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Software-Gebrauch und Wissensentwicklung am Beispiel dynamischer Geometrie-Software

Abstrakt: Im Vortrag wird davon ausgegangen, dass

1. ein Werkzeug nicht von selbst verständlich ist und die Aneignung spezifischer Verfahren zum adäquaten Gebrauch erfordert (Instrumentierung);
2. eine dynamische Geometrie-Software mathematisches Wissen enthält und ihre Benutzung auch mathematische Kenntnisse erfordert (Konzeptualisierung);
3. solche Umgebungen im Mathematikunterricht benutzt werden können, um den Aufbau von mathematischen Kenntnissen zu fördern.

Mittels einiger Beispiele in *Cabri-geomètre* wird gezeigt, wie Prozesse der Konzeptualisierung in Interaktion mit Prozessen der Instrumentierung durch geeignete Abfolgen von Aufgaben und durch die Vermittlung von Lehrern unterstützt werden. Der Fall des Erwerbs von instrumentellem Wissen für das Konzipieren mathematischer Aufgaben wird ebenfalls erörtert.

1. The nature of mathematical objects and the mediation of knowledge

As so often stated, the nature of mathematical objects is by essence abstract. Mathematical objects are only indirectly accessible through representations (D'Amore 2003 pp.39-43, Duval 2000). Since mathematical knowledge is mediated by representations on which the individual operates, representations can thus be considered as tools in the mathematical activity. The role of tools has been always received less attention in mathematics than in experimental sciences. But with the increasing use of technology, the mathematical activity is also mediated by this new kind of tools that have recourse to specific systems of representations to express mathematics. These systems may deeply differ from what they are in a paper and pencil environment. It is especially the case with dynamic geometry software that offers diagrams of a very specific nature: variable diagrams that can be continuously modified while keeping their geometrical properties when dragged. The direct manipulation of diagrams has a visible spatial effect but has also a mathematical counterpart. The operations performed on dynamic diagrams have a specific nature and this leads to two assumptions that are currently shared by various research works and supported by empirical research.

2. Mathematical knowledge and instrumental knowledge

Two main hypotheses underlie our analysis of the role of technology in the learning and teaching processes.

First hypothesis: A tool is not transparent and using a tool for doing mathematics not only changes the way to do mathematics but also requires a specific appropriation of the tool. In the last decade, some psychologists (Vérillon & Rabardel, 1995) have shown through empirical research, how the tool (also called artefact) itself gives rise to a mental construction by the learner using the tool to solve problems. The *instrument,* according to the terms of Vérillon and Rabardel, denotes this psychological construct of the user. The subject develops procedures and rules of actions when using the artefact and so constructs *utilization schemes* and simultaneously a representation of the properties of the tool. A scheme (used here in the Piagetian sense, Vergnaud 1990) must be understood as an invariant organisation of actions in a given class of situations. The notion of utilization scheme refers to an invariant organisation of actions involving the use of an artefact for solving a type of tasks.

Second hypothesis: Tools like those offered by information technology embed mathematical knowledge (as for example already visible in Cabri from the denominations of menu items, parallel line…) and their use requires the integration of both, knowledge of mathematics and of the tool.

An example of utilization schemes

Let us illustrate this claim with the example of the construction of a parallelogram in Cabri. Students are given two segments AB and AD and they are asked to construct the parallelogram ABCD. In a compass and ruler construction in paper and pencil environment, students would use a strategy based on the congruence of opposite sides. But in Cabri, almost all students use the strategy of constructing parallel lines to the given segments in order to obtain the fourth vertex C. It illustrates very clearly how much the preferred strategy is linked to the domain of efficiency of the tool. Constructing parallel lines in paper and pencil would be more tedious since the ruler and compass construction of a parallel line to a line is based on the construction of a parallelogram. In Cabri, the tool parallel line is available and students have a spontaneous recourse to it since the typical feature of a parallelogram for students is the parallelism of the sides. After parallel lines and point C are constructed (Fig.1), then the two additional sides (or the polygon) have to be constructed and parallel lines must be hidden (Fig.2). In this sequence of actions, called by Verillon and Rabardel (1995) *scheme of instrumented action*, are intertwined both mathematical knowledge and knowledge of how to use the tool for fulfilling the task to produce the dynamic diagram of a parallelogram. The use of the tool affects not only the choice of the construction strategy but also the actions to be done. In Cabri, segments have to be constructed since a segment cannot be obtained as a part of a line (Fig.3).

