lambda_t = u_{c,t}$, $\phi\beta E_t[\left(m_{t+1}^H + m_{t+1}^R\right)^{-\sigma_m}/\pi_{t+1}] = (1 - 1/R_t^{Euler})u_{c,t}$, $\theta n_t^{\sigma_n} = u_{c,t}mc_t\alpha y_t/n_t$, $1 = q_t[G_t + \frac{x_t}{x_{t-1}}G_t'] - E_t\beta(\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}(\frac{x_{t+1}}{x_t})^2 G_{t+1}')$, and $\varrho_t = y_t - w_t n_t - x_t$, which replace (61), (62), (64), (65) and (67). The parameters ϕ and σ_m in (23) are calibrated to get a velocity $\overline{y}/\overline{m}$ of 0.44 (see Christiano et al., 2005) and to replicate the impact output effect of policy rate shocks for the benchmark parametrization ($\phi = 14$ and $\sigma_m = 6.5$). The standard deviation of the productivity shock is again adjusted to match the observed standard deviation of detrended output ($st.dev.(\varepsilon_a) = 0.0064$), while the remaining parameter values are unchanged.

When money demand is induced by a money-in-the-utility-function (MIU) specification, the qualitative results with regard to interest rates and spreads are unchanged. Like be-

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[39]In the steady state, the multiplier on collateral constraint now satisfies $\eta/u_c = R^{Euler}[(1/R^m) - (1/R^{Euler})]$, such that $R^{Euler} > R^m$ again implies a binding collateral constraint.

fore, we compute selected moments of simulated time series taking both shocks, i.e. policy rate shocks and technology shocks, into account. The last column in Table 2 shows that all correlations exhibit the same sign and similar magnitudes as their empirical counterparts, though the negative correlation between the spread s_t and the policy rate is more pronounced. The standard deviation of the Euler spread again lies between the standard deviation of the Euler rate and the policy rate, while the latter ratio is almost four times larger than in the benchmark model. The impulse responses to an innovation to the policy rate are presented in Figure 2 (responses to productivity shocks are shown in Appendix B.2). They are consistent with previous results and broadly show the same pattern as the benchmark model (see 1). Given that money demand is less restrictive, aggregate demand and inflation are now affected by habit formation and investment adjustment costs in an intuitive way, while response of the Euler rate is virtually unchanged.

6 Conclusion

In this paper, we present a simple macroeconomic model where the rate of return on short-term treasuries is endogenously linked to the monetary policy rate and tends to be smaller than the rates on corporate borrowing, consistent with broad empirical evidence. We introduce monetary policy implementation via open market operations into a standard macroeconomic model, which gives rise to a liquidity premium on eligible assets, i.e. short-term treasuries, compared to non-eligible assets. The model predicts that this liquidity premium is negatively related to the ratio of bills to GDP, which accords to empirical evidence. While standard macroeconomic models typically assume that the (real) policy rate equals the rate of intertemporal substitution, we show that they differ and that the spread – also known as the Euler equation error – is negatively related to the policy rate and more volatile that the consumption Euler rate, which has been reported in several studies.

Although the existence of a liquidity premium substantially alters the monetary transmission mechanism, compared to a standard New Keynesian model for example, responses of real activity and inflation to monetary policy shocks are consistent with broad empirical evidence. Hence, the framework can be extended to medium-scale or large-scale models that can reasonably be estimated without neglecting the implementation of monetary policies beyond controlling the policy rate, which particularly regained interest during and in the aftermath of the recent financial crisis.

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A Appendix

A.1 Equilibrium conditions

Definition 1 *A rational expectations equilibrium (REE) is a set of sequences* $\{c_t,\ n_t,\ y_t,$
$w_t,\ m_t^R,\ m_t^H,\ mc_t,\ \varrho_t,\ v_t,\ R_t^q,\ R_t^{Euler},\ R_t,\ b_t,\ b_t^T,\ \pi_t,\ \tilde{Z}_t,\ s_t\}_{t=0}^{\infty}$ *satisfying*

