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Measuring asymmetry: A recommendation

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Abstract. A range of primary methods for dealing with price asymmetry, such as the approaches proposed by WOLFFRAM (1971) and HOUCK (1977), have been established in the literature, but consensus on which method should be preferred remains elusive. This note demonstrates that, theoretically, these two definitions are equivalent to a straightforward notion of asymmetry based on first differences. Using monthly data on gasoline prices and sales from the U.S., we illustrate, however, that, in practice, these approaches may yield divergent conclusions with respect to asymmetry. We argue that in such situations the asymmetry notion based on first differences should be preferred.

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1 Introduction

The estimation of so-called irreversible supply and demand functions that allow for asymmetric price responses has been a subject of ongoing research across a range of fields in economics, including agriculture (TRAILL, COLMAN, YOUNG, 1978), transport (DARGAY, 1992), and energy (GRIFFIN, SCHULMAN, 2005). While theoretical arguments in favor of asymmetric responses to rising or falling agricultural input prices were advanced by JOHNSON (1958), WOLFFRAM (1971) proposed one of the first empirical techniques to test for asymmetry. This technique, henceforth called the W technique, is based on cumulated price differences. It has become the most popular method of partitioning an explanatory variable to allow for the estimation of a non-reversible function (TRAILL, COLMAN, and YOUNG, 1978:528), and has since served as a foundation for more sophisticated approaches, such as error-correction models (for helpful surveys, FREY, MANERA, 2007).[1]

Despite the common belief of the superiority of the W technique, however, a number of articles have pointed to several weaknesses in its application, including its lack of intuition, the high dependence on the starting point of the data (GRIFFIN, SCHULMAN, 2005:7), and its proneness to multi-collinearity problems (SAYLOR, 1974; HOUCK, 1977). Using WOLFFRAM's (1971) stylized example, this note argues that the notion of asymmetry can be captured in a straightforward and highly intuitive manner in terms of first differences. We prove that in a deterministic context without stochastic influences, this asymmetry definition is equivalent to WOLFFRAM's alternative, but more readily interpretable. Using an empirical example originating from the real world, however, we demonstrate that, in practice, these approaches may yield divergent conclusions with respect to asymmetry. We argue that in such situations the asymmetry notion based on first differences should be preferred.

[1] Prior to WOLFFRAM (1971), TWEETEN and QUANCE (1969a, b) suggested a decomposition that employs two dummy variables to split up the price variable into two complementing explanatory terms capturing either increasing or decreasing input prices. In the aftermath of WOLFFRAM's (1971) seminal article, TWEETEN and QUANCE (1971:359) concede that their approach is inferior to the W technique.

2 WOLFFRAM's Example

To capture asymmetric price responses, in a seminal article, WOLFFRAM (1971) suggests the following decomposition of price variable x that is based on cumulated increases and decreases of the explanatory variable x, denoted here by w_i^+ and w_i^-, respectively. In detail, WOLFFRAM (2000:351-352) defines his approach by $w_1^+ = w_1^- := x_1$ and, for $i > 1$,

$$w_i^+ := w_{i-1}^+ + D_i^+ \cdot (x_i - x_{i-1}) = \sum_{k=2}^{i} (x_k - x_{k-1}) D_k^+, \quad (1)$$

$$w_i^- := w_{i-1}^- + D_i^- \cdot (x_i - x_{i-1}) = \sum_{k=2}^{i} (x_k - x_{k-1}) D_k^-, \quad (2)$$

where $D_i^+ = 1$ for $x_i > x_{i-1}$ and 0 otherwise, while $D_i^- = 1 - D_i^+$.

