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Optimal renewable-energy subsidies

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Optimal Renewable-Energy Subsidies*

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Abstract

We derive optimal subsidization of renewable energies in electricity markets. The analysis takes into account that capacity investment must be chosen under uncertainty about demand conditions and capacity availability, and that capacity as well as electricity generation may be sources of externalities. The main result is that generation subsidies should correspond to externalities of electricity generation (e.g., greenhouse gas reductions), and investment subsidies should correspond to externalities of capacity (e.g., learning spillovers). If only capacity externalities exist, then electricity generation should not be subsidized at all. Our results suggest that some of the most popular promotion instruments cause welfare losses.

Keywords: Peak-Load Pricing; Capacity Investment; Demand Uncertainty; Renewable Energy Sources; Optimal Subsidies; Feed-In Tariffs

JEL Codes: Q41; Q48; H23

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1 Introduction

Over the last two decades, many governments have introduced support schemes for electricity from renewable energy sources. According to the International Energy Agency's (IEA) World Energy Outlook 2011, \$ 44 billion were used for these subsidies in 2010. In many countries, it is a declared political goal to further raise renewables' market share, so the total amount of subsidies will probably rise. In the "New Policies Scenario" projections, which assume that the governments stick with their plans and serve as the baseline scenario, the IEA expects subsidies to reach almost \$ 180 billion per year in 2035 (IEA, 2011).

Without questioning whether this level of support is justified, the aim of this article is to derive optimal subsidy policies, so as to analyze whether currently popular promotion schemes are efficient. There is a large literature on reasons to promote renewable energies, of which the most important ones are learning spillover effects in manufacturing and second-best abatement of greenhouse gas emissions.[1] However, what has largely been neglected are the incentives of particular promotion policies regarding the supply behavior of renewable-energy capacity owners and their impact on welfare.

In this article, we close this gap and explicitly consider the policy implications of a distinguishing characteristic of electricity markets, namely, the difference between capacity and electricity generation. We analyze first-best subsidies for electricity generation technologies, taking external benefits of capacity and of electricity generation into account, of which zero externalities of either capacity or generation are special cases. By "first-best", we mean that the government has full information and it has access to non-distortive means of financing the subsidies (i.e., lump-sum taxes).

We find that marginal subsidies for electricity generation should equal its marginal external benefits, and marginal subsidies for capacity should cover marginal external benefits of capacity. While this seems trivial at first glance, it allows us to derive a number of policy-relevant conclusions. For example, it implies that if there are only

[1]See, for example, Rasmussen (2001), Jaffe et al. (2005), Bennear and Stavins, 2007, Kverndokk and Rosendahl (2007), Fischer and Newell (2008), Helm and Schöttner (2008), Gerlagh et al. (2009), Kalkuhl et al. (2013).

externalities of capacity (for instance, knowledge spillover effects of photovoltaic modules), then electricity generation should not be subsidized at all. Furthermore, only under very specific circumstances do optimal promotion schemes for renewable energy resemble the demand-independent "fixed feed-in tariffs" that are popular in many countries.

Our model uses the framework of a competitive peak-load pricing model – that is, the decision variable is the supply quantity, but in situations of high demand, supply can be limited by capacity. We are not aware of any literature that explicitly analyzes optimal subsidies in such a framework.[2] The model's unique characteristic is the separation of capacity and production as targets of subsidies. To focus on the basic principles shaping optimal subsidies, we assume that there is only one moment in which electricity generation and consumption take place and that there is only one electricity generation technology. While we think that neither of these assumptions changes the general insights of the model, we recognize that the full implications of these subsidies in a dynamic multi-technology market would have to be modeled explicitly.

While there is a large literature on renewable-energy promotion, as far as we know, only Bläsi and Requate (2010) and Reichenbach and Requate (2012) consider the implications of distinguishing between capacity and electricity generation. In these papers, learning spillover effects of capacity production are taken into account, making output subsidies for renewable-energy capacity producers optimal. However, in these models, electricity demand is deterministic and "capacity" is the number of firms, so that the distinction between capacity and electricity generation requires increasing marginal generation costs. By contrast, our model takes into account that at the moment of investment, demand and capacity availability are uncertain. Moreover, we model capacity as an explicit limit to electricity generation, which also allows to incorporate the case of constant marginal generation cost. In particular, this includes technologies like wind and solar power for which zero generation costs can be assumed.

[2]For an excellent survey of the theory of peak-load pricing, see Crew et al. (1995). For current applications of this model framework to electricity markets, see Borenstein and Holland (2005), Joskow and Tirole (2007) or, with renewable energy sources, see Chao (2011).

Implicitly, Newbery (2012) also distinguishes the different sources of positive externalities by stating that capacity rather than electricity generation should be promoted for the case of wind energy. We analyze this point in a general way using a formal model.

