A Macroeconomic Perspective on Asset Returns and Liquidity Premia

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Contents

List of Figures

List of Tables

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Chapter 1

Introduction

The essays to be comprised in this thesis are approaches to the problem of modelling the notion of aggregate liquidity as a potential driver of asset returns and of macroeconomic dynamics. The main thrust of this work is empirical, using methods ranging from econometric studies of the interconnection between asset returns and their degree of liquidity to estimating dynamic stochastic general equilibrium models with Bayesian methods to establish evidence for the effectiveness of quantitative easing policies.

In general, liquidity refers to the ease of trading an asset and to an asset's ability to be sold without having to accept a considerably large drop in the price or value. Therefore, bid-ask spreads are a common measure for an asset's degree of market liquidity. Authors like Canzoneri et al. (2013) point out that U.S. Treasuries might carry a liquidity premium which is induced by nonpecuniary returns to investors. Specifically, U.S. Treasuries are used to facilitate transactions in a number of ways: they serve as collateral in financial markets, banks hold them to manage the liquidity of their portfolios, individuals hold them in money market accounts that offer checking services, and importers and exporters hold them as transaction balances. Krishnamurthy and Vissing-Jorgensen (2012) formalize the notion that the investors obtain such liquidity services by assuming that holding U.S. Treasury securities directly contributes to their utility.

The essays presented in Chapter 2 and Chapter 3 of this thesis investigate whether there is empirical evidence for household preferences where liquidity services are gained not only from U.S. Treasuries but from a variety of liquid assets. Chapter 2 investigates whether an asset pricing model which is based on such investors' preferences can contribute to explain observed corporate-U.S. Treasury yield spreads. Chapter 3 seeks to provide a complete specification and parameterization of a utility function with liquidity services. This is done by providing a set of microfoundations, ranging from nonparametric hypothesis tests of revealed preference conditions for utility maximization, to parameter estimates for suitable specifications of utility functions. Chapter 4 (coauthored with Andreas Schabert and Roland Winkler) employs a monetary dynamic stochastic general equilibrium (DSGE) model to identify the effects of the U.S. Federal Reserve's (Fed) large-scale longer-term Treasury purchase program (LSAP 2) on the U.S. economy. In this model a bank sector relies on liquidity services which are gained from holdings of government bonds when providing financial intermediation between households and firms. Chapter 5 investigates whether liquidity premia can explain deviations from uncovered interest parity (UIP). For that purpose forward premium regression models are modified by assuming that investors value U.S. Treasuries' liquidity services which are induced by the U.S. dollar's role as a key currency. Chapter 6 concludes the thesis.

Empirical studies on determinants of corporate-U.S. Treasury bond yield spreads commonly rely on measures for default risk and the securities' degree of market liquidity (Collin-Dufresne, Goldstein, and Martin (2001)). A large number of contributions to the corporate finance literature find that these determinants have a rather limited explanatory power (e.g. Elton et al. (2001), Delianedis and Geske (2001), Huang and Huang (2003), Eom, Helwege, and Huang (2004)).

Chapter 2 investigates whether the unexplained share within corporate-U.S. Treasury bond yield spreads is related to an investors' valuation for liquidity services. Accordingly, this valuation should be priced separately from common measures for assets' market liquidity. The analysis builds on Krishnamurthy and Vissing-Jorgensen (2012) who show that changes in the supply of U.S. Treasury securities have a strong effect on corporate-Treasury bond yield spreads. In this model investors value certain features of U.S. Treasury securities, namely their liquidity and "absolute security of nominal return" as they directly contribute to investors's utility. This affects prices of Treasuries and hence, drives down their yields compared to assets that do not to the same extent share these features.

Chapter 2 presents an asset pricing model which is modified according to Poterba and Rotemberg (1986). In particular, investors' utility is a function of consumption and liquidity services which depend on the level of holdings of a set of liquid assets. The model is estimated using regression analysis while controlling for commonly employed measures for default risk and assets' degree of market liquidity (as summarized by Collin-Dufresne, Goldstein, and Martin (2001), Pflueger and Viceira (2012) and Longstaff (2004)).

Estimation results provide evidence in favor of the modified asset pricing model. Specifically, the unexplained share within corporate-U.S. Treasury bond yield spreads is to a significant extent driven by investors's valuation for liquid assets. While explanatory power remains low in previous studies, the model presented in Chapter 2 provides an improved empirical fit. Further, results point towards the existence of a demand function for liquidity services.

The model presented in Chapter 2 does not provide a complete specification of the investors' utility. In particular, in an ad hoc manner it is assumed that liquidity services are valued by some aggregator function which is a separate argument of the preference function. Chapter 3 seeks to provide a complete specification and parameterization of a representative agent's utility function which can rationalize the investors' behavior observed by Krishnamurthy and Vissing-Jorgensen (2012) and the study presented in Chapter 2.

The study presented in Chapter 3 checks Varian's (1982) necessary and sufficient revealed preference conditions for observed data on investors' liquid asset holdings and on the assets' prices. Consistency of the data with these conditions implies non-rejection of the hypothesis that investors are maximizing a utility function which is nonsatiated, continuos, concave and monotonic. The procedure proposed by Fleissig and Whitney (2003) is applied to test whether the data satisfy necessary and sufficient revealed preference conditions for weak separability between consumption and liquidity services. With the revealed preference tests done the question for a suitable specification of the representative investor's objective function arises. As the nonparametric testing routines applied in this essay do not provide guidance for that, Chapter 3 follows authors like Stock and Wright (2003), Hall (2005), and Holman (1998) by employing Generalized Method of Moments (GMM) to estimate coefficients of Euler equations which are derived from the investors' optimization problem under several proposed utility specifications.

Chapter 3 provides evidence from the nonparametric testing routines that necessary and sufficient conditions for utility maximization and weak separability are obtained for the dataset. However, results from GMM estimations imply rejection of almost all proposed utility specifications. Moreover, estimation results imply parameter values for the preferences which are not rejected that indicate misspecification.

The purpose of the essay presented in Chapter 4 is to identify the macroeconomic effects of LSAP 2. This nonstandard monetary policy measures was introduced by the U.S. Fed in response to the persistent negative effects of the 2008/2009 financial crisis. As outlined by Bernanke (2012), obtaining precise estimates of the effects of these operations on the broader economy is inherently difficult, as the counterfactual - how the economy would have performed in the absence of the Federal Reserve's actions - cannot be directly observed.

Recent studies on unconventional central bank balance sheet policies rely on non-monetary macroeconomic models (see Chen, Curdia, and Ferrero (2012), Del Negro et al. (2013), Gertler and Karadi (2013), and Gertler and Kiyotaki (2010)). To develop a framework for the macroeconomic analysis of the LSAP 2 program Chapter 4 extends a monetary DSGE model by Christoffel and Schabert (2013) which provides an explicit specification of the central bank's balance sheet options and its impact on financial intermediation. The model accounts for the specific role of government bonds to provide liquidity services to commercial banks. This is considered by modeling central bank monetary supply by an asset exchange in open market operations. Central bank money and reserves resp. are demanded by banks for liquidity management purposes when they provide intermediation between households and firms. In particular, costs of financial intermediation are specified in a stylized way following Curdia and Woodford (2011). The model is estimated for U.S. data using Bayesian techniques.

The analysis presented in Chapter 4 employs the model address the following questions: *First*, to what extent did shocks to financial intermediation and the Fed's unconventional balance sheet policy measures contribute to the volatility in real activity following the 2008/2009 financial crisis? *Second*, what are the macroeconomic effects of the LSAP 2 bond purchase program? To address the second issue, Chapter 4 follows Del Negro et al. (2013) by using the model to conduct a counterfactual policy simulation experiment.

Chapter 4 finds that shocks to financial intermediation significantly contribute to the evolution of U.S. key macroeconomic variables following the 2008/2009 crisis. In particular, the estimated model implies that in 2010:Q4, which is the quarter when LSAP 2 was initiated, roughly one third of the negative U.S. real per capita GDP trend deviation was attributed to the shock to financial intermediation. Further, the central bank can significantly alleviate adverse effects to the economy by easing the supply of reserves in exchange for log-term government bonds. The counterfactual policy simulation suggests that in the absence of LSAP 2 U.S. real per capita GDP would have dropped by additional 2.75 percentage points.

Chapter 5 addresses the empirical failure of UIP which has been documented by various evidence from forward premium regressions (see the survey article by Engel (2013)). The widely quoted result by Froot (1990) implies that the forward premium predicts future changes in the spot exchange rate which are inconsistent with UIP, in terms magnitude and in terms of the direction of the movement. This result is known as the forward premium puzzle.

Chapter 5 investigates whether liquidity premia can explain deviations from UIP, and can contribute to explain international interest rate differentials, namely the U.S.-U.K. Treasury yield spread. The analysis follows Canzoneri et al. (2013) by relaxing the assumption that risk-free U.S. domestic bonds and foreign bonds are perfect substitutes. Specifically, due to the role of the U.S. dollar as a key currency it is assumed that U.S. Treasuries provide unique liquidity services. Therefore, U.S. Treasuries will be held at a discount. To formalize this idea, following Krishnamurthy and Vissing-Jorgensen (2012) a representative agent asset-pricing model is modified by allowing investors to derive utility directly from holdings of U.S. Treasuries. From that model no-arbitrage conditions are derived for the international bond market which take into account for foreign exchange risk and price risk. Chapter 5 employs regression analysis to empirically test whether the model-implied no-arbitrage conditions can explain deviations from UIP for U.S. data and U.K. data. By following Fuhrer (2000) it is assumed that the households' forecasts regarding the dynamics of consumption, inflation, and the depreciation rate of the domestic currency can be described by an unconstrained vector autoregression.

Estimation results imply that investors' valuation for U.S. Treasuries' liquidity services contributes to explain deviations from UIP. Further, estimation results imply a positive association between the expected depreciation rate of the U.S. currency relative to the U.K. currency and the U.S.-U.K. Treasury yield spread or forward premium. However, the point estimate of the coefficient still is below unity.

Chapter 2

Investors' Valuation for Asset Liquidity and the Corporate-U.S. Treasury Yield Spread

2.1 Introduction

The study of determinants of corporate-U.S. Treasury bond yield spreads has been the subject of a large number of contributions to the corporate finance literature. Some recent papers by Elton et al. (2001), Delianedis and Geske (2001), Huang and Huang (2003), and Eom, Helwege, and Huang (2004) find that variables, i.e. default risk and measures for assets' market liquidity, that should in theory determine spreads between corporate bond yields and U.S. Treasury bond yields, have rather limited explanatory power. Longstaff, Mithal, and Neis (2005) use information from credit default swaps to estimate the share of corporate-Treasury yield spreads being explained by default risk, which they label as the default component. The residual is then labeled as nondefault component. The latter is found to be time-varying and strongly related to macroeconomic measures of bond market liquidity.[1] Krishnamurthy and Vissing-Jorgensen (2012) (KVJ) provide evidence that the nondefault component within the corporate-Treasury bond yield spread is to a significant extent driven by the total amount of U.S. Treasuries outstanding. They argue that investors value certain features of

[1] For example flows into money market mutual funds.

U.S. Treasury securities, namely their liquidity and "absolute security of nominal return" as they directly contribute to investors' utility. This affects prices of Treasuries and hence, drives down their yields compared to assets that do not to the same extent share these features.

In the present chapter[2] I investigate whether investors value liquidity only as a U.S. Treasury-specific feature, or whether investors value liquidity independently from the underlying asset. In particular, I ask whether there is evidence for a systematic pattern in investors' valuation for liquid assets which reflects the distinct degree to which that feature is prevalent. Empirical evidence for such a pattern being priced within the corporate-Treasury yield spread would point to the existence of a demand function for liquidity. Further, I check for robustness of the results presented by KVJ. For this purpose I modify a standard asset pricing model to allow for holdings of a certain group of liquid assets to directly contribute to investors' utility. Theoretical implications of that asset pricing model are then tested by employing regression analysis. Specifically, I compare the effects of changes in the aggregate holdings of assets which differ in their respective degree of perceived liquidity on alternative yield spread measures, while controlling for commonly employed measures for default risk and assets' market liquidity. Note that the study presented in this chapter is an extension of the approach by KVJ.

U.S. Treasuries are of high liquidity and are considered to be default-free. From a theoretical point of view this should be reflected in the interest differential between Treasuries and any other debt security with the same maturity length. As pointed out by Collin-Dufresne, Goldstein, and Martin (2001) the standard empirical approach to explain corporate-Treasury bond yield spreads is to find suitable controls proxying for spread determinants which are implied by Asset Pricing Theory's Consumption Capital Asset Pricing Model (CCAPM). Following Chen, Lesmond, and Wei (2007), those determinants are generally denoted as credit risk factors. Specifically, they are the expected loss in case of default on a corporate bond and the degree to which default states covary with the business cycle, commonly named as "expected default loss" and investors' demanded "risk premium" (see Elton et al. (2001)). Furthermore, authors like Amihud, Mendelson, and Pedersen (2005) and Acharya and Pedersen (2005) use controls which proxy for securities' level of market liquidity.[3] They argue that time-varying

[2] The chapter is based on Niestroj (2012).

[3] Following Amihud, Mendelson, and Pedersen (2005) market liquidity refers not only

differences in an asset's degree of market liquidity contribute to make returns, i.e. future expected payment streams, risky and therefore, induce an additional "liquidity-risk premium". For example, in times when investors would like to sell and the market liquidity of a corporate bond deteriorates, risk averse investors will demand an additional premium for holding these bonds, i.e. a market-liquidity induced risk premium. Chen, Lesmond, and Wei (2007) show that measures which control for CCAPM-implied spread determinants as well as for securities' degree of market liquidity can improve the ability of credit spread regressions to explain observed levels and variability of yield spreads. However, explanatory power still remains relatively low.[4]

KVJ find for U.S. data a strong negative correlation between corporate-Treasury bond yield spreads and the government Debt-to-GDP ratio (i.e. the ratio of the market value of publicly held U.S. government debt to U.S. GDP) over the period from 1926 to 2008. They argue that this reflects an investors' valuation for certain features of U.S. Treasury securities, i.e. a high degree of liquidity and a high degree of perceived safety. This valuation by the investors is further found to be priced separately from the commonly analyzed spread determinants, such as credit risk and assets' market liquidity. As theoretical rationale for the observed behavior KVJ assume that the holder of a U.S. Treasury security obtains some services and gains to the subjective level of well-being.[5] Those benefits are summarized as "convenience yield" which directly contribute to investors' utility and lead Treasuries to have significantly lower yields than they otherwise would have in a standard asset-pricing framework. The strong negative correlation they find, therefore reflects a Treasury demand curve, or more specifically an investors' demand for a certain feature of Treasuries. If the supply of Treasuries is low, the value that investors assign to the services offered by Treasuries is high. As a result the yields on Treasuries are low relative to the yields on corporate bonds. The opposite applies when the

to the ease of trading an asset but also to an asset's ability to be sold without having to accept a considerably large drop in the price or value. Therefore, in empirical studies bid-ask spreads are the commonly employed measures for an asset's degree of market liquidity.

[4]For an overview of regressions including standard controls see Collin-Dufresne, Goldstein, and Martin (2001).

[5]The assumption of an asset's feature providing specific services which are valued by investors is reminiscent of the money-in-the-utility-function model. For a complete elaboration of the rationale for investors' valuation for liquidity see KVJ.

supply of Treasuries is high.

In this chapter I test the hypothesis whether only U.S. Treasury-specific liquidity services are valued by investors, or whether investors have a general valuation for liquidity services which is independent from the underlying asset. The latter would point to the existence of a demand function for liquidity. For this purpose, I extend the asset pricing model developed by KVJ. Specifically, I employ a representative investor's utility function where not only Treasury holdings contribute directly to utility but also money balances as well as corporate debt security holdings. In particular, by following Poterba and Rotemberg (1986), utility is assumed to be a function of consumption and liquidity services which depend on the level of holdings of the assets under consideration. I analyze the modified asset pricing model's implications with regard to the effects of changes in the holdings of liquidity services providing assets on corporate-Treasury yield spreads. These implications are empirically tested by employing regression analysis. In addition to evaluating the effects of factors which should implied by the model drive corporate-Treasury yield spreads, the study presented in this chapter is also intended to conduct an exploratory analysis. This is done by regressing bond spreads on measures that capture the investors' perceived market-liquidity risk of corporate debt securities relative to U.S. Treasuries, and so called "flight-to-liquidity" episodes, where I follow Pflueger and Viceira (2012) and Longstaff (2004).

I find a significant negative association between changes in measures for aggregate money balances and near money holdings, measures for U.S. Treasury holdings, as well as measures for the holdings of corporate debt securities and corporate-U.S. Treasury yield spreads. Results indicate that yield spreads react the stronger, the higher the respective measure's degree of liquidity. Further, I find that this observation is robust across different model specifications including common measures for credit risk and assets' market-liquidity risk. Results imply that there is a systematic pattern in investors' valuation for liquidity services which points to the existence of a demand curve for liquidity.

The remainder of this chapter is organized as follows. Section 2.2 describes the model set-up and derives yield spread regression models. Section 2.3 provides results for empirical tests of model-implied hypotheses. Section 2.4 offers concluding remarks.

2.2 Theoretical framework

Testable corporate-Treasury yield spread regression models are derived from a theoretical framework which extends the standard asset pricing model by the concept of convenience yields. This approach is proposed by KVJ which is based on a the notion of a money-in-the-utility preference specification. Specifically, it is assumed that convenience yields as a function of Treasuries and some broad measures for the economy's wealth enter the utility function as a separate argument. I extend the asset-pricing model derived by KVJ by allowing for holdings of money, Treasuries and corporate debt securities to contribute to household's utility.[6] In a next step a theoretical asset-pricing model is derived from the household's optimization problem.

2.2.1 Utility function

Under the assumption that investors value liquidity services a representative agent's utility function, fulfilling the Inada conditions, is of the form:

$$u_t = u\left(c_t, \nu\left(\Theta_t, X_t, \xi_t\right)\right),$$
$$\text{with } \Theta_t = \Theta\left(m_{t,} b_t, s_t\right).$$

The argument c_t is the agent's consumption at date t and $\nu(\cdot)$ denotes the agent's gained convenience yield which is a function of a set of macroeconomic factors, denoted as X_t, and $\Theta(\cdot)$, a jet unspecified aggregator function of the real holdings of money m_t, Treasuries b_t, and corporate debt securities s_t. The term ξ_t is a preference shock which is intended to capture level-effects on the utility derived from asset holdings during times when exogenous shocks like a financial crisis, temporarily changes investors' valuation for liquidity services.[7]

Following KVJ the convenience yield function $\nu(\cdot)$ is assumed to capture unique services provided by liquid assets and a set macroeconomic factors

[6] This idea is based on Poterba and Rotemberg (1986) who use a utility function where so called liquidity services directly contribute to household's utility. The function's argument "liquidity services" is assumed to be a CES aggregate of demand deposits plus currency, short term savings deposits, and Treasury bill holdings.