Reiterating the stylized example that WOLFFRAM used to illustrate the W technique (Table 1), we now demonstrate with the help of Figure 1 that a more natural definition of asymmetry in terms of first differences $\Delta x_i := x_i - x_{i-1}$ and $\Delta y_i := y_i - y_{i-1}$ suggests itself: While for those parts of the graph with a positive slope the first differences of y and x are related by a factor $\beta^+ = 5$: $\Delta y_i = \beta^+ \Delta x_i$, the downward-sloping parts of the graph are linked by a factor $\beta^- = 3$: $\Delta y_i = \beta^- \Delta x_i$. These proportions also become apparent from Table 1 and the respective columns related to the first differences of x and y. Combining both the upward- and downward-sloping parts provides for a straightforward and highly intuitive definition of asymmetry: There is an asymmetric relationship between two variables x and y if the null hypothesis $H_0 : \beta^+ = \beta^-$ can be rejected for the following equation of first differences:

$$\Delta y_i = \beta^+ \Delta x_i D_i^+ + \beta^- \Delta x_i D_i^-. \quad (3)$$

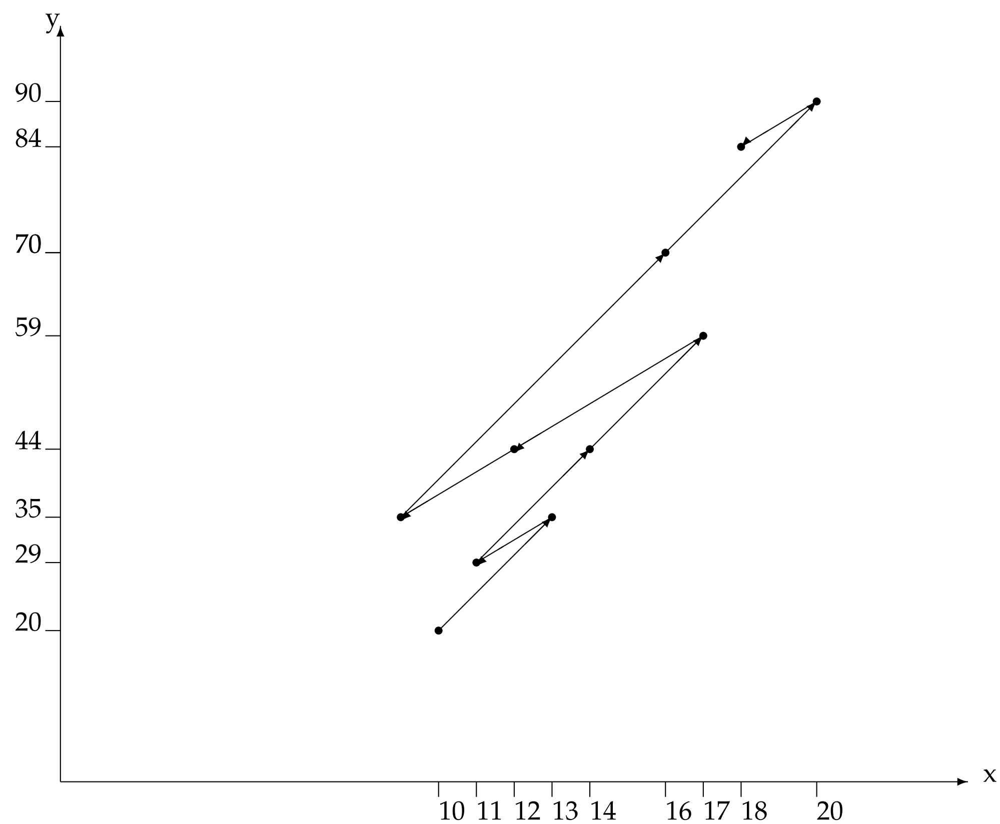
In case of symmetry, that is, in case that H_0 is true and, hence, $\beta := \beta^+ = \beta^-$, the relationship between y and x is also called reversible and simplifies to:

$$\Delta y_i = \beta \Delta x_i \underbrace{(D_i^+ + D_i^-)}_{=1} = \beta \Delta x_i. \quad (4)$$

Table 1: WOLFFRAM's Original Example.

