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IMathAS & automated Assessment of mathematical Proof

IMathAS is a web-based math assessment tool for delivery and automatic grading of math homework. In *IMathAS* electronic proofs (e-proofs) are not included by default as deductive arguments for a mathematical statement. The article will show how learners can be supported in building the arguments on justifications and previously established statements by application of *IMathAS*. Furthermore requirements and constraints are discussed for an e-proof to trace back to established statements.

What is *IMathAS*?

IMathAS (http://www.imathas.com/) is a web-based Internet Mathematics Assessment System which can be used within a browser. It has an integrated computer algebra system (CAS), which is not visible for the user. A gradebook is included into *IMathAS* to allow automatic grading of mathematical homework, tests and electronic assessments. The questions are algorithmically generated and numerical and mathematical expression answers can be generated by the computer. Furthermore, free text and essay environments can be included with manual grading by the teacher. *IMathAS* allows accurate display of mathematics and graphs. A randomizer-function allows individual questions for all students. Thereby, the questions are structurally equivalent. Thus, the results can not be cribbed, the students have to solve the task on their own. The biggest benefit for the teachers/authors is a shared joint repository of questions and tasks. The questions can be included in a public library accessible for all teachers/tutors in the community or the questions can be kept in a private library of single teachers, as well. Additionally, it is possible to modify and improve questions from other authors and to integrate them into your private libraries. An *IMathAS* installation on a Server should be planed for multiple educational institutions to support a philosophy of collaboration and sharing among teachers of different institutions.

Requirements and Constraints

Most educational facilities have financial constraints. Therefore it was a requirement for the web-based Learning Environment to be provided as OpenSource. Furthermore the cost of development for an e-proof-system is minimized if just the e-proof system is realized as a kind of plugin in an existing OpenSource solution. The aim to provide a solution within an existing OpenSource Solution has additional advantages, because new releases

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of the underlying mathematical assessment system will be available for the e-proof plugin as well. *IMathAS* was selected, because it provides a shared joint repository to a potentially large target group (universities, schools, etc.) in Rhineland-Palatinate. *IMathAS* provides an IT environment for collaborative support of teachers in a way, that the participating universities and schools can share their development of tasks by a One-Click-Solution with the community of authors. Furthermore, it was a requirement that the underlying OpenSource solution is capable to create individual tasks for all students that can be automatically graded with an integrated help system and a basic tutoring environment to adjust the level of support to the learners' problem solving skills. First of all *IMathAS* has all these features. Furthermore *IMathAS* is available for all regional universities in Rhineland-Palatinate. As a pilot version for electronic proofs the integration into *IMathAS* should be realized as a copy&paste solution for the authoring environment within the web-based mathematical assessment system.

Structure of an e-proof

There is a landscape of different types of proofs and support levels for the proof. We classify the considered prototype for an e-proof environment just in between

- understanding of a given proof of a certain theorem and
- the creation of an own proof for the same given theorem on a blank piece of paper.

An e-proof has e.g. some kinds of options where the students can select fragments of a proof or order them. Therefore, an e-proof is not as flexible as a proof on a blank piece of paper. The solution of a student of an e-proof task is checked against the correct solution which can include different pathways from the preconditions to the conclusions. The considered proofs for the e-proof environment can be decomposed into single fragments. Each fragment consists of three components, namely,

- the connection to the previous fragment,
- the description of the fragment itself and
- one or more justifications of the proof step.

This structure is depicted in the e-proof environment of *IMathAS*. In Fig. 1 the students view of the e-proof environment in *IMathAS* is visualized.