[7] Longstaff (2004) finds evidence for what he calls „flight to liquidity/quality" premium episodes by examining the spread between government agency bonds and U.S. Treasury bonds. In a flight to liquidity episode market participants suddenly prefer highly liquid securities, such as Treasuries, rather than less liquid securities.

which are valued by investors, where $\nu(\cdot)$ is concave with $\nu'(\cdot) > 0$, and $\nu''(\cdot) < 0$. For the purposes of the study presented in this chapter I follow KVJ by assuming that X_t is mainly driven by the U.S. Gross Domestic Product (GDP). Further, $\nu(\cdot)$ shall be homogeneous of degree one in GDP_t and Θ_t. Hence, $\nu(\cdot)$ can be transformed in the following manner:

$$\nu(\Theta_t, GDP_t, \xi_t) \equiv \nu\left(\frac{\Theta_t}{GDP_t}, \xi_t\right) GDP_t.$$

For simplicity, I further assume that for the unknown liquidity services aggregator function $\Theta(\cdot)$:

$$\frac{\Theta_t}{GDP_t} = \Theta\left(\frac{m_t}{GDP_t}, \frac{b_t}{GDP_t}, \frac{s_t}{GDP_t}\right).$$

The liquidity services function is concave as well, as I assume that $\Theta(\cdot)$ is increasing in $\frac{\theta_t}{GDP_t}$, with $\theta_t = \{m_t, b_t, s_t\}$, but the marginal benefit from holding another unit of liquid assets is decreasing in $\frac{\theta_t}{GDP_t}$. This captures the idea that holding more liquidity services providing assets reduces the marginal value of an extra unit of such assets. Further, $\nu(\cdot)$ shall have the property of $\lim_{\frac{\theta_t}{GDP_t} \to \infty} \nu'\left(\frac{\Theta_t}{GDP_t}, \xi_t\right) = 0$. Hence, the marginal value of a unit of $\frac{\theta_t}{GDP_t}$ approaches zero if the agent is holding a large amount of liquidity services providing assets. Moreover, under the hypothesis that investors value liquidity, holding one more unit of an asset that is more liquid compared to another asset should c.p. generate more utility than holding one more unit of the latter, therefore

$$\frac{\partial\Theta(\cdot)}{\partial m_t} > \frac{\partial\Theta(\cdot)}{\partial b_t} > \frac{\partial\Theta(\cdot)}{\partial s_t}.$$

KVJ point out that Treasuries with a high maturity length carry a higher interest rate risk and default risk compared to Treasuries with a short maturity length. Further, one could argue that Treasury bonds are less liquid than Treasury bills which is the reason for the interest rate on the latter to carry a liquidity premium. Therefore, the investors' perceived benefit in terms of utility from holding "short-term" Treasuries might differ from the benefit of holding "long-term" Treasury bonds. Hence, the marginal liquidity services of holding an additional unit of Treasury bonds will differ from the additional liquidity services of holding an additional unit of Treasury bills. This should be reflected in the true but unspecified parameterizations of $\nu(\cdot)$, and $\Theta(\cdot)$. However, for the study presented in this chapter it is sufficient to use this general specification to motivate the empirical analysis.

2.2.2 Household's problem

A representative household is assumed to maximize the expected sum of a discounted stream of utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, \nu\left(\Theta\left(m_t, b_t, s_t\right), GDP_t, \xi_t\right)\right), \tag{2.1}$$

subject to the budget constraint

$$\begin{aligned} & P_t c_t + P_t^M m_t + P_t^T b_t + P_t^S s_t \\ & \leq P_t y_t + P_t^M m_{t-1} + P_t^T b_{t-1} + P_t^S s_{t-1}\left(1-\delta_t\right), \end{aligned} \tag{2.2}$$

where E_0 is the expectation operator conditional on the information set in the initial period and $\beta \in (0,1)$, is the subjective discount factor. The household earns a real endowment income y_t and can carry wealth into the next period by investing into nominal holdings of money $P_t^M m_t$, Treasuries $P_t^T b_t$, and corporate debt securities $P_t^S s_t$. Assume for simplicity that the agent buys zero coupon discount bonds which pay out one unit of currency when being held to maturity.[8] The aggregate price level at date t is denoted by P_t. The nominal prices for one-period investments into money balances, Treasuries, and corporate debt securities are P_t^M, P_t^T, and P_t^S. Note that for the price of one unit of m_t it should hold that $P_t^M = 1$, which is one nominal unit of currency. An investment increases real holdings of convenience assets Θ_t by $\Theta'(\cdot)\frac{P_t^\theta}{P_t}$, where $P_t^\theta = \{P_t^M, P_t^T, P_t^S\}$. For a corporate debt security with face value of one the expected repayment is $(1-\delta_t)$ where δ_t is the expected default rate, which is $\delta_t = 0$, in the absence of default and $\delta_t > 0$, if there is default on the bond.

Maximizing the objective function (2.1) subject to the budget constraint (2.2) leads for given initial values and non-negativity constraints for m_t,

[8] Derivation of pricing expressions takes place for zero-coupon Treasury bonds and corporate bonds. In the empirical part coupon bonds are examined. However, KVJ argue that the impact of Treasury supply on coupon bond spreads is qualitatively similar to the effect on zero-coupon bond spreads.

b_t, and s_t, to the following first order conditions for consumption c_t, and investments into money balances m_t, Treasuries b_t, and corporate bonds s_t:

$$u'(c_t, \nu(\cdot)) = \lambda_t, \tag{2.3}$$

$$\lambda_t \frac{1}{P_t} = u'(c_t, \nu(\cdot))\, \nu'\left(\frac{\Theta_t}{GDP_t}, \xi_t\right) \frac{\partial \Theta(\cdot)}{\partial m_t} \frac{1}{P_t} + \beta E_t\left[\lambda_{t+1} \frac{1}{P_{t+1}}\right], \tag{2.4}$$

$$\lambda_t \frac{P_t^T}{P_t} = u'(c_t, \nu(\cdot))\, \nu'\left(\frac{\Theta_t}{GDP_t}, \xi_t\right) \frac{\partial \Theta(\cdot)}{\partial b_t} \frac{P_t^T}{P_t} + \beta E_t\left[\lambda_{t+1} \frac{P_{t+1}^T}{P_{t+1}}\right], \tag{2.5}$$

$$\begin{aligned} \lambda_t \frac{P_t^S}{P_t} =& \, u'(c_t, \nu(\cdot))\, \nu'\left(\frac{\Theta_t}{GDP_t}, \xi_t\right) \frac{\partial \Theta(\cdot)}{\partial s_t} \frac{P_t^S}{P_t} \\ &+ \beta E_t\left[\lambda_{t+1} \frac{P_{t+1}^S}{P_{t+1}} (1 - \delta_{t+1})\right], \end{aligned} \tag{2.6}$$

and (2.2) holding with equality, and the transversality conditions $\lim_{j\to\infty} \beta^j E_t\left(\lambda_{t+j} P_{t+j}^M m_{t+j}\right) = 0$, $\lim_{j\to\infty} \beta^j E_t\left(\lambda_{t+j} P_{t+j}^T b_{t+j}\right) = 0$, and $\lim_{j\to\infty} \beta^j E_t\left(\lambda_{t+j} P_{t+j}^S s_{t+j}\right) = 0$. Define the stochastic discount factor for nominal payoffs as $M_{t+1} = \beta \frac{u'(c_{t+1}, \nu(\cdot))}{u'(c_t, \nu(\cdot))} \frac{P_t}{P_{t+1}}$, so that, similar to KVJ, equations (2.4) - (2.6) can be expressed as

$$P_t^M = 1 = \frac{E_t[M_{t+1}]}{1 - \nu'(\Theta_t/GDP_t, \xi_t) \frac{\partial \Theta(\cdot)}{\partial m_t}}, \tag{2.7}$$

$$P_t^T = \frac{E_t\left[M_{t+1} P_{t+1}^T\right]}{1 - \nu'(\Theta_t/GDP_t, \xi_t) \frac{\partial \Theta(\cdot)}{\partial b_t}}, \tag{2.8}$$

$$P_t^S = \frac{E_t\left[M_{t+1} P_{t+1}^S (1 - \delta_{t+1})\right]}{1 - \nu'(\Theta_t/GDP_t, \xi_t) \frac{\partial \Theta(\cdot)}{\partial s_t}}. \tag{2.9}$$

Equations (2.7) - (2.9) require that under the assumption of liquidity services being an argument of the investor's utility function, increasing the amount of liquidity services providing assets held, will decrease their prices P_t^M, P_t^T and P_t^S. Specifically, increasing the holdings of liquidity services providing assets will lower the investor's willingness to pay for another unit of such assets. This is basically due to the assumption of $\nu(\cdot)$ and $\Theta(\cdot)$ being concave functions of m_t, b_t, and s_t. Further note, that by assuming $\frac{\partial\Theta(\cdot)}{\partial m_t} > \frac{\partial\Theta(\cdot)}{\partial b_t} > \frac{\partial\Theta(\cdot)}{\partial s_t}$, increasing the amount of m_t held should decrease liquidity services providing assets' prices P_t^M, P_t^T, and P_t^S stronger than increasing the amounts of b_t and s_t held. As a unit of money balances is assumed to provide more liquidity services than a unit of Treasuries or corporate debt securities, increasing the holdings of money lowers the investor's willingness to pay for another unit of liquidity services to a stronger extent, than increasing the holdings of Treasuries and corporate debt securities by the same amount. The same reasoning analogously holds for increasing the amount of b_t compared to increasing s_t. Therefore, one can interpret $\Theta'(\cdot)$ as a demand function for a certain feature of assets, namely their degree of liquidity. Further, the model implies that $P_t^M = 1 > P_t^T > P_t^S$, if c.p. holdings of m_t, b_t, and s_t are of the same size. Actually one would expect to find exactly this pattern for the assets' prices for most of the observations within the time period under consideration.

2.2.3 Corporate-Treasury yield spread model

In the following section the asset pricing model which was derived under the assumption that liquidity services are valued by investors is employed to explain spreads between the yields of corporate debt securities and U.S. Treasuries. This is done along the lines of KVJ. The goal is to obtain a model of spread determinants which can be empirically tested for its ability to explain observed corporate-U.S. Treasury yield spreads by using regression analysis.

Following KVJ and Elton et al. (2001)[9] the τ-period yields for U.S. Treasury debt securities $i_{t,\tau}^T$, and for corporate debt securities $i_{t,\tau}^S$ are computed by:

$$i_{t,\tau}^T = -\frac{1}{\tau} ln P_t^T, \text{ and } i_t^S = -\frac{1}{\tau} ln P_t^S,$$

where τ is the number of periods to maturity. By this, the price of a zero coupon bond is converted into a continuously compounded zero coupon

[9] This is a simplified version of Duffie and Singleton (1999). Specifically, I neglect the "recovery of market value" in case of default. Therefore, I do not account for losses in case of default as a fractional reduction of the bond's market value. Further, KVJ point out that the method of Duffie and Singleton (1999) reflects the standard approach in the corporate bond pricing literature.

bond yield. Therefore, for discount bonds with $P_{\tau}^{T}=P_{\tau}^{S}=1$, the corporate-Treasury yield spread for securities with any number of periods to maturity τ, can be expressed as:

$$i_{t,\tau}^{S}-i_{t,\tau}^{T}=\frac{1}{\tau}\left(lnP_{t}^{T}-lnP_{t}^{S}\right).$$

Now plug in (2.8) for P_{t}^{T}, and (2.9) for P_{t}^{S}

$$=\frac{1}{\tau}\left(ln\left(\frac{E_{t}\left[M_{t+\tau}\right]}{1-\nu'\left(\cdot\right)\frac{\partial\Theta(\cdot)}{\partial b_{t}}}\right)-ln\left(\frac{E_{t}\left[M_{t+\tau}\left(1-\delta_{t+\tau}\right)\right]}{1-\nu'\left(\cdot\right)\frac{\partial\Theta(\cdot)}{\partial s_{t}}}\right)\right)$$
$$\approx\frac{1}{\tau}\left(E_{t}\left[M_{t+\tau}\right]+\nu'\left(\cdot\right)\frac{\partial\Theta\left(\cdot\right)}{\partial b_{t}}-E_{t}\left[M_{t+\tau}\left(1-\delta_{t+\tau}\right)\right]-\nu'\left(\cdot\right)\frac{\partial\Theta\left(\cdot\right)}{\partial s_{t}}\right).$$

This approximation uses that $\ln\left(1+x\right)\approx x$, for small x. This approximation is regarded as sufficiently accurate to describe the corporate-Treasury yield spread model. Define the corporate-Treasury yield spread as $\Delta i_{t,\tau}=i_{t,\tau}^{S}-i_{t,\tau}^{T}$, and rearrange

$$\begin{aligned}\Delta i_{t,\tau}=&\frac{1}{\tau}E_{t}\left[M_{t+\tau}\right]E_{t}\left[\delta_{t+\tau}\right]+\frac{1}{\tau}cov_{t}\left(M_{t+\tau},\delta_{t+\tau}\right)\\&+\frac{1}{\tau}\nu'\left(\frac{\Theta_{t}}{GDP_{t}},\xi_{t}\right)\left(\frac{\partial\Theta\left(\cdot\right)}{\partial b_{t}}-\frac{\partial\Theta\left(\cdot\right)}{\partial s_{t}}\right).\end{aligned}\tag{2.10}$$

As Collin-Dufresne, Goldstein, and Martin (2001) point out, most empirical studies on determinants of corporate-Treasury bond yield spreads seek to find suitable proxies for the first two terms on the right hand side of equation (2.10). These models are generally derived from the standard approach of the Asset Pricing Theory's CCAPM. The first and the second term on the right hand side of (2.10) account for the factors that should implied by the CCAPM model drive corporate-Treasury yield spreads. The first term on the right-hand side of (2.10) reflects the expected losses in case of default on commercial papers and corporate bonds. The common label for this expression is "expected default losses". A higher expected probability of default in the business sector, leads investors to demand a higher premium and discount on prices resp. for corporate debt securities, and hence to a higher yield spread relative to Treasuries. The second term on the right-hand side reflects the so called "risk premium" which is related to variation in default probabilities. This premium investors demand, reflects in how far expected default rates covary with expected levels of the agent's marginal utility of consumption. The third term on the right hand side of (2.10) appears due to the modification of the standard asset pricing model by the assumption of investors to value assets' liquidity services. This term, which captures the marginal utility of holding another unit of money m_t, Treasuries b_t, and corporate bonds s_t, is a spread determinant which is not implied by the CCAPM. Due to the assumption that an additional unit of m_t offers more liquidity services than holding an additional unit of b_t and s_t, and that an additional unit of b_t offers more liquidity services than s_t, increasing the investors' holdings of m_t, b_t, and s_t should decrease bond yield spreads with the ordering of marginal impacts by

$$\left|\frac{\partial \Delta i_t\left(\cdot\right)}{\partial m_t}\right| > \left|\frac{\partial \Delta i_t\left(\cdot\right)}{\partial b_t}\right| > \left|\frac{\partial \Delta i_t\left(\cdot\right)}{\partial s_t}\right|.$$

For this model I assume that the shock parameter ξ_t captures so called "flight-to-liquidity" episodes and a securities' market-liquidity related risk premium. The term "flight-to-liquidity" was coined by Longstaff (2004) who defines this as an episode where one can observe on the markets, that some participants suddenly prefer to hold highly liquid securities, such as U.S. Treasuries rather than less liquid securities like corporate bonds and commercial papers. Therefore, in a flight-to-liquidity episode, investors will have an increased willingness to pay for another unit of b_t which will drive up Treasury prices and in turn decrease their yields and hence, decrease spreads relative to yields on corporate debt securities. Following Amihud, Mendelson, and Pedersen (2005) time-varying changes in an asset's market liquidity, like an increase of the time span of a transaction, as well as increasing bid-ask-spreads, contribute to make future expected payment streams risky. This is denoted as "liquidity risk" which would lead to a market-liquidity induced risk premium. For example, in times when investors would like to sell and the market liquidity of a corporate bond deteriorates, risk averse investors will demand a liquidity-risk premium for holding these bonds. Flight-to-liquidity episodes and market-liquidity related risk premia can therefore be interpreted as temporary shocks affecting investors' marginal convenience yields $\nu'\left(\cdot\right)$.

2.2.4 Estimation strategy

The empirical part of this chapter follows the lines of KVJ by estimating regression models derived from equation (2.10). This is done by using Ordinary Least Squares (OLS).[10] As pointed out by Longstaff, Mithal, and Neis (2005), empirical studies relying on regression models derived from the standard CCAPM approach find an unexplained share, the so called "nondefault" component, within corporate-U.S. Treasury yield spreads. The purpose of this analysis is to investigate whether the third term on the right-hand side of equation (2.10), can explain the observed nondefault component. Further, the present analysis poses a test of the hypothesis that liquidity services, which are assumed to be provided by a certain group of assets, are valued by investors. Specifically, I investigate whether investors' valuation for liquidity services is priced within corporate-U.S. Treasury yield spreads while controlling for proxies of spread determinants which are commonly used in the literature. On the one hand, these measures are basically intended to proxy for the spread determinants implied by the standard CCAPM model. These are namely the risk premium and the premium required to compensate for expected losses in case of default.[11] On the other hand, I employ proxies which have been used in recent studies to capture market-liquidity related risk premia and flight-to-liquidity episodes (see Pflueger and Viceira (2012) and Longstaff (2004)).[12] This is done for different yield spread measures as dependent variables, namely for a spread between yields on corporate debt securities and Treasuries with short maturities and a spread between yields of such securities with long maturities. In the following, I will refer to the former as the short-term spread and to the latter as the long-term spread.

The first part of the empirical analysis employs regression models based on the following specification:

$$\Delta i_t^{s,l} = \alpha + \beta_{1\theta} \log\left(\frac{\theta_t}{GDP_t}\right) + \beta_2 Vola_t + \beta_3 Slope_t + \varepsilon_t. \tag{2.11}$$

[10] OLS estimations are the common approach to analyze determinants of credit spreads in the empirical finance literature. See e.g. Collin-Dufresne, Goldstein, and Martin (2001), Chen, Lesmond, and Wei (2007), Eom, Helwege, and Huang (2004), Longstaff (2004), Longstaff, Mithal, and Neis (2005).

[11] For the choice of proxies for the risk premium and the expected default losses I follow KVJ and the survey of Collin-Dufresne, Goldstein, and Martin (2001).

[12] Note that due to data availability for these proxies the empirical study has to be divided into two parts.

