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Housing, collateral constraints, and fiscal policy

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Abstract

This paper studies the preferential tax treatment of housing that can be observed in many industrialized countries. It provides a rationale for it by means of an optimal taxation approach taking into account an important feature of housing, namely its usage as collateral. In a borrower-lender framework where private loans are assumed to be non-enforceable and have to be collateralized by housing, optimal fiscal policy should disburden constrained borrowers by subsidizing their housing.

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1 Introduction

Housing is subject to a preferential tax treatment in many industrialized countries. In the US, total housing subsidies added up to 220 billion dollars in 2011, corresponding to 1.5% of GDP (US Budget, 2011). Also in European countries the values of total housing subsidies in percent of GDP were in that range, e.g. 0.9% in Germany, 1.1% in France and 1.4% in Spain in 2000 (ECB, 2003).

The two most important housing subsidies are the deductibility of mortgage interest payments from income and the tax exemption of imputed rents on owner-occupied housing. In the US, the former amounted to 105 billion dollars while the latter added up to 38 billion dollars in 2011 (US Budget, 2011). These two subsidies accounted for 65% of total housing subsidies.

However, the view of economists on this preferential tax treatment of housing is controversial. On the one hand it is criticized by researchers like e.g. Poterba (1992) and Gervais (2002) among others, who argue that this treatment leads to a welfare loss since it distorts investment decisions of individuals towards housing. These studies are in line with Rosen who writes in the Handbook of Public Economics that "paternalism and political considerations seem to be the sources of this policy" (1985, p. 380).

On the other hand there are proponents of this treatment who argue that homeownership is accompanied with externalities which are internalized through these subsidies. For instance, Green and White (1997) stress the positive impact of homeownership on education of children and DiPasquale and Glaeser (1999) state that homeowners are "better citizens" in the sense that they are more involved in local organizations.

In contrast to these papers, this work gives a rationale for housing subsidies based on market imperfections. We assume that private loans are not enforceable and therefore have to be collateralized by housing. Looking at the data makes the importance of housing as a component of wealth and the relevance of its usage as collateral clear. First, housing makes up a large part of total household wealth as well as total national wealth. In the US, housing wealth accounts for half of total household wealth and is larger than annual GDP with an average ratio of housing wealth to GDP of about 1.5 from 1952-2008 (Iacoviello, 2009). Secondly, in 2010 residential mortgage debt amounted to 77% of GDP in the US and to 47% in Germany, to 41% in France and to 64% in Spain (Hypostat, 2010). To the best of our knowledge, this paper is the first one that studies optimal taxation of housing in the presence of collateral constraints.

The structure of the model is as follows. We consider a household sector that relates to Kiyotaki and Moore's (1997) model with two types of agents who differ in their discount factors, patient and impatient ones. Due to this difference in patience we get lenders, the patient agents, and borrowers, the impatient ones, in equilibrium. While for the former the collateral constraint is irrelevant in equilibrium, it is of importance for the latter. As in Iacoviello (2005) housing plays a dual role for households. First, it delivers utility to the agents together with consumption and leisure and secondly, private loans are collateralized by housing. The government, that is assumed to have access to a commitment technology, has exogenous expenditures that have to be financed by two taxes, a housing property tax that can differ for the two types of agents and a labor income tax. The different housing tax rates for the two types can be understood as follows. The patient households for whom the collateral constraint is irrelevant will always own a larger house than the impatient ones and therefore are taxed at another, a higher, rate than the impatient and hence wealth-poor agents.

The main result of this paper is that it provides a rationale for housing subsidies. In the presence of collateral constraints, optimal fiscal policy should subsidize housing of the impatient households, for whom the collateral constraint is relevant, to disburden them. This subsidy has to be financed to the largest extent by a housing tax on the patient households and to a smaller part by a labor income tax. Hence, this can be interpreted as redistribution from wealth-rich patient households with a higher housing stock to wealth-poor impatient households with a lower housing stock.

The main result of housing subsidies for impatient households is robust to several parameter variations and can be attributed for the most part to the collateral constraint. To illustrate this point, we analyze the effects of the discount rate difference of the types of agents on housing subsidies in comparison to the effects of the collateral constraint, with the result that the former plays a minor role.

We also consider a representative agent version of the model as reference case. Thereby we can understand how the inclusion of a durable good, housing, per se affects optimal fiscal policy compared to standard models. Furthermore, this allows us to compare the results of the representative agent version to existing literature. These results are in fact quite intuitive and in line with the principle of optimal taxation that goods with lower elasticities should be taxed at a higher rate. For the benchmark calibration, the housing tax rate is positive in the representative agent version as it is for patient households in the model with two types of agents.

The paper further relates to the work of Eerola and Määttänen (2009) that considers optimal taxation of housing in a dynamic representative agent model with fairly general preferences and an extended tax system compared to the model of this paper. However, the results of the representative agent version of our model are compatible with their results. Another closely related paper is Monacelli (2008) that considers a model with two types of agents with different patience rates and collateral constraints similar to the one of this paper. While Monacelli analyses optimal *monetary* policy in that framework, he points out that also the analysis of optimal fiscal policy in such a model would be of interest, which is done in this paper.

The rest of the paper is organized as follows. In section 2, the model with two types of agents, firms and the government is described, the Ramsey problem is set up and the equilibrium conditions for the steady state are derived. In section 3, the results for the full as well as the representative agent version are presented and a sensitivity analysis is given. The fourth and last section concludes.

2 The Model

In this section, we present the model with a household sector consisting of two types of agents, a production sector consisting of two types of firms and the government. Concerning the household sector, we follow Kiyotaki and Moore (1997), who pioneered the models with two types of agents, patient and impatient ones, resulting in an equilibrium with lenders and borrowers. We assume that private debt contracts are not enforceable and have to be collateralized by housing as in Iacoviello (2005). Therefore, a household can only borrow up to a fraction m of his expected end of period housing wealth. Additionally to its usage as collateral, housing delivers utility together with consumption and leisure.

Like in Favilukis et al. (2012), we consider a two-sector production side, such that both housing demand and supply are modeled explicitly. There are two types of firms, one of which produces non-durable consumption goods and the other durable housing.

The government levies a flat-rate tax on labor income and a housing property tax that can differ for the two types of agents and issues one-period bonds to finance an exogenous stream of government expenditures. It has no access to lump-sum taxes. The reason why housing tax rates can differ is that a patient household will own a larger house than an impatient one. Hence, rather than taxing degrees of patience differently, we can understand this as taxing the ones with a larger house at a higher rate than the ones with a smaller house. Due to the usage of housing as collateral that is only relevant for the borrowers, who will be the impatient agents in equilibrium, we will see that the housing tax rates will differ markedly.

2.1 Households

There is a continuum of households consisting of two types, patient and impatient ones. They differ in their discount factors $1 > \beta > \beta' > 0$ with β being the discount factor of patient and β' of impatient households. Henceforth, variables of patient (impatient) households are denoted without (with) a prime, while aggregate variables are denoted with a superscript T (e.g. c_t^T, for total consumption). The population share of patient households is s. Borrowing between the two types of households is modeled as follows. A household can borrow an amount $-\frac{b_t}{1+r_{t-1}}$ in period $t-1$ and has to pay back $-b_t$ in period t, where r_{t-1} is the real interest rate on loans between $t-1$ and t. Since we assume that private debt contracts are not enforceable, there is a limit on private debt given by a fraction m of the expected end of period housing wealth

$$b_{t+1}^{(\prime)} \geq -mp_{h,t+1}h_t^{(\prime)}, \tag{1}$$

where m denotes the exogenous pledgeable fraction of housing. As we will see below, this constraint will become relevant for impatient households, while it will be irrelevant for patient ones.

Both types of households derive utility from consumption $c_t^{(\prime)}$ and housing $h_t^{(\prime)}$ and disutility from labor $n_t^{(\prime)}$ and maximize the infinite sum of expected utility. Their objective is given by

$$\sum_{t=0}^{\infty} \beta^{(\prime)t} u(c_t^{(\prime)}, h_t^{(\prime)}, n_t^{(\prime)}). \tag{2}$$

We consider the following CRRA-specification of the utility function

$$u(c_t, h_t, n_t) = \frac{c_t^{1-\mu^c}}{1-\mu^c} + \frac{h_t^{1-\mu^h}}{1-\mu^h} - \frac{n_t^{1+\mu^n}}{1+\mu^n}, \tag{3}$$

where $\mu^{c(h)}$ denotes the inverse of the intertemporal elasticity of substitution in consumption (housing) and μ^n the inverse of the Frisch elasticity of labor supply.

2.1.1 Patient Households

The representative patient household generates income from working $w_t n_t$, with w_t being the real wage rate and the return of bond holdings b_t^g. Labor income is taxed at the rate τ_t^n. Every period the household can adjust its stock of housing according to $h_t - (1-\delta_h)h_{t-1}$ at the price of housing $p_{h,t}$, with δ_h being the depreciation rate of housing. The value of the housing stock owned by the household is taxed at the rate τ_t^h. Thus we consider a housing property tax, that is proportional to the value of the current housing stock and is paid every period. The budget constraint of the patient households is given by

$$\begin{aligned} & c_t + p_{h,t}\left[\left(1+\tau_t^h\right)h_t - (1-\delta_h)h_{t-1}\right] + \frac{b_{t+1}^g}{R_t^g} + \frac{b_{t+1}}{R_t} \\ &= \left(1-\tau_t^n\right) w_t n_t + b_t^g + b_t, \end{aligned} \tag{4}$$

where c_t denotes consumption spending, $\frac{b_{t+1}^g}{R_t^g}$ investment in new government bonds with the relating gross interest rate $R_t^g = 1 + r_t^g$ and b_t privately issued debt with the gross interest rate $R_t = 1 + r_t$. The patient household will hold positive amounts of $b_t^g > 0$ and $b_t > 0$ and hence be lender in equilibrium. That's why the collateral constraint (1) will be irrelevant for patient households: $b_{t+1} > 0 > -mp_{h,t+1}h_t$.

