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Pupils' competence in proofs and argumentation and their beliefs on mathematics – A comparative study between Korea and Germany

1. Introduction and Theoretical Background

Since the 1970s research on mathematical beliefs has become an important branch of research in mathematics education. Pehkonen (1995) indicated that problem solving competency depends not only on the students' mathematical knowledge or mathematical abilities, but also on their beliefs on mathematics. So beliefs on mathematics might be a reason for students' difficulties in solving mathematical tasks. Accordingly, beliefs on mathematics should be regarded as an explaining factor, which influences mathematical problem solving.

Törner and Grigutsch (1994) used the expression belief in the sense of a "mathematical world view", in accordance with the definition of Schoenfeld (1985). They have contributed significantly to the empirical research on mathematical beliefs in Germany. They concentrated on the various attitudes towards mathematics, related to four aspects of a mathematical world view. These aspects may be addressed as schema, formalism, process, and application. This paper will keep to the interpretation of Törner and Grigutsch.

2. Design of the Study

The data of 659 German 7th grade pupils in 27 classes and 189 Korean 7th grade pupils in 5 classes were collected[1]. These data related to competency in proof and argumentation on geometry questions and were taken from the completed questionnaires on beliefs about mathematics. We used a questionnaire designed by Törner and Grigutsch (1994) and revised by Klieme (2001). This questionnaire consists of 24 items, and was scaled between 1 (= totally agree) and -1 (= totally disagree).

2.1. Research questions

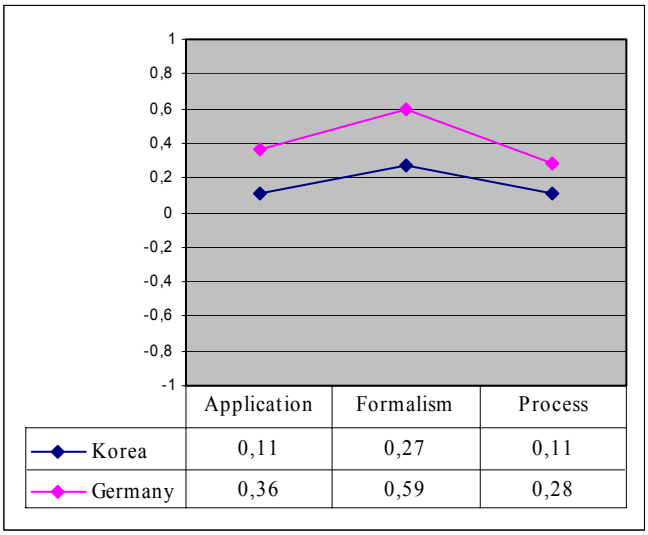
Our research aims are to compare pupils' beliefs about mathematics between Korea and Germany and to identify the relationship between beliefs and pupils' achievement. We address the following research question:

[1] This German data is from BIQUA Project which is in charged of Prof. Reiss and funded by DFG, on reasoning and proof in the geometry classroom (cf. Reiss, Hellmich & Thomas, 2001).

- Is there a correlation between beliefs and pupils' achievement? If then, which factors could influence on achievement?

3. Results

A factor analysis revealed three categories of mathematical beliefs, namely application, formalism, and process (see Kwak & Reiss, 2002). Interestingly, Korean and German pupils share similar model of views about mathematics as the below graph.



	Application	Formalism	Process
Korea	0,11	0,27	0,11
Germany	0,36	0,59	0,28

However, the ratings given by the German pupils are higher than those given by the Korean pupils. The German pupils agreed more strongly with the given statements than the Korean pupils (T-test: $p<0{,}001$ for all three factors). The aspect of formalism is the dominating factor for both groups of pupils (see Kwak & Reiss, 2002).

3.1. Correlation and associated p-values

The correlation coefficients and associated *p*-values among each of the three belief variables and the cognitive values for German pupils are presented in Table 3-1 and for Korean pupils in Table 3-2.

	Basic competence	Competence in proving	Metho. competence	Test
Application	.020 .479	.055 .052	.059* .036	.048 .081
Formalism	.006 .841	-.016 .561	-.039 .171	-.009 .730
Process	.053 .056	.046 .102	.052 .067	.056* .041

Table 3-1 correlation coefficients and p-values for German pupils (* p<.05. **p<.01)

The results show that there is a significant association between application and methodological competence for the German pupils, although the correlation is fairly weak. Moreover, there is also significant association between process and test, although the correlation is fairly weak. However, the correlation between formalism and competence in proving, and between formalism and methodological competence reported in the table is negative.

	Basic competence	Competence in proving	Metho. competence	Test
Application	.077 .143	.037 .487	.041 .036	.059 .252
Formalism	.135* .011	.072 .177	.046 .389	.101 .052
Process	.129* .016	.157** .003	.040 .459	.161** .000

Table 3-2 correlation coefficients and p-values for Korean pupils (* p<.05. **p<.01)

Unlike German result, there is a significant and positive correlation between formalism and basic competence for the Korean pupils. However, there is no significant correlation between application and methodological competence. Moreover, the results show that there is a significant and positive correlation between process and three cognitive variables (basic competence, competence in proving and test).

3.2. The relationship between the achievement test and beliefs about mathematics

From the results from the achievement test, three groups can be formed according to attainment in the test: a lower, a middle and an upper achievement group. I will examine the relationship between those groups with respect to their beliefs about mathematics.

The one-way analysis of variance (ANOVA) can be used to determine whether or not there were significant differences among the three groups. The results are as follows:

		Sum of squares	df	Mean square	F	p-value
Application	Between groups Within groups Total	.245 19.028 19.273	2 183 185	.123 .104	1.179	.310
Formalism	Between groups Within groups Total	.615 14.066 14.681	2 181 183	.307 .078	3.956	.021*
Process	Between groups Within groups Total	1.456 18.281 19.737	2 183 185	.728 .100	7.287	.001**

Table 3-3 the relationships between the Korean achievement groups with respect to the three belief factors

The results indicate the significant effect of process and formalism with F=7,287 (p=0.001) and F=3,956 (p=0.021) respectively. This means there is a significant difference between the three groups for formalism and process. To find out in which pairings of groups significant differences can be observed, the Scheffe Process can be applied. The result is as follows:

		Lower group	Middle group	High group
Formalism	Lower group		-.0314	-.1360*
	Middle group			-.1046
Process	Lower group		-.0094	-.1926*
	Middle group			-.1832*

Table 3-4 Scheffe Process for comparisons between groups (* p<.05.)

A significant difference between the effect of formalism on the lower group and on the upper group can be observed here. Moreover, a significant difference between the effect of process on the lower and on the upper group and between the effect of process on the middle group and on the upper group can also be observed.

4. Discussion

The relationship between beliefs and achievement groups can be summarised by saying that the Korean upper achievement group believed more strongly that mathematics is formalism-oriented than Korean lower group did. Also, the Korean upper achievement group attached greater importance to the process-oriented quality of mathematics than both the lower group and the middle group. However, any significant differences among the German achievement groups for all three beliefs are not observed. It is common fact that there is no significant difference among the Korean and German groups for the application-oriented belief. We may conclude that the application-oriented belief could not influence the pupils' achievement.

Reference

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