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Investigations in the mathematical classroom (open-ended approach)

1. Introduction

Educational system in Russia is at the stage of extensive reforming today. New goals for teachers are set by the recently introduced State educational standards of the second generation. Preparation of learners for fast changing environment, development of their ability to make independent decisions, extension of skills of critical thinking and gaining knowledge are becoming the priority aims of education.

Exploratory learning has been often used in the teaching practice in our country. In reality, there are some difficulties with the introduction of exploratory learning into the secondary school.

While analyzing international experience and tendencies of developmental education in our country, we have considered different approaches to organizing research activity of learners. As a result of the preliminary research, special interest was concentrated on the open approach developed in Japan in the 70th of the twentieth century. This method was essentially based on the use of problems with multiple solutions. After studying the ideology of this approach, as well as its didactic opportunities and similar Russian practices, we developed an effective methodical system of using open-ended problem approach for lessons of mathematics in the secondary school.

2. What makes open-ended problems so interesting?

What problems are open-ended? Open-ended problems are opposite to closed. We call a task closed if it is clearly defined and has only one correct solution. Open tasks give a pupil various possibilities in the statement of the question and in the solving of the problem. Naturally, the open-ended approach is a kind of non-routine pedagogical technology; its introduction into the educational process is connected with a variety of serious problems. First, it is very difficult for some teachers to realize that the task can have more than one right answer. It is quite possible that even the teacher cannot see other correct solutions. Second, the assessment system for solutions of open-ended tasks is not yet clear.

Taking into account these problems, we can state the main purpose of the developmental educational model based on open-ended tasks as follows: to create a transparent and understandable system of realizing open-ended ap-

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proach in mathematics classes. It would give an opportunity for teachers to use this technology in their own practice. Finally, we created a toolkit consisting of two blocks: a methodical manual for teachers and a booklet for pupils.

The methodical manual contains practical guidelines for using open-ended tasks in classes, a set of tasks for pupils of different age, examples of assessment systems and instructions for composing open-ended tasks.

However, the training of teachers on the basics of the open-ended approach is insufficient for achieving high educational results. Very important is also to develop the pupils' metacognitive awareness. That is why we elaborated the information booklet named "Introduction to the science".

Consider in detail some important features in our methodical recommendations that may help to make the open-ended approach more effective.

3. Classification of open-ended tasks

Open-ended tasks can be classified in different ways. The first way of classification is by the kind of "openness". Here, open-ended tasks can be divided into three groups: tasks with multiple right answers; tasks with multiple solution methods; assignments called "task-to-task" (while solving the original task, new questions and tasks may arise).

According to data source, semantic context of a task, or to the time necessary for the solving, it is possible to distinguish the following types: investigations (where a starting point is given or problem finding or problem formulating); real-life situations (which have their roots in the everyday life); problems without a question; problem variations ("what-if"-method); research projects (larger study entities, requiring independent working).

It is important to note that the same task can be attributed to the different groups depending on lesson's goals, level of knowledge of pupils and on wishes of a teacher.

4. Recommendations for composing open-ended tasks

As the source for open-ended tasks, one can use historical facts, popular scientific books, periodical documentaries etc. In order to learn how to compose an open-ended task, it is very convenient to do it in comparison with a corresponding "closed" task. In this case one needs just to reformulate a closed task for making it open-ended.

Table 1 describes the most important characteristics of open-ended tasks.

Table 1 – Characteristics of “open-ended” and “closed” tasks

Characteristics	«Closed» tasks	Open-ended tasks
Condition	Data is sufficient for solving a problem; there is no unnecessary data	Initial data for the task’s solutions may be insufficient or, conversely, excessive
Statement of question	Find out ... (value or an algorithm for constructing) prove... (the statement already known as right)	Is this statement true? What we can/cannot find out from the task’s condition? Whether it is possible to weaken the condition? Whether it is impossible to strengthen it?
The progress of solution	Theoretical knowledge of the pupil is sufficient for solving the task; there is only one or a limited number of strategies to address the task.	Knowledge for solving the problem is insufficient. Pupils need to solve subproblems accumulating the necessary experience; methods of solving problems can vary a lot
Result	There is only one solution of the problem	There are several «correct» solutions of the problem, depending on what direction of the research was selected.

The open-ended tasks differ from classical ones by higher degree of freedom. The most important feature in the ideology of open-ended approach is that pupils determine a goal of the research by themselves. During the solving of a problem, they can be confronted with another one.

4. The model of the system of assessment

Solving of open-ended task is a kind of research activity. The main goal of using such kind of tasks in classes is to improve pupils’ research skills. That is why the main criterion of efficiency of using open-ended approach is the development of pupils’ research competences.

Furthermore, the depth of the exploring the question and the originality of the ideas presented can be additionally appreciated.

4. Conducting lessons on the basis of open-ended problem approach

Note some of the most important stages in the lesson preparation.

Theoretical training of pupils

We elaborated a booklet for students named “Introduction to the science”. Basic recommendations for doing research are given: the algorithm of re-

search, overview of the main scientific methods, recommendations for the design and even advice on issues of public defense.

The optimal quantity of open-ended tasks

From the experience of schools using exploratory learning approach in their practice, solving open-ended tasks cannot be used more often than once a week, but at the same time such an activity will not be of any use if research assignments are not used at least once a month. Thus, conducting research lessons once a month is probably the optimal regularity.

Choosing the group of learners

It is necessary to decide if the exploration must be done by all the pupils equally in a class or the class will be divided into groups. It is still recommended to allow all the pupils to make research within one task.

Choosing the form of the presentation of results

The form of the presentation of results of solving open-ended tasks depends particularly on the task itself and on the amount of attention one wants to attract to it. There are four basic forms of results presentation: final report, public discussion, board sheets (posters) and science exhibition. Interchanging the forms of results presentation is recommended. But ultimately, the contents and difficulty of the task are a priority.

Results of the implementation

During the implementation of our approach, the efficiency of open-ended method was proved for pupils of the secondary school with classes of different profiles. As a result of using new approach, interviews of teachers and pupils revealed the following changes: increase of motivation to learn mathematics and of enthusiasm of pupils at a lesson; acquiring the experience of using mathematical knowledge in real life; stable positive dynamics of improvement of research competences; improvement of the psychological atmosphere in a classroom; reconsideration of the role of a teacher as a participant of the educational process in a classroom; acquisition of skills of work in a group.

The obtained results of our theoretical research and experimental work indicate that the elaborated model of education justifies stated purposes.

Literatur

Nohda, N. 1991. Paradigm of the "open-approach" method in mathematics teaching: Focus on mathematical problem solving. *International Reviews on Mathematical Education* 23 (2), 32–37ZDM, 23 (2), 32-37.