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Reasoning indicators – a case study

In the analysis of modelling processes, it is challenging to find out how reasoning processes are worked out by students. For this case study, a task on a predator-prey situation is designed with the purpose of encouraging argumentation. Written or verbal utterances of students aged 12 to 13 in group discussions are investigated with the aim to figure out in what way their logical reasoning is performed. It is examined how the students discuss and relate objects while modelling. Graphs are used to illustrate the intensity of students' discussions.

Framework

We assume that thinking leads to structures in the mind: a possible model for structures is a graph consisting of vertices that represents perception units or combinations of perception units, that on their turn can be linked, which is represented by arcs or edges. Arcs are ordered pairs of vertices, edges are just pairs of vertices. In discussions and notes of students as our empirical material, we consider words as the observable objects allowing us to infer that they are currently involved in a process of reasoning. In the theory of knowledge graphs [1], the basic idea is that certain substructures in the mind have been "framed and named". This process leads to words. Larger structures in the mind can be described by sentences. In principle every combination of words, e.g. in the form of sentences, is an indicator of a mental construct. We can roughly say that "thinking is linking" objects. In describing a problem before actually modelling, usually an exploration phase occurs in which the relevant concepts are gathered and represented. For this phase also specific indicators will be observable.

In the context of experiments in teaching mathematics we would like to have a clear picture of the kind of links that are made. This comes forward particularly in the investigation of reasoning processes involved in modelling.

The "Africa" task

We designed a predator-prey situation, where population of certain animals, under changing circumstances has to be worked out. For later use, we denote animals as following: A – antelopes, G – gnus, L – lions, M – monkeys, C – crocodiles, P – panthers, E – elephants, whereas the objects grass, trees and rain are denoted by X, Y and Z. The animals are living in three areas on two sides of a river. The possible food of every animal is explicitly stated. Among others, the following questions were posed:

"Suppose in the area with grass and trees there is no rain. By this, the amount of grass and the number of trees go down. Investigate what happens with all the species in the three areas. What happens if there is no more rain in the area where only grass grows? What happens if there is no more rain in the area where only trees grow?"

Methodological considerations

The 7th class students, having no special experience in modelling, were organised in teams of four and video-taped while working. They were free to form the teams by themselves, and had ninety minutes at their disposal for the entire task. Though general indications about how to manage their time according with each sub-task were given, students had in the end the freedom to arrange that on their own.

We studied the way students expressed their thinking by means of graphs. The use of graphs is well-known from knowledge representation theory. We refer to the vast literature on "semantic networks", see e.g. [2]. We counted the number of times an object A, … Z was mentioned and the number of times a specific link was established. This led to the following graphs with vertex set in {A, G, L, M, C, P, E, X, Y, Z}. The labels on the edges give the number of times a link appeared.

.Fig.1a Extracted graph of the 'white' team

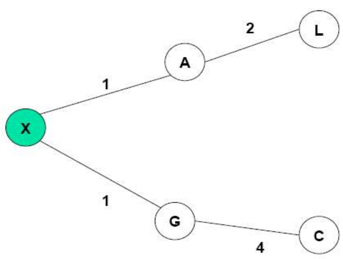


Fig.1b Extracted graph of the 'red' team

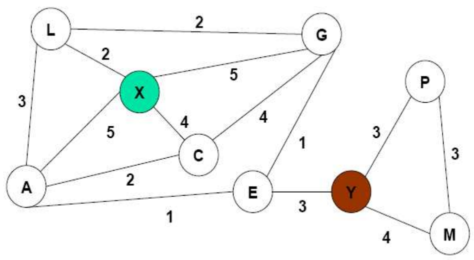
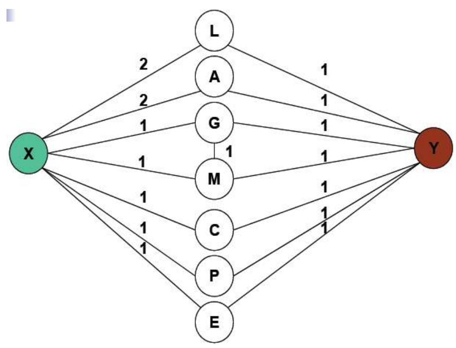