$$c_t = m_t^H + m_t^R,\ if\ R_t^{Euler} > 1, \tag{24}$$
$$or\ c_t \le m_t^H + m_t^R,\ if\ R_t^{Euler} = 1,$$
$$b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R,\ if\ R_t^{Euler} > R_t^m, \tag{25}$$
$$or\ b_{t-1}/\left(R_t^m \pi_t\right) \ge m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R,\ if\ R_t^{Euler} = R_t^m,$$
$$m_t^R = \Omega m_t^H, \tag{26}$$
$$\theta n_t^{\sigma_n} = u_{c,t} w_t / R_t^{Euler}, \tag{27}$$
$$w_t = mc_t \alpha y_t / n_t, \tag{28}$$
$$1/R_t^{Euler} = \beta E_t \left[u_{c,t+1}/\left(u_{c,t}\pi_{t+1}\right)\right], \tag{29}$$
$$E_t u_{c,t+1}\pi_{t+1}^{-1} = R_t E_t \left(R_{t+1}^m\right)^{-1} u_{c,t}\pi_{t+1}^{-1}, \tag{30}$$
$$\tilde{Z}_t \left(\varepsilon - 1\right)/\varepsilon = Z_t^1/Z_t^2, \tag{31}$$
$$1 = (1 - \phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{32}$$
$$s_t = (1 - \phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}, \tag{33}$$
$$y_t = a_t n_t^{\alpha}/s_t, \tag{34}$$
$$y_t = c_t, \tag{35}$$
$$b_t - b_{t-1}\pi_t^{-1} = (\Gamma - 1)b_{t-1}^T \pi_t^{-1} - R_t^m \left(m_t^H - m_{t-1}^H \pi_t^{-1}\right), \tag{36}$$
$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{37}$$
$$R_t^q = P_t \left(v_t + \varrho_t\right)/\left(P_{t-1}v_{t-1}\right), \tag{38}$$
$$1 = \beta E_t \left[\left(\lambda_{t+1}/\lambda_t\right) \cdot \left(R_{t+1}^q/\pi_{t+1}\right)\right], \tag{39}$$
$$\varrho_t = mc_t \left(y_t - w_t n_t\right), \tag{40}$$

(where $u_{c,t} = c_t^{-\sigma}$, $Z_t^1 = \lambda_t y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1$, $Z_t^2 = \lambda_t y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2$, $\lambda_t = \beta E_t \left[u_{c,t+1}/\pi_{t+1}\right]$, *and* $\widetilde{y}_t = a_t^{\frac{1+\sigma_n}{1+\sigma_n+\alpha(\sigma-1)}} \left(\alpha/\theta\right)^{\frac{\alpha}{1+\sigma_n+\alpha(\sigma-1)}}$*) the transversality conditions, for a monetary policy setting* $\{R_t^m \ge 1\}_{t=0}^{\infty}$ *according to (20),* $\Omega_t > 0$, *and* $\pi \ge \beta$, *given sequences* $\{a_t, \varepsilon_{r,t}\}_{t=0}^{\infty}$ *and initial values* $m_{-1}^H > 0$, $b_{-1} > 0$, $b_{-1}^T > 0$, *and* $s_{-1} \ge 1$.

If the collateral constraint is not binding, which would be the case when the policy rate equals the Euler rate, $R_t^m = R_t^{Euler}$, the model can be reduced to a standard sticky price model with a cash-credit good distortion, where Ricardian equivalence holds:

Definition 2 *Under a non-binding collateral constraint, a REE to a set of equilibrium sequences for* $\{c_t, n_t, y_t, w_t, mc_t, \pi_t, s_t, \tilde{Z}_t\}_{t=0}^{\infty}$ *given by (28),* $\theta n_t^{\sigma_n} = u_{c,t} w_t / R_t^m$, $1/R_t^m = \beta E_t \left[u_{c,t+1}/\left(u_{c,t}\pi_{t+1}\right)\right]$, *(where* $u_{c,t} = c_t^{-\sigma}$*) (31)-(35), for a monetary policy setting* $\{R_t^m \ge 1\}_{t=0}^{\infty}$ *according to (20),* $\pi \ge \beta$, *given sequences* $\{a_t, \varepsilon_{r,t}\}_{t=0}^{\infty}$ *and an initial value* $s_{-1} \ge 1$.

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A.2 Appendix to section 4

In this Appendix, we first define a *REE* under flexible prices for the simplified version, before we prove the claims made in proposition 1. Then, we examine the local determinacy properties under sticky prices, which will be used for the subsequent proof of proposition 2.

Definition 3 *For $\phi = 0$, $\Omega \to \infty$, $\sigma = 1$, and $\Gamma = \pi = 1$, a REE is a set of sequences $\{c_t,$ $n_t,\ y_t,\ w_t, m_t^R,\ \pi_t,\ R_t^{Euler},\ b_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying (27)-(29), (35),*

$$c_t = m_t^R,\ \text{if}\ \ R_t^{Euler} > 1,\ \ \text{or}\ c_t \le m_t^R,\ \text{if}\ \ R_t^{Euler} = 1, \tag{41}$$

$$b_{t-1}/\left(R_t^m \pi_t\right) = m_t^R,\ \text{if}\ \ R_t^{Euler} > R_t^m,\ \text{or}\ b_{t-1}/\left(R_t^m \pi_t\right) \ge m_t^R,\ \text{if}\ \ R_t^{Euler} = R_t^m, \tag{42}$$

$$b_t = \pi b_{t-1} \pi_t^{-1}, \tag{43}$$

$$y_t = a_t n_t^{\alpha}, \tag{44}$$

where $mc_t = (\varepsilon - 1)/\varepsilon$, the transversality conditions, for a monetary policy setting $\{R_t^m \ge 1\}_{t=0}^{\infty}$ according to (20), $\Omega_t > 0$, and $\pi \ge \beta$, given sequence $\{a_t, \varepsilon_{r,t}\}_{t=0}^{\infty}$ and an initial value $b_{-1} > 0$.