2.1.2 Impatient Households

The budget constraint of the representative impatient household analogously reads

$$c'_t + p_{h,t}\left[\left(1+\tau'^{h}_t\right)h'_t - (1-\delta_h)h'_{t-1}\right] + \frac{b'^{g}_{t+1}}{R^g_t} + \frac{b'_{t+1}}{R_t} \tag{5}$$
$$= \left(1-\tau^n_t\right)w_t n'_t + b'^{g}_t + b'_t.$$

Since we rule out short sales in government bonds, the impatient households will set $b'^{g}_{t+1} = b'^{g}_t = 0$. Furthermore, this type will be the private borrower in equilibirium, i.e. $b'_{t+1} = -\frac{s}{1-s}b_{t+1} < 0$, following from the market clearing condition for private debt $(1-s)\,b'_{t+1} + sb_{t+1} = 0$. Hence, the collateral constraint (1) will become relevant here. Therefore, there is a a limit on the obligations of impatient households which is given by $b'_{t+1} \geq -mp_{h,t+1}h'_t$.

2.2 Government

The government levies a flat-rate tax on labor income τ^n_t and a housing property tax $\tau^{(\prime)h}_t$ and issues one-period bonds ($b^{(\prime)g}_t \geq 0 \ \forall t \geq 0$) to finance an exogenous stream of government expenditures (g_t):

$$g_t - \frac{b^g_{t+1}}{R^g_t} + b^g_t = s\tau^h_t p_{h,t}h_t + (1-s)\,\tau'^{h}_t p_{h,t}h'_t + \tau^n_t w_t n^T_t, \tag{6}$$

where $n^T_t = sn_t + (1-s)\,n'_t$ denotes total labor supply. As mentioned before, the different housing tax rates τ^h_t and τ'^{h}_t can be understood as taxing the wealthier agents which will be the patient households in equilibrium at a rate that differs from (and will be higher than) the one for the wealth-poor impatient households which will own smaller houses in equilibrium.

2.3 Firms

The production side of the economy is characterized by two sectors, one of which produces consumption goods y_c and the other housing y_h. In both sectors there is a continuum of firms, which are assumed to produce with the same technology for simplicity. The representative firm of each sector produces its output with labor according to

$$y_{c,t} = n_{c,t}^T$$

$$y_{h,t} = n_{h,t}^T,$$

where total labor input in each sector is given by the weighted sum of labor input of the patient and impatient household in this sector $n_{c,t}^T = sn_{c,t} + (1-s)\,n'_{c,t}$ and $n_{h,t}^T = sn_{h,t} + (1-s)\,n'_{h,t}$. On the other hand, total labor supply $n_t^T = sn_t + (1-s)\,n'_t = n_{c,t}^T + n_{h,t}^T$ is splitted between the two types of firms. Labor is assumed to be totally mobile between the two sectors leading to a wage rate that is the same for both sectors.

2.4 Competitive Equilibrium

We now describe the competitive equilibrium of the private sector and then set up the Ramsey problem.

Patient Households

A patient household chooses the values of c_t, h_t, n_t, b_{t+1}^g and b_{t+1} to maximize

(2) subject to the budget constraint (4) leading to the first order conditions

$$h_t^{-\mu^h} = \left(1+\tau_t^h\right) p_{h,t} c_t^{-\mu^c} - \beta c_{t+1}^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} \tag{7}$$

$$n_t^{\mu^n} = \left(1-\tau_t^n\right) w_t c_t^{-\mu^c} \tag{8}$$

$$c_t^{-\mu^c} = \beta R_t^g c_{t+1}^{-\mu^c} \tag{9}$$

$$c_t^{-\mu^c} = \beta R_t c_{t+1}^{-\mu^c}. \tag{10}$$

Equation (7) describes housing demand. In the optimum, the marginal utility of current housing $h_t^{-\mu^h}$ equals the marginal utility of foregone consumption $c_t^{-\mu^c}$ at the gross price of housing $\left(1+\tau_t^h\right) p_{h,t}$ less the discounted marginal utility of next period's consumption $\beta c_{t+1}^{-\mu^c}$ achieved from selling the house after depreciation $(1-\delta_h)$ at the price $p_{h,t+1}$. Equation (8), that is fairly standard, describes labor supply of a patient household and equates the marginal rate of substitution between consumption and leisure $\frac{n_t^{\mu^n}}{c_t^{-\mu^c}}$ to the net real wage rate $\left(1-\tau_t^n\right) w_t$. Equations (9) and (10) are Euler equations with respect to public and private lending.

Impatient Households

An impatient household chooses the values of c_t', h_t', n_t' and b_{t+1}' to maximize (2) subject to the budget constraint (5) *and* the collateral constraint (1) leading to the first order conditions

$$h_t'^{-\mu^h} = \left(1+\tau_t^h\right) p_{h,t} c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} + \omega_t m p_{h,t+1} \tag{11}$$

$$n_t'^{\mu^n} = \left(1-\tau_t^n\right) w_t c_t'^{-\mu^c} \tag{12}$$

$$\omega_t = \frac{c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} R_t}{R_t} \tag{13}$$

and the complementary slackness conditions

$$\omega_t \left(b'_{t+1} + mp_{h,t+1} h'_t \right) = 0, \; b'_{t+1} + mp_{h,t+1} h'_t \geq 0, \; \omega_t \geq 0.$$

Equation (11) decribes housing demand of an impatient household. The term $\omega_t mp_{h,t+1}$ stems from the collateral constraint, with ω_t being the multiplier on this constraint. Equation (12) is the labor supply function of an impatient household. Equation (13) is the modified Euler equation resulting from the fact that the impatient household is borrowing constrained. In the steady state, the collateral constraint will be binding as we can see from (10) which becomes $\frac{1}{R} = \beta$ and (13) leading to $\omega = c'^{-\mu^c} \left(1/R - \beta'\right) = c'^{-\mu^c} \left(\beta - \beta'\right) > 0$. Finally, from the complementary slackness conditions we get $b' + mp_h h' = 0 \Leftrightarrow b' = -mp_h h'$.

Furthermore, the transversality conditions $\lim_{t\to\infty} \beta^t u_t^c \frac{-b_{t+1}^g}{R_t^g} = 0$ and $\lim_{t\to\infty} \beta^t u_t^c \frac{b'_{t+1}}{R_t} = 0$ must hold, of which the latter is redundant due to the collateral constraint that is more restrictive.

Firms

In both sectors the representative firm maximizes profits according to

$$\max_{n_{c,t}^T} \Pi_{c,t} = \max_{n_{c,t}^T} \left(n_{c,t}^T - w_t n_{c,t}^T \right)$$

in the final consumption goods sector and

$$\max_{n_{h,t}^T} \Pi_{h,t} = \max_{n_{h,t}^T} \left(p_{h,t} n_{h,t}^T - w_t n_{h,t}^T \right)$$

in the housing sector leading to the first order conditions

$$w_t = 1 \text{ and } p_{h,t} = 1.$$

Aggregate Ressource Constraint

Finally, due to identical production thechnologies and perfect mobility of labor between the two sectors, the aggregate ressource constraint is given by (see Appendix 5.1)

$$c_t^T + g_t + p_{h,t} h_t^T = y_{c,t} + p_{h,t} y_{h,t} + (1 - \delta_h) p_{h,t} h_{t-1}^T. \tag{14}$$

2.5 The Ramsey Problem

We assume that the government has access to a commitment technology and is able bind itself to its policy. The government chooses the values of h_t, c_t, n_t, h_t', c_t', n_t' and the tax rates τ_t^h, $\tau_t'^h$ and τ_t^n in order to maximize social welfare subject to the private sector equilibrium conditions, the ressource and the implementability constraint summarized in Appendix 5.2.1, while financing an exogenous stream of government expenditures $\{g_t\}_{t=0}^{\infty}$. Following Monacelli (2008), in this economy with two types of agents, social welfare is measured by the weighted sum of utility of the two types

$$\sum_{t=0}^{\infty} \beta^t s u(c_t, h_t, n_t) + \beta'^t (1 - s) u(c_t', h_t', n_t')$$

and the aggregate discount rate is defined as $\widetilde{\beta} = \beta^s \beta'^{(1-s)}$ to be used as discount rate for the constraints. For the mathematical formulation of the Ramsey problem see Appendix 5.2.2. The first order conditions of the Ramsey problem and the steady state are derived in Appendix 5.2.4.

3 Results

This section presents and discusses optimal taxation results of the model. First, as a natural starting point of the analysis, results for the representative agent version, which can be derived analytically, will be given. The relation of these results to existing literature on optimal taxation will be discussed. Lateron, numerical results for the full version of the model will be given and compared with the results of the representative agent version in order to point out the role of the collateral constraint. Finally, we will compare the role of the difference in discount rates against the role of the collateral constraint and present sensitivity analyses.

3.1 Representative Agent Version

By setting the discount rate of the impatient agents equal to the one of the patient agents, $\beta' = \beta$, the model collapses to a representative agent version. For this version, we can derive analytical solutions for the steady state tax rates which are the labor income tax τ^n and the housing property tax τ^h.

The optimal steady state tax rate on labor income is given by (see Appendix 5.3.1)

$$\tau^n = \frac{\phi\left(\mu^n + \mu^c\right)}{1 + \phi\left(1 + \mu^n\right)} > 0 \text{ for } \phi > 0,$$

and is positive for $\phi > 0$. It only depends on the multiplier on the implementability constraint $\phi \geq 0$ and the parameters μ^c and μ^h.

The optimal steady state tax rate on housing is given by (see Appendix 5.3.1)

$$\tau^h = \frac{\phi}{1-\phi} \underbrace{\frac{\mu^h - \mu^c}{\mu^h - 1}}_{(i)} \underbrace{(1 - \beta(1 - \delta_h))}_{(ii)}. \tag{15}$$

This equation reflects two features of housing: (i) can be attributed to the fact that housing delivers utility like consumption and (ii) to the durability of housing.