The paper proceeds as follows. We describe the model setting in Section 2.1, derive a social planner's solution in Section 2.2, and a decentralized solution in Section 2.3. Section 2.4 defines the optimal subsidies. As an application, we assess the promotion of renewables by fixed feed-in tariffs in Section 3. Finally, Section 4 discusses the results.

2 Model

2.1 The Market Environment

We consider a partial-equilibrium model of a market in which a good in the amount of q is traded. We assume that this good is electricity generated by some renewable energy source like biofuel, wind or photovoltaics.[3] Supply is limited by available capacity ak, where capacity is denoted by k and the capacity availability variable $0 \leq a \leq 1$ takes into account that at least some renewable energy technologies, in particular wind and solar energy, are not always completely available. $a = 0$ means that built capacity cannot be used, and $a = 1$ implies complete availability.

The model consists of two stages; the market stage follows the investment stage. In the investment stage, capacity k is built while the state of electricity demand, z, and the availability of capacity, a, are unknown. Thus, we have a setting of peak-load pricing under supply and demand uncertainty. In the market stage, a and z are drawn from a random distribution $F(a, z)$, with a density function $f(a, z)$. z is bounded between z_l and z_h, where $z_h > z_l$. Thus, z_h is the highest and z_l is the lowest state of demand. The price p is determined by electricity generation q and an inverse demand function

[3]The model is fairly general. It should be applicable to any sector in which a distinction between capacity and production is relevant and either of these can be the source of externalities. An important example is agriculture.

$p \equiv p(q,z)$.[4] For higher values of z, demand is higher for every quantity q, that is, the demand curve is shifted upwards: $\partial p(q,z)/\partial z > 0$.

There are two kinds of costs. Capacity k is built in the investment stage with an investment cost function $C(k)$. Generation costs of electricity are $c(q)$ and accrue in the market stage. Generation is limited by available capacity, $q \leq ak$. In both stages, there may be positive externalities, measured in money terms.[5] Capacity externalities (such as learning spillovers) are denoted by $B(k)$, and electricity generation externalities (such as carbon abatement) are $b(q)$.[6] Both are assumed to be concave functions, so they have non-increasing marginal effects.

In the second stage, consumer surplus v and producer surplus s are given by

$$v(q,z) \equiv \int_0^q p(\tilde{q},z)d\tilde{q} - p(q,z)q, \tag{1}$$

$$s(q,z) \equiv p(q,z)q - c(q). \tag{2}$$

The total market-stage welfare w is the sum of consumer surplus v, producer surplus s, and external benefits of production b:

$$w(q,z) \equiv v(q,z) + s(q,z) + b(q) = \int_0^q p(\tilde{q},z)d\tilde{q} - c(q) + b(q). \tag{3}$$

Letting $\mathbb{E}$ denote expectations, total expected welfare reads

$$\begin{aligned} W &\equiv \mathbb{E}\left[w(q(a,z),z)\right] + B(k) - C(k) \\ &= \int_{z_l}^{z_h}\int_0^1 f(a,z)w(q(a,z),z)\,da\,dz + B(k) - C(k), \end{aligned} \tag{4}$$

where $q(a,z)$ is the generated electricity in state (a,z).

[4]We assume that the functions have the typical characteristics, like a negative slope of the demand function, a non-negative slope of the marginal production costs function etc., and mention those that we deem important for clarification.

[5]The model could be reverted to analyze negative externalities of capacity and generation of electricity from fossil fuels.

[6]Both kinds of externalities are discussed in Section 4.

2.2 Socially Optimal Energy Generation and Investment Decisions

In this section we derive how a social planner, who can directly choose an optimal allocation of investment and generation, would maximize total welfare. We maximize the total expected welfare, (4), by maximizing the following Lagrangian:

$$\begin{aligned}\mathcal{L}_w = &\int_{z_l}^{z_h}\int_0^1 f(a,z)w(q(a,z),z)\,da\,dz + B(k) - C(k)\\ &+\int_{z_l}^{z_h}\int_0^1 \lambda_w(a,z)\left[ak - q(a,z)\right]\,da\,dz.\end{aligned} \tag{5}$$

$\lambda_w(a,z)$ is the Kuhn-Tucker multiplier for the capacity constraint in state (a,z), and thus the shadow price of capacity in that state. Generation cannot be negative, $q \geq 0$. Taking this non-negativity condition into account, the first-order conditions for the optimal electricity generation q in each state (a,z) are:[7]

$$\frac{\partial \mathcal{L}_w}{\partial q} = f(a,z)\frac{\partial w(q^*(a,z),z)}{\partial q} - \lambda_w^*(a,z) \leq 0, \quad q^*(a,z) \geq 0, \quad q^*(a,z)\frac{\partial \mathcal{L}_w}{\partial q} = 0. \tag{6a}$$

