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Additional geometrical disciplines in preparation of the teachers of mathematics

The Russian system of education is in the process of reforming nowadays. The general secondary school education will be replaced by the system of profile preparation, and the system of higher education will transform to the multilevel preparation of the students according to the Bologna process. The profile preparation of the pupils of school assumes their profound specialization in the senior classes on the appropriate scientific direction, in particular in the field of physical and mathematical education. It is supposed, that the profile school educational process will be conducted by masters of education (or mathematics). Therefore, in master programs at pedagogical universities, it is necessary to give masters - future teachers of mathematics deep mathematical knowledge and also skills necessary for successful work with the pupils of profile physical and mathematical classes. Such opportunity is provided by system of disciplines of choice of the students, and to such disciplines, according to the curriculum, up to 40 % of educational time is allocated. The offered disciplines will give the future teacher of mathematics a material that can be used for preparation of mathematical courses at the profile school.

Proceeding from the purposes and tasks of offered disciplines, contents and methods of teaching should satisfy to the following requirements.

First, the subject matter should be based on the school program and on the facts stated in the school textbooks, i.e. on the material which is accessible and can be mastered by the pupils. Second, the statement of a material should be in a sufficient measure strict and correct. Third, it is necessary to acquaint the students with methods of teaching of these disciplines, appropriate for the school complexity level. Fourth, it is necessary to connect the contents of these disciplines with elementary mathematics, to show, how to prepare the pupils of profile classes to understanding of ideas and methods of mathematical researches, to mastering university courses of higher mathematics.

As a matter of fact, the offered programs of geometrical disciplines of choice represent the adapted to school course sections of geometry, usually studied by students of Russian pedagogical universities. To such sections one can attribute constructive geometry, i.e. theory of solving tasks on construction figures on a plane by compasses and ruler, foundations of projective geometry, namely properties of the central projection and fol-

lowing from them model of a projective plane - extended Euclidean plane complemented by infinitely distant points, properties of axonometry and Monge's method of representing spatial bodies on a plane. Consider in more detail three syllabuses of offered disciplines.

Analytical methods at solving tasks of elementary geometry.

The program of a course is designed for 8 lectures and 10 practical lessons (exercises). In Russian school textbooks on geometry the foundations of vector algebra and analytical geometry are considered. In particular, properties of scalar product of vectors are considered, the equations of a straight line and a circle on a plane and planes and spheres in space are deduced. All reasonings are carried out in rectangular Cartesian system of coordinates. The large number of stereometric tasks is based on calculation of angles between straight lines and planes in space, i.e. tasks using properties of vector products of vectors. Therefore, in addition to the brief survey of a material of the school program close to the theme of the discipline, the basic attention should be given to the tasks aimed at overcoming these difficulties. It is offered to reduce tasks of finding the equation of a plane defined by three points to the determination of the nonzero solution of homogeneous system of three linear equations with four unknowns by coefficients of equations. For the solving of tasks on finding of angles between planes and straight lines and between two planes in the space it is necessary to prove (by methods stated in the school textbooks) the statement that the vector with coordinates equal to coefficients of the equation of a plane, is perpendicular to this plane. One of the basic skills, which should be mastered by the students on the exercises, is a choice of convenient rectangular Cartesian system of coordinates, on which the complexity of calculations depends.

Geometry of complex numbers.

The course is designed for 16 lectures and 8 practical lessons (exercises). The basic idea of a course is to acquaint the students and school pupils with geometrical interpretation of two-dimensional algebras of complex, dual and double numbers, and also with analytical methods of study of geometrical properties of figures by means of these algebras. According to the school program, the pupils are familiar with algebra of complex numbers of a kind $a+bi$, $i^2 = -1$. In the offered discipline future masters study properties of interpretation of complex numbers by points of the plane where by the known rule ($a+bi \Leftrightarrow M(a;b)$) to each complex number the point of the plane, namely of the so-called complex plane is put in

a correspondence. The questions of correspondence between geometrical transformations of the plane and appropriate properties of functions of complex variables are considered.

The concept of a dual number $a+b\varepsilon$, $\varepsilon^2=0$ is introduced. The properties of geometry of the plane of dual numbers are considered. On this plane the metrics of a Galilean plane is introduced, and the connection between geometry of this plane and properties of rectilinear movement in the mechanics of Galilei and Newton is established. The statements interpreting analogies and explaining distinctions between properties of figures on Galilean and Euclidean planes are proved. The algebraic properties of double numbers $a+be$, $e^2=1$ are studied. On a plane of double numbers the pseudoeuclidean metrics is introduced, and the properties of figures on this plane are considered. The properties of straight lines and circles, and also geometrical transformations by means of algebra of double numbers are studied.

Axiom of parallelism and elements of geometry of Lobachevsky.

The course is designed for 8 lectures and 8 practical lessons (exercises). The basic purpose of a course is to show the students and pupils a place and meaning of the axiom of parallelity in the logical construction of Euclidean planimetry, and also to acquaint them with some facts of geometry of Lobachevsky, or, that is the same, with hyperbolic geometry. It is well-known that the attempts of the proof of the fifth postulate of Euclid have very interesting history closely connected to the history of the development of the civilization. The following scheme of the construction of the discipline is offered. First, it is necessary to acquaint pupils with the history of attempts of the proof of the fifth postulate of Euclid. In these attempts, the statements equivalent to the axiom of parallelity were used. We list them:

1. The sum of angles of a triangle on a plane is equal to the straight angle.

2. The statement of Poseidonius: "On a plane, there exist three collinear points belonging to one half-plane and equidistant from a given straight line".

3. The statement of Wallis: "On a plane, there exist two similar but not equal triangles";

4. The statement of Farkas Bolyai: "Around any triangle, it is possible to circumscribe a circle";

5. The statement of Legendre: "The perpendicular to the side of an acute angle crosses its second side".

The proofs of equivalence of these statements to the axiom of parallelity are carried out. The axiom of parallelity of Lobachevsky is formulated, and it follows from it, that on a plane of Lobachevsky negations of above-mentioned statements are true. The pupils get acquainted with a model of Cayley-Klein where points of a plane of Lobachevsky are interpreted as internal points of a circle.

The experience of studying these disciplines by students and teaching in profile classes shows their efficiency. These courses acquaint students and pupils with a material connected with the school program, but extending its framework, stimulate their great interest to geometry.