Pál MAUS, Eötvös Loránd University Budapest

Problem solving in everyday life and in mathematics

The concepts of everyday life problem and mathematical problem

A lot of everyday life problem situations can be described and solved by using mathematical instruments, therefore these situations play significant role in the teaching of mathematics. However we use everyday life situations mainly in order to reformulate given mathematical problems instead of paying attention to the real life problems. In my opinion, solving real life problems, even without using mathematics, is as important part of teaching mathematics as using mathematics for modeling everyday life problem solving. For being sure, that we all think about the same if I say everyday life problem, I must talk a bit about this concept. The next examples occurred in a little survey, which I am going to talk about later. Here I list them only to illustrate what I mean as everyday life problem and mathematical problem.

1. You are in the staircase of a ten-storey house, where you can see 30 postboxes only with names of the owners, but there are no floor and flat numbers given. How can you find your friend's flat if you know only his name?
2. One morning a postman set off to deliver a few letters. After he delivered the 2/3rd of the letters he had to go back to the office for further 12 urgent letters. By 3 p.m. he delivered the half of the remaining letters, and then he went back to the office for 6 new letters. By the end of his working-time he could deliver the 5/6th of the leaving letters, but there were 3 letters that he could not. How many letters did the postman have at the beginning of the day?
3. A straight line divides the plane into 2 parts. Into how many parts do 2 straight lines divide the plane? What about 8 straight lines, supposing that each of them intersects every other and there are no three of them which have a common point?
4. Choose a few of the next numbers for which the sum of them is dividable by 30! Try to find as many solutions as possible!

   2589, 256, 2203, 569, 1111, 5604, 307, 582, 2545
5. Try to measure out 5l of water by using a 7l can and a 4l can.
6. We would like to measure the weight of a melon and the accuracy should be 1 dkg. Find out how to do it, if we have weights only of 1 kg, 10 dkg, and 1 dkg, but as many as we want?

If we have to find out, among the given six examples which are the

mathematical and everyday life problems, many of us would say that the 2. 3. 4. are mathematical and the 1. 5. 6. are everyday life problems. Some may argue with it, and it would be hard to decide who is right because mathematics deals mainly with describing and solving the problems of everyday life, therefore there is no strict line between the mathematical and everyday life problem solving. However, I will define these concepts, firstly to state that I will discuss exclusively problem solving itself, and secondly so that we see the similarities and the differences between the two mentioned kinds of problems.

We talk about *problem* when the alternation of a given starting situation is needed, but to attain this alternation, we have to use an unknown version of well-known routine procedures, or we need the usage of unknown procedures.

We talk about *mathematical problems* in the case when both the starting and ending states are defined with mathematical devices or it can be described with mathematical devices as well, and the needed processes are mathematical processes.

If the problematic states are everyday life situations (that can probably be described with mathematical devices), and the problem can be solved with resorts of everyday life (not mathematical), we talk about *everyday life problems.*

In exercise 1 it is possible to find the rule of ordering the postboxes, but this problem can be undoubtedly solved without any mathematical devices (by asking a neighbor, for example). That is why I listed it to the everyday life problems. However, the most important viewpoint is what the students' attitude is to these two types of problems.

The survey

The six listed problems occurred in a test, as I mentioned before. There were 100 students at age of 16-18 asked to solve the problems. The students I had chosen were not specially educated, their mathematical knowledge is below the average level, they were preparing to become kinder garden teachers, car mechanists or dressmakers. By the survey I wanted to examine if there are any differences between mathematical and everyday life problems for the students. The results can be formulated with the next table:

Exercises	**1.**	**2.**	**3.**	**4.**	**5.**	**6.**
Number of solutions	87	37	47	52	66	64
Number of right solutions	41	8	6	32	24	34