The first-order conditions for the optimal choice of capacity are:

$$\frac{\partial \mathcal{L}_w}{\partial k} = B'(k^*) - C'(k^*) + \int_{z_l}^{z_h}\int_0^1 a\lambda_w^*(a,z)\,da\,dz \leq 0, \quad k^* \geq 0, \quad k^*\frac{\partial \mathcal{L}_w}{\partial k} = 0, \tag{6b}$$

and, finally, the Kuhn-Tucker conditions for each state,

$$\frac{\partial \mathcal{L}_w}{\partial \lambda_w} = ak^* - q^*(a,z) \geq 0, \qquad \lambda_w^*(a,z) \geq 0, \qquad \lambda_w^*(a,z)\frac{\partial \mathcal{L}_w}{\partial \lambda_w} = 0, \tag{6c}$$

determine when to use full capacity. The asterisks denote (social) optimality of our choice variables q, k, and λ_w.

Firstly, we can derive an optimal supply rule. For this, we can distinguish three cases, consisting of combinations of the state and available capacity. Depending on the level of demand, it may either be optimal to generate at the capacity limit, $q = ak$, or to generate a positive amount below the capacity limit, $0 < q < ak$, or to generate nothing

[7]For the general methods of non-linear optimization see, for example, Chiang and Wainwright (2005).

at all, $q = 0$. In the following, we derive the conditions for the respective decision to be optimal.

In the first case, using all available capacity is optimal, $q^*(a, z) = ak > 0$.[8] By condition (6a), this implies

$$\lambda_w^*(a, z) = f(a, z)\frac{\partial w(ak, z)}{\partial q}, \tag{7}$$

which must be non-negative by (6c), i.e., $\partial w(ak, z)/\partial q \geq 0$. Thus, all available capacity is used if marginal welfare of generation is still non-negative at the capacity limit, and the shadow price of capacity for the respective state equals marginal welfare weighted by the state's density. In general, marginal welfare of generation is

$$\frac{\partial w(q, z)}{\partial q} = p(q, z) - c'(q) + b'(q), \tag{8}$$

so $q^*(a, z) = ak$ would imply that the price of electricity (that is, the marginal gross consumer surplus) and marginal externalities of generation at the capacity limit together are at least as high as marginal generation costs.

In the second case, a positive generation below the capacity limit is optimal, $ak > q^*(a, z) > 0$. By condition (6a) and (6c),

$$\lambda_w^*(a, z) = f(a, z)\frac{\partial w(q^*(a, z), z)}{\partial q} = 0. \tag{9}$$

Thus, generation is stopped below the capacity limit if marginal welfare is zero, and, hence, welfare cannot be increased by extending generation. In other words, if capacity is not scarce, its shadow price is zero.

The third and final case is that in which zero generation is optimal. Again, (6c) tells us that the shadow price of capacity in such states is zero. By (6a), we can see that zero generation is optimal if even the first produced unit does not yield positive marginal welfare.

[8] In the following discussion, we assume, without loss of generality, that $k > 0$.

Summarizing, we have the following *optimal supply rule*:

Proposition 1 (Optimal Supply)**.** *The welfare-maximizing supply is defined as follows:*

$$q^*(a,z) = \begin{cases} ak & \text{if } f(a,z)\frac{\partial w(ak,z)}{\partial q} = \lambda_w^*(a,z) \geq 0, \\ q^* \in [0, ak] & \text{if } f(a,z)\frac{\partial w(q^*(a,z),z)}{\partial q} = \lambda_w^*(a,z) = 0, \\ 0 & \text{if } f(a,z)\frac{\partial w(0,z)}{\partial q} \leq 0 = \lambda_w^*(a,z). \end{cases} \tag{10}$$

In the following, we just write q^*, λ_w^* to economize on notation. We can determine the impact of changes in the state of demand by differentiating the optimality condition for state (a, z) with respect to q and a and rearranging:

$$\frac{\partial q^*}{\partial z} = \begin{cases} \frac{\partial^2 w(q^*,z)/\partial q \partial z}{-\partial^2 w(q^*,z)/\partial q^2} = \frac{\partial p(q^*,z)/\partial z}{-\partial^2 w(q^*,z)/\partial q^2} & \text{if } q^*(a,z) \in (0, ak), \\ 0 & \text{else}, \end{cases} \tag{11}$$

where the first case derives from differentiating the first-order condition (9). The denominator in (11) is positive due to the usual assumptions about cost and benefit functions. Thus, the higher the demand, the more electricity should be generated as long as the capacity limit is not reached. Similarly,

$$\frac{\partial q^*}{\partial a} = \begin{cases} k & \text{if } q^*(a,z) = ak, \\ 0 & \text{else}. \end{cases} \tag{12}$$

This means that higher capacity availability raises electricity generation if capacity is the relevant limit. By contrast, if generation below the capacity limit is optimal, then an increase in available capacity does not change optimal generation.