Proof of proposition 1. Consider the model summarized in definition 3. Combining (27), (28), (35), and (44) leads to $y_t = a_t \left[(mc\alpha/\theta)\left(1/R_t^{Euler}\right)\right]^{\alpha/(1+\sigma_n)}$, such that a REE can be reduced to a set of sequences $\{y_t,\ \pi_t,\ R_t^{Euler},\ b_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying

$$y_t = a_t \left[(\mu/\theta)\left(1/R_t^{Euler}\right)\right]^{\alpha/(1+\sigma_n)}, \tag{45}$$

$$1/R_t^{Euler} = \beta y_t E_t[1/(y_{t+1}\pi_{t+1})], \tag{46}$$

$$y_t = b_{t-1}/\left(R_t^m \pi_t\right),\ \text{if}\ \ R_t^{Euler} > R_t^m,\ \text{or}\ y_t \le b_{t-1}/\left(R_t^m \pi_t\right),\ \text{if}\ \ R_t^{Euler} = R_t^m, \tag{47}$$

$$b_t = \pi b_{t-1} \pi_t^{-1} \forall t \ge 1, \tag{48}$$

and $\pi P_0 b_0 = B_{-1}$, where $\mu = \frac{\varepsilon-1}{\varepsilon}\alpha < 1$, for a monetary policy setting R_t^m for a given initial stock of treasuries $B_{-1} > 0$. Consider the case where the constraint (47) is binding, which requires $R_t^{Euler} > R_t^m$ to hold in equilibrium according to (22). Eliminating output in (46) with (47) for $R_t^{Euler} > R_t^m$, gives $1/R_t^{Euler} = \beta b_{t-1}(b_t R_t^m \pi_t)^{-1} E_t R_{t+1}^m$, and substituting out bonds by (48) leads to

$$1/R_t^{Euler} = (\beta/\pi)\, E_t R_{t+1}^m / R_t^m \tag{49}$$

$\Leftrightarrow R_t^{Euler}/R_t^m = (\pi/\beta)\left(1/E_t R_{t+1}^m\right)$. The latter implies that if $E_t R_{t+1}^m < \pi/\beta$, the Euler rate exceeds the policy rate, $R_t^{Euler} > R_t^m$, which is consistent with a binding collateral constraint, and that the spread R_t^{Euler}/R_t^m decreases with the expected policy rate. It further immediately follows from (47) and (48) that the policy rate is positively related to the bills-to-gdp ratio, $R_t^m = (b_t/y_t)\,\pi^{-1}$, such that the spread R_t^{Euler}/R_t^m is negatively related to the expected bills-to-gdp ratio $R_t^{Euler}/R_t^m = (\pi/\beta)\left[\pi/E_t\left(b_{t+1}/y_{t+1}\right)\right]$. $\blacksquare$

Under sticky prices, $\phi > 0$, and for $\Omega \to \infty$ and $\Gamma = \pi = 1$, the model can be reduced to a set of sequences $\{c_t, n_t, y_t, w_t, m_t^R, \pi_t, R_t^{Euler}, b_t, mc_t, Z_t, s_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying (27)-(29), (31)-(35), (41)-(43), the transversality conditions, for a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$, $\Omega_t > 0$, and $\pi > \beta$, given a sequence $\{a_t\}_{t=0}^{\infty}$ and initial values $b_{-1} > 0$ and $s_{-1} \geq 1$. Suppose that the average policy rate and the inflation target satisfy $R^m < \pi/\beta$ and $\pi > \beta \Rightarrow R^{Euler} > 1$, where steady state values exhibit not time index. Then, the collateral constraint is binding in the steady state. Log-linearizing the model at this steady state and assuming that shocks are sufficiently small such that the economy remains in the neighborhood of the steady state, we can define a RE equilibrium as follows. Note that $\widehat{x}_t$ denotes log-deviation from the steady state value $\widehat{x}_t = \log x_t/x$.