For $\phi > 0$, the sign of the tax rate, thus the question whether housing should be taxed or subsidized, depends on the parameters μ^h and μ^c. For the sign of τ^h, the term (ii) in (15) can be neglected since $1 - \underbrace{\beta(1-\delta_h)}_{\in(0,1)}$ is positive. Here, the analysis has to be restricted to values of $\phi < \phi^* = \frac{1}{\mu^h - 1}$, since for larger values the second derivatives become positive resulting in minima (see Appendix 5.3.2).

As mentioned before, the sign of τ^h only depends on the term (i) in (15). From principles of optimal taxation we know that goods with lower elasticities should be taxed at a higher rate. Since we do not consider a consumption tax at all, whether housing should be taxed or subsidized depends on whether its intertemporal elasticity of substitution is lower or higher than the one of consumption. There are three cases:

1. For $\mu^c = \mu^h$ housing and consumption should be treated identically due to indentical intertemporal elasticities of substitution, leading to an optimal tax rate on housing of zero.

2. If the elasticity of housing is smaller than the one of consumption, i.e. $\frac{1}{\mu^c} > \frac{1}{\mu^h} \Leftrightarrow \mu^c < \mu^h$, the optimal housing tax rate is positive.

3. For $\mu^c > \mu^h$ the optimal housing tax rate is negative since the elasticity of consumption is smaller than the one of housing.

These results are compatible with the ones of Eerola and Määttänen (2009) who consider a more general representative agent framework with capital and optimal taxation of capital in addition to housing.

While the term (ii) in (15) is irrelevant for the sign of τ^h, it has a large effect on the size of it. For the baseline calibration (see Table 1) for instance, it reduces the housing tax by more than 97%. However, the higher δ_h is, i.e. the lower the durability of housing is, the smaller is the impact of (ii) on the size of τ^h. Notice, that (ii) disappears for the case $\delta_h = 1$, where durability of housing is assumed away and housing fully depreciates within one period.

3.2 Results of the Full Version

Since analytical results are not available for the full version, we consider numerical results for the steady state, where the collateral constraint is binding, as we have seen before in section 2.4. For comparison, we also give numerical results for the representative agent version and the baseline calibration.

3.2.1 Calibration

In this section, the baseline calibration of the model is described. Following Iacoviello (2005), one time period is set to one quarter and the discount factor of patient households to $\beta = 0.99$ leading to a steady state gross real interest rate of $R = 1.01$, which is equivalent to an annual real interest rate of 4%. The discount factor of impatient households is set to $\beta' = 0.95$ by Iacoviello (2005) as a compromise of the estimates given in the literature, which is adopted here. However, in section 3.3, we will consider a variation in β' between 0.95 and 0.97 to see how this affects the result. In order to get a wage share of patient households equal to $\frac{swn}{swn+(1-s)wn'} = 0.64$ as in Iacoviello (2005), we set $s = 0.62$, while we will also show in the sensitivity analyses 3.4 how a variation in population shares alters the results. Furthermore, we set the pledgeable fraction of housing to $m = 0.55$ resulting from an estimation of Iacoviello (2005). Hence, an impatient agent can only borrow to 55% of the value of his house. We will also consider in section 3.3, how a variation in m between 0 and 1, which covers all relevant values for m, affects the results. The depreciation rate of housing is set following Davis and Heathcote (2005), who estimate an annual rate of 1.41%. According to this we set $\delta_h = 0.0035$ for a quarter.

In the calibration of the utility parameters μ^c and μ^n we follow King and Rebelo (1999), who say that the basic RBC model with log utility in consumption implies a labor supply elasticity of 4. Hence, we set $\mu^c = 1$ and $\mu^n = 1/4$, while we will also conduct robustness checks for both of these parameters in section 3.4.

Since the aim of the paper is to evaluate optimal taxation of housing, the utility parameter of housing μ^h is calibrated in order to match an empirical fact on housing. According to Iacoviello (2009), where some stylized facts about housing, that sould be matched when calibrating models of housing, are listed, total housing stock is on average 1.5 times as large as annual GDP in the US between 1952 and 2008. Therefore, we set the parameter μ^h in order to match this value. Since in the model

Table 1: Baseline Parameter Calibration.

Description	Source/Target	Parameter	Value
Discount factor patient households	Iaco. 2005	β	0.99
Discount factor impatient households	Iaco. 2005	β'	0.95
Pledgable fraction of housing	Iaco. 2005	m	0.55
Depreciation rate of housing	D&H 2005	δ_h	0.0035
Share of the patient households	wage share = 0.64	s	0.62
Inverse of Frisch elasticity	K&R 1999	μ^n	1/4
Inverse of IES in consumption	K&R 1999	μ^c	1
Inverse of IES in housing	$h^T/y = 6$	μ^h	1.75
Government expenditures	$g/y = 0.17$	g	0.172
Government debt	$b^g/y = 3$	b^g	3.1

one time period is one quarter and hence y in the notation of the model denotes quarterly GDP, we have to multiply this value by four and to match the ratio of total housing stock to quarterly GDP of $\frac{h^T}{y} = 6$. This is achieved by setting $\mu^h = 1.75$ leading to an elasticitiy of $\frac{1}{\mu^h} = 4/7$. Nevertheless, we will also give sensitivity results concerning the parameter μ^h in section 3.4.

For the calibration of governmental variables g and b^g we use data from the Worldbank (2012). In 2010, US general government final consumption expenditures amounted to 17% of annual GDP. Since both, government expenditures and GDP are flow variables, the ratio is the same for a time period of one quarter, $\frac{g}{y} = 0.17$. Moreover, US total central government debt made up 76.8% of annual GDP in 2010. Since government debt is a stock variable, this value again has to be multiplied by four. Hence the ratio we have to match in terms of quarterly GDP is given by $\frac{b^g}{y} = 3$. These values of the governmental variables are achieved by setting $g = 0.172$ and $b^g = 3.1$. The baseline parameter calibration is summarized in Table 1.

Given this parameter calibration we compute the steady state numerically, which delivers the optimal values of consumption, housing and labor for both types of agents as well as the optimal tax rates τ^h, τ'^h and τ^n.

3.2.2 Numerical Results

The results of the full and the representative agent version for the baseline calibration are summarized in Table 2. Notice, that the optimal tax rate on housing in the representative agent version is close to zero but still positive ($\tau^h = 0.2\%$), while for the full model we get two housing tax rates that differ both markedly from zero. The optimal housing tax rate for patient households is $\tau^h = 1.65\%$ and the one for impatient households $\tau'^h = -2.72\%$. Thus for the baseline calibration, it is optimal to subsidize housing of impatient and hence constrained households and to tax patient ones in the full version, while in the representative agent version housing is taxed at a rate close to zero. Hence the subsidy for impatient households results from the heterogeneity in patience rates and the collateral constraint, that are absent in the representative agent version.

To see how this subsidy optimally is financed, we consider the government budget (6) in the steady state

$$g + (1-\beta)\, b^g = \tau^n n^T + s\tau^h h + (1-s)\, \tau'^h h'. \tag{16}$$

Expenditures are given by $g + (1-\beta)\, b^g = 0.203$ and revenues by $\tau^n n^T + s\tau^h h + (1-s)\, \tau'^h h' = 0.1887 + 0.0668 - 0.0526 = 0.203$. We see, that the labor income tax finances government expenditures, while the housing subsidy for impatient households is financed for the most part by a housing tax on the patient households.

Therefore, the housing tax rate on the patient households is much larger than the tax rate on housing in the representative agent version. This point becomes clearer, when we consider the case $g = b^g = 0$ (last column of Table 2). For this case the left hand side of the government budget (16) is zero, $g + (1 - \beta) b^g = 0$ and a large decline in the labor income tax rate. On the right hand of (16) we have revenues from taxing labor income equal to $\tau^n n^T = 0.029$, revenues from taxing housing of patient households given by $s\tau^h h = 0.069$ and housing subsidies for impatient households equal $(1 - s) \tau'^h h' = -0.098$. Once again we see that the largest part, more than 70%, of housing subsidies are financed by taxing housing of patient households. Hence, this can be interpreted as a redistribution from wealth-rich, i.e. patient households with a higher housing stock ($h = 6.5$) to wealth-poor households with a lower housing stock ($h' = 5.1$).

To link these results to the empirical findings described in the introduction, we compute the ratio of total housing subsidies to GDP given by $\frac{-(1-s)\tau'^h h'}{swn+(1-s)wn'}$. For the baseline calibration we get a ratio of 5.24%. Hence, according to the model the granted subsidies in the US that added up to 1.5% of GDP in 2011 seem to be lower than what would be optimal. On the other hand, the model is likely to overestimate housing subsidies since it does not incorporate physical capital. Housing is the only component of wealth in the model, while in the US it accounts for half of total household wealth (see e.g. Iacoviello (2009)).

Table 2: Numerical Results - Comparison.

Version	Repr. Agent	Full Version	
Calibration	Baseline	Baseline	$g = b^g = 0$
c	0.8161	0.7999	0.9485
h	9.6310	6.5323	7.4249
n	1.0218	1.0630	1.0954
c'	–	0.8316	1.0136
h'	–	5.0929	6.6759
n'	–	0.9100	0.8398
τ^n	0.1795	0.1878	0.0296
τ^h	0.0020	0.0165	0.0149
τ'^h	–	−0.0272	−0.0388

3.3 Discounting vs. Collateral Constraint

The result of subsidizing impatient agents' housing is due to two features of the model, as we have seen in the previous section, the different discount rates of the two types and the collateral constraint, while the former is necessary for the latter. Without different discount rates the model collapses to the representative agent version where private borrowing and hence the collateral constraint are irrelevant.