We complete the socially optimal allocation by deriving the optimal investment rule. The first-order conditions for optimal capacity choice, (6b), can be summarized as follows:

Proposition 2 (Optimal Investment)**.** *Capacity is chosen so that marginal investment costs equal the sum of its direct marginal externalities and the sum of shadow prices of* *available capacity in all states, weighted by the share of capacity that is available,*

$$C'(k^*) = B'(k^*) + \int_{z_l}^{z_h} \int_0^1 a\lambda_w^*(a, z)\, da\, dz, \tag{13}$$

given that there is a non-negative amount of capacity for which both sides of this equation are also non-negative. Otherwise, zero capacity is built.

Proof. Evaluate (6b) for $k^* > 0$. □

From the discussion above, we know that the shadow price of capacity is positive only in those states in which available capacity limits generation: $q^* = ak$. As given in (13), marginal investment costs must pay off in these states. Moreover, if only a fraction a of capacity k is available in state (a, z), the shadow price is weighted by a – low availability makes it more expensive to have available capacity. For example, photovoltaic capacity is less valuable if sunshine is rare. Furthermore, states of high demand justify more capacity investment if it is probable that they actually occur. This follows from the fact that, by (9), the shadow price for a state equals marginal welfare weighted by the density of that state.

Note two implications of the welfare-maximizing allocation. Firstly, the larger capacity externalities $B(k)$, the more capacity is built, and the more states (a, z) in which there is idle capacity. If there are very large benefits of building capacity, it may even be true that full capacity should never be used. This can be seen by recognizing that (13) could be fulfilled even if the shadow price is zero for all states. Marginal capacity externalities $B'(k^*)$ must then equal marginal investment costs $C'(k^*)$.

Secondly, we can characterize which market price will be observed in state (a, z):

Proposition 3 (Optimal Market Price of Electricity). *Define the generation quantity $\hat{q}(z)$ by $p(\hat{q}, z) = c'(\hat{q})$.*

(i) Suppose that $q^* = ak \leq \hat{q}(z)$. Then,

$$p(ak, z) + b'(ak) \geq p(ak, z) \geq c'(ak) \geq 0$$

in optimum.

(ii) *Suppose that* $q^* \geq \hat{q}(z)$. *Then,* $c'(q^*) - p(q^*, z) = b'(q^*) \geq 0$ *and* $p(q^*, z) \gtreqless 0$.

Proof. (i) follows by definition from the facts that the price is determined by the demand function, that the demand function is downward sloping, that externalities are non-negative, and that the marginal cost function is non-negative and has a non-negative slope. (ii) follows from these facts and the fact that marginal welfare as defined in (8) must be zero. □

Point (i) of the proposition tells that if socially optimal generation is limited by capacity, it must be true that the price is above marginal generation costs. Point (ii) states the fact that in optimum, the price is lower than marginal generation costs if there are positive externalities and generation is not constrained by capacity. However, a welfare loss always results when the price is lower than marginal generation costs minus the marginal externalities from generation. Note that the price is always positive if every generated amount of electricity meets a positive willingness to pay. However, such an assumption about (short-run) electricity demand functions is not realistic, because every generated kWh has to be used in the same moment and cannot be stored (in relevant amounts). Thus, in electricity markets negative prices can occur.

2.3 Decentralized Allocation

We now consider the behavior of profit-maximizing firms under perfect competition. The aim is to derive how much they invest and generate, given a certain structure of subsidies, so that we can later derive optimal subsidies. To stick with the notion of producer surplus from (2), we define the operators' *second-stage profits* by

$$\pi(q, a, z) = p(a, z)q + \chi(q, a, z) - c(q), \tag{14}$$

which differs from producer surplus by the subsidy χ. The latter is a payment from the government to the firms, which may depend on the generated quantity, the state of demand, and the availability of capacity – this allows derivation of whether it is actually optimal to condition subsidies on these variables. We write $p(a, z)$ instead of $p(q, z)$ because firms take the price in state (a, z) as given; they are assumed to behave competitively and thus do not take their influence on the price into account. Total expected profit is given by

$$\begin{aligned}\Pi &\equiv \mathbb{E}\left[\pi(q(a,z),a,z)\right] + \sigma(k) - C(k)\\ &= \int_{z_l}^{z_h}\int_0^1 f(a,z)\pi(q(a,z),a,z)\,da\,dz + \sigma(k) - C(k),\end{aligned} \tag{15}$$

where $\sigma(k)$ is a subsidy for capacity installation. We assume for $\chi(q, a, z)$ and $\sigma(k)$ that they are continuous, concave functions of q and k, respectively – that is, marginal subsidies are constant or decreasing. The investors' Lagrangian is

