Ildar SAFUANOV, Moskau

Teaching prospective mathematics teachers to solve non-routine problems

George Polya (1981, p. xii) emphasized that prospective mathematics teachers should be taught the skill to solve mathematical problems. "…The solution of a non-routine mathematical problem is genuine creative work" (ibid.).

Furthermore, important is the fact that non-routine mathematical problems (tasks C1-C6) now constitute the essential part of the Unified State Examination (USE) in Mathematics.

The aim of this paper is to give implications for teaching prospective secondary mathematics teachers to solve non-routine mathematical problems including Olympiad problems and problems C1-C6 of the Unified State Examination in Mathematics. In particular, the contents of the new experimental course in solving non-routine mathematical problems for prospective mathematics teachers conducted with a group of ten 4-th year mathematics major students at the Moscow City Pedagogical University will be discussed.

Entertaining and non-standard mathematical problems have been used in teaching gifted pupils in many schools with mathematical bias in Soviet Union and Russia. Therefore, rich experience of teaching to solve mathematical problems was accumulated, and it is possible to apply this experience in teaching prospective teachers, too.

Wide known are the following methods of solving various kinds of problems (see, e.g., Kanel-Belov and Kovalji, 2004):

Invariants (in particular, parity, colorings);

Rule of extreme (in particular: testing the infinite case, small stirs method and infinite descending);

Proof by contradiction;

Mathematical induction;

Dirichlet principle;

Using properties of divisibility and remainders, Euclid's algorithm and congruences;

Graphs;

Rules of combinatorial analysis;

In J. Roth & J. Ames (Hrsg.), *Beiträge zum Mathematikunterricht 2014* (S. 1039–1042). Münster: WTM-Verlag

Symmetry reasons;

Backtracking;

"Divide and conquer" principle (restricting, binary search etc.);

Additional constructions (in geometry)

etc.

Some of these methods are different manifestations of more general methods and principles and also can be combined with each other. For example, Shapovalov (2006) argues that many of these methods and principles (Principle of the extreme, Dirichlet principle, invariants and colorings, infinite descending) are manifestation of the Principle of narrow places.

In our course, we used genetic approach (Safuanov, 2004). In particular, having solved a problem, we gradually establish its connection to serious mathematical theories. For example, considering naturally arising problem of Konigsberg bridges, we arrive to the important mathematical theory of Eulerian graphs.

Furthermore, note that G. Polya (1965, II, p.133) wrote that "the genetic principle may suggest the principle of consecutive phases…" The principle of consecutive phases distinguishes three phases in the solving process as well as in the development of mathematical concepts and theories: exploratory phase, phase of formalization and the phase of assimilation (ibid.)

The principle of concentrated teaching (Safuanov, 1999; Safuanov, 2003) manifests itself in our course in several directions. Knowledge of some mathematical topics was deepened. Some simple problems serve to the anticipation of more complex problems and mathematical theories.

The combination of functions was used: many problems serve not only to the raising the interest to studies (due to their entertaining character) but also promote the acquisition of new theoretical knowledge because they are connected to modern mathematical theories.

Finally, the means "linkage" is systematically used. One interesting problem leads to other, in some way connected with the former; thus the chains of problems are considered. For example, we offer chains of problems on weighing coins, chains of problems on checkered paper etc. Consider the list of topics in our course:

Using parity in problem solving.

Other invariants.

Rule of extreme.

Dirichlet principle.

Symmetry.

Graphs.

Problems on weighing coins.

Problems on crossings and transfusions.

Problems on checkered paper.

Problems of plain geometry (C4 of USE).

Problems of space geometry (C2).

Non-routine arithmetical and logical problems (C6).

For solving and discussing these problems, we used above-mentioned methods (invariants etc.).

However, it seems expedient to tell student teachers about more general approaches. Consider in more detail the analytic-synthetic activities in problem solving (Gusev and Safuanov, 2001).

The analysis and synthesis can be combined with each other.

S. L. Rubinshtein distinguished the important form of the analysis – one which is carried out through synthesis. The essence of such analysis is the following: "the object of thinking is being repeatedly included in new connections and thus it arises in new appearances, with new qualities fixed in new concepts; thus, new contents are repeatedly taken out of the object, it turns repeatedly to new sides; new properties of the object come to light" (Rubinshtein, 1958, p. 98-99).

Thus, the important means of thinking arises: "the analysis through synthesis". Its role in psychology is connected with the detection of new qualities, sides and properties of objects.

Consider an example (from plane geometry) of application of this means.

Problem (C4 of USE, see Yashchenko et al., 2014).

AM is a median of the triangle ABC. AB=10. AC=12. AM=5. What is the area of the triangle ABC?

Solution.

We begin with the analysis. In order to find the area of the triangle ABC, it is necessary to know, e.g., the lengths of all three sides. However, we know the lengths of only two sides and of a median. What can we do in this situation? The idea is to construct a new triangle with the same area and known lengths of all three sides. This is a synthetic reason. So, we construct the continuation of the median AM by a segment MD so that MD=AM=5. It easy to see that triangles ACM and BDM are equal. Therefore, the area of the triangle ABD is equal to the area of the triangle ABC. Furthermore, BD=AC=12. Thus, we know the lengths of all three sides of the triangle ABD. One can easily find (e.g., by Heron's formula) the area of the triangle: 48.

The solution of this problem is a vivid example of the application of the analysis through synthesis. The analysis leads us here to the necessity of the additional construction.

The problem C6 usually deals with arithmetic of integer numbers. Therefore, it is connected with number theory course the student study in the university. Here also the analysis through synthesis can be applied as one see, e.g., in the solution of Problem 4 of the article by Gusev and Safuanov (2001).

First outcomes of the implementation of our course demonstrated the positive changes in prospective mathematics teachers' skills in solving non-routine problems as well as in their beliefs about the problem solving.

Literatur

Gusev, V.A., & Safuanov, I.S. (2001). Analytic-synthetic activities in the learning of mathematics. In M. van den Heuvel-Panhuizen (Ed.), Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education (Vol. 3. pp. 73-80). Utrecht, Netherlands: PME.

Kanel-Belov, A., and Kovalji, A. (2008). How to solve non-routine problems (in Russian). M.: MCNMO.

Polya, G. (1981). Mathematical Discovery. New York: Wiley.

Rubinshtein, S. L. (1958). On the thinking and methods of its study (in Russian). M. - L.

Safuanov, I. (1999). On some under-estimated principles of teaching undergraduate mathematics. – In O. Zaslavsky (Ed.), Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education (Vol. 3, pp. 153-160). Haifa, Israel: Technion.

Safuanov, I. (2003). Applications of the principle of the concentrated teaching in the design of the mathematical course at the university. – Beitraege zum Mathematikunterricht 2003. Hildesheim: Franzbecker, S.557-560.

Safuanov I S. (2004). Psychological Aspects of Genetic Approach to Teaching Mathematics. In M. J. Hψines & A. B. Fuglestad (Eds.), Proc. 28th Conf. of the Int. Group for the Psychology of Mathematics Education (Vol. 4, pp. 153-160). Bergen, Norway: PME.

Shapovalov (2006). The principle of narrow places. M.: MCNMO.

Yashchenko, I., Shestakov, S., Trepalin, A. and Zakharov, P. (2014). Preparation to USE in Mathematics (in Russian). M.: MCNMO.