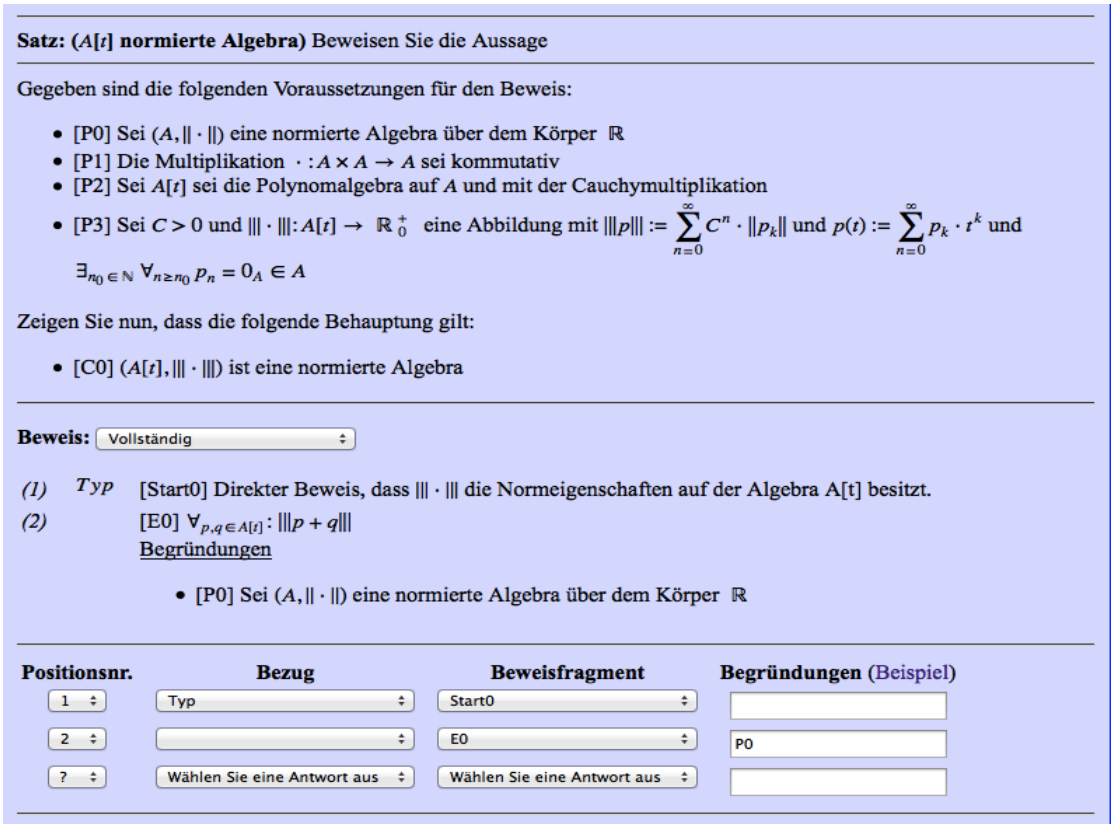

Fig. 1: Student's view of the e-proof environment in IMathAS

Additionally, it is possible to give the students the opportunity to define their own proof steps with the application of an integrated editor in *IMathAS* and to use these proof steps in the e-proof environment. This method is closer to creating a proof on a blank piece of paper. Furthermore, pre-defined and self-defined proof steps can be combined in the e-proof environment. The consequence of allowing self-defined proof steps is that the teacher needs to correct the students' electronic solutions manually.

Authoring Support within *IMathAS*

```
//-[0]Previous_Step---[1]StepID---[2]Connection---[3]necessary_Justification---[4]optional_Justification--
$so=0
$SolutionStep[$so]=array(" ","MY1"," ",array("P0","P3"),array())
$so+=1
$SolutionStep[$so]=array(" ","E0"," ",array(),array())
$so+=1
$SolutionStep[$so]=array("E0","MY2","",array("MY1"),array())
$so+=1
$MinimalProofSteps = $so
```

Fig. 2: Exemplary Source Code for Solution Steps

It should be possible for an author to create e-proof-tasks in *IMathAS* without knowing the correct syntax of the system. Therefore, a solution has to be created by allowing the user to select proof steps and justifications to create a new e-proof-task. The source code for the solution steps (cf Fig.2) will then be generated automatically. The teacher creates all needed proof

steps and performs the correct students' activity for problem solving in the student's view (Fig. 1). Along with this teacher's interaction the solution code will appear in the authoring mode of the e-proof-system. The only step the teacher/author of the proof is to copy the generated code into the e-proof definition in *IMathAS* (i.e. Common Control of a task).

Conclusions and Next Steps

An OpenSource environment, namely *IMathAS*, was selected, because it provides all the required features for a shared task development and the system was installed for all universities and educational facilities in Rhineland-Palatinate. This is a good opportunity to have collaboration on a joint repository, that the provided tasks can be adapted to the individual needs of the universities or educational facilities and learn from good case studies of tasks which are used in lectures and seminars. The randomizers within the system allow to provide individual tasks for all students that can be automatically graded, even for written examinations, to prevent cribbing. There is an integrated help system in *IMathAS* which allows to adjust the level of support for learners according to the problem solving skills of a student. The support and help system is currently in a prototype version and needs additional research input for specific proofs. Furthermore, the prototype of an e-proof-system was provided and integrated for e-proofs in *IMathAS* by copy&paste-integration as a task. Flexibility is included by allowing the students to generate self-defined proof steps in addition to predefined proof steps. The current prototype can be characterized as proof of concept and as a technological solution that provides an option in between the two pillars "understanding a given proof" and "create your own proof on a blank piece of paper". The core question is: Does the e-proof environment support the skills of logical deduction of a proof? This question still remains unanswered. The objective is to support learners in creating their own proof on a blank piece of paper. The supportive mechanism of an e-proof environment has to be evaluated with an empirical study to answer the questions how and why an e-proof environment should be used to support learners in creating their own proof on a blank piece of paper.

Literatur

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