The aim of this section is to analyze how these two features affect housing subsidies. Therefore we first define the two effects related to these two features. Housing subsidies stemming from the collateral constraint that are granted by the Ramsey planer in order to soften the constraint and hence originate from the market friction are attributed to the *collateral effect*, whereas housing subsidies that purely stem from the difference in discounting and hence are based on preferences are attibuted to the *discount rate effect*. To identify how housing subsidies are affected by these two effects, we conduct the following experiment. First, we consider a variation in

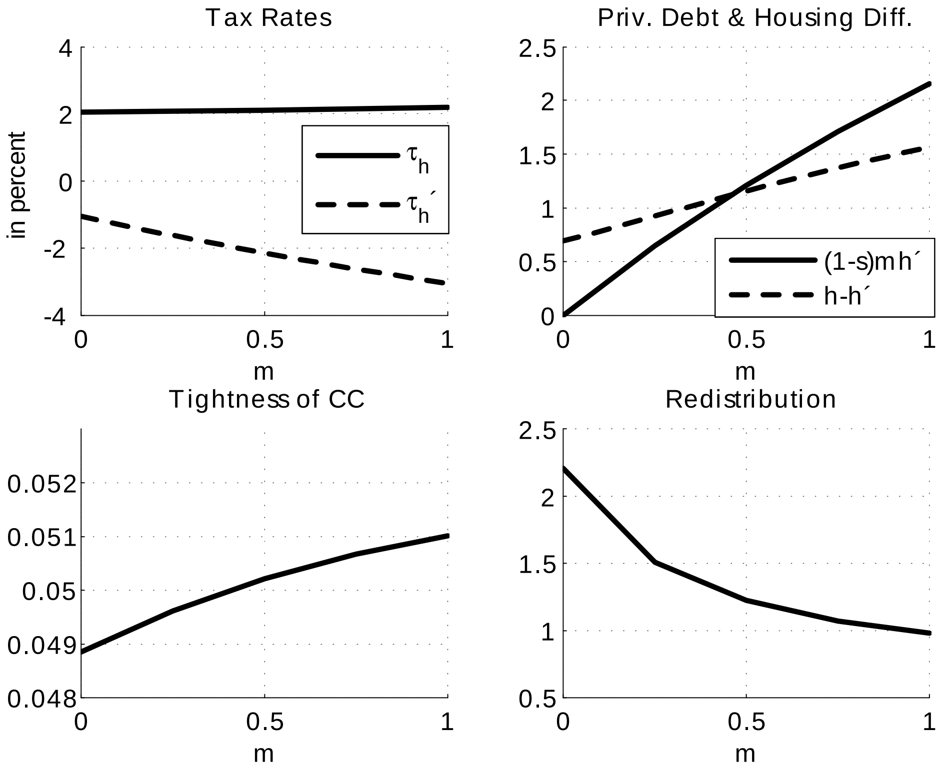

Figure 1: Effects of varying the pledgeable fraction of housing m for the baseline calibration with $s = 0.5$.

the pledgeable fraction of housing, m, reaching from 0 to 1 and illustrate in Figure 1 how this affects the housing tax rates τ^h and τ'^h, private debt given by $(1-s)\,mh'$, the difference in housing stocks of the two agents, $h - h'$, the tightness of the collateral constraint measured by $\omega = c'^{-\mu^c}(\beta - \beta')$ (see (13)) and redistribution measured by the ratio of revenues from taxing housing of the patient agents to the subsidies that impatient agents receive: $Red = -\frac{sh\tau^h}{(1-s)h'\tau'^h}$. The plots are given for the benchmark calibration with equal shares, $s = 0.5$, for convenience in aggregation. Then we do the same for a variation in the borrowers' discount rate between $\beta' = 0.95$ and $\beta' 0.97$.

First, consider the lower limit $m = 0$, where private borrowing and hence the collateral effect is shut down (see Iacoviello (2005) for a similar experiment). Since the link between borrowing and housing of the impatient household is cut off, in this case the resulting level of subsidies is only due the discount rate effect. Then the variation in m between the lower and upper limit $m = 1$, where housing is fully pledgeable, illustrates the role of the collateral effect compared to the discount rate effect for a given $\beta' = 0.95$. Figure 1 shows that a higher pledgeable fraction of housing leads to a larger amount of private debt (panel 2) and hence to a tighter collateral constraint (panel 3) resulting in a higher level of housing subsidies for the constrained households (panel 1, dashed line), whereas the tax rate on the patient agents does not change much (panel 1, solid line). This is due to the fact that the collateral constraint and hence the parameter m is not directly relevant for the patient agents. Thus, the level of redistribution (panel 4), as it is measured here, decreases in m since housing subsidies to impatient agents rise faster than housing tax revenues from patient ones do.

For $m = 0$, where the collateral channel is shut down, the resulting subsidy is $\tau'^h = -1.04\%$, whereas for the baseline case of $m = 0.55$ it more than doubles to $\tau'^h = -2.24\%$. This makes clear that housing subsidies not only result from a difference in preference parameters but are also due to the market friction, the collateral constraint. Regarding the rates just mentioned and taking into account that the discount rate channel dampens the effect of the collateral channel, which is discussed below, more than half of the resulting subsidies can be attributed to the

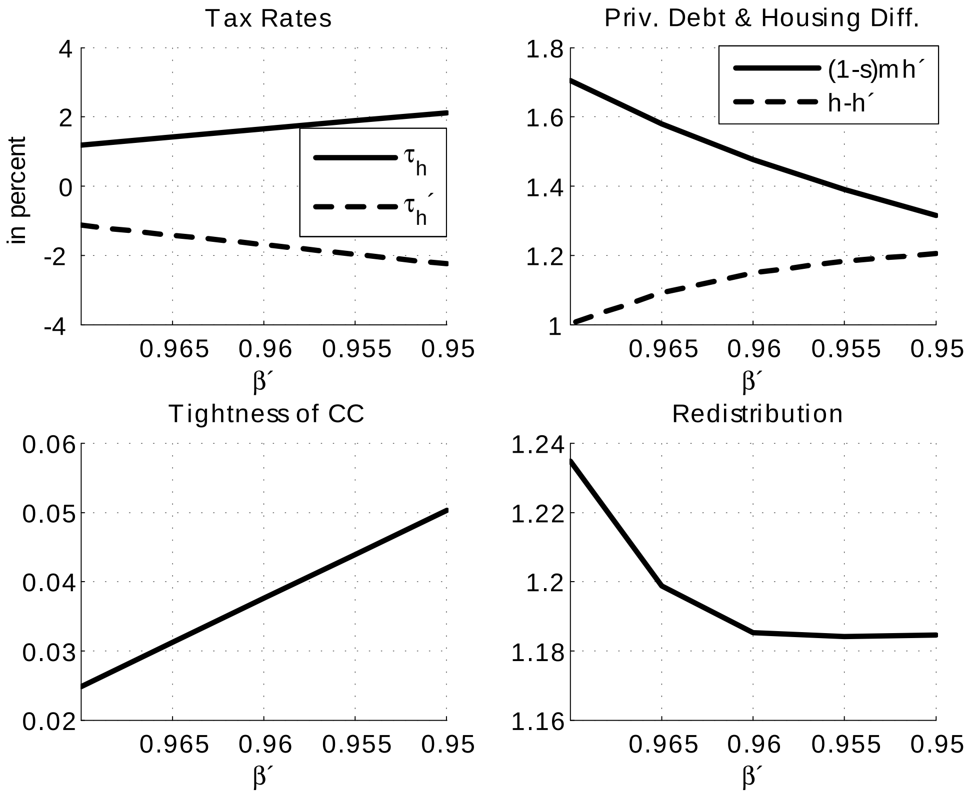

Figure 2: Effects of varying the impatient agents' discount rate β' for the baseline calibration with $s = 0.5$.

collateral constraint in the baseline calibration.

Figure 2 plots the results for a variation in β'. Notice that β' decreases, i.e. the difference in discount rates increases from left to right on the abscissa. The higher this difference is, the larger is the housing subsidy for impatient agents τ'^h (panel 1, dashed line) and the housing tax for patient agents τ^h (panel 1, solid line). In contrast to the variation in m the variation in β' affects both rates equally. As for a higher m the level of redistribution (panel 4) decreases in the difference in discount rates for the same reason. In contrast, unlike a higher m leading to

Table 3: Numerical Results - Robustness.

	Baseline Calibration, except			
	–	$\mu^h = 1.5$	$\mu^c = 2$	$s = 0.5$
c	0.7999	0.7933	0.8713	0.7950
h	6.5323	9.2023	6.8915	5.9908
n	1.0630	1.0965	1.1581	1.0595
c'	0.8316	0.8396	0.8945	0.8198
h'	5.0929	6.8126	5.6412	4.7861
n'	0.9100	0.8739	0.9383	0.9370
τ^n	**0.1878**	**0.1882**	**0.2124**	**0.1935**
τ^h	0.0165	0.0150	0.0124	0.0212
τ'^h	**−0.0272**	**−0.0281**	**−0.0366**	**−0.0224**

higher borrowing, a larger discount rate difference lowers borrowing since it reduces housing of the impatient agents. Hence, we can conclude that the discount rate effect dampens the collateral effect in reducing private borrowing.

3.4 Sensitivity Analyses

In the previous section, we have seen that the main result of optimality of housing subsidies to impatient agents is robust to variations in the parameters m and β'. In this section, we will check whether it is also robust to changes in the parameters μ^c, μ^h and s. Two interesting questions come here in mind. The first question is, what happens if the intertemporal elasticities are changed, i.e. if $\mu^h < \mu^c$. Since we have seen that this changed the sign of the housing tax in the representative agent version, it is interesting to see how this change in the parameters will affect optimal taxation in the full version. Another question we will explore is what happens, if the share of lenders s is changed. We will consider the case where both types have equal shares $s = 0.5$. Table 3 summarizes the results.

First of all, we can conclude from Table 3, that for every parameter variation we consider, it remains optimal to subzidize housing of impatient households and to tax housing of patient ones.

In the third column where we lower μ^h, housing demand rises and both types have higher housing stocks ($\frac{h^T}{y} \approx 8.2$) compared to the baseline calibration in column 2 of Table 3. Athough τ^h is lower, tax revenues from taxing housing of patient agents are higher due to their higher housing stock $h = 9.2$. Therefore, subsidies for impatient households can increase slightly.