Original Values				W technique			
y	x	Δy	Δx	w^+	w^-	Δw^+	Δw^-
20	10	–	–	10	10	–	–
35	13	15	3	13	10	3	0
29	11	-6	-2	13	12	0	2
44	14	15	3	16	12	3	0
59	17	15	3	19	12	3	0
44	12	-15	-5	19	17	0	5
35	9	-9	-3	19	20	0	3
70	16	35	7	26	20	7	0
90	20	20	4	30	20	4	0
84	18	-6	-2	30	22	0	2

Figure 1: Illustration of WOLFFRAM's Example



By recursive iteration, the following equation for y_i in levels can be derived from reversible relationship (4):

$$\begin{aligned} y_i &= y_{i-1} + \beta \cdot (x_i - x_{i-1}) \qquad \text{(if } i > 2\text{)} \\ &= y_{i-2} + \beta \cdot (x_{i-1} - x_{i-2}) + \beta \cdot (x_i - x_{i-1}) \qquad \text{(if } i > 3\text{)} \\ &= y_{i-2} - \beta \cdot x_{i-2} + \beta \cdot x_i = \ldots \\ &= y_1 - \beta \cdot x_1 + \beta \cdot x_i. \end{aligned}$$

In short, from reversible relationship (4) it follows that $y_i = \beta \cdot x_i$ for all $i \geq 1$.

In a similar vein, a representation for y_i can be gained from asymmetry definition (3) for $i > 1$:

$$\begin{aligned} y_i &= y_{i-1} + \beta^+ \cdot (x_i - x_{i-1}) D_i^+ + \beta^- \cdot (x_i - x_{i-1}) D_i^- \\ &= \ldots \\ &= y_1 + \beta^+ \underbrace{\sum_{k=2}^{i} (x_k - x_{k-1}) D_k^+}_{= w_i^+ - w_1^+} + \beta^- \underbrace{\sum_{k=2}^{i} (x_k - x_{k-1}) D_k^-}_{= w_i^- - w_1^-} \\ &= y_1 - \beta^+ w_1^+ - \beta^- w_1^- + \beta^+ w_i^+ + \beta^- w_i^-. \end{aligned}$$

Hence, adopting asymmetry definition (3) implies that y_i can be decomposed according to the W technique proposed by WOLFFRAM:

$$y_i = \beta^+ w_i^+ + \beta^- w_i^-. \tag{5}$$

In short, both definitions (3) and (5) of asymmetry are equivalent in theory. Using OLS methods, this equivalence can also be easily confirmed for WOLFFRAM's empirical example presented in Table 1, for which one gets the following estimates: $\widehat{\beta}^+ = 5$ and $\widehat{\beta}^- = -3$ for definition (3) and $\widehat{\beta}^- = 3$ for definition (5), respectively, while standard errors are vanishing for both coefficients.

HOUCK (1977:570) proposes an alternative approach that "is consistent with the WOLFFRAM technique but is operationally clearer." In fact, from a theoretical point of view, his approach is even equivalent to WOLFFRAM's technique given by Equation (5), as will be shown now. From WOLFFRAM's asymmetry specification (5) and, specifically, $y_1 = \beta^+ w_1^+ + \beta^- w_1^-$, it follows that

$$y_i - y_1 = \beta^+(w_i^+ - w_1^+) + \beta^-(w_i^- - w_1^-) \qquad \text{for } \ i > 1. \tag{6}$$

By defining a new dependent variable $y_i^* := y_i - y_1$, Equation (6) reads:

$$y_i^* = \beta^+(\sum_{k=2}^{i}(x_k - x_{k-1})D_k^+) + \beta^-(\sum_{k=2}^{i}(x_k - x_{k-1})D_k^-) \qquad (i > 1), \tag{7}$$

where, in contrast to WOLFFRAM's specification (5), the right-hand side is purged from any initial values.[2] (In fact, instead of (7), HOUCK (1977:570) suggests a specification including a deterministic trend αt. This trend is dropped here for the sake of simplicity, but included in the empirical example presented in the next section.) Again, using OLS methods, the equivalence of both HOUCK's and WOLFFRAM's definitions can be confirmed for WOLFFRAM's empirical example, for which the estimates for the slope coefficients β^+ and β^- turn out to be the same, respectively.