Following KVJ the regression model (2.11) is estimated for a long-term spread and a short-term spread as dependent variables. The dependent variable $\Delta i_t^{s,l}$ in each of the corporate-Treasury spread regressions, is a monthly yield spread measured in percentage points and ε_t denotes an error term.[13] The long-term spread Δi_t^l is the difference between an index number on Aaa-rated long maturity corporate bond yields and an index on long maturity Treasury yields. The short-term spread Δi_t^s is the spread between a commercial paper yield index and a Treasury bills yield index. The third term on the right hand side of (2.10) is for the regression model (2.11) captured by the log of (θ_t/GDP_t), with $\theta_t = \{m_t, b_t, s_t\}$. The variable m_t/GDP_t is proxied by the empirical measure for the holdings of money balances, which is the monetary base aggregate, scaled by U.S. GDP. The respective proxy is denoted as (MB_t/GDP_t). Following KVJ the variable b_t/GDP_t is proxied by the face value of the outstanding stock of U.S. Treasuries, which is scaled by U.S. GDP, and is named as $Debt_t/GDP_t$. The face value of corporate bonds and commercial papers outstanding, scaled by U.S. GDP, is the proxy for s_t/GDP_t. It is denoted as (CD_t/GDP_t). Following KVJ a log functional form is used because it provides a good fit and requires estimation of only one parameter.[14] Further, the interpretation of a regression coefficient for a log independent variable, which is expressed as a share, on a dependent variable denoted in percentage points is more convenient.

To control for the premium associated with the expected default losses, which is captured by the first term on the right-hand side of (2.10), I follow KVJ and use a measure for stock return volatility, named $Vola$. The volatility measure for a given month is computed as the standard deviation of weekly log returns on the value-weighted S&P 500 index up to the end of a month. Then, this is multiplied by the square root of 4 to derive the standard deviations on a monthly basis. The proxy $Vola$ is commonly used in the corporate finance literature as a measure for aggregate expected default losses as Collin-Dufresne, Goldstein, and Martin (2001) point out. An increased stock market volatility is generally regarded as implying an increasing probability of defaults in the economy's private firms sector. Hence, investors will demand a higher premium for holding corporate debt securities. Therefore, one can expect corporate-Treasury yield spreads to increase with $Vola$.

[13] See Appendix A.1 for data description.

[14] For quarterly and monthly time series data the Debt-to-GDP ratio is non-stationary but the log of the variable is stationary.

To proxy for the risk premium, which is captured by the second term on the right-hand side of (2.10), I follow KVJ by employing the slope of the yield curve. The proxy *Slope* is measured as the spread between the 10-year Treasury yield and the 3-month Treasury yield. As Collin-Dufresne, Goldstein, and Martin (2001) point out, the slope of the yield curve is regarded as a measure for the state of the business cycle. KVJ assert that the slope of the yield curve is known to predict the excess returns on stocks and captures time-varying risk premia on corporate bonds. For example, if investors are more risk averse in a recession, when *Slope* is high, they will demand a higher risk premium for holding corporate bonds. Thus, the slope of the yield curve serves as a measure for the variation in the risk premium component of the bond spread, i.e. the term involving $cov_t(\cdot)$ in (2.10). Further, KVJ note that to the extent that corporate default risk is likely to vary with the business cycle, the *Slope* variable can furthermore contribute to control for the default risk in the yield spread.

By estimating (2.11) for the two regression model specifications the study presented in this chapter is intended to test the following hypotheses:

Hypothesis 1 *The yield spread model (2.10) requires that an increase in the proxies for the holdings of liquidity services providing assets,* MB_t/GDP_t*,* $Debt_t/GDP_t$*, and* CD_t/GDP_t *decreases the observed spread measures. Hence, the regression results would provide support in favour of the yield spread model (2.10) if point estimates for the coefficients would imply that* $\beta_{1\theta} < 0$*.*

A priori, one can assume that the three groups of assets under consideration can be ordered by their postulated degree of liquidity services provision in the following manner:

$$\begin{array}{rl} MB_t/GDP_t & \text{most liquid} \\ Debt_t/GDP_t & \\ CD_t/GDP_t & \text{least liquid} \end{array}$$

Hypothesis 2 *Further, the yield spread model (2.10) implies that c.p. increasing the proxy for the holdings of the most liquid asset will decrease spreads to a larger extent than increasing the proxy for the holdings of the* *least liquid asset. Therefore, empirical evidence in favour of the yield spread model (2.10) would require* $|\beta_{1m}| > |\beta_{1b}| > |\beta_{1s}|$.

Testing both hypotheses is done by estimating whether changes in the aggregate holdings of assets that are presumed to bear less or more liquidity services than Treasuries will drive spreads in the predicted way. Therefore, I test whether an increase in the holdings of assets that are more (less) liquid than Treasuries reduces observed spreads to a stronger (weaker) extent than an increase in the holdings of Treasuries.

The second part of the empirical analysis employs regression models based on the following specification:

$$\begin{aligned}\Delta i_t^{s,l} = \alpha + \beta_{1\theta} \log\left(\frac{\theta_t}{GDP_t}\right) + \beta_2 Vola_t + \beta_3 Slope_t \\ + \beta_4 ASW_t + \beta_5 Agency_t + \varepsilon_t. \qquad (2.12)\end{aligned}$$

Longstaff (2004) provides evidence for a "flight-to-liquidity" premium in the prices for U.S. Treasuries. This is captured by the spread between yields of bonds issued by Resolution Funding Corporation (Refcorp), a U.S. government agency which is guaranteed by the Treasury, and U.S. Treasury bonds. By full repayment being guaranteed, Refcorp bonds therefore have literally the same default risk as Treasuries. Since Treasuries are more liquid and more popular than Refcorp bonds, a widening (deterioration) of this yield spread reflects investors' preference to hold more (less) highly liquid assets. The reason behind such changes in preferences lies in changing conditions of financial markets, e.g. financial market turmoil would suddenly increase investors' preference for highly liquid assets. Therefore, I use the spread between Refcorp bond yields and U.S. Treasury bond yields to control for flight-to-liquidity episodes. This variable is named *Agency*.

To proxy for market-liquidity related risk premia I follow Pflueger and Viceira (2012) by employing the difference between asset-swap spreads (ASW) for corporate debt securities and Treasury securities. Consider an investor owning a bond and entering into an an asset swap contract. The payer of the bond cash flows can hedge by holding the bond and financing the position on the short term debt market. Hence, the asset-swap spread reflects the current and expected financing costs of holding the long bond position. Therefore, the difference between the asset-swap spreads for corporate bonds, and commercial papers resp., and Treasuries, is a measure for the relative cost of financing a long position in the corporate debt securities market versus financing a long position in the Treasuries market. A widening of this difference indicates a decreasing relative liquidity of corporate debt securities. Hence, an increase in *ASW* should have a positive impact on corporate-Treasury yield spreads.

Inclusion of those two proxies to the estimation model is actually not backed by the theoretical yield spread model (2.10). Both covariates are intended to measure an asset's degree of market liquidity. Therefore, they are in a certain way different from the proxies for the holdings of liquidity services providing assets which are derived within the present asset pricing model setting. The purpose of including those two new controls into the regression model is to investigate whether estimations are robust across the specifications (2.11) and (2.12) with regard to the coefficients on the measures for the holdings of liquidity services providing assets. If Hypothesis 1 and Hypothesis 2 are not rejected for estimation model (2.12) this would provide further support in favour of the modified asset pricing model which assumes investors to value liquidity services. Hence, the following hypothesis can be derived:

Hypothesis 3 *Adding proxies to the estimation model which control for securities' market-liquidity risk and flight-to-liquidity premia (ASW,Agency), will not change the model-implied restrictions on the estimated coefficients and their required ordering:* $\beta_{1\theta} < 0$ and $|\beta_{1m}| > |\beta_{1b}| > |\beta_{1s}|$.

To retain comparability of empirical results, the same data as in KVJ are used for construction of model variables.[15] Differently from KVJ, for the study presented in this chapter I use data at a monthly frequency. Increasing the number of observations within the data set will make regression results more precise and more sound. Further, the use of monthly data is expected to lead to a stronger emphasis of coefficients measuring market volatility. Therefore, if the estimated impact of the U.S. Debt-to-GDP ratio on corporate-Treasury yield spreads is robust across annual and monthly data, this would pose evidence in favor of the present approach to modify the standard asset pricing model. Further to note is that short-term and long-term spreads might not be in the same way affected by changes of the proxies described above. Hence, estimated coefficients on the logs of

[15]Except for MB_t/GDP_t, CD_t/GDP_t *Agency*, and *ASW* as those proxies do not appear in KVJ.

(θ_t/GDP_t) might differ between the regressions with Δi_t^s and Δi_t^l as dependent variables. Note that short-term and long-term spreads can be regarded as different price measures. This reflects a distinct investors' valuation for the spread determinants with regard to the respective long-term and short-term assets to be priced. Therefore, different estimated coefficients on the logs of (θ_t/GDP_t) for short-term and long-term spreads, point to the existence of a differently priced value of short-term and long-term liquidity services. Note that the present asset pricing model still leaves open the possibility for such a specification of $\nu(\cdot)$ and $\Theta(\cdot)$.

2.3 Empirical results

Since data on the securities' market-liquidity related risk measure *ASW* and on the measure for flight-to-liquidity episodes *Agency* are only available from 1987 onwards, the empirical results are split into two parts: In the first part the standard CCAPM implied credit spread regression model is augmented by the measures for the holdings of liquidity services providing assets. The dependent variables are long-term and short-term bond yield spreads. For that estimation monthly time series data are employed ranging from April 1971 to September 2008. This data sample is chosen for the estimation as it covers a period with a presumably constant pattern in investors tastes and as it leaves out the recent financial market turmoil. In the second part of the empirical study the covariates *ASW* and *Agency* are included where only a short-term yield spread is the dependent variable.[16] To derive monthly GDP data I used a cubic spline interpolation on the time series of quarterly U.S. GDP.

Further note that for Tables 2.1-2.4 report t-statistics with adjusted standard errors, after finding an AR(1), AR(2), and AR(3) error structures in most regressions. Following KVJ the AR(n) structure is motivated by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The first-order AR coefficients for each estimation are presented in the respective tables. Serial correlation is especially pronounced in the long-term spread regressions. I use the Newey-West estimator to correct the t-statistics and standard errors for autocorrelation in the error terms.

[16] The covariates *Agency* and *ASW* are expected to capture effects which only affect asset prices in the short-run. In fact, regression results do not show a significant impact on long-term spreads.

2.3.1 Impact of liquid asset supply changes on price measures

Table 2.1 presents results for estimating (2.11) with the long-term spread and the short-term spread being regressed on the measure for Treasury holdings, $\log(Debt_t/GDP_t)$, the measure for expected default losses, $Vola$, and the proxy for the risk premium, *Slope*. A constant term is included as well. This estimation model basically reproduces KVJ. However, the study presented in this chapter uses monthly data. Panel A summarizes the coefficient estimates for the long-term spread as dependent variable, which is in this case the spread between the yields on Aaa-rated corporate bonds and the yields on U.S. Treasury bonds. The mean value of the Aaa-Treasuries spread is at 96 basis points (bp) for the period from April 1971 to September 2008. The coefficient of -0.784 on the $\log(Debt_t/GDP_t)$ variable implies that a decrease of one standard deviation in the Debt-to-GDP ratio, from its mean value of 0.498 to 0.364, increases the Aaa-Treasury spread on average by 25 bp. This is consistent with Hypothesis 1 and statistically significant. From the perspective of the asset pricing model (2.10) one would argue that such a decrease in liquid asset holdings increases the investors' valuation, or willingness to pay resp. for liquidity services. Note that KVJ find for the same time period with annual data an average increase of 22 bp. Further, $Vola$ is found to be significantly related to the spread. The magnitude of the respective regression coefficient β_2 implies that expected default losses are an important driver of long term bond spreads. While KVJ estimate for a one standard deviation increase in their default risk measure an increase of 10 bp in the Aaa-Treasuries spread, the present study finds an increase by 13 bp and a regression coefficient of 5.588.[17] Though, evidence presented in Panel A of Table 2.1 indicates that *Slope* does not exhibit a significant impact on the Aaa-Treasuries spread.

In Panel B of Table 2.1 results are shown for estimating the regression model (2.11) with a short-term bond spread as dependent variable. This spread is the difference between the yields of highest rated commercial paper and Treasury bills, both with 3-month maturity length. Changing holdings of liquidity services providing assets might be priced differently within short-term and long-term spreads. Hence, it should not be expected to find estimated coefficients on $\log(Debt_t/GDP_t)$ to be the same across the two panels. Nonetheless, the effect of changes in aggregate Treasury holdings

[17] For the sample of 1971-2008 this study finds a mean value of 0.035 and a standard deviation of 0.023 for $Volatility$.

on the short-term spread is estimated to be of fairly similar magnitude as the effect on long-term spreads. The mean value of the commercial paper-Treasury bills spread is at 62 bp for the period from April 1971 to September 2008. A decrease of one standard deviation in the Debt-to-GDP ratio from its mean, is found to increase the commercial paper-Treasury bills spread an average by 22 bp. Compared to that KVJ estimate an average increase of 23 bp. Further, this study finds evidence for a statistically significant impact of $Vola$ on short-term spreads. An increase of $Vola$ by one standard deviation increases the respective spread by 13 bp. KVJ however, find no significant effect of their default risk measure on short-term spreads for annual data within the period from 1969 to 2007. Panel B further shows that the measure for the risk premium *Slope* exhibits a significant but rather small impact on the commercial paper-Treasury bills spread.[18]

The estimation results presented in Table 2.1 imply that Hypothesis 1 can be regarded as being not rejected. This poses evidence in favor of the predictions made by the theoretical pricing model (2.10). Prior results by KVJ are found to be confirmed. Increasing the number of observations, by changing the data frequency from an annual basis to a monthly basis, leads for the same sample period to similar results regarding the estimated coefficients on $\log{(Debt_t/GDP_t)}$, and $Vola$. Note that for monthly data the nondefault component proxied by the Debt-to-GDP ratio, as well as the default risk component proxied by stock market volatility, play a more pronounced role compared to the respective measures estimated on annual data.

Table 2.2 presents results for estimating (2.11) with the long-term spread as the dependent variable. In the first column estimated coefficients for the regression of the AAA-Treasury yield spread on $\log{(CD_t/GDP_t)}$, $Vola$ and *Slope* are shown. In the second column estimation output is shown for the regression model where the $\log{(Debt_t/GDP_t)}$ regressor replaces the proxy for the holdings of corporate debt securities. Note that for reasons of comparability of results, here the same information as in Table 2.1, Panel A is provided. Results presented in the third column refer to the estimation where the proxy for money balances, $\log{(MB_t/GDP_t)}$, replaces the

[18] These regressions were also conducted for quarterly data but are not provided for reasons of brevity. Results imply that a decrease in the Debt-to-GDP ratio by one standard deviation increases the long term spread by 26 bp and the short-term spread by 21 bp. An increase in $Volatility$ by one standard deviation increases the long-term spread by 14 bp and the short-term spread by 17 bp.

former proxies. The corporate-Treasury yield spread model described in Section 2.2 implies that under the hypothesis of liquidity services being a priced attribute, estimated coefficients would be in absolute terms ordered by $|\beta_{1m}| > |\beta_{1b}| > |\beta_{1s}|$. This is basically outlined by Hypothesis 2.

In Table 2.2 the coefficient on the proxy for the aggregate holdings of corporate debt securities is estimated to be in absolute terms smaller than the coefficient on the proxy for the aggregate Treasuries holdings. This result is in line with Hypothesis 2. The respective coefficient implies an increase of 7 bp in the Aaa-Treasuries yield spread due to a decrease of the corporate debt-to-GDP ratio by one standard deviation from its mean value. However, the coefficient on $\log(MB_t/GDP_t)$ is found to be insignificant while the regression model has a relatively low R^2. Finding an insignificant coefficient on the proxy for money balances does not seem to support the model implication that changes in the holdings of assets that should deliver more liquidity services than Treasuries will cause a stronger impact on long-term bond spreads than changes in holdings of the latter. However, Treasury bond holdings and corporate bond holdings are with regard to their long runtime generally motivated by matching investor's long-term objectives. In contrast to that the investor's decision regarding holdings of money balances is affected by short-term investment objectives. Hence, for those two groups of assets there are different underlying investment motives. As pointed out by KVJ, assets do not only may provide liquidity services to a different degree, there might also be a difference between short-term and long-term liquidity services. Therefore, it should not be surprising that the estimated effect of changing money holdings on the long-term spread is insignificant. Including a different measure for the most liquid assets, namely the difference of M3-M2 scaled by GDP, instead of monetary base scaled by GDP, yields a regression coefficient which is in line with Hypothesis 2. Column 4 of Table 2.2 reports an estimated coefficient of -1.718 on the measure $\log((M3_t - M2_t)/GDP_t)$ which implies an average increase in the Aaa-Treasuries spread by 75 bp following a decrease in the M3-M2-to-GDP ratio by one standard deviation from its mean value.[19] Note that M3-M2 covers the positions of large time deposits, institutional money market funds, repurchase agreements and other larger liquid assets. Investors hold these highly liquid near money assets mostly for long-term investments horizons. Hence, the insignificance of $\log(MB_t/GDP_t)$ seen against the background of evidence presented in the fourth column of Ta-

[19] The last available observation on the M3 aggregate is January 2006.

ble 2.2 points to a difference between long-term and short-term liquidity services. Further, this would imply the existence of market segmentation for long-term and short-term liquidity services providing assets. All other variables included in the regression models, but the four discussed hare, provide roughly the same evidence as explained in the paragraph above.

Table 2.3 presents results for estimating (2.11) with the short-term spread as the dependent variable. In Column 1 of Table 2.3 output is reported for the regression of the short-term spread on the $\log(CD_t/GDP_t)$, $Volatility$ and $Slope$ measures. In Column 2 the proxy for the aggregate holdings of corporate debt securities is replaced by the proxy for the holdings of Treasury debt, $\log(Debt_t/GDP_t)$, and in Column 3 by the proxy for money balances, $\log(MB_t/GDP_t)$. Results presented in Table 2.3 are in line with Hypothesis 1 and Hypothesis 2, and are statistically significant. Specifically, it is found that $|\beta_{1m}| > |\beta_{1b}| > |\beta_{1s}|$. Expressed in terms of basis points, coefficients imply a 26 bp, 22 bp, and 5 bp increase of the commercial paper-Treasury bills spread by decreasing of the respective asset-to-GDP ratios by a one standard deviation from their means. Further comparing the magnitudes of the coefficients on the proxies for money holdings in column 3 and 4 with the accordant results from Table 2.1, there is further support for the implication of market segmentation and differently priced short-term and long-term liquidity services. Coefficients on $\log((M3_t - M2_t)/GDP_t)$ and $\log(MB_t/GDP_t)$ imply that changes in the holdings of long-term near money assets do not cause such a strong impact on short-term yields as changes in the base money measure. All other variables included here, again provide the same evidence as explained in the paragraphs above.

Table 2.4 reports results for estimations of (2.12) which are including the market-liquidity risk measure and the measure for flight-to-liquidity episodes. The dependent variable here is only the short-term spread. The data sample covers the period from April 1987 to September 2008. In Column 1 estimated coefficients of the commercial paper-Treasury bills yield spread regression on $\log(Debt_t/GDP_t)$, $Vola$, $Slope$ and a constant are shown. Column 2 reports results for an estimation where the covariates $Agency$ and ASW are added to the regression model. In the same manner, regressions are estimated for specifications of (2.11) and (2.12) that employ $\log(CD_t/GDP_t)$ and $\log(MB_t/GDP_t)$ instead of $\log(Debt_t/GDP_t)$. Accordant results are summarized in Columns 3 and 4, and Columns 5 and 6.