In column 4, we set $\mu^c = 2 > \mu^h = 1.75$ and we see that, in contrast to the respresentative agent version, there is no important change in the tax rates. Moreover, τ'^h becomes larger while τ^h decreases, since households attach a higher value to housing compared to consumption. As a result, both types work more to own a larger house, while the labor income tax increases to finance the subsidies.

In column 5 the share of lenders in the economy is lower than in the baseline calibration. This means that there are less wealth-rich households in the economy who bear the tax burden. Therefore, the tax rates τ^n and τ^h are higher while the subsidy τ'^h is lower. As a result both types of households have lower consumption and housing levels.

Summing up, in every variation we considered, m, β', μ^h and s, the main result of the paper holds: it is optimal to disburden the impatient and hence constrained households by subsidizing their housing.

4 Conclusion

Housing subsidies that can be observed in many industrialized countries have been subject to macroeconomic research since many years. Nevertheless, there is no definite conclusion one can draw from this research. While the opponents point at inefficiencies resulting from housing subsidies due to distortions in investment decisions of agents, the proponents argue that subsidies internalize externalities accompanied with homeownership.

This paper, where we have studied optimal taxation of housing in a borrower-lender framework resulting from different discount rates with housing being used as collateral for private loans, provides results in favor of housing subsidies. The main result of this paper is that in such an economy, optimal fiscal policy should disburden impatient borrowers by subsidizing their housing in the presence of collateral constraints. This subsidy has to be financed to the largest extent by a housing tax on the patient and unconstrained households and to a smaller part by a labor income tax. Hence, redistribution from patient/unconstrained households to impatient/constrained ones takes place.

In this framework housing subsidies result from two features of the model, the different discount rates of the two types of agents and the collateral constraint. We have seen that for the baseline calibration more than half of the subsidy can be attributed to the collateral constraint. Hence, housing subsidies not only result from the difference in preference parameters but are also due to the market friction in our model. Moreover, the sensitivity analyses showed that the main result of housing subsidies for constrained households is robust to several parameter variations.

Furthermore, we considered a representative agent version of the model the results of which are quite intuitive and in line with the principles of optimal taxation. For the baseline calibration however, it was not optimal to subsidize housing in this environment unlike for the model with two types and collateral constraints.

This paper gives a rationale for housing subsidies other than externalities that have been focused on in previous literature and indicates a new path for further research. An extension of the model could be the addition of inter-generational heterogeneity in an overlapping generations model as in Gervais (2002). The life cycle behavior of agents could also have substantial implications and should also be accounted for when trying to measure the effects of housing subsidies on social welfare. This is left for future research.

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/countries on December 12, 2012.

5 Appendix

5.1 Aggregate Ressource Constraint

Consolidation of the budget constraints (4), (5) and (6) delivers

$$
\begin{aligned}
& sc_t + (1-s)\, c'_t + sp_{h,t} \left[\left(1+\tau^h_t\right) h_t - (1-\delta_h) h_{t-1} \right] \\
& + (1-s)\, p_{h,t} \left[\left(1+\tau'^h_t\right) h'_t - (1-\delta_h) h'_{t-1} \right] + g_t \\
= \; & s\left(1-\tau^n_t\right) w_t n_t + (1-s)\left(1-\tau^n_t\right) w_t n'_t \\
& + s\tau^h_t p_{h,t} h_t + (1-s)\, \tau'^h_t p_{h,t} h'_t,
\end{aligned}
$$

since the terms b_t, b'_t and b^g_t cancel out. With $x^T_t = sx_t + (1-s)\, x'_t$ for aggregate variables this becomes to

$$
\begin{aligned}
& c^T_t + p_{h,t} \left[\left(1+\tau^h_t\right) h^T_t - (1-\delta_h) h^T_{t-1} \right] + g_t \\
= \; & \left(1-\tau^n_t\right) w_t n^T_t + \tau^h_t p_{h,t} h^T_t,
\end{aligned}
$$

which can further be simplified to

$$
\begin{aligned}
& c^T_t + p_{h,t} h^T_t + g_t \\
= \; & w_t n^T_t + p_{h,t}(1-\delta_h) h^T_{t-1}.
\end{aligned}
$$

Inserting the production functions, we get (14).

5.2 Solution of the Full Version

5.2.1 Summary of Private Sector Equilibrium Conditions

Summarizing the private sector equilibrium conditions delivers

$$h_t^{-\mu^h} = \left(1+\tau_t^h\right) p_{h,t} c_t^{-\mu^c} - \beta c_{t+1}^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1}$$

$$n_t^{\mu^n} = \left(1-\tau_t^n\right) w_t c_t^{-\mu^c}$$

$$c_t^{-\mu^c} = \beta R_t c_{t+1}^{-\mu^c}$$

$$c_t^{-\mu^c} = \beta R_t^g c_{t+1}^{-\mu^c}$$

$$h_t'^{-\mu^h} = \left(1+\tau_t'^h\right) p_{h,t} c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} + \omega_t m p_{h,t+1}$$

$$n_t'^{\mu^n} = \left(1-\tau_t^n\right) w_t c_t'^{-\mu^c}$$

$$\omega_t = \frac{c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} R_t}{R_t}$$

$$c_t' + \left(1+\tau_t'^h\right) p_{h,t} h_t' = \left(1-\tau_t^n\right) w_t n_t' + (1-\delta_h) p_{h,t} h_{t-1}' - \frac{b_{t+1}'}{R_t} + b_t'$$

$$0 = \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \right) \left[g_t - s\tau_t^h p_{h,t} h_t - (1-s)\,\tau_t'^h p_{h,t} h_t' - \tau_t^n w_t n_t^T \right] + b_0^g$$

$$y_{c,t} = n_{c,t}^T, \quad y_{h,t} = n_{h,t}^T, \quad w_t = 1, \quad p_{h,t} = 1$$

$$h_t^T = s h_t + (1-s)\, h_t', \quad n_t^T = s n_t + (1-s)\, n_t', \quad c_t^T = s c_t + (1-s)\, c_t'$$

$$n_t^T = n_{c,t}^T + n_{h,t}^T, \quad b_{t+1}' \geq -m p_{h,t+1} h_t',$$

$$c_t^T + g_t + p_{h,t} h_t^T = y_{c,t} + p_{h,t} y_{h,t} + (1-\delta_h) p_{h,t} h_{t-1}^T$$

Eliminating prices and using $\frac{\beta^t c_t^{-\mu^c}}{c_0^{-\mu^c}} = \prod_{i=0}^{t-1} \left(R_i^g\right)^{-1}$ the conditions above can be reduced to

$$
\begin{aligned}
h_t^{-\mu^h} &= \left(1+\tau_t^h\right) c_t^{-\mu^c} - \beta\left(1-\delta_h\right) c_{t+1}^{-\mu^c} \\
n_t^{\mu^n} c_t^{\mu^c} &= \left(1-\tau_t^n\right) \\
R_t^g = R_t &= \frac{c_t^{-\mu^c}}{\beta c_{t+1}^{-\mu^c}} \\
h_t'^{-\mu^h} &= \left(1+\tau_t'^h\right) c_t'^{-\mu^c} - \beta'\left(1-\delta_h\right) c_{t+1}'^{-\mu^c} + m\left[\frac{c_t'^{-\mu^c}}{c_t^{-\mu^c}} \beta c_{t+1}^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c}\right] \\
n_t'^{\mu^n} c_t'^{\mu^c} &= \left(1-\tau_t^n\right) \\
c_t' + \left(1+\tau_t'^h\right) h_t' &= \left(1-\tau_t^n\right) n_t' + (1-\delta_h) h_{t-1}' + \frac{m h_t'}{c_t^{-\mu^c}} \beta c_{t+1}^{-\mu^c} - m h_{t-1}' \\
0 &= \sum_{t=0}^{\infty}\left(\frac{\beta^t c_t^{-\mu^c}}{c_0^{-\mu^c}}\right)\left[g_t - s\tau_t^h h_t - (1-s)\tau_t'^h h_t' - \tau_t^n n_t^T\right] + b_0^g \\
& s c_t + (1-s) c_t' + g_t + s h_t + (1-s) h_t' \\
& \quad = s n_t + (1-s) n_t' + (1-\delta_h)\left(s h_{t-1} + (1-s) h_{t-1}'\right)
\end{aligned}
$$

given $b_0^g > 0$ and $b_0 > 0$.

5.2.2 The Ramsey Problem

The Ramsey problem reads

$$J=\sum_{t=0}^{\infty}\left\{\begin{array}{c}\beta^{t}su(c_t,h_t,n_t)+\beta'^{t}\left(1-s\right)u(c'_t,h'_t,n'_t)\\ +\widetilde{\beta}^{t}\lambda_{t,1}\left[h_t^{-\mu^h}-\left(1+\tau_t^h\right)c_t^{-\mu^c}+\beta\left(1-\delta_h\right)c_{t+1}^{-\mu^c}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,2}\left[n_t^{\mu^n}c_t^{\mu^c}-1+\tau_t^n\right]+\widetilde{\beta}^{t}\lambda_{t,3}\left[n_t'^{\mu^n}c_t'^{\mu^c}-1+\tau_t^n\right]\\ +\widetilde{\beta}^{t}\lambda_{t,4}\left[\begin{array}{c}h_t'^{-\mu^h}-\left(1+\tau_t'^h\right)c_t'^{-\mu^c}+\beta'\left(1-\delta_h\right)c_{t+1}'^{-\mu^c}\\ -m\left(c_t'^{-\mu^c}c_t^{\mu^c}\beta c_{t+1}^{-\mu^c}-\beta' c_{t+1}'^{-\mu^c}\right)\end{array}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,5}\left[\begin{array}{c}-c'_t-\left(1+\tau_t'^h\right)h'_t+\left(1-\tau_t^n\right)n'_t+(1-\delta_h)h'_{t-1}\\ +\frac{mh'_t}{c_t^{-\mu^c}}\beta c_{t+1}^{-\mu^c}-mh'_{t-1}\end{array}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,6}\left[\begin{array}{c}-sc_t-\left(1-s\right)c'_t-g_t-sh_t-\left(1-s\right)h'_t\\ +sn_t+\left(1-s\right)n'_t+\left(1-\delta_h\right)\left(sh_{t-1}+\left(1-s\right)h'_{t-1}\right)\end{array}\right]\\ +\beta^{t}\lambda_7\frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\left[g_t-s\tau_t^h h_t-\left(1-s\right)\tau_t'^h h'_t-\tau_t^n\left(sn_t+\left(1-s\right)n'_t\right)\right]+\beta^{t}\lambda_7 b_0^g\end{array}\right\}$$

where $\lambda_{t,i}$ denotes the Langrange multiplier on constraint i in period t, while the multiplier λ_7 on the implementabilitiy constraint, which is derived in Appendix 5.2.3, has no time index since it is an intertemporal constraint. The first order conditions of the Ramsey problem are derived in Appendix 5.2.4, where also the steady state of the problem is given.