Fig.1c Extracted graph of the 'grey' team



Findings

The graphs in Fig. 1 were produced for three of the participating students' teams. Note that the graphs corresponding to the 'red' and 'grey' team contain almost all vertices. The graph in Fig. 1b is rather complex, with considerably long paths and high weights on the edges, which indicates that the students explored and found many links. In other cases teams considered fewer objects, see Fig. 1a. Also quite messy structures appeared, like in Fig. 1b.

In the following excerpt of a protocol we mention indicators:

41:29 Mara: *If* the trees dry up, *then* monkeys die off, *then* the elephant dies... (mumbling...) *because* the panther does not have food anymore...

41:39 Jana: Gnus and monkeys die off.

41:40 Susi: No!

41:42 Mara: Antelopes... and *thus* die off...

41:43 Susi: No!!!... (mumbling)

41:47 Mara: Yes, that's why!

41:51 Jana (to Susi): *If* the grass dries up ... (in the meantime Mara talks to Lea: we are at trees, they (Jana and Susi) are at grass). *If* Gnus and Antelopes are dead, *then* the crocodiles and lions are dying off...

42:26 Mara: *Then* have... *then* the elephants do not have food anymore...

The reasoning indicators we encounter for situations where links between objects are established are those expressing logical connections: "if... then... " and "because". Our analysis led to the following set of indicators for links between objects: *because*, *therefore*, *thus*, *fewer... fewer*, *if... then*, etc. Other types of indicators occur each time a statement about an object is made. This can involve a process, e.g. *die off*, or any other statement which clearly indicates that reasoning processes are performed. For the moment we make the distinction just between reasoning indicators present in statements about relations between objects and indicators to be found in simply statements about objects. We do not give here a list of indicators of the second type, since such a list would be specific to our case study. Each particular situation yields to a set of this type of indicators, but no generalisation is to be made about it.

The graphs as way to illustrate students' discussions contain much information in two dimensions: on one hand they consist and depict structural aspects of the argumentation, by the links established between objects, on the other hand provide semantic information, through the objects units students build on.

Our experiment has shown that there is ample use of reasoning indicators. An obvious research project it to collect and classify such indicators. Ideally they should be partitioned according to the four types of relationships between the real world and the mathematical world, as well as within these worlds, as we distinguished in [3]. A difficulty here is the fact that students use language to express relationships in different ways. A related didactical project would therefore be to train students to use 'proper' description.

Discussions

We can only shortly discuss some aspects of our use of graphs for representing the modelling process. The extracted graphs give a good picture of the outcome of the discussions in the teams. It also shows striking difference in the quality of graphs. On one side there is a complete coverage of the posed problems and quite rich structure, whereas on the other side only partial coverage is visible and the graph structure is rather simple.

Each graph is composed of vertices and edges, that were mentioned by participants of the team. This means that each participant contributed a subgraph of the total graph, that in a way expresses the role played by the participant in the discussions. For each element of the graph the frequency with which the element was mentioned by a participant is known from the protocols and this too sheds a light on the role played.

Finally, we should stress that in our graph theoretical analysis no distinction between types of links was made. Concepts were linked and no reference to the indicator was made. A more refined analysis could contain such references, which would enable to see where different types of logical reasoning were used and by whom.

Literature

1. P. James: Knowledge Graphs. In R.P. van der Riet & R.A. Meersman, ed.: Linguistic Instruments in Knowledge Engineering, (1992) 97-117.

2. J. F. Sowa: Knowledge Representation: Logical, Philosophical, and Computational Foundations, Brooks/Cole, (2000), ISBN 0 534 949657.

3. R. Grigoraş and S. Halverscheid: Modelling the Travelling Salesman Problem: Relations between the World of Mathematics and the Rest of the World. Submitted, (2008).