Definition 4 *For $\Omega \to \infty$, $\Gamma = \pi = 1$, $R^m \in [1, 1/\beta)$, a REE is a set of convergent sequences $\{\widehat{y}_t, \pi_t, \widehat{b}_t, \hat{R}_t^{Euler}, \hat{R}_t^m\}_{t=0}^{\infty}$ satisfying*

$$\widehat{y}_t = \widehat{b}_{t-1} - \widehat{\pi}_t - \hat{R}_t^m, \tag{50}$$

$$\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \hat{R}_t^{Euler} + E_t\widehat{\pi}_{t+1}, \tag{51}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi(\varpi - 1)\widehat{y}_t - \chi(1 + \sigma_n)\alpha^{-1}\widehat{a}_t + \chi\hat{R}_t^{Euler}, \tag{52}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{53}$$

$$\widehat{R}_t^m = \rho_R\widehat{R}_{t-1}^m + \rho_\pi(1 - \rho_R)\widehat{\pi}_t + \rho_y(1 - \rho_R)(\widehat{y}_t - \varsigma\widehat{a}_t) + \varepsilon_{r,t}, \tag{54}$$

where $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1$, $\chi = (1 - \phi)(1 - \beta\phi)/\phi$, and $\varsigma = \frac{1+\sigma_n}{1+\sigma_n+\alpha(\sigma-1)}$, given $b_{-1} > 0$.

The following lemma describes local determinacy for the REE given in definition 4.

Lemma 1 *Suppose that the central bank sets the policy rate according to (20) with $\rho_R = 0$. The REE as given in definition 4 is uniquely determined if*

$$(\rho_\pi + 1/2)\left[(1 + \sigma_n)\alpha^{-1} - (1 + \sigma)\right] > -(1 + \rho_y)(1 + \beta + \chi)\chi^{-1}, \tag{55}$$

Proof. Consider the model given in definition 4, which can – by eliminating the Euler rate with (51) – be further reduced to (50), (53), and

$$\widehat{\pi}_t = (\beta + \chi)E_t\widehat{\pi}_{t+1} + \chi(\varpi - 1 - \sigma)\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1}. \tag{56}$$

Abstracting from shocks, $a_t = 1$ and $\varepsilon_{r,t} = 0$, for simplicity, and using the policy rule $\widehat{R_t}^m = \rho_\pi\widehat{\pi}_t + \rho_y\widehat{y}_t$, condition (50) implies $\widehat{y}_t = \frac{1}{1+\rho_y}\widehat{b}_{t-1} - \frac{1+\rho_\pi}{1+\rho_y}\widehat{\pi}_t$. Substituting out output with the latter, (56) can together with (53) be written as $[(\beta + \chi) - \chi\sigma\frac{1+\rho_\pi}{1+\rho_y}]E_t\widehat{\pi}_{t+1} + \frac{\chi(\varpi-1)}{1+\rho_y}\widehat{b}_t = \widehat{\pi}_t[1 + \chi(\varpi - 1 - \sigma)\frac{\rho_\pi}{1+\rho_y}]$. Hence, the model can be reduced to a two-dimensional system in $\widehat{b}_t$ and $\widehat{\pi}_t$, which exhibits the characteristic polynomial $F(X) = X^2 - \left(\frac{\zeta_2}{\zeta_1} + \frac{\zeta_3}{\zeta_1} + 1\right)X + \frac{\zeta_3}{\zeta_1}$, where

$\zeta_1 = (\beta + \chi) - \chi\sigma\frac{1+\rho_\pi}{1+\rho_y}$, $\zeta_2 = \frac{\chi(\varpi-1)}{1+\rho_y} > 0$, $\zeta_3 = 1 + \chi(\varpi - 1 - \sigma)\frac{\rho_\pi}{1+\rho_y} > 0$, and $\zeta_4 = \frac{\chi(\varpi-1-\sigma)}{1+\rho_y}$. Hence, $F(0) = \frac{\zeta_3}{\zeta_1}$ and $F(1) = -\frac{\zeta_2}{\zeta_1}$, implying $signF(0) = -signF(1)$, and there exists at least one real stable eigenvalue between zero and one. Further, $F(X)$ at $X = -1$ is given by $F(-1)\zeta_1/2 = (\beta + 1 + \chi) + \chi\frac{(1/2)+\rho_\pi}{1+\rho_y}[\frac{1+\sigma_n}{\alpha} - (1+\sigma))]$. For $F(-1)\zeta_1/2 > 0$, such that $signF(0) = signF(-1)$, there exists exactly one stable eigenvalue, between zero and one, and one unstable eigenvalue, indicating local determinacy. Hence, the equilibrium is uniquely determined if (55) is satisfied. ∎

Condition (55) is hardly restrictive for a reasonable choice of parameter values. If, for example, the Frisch labor supply elasticity is not too small, i.e. $\sigma_n > \alpha(1+\sigma) - 1$, (55) is always satisfied. The following proof examines the simplified version for flexible prices and for sticky prices, where (55) is satisfied and determinacy is guaranteed by $\sigma = 1$ and $\rho_{\pi,y} = 0$.

Proof of proposition 2. Consider a simplified version of the model with $\phi \geq 0$, $\Omega \to \infty$, $\sigma = 1$, and a monetary policy satisfying (20) with $\rho_\pi = \rho_y = 0$, $\rho_R > 1/2$, $E_t R_{t+1}^m < \pi/\beta$, and $\pi = 1$.