It is of interest to note that HOUCK (1977:570) additionally suggests a specification that includes only first differences of the increasing and decreasing phases of x without summing these up, as in Equation (7):

$$\Delta y_i = \alpha + \beta^+ \Delta x_i D_i^+ + \beta^- \Delta x_i D_i^-. \tag{8}$$

Apart from constant α, with this specification, HOUCK, in fact, proposes testing asymmetry according to asymmetry definition (3). It is unclear, though, whether HOUCK is aware that his proposed approach is theoretically identical to both the W technique and that based on first differences, or the implications arising from the application of these alternatives to real data, as the matter is not taken up his paper.

In sum, while numerous approaches have been suggested in the economic literature to capture asymmetry, this section has demonstrated that, theoretically and for contrived examples, such as WOLFFRAM's, in which stochastic disturbances are absent, both WOLFFRAM's and HOUCK's approaches are equivalent to the asymmetry

[2]The same goal could be achieved by setting $w_1^+ = w_1^- = 0$, rather than $w_1^+ = w_1^- = x_1$, as is suggested by WOLFFRAM (1971:358).

definition (3), which is based on first differences. However, for empirical examples originating from the real world, such as that presented in the subsequent section, we now demonstrate that WOLFFRAM's and HOUCK's approaches and the definition based on first differences may yield contrary answers to the question of asymmetry.

3 Empirical Illustration

To illustrate this point, we present an empirical application that regresses logged monthly gasoline sales, $\log y$, on logged monthly gasoline price ($\log x$) data retrieved from the Energy Information Administration (EIA, 2013a, b) for the period spanning January 1983 through August 2013. The gas price is deflated using a consumer price index obtained from the Federal Reserve Bank of St. Louis (FRED, 2013). For keeping the example simple and as close as possible to the theoretical discussion of the previous section, we abstain from using more sophisticated methods such as error-correction models (ECMs), but note that our recommended approach is sufficiently flexible to readily incorporate various extensions.

The empirical results obtained from the W decomposition given by Equations 1-2 are compared in Table 2 to those received from the estimation of asymmetry definition (3), as well as those from HOUCK's approach, for which the key explanatory variables are defined as follows: $h_i^+ := \sum_{k=2}^{i}(x_k - x_{k-1})D_k^+ = w_i^+ - w_1^+$ and $h_i^- := \sum_{k=2}^{i}(x_k - x_{k-1})D_k^- = w_i^- - w_1^-$. To account for potential month effects, we have added dummy variables to the model specification, the results of which have been suppressed in Table 2 for the sake of conciseness.

Several outcomes bear highlighting: First, apart from the constants, the empirical results of WOLFFRAM's and HOUCK's specifications are identical. This is due to the fact that both the dependent variables y_i and y_i^* and the key explanatory variables h_i^+, h_i^- and w_i^+, w_i^- differ merely by constants. In other words, WOLFFRAM's and HOUCK's approaches are not only theoretically equivalent, as has been shown in the previous section, but are also identical from an empirical point of view. Second, while all key explanatory variables show the expected signs, yet are not always statistically significant, F tests clearly reject the null hypothesis of symmetry for the WOLFFRAM (= HOUCK) approach, but not for the approach based on first differences.[3] This divergence raises the question as to which approach should be preferred when conclusions are drawn with respect to asymmetry.

Table 2: Empirical Comparison of Asymmetry Approaches.

