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Digital technology in mathematics education: a reflective look into the mirror

1. Introduction

Nowadays, we are surrounded by a diversity of digital tools and devices. Digital technology includes visible tools, such as smartphones and tablets, as well as technology embedded less visibly in, for example, cars or medical equipment; in both cases, however, digital technology drastically affects daily life as well as professional practice. As a consequence, one might expect education to be in a process of transformation, too: on the one hand, education should prepare for a technology-rich future, and on the other, it might benefit from the opportunities that digital technology offers.

But is this really the case? Is education, and in our case mathematics education in particular, involved in a fundamental process of change due to the availability of digital tools? And, if the answer is yes, do we have evidence that this change leads to improvements in mathematics achievement? These are the questions that we want to reflect upon in this contribution.

2. Inversion as an example

It is beyond any doubt that digital tools may invite, or at least can be used for, interesting mathematical activities. As an example, we look at a well-known lithograph, made in 1935 by the Dutch artist M.C. Escher, entitled 'Hand with reflecting sphere' (see http://en.wikipedia.org/wiki/Hand_with_Reflecting_Sphere). Inspired by this spherical self-portrait I set up a two-dimensional variant using Geogebra (http://www.geogebra.org/). Figure 1 shows a very rough sketch of a face, as well as its image under a kind of reflection in the unit circle with centre M. More precisely, this mapping is called an inversion, and the im-age A' of a point A (not M) lies on the ray starting in M through A, such that MA' equals $1/MA$.

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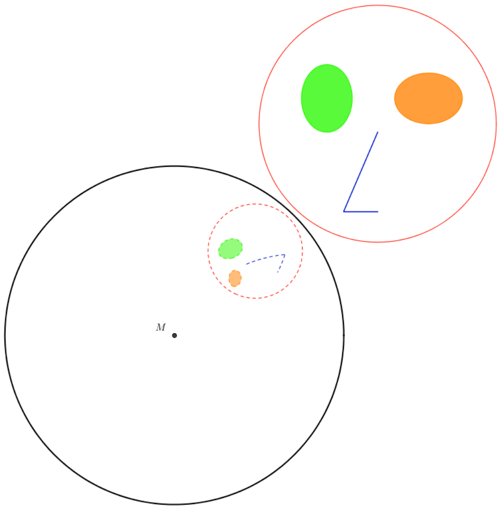

Figure 1. Inversion in the unit circle

A careful look at the image of the nose reveals that straight line segments do not remain straight under inversion. We can understand that at least something needs to change: if a straight line lies completely outside the unit circle, its inversion will be completely inside the circle, so it cannot be a straight line anymore. In Figure 2, we investigate the inverse of a straight line, which appears to be a circle through the centre *M*. Of course, as the distance *MA* is approaching infinity while point *A* on the line is moving away, the distance *MA'* will approach 0. This explains *M* being on the image of the line. The proof that lines are mapped into these circles (and vice versa) can be obtained through algebra.

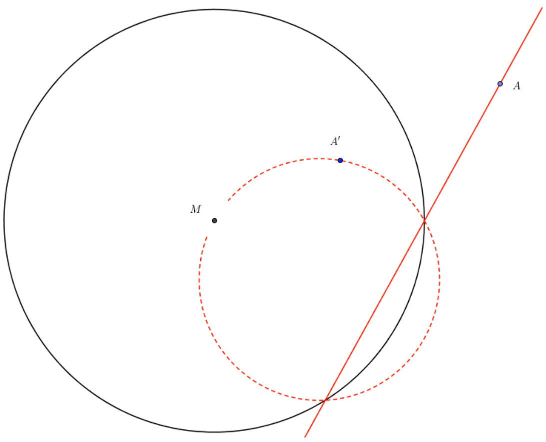

Figure 2. Inversions of lines and circles

Finally, we investigate the inversions of more complicated curves: Pascal's limacons or snail curves. If we use $R = 1 + a \cdot \cos(\theta)$ as polar equations of these curves, Figure 3 shows these curves for $a>1$, $a=1$, and $a<1$, respectively. The images suggest that the inversions are the conics. Indeed, $R = 1/(1 + a \cdot \cos(\theta))$ is a polar equation of the conics!

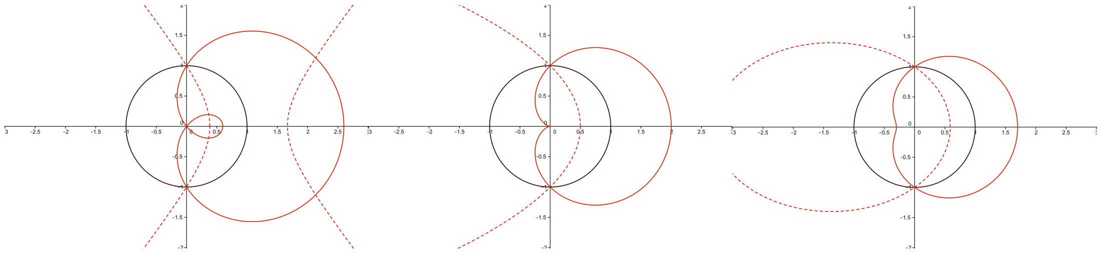

Figure 3. Pascal's snail curves and conics as inversions

This example, which I previously used with students (Drijvers, 1992), shows how exploration with digital technology, in this case a dynamic geometry tool, may lead to conjectures, invite proof and provide insight in mappings such as inversions, which without such a digital tool would have been less accessible.