Comparison of estimated coefficients on the proxies for money balances, Treasury holdings and the holdings of corporate debt securities in columns 1, 3, and 5, indicates that the ordering of $|\beta_{1m}| > |\beta_{1b}| > |\beta_{1s}|$ is preserved for the shorter data sample. Including the liquidity-risk proxy ASW, and the proxy for flight-to-liquidity episodes *Agency*, yields for all regression models statistically significant regression coefficients β_4, and β_5, with the expected signs. For the regression results reported in Column 2 of Table 2.4, where $\log(Debt_t/GDP_t)$ is included as a covariate, an increase of ASW by one standard deviation from its mean of 0.441 to 0.644, increases the short term yield spread on average by 22 bp. This confirms the view that an investors' demanded market-liquidity related risk premium is an important driver of the commercial paper-Treasury bill spread. Further, if the measure *Agency* increases by one standard deviation from its mean of 1.054 to 1.997, the short-term spread decreases by 20 bp, which provides evidence for a flight-to-liquidity premium in the commercial paper-Treasury bill spread. The corresponding estimated effects implied by regression output presented in the fourth column of Table 2.4 are 23 bp and 21 bp resp., and for the output depicted in Column 6 I calculate 20 bp and 15 bp resp. Compared to the results shown in columns 1, 3, and 5, coefficients of the proxies for Treasury holdings, money balances and the holdings of corporate debt securities decrease sharply. The sizes of coefficients now imply that decreases of the respective measure by one standard deviation from its mean value, increase spreads by only 3, 8, and 1 bp resp.[20] This implies that in regression models excluding measures for securities' market-liquidity risk and flight-to-liquidity episodes, the coefficients on the proxies for liquidity services providing assets capture sizeable information which should actually be attributed to the former measures. However, seen against the background of the commercial paper-Treasury bills spread's mean being at 46 bp for the period of April 1987 to September 2008, still priced liquidity services can be regarded as a significant driving force. In addition to that, the model-implied ordering of estimated coefficients is still preserved with $|\beta_{1m}| > |\beta_{1b}| > |\beta_{1s}|$, while controlling for market-liquidity risk and flight-to-liquidity episodes. Hence, one can regard Hypothesis 3 to be not rejected. Further, R^2 measures for all three regression models rise to val-

[20] For the time period April 1987 to September 2008 the mean of $Debt_t/GDP_t$ is 0.603 with a standard deviation of 0.051. For the same period MB/GDP has a mean of 0.056 and a standard deviation of 0.005. The mean of CD/GDP is 0.225 and the standard deviation is 0.031.

ues of roughly 0.8, and Durbin-Watson statistics increase significantly by including market-liquidity risk measures and measures for flight-to-liquidity episodes. This points to a better model fit and a lager share of the spread's variance being explained by the regression models. Therefore, one can argue that the study presented in this chapter yields a significant contribution to explain the nondefault component which appears to be found within corporate-Treasury yield spreads.

2.4 Conclusion

This chapter provides empirical evidence which is supporting the notion that the nondefault component within the corporate-Treasury bond yield spread is to a significant extent driven by the investors' valuation for liquid assets. Estimation results imply that changes in the aggregate holdings of assets that are presumed to bear less or more liquidity services than U.S. Treasuries will affect corporate-Treasury yield spreads in the way predicted by an asset pricing model which allows for holdings of liquid assets to contribute to investors' utility. Specifically, results imply that investors value liquidity independently from the underlying asset. Finding this systematic pattern points to the existence of a demand function for liquidity attributes. Several regression model specifications are estimated using different data samples and datasets where results are found to be robust. Estimation results show that investors price liquidity services separately from measures for credit risk, market-liquidity related risk, and flight-to-liquidity episodes. Moreover, results imply market segmentation for long-term and short-term assets as a difference in the valuation for long-term and short-term liquidity services points to the existence of different investment motives. Compared to commonly employed corporate-Treasury bond yield spread regression models, the study presented in this chapter uses model specifications which yield a better empirical fit and explain a lager share of the observed yield spreads' variation. Further, finding empirical evidence for liquidity services provision being priced by investors poses a challenge to standard asset pricing theory models.

Table 2.1: Impact of Treasury supply on corporate-U.S. Treasury yield spreads

Period	Apr 1971 - Sep 2008	
	Panel A: Aaa-Treasury	Panel B: CP-Bills
$log(Debt/GDP)$	−0.784	−0.620
	[−7.596]	[−8.732]
Volatility	5.588	5.819
	[4.005]	[6.567]
Slope	0.006	−0.136
	[0.327]	[−8.825]
Intercept	0.387	0.156
	[5.319]	[2.790]
R^2	0.419	0.350
Durbin-Watson	0.282	0.536
ρ	0.646	0.658
N	453	453

Notes: t-statistics with adjusted standard errors are reported in the brackets. AR(n) structures are found by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The first-order AR coefficients for each OLS estimation are denoted as ρ. *The Newey-West estimator is employed to correct the t-statistics and standard errors for autocorrelation in the error terms.*

Table 2.2: Impact of MB/GDP, Debt/GDP, CD/GDP on Aaa-Treasury yield spread

Period	Apr 1971 - Sep 2008			Apr 1971 - Jan 2006
	(1)	(2)	(3)	(4)
$log(MB/GDP)$			0.303	
			[1.122]	
$log((M3-M2)/GDP)$				−1.718
				[−4.018]
$log(Debt/GDP)$		−0.784		
		[−7.596]		
$log(CD/GDP)$	−0.277			
	[−4.369]			
Volatility	6.554	5.588	6.337	5.879
	[4.803]	[4.005]	[4.596]	[4.007]
Slope	−0.017	0.006	−0.027	−0.038
	[−0.799]	[0.327]	[−1.309]	[−1.767]
Intercept	−0.741	0.387	1.846	1.399
	[−1.902]	[5.319]	[2.329]	[2.352]
R^2	0.288	0.458	0.140	0.240
Durbin-Watson	0.287	0.282	0.225	0.239
ρ	0.589	0.646	0.611	0.615
N	453	453	453	453

Notes: t-statistics with adjusted standard errors are reported in the brackets. AR(n) structures are found by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The first-order AR coefficients for each OLS estimation are denoted as ρ. The Newey-West estimator is employed to correct the t-statistics and standard errors for autocorrelation in the error terms.

Table 2.3: Impact of MB/GDP, Debt/GDP, CD/GDP on CP-Bills yield spread

Period	Apr 1971 - Sep 2008			Apr 1971 - Jan 2006
	(1)	(2)	(3)	(4)
$log(MB/GDP)$			−1.267	
			[−4.216]	
$log((M3 - M2)/GDP)$				−0.254
				[−4.096]
$log(Debt/GDP)$		−0.620		
		[−6.101]		
$log(CD/GDP)$	−0.198			
	[−4.672]			
Volatility	6.581	5.819	7.339	5.748
	[3.828]	[3.482]	[4.202]	[2.799]
Slope	−0.155	0.136	−0.158	−0.177
	[−5.941]	[−5.709]	[−5.856]	[−6.139]
Intercept	−0.607	0.156	−3.085	0.132
	[−2.456]	[2.462]	[−3.509]	[1.120]
R^2	0.288	0.350	0.308	0.295
Durbin-Watson	0.521	0.536	0.566	0.486
ρ	0.646	0.658	0.624	0.674
N	453	453	453	453

Notes: t-statistics with adjusted standard errors are reported in the brackets. AR(n) structures are found by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The first-order AR coefficients for each OLS estimation are denoted as ρ. *The Newey-West estimator is employed to correct the t-statistics and standard errors for autocorrelation in the error terms.*

Table 2.4: Impact of MB/GDP, Debt/GDP, CD/GDP, market liquidity measures on CP-Bills yield spread

Period	Apr 1987 - Sep 2008					
	(1)	(2)	(3)	(4)	(5)	(6)
$log(MB/GDP)$			−2.313	−0.953		
			[−8.713]	[−4.356]		
$log(Debt/GDP)$	−1.495	−0.379				
	[−5.077]	[−3.436]				
$log(CD/GDP)$					−1.029	−0.068
					[−6.908]	[−1.430]
Volatility	2.862	2.104	4.007	2.459	5.216	2.737
	[1.916]	[3.266]	[3.711]	[3.822]	[4.671]	[4.646]
Slope	−0.050	0.077	−0.045	0.054	−0.069	0.079
	[−2.734]	[5.240]	[−3.156]	[3.549]	[−4.788]	[5.342]
ASW		1.095		0.945		1.146
		[9.094]		[9.658]		[13.609]
Agency		−0.209		−0.161		−0.227
		[−12.144]		[−8.190]		[−13.331]
Intercept	−0.301	−0.175	−6.204	−2.534	−1.170	−0.198
	[−2.064]	[−1.320]	[−8.192]	[−4.092]	[−4.977]	[−0.716]
R^2	0.287	0.671	0.505	0.715	0.402	0.684
Durbin-Watson	0.396	0.762	0.610	0.862	0.547	0.816
ρ	0.762	0.602	0.685	0.566	0.679	0.535
N	258	258	258	258	258	258

Notes: t-statistics with adjusted standard errors are reported in the brackets. AR(n) structures are found by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The first-order AR coefficients for each OLS estimation are denoted as ρ. The Newey-West estimator is employed to correct the t-statistics and standard errors for autocorrelation in the error terms.

Chapter 3

An Empirical Study on Investors' Preferences for Liquid Assets

3.1 Introduction

Recent empirical studies on determinants of corporate-U.S. Treasury bond yield spreads find that Investors value the liquidity of U.S. Treasury bonds. At the same time U.S. Treasuries' degree of liquidity, which might be perceived as an inherent feature, or being driven by changing market conditions, is found to be priced separately from commonly studied spread determinants which are implied by the standard Asset Pricing Theory's Consumption Capital Asset Pricing Model (CCAPM).[21]

A recent study by Krishnamurthy and Vissing-Jorgensen (2012) (KVJ) finds a significant negative association between the aggregate supply of U.S. Treasuries and corporate-U.S. Treasury bond yield spreads. They argue that this reflects a demand function for what they call U.S. Treasury-specific liquidity services or "convenience yields". Therefore, the high level of liquidity services offered by Treasuries would drive down their yields compared to assets that do not to the same extent share this feature. Further, when the supply of Treasuries is low, the value that investors assign to the liquidity

[21] see Longstaff (2004), Longstaff, Mithal, and Neis (2005), Amihud, Mendelson, and Pedersen (2005), Acharya and Pedersen (2005), Chen, Lesmond, and Wei (2007), Pflueger and Viceira (2012).

services offered by Treasuries is high, implying increasing (decreasing) Treasury prices (yields) and in turn, increasing corporate-Treasury bond yield spreads. The same argument applies in the opposite direction when the supply of treasuries is high. Moreover, Niestroj (2012) (NIE) finds evidence that the notion of priced liquidity services is not only U.S. Treasury-specific. In particular, it is shown that investors value liquidity services which can be provided by a variety of assets - while each asset might be featuring this attribute to a different degree. This does not only support the view that investors in general value the attribute of liquidity services provision when pricing assets, but also this points to the existence of a demand function for liquidity.

However, both approaches derive asset pricing models under the ad hoc assumption that asset holdings directly contribute to investors' utility. Specifically, liquidity services are derived via an unknown aggregator function which is a separate argument of investors' utility. Neither approach provides a complete specification of the underlying preference and aggregator functions but defines a set of requirements to them.

This chapter seeks to fill this gap by providing a complete specification and parameterization of a representative agent's utility function which can rationalize the investors' behavior observed by KVJ and NIE. For that purpose in this chapter I first use nonparametric testing routines to examine whether a preference maximization model cannot be rejected where liquidity services directly contribute to investors utility. Specifically, I check Varian's (1982) necessary and sufficient revealed preference conditions for monthly data on consumption, money holdings, Treasury holdings and prices, and on corporate debt securities holdings and prices. Consistency of the data with these conditions means non-rejection of the hypothesis that investors are maximizing a utility function which is nonsatiated, continuos, concave and monotonic.[22] Further, I test whether the data satisfy necessary and sufficient revealed preference conditions for weak separability between several groupings of the liquidity services providing assets and consumption. This is done by applying the procedure proposed by Fleissig and Whitney (2003). The reason for employing this second nonparametric test is that KVJ and NIE implicitly assume weak separability for their analyses. As

[22]The data used for the present analysis are basically the same as in KVJ and NIE, while it is assumed that money, Treasuries, and corporate debt securities are providing liquidity services. The reason for the choice of the data set lies in the intention of this chapter to carry forward the analysis of the previous authors.

pointed out by Swofford and Whitney (1987) weak separability is a convenient feature as it keeps the subsequent theoretical analysis analytically tractable and for the empirical part of the study, it reduces data requirements and conserves statistical degrees of freedom. If both hypotheses are not rejected, the question for a suitable specification of the investor's utility function arises. As the nonparametric testing routines applied in this chapter do not provide much guidance for that, Generalized Method of Moments (GMM) is employed to estimate coefficients of Euler equations which are derived from the investors' optimization problem under several proposed utility specifications. This further poses an indirect test of the asset pricing models employed by KVJ and NIE. This is due to the fact that only a subset of the proposed utility function specifications meets the requirements which are imposed on investors' demand behavior by their modified asset pricing model.

This chapter provides evidence from the nonparametric testing routines that necessary and sufficient conditions for utility maximization and weak separability are obtained for the dataset. However, results from GMM estimations imply rejection of almost all proposed utility specifications. Only the model based on the specification proposed by Poterba and Rotemberg (1986) is not rejected. Estimation results however, imply parameter values which indicate misspecification.

Estimating parameters of utility functions which include consumption and an aggregator function of near monies holdings, which is denoted as "liquidity services", goes back to Poterba and Rotemberg (1986). The aim of this study was to examine the impact of open market operations on short-term interest rates. Analyzing the effects of nonstandard monetary policy operations such as large-scale asset purchase programs (LSAPs) has recently become an active field of macroeconomic research. These measures have been introduced to provide liquidity in exchange for private sectors' assets. From a theoretical perspective, it is generally expected that such nonstandard open market operations in private assets do not exhibit an effect on real variables. In particular, as shown by Eggertsson and Woodford (2003), this irrelevance result by Wallace (1981) applies to the canonical New Keynesian macroeconomic model approach. Hence, the model framework for monetary policy analysis as summarized by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) does not provide a suitable approach for the evaluation of the effectiveness of this policy. Contributions like Chen, Curdia, and Ferrero (2012), Gertler and Karadi (2013), Gertler and Kiyotaki (2010), and Del Negro et al. (2013) rely on investors' heterogenous preferences and on financial market frictions to make the relative supply of liquid assets have an effect on the equilibrium allocation. Schabert and Reynard (2009), Schabert and Hörmann (2011), and Christoffel and Schabert (2013) assume for this purpose liquidity constraints and model explicitly the central bank's balance sheet policy options. Others like Krishnamurthy and Vissing-Jorgensen (2011), Swanson (2011), and Peersman (2011) employ econometric approaches by using event studies, and in the last case, a VAR analysis.

So far, macroeconomic models that analyze the effects of nonstandard monetary policy measures do not rely on household preferences which account for assets liquidity services. In particular, such an approach would be criticized as being ad hoc. The study presented in this chapter investigates whether there is evidence in favor of such an approach by deriving a first proposal for a suitable utility function. This is done by providing a set of microfoundations, starting with nonparametric hypothesis testing, and then by moving on to parameter estimations.

Varian's (1982) Generalized Axiom of Revealed Preference (GARP) provides testable necessary and sufficient conditions for a finite number of observations on consumer behavior to be consistent with the preference maximization model. Varian (1982 and 1983) point out that the standard approach is to postulate a possible parametric form for demand functions and fitting them to observed data, and then to test for the hypothesis under consideration. In contrast to that, employing the test for GARP does not require an ad hoc specification of functional forms and is therefore completely nonparametric. Specifically, instead of testing the joint hypothesis that demand behavior can be described by some parametric form and the restriction one wants to test for, Varian (1982) provides a complete test of the hypothesis in question alone. As further pointed out by Varian (1982 and 1983), this test relies on algebraic conditions on a finite body of data to be consistent with the maximization hypothesis. These are denoted as "revealed preference" conditions and provide a complete test on the restriction imposed by maximizing behavior, in the sense that every maximizing consumer's demand behavior must satisfy these conditions, and that all behavior that satisfies these conditions can be viewed as maximizing behavior.[23]

[23] Note that revealed preference analysis was not only at the time of its introduction a major contribution to the theory of consumer behavior but still is an active field

Under the assumption that assets are held for real nonpecuniary returns from such attributes as their perceived liquidity, it is a convenient feature of the utility function under consideration to be weakly separable in the arguments of consumption and so called liquidity services which are derived from liquid asset holdings. As pointed out by Swofford and Whitney (1987), weak separability implies two-staged budgeting. Households first decide upon the allocation of expenditures between consumption and asset holdings. In the second stage Households allocate expenditures among the goods within each subgroup based only on the relative prices of the goods in that group. Weak separability has the necessary and sufficient condition that the marginal rate of substitution (MRS) between any two goods within a group is independent of the goods outside the group. Hence, for the determination of the utility's functional form it can be assumed that liquidity services are derived by a jet unknown aggregator function of the liquid asset holdings, which represents a separate argument of the utility function. To test for weak separability I use the approach by Fleissig and Whitney (2003). Employing this procedure comes with the advantage that in case of nonrejection of the weak separability hypothesis one can easily revert to well known functional forms for the aggregator like constant-elasticity-of-substitution (CES) and Cobb-Douglas. If weak separability does not obtain, two stage budgeting procedure is not an accurate description of consumer behavior. Moreover, this would pose evidence against the asset pricing models proposed by KVJ and NIE.

Swofford and Whitney (1987) further point out, that the second stage of the two-staged budgeting process is the focus of mainly microeconomists examining how households allocate total consumption expenditure among various categories of goods and services. In particular, this involves estimating parameters of demand functions and utility functions. Monetary economists like Barnett (1980), Feldstein and Stock (1996), and Drake and Mills (2005) have used this approach to obtain estimates of elasticities of substitution between narrow transaction balances and less liquid near monies in attempts to clarify the appropriate definition for money balances. Nonparametric tests to evaluate if groups of monetary assets are weakly separable from other goods have been used, among others, by Barnett, Fisher, and

of research. E.g. see Blundell (2005), Blundell, Browning, and Crawford (2003), and Andreoni and Miller (2002). The basic idea has been applied in a number of other areas of economics. A recent application can be found in the field of agricultural economics on revealed preferences for organic and cloned milk. See Brooks and Lusk (2010).

Serletis (1992), Belongia (1996), Swofford and Whitney (1987, 1988), and Drake and Chrystal (1994, 1997) while the construction of the monetary aggregates is neglected.