	WOLFFRAM		HOUCK		First Differences	
	$\log y$		$\log y^*$		$\Delta \log y$	
	Coeffs.	Errors	Coeffs.	Errors	Coeffs.	Errors
$\log w^+$	**-1.304	(0.1174)	–	–	–	–
$\log w^-$	0.196	(0.1274)	–	–	–	–
$\log h^+$	–	–	**-1.304	(0.1174)	–	–
$\log h^-$	–	–	0.196	(0.1274)	–	–
$D^+\Delta \log x$	–	–	–	–	-0.068	(0.0555)
$D^-\Delta \log x$	–	–	–	–	-0.031	(0.0476)
year	** 0.061	(0.0070)	** 0.061	(0.0070)	–	–
const.	**-108.246	(13.6023)	** -119.944	(13.7010)	**-0.066	(0.0054)
Adj. R^2	0.4703		0.4703		0.4380	
Correlation	$(w^+, w^-) : 0.993$		$(h^+, h^-) : 0.993$		$(D^+\Delta x, D^-\Delta x) : 0.232$	
F test statistics	$F(1, 353) =$ **91.13		$F(1, 353) =$ **91.13		$F(1, 353) = 2.34$	
DF test statistics	$Z(t) = 0.131$		$Z(t) = 0.131$		$Z(t) =$ ** -22.589	
Number of observations: 368						

We argue that, for at least three reasons, the asymmetry definition based on first differences should be preferred. First, while it is equivalent to WOLFFRAM's decomposition in a deterministic context, but is generally different in empirical examples with a limited number of observations,[4] the basic principle of asymmetry is reflected in a highly transparent manner only by definition (3).

[3]Estimating the symmetric specification yields the following result:

$$\Delta \log y = -0.066(0.0053) - 0.047(0.0316)\Delta \log x,$$

where standard errors are in parentheses, month dummies are again dropped, and a DICKEY-FULLER test statistic of $Z(t) = -23.437$ does not lead to the rejection of a co-integration relationship.

[4]Using a simulation and a modification of WOLFFRAM's example that includes normally distributed

Second, beyond this theoretical argument, due to its formulation in first differences, definition (3) is also adequate in cases when the variables involved are integrated of order one, I(1), as in our example, in which DICKEY-FULLER tests indicate that we cannot reject the null hypothesis that both the price and sales variables are I(1), whereas we can reject the null hypotheses that $D^+\Delta x$, $D^-\Delta x$, and Δy are integrated. In contrast, as this empirical example illustrates, the W technique may suffer from spurious correlation: DICKEY-FULLER tests indicate that (1) we cannot reject the null hypotheses that both variables, w^+ and w^-, are I(1) and (2) there is no co-integration relationship between y, w^+ and w^- (see the statistics reported in the penultimate row of Table 2).

Third, the W technique is highly prone to multi-collinearity, as exemplified by Table 2: the correlation coefficient between w^+ and w^- amounts to about 0.993, whereas the correlation between $D^+\Delta x$ and $D^-\Delta x$ is substantially lower at 0.232.

4 Summary and Conclusion

This paper has demonstrated that WOLFFRAM's (1971) method for dealing with asymmetry, which has established itself as a standard within the field of agricultural economics and other economic disciplines, is theoretically consistent with an alternative definition of asymmetry that is based on first differences and highlighted here. While both approaches yield the same results for the stylized example given by WOLFFRAM (1971), using an empirical example originating from the real world in which the data generation process is characterized by a stochastic component and the number of observations is limited, we have illustrated that both definitions may yield contrary answers to the question of asymmetry.

This divergence raises the question as to which approach should be preferred

error terms, we find indistinguishable coefficient estimates for both approaches for 10,000 observations, but substantially divergent estimates for only 100 observations. In this case, we also receive contradictory results for the issue of asymmetry.

when conclusions are drawn with respect to asymmetry. On the basis of our theoretical discussion, we argue that in such situations the definition of asymmetry based on first differences should be preferred for several reasons, not least because it is more easy to grasp than WOLFFRAM's W technique to capture asymmetry. In fact, the W technique incorporates the history of the price trajectory by splitting up the price variable x into two complementary variables w^+ and w^- that reflect either cumulated price increases or decreases, respectively. This technique comes at some cost of intuition: Because the W technique implies that the *level* of dependent variable y is supposed to be explained by cumulated *changes* of an explanatory variable x, it is not immediately clear how to interpret the coefficients. Beyond this, as our empirical example has illustrated, the W technique may be more prone to spurious correlation, as well as multi-collinearity problems.

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