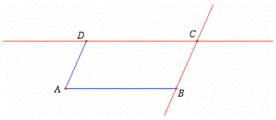

Fig.1

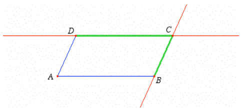

Fig.2

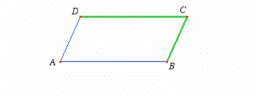

Fig.3

The mathematical knowledge of the user is another critical factor affecting the type of strategy that is used. Students are often successful in constructing a parallelogram in Cabri by obtaining the fourth vertex as the intersecting point of lines parallel to the given segments. But the teacher may expect another strategy, the use of a central symmetry around the midpoint of segment BD. This latter strategy is valid even when the parallelogram is “flat” whilst the former one would not provide a flat parallelogram. The central symmetry strategy is shorter than the parallel line strategy in number of operations. The scheme of instrumented action in Cabri attached to this strategy differs from the preceding one and clearly depends on mathematical knowledge. It involves the invariance of a parallelogram under central symmetry, a geometrical property which is not operational in a paper and pencil environment for constructing a figure. It is generally not proposed by students and must be introduced by the teacher. An instrumented task can thus be the source of reinforcing or introducing knowledge. This is an important issue related to the integration of technology into teaching.

As described above, a scheme of instrumented action involves actions directly linked to a specific use of the artefact. For example, in Cabri, in order to construct a parallel line to a segment, the user has to perform a sequence of elementary actions, selecting a menu, pulling down it, selecting the tool in the menu, showing a point and a line. Each of these actions requires the move of the cursor by using the mouse and clicking. The user has also to construct an invariant organisation attached to the sequence of these elementary actions called *scheme of usage* by Vérillon and Rabardel. The design of interface certainly affects the construction of schemes of usage. However mathematical knowledge is also involved in a scheme of usage. Below are presented two schemes of usage requiring a functional view of a geometrical object, i.e. a conception of geometrical objects as a function of other objects. At middle school or even high school, students do not have such a conception and therefore may encounter difficulties in using tools of DGE[1]. Constructing parallel lines in DGE requires for example designating with the mouse two elements of which a parallel line is function of, the direction (i.e. a line) and a point through which the parallel line is passing. Very often the students working with Cabri we could observe at middle school or even high school showed the direction and were waiting for the parallel line to be drawn and did not understand why the computer did nothing. After a while they clicked anywhere in the screen and more than often they clicked on the line giving the direction. The obtained parallel line was coinciding with the line and could not be seen unless the cursor came near the line and an ambiguity message was displayed "What object?". Understanding this message requires being familiar with the ambiguity notion in Cabri, i.e. a sophisticated knowledge of the tool. Very often students do not understand the situation they have created and a solution can be found only with the help of the teacher.

The interface can make students aware of the necessity of showing the variable elements the object to be constructed is function of. Taking into account this difficulty of students, the designers of Cabri-junior (on the TI 83 Plus or Silver and TI 84) decided to display under the form of a dotted object the temporary spatial position of the geometrical object to be constructed. This temporary position is determined by the first variable element already shown by the user and the current position of the cursor. When the user clicks the final variable element defining the object to be constructed, this latter object is displayed in the usual way (Fig.4). This new interface continuously informs the user.

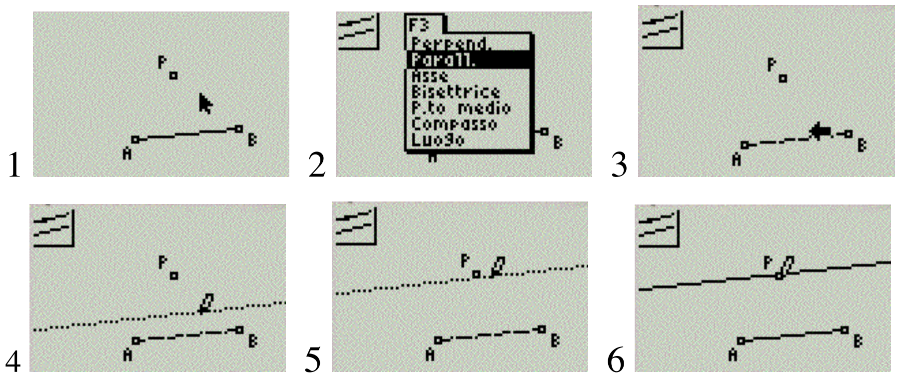

Fig.4 – The interface of the construction of a parallel line to a line in Cabri Junior

Briefly speaking, solving mathematical tasks in a technological environment requires two kinds of intertwined knowledge, mathematical and instrumental.