With the revealed preference tests done the question for a suitable specification of the representative investor's objective function arises. The approaches by KVJ and NIE impose several requirements to its unspecified underlying utility function, whereas the nonparametric revealed preference tests which are applied in this chapter provide support for a wide set of possible utility function specifications. To find the most suitable one, GMM is employed which is frequently used to estimate and test asset pricing models.[24] In this chapter I estimate coefficients of Euler equations which are derived from the investors' optimization problem under different utility specifications. In particular, I consider Cobb-Douglas and CES aggregator functions for the liquidity services measure. The aggregator functions are nested in utility functions with each displaying constant relative risk aversion. As it can be shown, the investors' demand behavior observed by KVJ and NIE is consistent with the aggregator function to be CES, and the utility's arguments of consumption and liquidity services to be not additively separable. Hence, this empirical analysis of different utility specifications can be regarded as an additional test of the models by KVJ and NIE.

The chapter proceeds as follows: Section 3.2 describes the nonparametric testing routines which are applied in the present chapter and the data. Further, results from hypothesis testing are presented. Section 3.3 discusses the representative agent's utility maximization problem and derives Euler equations under alternative specifications of the investor's utility function. Results from GMM estimations are presented here as well. Section 3.4 concludes.

3.2 Nonparametric tests for utility maximization and weak separability

3.2.1 Testing the maximization hypothesis

This analysis employs Varian's (1982 and 1983) nonparametric approach to demand analysis. Varian (1982) shows that observed demand behavior can be rationalized by a nonsatiated, continuous, concave, monotonic utility

[24] For a theoretical treatment of this method see Hansen (1982). For recent empirical studies employing GMM on asset pricing models see e.g. Stock and Wright (2003), Yogo (2004), and Hall (2005).

function if one of several equivalent conditions is met. The easiest one to test is Varian's (1982) GARP.[25]

To recover investors' preferences from a finite number of observations of k-vectors of prices and quantities (p^i, x^i), $i = 1, ..., n$, with $p^i = (p_1^i, ..., p_k^i)$, and $x^i = (x_1^i, ..., x_k^i)$, consider the following definitions:

Definition (1): A utility function rationalizes the data (p^i, x^i), $i = 1, ..., n$, if $u(x^i) \geq u(x)$, for all x such that $p^i x^i \geq p^i x$, for $i \geq 1, ..., n$.

Definition (2): An observation x^i is directly revealed preferred to a bundle x, written $x^i R^0 x$, if $p^i x^i \geq p^i x$. An observation x^i is revealed preferred to a bundle x, written $x^i R x$, if there is some sequence of bundles $(x^j, x^k, ..., x^l)$ such that $x^i R^0 x^j$, $x^j R^0 x^k$, ..., $x^l R^0 x$. Then R is the transitive closure of the resolution R^0.

Definition (3): The data satisfies the Generalized Axiom of Revealed Preference (GARP) if $x^i R x^j$ implies $p^j x^j \leq p^j x^i$.

Varian (1982) points out that the third definition demands that there are no cyclical inconsistencies if x^i is preferred to all other affordable bundles. Then x^i is better than any bundle x^j chosen at all prices p^j. It is further pointed out, that the advantage of GARP is that it is an easily testable condition, and as Afriat's Theorem demonstrates it is a necessary and sufficient condition for utility maximization:

Afriat's Theorem (Afriat 1967, Diewert 1973, Varian 1982)
The following conditions are equivalent:
(1) *there exists a nonsatiated utility function that rationalizes the data.*
(2) *the data satisfies GARP.*
(3) *there exist numbers* $U^i, \lambda^i > 0, i = 1, ..., n$ *that satisfy the Afriat inequalities:* $U^i \leq U^j + \lambda^i p^i (x^i - x^j)$, *for* $i, j = 1, ...n$.
(4) *there exists a concave, monotonic, continuous, nonsatiated utility function that rationalizes the data.*

Where U^i is the utility level and λ^i the measure for marginal utility of income at observed demands. Varian (1982) points out, that the equivalence of conditions (1) and (4) shows that if some data can be rationalized

[25] The remainder of this subsection is taken from Varian (1982 and 1983).

by any nondegenerate utility function at all, this utility function has desirable properties. Or put differently, violations of continuity, concavity, and monotonicity cannot be detected within a finite number of observations. Further, conditions (2) and (3) give directly testable conditions that the data must satisfy if it is to be consistent with the maximization model. Condition (3) asks for a nonnegative solution to a set of linear inequalities. Condition (2) is more convenient for computation. As pointed out by Varian (1983), Afriat (1967) derived Afriat's Theorem first with a different but equivalent version of condition (2) which demanded that data satisfies "cyclical consistency" and Diewert (1973) provided a different proof, omitting a consideration of condition (2). As further pointed out, Varian (1982) showed that GARP is equivalent to Afriat's (1967) cyclical consistency condition. GARP is to be preferred as it is much easier to evaluate in practice.[26]

To test whether GARP is satisfied Varian (1982) proposes to use the dataset to construct an n by n matrix M, with the $i-j$ entry is given by

$$m_{ij} = \begin{Bmatrix} 1, \text{if } p^i x^i \geq & p^i x^j, \text{ that is } x^i R^0 x^j \\ 0, & \text{otherwise.} \end{Bmatrix}$$

The matrix M summarizes the relation R^0. Hence, once R - the transitive closure of the directly revealed preference relation R^0 - is known one can test whether GARP satisfied. For that purpose Varian (1982) proposes Warshall's algorithm which operates on M to create the matrix MT, where

$$mt_{ij} = \begin{Bmatrix} 1, & \text{if } x^i R x^j \\ 0, & \text{otherwise.} \end{Bmatrix}$$

MT represents the relation R. Hence, to test for consistency with GARP one has to look at each element $mt_{ij} = 1$, and check if $p^j x^j > p^j x^i$, for some i and j. If that is the case then there is a violation of GARP detected.

3.2.2 Testing the weak separability hypothesis

To check whether goods can be either combined or have to be studied independently, a test for weak separability is performed. Fleissig and Whitney (2003) propose a new method to evaluate the separability conditions from the revealed preference theory of Varian (1983).[27] Following Varian

[26] For a detailed discussion see Varian (1982).

[27] The remainder of this subsection is taken from Fleissig and Whitney (2003).

(1983) partition the data in two sets of goods and prices (p^i, x^i), (q^i, y^i) with $i = 1, ..., n$.

Definition (4): A utility function $u(x)$ is (weakly) separable in the y goods if there exists a subutility function $\nu(y)$ and a macro function $u^*(x, \nu(y))$ which is continuous and monotonically strictly increasing in $\nu(y)$, such that $u(x, y) \equiv u^*(x, \nu(y))$.

Varian (1983) points out that the necessary condition for weak separability demands that the subdata must satisfy GARP because each observation must solve the problem

$$max\ \nu(y) \tag{3.1}$$
$$s.t.\ q^i y \leq q^i y^i. \tag{3.2}$$

The necessary and sufficient conditions for separability are summarized by the following theorem of Varian (1983):

Varian's Separability Theorem (Varian 1983) *The following conditions are equivalent:*
(i) *There exists a weakly separable concave, monotonic, continuous nonsatiated utility function that rationalizes the data.*
(ii) *There exist numbers* $U^i, V^i, \lambda^i, \mu^i > 0$, that satisfy separability inequalities for $i, j = 1, ..., n$:

$$U^i \leq U^j + \lambda^j p^j \left(x^i - x^j\right) + \lambda^j/\mu^j p^j \left(V^i - V^j\right), \tag{a}$$
$$V^i \leq V^j + \mu^j p^j \left(y^i - y^j\right). \tag{b}$$

(iii) *The data* (q^i, y^i) *and* $(p^i, 1/\mu^i, x^i, V^i)$ *satisfy GARP for some choice of* (V^i, μ^i) *that satisfies the Afriat inequalities.*

Fleissig and Whitney (2003) point out that condition (ii) provides a direct way for testing the necessary and sufficient conditions. However, one would need to check for a solution to a system of $2n\,(n-1)$ equations of which half of them are nonlinear. Fleissig and Whitney (2003) assert that condition (iii) is equivalent to evaluating GARP with $1/\mu^i$ as a "sub group price index" and V^i as a "sub group quantity index" for the separable goods y^i. Varian's (1983) NONPAR algorithm is based on condition (iii), where indices are calculated to satisfy the inequality constraint (b). If the data pass the test for GARP fulfilling condition (iii), then from condition (i), it follows that the observed data are consistent with weakly separable preferences. Fleissig and Whitney (2003) point out, if NONPAR does not find that the data on the y-goods pass the GARP test, weak separability cannot be rejected since there might be other values for the quantity and price indices that may satisfy the inequalities of (b).[28] Hence, Varian's (1983) NONPAR approach tests the sufficient but not necessary conditions for weak separability.

Fleissig and Whitney (2003) propose a new approach to evaluate Varian's separability condition (iii) by using an alternative method to estimate V^i and μ^i. The aim is to find an estimate of an unknown aggregator function $\nu(y, q)$ which is a function of prices and quantities of the y-goods. This approach relies on Diewert (1976, 1978) who shows that a certain class of statistical index numbers provides a second-order approximation to an arbitrary or unknown, twice-differentiable linear homogeneous aggregator function. This class of index number is denoted as "superlative index". Two of those Indices which posses the property of being superlative are for example the Fisher Index and the Törnqvist-Theil Index. Therefore, Fleissig and Whitney (2003) build on calculating superlative index numbers to obtain estimates for V^i and a corresponding range of μ^i that satisfy Varian's conditions.

The first step of the procedure proposed by Fleissig and Whitney (2003) uses a superlative index number $QV^i = f(q, y)$, which is a function of prices and quantities from the y-goods as an initial estimate for V^i in the inequality (b). The objective is to find how close the superlative index QV^i is to providing a solution. By adding a positive number QV^i_p or a negative number QV^i_n to QV^i the superlative index number with error

$$QV^{i*} = QV^i + QV^i_p - QV^i_n,$$

will provide a solution if one exists. If $QV^i_p = 0$, and $QV^i_n = 0$, for $i = 1, ..., n$, the superlative index without error provides a solution. Under the assumption that the superlative index number with error gives a solution to the separability inequalities, (b) can be written as

[28] Fleissig and Whitney (2003) state that Barnett and Choi (1989) find that for this reason Varian (1983) fails to find weak separability even for data generated by Cobb-Douglas utility functions.

$$QV^i + QV_p^i - QV_n^i \leq QV^j + QV_p^j - QV_n^j + \mu^j p^j \left(y^i - y^j\right). \qquad (3.3)$$

In the next step the deviations around the superlative index QV^i are minimized by making QV_p^i and QV_n^i as small as possible. If there exists a superlative index number with error QV^{i*} and corresponding $\mu^i > 0$, that satisfy (3.3) then QV^{i*} can be regarded as the "group quantity index" and $1/\mu^i$ as the "group price index" for the separable goods y. These are used by Fleissig and Whitney (2003) to solve the separability conditions (iii) of Varian (1983). As a vector QV^{i*} might give a large range of values for μ^i that satisfy the separability conditions, the budget constraint from the separable y-data is used to find values for μ^i without restricting the solution. Taking the group quantity index QV^i and group price index $1/\mu^i$, it is required that

$$\left(1/\mu^i\right) QV^i = inc^{y,i},$$

where $inc^{y,i}$ is the expenditure on the y-goods in period i. See equations (3.1) and (3.2). Solving for μ^i gives

$$\mu^i = QV^i / inc^{y,i}. \qquad (3.4)$$

Hence, the aim is to keep μ^i the inverse of the group price index as close as possible to $QV^i/inc^{y,i}$ and thus to minimize deviations from adding up

$$\mu^i = QV^i / inc^{y,i} + \mu_p^i - \mu_n^i,$$

where μ_p^i are positive deviations and μ_n^i are negative deviations around $QV^i/inc^{y,i}$. When QV_p^i, QV_n^i, μ_p^i, μ_n^i are close to 0, then the superlative index (with some error) provides a solution to the Afriat inequalities with adding up (closely approximated). Fleissig and Whitney (2003) note that to find a solution to the problem, Varian's (1983) separability theorem requires the inverse of the group price index to be nonnegative. Additionally the group quantity indices with and without errors are required to be nonnegative to retain an economic interpretation of the solution.

Fleissig and Whitney (2003) use linear programming to find a solution to the problem described above. Under the constraints (3.3) and (3.4) as well as the respective nonnegativity constraints the objective is to minimize the deviations of QV^{i*} for the calculated superlative index QV^i and to minimize the deviations of $(1/\mu^i)\, QV^i$ from the expenditures on the y-separable goods. These equations can be represented in the form of a linear program (LP) model

$$\min \{c'x \; Ax \leq b, x \geq 0\},$$

where A is the coefficients matrix, c is the objective vector and b is the right-hand side vector. Note that this approach requires conversion of nonnegativity constraints to weak inequalities. The LP model by Fleissig and Whitney (2003) finds a solution to the Afriat inequalities, if it exists, by minimizing the deviations around QV^i and $QV^i/inc^{y,i}$.[29]

3.2.3 Data description

Aggregate time series data on four categories of goods are used for the study presented in this chapter. These categories cover data on consumption, money balances, corporate debt securities, and Treasury debt securities. The latter two are divided into two subclasses, namely short-term securities and long-term securities. In this chapter I use 474 monthly per capita observations from January 1969 to June 2008. The goods and assets which are subject to the testing procedures are labeled as:[30]

CnDUR: real average personal consumption of nondurables.
M: money balances, currency component of M1 plus demand deposits.
TrBi: holdings of Treasury bills.
TrBo: holdings of Treasury bonds**.**
CP: holdings of commercial paper, P1 rated.
CB: holdings of private sector issued bonds, Aaa rated.

The data series are deflated using a 2005 price index and are calculated as per capita values by dividing through total population. As pointed out by Swofford and Whitney (1987), the way per capita asset holdings are calculated here, might be subject to some criticism. The "household sector" in this study includes asset holdings by institutional investors, personal trusts and nonprofit organizations. These organizations probably hold little cash and a small amount of demand deposits but holdings of corporate debt securities and Treasuries are substantial. Further, deriving per capita data

[29] See Fleissig and Whitney (2003), pp. 135 - 136, 141.
[30] See Appendix B.1 for data and variables description.

by dividing through total population may also cause problems. Of course consumption is done by individuals under the age of eighteen acting on their own behalf. However, here the six year old is treated as an independent agent.

Securities' prices are derived from returns on Treasury bills, Treasury bonds, commercial paper, and corporate bonds. These are treated as holding period returns of zero coupon discount bonds. Returns on equity are derived from Standard and Poors' S&P 500 Index.

3.2.4 Nonparametric tests: results

First, the tests to check for the utility maximization hypothesis are performed. The data are not consistent with utility maximization if they violate GARP. The approach I pursue in this chapter, is to first check which grouping of goods can be rationalized by a well behaved utility function and then to test the feasible groupings for weak separability. I apply the proposed test for violations of GARP by Varian (1982) on the following groupings of goods:

A. $CnDUR, M, TrBi, CP,$
B. $CnDUR, M, TrBo, CB,$
C. $CnDUR, M, TrBi, TrBo.$

To keep the study presented in this chapter close to the framework of KVJ and NIE in each grouping only three liquid assets are included. Set A includes Treasury bills and commercial paper which have a rather short maturity length compared to the Treasury bonds and corporate bonds grouped in B. Analyzing set A separately from set B reflects the assumption that investors have a different valuation of short-term and long-term liquidity services. Set C captures the notion that an assets-in-the-utility specification might be suitable to explain a connection between changes in the slope of the yield curve and household demand behavior.

Results from testing A, B, and C for consistency with GARP are presented in Table 3.1. For grouping A, 97.5 percent of the data, for B, 96.7 percent of the data, and for C, 96.3 percent of the data satisfy GARP. As violations of GARP make up a very low share among the 474 observations, I regard the testing results for all groupings as not rejecting the utility maximization hypothesis for the data sample.[31] Hence, per capita data on nondurable consumption and money balances, combined with U.S. Treasury holdings and corporate debt security holdings can be regarded as rationalized by a well-behaved utility function. Violations of GARP among data on asset prices and asset holdings might be found in times when unforeseen price movements are strong enough to make the holding of a once preferable portfolio to suddenly contradict the utility maximization principle.[32]

With the maximization hypothesis being not rejected for sets A, B, and C, next is to test for weak separability using the procedure by Fleissig and Whitney (2003). Under the assumption that Treasury bills, Treasury bonds, commercial paper and corporate bonds yield nonpecuniary returns to the investor, such as liquidity services, I test the hypotheses whether the following sets of money and asset holdings can be regarded as an argument of investors' utility which is weakly separable from consumption:

$$U_t = U\left[CnDUR; V\left(M, TrBi, CP\right)\right], \tag{3.5}$$
$$U_t = U\left[CnDUR; V\left(M, TrBo, CB\right)\right], \tag{3.6}$$
$$U_t = U\left[CnDUR; V\left(M, TrBi, TrBo\right)\right]. \tag{3.7}$$

Results are presented in Table 3.2. For the case of assumed 1 percent measurement error, among all of the groupings (3.5), (3.6), and (3.7) no violations of weak separability conditions are found. For the case of 5 percent measurement error only for the most liquid grouping (3.5), no violations of the separability conditions are detected. Testing (3.6) at 5 percent measurement error finds violations of separability conditions for 28 percent of the observations, and for testing (3.7) at 5 percent measurement error the share of violations makes up 16 percent. However, I neglect the results for assumed 5 percent measurement errors as I regard U.S. data as being measured accurately enough for the purposes of the present study. Hence, for the purpose of the further empirical analysis I regard necessary and sufficient conditions for weakly separable utility maximization as being obtained for nondurables

[31] Varian (1991) proposes a statistical test for the size and number of violations of GARP. However, due to the very low share of violations in this chapter I refrain from further investigation.

[32] During the Great Moderation there might have been few situations where financial markets saw such strong price movements. However, events like the Oil crisis, the Volcker disinflation, or Black Monday 1987 might have had an effect on securities prices which was forceful enough.

consumption and money balances, along with liquidity services associated with Treasury bills and commercial paper holdings, Treasury bonds and corporate bonds holdings, and Treasury bills and Treasury bonds holdings. Further, note that the procedure by Fleissig and Whitney (2003) evaluates separability conditions by using a superlative index to aggregate quantities of separable goods. For the necessary and sufficient separability conditions being met, this implies that the underlying unknown aggregator function can be regarded as linearly homogeneous and twice-differentiable. Hence, for the second part of the empirical analysis it can be assumed that CES or Cobb-Douglas aggregator functions might be suitable candidates for the separate argument within the utility function which captures liquidity services.

Table 3.1: Violations of the utility maximization hypothesis

Goods category	(1)	(2)	(3)
CnDUR	X	X	X
M	X	X	X
TrBi	X		X
TrBo		X	X
CP	X		
CB		X	
Share of violations	0.025	0.033	0.037
Share consistent with GARP	0.975	0.967	0.963

Notes: This table presents results for testing sets A, B, and C for consistency with GARP.

