Ildar SAFUANOV, Moskau

Genetic approach to the teaching of discrete mathematics

As noted earlier (Safuanov, 2005), the principle of the genetic approach in teaching mathematics requires that the method of the teaching of a subject should be based, as far as possible, on natural ways and methods of knowledge inherent in the science. The teaching should follow the path of the development of knowledge. We will call the teaching of a mathematical discipline *genetic* if it follows natural ways of the creation and application of the mathematical theory. Genetic teaching gives the answer to the question: how can the development of the contents of the mathematical theory be explained?

Genetic teaching of mathematics at universities should have the following properties:

It should be based on students' previously acquired knowledge, experience and level of thinking;

For the study of new themes and concepts, the problem situations and wide contexts (matching the experience of students) of non-mathematical or mathematical content should be used;

In teaching, various problems and naturally arising questions are widely used, which should be answered by students independently with minimal necessary effective help of the teacher;

Strict and abstract reasonings should be preceded by intuitive or heuristic considerations; construction of theories and concepts of a high level of abstraction can be properly carried out only after accumulation of sufficient (determined by thorough analysis) supply of examples, facts and statements at a lower level of abstraction;

Students’ mental and cognitive activity should be stimulated: they should be constantly motivated;

The gradual enrichment of mathematical objects through the study of interrelations with other objects, through consideration of the objects themselves and by looking at results from new angles and in new contexts should be undertaken.

The genetic approach to the teaching of mathematical disciplines has various aspects: historical, logical-epistemological, psychological, sociocultural. The basic themes of a course of discrete mathematics for pedagogical universities provides rich opportunities for using all these aspects, beginning with a historical one (see, e.g., Barnett et al., 2009). Further more, one can also apply the principle of the concentrated teaching (Safuanov, 2003) in this course

The program of a course includes the following themes:

Fibonacci numbers. Transformations of the sums of degrees of natural numbers. The Euler's summation formula. Elements of coding theory. Codes detecting and correcting errors. Hamming codes. Elements of combinatorics. Newton's binomial theorem. Pascal's triangle. The basic concepts of the graph theory. Eulerian graphs. Hamiltonian graphs. Planar graphs. The theorem of Euler about polyhedra. The nonplanar graphs of Kuratowski - Pontryagin. Bipartite graphs. The theorem of Koenig. The problem of Four colours.

It is expedient to begin the course with acquainting students with the method of mathematical induction. One can introduce the principle and method of mathematical induction informally, proceeding from reasons of common sense.

It is appropriate to demonstrate the first applications of the method of mathematical induction on the example of summation of consecutive natural numbers (or of their degrees). From these tasks, one can proceed to the proof of the Euler's summation formula (Graham, Knuth and Patashnik, 1994, pp.455-461).

As other applications of the method of mathematical induction it is possible to consider Catalan numbers, elements of combinatorics with Newton's binomial theorem and Pascal's triangle and, finally, Fibonacci numbers. The Fibonacci numbers have huge number of beautiful properties, many connections to other sections of mathematics; they have a lot of applications. The applications go as far as to the theoretical foundations of the stock and currencies exchange activities. The connections of Fibonacci numbers with gold section allow to consider the numerous applications in art – in architecture, music, together with applications in other spheres of the human activity - construction, botany etc.

Certainly, such concepts connected with names of famous scientists, as the Fibonacci numbers, the Pascal triangle, Newton's binomial formula, allow us to consider in detail both historical sources and preconditions of their origination. Therefore, it is possible to create fruitful problem situations for introduction and construction of these concepts. These problem situations will promote also the development of motivation of learning.

Elements of the coding theory (codes correcting errors, Hamming codes) constitute an applied theme intensively developed mainly during the second half of twentieth century. One can easily find both practical tasks, which have resulted in creation of the appropriate theories, and connections of these theories with such important sections of theoretical mathematics as linear algebra, theory of groups, theory of polynomials and theory of fields.

The great place in the course of discrete mathematics is occupied by elements of graph theory – one of major theoretical bases of modern applied mathematics which is distinct from traditional sections connected with concepts of limit and continuity. The graph theory also has been intensively developed mainly in 20-th century. However, the first sources of graph theory are in 18-th century, when Leonhard Euler was the first to pose and solve the famous problem on Seven Bridges of Koenigsberg. This problem, which until now has not lost its importance as an entertaining task for the capable pupils, results in the important concept of the Eulerian graph (graph containing an Eulerian cycle that is a cycle in a graph which visits each edge exactly once). It is possible to introduce also other concepts and results of graph theory (for example, Hamiltonian, planar and bipartite graphs) using a history of their origination from practical tasks and even from entertaining puzzles. For example, the concepts of planar and nonplanar graphs can be developed from the consideration of the famous problem of Three Houses and Three Wells, On the other hand, the famous Four Colour problem ("whether four coluors are sufficient to colour every planar map in such a way that regions sharing a boundary are coloured in different colours") gives rise to the demonstration of the complexity of graph theory. The simplified version of the problem where the number of colours is five can be solved more elementarily and its solution can be presented in a class.

Our experience of teaching discrete mathematics has contained rich manifestations of principle of concentrated teaching (Safuanov, 2003): anticipation, repetition, combination of functions and linkage (Verzahnung). For example, such topics as recurrences and coding theory are linked to many concepts of fundamental mathematics such as infinite series, groups, polynomials and matrices. Throughout the course, the ideas of induction and recurrence are repeatedly developed and used. Entertaining problems (Koenigsberg Bridges, Three Houses and Three Wells, Horse Tour on Chessboard) anticipate important concepts of graph theory. More globally, the course of discrete mathematics as a whole anticipates further, more sophisticated courses in Computer Science.

Thus, it is possible and useful to apply the genetic approach (combined with the principle of concentrated teaching) to the teaching of discrete mathematics

Literatur

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