3. The design of teaching sequences with Cabri

The intertwining of mathematical and instrumental knowledge in the use of Cabri led to the design of teaching sequences based on two different theoretical principles that were combined.

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[1] In the paper DGE denotes Dynamic Geometry Environments

On the one hand, students were faced with problem situations, the solution of which requires the construction of solving means corresponding to a new knowledge item to be learned. Cabri was thus used as a component of the adidactic milieu (Brousseau 1997) in these situations, i.e. an antagonistic system interacting with the student in a problem situation. Cabri offers a wide range of possible actions in the construction or exploration of diagrams as well as feedback, that is often a combination of the use of Cabri tools and of the drag mode.

On the other hand, teaching contributed to the internalization process from the external tools experienced in Cabri towards the construction of the meaning of the mathematical concept. The teacher or the designer of the teaching sequence organized tasks in Cabri, in which the actions based on the use of Cabri tools to solve the task are the counterpart of the theoretical mathematical knowledge whose learning was intended. Then the teacher through social means, in particular through collective discussion and interventions favored the move from external tools to internal tools (Bartolini Bussi and Mariotti1999). The idea of internalization process is not new and is present in the Vygostkian theory of semiotic mediation and of tool. Examples of such internalization processes built around a Cabri tool have been experimented (cf. Sträßer 2002, Jahn 2002 about the notion of pointwise transformation through the tools Trace and Locus) at different school levels

An example: construction tasks for 8 year-old children

The didactical principle was to use geometrical models of real contexts with which children were familiar. It is important to note that children worked with these models in both paper and pencil environment and Cabri. Knowledge that children had about those contexts allowed them to control their solutions and to interpret the feedback received from Cabri.

Dynamic figures representing moving real objects were used. Children were given the Cabri representation of a car or of a boat with a missing element to construct : the mast or the wheel (Fig.5). In the motion, some spatial relations must be preserved. The mast of a boat must stay perpendicular to the deck of the boat in the movement of the waves. The wheel of a car must stay centered at the midpoint of two points of the frame by staying attached to the car in its motion. The real context was chosen to give meaning to the dragging of the diagram[2].

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[2] The new possibility of Cabri II plus of importing pictures and attaching them to geometrical diagrams allows emphasizing the modeling role of real situations that geometry can play.

Fig.5 A boat and a car with a missing element to construct in Cabri

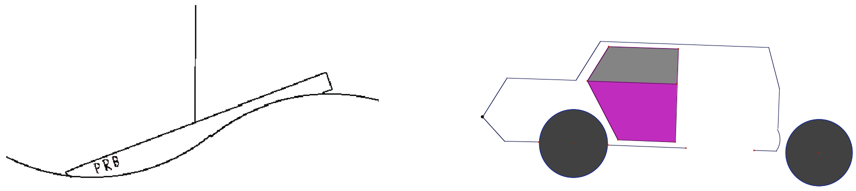

Fig. 6 Invalidation of construction by eye in the drag mode

If children draw by eye the missing element, their solution is obviously invalidated as soon as they drag the diagram (Fig.6). Children immediately were aware of their mistake. They tried to find a means to “link” the element to the diagram. But this was not easy. For example, they were able to describe with gestures the position of the mast with an oblique deck but had difficulties to tell it with words. For the mast, the difficulty lies in identifying the Cabri tool which provides the relevant spatial invariant. Although this situation came at the end of a sequence of tasks about perpendicularity and right angles, it was not easy for children to identify the same spatial relation in the boat task as that one encountered in paper and pencil environment. This was why the role of the teacher was critical to help children express what they should obtain. The difficulty in the missing wheel situation was not to find the tool “circle” whose shape is easily identified as identical to the shape of the wheel but to construct the centre of the circle as the midpoint of two points. In both cases, the geometrical concept is used to model a spatial invariant and this geometrical concept is mediated by the Cabri tool having the same denomination as the geometrical concept. Finding the right tool requires the identification of the geometrical concept.

But instrumental knowledge was also required in the boat situation. When children constructed a perpendicular line to the deck, they were not satisfied with what they obtained. A mast is not going down under the boat. It should be of limited magnitude and only at the top of the boat. The instrumental knowledge of constructing a segment on a line was necessary and no child had constructed this scheme of usage. Therefore intermediate activities aimed at installing this scheme of use were considered by the designers of the sequence as useful to give before the problem situation of the boat. This sequence of activities is currently experimented with several other classes[3].