3. What is known about the effect of using ICT on mathematical achievement?

If we have had inspiring examples around for many years, one might expect that they are intensively used in mathematics education and that this has led to students' improved understanding and mathematical achievement. Is this impression backed up by research data?

This is only the case to a limited extent. In spite of the impressive number of research studies devoted to the use of technology in mathematics education, there are not so many review studies or publications that synthesize the findings in a somewhat generic way. I found three recent review studies (Cheung and Slavin, 2011; Li and Ma, 2010; Rakes, Valentine, McGatha, and Ronau, 2010). These three studies report positive effects of the use of digital technology in mathematics education. Rakes, Valentine, McGatha, and Ronau (2010) speak about small but significant positive effects, specifically for algebra. For mathematics in general, Li and Ma's (2010) review includes 41 studies and similarly report "… a moderate but significant positive effect of computer technology on mathematics achievement" (Li & Ma, 2010, p. 232). Cheung and Slavin in their 2011 study set the criteria for studies to be included higher, including an experimental design and long duration. Their final conclusion refers to a modest difference: "Educational technology is making a modest difference in learning of mathematics. It is a help, but not a breakthrough." (Cheung & Slavin, 2011, p. 20). In short, the effect of the use of digital resources on mathematics achievement seems to be positive, but modest. This is not an overwhelming result.

If we consider these review studies in more detail, it is interesting to notice that the effect sizes reported in the different research reports did not significantly increase over time. This suggests that, even if digital tools became more sophisticated and more widespread, and while teachers became more used to exploiting them in their courses, the benefits for student achievement did not seem to increase. Other findings from the reviews studies are that small-scale studies have bigger effect sized than large-scale ones – which makes sense as small-scale studies are easier to control – and that short interventions do not necessarily lead to smaller effect sizes. The Rakes et al. (2010) review study also claims that the largest effect sizes are found in studies on conceptual understanding rather than on procedural skill acquisition (Rakes et al., 2010). The latter finding is interesting, as it has often been suggested that digital tools are particularly efficient for practicing procedural skills.

All in all, we conclude that there is modest support for the claim that the use of digital technology may have a positive effect on student achievement. However, little is known about decisive factors that explain these effects, or about successful approaches in teaching that may optimize the possible benefits.

As an aside, we notice that the above review studies only included experimental quantitative studies, whereas an important body of research concerns qualitative, often explorative studies. It is a challenge to researchers to design studies on the use of digital technology in mathematics education that combine the affordances of both methodological paradigms.

4. Promising perspectives

The previous section shows that our knowledge on fruitful integration of technology in mathematics education is limited. What perspectives can help us to further proceed in this direction? In the following, we briefly address the notions of instrumental genesis and instrumental orchestration, and the issue of digital assessment.

Instrumental genesis

The interplay between the user and the digital tool for solving a mathematical task is a subtle bi-directional phenomenon: the user decides on howway he wants to use the tool, but the opportunities and constraints of the tool also guide how the user will use it. Techniques for using the tool will co-emerge with the user's mathematical thinking. This process of co-emergence is called instrumental genesis (Artigue, 2002). It is based on the distinction between an artefact, the (in our case technological) object that the students use, and an instrument, i.e., the artefact together with the mental schemes that the user puts into action while using the artefact (Verillon and Rabardel, 1995). Examples of such schemes can be found in literature (Drijvers, 2003; Drijvers and Gravemeijer, 2004).

The reason why I consider the notion of instrumental genesis important is that it offers a lens to observe and to become aware of the interplay between the techniques to use a digital tool and the mathematical thinking involved. If we, as teachers, educators or researchers get our fingers behind this interaction, we will be better able to understand student learning with technology and to exploit the potential digital tools offer for learning. As soon as we really understand the process of instrumental genesis in specific situations (i.e., for specific tools and tasks), we can generalize this knowledge towards the relationship between tool use and mathematical learning in general.

Instrumental orchestration

Instrumental genesis is an important lens to look at student learning with technology. However, students' instrumental genesis usually needs guidance by a teacher. For this, Trouche (2004) introduced the notion of instrumental orchestration. An instrumental orchestration is the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment in a given mathematical task situation, in order to guide students' instrumental genesis. An instrumental orchestration consists of different layers: the layer of didactical configuration, the layer of exploitation mode, and the layer of didactical performance (Drijvers et al., 2010). The model has been used to set up a tentative taxonomy of ways in which teachers can use digital tools in their teaching (Drijvers et al., 2013).