$$\begin{aligned}\mathcal{L}_\pi = &\int_{z_l}^{z_h}\int_0^1 f(a,z)\pi(q(a,z),a,z)\,da\,dz + \sigma(k) - C(k)\\ &+\int_{z_l}^{z_h}\int_0^1 \lambda_\pi(a,z)\left[ak - q(a,z)\right]\,da\,dz,\end{aligned} \tag{16}$$

and the first-order conditions are

$$\frac{\partial \mathcal{L}_\pi}{\partial q} = f(a,z)\frac{\partial \pi(q^{\#}(a,z),a,z)}{\partial q} - \lambda_\pi^{\#}(a,z) \le 0, \qquad q^{\#}(a,z) \ge 0,$$
$$q^{\#}(a,z)\frac{\partial \mathcal{L}_\pi}{\partial q} = 0 \tag{17a}$$

for profit-maximizing electricity generation in each state (a, z),

$$\frac{\partial \mathcal{L}_\pi}{\partial k} = \sigma'(k^{\#}) - C'(k^{\#}) + \int_{z_l}^{z_h}\int_0^1 a\lambda_\pi^{\#}(a,z)\,da\,dz \le 0, \quad k^{\#} \ge 0, \quad k^{\#}\frac{\partial \mathcal{L}_\pi}{\partial k} = 0 \tag{17b}$$

for the firms' choice of capacity, and, finally, the Kuhn-Tucker conditions of each state (a, z),

$$\frac{\partial \mathcal{L}_\pi}{\partial \lambda_\pi} = ak^{\#} - q^{\#}(a,z) \ge 0, \qquad \lambda_\pi^{\#}(a,z) \ge 0, \qquad \lambda_\pi^{\#}(a,z)\frac{\partial \mathcal{L}_\pi}{\partial \lambda_\pi} = 0, \tag{17c}$$

where $\#$ denotes profit maximization. The interpretation is exactly the same as in the case of the social planner's optimality conditions (6), except that marginal profit takes the place of marginal welfare. For equilibrium generation $q^{\#}(a,z)$, marginal profit is given by

$$\frac{\partial \pi(q^{\#}(a,z),a,z)}{\partial q} = p(q^{\#}(a,z),z) + \frac{\partial \chi(q^{\#}(a,z),a,z)}{\partial q} - c'(q^{\#}(a,z)). \tag{18}$$

We can solve for the firms' supply behavior along the lines of solving (17), and summarize:

Proposition 4. *The profit-maximizing supply is defined as follows:*

$$q^{\#}(a,z) = \begin{cases} ak & \text{if } f(a,z)\frac{\partial \pi(ak,a,z)}{\partial q} = \lambda_{\pi}^{\#}(a,z) \geq 0, \\ q^{\#} \in [0, ak] & \text{if } f(a,z)\frac{\partial \pi(q^{\#}(a,z),a,z)}{\partial q} = \lambda_{\pi}^{\#}(a,z) = 0, \\ 0 & \text{if } f(a,z)\frac{\partial \pi(0,a,z)}{\partial q} \leq 0 = \lambda_{\pi}^{\#}(a,z). \end{cases} \tag{19}$$

The supply rule says that the firms use all available capacity, $q^{\#} = ak$, if the market's willingness to pay for electricity plus the marginal generation subsidy is at least as large as marginal generation costs at the capacity limit. A positive amount of electricity below the capacity limit, $ak > q^{\#} > 0$, is generated if this sum is zero, and none is generated if even the first generated unit of electricity does not yield a profit.

Likewise, we can solve (17b) for the firms' investment rule and summarize:

Proposition 5 (Profit-Maximizing Investment)**.** *Capacity is chosen so that marginal investment costs equal the sum of marginal capacity subsidies and the sum of shadow prices of available capacity in all states, weighted by the share of capacity that is available,*

$$C'(k^{\#}) = \sigma'(k^{\#}) + \int_{z_l}^{z_h} \int_0^1 a\lambda_{\pi}^{\#}(a,z)\, da\, dz, \tag{20}$$

given that there is a non-negative amount of capacity for which both sides of this equation are also non-negative. Else, zero capacity is built.

2.4 Decentralizing the First-Best Solution: Optimal Subsidies

A welfare-maximizing government can use subsidies to reproduce the first-best allocation. Firms choose the socially optimal allocation if the following conditions are met.