To establish the claims made in the <u>first</u> part of the proposition, we separately examine the flexible price case and the sticky price case. Consider the model summarized in definition 3. The equilibrium sequences $\{y_t, \pi_t, R_t^{Euler}, b_t\}_{t=0}^\infty$ are then characterized by (45)-(46), (48), $\pi P_0 b_0 = B_{-1}$, and $y_t = b_{t-1}/(R_t^m \pi_t)$. Using that (49) then holds (see proof of proposition 1) and (20), leads to

$$R_t^{Euler} = (R_t^m)^{1-\rho_R} \cdot (1/R^m)^{1-\rho_R}(\pi/\beta)/\exp[(1/2)var(\varepsilon_{r,t}^{\rho_R})], \tag{57}$$

where we used that $E_t \exp(\varepsilon_{t+1}^{\rho_R}) = \exp[(1/2)var(\varepsilon_{r,t}^{\rho_R})]$. Next, substitute out the Euler rate in (45) with (57), to get

$$y_t = (1/R_t^m)^{\frac{(1-\rho)\alpha}{1+\sigma_n}} a_t\Theta, \tag{58}$$

where $\Theta \equiv [(\mu/\theta)(\beta/\pi)R^m \exp[(1/2)var(\varepsilon_{r,t}^{\rho_R})]]^{\alpha/(1+\sigma_n)}$. Further, substitute out output with (58) in $\pi_t = b_{t-1}/(R_t^m y_t)$ (see 47), which leads to

$$\pi_t = (1/R_t^m)^{\frac{1+\sigma_n-\alpha(1-\rho_R)}{1+\sigma_n}} b_{t-1}/(a_t\Theta). \tag{59}$$

The solutions (57)-(59) imply that output and inflation decrease with the policy rate and that the Euler rate increases with the policy rate.

Now consider the sticky price case, summarized in definition 4 with $\sigma = 1$. Given that the policy rate is exogenous, condition (55) reduces to $\chi(1/2)(1+\sigma_n)\alpha^{-1} > -(1+\beta)$,

which is obviously satisfied, implying that the equilibrium is locally determined and the stable eigenvalue is strictly positive (see proof of lemma 1). Hence, the unique solution to the system (50)-(53), is given by the generic form $\widehat{\pi}_t = \delta_1 \widehat{b}_{t-1} + \delta_2 \hat{R}_t^m + \delta_5 \widehat{a}_t$, $\widehat{y}_t = \delta_3 \widehat{b}_{t-1} + \delta_4 \hat{R}_t^m + \delta_6 \widehat{a}_t$, and $\widehat{b}_t = (1 - \delta_1) \widehat{b}_{t-1} - \delta_2 \hat{R}_t^m$, where the stable eigenvalue is $1 - \delta_1 \in (0, 1)$ (see 1). Inserting these solutions into the two-dimensional system (53) and (56) for $\rho_{\pi,y} = 0$ leads to the following conditions for the coefficient δ_2 and δ_4 :

$$\partial \widehat{\pi}_t / \partial \hat{R}_t^m = \delta_2 = -\chi \left((1 + \sigma_n) \alpha^{-1} - (1 - \rho_R) \right) / \Psi < 0,$$
$$\partial \widehat{y}_t / \partial \hat{R}_t^m = \delta_4 = - \left[1 + \delta_1 \beta + \chi (1 - \rho_R) \right] / \Psi < 0,$$

where $\Psi = 1 + \delta_1 \beta + \chi (1 + \sigma_n) \alpha^{-1} > 0$. Hence, in response to a monetary contraction, $\hat{R}_t^m > 0$ inflation and output decline, while the Euler rate, which satisfies (57) and thus $\hat{R}_t^{Euler} = (1 - \rho_R)\hat{R}_t^m$, increases.

Turning to the <u>second</u> part of the proposition, we use that the solution to the Euler rate (57) holds regardless of the degree of price flexibility. It implies the following solution for the spread R_t^{Euler}/R_t^m :

$$R_t^{Euler}/R_t^m = (R_t^m)^{-\rho_R} \cdot (1/R^m)^{1-\rho_R} (\pi/\beta)/\exp[(1/2)var(\varepsilon_{r,t}^{\rho_R})]. \tag{60}$$

According to (60) the ratio R_t^{Euler}/R_t^m decreases with the policy rate and with its variance, while $R_t^m = \pi b_t/y_t$ (or in log-linearized terms $\hat{R}_t^m = \widehat{b}_t - \widehat{y}_t$) implies that R_t^{Euler}/R_t^m is negatively related to the bills-to-gdp ratio. The solutions (57) and (60) further imply that the variance of the ratio R_t^{Euler}/R_t^m is larger than the variance of the Euler rate R_t^{Euler} for a sufficiently large autocorrelation of the policy rate, $\rho_R > 1/2$. Using the approximations $\log \left(R_t^{Euler}/R_t^m \right) \approx s_t^{Euler}$ and $\log R_t^{Euler} \approx R_t^{Euler} - 1$, establishes the claims made in the part 2 of the proposition. ∎

A.3 Equilibrium conditions of the quantitative version

In this Appendix, we present the full set of equilibrium conditions for the quantitative version of the model (as presented in Section 5.1).