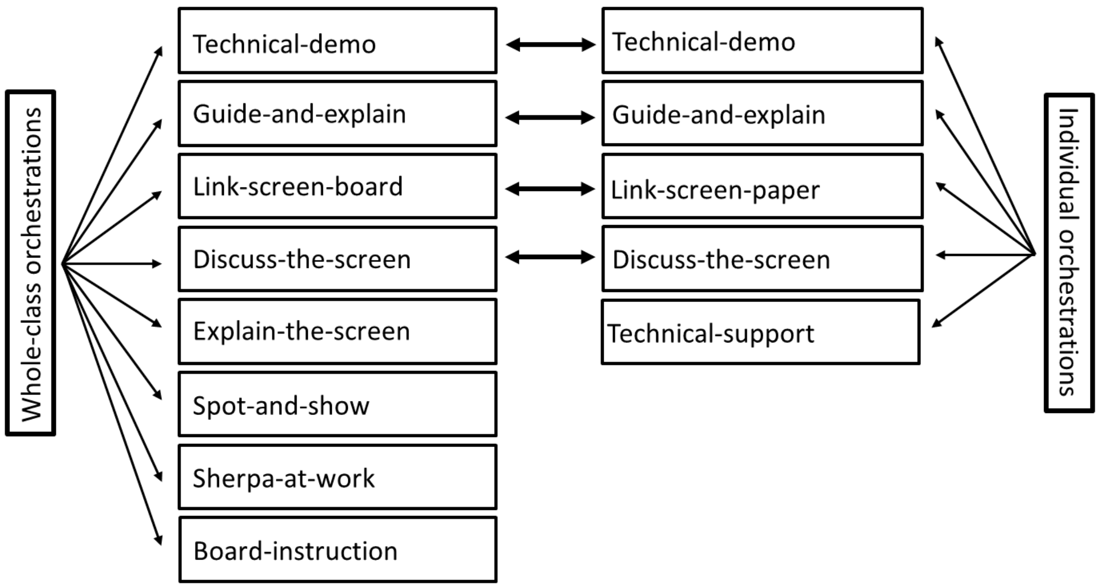

Figure 4. Tentative taxonomy of orchestrations (Drijvers et al., 2013)

The reason why I consider the notion of instrumental orchestration important is that the integration of digital tools in mathematics teaching is far from straightforward to teachers. The orchestration model offers a means to observe and classify teaching arrangements, and to use this lens for teacher professional development. If teachers become aware of their orchestrational decisions, this may help them to make appropriate choices in the future and to develop ways of teaching that match their preferences and that make optimal use of the opportunities digital tools offer.

Digital assessment

The final important perspective we now discuss is digital assessment. If the availability of digital tools affects mathematics learning and teaching, it will certainly also affect assessment. Digital assessment offers opportunities for formative and summative tests that are flexible in time, adaptive, and can be scored automatically. In the nearby future, we expect assessment to be more and more to be delivered through digital means.

However, the point to make here is that we should put high demands on environments for digital assessment, so that we are sure not to lose the advantages of paper-end-pencil assessment, including manual grading by the teacher:

> Yet there is much more to mathematics than producing such simple responses: ideally assessment across the full bandwidth of mathematics should deal with multiple-step calculations, checking each step as a teacher might, analysing arguments and explanations, and certainly, as in the example above, providing full credit for solutions that are mathematically correct but differ in mathematical form from that expected by the setter of the question. (Stacey and Wiliam, 2013, p. 729)

As a consequence, environments for digital assessments should

- provide students with appropriate mathematical tools for entering equations, making tables, drawing graphs, and making geometrical constructions;
- provide students with means to show their problem solving strategy and to explain their reasoning;
- provide intelligent means of scoring, i.e., provide partial credits for partial solutions and full credits for solutions that are correct but unforeseen, or equivalent to the correct solution.

If environments for digital assessment have important weaknesses on the above features, one might wonder if digital assessment is appropriate. It is my conviction that we should insist on environments that offer full mathematical functionality and to support this development.

5. Conclusion

In this paper we set out to investigate whether mathematics education is involved in a fundamental process of change due to the availability of digital tools, and, if the answer is yes, whether we have evidence that this change leads to improvements in mathematics achievement.

Even if we saw that technological tools may support an explorative inquiry of a mathematical situation, we conclude that changes in mathematics education seem to take place only slowly. There is modest support for a positive effect of using ICT on mathematics achievement, but we do not know enough about decisive factors and successful approaches.

Instrumental approaches should inform us in more detail about the subtle relationship between tool use and mathematical thinking. Instrumental orchestrations may help us to identify productive ways to exploit this relationship in teaching. Digital assessment seems a logical next step, but requires a further development of appropriate environments.

As a tentative research agenda, therefore, we should work on generalizable and replicable results in the domain of ICT use in mathematics education; on suitable transformations of research findings into teaching approaches and educational policies; and on a critical scientific attitude within our community.

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