Table 3.2: Test for weak separability

Goods category	(1)		(2)		(3)	
CnDUR	X		X		X	
M	X		X		X	
TrBi	X				X	
TrBo			X		X	
CP	X					
CB			X			
Measurement error	1%	5%	1%	5%	1%	5%
Share of violations	0	0	0	0.28	0	0.16
Share consistent with GARP/Sep	1	1	1	0.72	1	0.84

Notes: This table presents results for the Fleissig and Whitney (2003) test for weak separability.

3.3 Determination of the utility's parametric representation

With nonrejection of the hypotheses of utility maximization and weak separability, this chapter now turns to estimating preference parameters for a set of proposed utility functions. Nonparametric testing routines and evidence from KVJ and NIE support a liquidity services-in-the-utility formulation for empirical work on households' asset demand. For the further proceeds of the analysis utility functions are assumed which include as arguments both, consumption, and liquidity services while the latter capture non-pecuniary returns to the investor by holdings of a certain group of assets. Liquidity services are derived by an aggregator function which has holdings of money, Treasuries and corporate debt securities as arguments. It is required that those assets are not perfect substitutes in terms of utility. Otherwise there would be no need for an aggregator function - a simple sum aggregate would suffice.

3.3.1 Modified asset pricing model

Following KVJ and NIE assume that under the premise that investors value liquidity services a representative agent's utility function which shall fulfill the Inada conditions is of the form:

$$u_t = u\left(c_t, \nu\left(m_t, b_t, s_t\right)\right), \tag{3.8}$$

where c_t is the agent's consumption at date t and $\nu(\cdot)$ denotes the measure for liquidity services which is an (unknown) aggregator function of the real holdings of money m_t, Treasuries b_t, and corporate debt securities s_t. The liquidity services function $\nu(\cdot)$ is assumed to capture unique services provided by liquid assets which are valued by investors, where $\nu^{'}(\cdot) > 0$, and $\nu^{''}(\cdot) < 0$. The liquidity services function is concave as it is assumed that $\nu(\cdot)$ is increasing in m_t, b_t and s_t, but the marginal benefit derived from liquidity services is decreasing in m_t, b_t and s_t. Further, it has the property of $\lim_{m_t,b_t,s_t\to\infty} \nu^{'}(\cdot) = 0$. This reflects the assumption that holding more liquidity services providing assets reduces the marginal value of an extra unit of such an assets. Further, this marginal value approaches zero if the agent is holding a sufficiently large amount of liquidity services providing assets. Moreover, under the assumption that investors value liquidity, holding one more unit of an asset that is the most liquid should c.p. generate more utility than holding one more unit of the least liquid i.e.

$$\frac{\partial \nu(\cdot)}{\partial m_t} > \frac{\partial \nu(\cdot)}{\partial b_t} > \frac{\partial \nu(\cdot)}{\partial s_t}.$$

From the first order conditions of the household's utility maximization problem moment conditions for the GMM estimation are derived. These will be used in section 3.3.3 to estimate parameters of the implied demand functions. The representative household is further assumed to maximize the expected sum of a discounted stream of utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right), \tag{3.9}$$

subject to the budget constraint

$$P_t c_t + M_t + \frac{B_t}{R_t^b} + \frac{S_t}{R_t^s} + \frac{D_t}{R_t^d} \leq P_t y_t + M_{t-1} + B_{t-1} + S_{t-1} + D_{t-1}, \quad (3.10)$$

where E_0 is the expectation operator conditional on the information set in the initial period and $\beta \in (0,1)$, is the subjective discount factor. The price of one unit of consumption at date t is denoted by P_t. The household gains a real endowed income y_t and carries wealth into the next period by investing in nominal holdings of money M_t, Treasuries B_t, and corporate debt securities S_t. Nominal equity holdings D_t provide the numeraire asset in defining preferences. Further, the agent is assumed to hold only zero coupon discount bonds which pay out one unit of currency when being held to maturity. The gross returns on money, Treasuries, corporate debt securities, and equity are $R_t^m = 1$, R_t^b, R_t^s, and R_t^d. Maximizing the objective (3.9) subject to the budget constraint (3.10) leads for given initial values and non-negativity constraints to the following first order conditions for real consumption and real holdings of money, equity, Treasuries, and corporate debt securities:

$$\frac{\partial u}{\partial c_t} = \lambda_t, \quad (3.11)$$

$$\frac{\partial u}{\partial m_t} + \beta E_t \left[\frac{P_t \lambda_{t+1}}{P_{t+1}} \right] = \lambda_t, \quad (3.12)$$

$$\beta E_t \left[\frac{P_t \lambda_{t+1}}{P_{t+1}} \right] = \frac{\lambda_t}{R_t^d}, \quad (3.13)$$

$$\frac{\partial u}{\partial b_t} + \beta E_t \left[\frac{P_t \lambda_{t+1}}{P_{t+1}} \right] = \frac{\lambda_t}{R_t^b}, \quad (3.14)$$

$$\frac{\partial u}{\partial s_t} + \beta E_t \left[\frac{P_t \lambda_{t+1}}{P_{t+1}} \right] = \frac{\lambda_t}{R_t^s}, \quad (3.15)$$

and (3.10) holding with equality and the accordant transversality conditions.[33] Define the stochastic discount factor for nominal payoffs as $M_{t+1} = \beta \frac{\partial u / \partial c_{t+1}}{\partial u / \partial c_t} \frac{P_t}{P_{t+1}}$, so that from (3.12), (3.14), and (3.15) the optimality conditions for holdings of money, Treasuries, and corporate debt securities can

[33] The transversality conditions for holdings of Treasuries, corporate debt securities, and equity are given by: $\lim_{j\to\infty} \beta^j E_t \left(\lambda_{t+j} b_{t+j} / R_{t+j}^b \right) = 0$, $\lim_{j\to\infty} \beta^j E_t \left(\lambda_{t+j} s_{t+j} / R_{t+j}^s \right) = 0$, and $\lim_{j\to\infty} \beta^j E_t \left(\lambda_{t+j} d_{t+j} / R_{t+j}^d \right) = 0$.

be derived which can be interpreted as pricing equations:

$$\frac{\partial u}{\partial m_t} / \frac{\partial u}{\partial c_t} + E_t\left[M_{t+1}\right] = 1, \tag{3.16}$$

$$\frac{\partial u}{\partial b_t} / \frac{\partial u}{\partial c_t} + E_t\left[M_{t+1}\right] = \frac{1}{R_t^b}, \tag{3.17}$$

$$\frac{\partial u}{\partial s_t} / \frac{\partial u}{\partial c_t} + E_t\left[M_{t+1}\right] = \frac{1}{R_t^s}. \tag{3.18}$$

The first term on the left hand sides of equations (3.16), (3.17), and (3.18) captures the modification of the standard asset pricing model by the assumption of liquidity services.[34] The marginal utility from holding money m_t, Treasuries b_t, and corporate bonds s_t induces a liquidity services premium on each asset's price. Increasing the investors' holdings of m_t, b_t, and s_t should decrease liquid assets' prices which is due to the assumption of $\nu(\cdot)$ being concave. Hence, equations (3.16) - (3.18) reflect that under the assumption of liquidity services being an argument of the investor's utility function, increasing the amount of liquid assets held will lower the investor's willingness to pay for another unit of such assets.

Note that the assumed functional form of the aggregator $\nu(\cdot)$ is crucial. KVJ and NIE do not fully specify the functional form of $\nu(\cdot)$ but implicitly define a set of requirements to it. It can be shown that employing a CES aggregator nested in a CRRA utility would match those requirements.[35] Specifically, in this case each asset's liquidity services premium is not only driven by the level of holdings of the respective asset, but in addition to that, it is driven by the total holdings of liquidity services providing assets. Moreover, by assuming $\frac{\partial \nu(\cdot)}{\partial m_t} > \frac{\partial \nu(\cdot)}{\partial b_t} > \frac{\partial \nu(\cdot)}{\partial s_t}$, increasing the holdings of m_t should c.p. decrease asset prices to a larger extent than increasing the holdings of b_t and s_t. Analogously, the same applies c.p. for increasing the holdings of b_t compared to increasing holdings of s_t. This requirement can be fulfilled by making use of the CES aggregator. In contrast to that, employing a Cobb-Douglas aggregator nested in a CRRA utility implies that each asset's liquidity services premium is driven by the level of holdings of the respective asset but not by the total holdings of liquidity services providing assets. E.g. the liquidity premium on R_t^s would be a function of s_t, and not of m_t and b_t. This implication however, is not in line with

[34] Please note that for simplicity default risk is neglected.

[35] See Appendix B.2.

KVJ and NIE. Further, assuming additive separability between the utility function's arguments of consumption and liquidity services, as well yields implications which are not in line with the investors' behavior observed by KVJ and NIE. Specifically, for the case of a CES aggregator together with additive separability, each asset's liquidity services premium would not be a decreasing function in total holdings of liquid asset. For the Cobb-Douglas aggregator, the liquidity premium would be decreasing in holdings of the asset under consideration but increasing in overall asset holdings. Note that the CES function degenerates to a Cobb-Douglas aggregator if the elasticity of substitution is unity. One could estimate the model under a parameter restriction on the elasticity of substitution. However, the present chapter's approach is to first estimate a broad variety of possible model specifications and then to select the most suitable one.

3.3.2 Moment conditions

Set (3.13) equal to (3.12), (3.14), and (3.15). Then plug in (3.11) for the shadow price of income $\lambda_t \geq 0$, then

$$1 = E_t \left[\frac{\partial u}{\partial c_t} - \beta \frac{P_t \left(1 + r_t^d\right)}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right], \tag{3.19}$$

$$0 = E_t \left[\frac{\partial u}{\partial m_t} - \beta \frac{P_t r_t^d}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right], \tag{3.20}$$

$$0 = E_t \left[\frac{\partial u}{\partial b_t} - \beta \frac{P_t \left(r_t^d - r_t^b\right)}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right], \tag{3.21}$$

$$0 = E_t \left[\frac{\partial u}{\partial s_t} - \beta \frac{P_t \left(r_t^d - r_t^s\right)}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right], \tag{3.22}$$

which yields Euler equations and implied demand functions for consumption (3.19) money (3.20), Treasuries (3.21), and corporate debt securities (3.22). Here I already used the representation of the equations as conditional moment conditions. Equations (3.19) - (3.22) are now written in terms of excess returns. Note that r_t^d, r_t^b, and r_t^s denote net returns on equity, Treasuries, and corporate debt securities. The Euler equation for consumption has the well known interpretation like in the standard case without assets in the utility. The Euler equation for money holdings (3.20) requires that in equilibrium utility cannot be increased by holding one unit of money less at time t, investing it in equities, and consuming the payoff at time $t+1$. The forgone utility associated with a one unit reduction in money holdings is $\frac{\partial u}{\partial m_t}\frac{1}{P_t}$. Transferring one unit of money to equities at time t increases real wealth at $t+1$ by r_t^d, since money yields no nominal return while equity does. The gain in utility if these higher proceeds are consumed in period $t+1$ is $\beta E_t\left[\frac{r_t^d}{P_{t+1}}\frac{\partial u}{\partial c_{t+1}}\right]$. Equating this to foregone utility yields (3.20). Analogously Euler equations (3.21) and (3.22) equate the costs and benefits in terms of utility of transferring one unit of currency from Treasuries or corporate debt securities into equities for one period.

3.3.3 GMM estimation models

Methodologically, this chapter follows previous authors like Holman (1998). For each proposed specification of utility function (3.8) correspondent sets of Euler equations are derived from (3.19) to (3.22) which are then estimated by using GMM. The Euler equations state that in equilibrium the representative agent's expectations are orthogonal to all of the variables within the information set at the time predictions are made. Specifically, they imply population orthogonality conditions which are a function of the observed data and the preference parameters. The GMM estimator is a nonlinear instrumental variable estimator of the population parameters that tries to make the sample orthogonality conditions close to zero by minimizing a distance function.[36] According to Verbeek (2000) there are several advantages of this method when estimating asset pricing models. One is that GMM does not require distributional assumptions or assumptions regarding data generating processes. Further it can allow for heteroskedasticity of unknown form which is a convenient feature when working with data on asset returns. Importantly it can estimate parameters even if the model cannot be solved analytically from the first order conditions. This is especially useful for the present asset pricing models which are comprised of the nonlinear Euler equations (3.19) to (3.22).

I use the same time series data on consumption and holdings of money, Treasuries and corporate debt securities, as well as the same data on prices and returns as in Section 3.2.3 of this chapter. Further, three different sets of instruments are employed, depending on whether Treasury bills and commercial paper holdings, Treasury bills and Treasury bond holdings, or

[36] Hansen (1982) provides the conditions under which the GMM estimator is consistent, asymptotically normal, and efficient.

Treasury bond and corporate bond holdings, are arguments of (3.8). Following the approach of Hall (2005) each set of instruments includes a constant term, two lagged values of the real returns on equity, and the real returns on the correspondent assets included in each of the estimation models, as well as the past two growth rates of real per capita consumption and real per capita asset holdings. The reason for this choice is that lagged values and lagged growth rates can be assumed to be uncorrelated with current innovations.

Note that there are more instruments than parameters. Hence, the system of Euler equations is overidentified. The J-test of overidentifying restrictions by Hansen (1982) and Hansen and Singleton (1982) is used to conduct a joint test of the specification of the asset pricing model and the validity of the instrument set.

The following six specifications of investor's utility are proposed:

I. Poterba Rotemberg Utility: Poterba and Rotemberg (1986) use a nested CES preferences specification:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t^{\alpha} L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)^{1-\alpha}\right\}^{\sigma},$$

where L_t captures liquidity services derived from a CES aggregator function:

$$L_t = \left[\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma} + \delta_B\left(\frac{B_t}{P_t}\right)^{\gamma} + (1-\delta_M-\delta_B)\left(\frac{S_t}{P_t}\right)^{\gamma}\right]^{\frac{1}{\gamma}}.$$

This utility function exhibits constant relative risk aversion. Further, utility is Cobb-Douglas in consumption and liquidity services, ensuring that more consumption raises the marginal utility of liquidity services and vice versa. The liquidity measure is a CES function of real money balances, Treasuries holdings and corporate debt securities holdings. It must be pointed out that these preferences are quite restrictive. In particular, they impose homogeneity and require separability between its arguments. Further, the elasticity of substitution between consumption and liquidity services is assumed to be equal to one.

With these preferences, from equations (3.19) to (3.22) the following moment conditions can be derived:

$$1 = E_t\left[\beta\frac{P_t\left(1+r_t^d\right)}{P_{t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma\alpha-1}\left(\frac{L_{t+1}}{L_t}\right)^{\sigma(1-\alpha)}\right],$$

$$0 = E_t\left[\begin{array}{c} c_t^{\sigma\alpha}L_t^{\sigma(1-\alpha)-1}\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma-1} \\ -\frac{\alpha\beta}{1-\alpha}\frac{P_t r_t^d}{P_{t+1}}c_{t+1}^{\sigma\alpha-1}L_t^{\sigma(1-\alpha)} \end{array}\right],$$

$$0 = E_t\left[\begin{array}{c} c_t^{\sigma\alpha}L_t^{\sigma(1-\alpha)-1}\delta_B\left(\frac{B_t}{P_t}\right)^{\gamma-1} \\ -\frac{\alpha\beta}{1-\alpha}\frac{P_t\left(r_t^d-r_t^b\right)}{P_{t+1}}c_{t+1}^{\sigma\alpha-1}L_t^{\sigma(1-\alpha)} \end{array}\right],$$

$$0 = E_t\left[\begin{array}{c} c_t^{\sigma\alpha}L_t^{\sigma(1-\alpha)-1}\left(1-\delta_M-\delta_B\right)\left(\frac{S_t}{P_t}\right)^{\gamma-1} \\ -\frac{\alpha\beta}{1-\alpha}\frac{P_t\left(r_t^d-r_t^s\right)}{P_{t+1}}c_{t+1}^{\sigma\alpha-1}L_t^{\sigma(1-\alpha)} \end{array}\right].$$

I report estimates of the parameters $\{\sigma, \beta, \alpha, \gamma, \delta_M, \delta_B\}$ in Section 3.3.4. For a first round of estimations I constrain the utility function parameters. I require that δ_M, δ_B, and $(1-\delta_M-\delta_B)$ be positive and sum up to one. Further, I require α, β, and γ to be positive between zero and one, and σ to be less than zero. The validity of the model and the restrictions are checked by using the J-test of overidentifying restrictions. The constraints are successively relaxed if the J-test rejects the model together with the restrictions in place. If α was equal to one, convenience yields and liquidity services can not be regarded as a source of utility. Note that if γ was estimated to be exactly zero, L_t would degenerate to a Cobb-Douglas aggregation. If the elasticity of substitution γ is close to one, linear aggregation would be implied. In the latter case money, Treasuries, and corporate debt securities would be close substitutes in terms of utility.

II. Nested CES Liquidity Services: In contrast to Poterba and Rotemberg (1986), in this case utility is not Cobb-Douglas in consumption and liquidity services:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right\}^{\sigma},$$

where L_t captures liquidity services derived by the CES aggregator function

$$L_t = \left[\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma} + \delta_B\left(\frac{B_t}{P_t}\right)^{\gamma} + \left(1-\delta_M-\delta_B\right)\left(\frac{S_t}{P_t}\right)^{\gamma}\right]^{\frac{1}{\gamma}}.$$

This utility function as well exhibits constant relative risk aversion. These preferences are less restrictive compared to Poterba and Rotemberg (1986) as a unitary elasticity of substitution between consumption and liquid assets is not demanded. Following (3.19) to (3.22) moment conditions are then given by

$$1 = E_t\left[\beta\frac{P_t\left(1+r_t^d\right)}{P_{t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma-1}\left(\frac{L_{t+1}}{L_t}\right)^{\sigma}\right],$$

$$0 = E_t\left[c_t^{\sigma}L_t^{\sigma-1}\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma-1} - \beta\frac{P_t r_t^d}{P_{t+1}}c_{t+1}^{\sigma-1}L_t^{\sigma}\right],$$

$$0 = E_t\left[c_t^{\sigma}L_t^{\sigma-1}\delta_B\left(\frac{B_t}{P_t}\right)^{\gamma-1} - \beta\frac{P_t\left(r_t^d - r_t^b\right)}{P_{t+1}}c_{t+1}^{\sigma-1}L_t^{\sigma}\right],$$

$$0 = E_t\left[c_t^{\sigma}L_t^{\sigma-1}\left(1-\delta_M-\delta_B\right)\left(\frac{S_t}{P_t}\right)^{\gamma-1} - \beta\frac{P_t\left(r_t^d - r_t^s\right)}{P_{t+1}}c_{t+1}^{\sigma-1}L_t^{\sigma}\right].$$

I report estimates of the parameters $\{\sigma, \beta, \gamma, \delta_M, \delta_B\}$ in Section 3.3.4. For the estimation I constrain the utility function parameters. I require that δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ be positive and sum up to one. Further, I require β and γ to be positive and between zero and one, and σ to be less than zero.