Definition 5 *A REE of the quantitative version of the model for $R_t^{Euler} > R_t^m$ is given by a set of sequences* $\{c_t, y_t, n_t, x_t, k_t, w_t, \varrho_t, v_t, q_t, \lambda_t, m_t^R, m_t^H, b_t, b_t^T, mc_t, \check{Z}_t, s_t, \pi_t, R_t,$

$R_t^{Euler} = R_t^L$, $R_t^q\}_{t=0}^\infty$ *satisfying (26), (28)-(33), (36)-(39),*

$$\lambda_t = \beta E_t \left[(u_{c,t+1}/\pi_{t+1}) \right], \tag{61}$$

$$c_t + x_t = m_t^H + m_t^R, \tag{62}$$

$$b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \tag{63}$$

$$\theta n_t^{\sigma_n} R_t^{Euler} = u_{c,t} w_t, \tag{64}$$

$$R_t^{Euler} = q_t \left[G_t + (x_t/x_{t-1}) G_t' \right] - E_t \beta \left(\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left(x_{t+1}/x_t \right)^2 G_{t+1}' \right), \tag{65}$$

$$q_t = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}(1-\alpha)mc_{t+1}\left(y_{t+1}/k_t\right) + (1-\delta)\frac{\lambda_{t+1}}{\lambda_t}q_{t+1} \right), \tag{66}$$

$$\varrho_t = y_t - w_t n_t - x_t \left(2R_t^{Euler} - 1 \right) / R_t^{Euler}, \tag{67}$$

$$y_t = a_t n_t^\alpha k_{t-1}^{1-\alpha}/s_t, \tag{68}$$

$$y_t = c_t + x_t, \tag{69}$$

$$k_t = (1-\delta) k_{t-1} + x_t G_t, \tag{70}$$

(where $u_{c,t} = (c_t - hc_{t-1})^{-\sigma}$, $G_t = 1 - \xi\frac{1}{2}\left(x_t/x_{t-1} - 1\right)^2$ and $G_t' = -\xi\left(x_t/x_{t-1} - 1\right)$) as well as the transversality conditions, a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^\infty$, $\Omega_t > 0$, and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$, for a given sequences $\{a_t, \varepsilon_t\}_{t=0}^\infty$ and $\{\widetilde{y}_t\}_{t=0}^\infty$ (see below), and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, $k_{-1} > 0$, $x_{-1} > 0$, and $s_{-1} \geq 1$.

The efficient output level $\widetilde{y}_t$, which required for the interest rate rule (20), is jointly determined with the efficient allocation of $\{\widetilde{y}_t, \widetilde{n}_t, \widetilde{c}_t, \widetilde{k}_t, \widetilde{x}_t, \widetilde{q}_t\}_{t=0}^\infty$ satisfying

$$\theta\widetilde{n}_t^{1+\sigma_n} = \widetilde{u}_{ct}\alpha\widetilde{y}_t, \quad \widetilde{y}_t = a_t\widetilde{n}_t^\alpha\widetilde{k}_{t-1}^{1-\alpha}, \quad \widetilde{y}_t = \widetilde{c}_t + \widetilde{x}_t, \quad \widetilde{k}_t = (1-\delta)\widetilde{k}_{t-1} + \widetilde{x}_t G\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right),$$

$$1 = \widetilde{q}_t \left[G\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right) + \left(\widetilde{x}_t/\widetilde{x}_{t-1}\right) G'\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right) \right] - E_t\beta \left(\frac{\widetilde{u}_{ct+1}}{\widetilde{u}_{ct}}\widetilde{q}_{t+1}\left(\widetilde{x}_{t+1}/\widetilde{x}_t\right)^2 G'\left(\widetilde{x}_{t+1}/\widetilde{x}_t\right) \right),$$

$$\widetilde{q}_t = \beta E_t \left(\frac{\widetilde{u}_{ct+1}}{\widetilde{u}_{ct}} \left[(1-\alpha)\left(\widetilde{y}_{t+1}/\widetilde{k}_t\right) + (1-\delta)\widetilde{q}_{t+1} \right] \right),$$

where $\widetilde{u}_{ct} = (\widetilde{c}_t - h \cdot \widetilde{c}_{t-1})^{-\sigma}$ given $\widetilde{x}_{-1} > 0$ and $\widetilde{k}_{-1} > 0$.

