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Getting started well: The training "VorMath" as a tool to improve mathematical precursor skills and mathematical thinking before school

1. Introduction

Since children have various backgrounds and experiences concerning fundamental mathematical fields, the skills of children, when starting school, are very varied. Some children are able to tell what one fewer than one hundred is, while others fail to recognize the number of three objects at once. It is a tough challenge for teachers of children at school-entry level to meet the requirements of all those in their class, so that it hardly ever can be achieved. How can those in the field of education help to improve the abilities of lower achieving children before they start school, so that all have at least a minimum standard of mathematical ability? To answer this question, Schwank and her colleagues developed a training project called VorMath which is an abbreviation for "Vorschul-Mathematik" – preschool mathematics. The playful training aims to facilitate mathematical precursor skills and functional thinking, the description of which follows.

2. Mathematical precursor skills

Krajewski (2003, 2005) investigated how various different skills of preschool children related to their mathematical abilities during the first and second year of schooling. She found specific and unspecific skills that can predict later mathematical performance to a certain extent. Unspecific precursor skills are intelligence, capacity of memory, and number speed. They have only indirect effects on mathematical competencies through specific mathematical precursor skills and influence other competencies as well. However, the concept of Krajewski focuses not on the unspecific, but on two specific mathematical precursor skills: set prior knowledge and number prior knowledge. The term *set prior knowledge* is used for an understanding of sets, and relations between sets, and includes skills like seriation, comparison of sets, and comparison of lengths. The term *number prior knowledge* refers to knowledge of counting, and of the order of number words, and includes knowing the number signs, counting, and simple arithmetics. Set prior knowledge affects mathematical competencies indirectly through number prior knowledge, which is the only factor with a direct effect on mathematical performance.

Both of these specific skills are specific risk factors, indicating that a low level in these skills or knowledge is related to a development of mathemat-

ical difficulties or even developmental dyscalculia (cf. Krajewski, 2003). So training that can encourage mathematical precursor skills should minimize the risk of developing mathematical difficulties.

3. The relationship between basic arithmetic principles and functional thinking

Many basic arithmetic principles are founded on functions and operations. The principle of ordination, for instance, follows from the order of the natural numbers, which is founded on the function of creating the successor of every natural number by adding one more (cf. Dedekind, 1901, pp. 24, 33 - 34). To understand such principles, a special way of thinking is beneficial, a way of thinking where different (mathematical) items are conceived in terms of functions or operations. Such a thinking is accompanied by a rather dynamic internal representation, and takes into account how something has been changed or transformed. This way of thinking was discovered by Schwank (e. g. 1993, 2005b), and is called *functional thinking*.

Alternatively, different mathematical items can be compared in terms of similarities, relations, or features. Here an internal representation is more static, and considers what is to be seen. This predicative thinking makes it difficult to consider functions and operations, and therefore to understand basic arithmetic principles such as ordination. Consequently, it can be seen that strategies to solve mathematical tasks can vary significantly. See Schwank (2005b, p. 119; 2005a, p. 34) for a fine example of an effect of these different ways of thinking, when solving a mathematical task.

Although many people are able to think in both ways, especially when tasks are simple, they often have an unconscious preference for one of the two ways of thinking, which can lead to poor skills when the originally disregarded way is required. This is especially the case when tasks are of a complex matter. In conclusion, training in functional thinking enhances the understanding of basic arithmetic principles of children, especially those who favor the predicative thinking.

4. The training VorMath

We have seen so far that it is promising to train mathematical precursor skills and functional thinking. The question is whether this is possible, and whether both aims might be achieved through one single clearly laid out training.

Schwank and her colleagues have been attempting to create such a training project for the participation of pre-school children, which has consequently been labelled VorMath (Vorschul-Mathematik). Based on the fundamental

ideas of the playful training project, it has been developed further to a training project, which is constituted of eight playful sessions, and has a frame story about a storyteller whose stories are about animals.

To facilitate mathematical precursor skills, tasks with various premathematical contents were included. These contents were, for example, seriation, comparing sets and length, and counting. For simple counting, an example of a task is given in figure 1. It shows the main material of the training: the *spiral staircase* (Schwank, Aring & Blocksdorf, 2005). This material is made of ten columns of small wooden balls, where every column has one ball more than the previous one, starting with zero balls up to nine balls per column. In the given case, a squirrel figure is standing on the starting position, the column with zero balls. The children have to move the squirrel forward from one column to the next, and count out loud the number of jumps the squirrel makes.

The position after doing a jump arises from the function $h \rightarrow h + 1$, where h represents the height of the previous position. This can be seen as an opportunity to facilitate functional thinking, since it seems to be easy to reconstruct this operation and build an adequate internal functional representation, for instance through imaging the squirrel jumping. To sum up, this task can probably facilitate both, mathematical precursor skills and functional thinking.

Additionally, one focus of VorMath lies on the "number construction sense." The term "number sense" was introduced by Dehaene (1997) and "represents the universal ability to represent and manipulate numerical magnitudes nonverbally on a spatially oriented mental number line" (Aster & Shalev, 2007, p. 863). Schwank has observed (2005b, p. 122) that this definition omits constructional processes. Therefore the word construction has been added here.

Figure 1: First three steps of a simple counting task using the spiral staircase. While counting aloud children shall move an animal from one column to the next.

In later sessions of the training simple addition and subtraction tasks are contents of VorMath. With the moving squirrel and other animal figures, it is easy to show that subtraction is the inverse operation of addition, and vice-versa: the animals move higher (forward) in the case of addition and lower (backward) in the case of subtraction.

Finally, VorMath is bound to common principles of education like discussing solutions of tasks, asking the children to state reasons, and Bruner's (e. g. 1974)

principle of different representative modes. The above mentioned examples are enactive representations for a number of "jumps" or a concrete addition operation. But iconic and symbolic representations are also included. Iconic modes, for example, were realized through the use of worksheets with a 2-dimensional image of the spiral staircase and varying positions of different animals. Children, after learning how to draw arrows, have to draw the movement of the animal in question with the length of an arrow indicating how far the "jump" is and the arrowhead pointing in the direction of the movement (e. g. counting forward / addition vs. counting backward / subtraction).

5. Summary and direction of future developments

The promising training VorMath embraces many relevant arithmetic contents, follows basic educational principles, and has been developed to improve mathematical precursor skills and functional thinking. Currently, a study is being carried out to evaluate VorMath with quantitative and qualitative methods. This study intends to investigate whether the training project attains its main goals.

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