A design principle emerging from those teaching experiments lies in the intertwining of problem situations and intermediate activities. Problem situations are aimed at creating a break in which new mathematical knowledge has to be constructed in order to find a solution. Intermediate activities are aimed at anchoring the newly constructed mathematical knowledge by linking it to already available mathematical knowledge and at constructing instrumental knowledge (Assude & Gelis 2002). Often at the beginning of the use of ICT in mathematics education, students were faced with complex open-ended problems in the computer environment because of the implicit idea that technology was facilitating the task. The identification of the importance of instrumental knowledge made clear the oversimplification of this view about ICT and the necessity of organizing a gradualness even when using ICT. Nevertheless the importance of situations posing a problem for students is not rejected in our perspective. Such situations must be carefully organized at well-chosen moments of the sequence that allow students to concentrate on the mathematical problem.

4. Instrumentation of Cabri by teachers

Following an instrumental approach, we claim a third hypothesis: Teachers must develop not only utilization schemes for solving tasks with technology but also additional *specific instrumentation schemes for the design of tasks taking advantage of technology*. We call these schemes *second order utilization schemes.*

In a 3 years project teachers had to design teaching scenarios with Cabri for high school (Laborde 2001). From the evolution over time of the type of scenarios, we could distinguish four types of tasks in their order of occurrence:

- tasks in which Cabri is used a s a provider of data, or a facilitator of material actions such as obtaining a set of measures of the angles of a quadrilateral for different positions of its vertices;
- tasks in which Cabri facilitates a mathematical analysis of the problem to be solved or of the situation to be explored thanks t the combination of drag mode and Cabri tools
- tasks that have a paper and pencil counterpart but that can solved differently in Cabri (for example, the construction of a parallelogram by using point symmetry)

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[3] This is done within the research team about technology in Education MAGI "Mieux apprendre la géométrie avec l'informatique" supported by the Ministry of Education

- tasks that could not be posed without the mediation of Cabri.

In the two first kinds of tasks, Cabri is used as an “amplifier” in terms of Pea (1985) whilst in the two others Cabri is used as a “conceptual reorganizer” (Pea, *ibid*.) It must be noticed that the instrumentation of Cabri constructed by teachers, and especially by novice, teachers, gave rise first to the first kinds of tasks and that it is only later that the two other kinds of tasks were designed. The fourth kind was indeed introduced after exchanges with researchers. If we reconsider the evolution of the choices of the teachers, we could interpret the early behaviour of both novice teachers as an unconscious way of trying to avoid interaction of Cabri with the core of the didactic system (anecdotal tasks whose content was not essential for the curriculum) and to limit the scope of the schemes of instrumented actions developed by students only to dragging, observing and measuring.

References

Assude T., Gelis J.-M. (2002) La dialectique ancien-nouveau dans l'intégration de Cabri-géomètre à l'école primaire *Educational Studies in mathematics*, *50.3*, 259-287

Bartolini Bussi M.-L. & Mariotti M.-A. (1999) Semiotic mediation: from history to the mathematics classroom *For the learning of mathematics* 19(2), pp. 27-35

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics.* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Trans. and Eds.). Kluwer, Dordrecht.

D’Amore B. (2003) *Le basi filosofische, pedagogiche, epistemologiche e concettuali della Didattica della Matematica*, Pitagora Editrice, Bologna, Italy

Duval, R. (2000) Basic issues for research in mathematics education, In T. Nakahara, M. Koyama (Eds.), *Procedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, (Vol 1, pp. 55-69) Hiroshima University, Hiroshima, Japan

Jahn, A. P. (2002). "Locus" and "Trace" in Cabri-géomètre: relationships between geometric and functional aspects in a study of transformations. *Zentralblatt für Didaktik der Mathematik*, 34(3), 78-84.

Laborde C. (2001) Integration of technology in the design of geometry tasks with Cabri-geometry *International Journal of Computers for Mathematical Learning* 6, 283-317

Pea, R. (1985). Beyond amplification: Using the computer to reorganise mental functioning. *Educational Psychologist, 20*(4), 167-182.

Sträßer, R. (2002). Research on Dynamic Geometry Software (DGS) - an introduction. *Zentralblatt für Didaktik der Mathematik*, 34(3), 65.

Vergnaud G. (1990) La théorie des champs conceptuels, *Recherches en Didactique des Mathématiques,* vol.10 n°2-3, 133-170

Vérillon P. & Rabardel P. (1995) Cognition and artifacts. A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education* 10(1), 77-101.