III. Nested CES Liquidity Services, additively separable: Utility is not Cobb-Douglas in consumption and liquidity services but assumed to be additively separable in its arguments:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t + L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right\}^{\sigma},$$

where L_t captures liquidity services derived by the CES aggregator function

$$L_t = \left[\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma} + \delta_B\left(\frac{B_t}{P_t}\right)^{\gamma} + (1-\delta_M-\delta_B)\left(\frac{S_t}{P_t}\right)^{\gamma}\right]^{\frac{1}{\gamma}}.$$

Here equations (3.19) to (3.22) become

$$
\begin{aligned}
1 &= E_t \left[\beta \frac{P_t \left(1 + r_t^d\right)}{P_{t+1}} \left(\frac{c_{t+1+L_{t+1}}}{c_t + L_t} \right)^{\sigma - 1} \right], \\
0 &= E_t \left[\begin{array}{c} \left(c_t + L_t\right)^{\sigma-1} L_t^{\frac{1}{\gamma}-1} \delta_M \left(\frac{M_t}{P_t} \right)^{\gamma - 1} \\ -\beta \frac{P_t r_t^d}{P_{t+1}} \left(c_{t+1+L_{t+1}} \right)^{\sigma-1} \end{array} \right], \\
0 &= E_t \left[\begin{array}{c} \left(c_t + L_t\right)^{\sigma-1} L_t^{\frac{1}{\gamma}-1} \delta_B \left(\frac{B_t}{P_t} \right)^{\gamma - 1} \\ -\beta \frac{P_t \left(r_t^d - r_t^b\right)}{P_{t+1}} \left(c_{t+1+L_{t+1}} \right)^{\sigma-1} \end{array} \right], \\
0 &= E_t \left[\begin{array}{c} \left(c_t + L_t\right)^{\sigma-1} L_t^{\frac{1}{\gamma}-1} \left(1 - \delta_M - \delta_B\right) \left(\frac{S_t}{P_t} \right)^{\gamma - 1} \\ -\beta \frac{P_t \left(r_t^d - r_t^b\right)}{P_{t+1}} \left(c_{t+1+L_{t+1}} \right)^{\sigma-1} \end{array} \right].
\end{aligned}
$$

I report estimates of the parameters $\{\sigma, \beta, \gamma, \delta_M, \delta_B\}$ in section 3.3.4. For the estimation I constrain the utility function parameters. I require that δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ be positive and sum up to one. Further, I require β and γ to be positive between zero and one, and σ to be less than zero.

IV. Cobb-Douglas Utility with Cobb-Douglas liquidity services: Utility is assumed to be Cobb-Douglas in consumption and liquidity services.

$$
u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma} \left\{ c_t^{\alpha} L_t \left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)^{1-\alpha} \right\}^{\sigma},
$$

where L_t denotes liquidity services which are as well derived by a Cobb-Douglas aggregator function

$$
L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)}.
$$

Again these preferences are quite restrictive. The utility function exhibits constant relative risk aversion. It imposes homogeneity and separability. Further, a unitary elasticity of substitution between consumption and liquidity services is assumed.

From (3.19) to (3.22) moment conditions are then given by

$$1 = E_t \left[\beta \frac{P_t \left(1 + r_t^d\right)}{P_{t+1}} \left(\frac{c_{t+1}}{c_t}\right)^{\sigma\alpha - 1} \left(\frac{L_{t+1}}{L_t}\right)^{\sigma(1-\alpha)} \right],$$

$$0 = E_t \left[\begin{array}{c} c_t^{\sigma\alpha} \delta_M \left(\frac{M_t}{P_t}\right)^{\delta_M \sigma(1-\alpha)-1} \left(\frac{B_t}{P_t}\right)^{\delta_B \sigma(1-\alpha)} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)\sigma(1-\alpha)} \\ -\frac{\alpha\beta}{1-\alpha} \frac{P_t r_t^d}{P_{t+1}} c_{t+1}^{\sigma\alpha-1} L_t^{\sigma(1-\alpha)} \end{array} \right],$$

$$0 = E_t \left[\begin{array}{c} c_t^{\sigma\alpha} \left(\frac{M_t}{P_t}\right)^{\delta_M \sigma(1-\alpha)} \delta_B \left(\frac{B_t}{P_t}\right)^{\delta_B \sigma(1-\alpha)-1} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)\sigma(1-\alpha)} \\ -\frac{\alpha\beta}{1-\alpha} \frac{P_t\left(r_t^d - r_t^b\right)}{P_{t+1}} c_{t+1}^{\sigma\alpha-1} L_t^{\sigma(1-\alpha)} \end{array} \right],$$

$$0 = E_t \left[\begin{array}{c} c_t^{\sigma\alpha} \left(\frac{M_t}{P_t}\right)^{\delta_M \sigma(1-\alpha)} \left(\frac{B_t}{P_t}\right)^{\delta_B \sigma(1-\alpha)} (1 - \delta_M - \delta_B) \left(\frac{S_t}{P_t}\right)^{\substack{(1-\delta_M-\delta_B) \\ \cdot\sigma(1-\alpha)-1}} \\ -\frac{\alpha\beta}{1-\alpha} \frac{P_t\left(r_t^d - r_t^s\right)}{P_{t+1}} c_{t+1}^{\sigma\alpha-1} L_t^{\sigma(1-\alpha)} \end{array} \right].$$

I report estimates of the parameters $\{\sigma, \beta, \alpha, \delta_M, \delta_B\}$ in Section 3.3.4. For the estimation I constrain the utility function parameters. I require δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ to be positive and sum up to one, and α and β to be positive between zero and one, and σ to be less than zero.

V. Nested Cobb-Douglas Liquidity Services: Utility is not Cobb-Douglas in consumption and liquidity services

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right\}^{\sigma},$$

where L_t captures liquidity services derived from a Cobb-Douglas aggregator function

$$L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)}.$$

Then equations (3.19) to (3.22) imply

$$1 = E_t\left[\beta\frac{P_t\left(1+r_t^d\right)}{P_{t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma-1}\left(\frac{L_{t+1}}{L_t}\right)^{\sigma}\right],$$

$$0 = E_t\left[\begin{array}{c} c_t^{\sigma}\delta_M\left(\frac{M_t}{P_t}\right)^{\delta_M\sigma-1}\left(\frac{B_t}{P_t}\right)^{\delta_B\sigma}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)\sigma} \\ -\beta\frac{P_t r_t^d}{P_{t+1}}c_{t+1}^{\sigma-1}L_t^{\sigma}\end{array}\right],$$

$$0 = E_t\left[\begin{array}{c} c_t^{\sigma}\left(\frac{M_t}{P_t}\right)^{\delta_M\sigma}\delta_B\left(\frac{B_t}{P_t}\right)^{\delta_B\sigma-1}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)\sigma} \\ -\beta\frac{P_t\left(r_t^d-r_t^b\right)}{P_{t+1}}c_{t+1}^{\sigma-1}L_t^{\sigma}\end{array}\right],$$

$$0 = E_t\left[\begin{array}{c} c_t^{\sigma\alpha}\left(\frac{M_t}{P_t}\right)^{\delta_M\sigma}\left(\frac{B_t}{P_t}\right)^{\delta_B\sigma}(1-\delta_M-\delta_B)\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)\sigma-1} \\ -\beta\frac{P_t\left(r_t^d-r_t^s\right)}{P_{t+1}}c_{t+1}^{\sigma-1}L_t^{\sigma}\end{array}\right].$$

I report estimates of the parameters $\{\sigma, \beta, \delta_M, \delta_B\}$ in Section 3.3.4. For the estimation I constrain the utility function parameters. I require that δ_M, δ_B, and $(1-\delta_M-\delta_B)$ be positive and sum up to one. Further, I require β to be positive between zero and one, and σ to be less than zero.

VI. Nested Cobb-Douglas Liquidity Services, additively separable: Utility is additively separable in consumption and liquidity services:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t + L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right\}^{\sigma},$$

where L_t captures liquidity services derived from a Cobb-Douglas aggregator function

$$L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M}\left(\frac{B_t}{P_t}\right)^{\delta_B}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)}.$$

This utility function exhibits constant relative risk aversion. Equations (3.19) to (3.22) imply the following moment conditions

$$1 = E_t \left[\beta \frac{P_t \left(1 + r_t^d\right)}{P_{t+1}} \left(\frac{c_{t+1+L_{t+1}}}{c_t + L_t} \right)^{\sigma-1} \right],$$

$$0 = E_t \left[\begin{array}{c} (c_t + L_t)^{\sigma-1} \delta_M \left(\frac{M_t}{P_t}\right)^{\delta_M - 1} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)} \\ -\beta \frac{P_t r_t^d}{P_{t+1}} \left(c_{t+1+L_{t+1}}\right)^{\sigma-1} \end{array} \right],$$

$$0 = E_t \left[\begin{array}{c} (c_t + L_t)^{\sigma-1} \delta_B \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B - 1} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)} \\ -\beta \frac{P_t\left(r_t^d - r_t^b\right)}{P_{t+1}} \left(c_{t+1+L_{t+1}}\right)^{\sigma-1} \end{array} \right],$$

$$0 = E_t \left[\begin{array}{c} (c_t + L_t)^{\sigma-1} (1 - \delta_M - \delta_B) \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)-1} \\ -\beta \frac{P_t\left(r_t^d - r_t^b\right)}{P_{t+1}} \left(c_{t+1+L_{t+1}}\right)^{\sigma-1} \end{array} \right].$$

I report estimates of the parameters $\{\sigma, \beta, \delta_M, \delta_B\}$ in Section 3.3.4. For the estimation I constrain the utility function parameters. I require that δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ be positive and sum up to one. Further, I require β to be positive between zero and one, and σ to be less than zero.

3.3.4 GMM estimation results

In the following subsections of this chapter GMM estimation results are discussed for the moment condition sets I to VI. For each Table presenting estimation results, in the first column estimates are shown for the dataset including nondurables consumption, money balances, and data on returns and holdings of Treasury bills and commercial paper. Columns 2 and 3 present results for the sets including data on Treasury bills and Treasury bonds, and on Treasury bonds and corporate bonds.

I. Poterba Rotemberg Utility: Table 3.3 presents the estimates of the parameters from specification I. with corresponding moment conditions. Three sets of estimates corresponding to each dataset are reported. For all of the three data sets the J-test of overidentifying restrictions indicates rejection of model I. at the 5 percent level with all constraints on σ, β, α, γ, δ_M, and δ_B in place. The model is rejected as well at the 5 percent level for relaxing the restriction on β and subsequently relaxing the restriction on γ.

The model is not rejected at the 5 percent significance level if the restrictions on σ, δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ are in place for all data sets. The estimated coefficient values are remarkably similar across the three model estimations. Further, in the fourth column I report results from Poterba and Rotemberg (1986).[37] Note that in this case liquidity services are derived from a different set of assets. For their estimation the authors use quarterly data from 1959:Q1 to 1981:Q3 on nondurables consumption, money, time deposits, and Treasury bills. Still, it is notable that estimation results of the present study for the utility specification I. are close to the results of Poterba and Rotemberg (1986). This is found in spite of employing different data sets, different instrument sets, and a different data frequency. For the present model the intertemporal elasticity of substitution σ is expected to be larger than zero in absolute terms. If $\sigma = 0$, then the nested function degenerates to a logarithmic function of consumption and liquidity services. Poterba and Rotemberg (1986) estimate a range of σ between -6.5 and -5.6, whereas the present study finds a range between -7.7 and -6.0. Across all three datasets the estimated discount rate β is greater than zero and below one. However, unity is not excluded from the 95 percent confidence intervals in columns 1 and 2. Still the point estimates are slightly smaller than those which are found in other studies.[38] The estimated share of expenditures devoted to consumption α is for all estimations significantly greater than zero at the 1 percent level and lies between 0.68 and 1.07. For the estimations presented in columns 1 and 2 of Table 3.3, results for α suggest that liquidity services are not a direct source of utility. Only for the estimation model including data on Treasury bonds and corporate bonds, the size of α implies that convenience yield is a source of utility. However, the size of the point estimate suggests that an implausibly large share of households expenditures is devoted to Treasury bond and corporate bond holdings. The main backdraw of the study presented in this chapter is that the nonparametric testing procedures provide little guidance about the functional form of the household's utility and about the complete set of liquidity services providing assets. Hence, the three data groupings considered for this study

[37] Poterba and Rotemberg (1986) further provide estimation results for tax adjusted data and different sets of instruments. In Table 3.3 results are taken from an estimation which is closest to the setting which is analyzed in this chapter.

[38] Hansen and Singleton (1983) estimate that β lies between 0.995 and 1.096. Poterba and Rotemberg (1986) find that for their utility specification the discount factor is larger than unity.

might leave out further assets which yield liquidity services.[39] The inverse of the elasticity of substitution between liquidity services yielding assets γ is significantly larger than zero and lies between 0.5 and 0.58. If γ was estimated to be 1 a linear aggregator function would be implied. Assets would be one for one substitutes in terms of utility. If γ was equal to zero, the aggregator would reduce to the Cobb-Douglas specification. The hypothesis that $\gamma = 1$ is rejected for all three estimations. Within the convenience yield aggregator the coefficients δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ are estimated with relatively wide confidence intervals. Further, for the estimation model presented in the second column of Table 3.3, δ_B is not significantly different from zero at the 5 percent level. However, for all three estimations the following pattern of point estimates is observed: $\delta_M > \delta_B > (1 - \delta_M - \delta_B)$. Assuming that real asset holdings were of equal size, this result implies that marginal utility of another unit of real money balances would exceed that from another unit of Treasuries or corporate debt securities.

II. Nested CES Liquidity services The estimates of the parameters from specification II are shown in Table 3.4. Notably the J-test only does not reject the validity of the model which is estimated for data on Treasury bills and Treasury bonds. For this estimation result presented here, only the restrictions on σ, δ_M, δ_B, and $(1 - \delta_M - \delta_B)$ are in place. Note that the estimated share of expenditures devoted to consumption α is larger that unity implying that liquidity services are not a direct source of utility. The estimation model including data on Treasury bills and commercial paper as well as the estimation model including data on Treasury bonds and corporate bonds are rejected. Further, point estimates for the three datasets are not as similar among each other as for utility specification I. The most striking difference is that estimates for σ range between -3.19 and -11.83 and estimates for γ range between 0.09 and 1.00, implying perfect substitutability of liquid assets for the latter case. Specification II. compared to specification I. does not require the elasticity of substitution between consumption and the liquid assets' aggregate to be unity. As this requirement seems to be quite restrictive and there is no theoretical guidance about the size of the elasticity of substitution for this model, one would not a priori expect that estimating model II yields J-tests of overidentifying restrictions which indicate rejection of two of the three estimation models.

[39] Note that for the study presented in this chapter three datasets were employed which are intended to closely match the datasets used in KVJ and NIE.

III. - VI. Specifications The GMM estimation routine does not find a solution for the minimization problem. This is found for all of the three different data sets with the corresponding instrument sets.

3.4 Conclusion

This paper presents a model of aggregate demand for consumption, money balances, U.S. Treasuries, and corporate debt securities where the asset holdings directly contribute to the investors' utility. The first part of the analysis applies Varian's (1982) nonparametric testing procedure on monthly per capita data on nondurable consumption, money balances, U.S. Treasuries, and corporate debt securities. As violations of GARP can only be detected for a very low share of the observations, results for the nonparametric testing routines can be seen as not rejecting the utility maximization hypothesis. Therefore, all of the three groupings can be regarded as being rationalized by a well-behaved utility function. Additionally, necessary and sufficient conditions for weakly separable utility maximization are obtained for monthly per capita data on nondurables consumption and money balances, along with liquidity services derived from Treasury holdings and corporate debt securities holdings. In the second part of the analysis Euler equations implied by the modified asset-pricing model are estimated under alternative utility specifications. Surprisingly, only the restrictive utility specification proposed by Poterba and Rotemberg (1986) yields parameter estimates which are relatively robust to the choice of data. Estimation results however, imply parameter values which indicate misspecification.

In the presence of many assets which might provide liquidity services, a more complete modelling of the financial sector is needed. The paper makes a step in that direction. However, the analysis suffers from several shortcomings. These are primarily limitations of the particular functional form and parameterization of the utility function and the data choice. Eventually, the approach should be extended to incorporate a broader range of assets. However, the nonparametric testing routines provide little guidance about the true functional form and the true set of liquidity services providing assets. A second issue is that the menu of important assets changes over time as Poterba and Rotemberg (1986) noted before. E.g. financial innovations like the increasing importance of money market mutual funds allow assets to be repackaged to yield different degree of liquidity services.

Table 3.3: Parameter estimates of specification I. Poterba and Rotemberg (1986) utility

Period	Jan 1969 - Jun 2008			Q1 1959 - Q3 1981
	(1)’	(2)’	(3)’	Poterba and Rotemberg (1986)
Parameter	M, TrBi, CP	M, TrBi, TrBo	M, TrBo, CB	Money, STD, TrBi
σ	-6.044**	-6.743**	-7.734**	-6.091
	$[-6.956; -5.934]$	$[-7.576; -5.912]$	$[-7.876; -7.590]$	
β	0.999**	0.999**	0.997**	1.007
	[0.997; 1.011]	[0.997; 1.001]	[0.995; 0.998]	
γ	0.499**	0.578**	0.565**	0.269
	[0.446; 0.552]	[0.536; 0.620]	[0.556; 0.574]	
α	1.074**	1.046**	0.681**	0.965
	[1.071; 1.077]	[1.043; 1.049]	[0.679; 0.682]	
δ_M	0.538**	0.532**	0.496**	0.316
	[0.405; 0.670]	[0.454; 0.611]	[0.437; 0.554]	
δ_B	0.305*	0.307	0.429**	0.515
	[0.122; 0.427]	[0.055; 0.559]	[0.355; 0.503]	
$1 - \delta_M - \delta_B$	0.157*	0.160	0.075**	0.168
J-Test (p-val)	0.426	0.381	0.380	
N	474	474	474	109

Notes: ’ indicates that only restrictions on σ, δ_M, δ_B, and $1 - \delta_M - \delta_B$ are in place.
** Significant at the 1 percent level.
* Significant at the 5 percent level.
Confidence intervals are provided within the brackets.
STD denotes "Small Time Deposits".

Table 3.4: Parameter estimates of specification II. nested CES liquidity services

Period	Jan 1969 - Jun 2008		
	(1)	(2)'	(3)
Parameter	M, TrBi, CP	M, TrBi, TrBo	M, TrBo, CB
σ	-3.189**	-11.831**	-4.838**
	$[-3.197; -3.180]$	$[-11.853; -11.812]$	$[-4.857; -4.819]$
β	0.995**	1.007**	1.008**
	$[0.993; 0.996]$	$[0.999; 1.014]$	$[1.005; 1.012]$
γ	1.001**	0.090**	0.478**
	$[1.001; 1.001]$	$[0.089; 0.090]$	$[0.478; 0.478]$
α	1.074**	1.046**	0.681**
	$[1.071; 1.077]$	$[1.043; 1.049]$	$[0.679; 0.682]$
δ_M	0.743**	0.926**	0.591**
	$[0.707; 0.781]$	$[0.888; 0.964]$	$[0.531; 0.651]$
δ_B	0.256**	0.066	0.292**
	$[0.248; 0.289]$	$[.0.041; 0.173]$	$[0.263; 0.320]$
$1 - \delta_M - \delta_B$	0*	0.008	0.117**
J-Test (p-val)	0.038	0.073	0.01
N	474	474	474

Notes: ' indicates that only restrictions on σ, δ_M, δ_B, and $1 - \delta_M - \delta_B$ are in place.
** Significant at the 1 percent level.
* Significant at the 5 percent level.
Confidence intervals are provided within the brackets.

