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The 12 Math-Blunders of Killer-Mathematics

Mathematics itself avoids blunders by being well defined and well proved. However, mathematics education fails its goal by making blunder after blunder at all levels from grade 1 to 12. This paper uses the techniques of natural learning and natural research to separate natural mathematics from killer-mathematics. Two digit numbers, addition, fractions, balancing equations, and calculus are examples of mathematics that has been turned upside down creating the 'metamatism' that killed mathematics and turned natural Enlightenment mathematics into modern missionary set-salvation.

Math-Blunder 1, Teaching Both Numbers and Letters as Symbols

In primary school both numbers and letters are treated as symbols. However, numbers are not symbols, but icons representing different degrees of multiplicity. If written in a less sloppy way it becomes clear that there are four strokes in the icon 4, five in the icon 5 etc. A letter is not representing a distinct sound in nature. On the contrary, a letter constructs and installs a sound to be distinct. Treating letters and numbers alike makes it difficult later to distinguish between the truth-values of number-statements and word-statements, i.e. between nature and choice.

Math-Blunder 2, Teaching 2digit Numbers Before Decimal Numbers

Mathematics can be introduced using 1digit numbers alone. However, the traditional curriculum introduces two digit numbers from the beginning thus creating problems to many students. The traditional way of making sense of 2digit numbers is 32 = 3*10 + 2*1. But then we cannot make sense of the number 10 since defining ten as 10 = 1*10 + 0*1 is a circular self-reference. Ten is the only number with a name and without a symbol.

Recounting the total 8 in 3s produces 2 leftovers: T = (8/3) *3 = 2*3 + 2*1. When stacking, we choose between two options: we can count the 2 left-overs in 3s (2 = 2/3 * 3) and put them on top of the existing 'single-stack' of 3-bundles; or we can place the 2 leftovers as a separate stack next to the existing stack of 3-bundles, thus producing a 'multi-stack' of 3s and 1s. Here 2digit-numbers occur as decimal numbers 2.2 or 'cup-numbers' 2)2) avoiding the self-reference using the number ten: 2)2) = 2.2*3 = 2*3 + 2*1.

Math-Blunder 3, Teaching Fractions Before Decimals

In the traditional curriculum decimal numbers are examples of fractions, thus being postponed until fractions are taught around grade 4. In a natural approach both fractions and decimals occur together in grade 1 as different ways of placing the leftovers as shown above. After that, fraction should rest until they reoccur as per-numbers in double-counting (se below).

Math-Blunder 4, Teaching Addition Before Division

The traditional curriculum introduces addition as the first operation, which leads directly to the use of 2digit numbers, and thus to Math-Blunder 2. In a natural approach, the first thing to do is to count multiplicity by bundling and stacking: T = 3 4s = 3*4, predicted by the 'recount-equation' T = (T/b)*b. So counting leads to division and multiplication. Then subtraction is introduced as the idea of carrying when internal trade is needed to sell 0.3 4s from a stock of 2.1 4s:

T=2.1 4s= 2)1)= 2-1)4+1)= 1)5)= 1)5-3) &)3)= 1)2) &)3)=1.2 4s & 0.3 4s

Later, when the students are accustomed to decimal numbers through recounting and internal trade and cup-writing, it is time to recount in tens and to introduce addition and traditional ten-based multiplication 3*4=12.

Math-Blunder 5, Forgetting the Units

The traditional curriculum treats numbers without units. In a natural approach where counting produces stacks, numbers always carry units: 2 3s = 2*3. Adding numbers without their units leads to 'mathematism', i.e. mathematics that is true in the library, but not in the laboratory. Thus 2+3 = 5 is seldom true while 2*3 = 6 is always true: 2weeks+3days = 17days, 2m+3cm = 203cm etc., while 2 3s always can be recounted as 6 1s. Also the integration formula tells directly that the per-number f must be multiplied with its unit 'dx' before being added: $\Delta F = \int f\,dx$.

Math-Blunder 6, Teaching Fractions Before PerNumbers

The traditional curriculum only talks about per-numbers in connection with percentages, taught as examples of fractions, thus having to wait until fractions are taught around grade 4. In a natural approach fractions first occur as 'proto-fractions' when recounting in number-units: 2 = (2/3) *3. Later fractions occur as 'per-numbers' when double-counting in two different units creates a 'guide-equation' 4kg = 5$, which is re-described as 'per-numbers': 4kg per 5$ = 4kg/5$ = 4/5 kg/$.