Proposition 6 (Optimal Subsidies: Necessary Conditions)**.**

(i) Consider the states (a, z) *for which the optimal supply rule from Proposition 1 implies using all available capacity,* $q^*(a, z) = ak$*. For these states it must hold that*

$$p(ak, z) + \frac{\partial \chi(ak, a, z)}{\partial q} - c'(ak) \geq 0. \tag{21}$$

(ii) Consider the states (a, z) *for which the optimal supply rule from Proposition 1 yields an interior solution,* $q^*(ak, z) \in (0, ak)$*. For these states it must hold that*

$$\frac{\partial \chi(q^*, a, z)}{\partial q} - b'(q^*) = 0. \tag{22}$$

(iii) Consider the states (a, z) *for which the optimal supply rule from Proposition 1 yields zero generation,* $q^*(a, z) = 0$*. For these states it must hold that*

$$p(0, z) + \frac{\partial \chi(0, a, z)}{\partial q} - c'(0) \leq 0. \tag{23}$$

(iv) For firms' investment according to Proposition 5 to be optimal as described in Proposition 2, it must hold that

$$B'(k^*) - \sigma'(k^*) + \int_{z_l}^{z_h} \int_0^1 a \left[\lambda_w^*(ak, z) - \lambda_\pi^\#(ak, a, z)\right] \, da \, dz = 0. \tag{24}$$

Proof. We start with the inner solution, case (ii). For a positive generation below capacity, $q = q^*(a, z) < ak$, to be both socially optimal and profit-maximizing, we need

$$p(q^*, z) - c'(q^*) + b'(q^*) = 0,$$
$$p(q^*, z) + \frac{\partial \chi(q^*, a, z)}{\partial q} - c'(q^*) = 0$$

from (6) and (8), (17) and (18). Solving these equations, we see that (22) must hold. For (iii) and (i), the exact level of generation subsidies is not important because firms cannot use more than all available capacity or less than none – see the firms' optimal supply rule in Proposition 4. (iv) is implied by equating (13) and (20) and rearranging. □

In particular, the following subsidy scheme fulfills these necessary conditions:[9]

Proposition 7 (Optimal Subsidies: Sufficient Conditions)**.** *The subsidy scheme (consisting of generation subsidies and capacity subsidies) is optimal if for all states* (a, z)

$$\frac{\partial \chi(q^*, a, z)}{\partial q} = b'(q^*), \tag{25}$$

and if

$$\sigma'(k^*) = B'(k^*). \tag{26}$$

Proof. If the shadow prices in the first-order condition sets of welfare maximization, (6) and profit maximization, (17) are identical for the same capacity level, then generation decisions must be identical as well. This, in turn, is true if marginal welfare (8) and marginal profit (18) of q^* coincide for the same available capacity. Substituting (25) in these equations shows that they always do. (24) then implies (26). □

This yields a simple rule for optimal subsidies. Firstly, marginal generation subsidies should equal marginal external benefits of generation in every state of demand and capacity availability. Secondly, marginal capacity subsidies should equal marginal external benefits of capacity. We show in detail what the optimal-subsidy rules imply for the dependence of optimal subsidies on the state in Appendix A.1.

[9]There can be other subsidy schemes fulfilling the necessary conditions. Yet, these schemes are complex and, presumably, unrealistic for a real-world application. For example, for states in which all capacity is to be used, subsidies can be arbitrarily high as long as (21) is fulfilled. However, this implies that the shadow price of capacity for the firms exceeds the social shadow price. To still induce optimal investment, (24) then implies that marginal capacity subsidies must be lower than marginal capacity externalities to counterbalance this difference.

3 Application: Assessing the Promotion of Renewables by Fixed Feed-in Tariffs

Under a fixed feed-in tariff system, an operator of renewable-energy capacity receives a certain amount of money for every unit of electricity that is generated and fed into the grid. Such subsidies are one of the most widely applied instruments to promote renewable energies (cf. IEA/IRENA, 2014). In this section we demonstrate that they can be optimal only under very specific assumptions.

If the instrument to promote renewables is a fixed tariff ϕ per unit of electricity, then the tariff ϕ itself does not depend on the generated quantity q. The firms' second-stage profit (14) becomes:

$$\pi(q) = \phi q - c(q). \tag{27}$$

The firms' Lagrangian is

$$\begin{aligned}\mathcal{L}_\phi = &\int_{z_l}^{z_h}\int_0^1 f(a,z)\left[\phi q - c(q)\right]\, da\, dz - C(k)\\ &+ \int_{z_l}^{z_h}\int_0^1 \lambda_\phi(a,z)\left[ak - q(a,z)\right]\, da\, dz,\end{aligned} \tag{28}$$

so that the first-order conditions corresponding to (17) are

$$\frac{\partial \mathcal{L}_\phi}{\partial q} = f(a,z)\left[\phi - c'(q)\right] - \lambda_\phi^{\#}(a,z) \le 0, \quad q^{\#}(a,z) \ge 0, \quad q^{\#}(a,z)\frac{\partial \mathcal{L}_\phi}{\partial q} = 0 \tag{29a}$$

for profit-maximizing electricity generation in each state (a,z),

$$\frac{\partial \mathcal{L}_\phi}{\partial k} = -C'(k^{\#}) + \int_{z_l}^{z_h}\int_0^1 a\lambda_\phi^{\#}(a,z)\, da\, dz \le 0, \quad k^{\#} \ge 0, \quad k^{\#}\frac{\partial \mathcal{L}_\phi}{\partial k} = 0 \tag{29b}$$

for the firms' choice of capacity, and, finally, the Kuhn-Tucker conditions of each state