5.2.3 Intertemporal Government Budget Constraint

The intertemporal government budget constraint is derived as follows. We write the government budget (6) for $t+1$ and solve for

$$b_{t+1}^g=s\tau_{t+1}^h p_{h,t+1}h_{t+1}+\left(1-s\right)\tau_{t+1}'^h p_{h,t+1}h'_{t+1}+\tau_{t+1}^n w_{t+1}n_{t+1}^T-g_{t+1}+\frac{b_{t+2}^g}{R_{t+1}^g}$$

and insert this in the one for t

$$g_t - \frac{1}{R_t^g}\left[s\tau_{t+1}^h p_{h,t+1} h_{t+1} + (1-s)\,\tau_{t+1}'^h p_{h,t+1} h_{t+1}' + \tau_{t+1}^n w_{t+1} n_{t+1}^T - g_{t+1} + \frac{b_{t+2}^g}{R_{t+1}^g}\right] + b_t^g$$
$$= s\tau_t^h p_{h,t} h_t + (1-s)\,\tau_t'^h p_{h,t} h_t' + \tau_t^n w_t n_t^T.$$

This can be rewritten as

$$\begin{aligned} g_t + \frac{g_{t+1}}{R_t^g} - \frac{b_{t+2}^g}{R_t^g R_{t+1}^g} + b_t^g &= s\tau_t^h p_{h,t} h_t + (1-s)\,\tau_t'^h p_{h,t} h_t' \\ &+ \frac{s\tau_{t+1}^h p_{h,t+1} h_{t+1} + (1-s)\,\tau_{t+1}'^h p_{h,t+1} h_{t+1}'}{R_t^g} + \tau_t^n w_t n_t^T + \frac{\tau_{t+1}^n w_{t+1} n_{t+1}^T}{R_t^g}. \end{aligned}$$

Iterating on this we get with the transversality condition on government debt the intertemporal government budget constraint

$$\begin{aligned} &\sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}(R_i^g)^{-1}\right) g_t + b_0^g \\ &= \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}(R_i^g)^{-1}\right) s\tau_t^h p_{h,t} h_t + (1-s)\,\tau_t'^h p_{h,t} h_t' + \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}(R_i^g)^{-1}\right)\tau_t^n w_t n_t^T \\ &\Leftrightarrow \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}(R_i^g)^{-1}\right)\left[g_t - s\tau_t^h p_{h,t} h_t - (1-s)\,\tau_t'^h p_{h,t} h_t' - \tau_t^n w_t n_t^T\right] + b_0^g = 0. \end{aligned}$$

5.2.4 First Order Conditions and Steady State

The first order conditions of the Ramsey problem can be summarized by:

$$\lambda_{t,1}c_t^{-\mu^c} + \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} sh_t = 0$$

$$\lambda_{t,2} + \lambda_{t,3} - \lambda_{t,5}n_t' - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} n_t^T = 0$$

$$\lambda_{t,4}c_t'^{-\mu^c} + \lambda_{t,5}h_t' + \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} (1-s)\, h_t' = 0$$

$$\begin{aligned}
\widehat{\beta}^t \frac{sc_t}{\mu^c} + \lambda_{t,1}\left(1+\tau_t^h\right) + \lambda_{t,2}n_t^{\mu^n} c_t^{2\mu^c} - \lambda_{t,4}mc_t'^{-\mu^c}\beta c_{t+1}^{-\mu^c} c_t^{2\mu^c} \\
+\lambda_{t,5}mh_t'\beta c_{t+1}^{-\mu^c} c_t^{2\mu^c} - \lambda_{t,6}\frac{sc_t^{\mu^c+1}}{\mu^c} - \widehat{\beta}^t \frac{\lambda_7}{c_0^{-\mu^c}}\left[g_t - s\tau_t^h h_t - (1-s)\,\tau_t'^h h_t' - \tau_t^n n_t^T\right] \\
-\widehat{\beta}\lambda_{t-1,1}(1-\delta_h) + \widehat{\beta}\lambda_{t-1,4}mc_{t-1}'^{-\mu^c} c_{t-1}^{\mu^c} - \widehat{\beta}\lambda_{t-1,5}mh_{t-1}' c_{t-1}^{\mu^c} = 0
\end{aligned}$$

$$\widehat{\beta}^t h_t^{-\mu^h} - \lambda_{t,1}\frac{\mu^h}{s}h_t^{-\mu^h-1} - \lambda_{t,6} - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^h + \lambda_{t+1,6}\widetilde{\beta}(1-\delta_h) = 0$$

$$-\widehat{\beta}^t n_t^{\mu^n} + \lambda_{t,2}\frac{\mu^n}{s}c_t^{\mu^c} n_t^{\mu^n-1} + \lambda_{t,6} - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^n = 0$$

$$\begin{aligned}
\overline{\beta}^t \frac{(1-s)\,c_t'}{\mu^c} + \lambda_{t,3}n_t'^{\mu^n} c_t'^{2\mu^c} + \lambda_{t,4}\left[\left(1+\tau_t'^h\right) + mc_t^{\mu^c}\beta c_{t+1}^{-\mu^c}\right] - \frac{\lambda_{t,5}}{\mu^c c_t'^{-\mu^c-1}} \\
-\frac{\lambda_{t,6}(1-s)}{\mu^c c_t'^{-\mu^c-1}} - \overline{\beta}\lambda_{t-1,4}[1-\delta_h+m] = 0
\end{aligned}$$

$$\begin{aligned}
\overline{\beta}^t h_t'^{-\mu^h} - \lambda_{t,4}\frac{\mu^h}{(1-s)}h_t'^{-\mu^h-1} - \frac{\lambda_{t,5}}{(1-s)}\left[\left(1+\tau_t'^h\right) - mc_t^{\mu^c}\beta c_{t+1}^{-\mu^c}\right] - \lambda_{t,6} \\
-\widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t'^h + \widetilde{\beta}\lambda_{t+1,5}\frac{(1-\delta_h-m)}{(1-s)} + \widetilde{\beta}\lambda_{t+1,6}(1-\delta_h) = 0
\end{aligned}$$

$$-\overline{\beta}^t n_t'^{\mu^n} + \lambda_{t,3}\frac{\mu^n}{(1-s)}c_t'^{\mu^c} n_t'^{\mu^n-1} + \lambda_{t,5}\frac{(1-\tau_t^n)}{(1-s)} + \lambda_{t,6} - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^n = 0,$$

with $\widehat{\beta}^t = \frac{\beta^t}{\widetilde{\beta}^t} = \left(\frac{\beta}{\beta^s \beta'^{(1-s)}}\right)^t = \left[\left(\frac{\beta}{\beta'}\right)^{1-s}\right]^t$ and $\overline{\beta}^t = \frac{\beta'^t}{\widetilde{\beta}^t} = \left(\frac{\beta'}{\beta^s \beta'^{(1-s)}}\right)^t = \left[\left(\frac{\beta'}{\beta}\right)^s\right]^t$.

Assuming that we are initially in the steady state ($c_0 = c$ for $t = 0$), where variables without subscript denote steady state values henceforth, these conditions read in the steady state

$$\lambda_1 c^{-\mu^c} + \lambda_7 sh = 0 \tag{17}$$

$$\lambda_2 + \lambda_3 - \lambda_5 n' - \lambda_7 n^T = 0 \tag{18}$$

$$\lambda_4 c'^{-\mu^c} + \lambda_5 h' + \lambda_7 (1-s) h' = 0 \tag{19}$$

$$\frac{sc}{\mu^c} + \lambda_1 \left[1 + \tau^h - \widehat{\beta}(1-\delta_h)\right] + \lambda_2 (1-\tau^n) c^{\mu^c}$$
$$+\lambda_4 m c'^{-\mu^c} c^{\mu^c} \left(\widehat{\beta} - \beta\right) + \lambda_5 m h' c^{\mu^c} \left(\beta - \widehat{\beta}\right) - \lambda_6 \frac{s c^{\mu^c+1}}{\mu^c} \tag{20}$$

$$-\lambda_7 c^{\mu^c} \left[g - s\tau^h h - (1-s)\tau'^h h' - \tau^n n^T\right] = 0 \tag{21}$$

$$h^{-\mu^h} \left(1 - \lambda_1 \frac{\mu^h}{sh}\right) + \lambda_6 \left[\widetilde{\beta}(1-\delta_h) - 1\right] - \lambda_7 \tau^h = 0 \tag{22}$$

$$-n^{\mu^n} + \lambda_2 \frac{\mu^n (1-\tau^n)}{sn} + \lambda_6 - \lambda_7 \tau^n = 0 \tag{23}$$

$$\frac{(1-s) c'}{\mu^c} + \lambda_3 (1-\tau^n) c'^{\mu^c} + \lambda_4 \left[1 + \tau'^h + m\beta - \overline{\beta}(1-\delta_h+m)\right]$$
$$-\frac{\lambda_5}{\mu^c c'^{-\mu^c-1}} - \frac{\lambda_6 (1-s)}{\mu^c c'^{-\mu^c-1}} = 0 \tag{24}$$

$$h'^{-\mu^h} \left(1 - \lambda_4 \frac{\mu^h}{(1-s) h'}\right) + \lambda_5 \left[\frac{\widetilde{\beta}(1-\delta_h-m) - 1 - \tau'^h + m\beta}{1-s}\right]$$
$$+\lambda_6 \left[\widetilde{\beta}(1-\delta_h) - 1\right] - \lambda_7 \tau'^h = 0 \tag{25}$$

$$-n'^{\mu^n} + \lambda_3 \frac{\mu^n (1-\tau^n)}{(1-s) n'} + \lambda_5 \frac{(1-\tau^n)}{(1-s)} + \lambda_6 - \lambda_7 \tau^n = 0. \tag{26}$$