Math-Blunder 7, Teaching Proportionality Instead of DoubleCounting

The traditional curriculum sees proportionality as an example of a function thus having to wait until functions are taught around grade 8. In a natural approach proportionality is just another name for 'double-counting' occurring when a quantity is counted in two different units. Double-counting takes place already in grade 1 where a total can be counted both in

2s and in 3s raising questions as T = 5 2s = ? 3s. Later when double-counting in kg and $, we get a 'guide-equation' like 4kg = 5$. To answer questions as 10 kg = ?$ we just recount the 10 kg in 4s:

T= 10 kg = (10/4)* 4 kg = (10/4)* 5 $ = 12.5 $

Math-Blunder 8, Teaching Balancing Instead of Backward Calculation

The traditional curriculum sees an equation '2+3*x = 14' as an example of a statement that can be changed by performing identical operations on both sides of the equation sign until the 2 and the 3 have been neutralised. In a natural approach an equation is just another name for reversing a calculation where x is multiplied by 3 and added 2 to get 14, so 14 must have 2 subtracted and be divided by 3 to get back to x. This method is identical to the old 'move & reverse calculation sign' method.

Math-Blunder 9, Killer Equations Instead of Grounded Equations

The traditional curriculum doesn't mind 'killer-equations' only living inside classrooms where they kill the interest of the students. In a natural approach an equation is grounded as an abstraction form a real life situation, typically a word problem as e.g. '2$ plus 3kg at ?$/kg total 14$' leading to the equation '2 + 3*x = 14'.

Math-Blunder 10, Teaching Geometry Before Trigonometry

The traditional curriculum presents geometry as statements deduced from metaphysical axioms. A natural approach respects the original Greek meaning of geometry: measuring earth. Earth can be divided into triangles, which again can be divided into right-angled triangles. The Greeks provided two and the Arabs three equations to predict the three unknowns.

Math-Blunder 11, Postponing Calculus

The traditional curriculum sees calculus as two examples of limits, the gradient and the integral. Thus calculus cannot be introduced before the real numbers and the concepts of functions, limits and continuity are introduced late in secondary school. In a natural approach calculus means adding variable per-numbers. This takes place from grade 1, where 4 3s and 2 5s can be added as 8s. Asking '2*5 + 4*3 = ?*8' is integration; and asking '2*5 + ?*3 = 4*8' is differentiation. Later when blending tea, asking '5 kg at 2$/kg + 3 kg at 4$/kg = 8kg at ? $/kg' is integration; and asking '5 kg at 2$/kg + 3 kg at ? $/kg = 8kg at 4 $/kg' is differentiation.

Math-Blunder 12, the 5 Meta-Blunders of Mathematics Education

1. The Preclusion of Prediction. In the 1600s the predicting ability of mathematics was used to replace political correctness with natural correctness by showing that the Pope was wrong when claiming that a falling object obeys a metaphysical will that is unpredictable. Instead Brahe, Kepler and Newton used the predicting ability of mathematics to prove that physical things move according to a physical will, a force, that is predictable since it can be described in numbers and formulas. The predicting ability of mathematics thus laid the foundation of the Enlightenment and its two democracies in America and in France.

2. Interchanging Product and Process. The tradition presents concepts as examples of abstractions. This is turning mathematics upside down, having developed through thousands of years as abstractions from examples.

3. Interchanging Goal and Means. Mathematics should be a means to an outside goal, i.e. a number-language enabling us to predict the world by numbers and calculations. However, this relationship is turned upside down, so mathematics has become the goal and the world a means. Using the phrasing 'the world applies math' instead of the phrasing 'the world creates math' enforces this mathematical somersault.

4. Funding Library Research Instead of Laboratory Research. Math education research has an 'irrelevance paradox' since the number of research articles increase with the number of problems they try to solve. Examples of 'irrelevant' research are 'lackey-research' accepting the hidden choices of math education and wanting to understand the problems instead of solve them; 'ghost-research' setting up hypotheses based upon library concepts that by being non-operationalizable have to be installed as 'ghosts' in order to be studied; and 'mirror-research' researching mathematics education research instead of mathematics education itself.

5. Turning Natural Mathematics into Metamatism. Transforming natural multiplicity-based mathematics into 'metamatism' by turning it upside down to set-based 'metamatics' not able to tell predicting mathematics from 'mathematism', is the mother of all meta-blunders. The MATHeCADEMY.net solves this by offering a natural approach to math.

Literature

Tarp, A (2006) *The 12 Math-Blunders of Killer-Mathematics*, http://www.mathecademy.net/Papers.htm

Zybartas, S. & Tarp, A. (2004). *One Digit Mathematics,* http://www.mathecademy.net/Papers.htm