(a, z),

$$\frac{\partial \mathcal{L}_\phi}{\partial \lambda_\phi} = ak^{\#} - q^{\#}(a, z) \geq 0, \qquad \lambda_\phi^{\#}(a, z) \geq 0, \qquad \lambda_\phi^{\#}(a, z)\frac{\partial \mathcal{L}_\phi}{\partial \lambda_\phi} = 0. \tag{29c}$$

Let us firstly consider technologies with constant marginal generation costs. In particular, this includes those renewable-energy technologies that have zero marginal generation costs: Wind energy and photovoltaics. The difference between the tariff and marginal generation costs, $\phi - c'(q)$, then is the same in all states (a, z). This is also true for the shadow price $\lambda_\phi^{\#}$ of capacity. If $\phi - c'(q)$ is positive, then all available capacity is always used. If it is zero or negative, then no capacity is built. Thus, a fixed feed-in tariff can only be an optimal way of promoting such a technology if available capacity should always be used completely.

What about technologies with increasing marginal generation costs (for which biofuel plants may be an example)? Again, if it is socially optimal always to use all available capacity, then implementing this behavior is possible with a fixed feed-in tariff that is high enough to make the firms' shadow price always positive. However, if this is not the case, (29a) and (29c) show that generation below the capacity limit can be incentivized if the tariff equals marginal generation cost for that quantity and, thus, the shadow price of capacity is zero:

$$\phi - c'(q) = \lambda_\phi^{\#}(a, z) = 0. \tag{30}$$

Suppose that such an interior solution is chosen for one state (a, z). Because ϕ is the same in all states, the same generation quantity must be profit-maximizing for all other demand states. Thus, any capacity larger than the amount necessary to fulfill (30) would never be used and cannot pay off. But then (30) cannot describe an interior solution if firms invest according to (29b). Therefore, we can conclude that a fixed feed-in tariff can only incentivize to build an amount of capacity that is used whenever it is available. In brief, a fixed feed-in tariff can, in general, be optimal only if it is always optimal to use all available capacity.

To analyze whether this can be true for actual electricity markets, note that using full capacity is optimal if the price for electricity plus the marginal externalities of generation at the capacity limit always (at least) cover marginal generation costs. Because marginal generation costs and externalities are not directly observable, it is difficult to demonstrate welfare losses in empirical data. Yet we think that there are at least some indications to welfare losses due to fixed feed-in tariffs.

In Germany, we occasionally observe hours with negative electricity prices, but, at the same time, electricity from renewables is generated. Even if we assume the lowest marginal generation costs, zero, which is plausible for wind and solar power, fixed feed-in tariffs can only be optimal when there are marginal generation externalities that are equivalent to the negative prices. Given that there have been hours in which the wholesale price for electricity in Germany was around –500 € per MWh, it does not seem likely that there are generation externalities that justify this.[10]

Note that strongly negative prices are only the most obvious sign that fixed feed-in tariffs may induce welfare losses. For renewables with positive marginal generation costs, like biomass, welfare losses can also occur when the price is positive, namely, whenever the price falls below marginal generation costs minus marginal generation externalities. If there are no marginal generation externalities (which we argue in the discussion below), then welfare losses occur whenever the price falls below marginal generation costs.

Summarizing, fixed feed-in tariffs are suboptimal as soon as it is not optimal to use all available capacity, i.e., as soon as renewables should respond to demand. In electricity markets with a small share of renewables capacity, this might be of minor relevance. However, in electricity markets with a considerable capacity of renewables (in relation to demand), these considerations are important. For instance, in Germany the cumulative capacity of photovoltaics and wind energy together, given full availability, account for more than 60 GW, while demand usually is between 30 GW and 80 GW (see ENTSO-E, 2014). Thus, it is obvious that renewables are not a niche product

[10]For illustrations and discussions of negative-price occurrences in Germany see, for example, Andor et al. (2010) or Brandstätt et al. (2011).

anymore in the German electricity market, but have a considerable market share and thus should respond to demand.

4 Discussion

This article has shown that defining the rationale to support renewable energies is crucial to identify the optimal promotion instrument. If positive externalities arise from the production and installation of renewable energies capacity, but not from the generation of electricity, then generation-based instruments (like all kinds of feed-in tariffs as well as renewable portfolio standards) are suboptimal. They distort the supply decision, and thus cause welfare losses. Instead, a capacity subsidy that is equivalent to the externality would maximize welfare.