B Further Appendices

## B.1 Parameter values

| Table A1 Benchmark parameter values	
Subjective discount factor	$\beta = 0.993$
Inverse of intertemporal substitution elasticity	$\sigma = 1$
Inverse of Frisch elasticity of labour supply	$\sigma_n = 1$
Substitution elasticity	$\varepsilon = 13$
Steady state working time	$n = 0.33$
Labour share	$\alpha = 0.66$
Investment adjustment cost	$\vartheta = 0.065$
Rate of depreciation of capital stock	$\delta = 0.025$
Habit parameter	$h = 0.7$
Fraction of non-price adjusting firms	$\phi = 0.8$
Steady state interest rate	$R^m = 1.0159$
Share of repos to outright purchases	$\Omega = 12$
Steady state inflation	$\pi = 1.0113 \ (= \Gamma)$
Policy rule coefficients	$\rho_R = 0.73, \ \rho_\pi = 1.5, \ \rho_y = 0.78$
Standard deviation of policy rate shocks	$sd(\varepsilon_{R,t}) = 0.003$
Autocorrelation of tfp-shocks	$\rho_A = 0.8$
Standard deviation of tfp-shocks	$sd(\varepsilon_{A,t}) = 0.0173$

B.2 Additional Figures
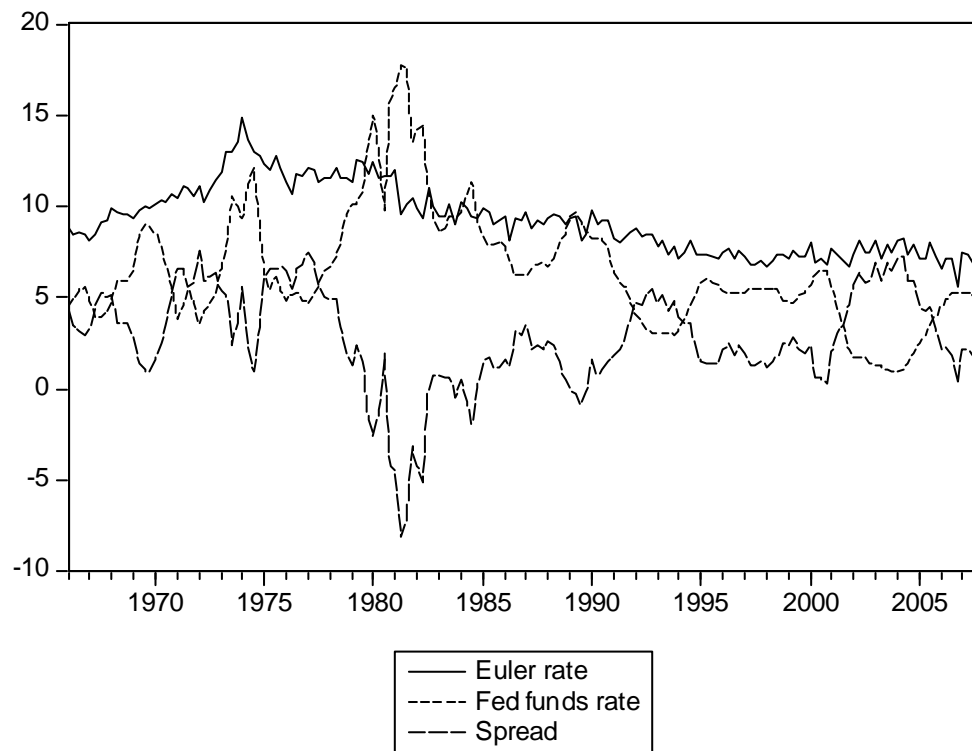
Computed Euler rate and Federal Funds rates
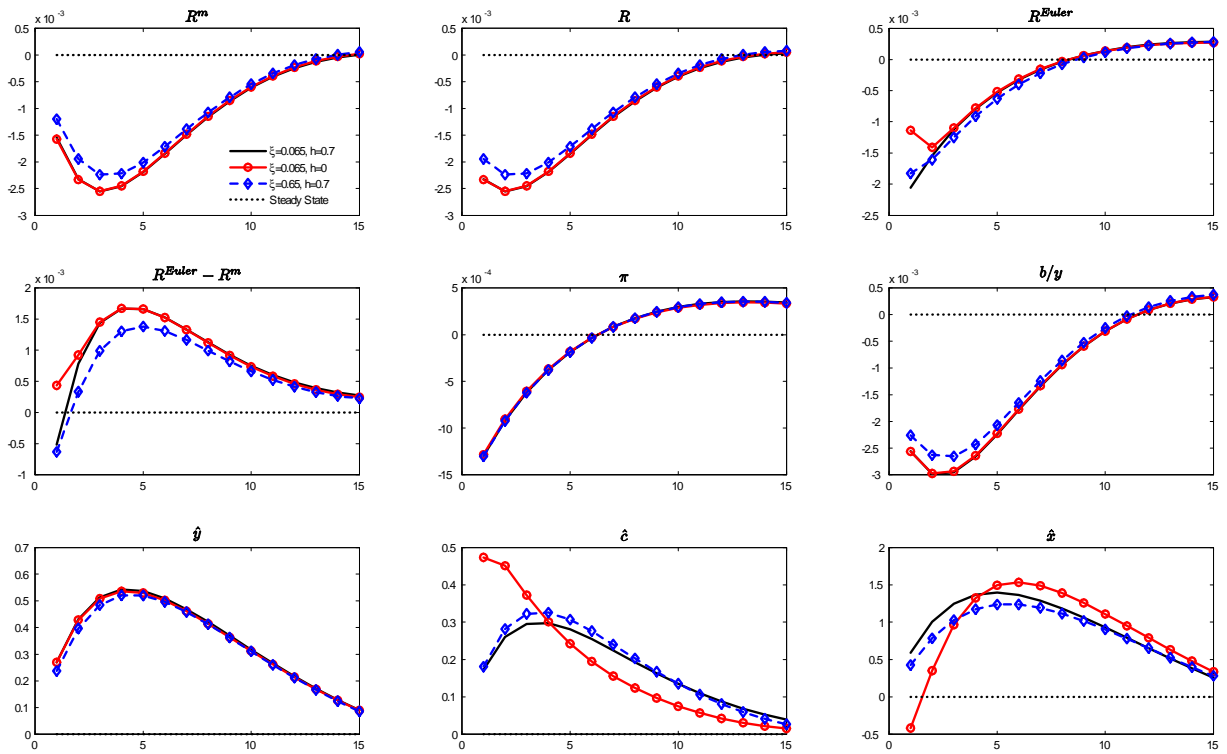
Impulse responses to a positive TFP shock
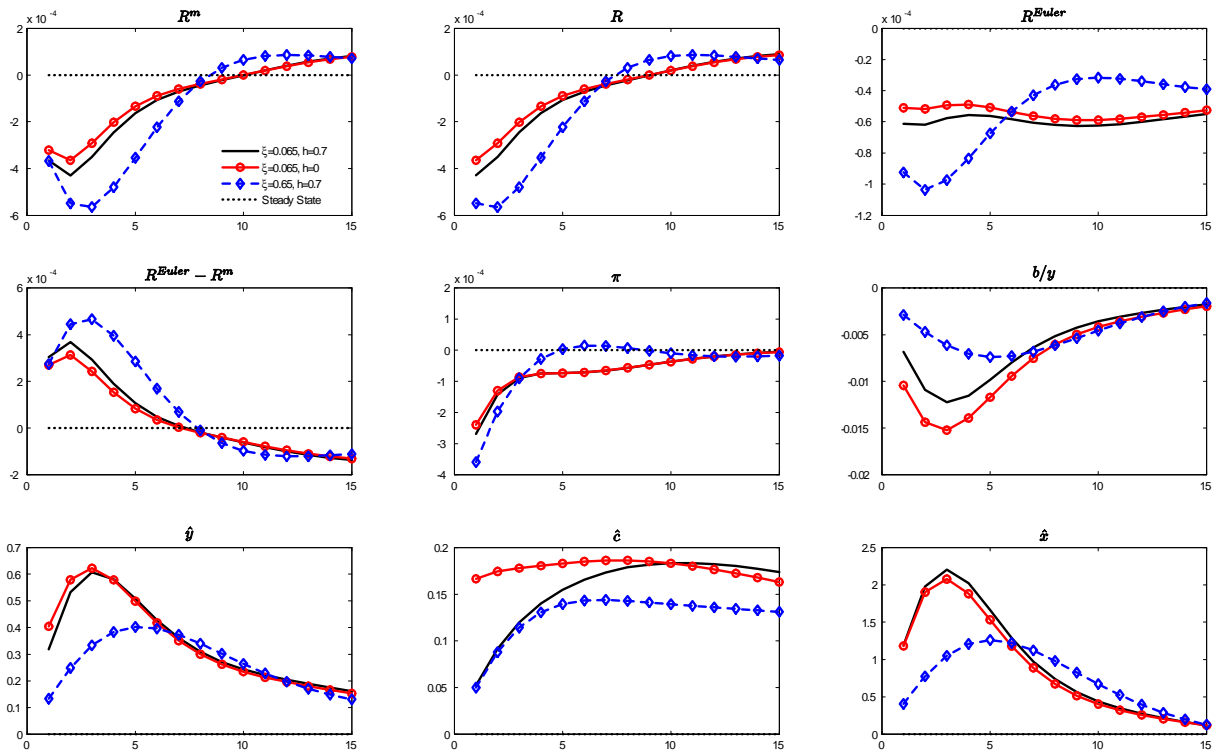
Impulse responses to a positive TFP shock for MIU