# Master Programs for future mathematics teachers in Russian Federation

### Introduction

Serious innovations in educational system and, generally, the steps to the educational reforms were caused by crucial changes in political and social life during the last 15-20 years. The differentiation of education is the central part of discussions on the reforms, which continue since 1990 till today. Two kinds of the differentiation are being discussed:

- 1) w.r. to the amount of mathematics to be studied;
- 2) w.r. to the inclination of classes: mathematical, for engineering and natural sciences, for the humanities.

Now the process of the differentiation of schools and higher education takes place. Generally, approaches to the higher (university) education have changed. In some of pedagogical universities, two-stage curricula have been elaborated: after first 4 years, students become Bachelors and may teach at lower secondary schools. After 2 years of additional studies, they become Masters and have the right to teach at upper secondary schools. Obligatory assignments to institutes for annual production of young teachers, percent of satisfactory marks etc. are abolished. Institutes can almost independently work out their curricula.

#### 2. The structure of curricula.

Generally, in standards recommended by the Ministry of Education in 1995, the amount of classroom hours for the whole course of pre-service teacher education program is about 4000, half of which (about 2000) are devoted to mathematical disciplines, one quarter (about 1000) to general cultural (e. g. social, philosophical and medical) sciences and one quarter (also about 1000) to psychological and pedagogical (including mathematics education) disciplines. However, the total amount of mathematics education is usually only about 170 hours (i.e. less than 5% of the whole program). The standards are used both in traditional 5-year course and in new two-level 6-year courses completing with master's degree.

During the last years some modifications in this preparation are taking place connected with multilevel structure of higher education: two-years' incomplete education, four-years' study for the Bachelor's degree; additional two-years' preparation to the Master's degree; alternative (traditional) way is five years' professional training for obtaining teacher's diploma. Besides, the rather large role belongs to three-years' graduate study, three-years' postgraduate study, and also to the ramified system of in-service professional training of each school or university teacher (ideally - once every five years of work).

The new system of continuous education for teachers is only arising, but both in traditional and modern systems there is a lot of unsolved problems, which are caused by the lack of the complex approach to the development of the professional competence of teachers.

We will consider the new approaches to teacher education for mathematical profile classes in a Master program in the Moscow City Pedagogical University.

# 3. The Master program in Mathematics Education in the Moscow City Pedagogical University.

### 3.1. Overview.

In the Moscow City Pedagogical University, a new Master Program on mathematics education devoted to preparing teachers for the work in profile classes with mathematical bias also has been prepared. The curriculum includes advanced courses in Mathematics Education as well as in Mathematics such as «Geometry of complex and dual numbers» or «Abstract algebraic systems».

Proceeding from the purposes of offered mathematical disciplines, subject matter and methods of its teaching should satisfy to the following requirements.

The subject matter should be based on the school curriculum and on the facts stated in the school textbooks.

The statement of a material should be in a sufficient measure strict and correct.

It is necessary to acquaint the students with methods of teaching of these disciplines, appropriate for the school complexity level.

It is necessary to connect the contents of these disciplines with elementary mathematics, to show, how to prepare the pupils of profile classes to understanding of ideas and methods of mathematical researches, to mastering university courses of higher mathematics.

For example, the offered syllabuses of geometrical disciplines of choice represent the adapted to the school level geometry courses, usually studied by students of Russian pedagogical universities. One can choose the following sections:

constructive geometry, i.e. theory of solving tasks on construction figures on a plane by compasses and ruler;

foundations of projective geometry, namely properties of the central projection and of the model of a projective plane i.e. extended Euclidean plane complemented by infinitely distant points;

properties of axonometric and Monge's method of representing spatial bodies on a plane.

analytical methods in solving tasks of elementary geometry;

geometry of complex numbers;

axiom of parallelism and elements of geometry of Lobachevsky (hyperbolic geometry).

Consider in more detail three syllabuses of offered geometrical disciplines.

### 3.2. Syllabuses of offered geometrical disciplines.

## 3.2.1. Analytical methods at solving tasks of elementary geometry.

Russian school textbooks of geometry and foundations of vector algebra and analytical geometry are considered. In particular, properties of inner product of vectors are considered, the equations of a straight line and a circle on a plane and planes and spheres in space are deduced. One of the basic skills, which should be mastered by the students on the exercises, is a choice of convenient rectangular Cartesian system of coordinates, on which the complexity of calculations depends.

# 3.2.2. Geometry of complex numbers.

The basic idea of this course is to acquaint the students with geometrical interpretation of two-dimensional algebras of complex, dual and double numbers, and also with analytical methods of study of geometrical properties of figures by means of these algebras. Future masters study properties of interpretation of complex numbers by points of a plane where by a known rule to each complex number the point of a plane, namely the so-called complex plane is put in a correspondence. The questions of correspondence between geometrical transformations of a plane and appropriate properties of functions of complex variables are considered.

The concept of a dual number is introduced. Properties of geometry of a plane of dual numbers are considered. Algebraic properties of double numbers are studied. On the plane of double numbers the pseudoeuclidean metrics is introduced, and the properties of figures on this plane are considered. Properties of straight lines and circles, and also geometrical transformations are studied by means of algebra of dual numbers.

# **3.2.3.** Axiom of parallelism and elements of geometry of Lobachevsky (hyperbolic geometry).

The basic purpose of a course is to show the students and pupils a place and meaning of the axiom of parallelism in the logical construction of Euclidean plane geometry, and also to acquaint them with some facts of hyperbolic geometry. It is well known that the attempts of the proof of the fifth postulate of Euclid have very interesting history closely connected to the history of development of a civilization. It is offered the following scheme of the construction of discipline. First, it is necessary to acquaint the pupils with the history of attempts of the proof of the fifth postulate of Euclid. In these attempts, the statements equivalent to the axiom of parallelism were used.

The proofs of equivalence of these statements to this axiom of parallelism are carried out. The axiom of parallelism of Lobachevsky is formulated, and it follows from it, that on a plane of Lobachevsky is not hold abovementioned statements. The pupils get acquainted with a model of Cayley-Klein where points of a plane of Lobachevsky are interpreted as internal points of a circle.

The experience of studying these disciplines by students and teaching in profile classes shows their efficiency. These courses acquaint students and pupils with a material having connected with the school program, but extending its framework, stimulate the great interest to geometry.

#### Literatur

Dorofeev, G.V., Kuznecova, L.V., Suvorova, S.B., Firsov, V.V. (1990). Differentiation in Mathematical education. Matematika v shkole, No. 5, 6-19.