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## Technology related Arithmetic

*Technology has changed daily life and mathematics teaching in upper classes. But in most countries all over the world we still waste more than 50% of the time in mathematics lessons in grades 3 and 4 in primary schools (Schipper 1998, Schmassmann 2007) to teach and train paper & pencil techniques which later on will not be used any longer, neither inside nor outside from schools. In this paper we will present ideas how to get a more technology oriented arithmetic curriculum for primary schools.*

*We will **NOT** discuss here the use of calculators<sup>1</sup>. We will concentrate on traditional goals and how they should be changed, goals like number sense, mental arithmetic, estimation abilities, rounding and approximation skills.*

### Brief Analysis

Paper & pencil techniques were developed to get a tool for computing with big numbers. Other tools were logarithms or the slide rule (*Rechenschieber* in German) or different types of an abacus or different types of tables or sticks and other mathematical devices. Our modern tools for computing with big numbers now are calculators or computers. Using these machines is much more comfortable than practicing paper & pencil techniques, they are quicker and safer to get the correct result. Thus paper & pencil techniques are not practiced any longer outside from schools.

Teaching and learning paper & pencil techniques in primary schools is combined with two problems. First, the numbers get reduced into sequences of digits (*Rechenzahl* in German). The original meaning of the number as a measurement number (*Groesse*) or a quantity number (*Masszahl*) gets lost. Of course, then also the computation result is just a sequence of digits and often for the student it is not easy to check if all the digits are correct. Second, young children get confronted with the burden to learn different techniques to manipulate sequences of digits. These syntactical activities are quite different from the semantic activities they had practiced before. Very often a necessary understanding is missing.

These two problems are more fundamental than usually discussed. Introducing paper & pencil skills demands a change from common sense experiences on to an algorithmic thinking. Distinguishing these two types of mental processes we refer to a polarity in thinking which already was discussed before by many other authors. Vygotski talks about spontaneous and

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<sup>1</sup> But there are references below which discuss the use of calculators in more detail.

scientific concepts, Ginsburg (1977) compares informal work and written work, or Strauss (1982) discusses a common sense knowledge vs. a cultural knowledge. Strauss especially has pointed out that these two types of knowledge are quite different by nature, that they develop quite differently, and that sometimes they interfere and conflict (“U-shaped” behavior).

Via the *Dual Process Theories* we get an explanation: Our cognition operates in two quite different modes which we will call here *System 1* and *System 2* (for more details see Kahneman/Frederick 2005 and Leron/Hazzan 2006). Working on a mathematical problem will happen in parallel where a spontaneous or intuitive thinking (*System S1*) may interfere with an analytical or reflective thinking (*System S2*). S1 processes are fast and automatic and need not much working memory, but they are very resistant against changes. To transform or to coordinate S1 experiences into appropriate and more flexible S2 experiences the processes must become conscious. Discussions are an important tool to bring unconscious processes into consciousness.

## **Number Sense**

In the German language there are two words with a different meaning. *Zahlbegriff* is the term for describing the analytical and logical aspects. With *Zahlgefuehl* or *Zahlengefuehl* the intuitive and emotional aspects are summarized. It is interesting to see that most of the German literature concentrates on *Zahlbegriff* only. Most authors avoid discussing aspects of *Zahlgefuehl* (except the dissertation Lange 1984). In the English literature however there is only the one word “number sense” and it describes a more balanced view. Here the term includes both aspects, common sense and analytical components: „Number sense refers to an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect arithmetical errors, and a common sense approach to using numbers. ... Above all, number sense is characterized by a desire to make sense of numerical situations (Reys 1991, cited in Sowder/ Schappelle, 1994, p. 342).

Number sense not only refers to numbers but also to both, to conscious and to unconscious techniques to manipulate numbers, and it also includes a feeling about possible outcomes of these techniques. With a good number sense we roughly can predict the result of calculations, sometimes spontaneously (intuitively) and sometimes consciously (by approximating). Especially we can develop an intuitive feeling for additive and multiplicative structures. We can get a feeling for computation results like skilled painters

sometimes have when they decide without any measuring how much wall-paper they will need for that room.

### **Number Sense and Technology**

Number sense as described above is necessary to remain mentally independent up to a certain degree from the use of technology. A good number sense allows us (intuitively in *System S1*) to control the results or to discover computation mistakes. And there are possibilities to use technology to further the development of that number sense, see (\*)-references below.

Using technology consciously is done in *System S2*. In a technology related arithmetic we therefore need a balance to remain mentally independent up to a specific degree from just pressing buttons. We need spontaneous reactions in *System S1* if something goes wrong. These reactions will depend on the size and the order of numbers used and on the operations performed.

### **Number Sense in Different Number Spaces**

In daily life not all numbers are equally important. Often rounded numbers are used, e.g. for the size of a swimming pool or a garbage container, for the distance between two cities or between the earth and the moon, or for the weight of an elephant or a lion, etc. “To make sense of a numerical situation” most of the multi digit numbers are unimportant, in daily life they are replaced by rounded numbers.

This means we intuitively distinguish (in *System S1*) between unimportant (multi digit) numbers and important (rounded) numbers. Putting all these “important” numbers on the number line we do not get an equidistant pattern but a pattern which looks more like a logarithmic pattern. It seems as if we determine the importance of numbers in a similar way as we perceive the intensity of light or of sounds (Weber-Fechner law). This would mean that especially in large number spaces there are only a few “important” numbers. The larger the number space is the more unimportant numbers it will have. Do we still need then the paper & pencil techniques for all these unimportant numbers, when we have calculators?

### **Future of Paper & Pencil Skills**

We argue paper & pencil techniques should disappear in a technology related curriculum. Instead we suggest introducing two new strands. On the one hand we recommend using a calculator when we need quick results or when it can be used as a methodological tool. The other strand must expand the traditional mental arithmetic curriculum. Here we round big numbers

and do mental computations then with the rounded numbers. We must further more intensively traditional activities like skill training (addition tables, multiplication tables, stimulus response knowledge, etc.), applying commutative, associative, distributive law and other rules, applying consciously known properties, training of tricks, etc. There also are possibilities to train mental arithmetic via the use of calculators (Meissner 2006).

The new curriculum strand *Mental Arithmetic* also must include activities how to detect arithmetical errors and calculator activities to develop a number sense for additive and multiplicative structures. For more details see the “Discussion Forum” web page below.

## References<sup>2</sup>

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## See also the web pages

- (\*) Discussion Forum on the Future of Paper & Pencil Skills:  
<http://wwwmath1.uni-muenster.de/didaktik/u/meissne/WWW/Forum-P&P.htm>
- (\*) Investigations with Calculators in Primary Schools:  
<http://wwwmath1.uni-muenster.de/didaktik/u/meissne/WWW/TRint.htm>
- (\*) Summary report on TIM Calculator Projects in Germany:  
<http://wwwmath1.uni-muenster.de/didaktik/u/meissne/WWW/TR.htm>

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<sup>2</sup> References marked with an asterix (\*) also discuss possibilities how and why to use calculators in a technology related arithmetic education