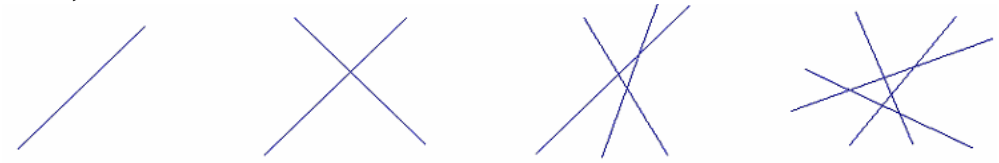
The average of the number of solutions within the three mathematical problems (2. 3. 4.) is 1,36 and the average of the number of solutions

within the three everyday life problems (1. 5. 6.) is 2,17.
The average of the number of right solutions within the three mathematical problems (2. 3. 4.) is 0,46 and the average of the number of right solutions within the three everyday life problems (1. 5. 6.) is 0,99.
Two very important conclusions can be drawn up from these data: students are more familiar with everyday life problems, than mathematical problems, and they have more success in solving everyday life problems, than in mathematics.
This is something like I have expected before the survey, so it is not a surprising fact, but this can be a warning for teachers, that in the students' mind there is a gap between mathematics and everyday life.
In the textbooks there are more and more problems, which seem to be everyday life problems, but actually most of them are mathematical problems in everyday life context so they are not suitable for restricting the gap. Using real everyday life problems in teaching mathematics has its danger, that the students' solutions will not be mathematical solutions, but in the survey many of the students tried to think mathematically even in the given everyday life problems, and non mathematical solutions can also be valuable from the aspect of problem solving.

Strategies in mathematical and in everyday life problem solving

One of the most complicated segments of the process of problem solving is making the plan of solution. It is typical for both the everyday life and the mathematical problems that when we make the plan of solution, we think according to certain patterns. These patterns are called problem solving strategies. Sometimes we use them unconsciously, we must pay special attention to them when we teach mathematics. In the literature related to mathematical problem solving we can find the most frequent strategies many times, and when we solve the mathematical problems that are given as examples, we also use some of them. If we examine the solving processes of the three everyday life problems, it turns out that we use the same strategies that we use for solving certain mathematical problems.

Solution of task 3: If we examine the cases of 1, 2, 3 and 4 straight lines we can find out that the rule of creating the number of plane parts is to add one more, than before.



2 4 7 11

Solution of task 1: After we realised that the order of the post boxes is the same as the order of the flats, and that there are 3 flats on each floors, there is only one thing left: we have to examine which group of 3 contains the flat of our friend, and which member of the group is it.

Solution of task 2: Let's use backward thinking. If there are 3 letters left at the end of the day, then there must have been 18 letters before delivering the 5/6 part, because the 3 remainder equals the 1/6 part. If we subtract the 6 formerly taken letters from the 18, the remainder 12 is the half of the amount that he had after taking over the 12 letters in the office. That means that after delivering the 2/3 part, he also had 12 letters left. Consequently, he set off with 36 letters in the morning.

Solution of task 5: It is practical to use backward thinking again. How can we get 5l of water? It seems obvious to divide it into a 4l and a 1l part, because 4l can be easily measured out. Afterwards we can find out in the similar way how to measure out 1l by subtracting 3l from 4l, and then to measure out 3l by subtracting 4l from 7l.

Solution of task 4: There are more then 500 groups to try out, but it is worth only to check the ones in which the sum of the last digits equals 0. Afterwards we can easily choose those cases, where the sum of the digits can be divided by 3.

Solution of task 6: Here it is also worth to use the strategy of intelligent guessing and testing. First we determine the approximate weight of the melon by using only the 1 kg weights, and then we can use the 10 dkg weights to make the measuring more precise, and finally we can achieve the wanted exact result by using the 1 dkg weights.

Conclusions and open questions

We could see that everyday life problems take a distinguished position in the students' conception of problem solving. By keeping this fact in view, we can say that this kind of problems should play significant role in the teaching of mathematics. There were other reasons shown, why everyday life problems are useful for mathematical problem solving, but a very important question is, what is the most efficient way of using them in the teaching process.

Beside the question of planning, how to use everyday life problems in teaching mathematics, there is one more interesting question: Shall we use everyday life problems for developing students' mathematical problem solving skills or on the contrary?