The private sector equilibrium conditions that determine the steady state together with the first order conditions of the Ramsey problem (17)-(26) are given by

$$h^{-\mu^h} = c^{-\mu^c} \left[\left(1+\tau^h\right) - \beta \left(1-\delta_h\right) \right] \tag{27}$$

$$n^{\mu^n} c^{\mu^c} = (1-\tau^n) \tag{28}$$

$$R^g = R = \frac{1}{\beta}$$

$$h'^{-\mu^h} = c'^{-\mu^c} \left[\left(1+\tau'^h\right) - \beta' \left(1-\delta_h\right) + m\left(\beta - \beta'\right) \right] \tag{29}$$

$$n'^{\mu^n} c'^{\mu^c} = (1-\tau^n) \tag{30}$$

$$c' = n' \left(1-\tau^n\right) + h' \left[m\left(\beta-1\right) - \delta_h - \tau'^h \right] \tag{31}$$

$$g + (1-\beta) b^g = s\tau^h h + (1-s) \tau'^h h' + \tau^n \left(sn + (1-s) n'\right) \tag{32}$$

$$sc + (1-s) c' + g = sn + (1-s) n' - \delta_h sh - \delta_h (1-s) h'. \tag{33}$$

5.3 Representative Agent Version

5.3.1 Solution

The first order conditions of the representative household are given by (with $u_t^x = \frac{\partial u}{\partial x_t}$)

$$u_t^h + \beta u_{t+1}^c \left(1-\delta_h\right) p_{h,t+1} = u_t^c \left(1+\tau_t^h\right) p_{h,t} \tag{34}$$

$$u_t^n = -u_t^c w_t \left(1-\tau_t^n\right) \tag{35}$$

$$u_t^c = u_{t+1}^c \beta R_t^g \tag{36}$$

and the transversality condition on bonds holds $\lim_{t\to\infty} \beta^t u_t^c \frac{b_{t+1}^g}{R_t^g} = 0$. Inserting (36) in (34) delivers the relationship bewteen the marginal utilities of housing and consumption

$$\frac{u_t^h}{u_t^c} = \left(1+\tau_t^h\right) p_{h,t} - \frac{(1-\delta_h)}{R_t^g} p_{h,t+1}. \tag{37}$$

The first order conditions of the firms lead to the real wage rate $w_t = 1$and the price of housing $p_{h,t} = 1$ and the aggregate ressource constraint reads

$$c_t + g_t + h_t - (1-\delta_h)h_{t-1} = n_t \tag{38}$$

The Ramsey problem is to maximize social welfare subject to the aggregate ressource contraint (38) and the implementability constraint (46), which is derived in the appendix 5.3.3, and can be written as

$$J = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t, h_t, n_t) + \phi\left[u_t^c c_t + u_t^h h_t + u_t^n n_t\right] \\ +\rho_t\left[-c_t - g_t - h_t + n_t + (1-\delta_h)h_{t-1}\right] \end{array} \right\} + \phi\left[(1-\delta_h)p_{h,0}h_{-1} + b_0^g\right].$$

Insertion of the marginal utilities leads to

$$J = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t, h_t, n_t) + \phi\left\{c_t^{1-\mu^c} + h_t^{1-\mu^h} - n_t^{1+\mu^n}\right\} \\ +\rho_t\left[-c_t - g_t - h_t + n_t + (1-\delta_h)h_{t-1}\right] \end{array} \right\} + \phi\left[(1-\delta_h)p_{h,0}h_{-1} + b_0^g\right].$$

The first order conditions of the Ramsey problem are given by

$$\frac{\partial J}{\partial c_t} = 0 \Rightarrow \rho_t = c_t^{-\mu^c}\left[1+\phi\left(1-\mu^c\right)\right] \tag{39}$$

$$\frac{\partial J}{\partial n_t} = 0 \Rightarrow \rho_t = n_t^{\mu^n}\left[1+\phi\left(1+\mu^n\right)\right]. \tag{40}$$

Equalizing (39) and (40) we get the optimal labor income tax

$$\left(1-\tau_t^n\right) =^{(35)} \frac{n_t^{\mu^n}}{c_t^{-\mu^c}} = \frac{\left[1+\phi\left(1-\mu^c\right)\right]}{\left[1+\phi\left(1+\mu^n\right)\right]} \tag{41}$$

$$\Rightarrow \tau^n = 1 - \frac{\left[1+\phi\left(1-\mu^c\right)\right]}{\left[1+\phi\left(1+\mu^n\right)\right]} = \frac{\phi\left(\mu^n+\mu^c\right)}{1+\phi\left(1+\mu^n\right)} > 0 \text{ for } \phi > 0.$$

As we can see in

$$\frac{\partial \tau^n}{\partial \phi} = \frac{\mu^n + \mu^c}{\left[1 + \phi\left(1 + \mu^n\right)\right]^2} > 0,$$

the labor income tax is increasing in ϕ and is concave for $\mu^c \geq 1$ (see appendix 5.3.2).

The first order condition with respect to housing is given by

$$\begin{aligned}
\frac{\partial J}{\partial h_t} = 0 &\Rightarrow h_t^{-\mu^h} + \phi\left(1 - \mu^h\right) h_t^{-\mu^h} - \rho_t + \beta \rho_{t+1}\left(1 - \delta_h\right) = 0 \\
&\Leftrightarrow \rho_t = h_t^{-\mu^h} + \phi\left(1 - \mu^h\right) h_t^{-\mu^h} + \rho_{t+1}\beta\left(1 - \delta_h\right) \\
&\Rightarrow^{(39)} c_t^{-\mu^c}\left[1 + \phi\left(1 - \mu^c\right)\right] = h_t^{-\mu^h} + \phi\left(1 - \mu^h\right) h_t^{-\mu^h} \\
&\quad + c_{t+1}^{-\mu^c}\left[1 + \phi\left(1 - \mu^c\right)\right]\beta\left(1 - \delta_h\right) \\
&\Rightarrow \left[1 + \phi\left(1 - \mu^c\right)\right] = \underbrace{\frac{h_t^{-\mu^h} + c_{t+1}^{-\mu^c}\beta\left(1 - \delta_h\right)}{c_t^{-\mu^c}}}_{\left(1 + \tau_t^h\right)} \\
&\quad + \phi\left(1 - \mu^h\right)\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} + \underbrace{\frac{c_{t+1}^{-\mu^c}\beta}{c_t^{-\mu^c}}}_{1/R_t^g}\phi\left(1 - \mu^c\right)\left(1 - \delta_h\right).
\end{aligned}$$

Hence, the optimal tax rate on housing can be written as

$$\begin{aligned}
\tau_t^h &= \phi\left(1-\mu^c\right) - \phi\left(1-\mu^h\right)\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} - \frac{\phi\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g} \\
&= \phi\left[1-\mu^c-\left(1-\mu^h\right)\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} - \frac{\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&=^{(43)} \phi\left[1-\mu^c-\left(1-\mu^h\right)\left[\left(1+\tau_t^h\right)-\frac{\left(1-\delta_h\right)}{R_t^g}\right] - \frac{\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&= \phi\left[1-\mu^c-\left(1-\mu^h\right)\left(1+\tau_t^h\right)+\frac{\left(1-\mu^h\right)\left(1-\delta_h\right)}{R_t^g} - \frac{\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&= \phi\left[1-\mu^c-1+\mu^h-\left(1-\mu^h\right)\tau_t^h+\frac{\left(1-\mu^h-1+\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&\Rightarrow \tau_t^h\left[1+\phi\left(1-\mu^h\right)\right] = \phi\left[\mu^h-\mu^c-\frac{\left(\mu^h-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
\tau_t^h &= \frac{\phi}{1+\phi\left(1-\mu^h\right)}\left(\mu^h-\mu^c\right)\left(1-\frac{\left(1-\delta_h\right)}{R_t^g}\right),
\end{aligned}$$

with $\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} = \left(1+\tau_t^h\right)-\frac{(1-\delta_h)}{R_t^g}$ following from (43). In the steady state with $R_t^g = \beta^{-1}$ the optimal housing tax rate is given by

$$\tau^h = \frac{\phi\left(\mu^h-\mu^c\right)}{1-\phi\left(\mu^h-1\right)}\left(1-\beta\left(1-\delta_h\right)\right). \tag{42}$$

The housing tax rate is increasing in ϕ for $\mu^c < \mu^h$ and decreasing for case $\mu^c > \mu^h$, as

$$\frac{\partial\tau^h}{\partial\phi} = \frac{\left(\mu^h-\mu^c\right)}{\left[1-\phi\left(\mu^h-1\right)\right]^2}\left[1-\beta\left(1-\delta_h\right)\right]$$

shows. Furthermore, τ^h is convex for case 2 and $\mu^h \geq 1$ (see Appendix 5.3.2).