In contrast, if externalities arise from the generation of electricity, but not from capacity, a generation-based subsidy that reflects this benefit would maximize welfare. However, fixed feed-in tariffs are most probably not the adequate instrument. Instead, a subsidy that is paid on top of the market price and reflects the generation externality should be used. Finally, if there are externalities from both renewable capacity and electricity generation by renewables, then a combination of a generation subsidy and a capacity subsidy is optimal. It should be clear that the magnitudes of the subsidies should correspond to the specific externalities.

Thus, economists should carefully identify the reasons for and the aims of promoting renewable energies. Once they are clear, we can systematically derive the optimal promotion of renewables. To be concrete, we briefly describe normative conclusions that arise from this model based on assumptions that we deem plausible.

Basic economic theory suggests direct caps on or prices for emitting greenhouse gases as first-best instruments for internalizing this negative externality of fossil fuel. Therefore, we do not see them as a reason to promote electricity generation from renewable energies. In contrast, learning spillovers seem to be a plausible reason to promote renewables (see, for instance, Fischer and Newell, 2008 and Gerlagh et al., 2009). However, while the production and installation of renewable capacity will very likely engender beneficial external learning effects,[11] it is difficult to conceive of such positive externalities from electricity generation. Based on these assumptions, the optimal promotion scheme for renewable energies is a capacity subsidy.[12] Generation-based subsidies, instruments that are currently the most common promotion schemes, then are likely even to be harmful.

However, some authors argue for subsidizing electricity from renewable-energy sources as a second-best policy (cf. Bennear and Stavins, 2007, Kalkuhl et al., 2013). According to this line of argument, the implementation of an efficient first-best instrument – e.g., a correctly adjusted carbon tax – is impossible (or at least very unrealistic) due to political (or other) constraints, but alternative second-best approaches can be a pragmatic solution. The positive externalities of renewables then arise from the generation of electricity because renewables substitute fossil fuels. If this is deemed to be the real cause to promote renewables, a generation-based subsidy may be welfare-maximizing.If the short-run marginal benefits of abating carbon emissions are about constant (cf. McKibbin and Wilcoxen, 2002), and if one additional kWh of electricity from renewable energy sources approximately replaces one kWh of electricity from fossil fuels (because the elasticity of electricity demand is low), then the marginal positive externality of electricity from renewables is about constant, and we would suggest a constant per-kWh subsidy.

[11]Positive externalities of capacity manufacturing arise from spillovers of learning-by-doing. Additional positive externalities may stem from research and development (R&D) spillovers. However, these would be independent from the amount of manufactured capacity and are therefore not part of our model. If these exist, an additional subsidy for R&D could be optimal, see, for instance, Fischer and Newell, 2008.

[12]The level of optimal subsidies would depend on estimates of learning-spillover effects and, thus, is an empirical question. Such estimates would have to be conducted separately for each technology.

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A Appendix

A.1 Dependence of Optimal Subsidies on the State of Demand and the Availability of Supply

In the following proposition, we derive whether the optimal (marginal) subsidy should change with the state of the world:

Proposition A.1 (Dependence of Optimal Subsidies on the State)**.** *Consider the states (a, z) for which the optimal supply rule from Proposition 1 yields an interior solution, $q^*(z, ak) \in (0, ak)$, or assume that the subsidy scheme has the form of Proposition 7. Then it must hold that*

$$\frac{\partial^2 \chi(a, z, q^*)}{\partial z \partial q} + \left[\frac{\partial^2 \chi(a, z, q^*)}{\partial q^2} - b''(q^*)\right]\frac{\partial q^*}{\partial z} = 0, \tag{A.1}$$

$$\frac{\partial^2 \chi(a, z, q^*)}{\partial a \partial q} + \left[\frac{\partial^2 \chi(a, z, q^*)}{\partial q^2} - b''(q^*)\right]\frac{\partial q^*}{\partial a} = 0. \tag{A.2}$$

Proof. Differentiate (22). □

To illustrate, suppose that subsidies take a particular (but typical) form:

Proposition A.2 (Per-Unit Subsidies)**.** *Suppose that subsidies are a fixed (but possibly* *state-dependent) per-unit payment on top of the market price:*

$$\eta(a,z) = \frac{\partial \chi(a,z,q)}{\partial q}. \tag{A.3}$$

Then the dependence of subsidies on the state is given as follows:

$$\frac{\partial \eta(a,z)}{\partial z} = b''(q^*)\frac{\partial q^*}{\partial z}, \qquad \frac{\partial \eta(a,z)}{\partial a} = b''(q^*)\frac{\partial q^*}{\partial a}. \tag{A.4}$$

Proof. Substitute (A.3) in (A.1) and (A.2). □

Thus, if marginal benefits are constant, the per-unit subsidy should be constant in all states. By contrast, if marginal benefits are decreasing in q, (11) and (A.4) imply that an increase in the strength of demand should lead to a lower per-unit subsidy if the capacity limit is not binding. Likewise, by (12) and (A.4) an increase in capacity availability should lower the per-unit subsidy if the capacity limit *is* binding.