5.3.2 Additionals on Optimal Tax Rates

Second Derivatives

The second derivatives wrt housing are given by

$$\frac{\partial^2 J}{\partial h_t^2} = -\underbrace{\mu^h h_t^{-1-\mu^h}}_{>0} \left[1 - \phi\left(\mu^h - 1\right)\right].$$

Whether this expression is positive or negative depends on the last term. We get $\frac{\partial^2 J}{\partial h_t^2} < 0$ and thus a maximum for

$$1 - \phi\left(\mu^h - 1\right) > 0 \Leftrightarrow \phi < \frac{1}{\left(\mu^h - 1\right)}.$$

For c and n the second derivatives are always positive for $1 \leq \mu^c \leq 2$, which is the case in the benchmark calibration,

$$\begin{aligned} \frac{\partial^2 J}{\partial c_t^2} &= -\underbrace{\mu^c c_t^{-1-\mu^c}}_{>0} \left[1 - \phi\left(\mu^c - 1\right)\right] < 0 \\ &\Leftrightarrow \phi < \frac{1}{\left(\mu^c - 1\right)} \geq 1 \text{ for } 1 \leq \mu^c \leq 2, \\ \text{and } \frac{\partial^2 J}{\partial n_t^2} &= -\underbrace{\mu^n n_t^{-1+\mu^n}}_{>0} \left[1 + \phi\left(\mu^n + 1\right)\right] < 0. \end{aligned}$$

Shape of the Tax Rates

The labor income tax in conave in ϕ for $\mu^c \geq 1$ since

$$\begin{aligned} \frac{\partial^2 \tau^n}{\partial \phi} &= \frac{1 + \phi + \phi\mu^n - 2\mu^n - 2\mu^c - 2\left(\mu^n\right)^2 - 2\mu^c\mu^n}{\left[1 + \phi\left(1 + \mu^n\right)\right]^3} \\ &= \frac{\overbrace{1 + \phi - 2\mu^c}^{<0 \text{ for } \mu^c \geq 1} + \overbrace{\phi\mu^n - 2\mu^n}^{<0} - 2\left(\mu^n\right)^2 - 2\mu^c\mu^n}{\left[1 + \phi\left(1 + \mu^n\right)\right]^3} < 0. \end{aligned}$$

The housing tax is convex for $\mu^c < \mu^h$ and $\mu^h > 1$ due to

$$\frac{\partial \tau^h}{\partial \phi} = \frac{\left(\mu^h - \mu^c\right)}{\left[1 - \phi\left(\mu^h - 1\right)\right]^2}\left[1 - \beta\left(1 - \delta_h\right)\right]$$

$$\frac{\partial^2 \tau^h}{\partial \phi^2} = \frac{\left(\overbrace{1 - \phi\left(\mu^h - 1\right)}^{>0 \text{ for } \phi < \frac{1}{(\mu^h - 1)}} + \overbrace{2\left(\mu^h - 1\right)}^{>0 \text{ for } \mu^h > 1}\right)\left(\mu^h - \mu^c\right)\left[1 - \beta\left(1 - \delta_h\right)\right]}{\left[1 - \phi\left(\mu^h - 1\right)\right]^3} > 0.$$

5.3.3 Derivation of the Implementability Constraint

The implementability constraint is derived as follows. We write (37) in the form

$$R_t^g = \frac{(1 - \delta_h) p_{h,t+1}}{\left(1 + \tau_t^h\right) p_{h,t} - \frac{u_t^h}{u_t^c}} \tag{43}$$

and rewrite condition (36):

$$u_t^c = u_{t-1}^c \left(\beta R_{t-1}^g\right)^{-1} \text{ and } u_{t-1}^c = u_{t-2}^c \left(\beta R_{t-2}^g\right)^{-1} \Rightarrow u_t^c = u_{t-2}^c \beta^{-2} \left(R_{t-1}^g\right)^{-1} \left(R_{t-2}^g\right)^{-1}.$$

Iterating forward we get

$$\beta^t u_t^c = u_0^c \prod_{i=0}^{t-1} \left(R_i^g\right)^{-1}. \tag{44}$$

Thus we can rewrite the transversality condition as (with $u_0^c > 0$)

$$\lim_{t \to \infty} \prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \frac{b_{t+1}^g}{R_t^g} = 0.$$

Now we solve the household budget constraint for period $t+1$ for b_{t+1}^g

$$b_{t+1}^g = c_{t+1} + \left(1 + \tau_{t+1}^h\right) p_{h,t+1} h_{t+1} + \frac{b_{t+2}^g}{R_{t+1}^g} - \left(1 - \tau_{t+1}^n\right) w_{t+1} n_{t+1} - (1 - \delta_h) p_{h,t+1} h_t$$

and insert this in the one for period t to get

$$c_t + \left(1+\tau_t^h\right) p_{h,t} h_t + \frac{1}{R_t^g} \left[\begin{array}{c} c_{t+1} + \left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1} + \frac{b_{t+2}^g}{R_{t+1}^g} \\ - \left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1} - (1-\delta_h) p_{h,t+1} h_t \end{array} \right] - b_t^g$$
$$= \left(1-\tau_t^n\right) w_t n_t + (1-\delta_h) p_{h,t} h_{t-1}.$$

This can be rewritten as

$$c_t + \frac{c_{t+1}}{R_t^g} + \left(1+\tau_t^h\right) p_{h,t} h_t - \frac{(1-\delta_h) p_{h,t+1} h_t}{R_t^g} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} + \frac{b_{t+2}^g}{R_t^g R_{t+1}^g}$$
$$= \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g.$$

We now collect the terms with h_t, factor out h_t and insert (43)

$$\begin{aligned} & h_t \left[\left(1+\tau_t^h\right) p_{h,t} - \frac{(1-\delta_h) p_{h,t+1}}{R_t^g} \right] \\ &=^{(43)} h_t \left[\left(1+\tau_t^h\right) p_{h,t} - \frac{(1-\delta_h) p_{h,t+1}}{(1-\delta_h) p_{h,t+1}} \left(\left(1+\tau_t^h\right) p_{h,t} - \frac{u_t^h}{u_t^c} \right) \right] \\ &= h_t \left[\left(1+\tau_t^h\right) p_{h,t} - \left(\left(1+\tau_t^h\right) p_{h,t} - \frac{u_t^h}{u_t^c} \right) \right] = h_t \frac{u_t^h}{u_t^c}. \end{aligned} \tag{45}$$

Thus we can rewrite the buget constraint again to get

$$c_t + \frac{c_{t+1}}{R_t^g} + h_t \frac{u_t^h}{u_t^c} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} + \frac{b_{t+2}^g}{R_t^g R_{t+1}^g}$$
$$= \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g.$$

Inserting the budget constraint of $t+2$ then delivers

$$c_t + \frac{c_{t+1}}{R_t^g} + h_t \frac{u_t^h}{u_t^c} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g}$$
$$+ \frac{1}{R_t^g R_{t+1}^g} \left[c_{t+2} + \left(1+\tau_{t+2}^h\right) p_{h,t+2} h_{t+2} + \frac{b_{t+3}^g}{R_{t+2}^g} - \left(1-\tau_{t+2}^n\right) w_{t+2} n_{t+2} - (1-\delta_h) p_{h,t+2} h_{t+1} \right]$$
$$= \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g.$$

We can rewrite this as

$$
\begin{aligned}
&c_t + \frac{c_{t+1}}{R_t^g} + \frac{c_{t+2}}{R_t^g R_{t+1}^g} + h_t \frac{u_t^h}{u_t^c} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} - \frac{(1-\delta_h) p_{h,t+2} h_{t+1}}{R_t^g R_{t+1}^g} \\
&+ \frac{\left(1+\tau_{t+2}^h\right) p_{h,t+2} h_{t+2}}{R_t^g R_{t+1}^g} + \frac{b_{t+3}^g}{R_t^g R_{t+1}^g R_{t+2}^g} = \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} \\
&+ \frac{\left(1-\tau_{t+2}^n\right) w_{t+2} n_{t+2}}{R_t^g R_{t+1}^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g.
\end{aligned}
$$

Repeating the steps above in (45) we get

$$
\begin{aligned}
&c_t + \frac{c_{t+1}}{R_t^g} + \frac{c_{t+2}}{R_t^g R_{t+1}^g} + h_t \frac{u_t^h}{u_t^c} + \frac{h_{t+1}}{R_t^g} \frac{u_{t+1}^h}{u_{t+1}^c} + \frac{\left(1+\tau_{t+2}^h\right) p_{h,t+2} h_{t+2}}{R_t^g R_{t+1}^g} + \frac{b_{t+3}^g}{R_t^g R_{t+1}^g R_{t+2}^g} \\
&= \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + \frac{\left(1-\tau_{t+2}^n\right) w_{t+2} n_{t+2}}{R_t^g R_{t+1}^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g.
\end{aligned}
$$

Iterating on this and using the transversality conditions, we get the intertemporal budget constraint with the initial endowments of h_{-1} and b_0^g

$$
\begin{aligned}
&\sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \right) c_t + \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \right) h_t \frac{u_t^h}{u_t^c} \\
&= \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \right) \left(1-\tau_t^n\right) w_t n_t + (1-\delta_h) p_{h,0} h_{-1} + b_0^g,
\end{aligned}
$$

which can be rewritten as

$$
\sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \right) \left[c_t + h_t \frac{u_t^h}{u_t^c} - \left(1-\tau_t^n\right) w_t n_t \right] + (1-\delta_h) p_{h,0} h_{-1} + b_0^g = 0.
$$

By eliminating prices with (44) and (35) we get the implementability constraint

$$
\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \frac{u_t^c}{u_0^c} \left[c_t + h_t \frac{u_t^h}{u_t^c} + \frac{u_t^n}{u_t^c} n_t \right] + (1-\delta_h) p_{h,0} h_{-1} + b_0^g = 0. \\
\Leftrightarrow^{u_0^c > 0} &\sum_{t=0}^{\infty} \beta^t \left[u_t^c c_t + u_t^h h_t + u_t^n n_t \right] + (1-\delta_h) p_{h,0} h_{-1} u_0^c + b_0^g u_0^c = 0. \qquad (46)
\end{aligned}
$$