Chapter 4

The Effects of Large-Scale Asset Purchases in an Estimated DSGE Model of the U.S. Economy

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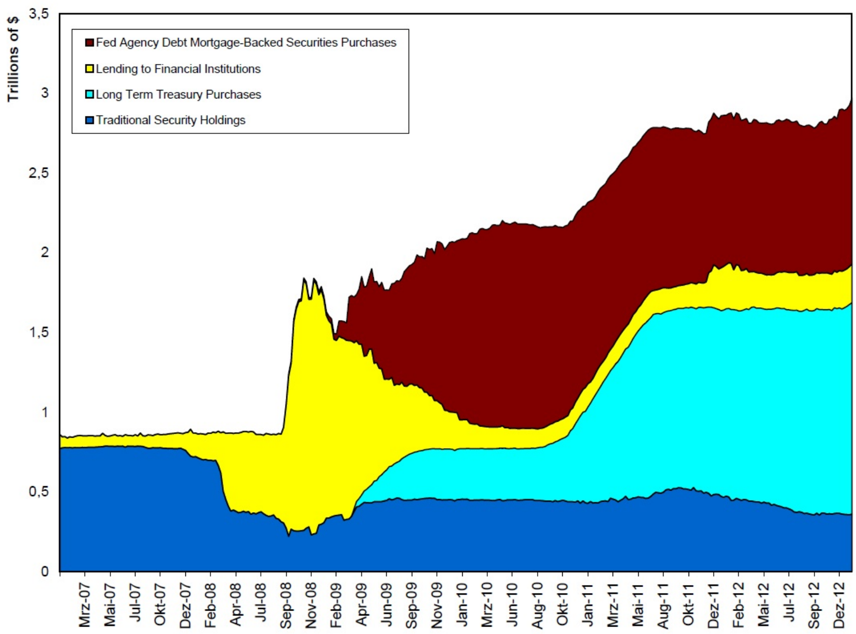
4.1 Introduction

In response to the 2008/2009 financial crisis the U.S. Federal Reserve (Fed) conducted monetary policy operations that go beyond the standard interest rate policies. Besides providing guidance about the likely future path of key interest rates and the set up of new lending facilities, the Fed most prominently embarked on quantitative easing measures which were implemented by large-scale asset purchase programs (LSAP). By the means of these unconventional policy measures the Federal Reserve System's holdings of domestic securities increased to approximately $2.6 trillion until the end of 2012 (see Figure 4.1). To a large extent these increases were caused by the Fed's LSAP 1 and LSAP 2 programs. LSAP 1 embodied purchases of mortgage-backed securities (MBS), agency securities, and U.S. Treasury securities between late 2008 and early 2010.[40] In contrast to that LSAP 2

[40] Further, reinvestment of principal payments from agency debt and MBS into longer-term securities took place to keep constant the Fed's holdings of securities at their current level.

Figure 4.1: Federal Reserve's asset holdings



Notes: The Federal Reserve System's holdings of domestic securities.
Source: Federal Reserve Board of Governors.

was primarily established as a purchase program of longer-term U.S. Treasury securities. From November 12, 2010 through June 30, 2011 the Federal Reserve's Open Market Trading Desk conducted $767 billion of purchases.[41]

LSAP programs expanded the monetary base by the provision of central bank money in exchange for specified groups of commercial banks' assets, i.e. long-term U.S. Treasuries in the case of LSAP 2. As outlined by Bernanke (2012) these nonstandard open market operations have been introduced to enhance credit supply by alleviating financial intermediation, and to support the transmission mechanism of monetary policy. Specifically, with the monetary policy rate at the zero lower bound (ZLB), large-scale purchases of longer-term securities are intended to lower their interest rates. These effects should be transmitted to private borrowing rates, particularly at longer maturities, and finally stimulate economic activity. However, as-

[41]After June 2011 principal payments on all domestic securities were reinvested in Treasury securities to maintain the Fed's holdings of domestic securities at approximately $2.6 trillion.

sessing the qualitative and quantitative effects of LSAP programs on key macroeconomic variables poses to be a rather complicated task, as further pointed out by Fed Chairman Ben Bernanke at Jackson Hole Meeting 2012:

> "While there is substantial evidence that the Federal Reserve's asset purchases have lowered longer-term yields and eased broader financial conditions, obtaining precise estimates of the effects of these operations on the broader economy is inherently difficult, as the counterfactual - how the economy would have performed in the absence of the Federal Reserve's actions - cannot be directly observed."

In this chapter it is our aim is to identify the effects of the LSAP 2 longer-term Treasury purchase program on the U.S. economy. To develop a framework for the macroeconomic analysis of the LSAP 2 program we extend the monetary DSGE model by Christoffel and Schabert (2013) which provides an explicit specification of the central bank's balance sheet options. The model is estimated for U.S. data using Bayesian techniques. We employ the model to simulate a crisis scenario which matches the deviation of real per-capita output from its long-run trend in 2010:Q4, which was the quarter when LSAP 2 was launched. Then we conduct a counterfactual policy simulation to identify the effects of this balance sheet policy measure.

Several contributions to the empirical literature find evidence that LSAP programs have indeed been effective in reducing long-term U.S. Treasury rates. Krishnamurthy and Vissing-Jorgensen (2011) estimate that LSAP 2 reduced the ten-year Treasury Note yield by 33 basis points, likewise D'Amico and King (2013) estimate a reduction of 55 basis points.[42] However, the lack of experience with these balance sheet policies has raised questions on the transmission to the real economy and fueled discussions about possible inflationary effects (see Borio and Disyatat (2009)). Still, there have been only few contributions to the academic research providing theoretical guidance for the evaluation of the effectiveness of this policy. As Christoffel and Schabert (2013) point out, one reason for this is that standard macroeconomic models which are typically used for monetary analysis, assume that central banks directly control the private sector savings rate (i.e. the consumption Euler equation rate). By this assumption, important

[42] For surveys of estimates regarding the impact of LSAP 1 on the ten-year Treasury yield see Chen, Curdia, and Ferrero (2012) and Bernanke (2012).

aspects of the monetary transmission mechanism are neglected and severe limitations to the analysis emerge once the zero lower bound on interest rates is reached.[43] A recent study on the macroeconomic effects of the U.S. Fed's LSAP 2 program by Chen, Curdia, and Ferrero (2012) assumes segmented bond markets along the lines of Andres, Lopez-Salido, and Nelson (2004) in a non-monetary macroeconomic model. Specifically, investors are assumed to have heterogenous preferences for assets of different maturities. This idea reflects a "preferred habitat" motive similar to Vayanos and Villa (2009). This analysis however, estimates that the LSAP 2 program increased the growth of U.S. Gross Domestic Product (GDP) by only less than half of a percentage point.

This chapter applies the macroeconomic model by Christoffel and Schabert (2013) which accounts for central bank asset acquisition and its impact on financial intermediation. Specifically, monetary transmission is based on financial intermediation and an endogenous pass through of policy rate changes. In this chapter we consider multiple assets that differ with respect to their ability to be exchanged for central bank money. This leads to a spread between interest rates on non-eligible and eligible assets, which reflects basically a liquidity premium. Central bank balance sheet policy therefore can be effective by influencing the liquidity premium. The model by Christoffel and Schabert (2013) accounts for the specific role of government bonds to provide liquidity services to commercial banks. This is considered by modeling central bank monetary supply by an asset exchange in open market operations, as it is common practice (i.e. repurchase agreements). Central bank money and reserves resp. are demanded by banks for liquidity management purposes when they provide intermediation between households and firms. In particular, costs of financial intermediation are

[43]Further, by the irrelevance result of Wallace (1981) it is generally expected from a theoretical point of view that nonstandard open market operations in private assets do not exhibit an effect on real variables. As shown by Eggertsson and Woodford (2003) this result also applies for nonstandard open market operations conducted in models with nominal frictions, money in the utility, and where the interest rate is at its ZLB. In this regard Walsh (2010) points out that private agents will demand money up to satiation if the ZLB is reached. In particular, by the assumption that assets are perfect substitutes there is no portfolio-balance effect. In case of LSAPs, market participants will take advantage of arbitrage opportunities, which will make decisions about size and composition of the central bank's balance sheet irrelevant for the real variables' allocation. This result even applies to New Keynesian models with credit frictions, such as Curdia and Woodford (2011), as long as assets purchased (short-term bonds) by the central bank will be perceived as equivalent to reserves.

specified in a stylized way following Curdia and Woodford (2011). Banks further hold government short-term bonds and long-term bonds where only the former in absence of unconventional policy measures can be used to acquire reserves in open market operations. LSAP 2 is implemented by central bank purchases of a certain fraction of long-term bonds held by the commercial banks. As the purpose of the analysis which is presented in this chapter is to provide quantitative results, we estimate the model for the U.S. economy.

The model is applied to address the following questions: *First*, to what extent did shocks to financial intermediation and central bank's unconventional open market operation measures contribute to the volatility in real activity following the 2008/2009 financial crisis? *Second*, what are the macroeconomic effects of the U.S. Fed's LSAP 2 large-scale long-term Treasury bond purchase program?

We find that shocks to financial intermediation significantly contribute to the evolution of U.S. key macroeconomic variables following the 2008/2009 crisis. In particular, our estimated model implies that in 2010:Q4, which is the quarter when LSAP 2 was initiated, roughly one third of the negative U.S. real per capita GDP trend deviation was attributed to the shock to financial intermediation. Further, the central bank can significantly alleviate adverse effects to the economy by easing the supply of reserves in exchange for log-term government bonds. Our counterfactual policy simulation finds that in the absence of LSAP 2 real per capita U.S. GDP would have dropped by additional 2.75 percentage points.

Besides the contributions by Chen, Curdia, and Ferrero (2012) and Curdia and Woodford (2011) there are few recent studies on unconventional central bank policies such as LSAPs which are employing macroeconomic models with financial market imperfections. However, these studies rely on non-monetary macroeconomic models. Del Negro et al. (2013) incorporate credit market frictions a la Kiyotaki and Moore (2012) into a standard DSGE model. Firms facing investment opportunities can issue own debt securities only up to a certain fraction of illiquid assets on their balance sheet in each period. In contrast to that, government securities are not subject to such resaleability constraints which gives government bonds the role of liquidity in the model. Gertler and Karadi (2013) incorporate financial intermediaries within an otherwise standard macroeconomic model where the condition of the intermediary's balance sheet influences the overall flow of credit. Non-conventional monetary policy is introduced in this environment by the central bank acting as a financial intermediary. Specifically, the central bank borrows funds from savers and lends them to investors in times when private sector financial intermediation is interrupted by a financial crisis. Gertler and Kiyotaki (2010) assume that financial intermediaries face an endogenous balance sheet constraint and that rates of return on investments differ across a segmented market. This induces demand for funds which are traded on an interbank market. It is shown that a disruption in financial intermediation together with financial frictions can increase the severity of a recession. Under this setting direct central bank lending can be effective as it targets the distressed interbank markets.

The remainder of this chapter is organized as follows. Section 4.2 presents the model. Section 4.3 discusses empirical implementation, estimation results, and model-implied variables' dynamics. Specifically, we discuss the data and priors for the estimation and analyze the model's empirical performance in terms of implied business cycle moments, variance decomposition, and forecasting performance, and calculate the model-implied contributions of macroeconomic shocks to the historical variations of output. Furthermore, we analyze the model's impulse responses to exogenous shocks. Section 4.4 conducts a counterfactual policy experiment to analyze the effects of a LSAP 2 scenario on the estimated model. Section 4.5 concludes.

4.2 The model

The macroeconomic model employed in this chapter builds on Christoffel and Schabert (2013). The model contains five sectors: The household sector and the firm sector are close to the formulation of Smets and Wouters (2007). The financial intermediation sector follows the formulation of Curdia and Woodford (2011). The government and the central bank sectors are enriched and modified to allow for balance sheet policies. Banks provide financial intermediation between households and borrowing firms. They further hold short-term bonds and long-term bonds issued by the government, while the former in the absence of unconventional monetary policy measures serve as only eligible asset for an exchange against central bank money in open market operations. Firms borrow from banks to finance the payment of wages in the production process, are monopolistic competitive suppliers of differentiated goods, face price adjustment cost, invest into physical capital, face investment adjustment cost a la Christiano, Eichenbaum, and Evans (2005), and decide on the level of capital utilization. Households consume, are monopolistic suppliers of differentiated labor, face wage adjustment cost, exchange state contingent contracts in zero-net-supply among themselves, and deposit funds at the financial intermediaries. The government purchases goods, raises lump-sum taxes, and issues short-term bonds and long-term bonds, where the latter are modeled as perpetuities. The central bank sets the main refinancing rate according to a Taylor-type rule, supplies money in exchange for eligible assets, and decides on the size and the composition of its balance sheet.

4.2.1 Households

The household sector of the model economy is comprised of a continuum of infinitely lived and identical households, indexed with $i \in [0, 1]$. Their utility increases with households' real consumption and decreases with working time. Further, it is assumed that beginning-of-period holdings of deposits provide utility, which serves as a convenient short-cut for modelling transaction services of deposits (just like the textbook specification of money-in-the-utility function). Household i maximizes the expected sum of a discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u\left(c_{i,t}, n_{i,t}, \frac{d_{i,t-1}}{\pi_t}\right), \tag{4.1}$$

where E_0 is the expectation operator conditional on the time 0 information set and $\beta \in (0, 1)$, is the subjective discount factor. The term ξ_t is a stochastic preference parameter.[44] It is assumed that ξ_t evolves according to a stationary AR(1) process in logs. The household's real value of bank deposits is denoted as $d_{i,t}$, with $d_{i,t} = \frac{D_{i,t}}{P_t}$, and P_t representing the price of the wholesale final goods. Inflation $\frac{P_t}{P_{t-1}}$ is labeled as π_t. The instantaneous utility u is assumed to be increasing in household consumption $c_{i,t}$ and real deposits $d_{i,t}$, and decreasing in working time $n_{i,t}$. Household preferences further allow for external habits in consumption. Under these assumptions the households' instantaneous utility function is specified in the following

[44] Christoffel and Schabert (2013) point out that stochastic preference parameters have been used in several studies on policy options at the ZLB (see e.g. Eggertsson and Woodford (2003), or Eggertsson (2011)).

way:

$$u(c_{i,t},n_{i,t},d_{i,t}) = \frac{1}{1-\sigma}\left(c_{i,t}-h\cdot c_{t-1}\right)^{1-\sigma} + \varrho\frac{1}{1-\varphi_d}\left(\frac{d_{i,t-1}}{\pi_t}\right)^{1-\varphi_d} - \nu\frac{1}{1+\upsilon}n_{i,t}^{1+\upsilon},$$

such that $u_{c,i,t} = (c_{i,t}-hc_{t-1})^{-\sigma}$, $u_{d,i,t} = \varrho\pi_t^{-1}\left(\frac{d_{i,t-1}}{\pi_t}\right)^{-\varphi_d}$, $u_{n,i,t} = -\nu n_{i,t}^{\upsilon}$, with $h>0$, $\varrho>0$, $\nu>0$, $\sigma\geq 1$, $\varphi_d\geq 0$, and $\upsilon\geq 0$, where $u_{c,i,t}$ denotes household i's marginal utility of consumption, $u_{d,i,t}$ denotes marginal utility of services gained from deposit holdings, and $u_{n,i,t}$ denotes marginal (dis-)utility from working time. The flow budget constraint for each household reads:

$$P_t w_{i,t}n_{i,t} + P_t\Pi_{i,t} \geq P_t c_{i,t} + P_t WAC_{i,t} + P_t\tau_{i,t} + \frac{D_{i,t}}{R_t^D} - D_{i,t-1} + E_t[\varphi_{t,t+1}S_{i,t}] - S_{i,t-1}. \quad (4.2)$$

Household i supplies labor against the real wage rate $w_{i,t}$, invests in deposits, and state contingent claims $S_{i,t}$, where R_t^D denotes the risk-free rate of return on deposits, and $\varphi_{t,t+1}$ the stochastic discount factor. Profits from the production sector $\Pi_{i,t}$, are distributed to the households. The household further has to pay a lump-sum tax which is denoted as $\tau_{i,t}$. We assume that households have to set their individual wage rate $w_{i,t}$. Wage rigidities are introduced by constraining the optimal choice of $w_{i,t}$ by wage adjustment cost $WAC_{i,t}$ which each household individually faces when adjusting the nominal wage rate.[45] These costs are specified in the following way:

$$WAC_{i,t} = \frac{\omega_W}{2}\left(\frac{P_t w_{i,t}}{P_{t-1}w_{i,t-1}\left(\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}\right)} - 1\right)^2 y_t, \quad (4.3)$$

where $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$ denotes past inflation and $\bar{\pi}$ denotes the steady state inflation rate. The parameter $\omega_W > 0$ measures the degree of wage stickiness and the degree of wage indexation to past inflation is measured by $\iota_w \in (0,1)$. Each household has to pay an increasing and convex cost which is zero at the steady state. This cost is measured in terms of aggregate output

[45] The specification of wage adjustment cost we use here is similar to Kim (2000), and Ireland (2007) which is based on Rotemberg's (1982) quadratic cost of price adjustment.

when the pace of its nominal wage changes deviates from an index number of the steady state inflation rate and the past inflation rate.

Household $i's$ borrowing is restricted by the following no-Ponzi game condition $\lim_{s\to\infty} E_t\varphi_{t,t+s}S_{i,t+s} \geq 0$, as well as by the non-negativity constraint for deposit holdings $D_{i,t} \geq 0$. Maximizing the objective (4.1) subject the budget constraint (4.2) and the two aforementioned borrowing constraints, leads for given initial values $D_{i,-1}$, $S_{i,-1}$, $c_{i,-1}$, $w_{i,-1}$, $\pi_{-1} > 0$, to the following first order conditions for consumption, investments in deposits, and contingent claims,

$$\xi_t u_{c,i,t} = \lambda_{i,t}, \tag{4.4}$$

$$\frac{\lambda_{i,t}}{R_t^D} = \beta E_t \frac{\lambda_{i,t+1}}{\pi_{t+1}} + \beta E_t \frac{u_{d,i,t+1}}{\pi_{t+1}}, \tag{4.5}$$

$$\varphi_{t,t+1} = \frac{\beta}{\pi_{t+1}} \frac{\lambda_{i,t+1}}{\lambda_{i,t}}, \tag{4.6}$$

and the budget constraint (4.2) holding with equality, as well as the transversality conditions, where $\lambda_{i,t} \geq 0$ denotes the multiplier on the budget constraint.

Households are assumed to be monopolistically supplying differentiated labor services $n_{i,t}$. Perfectly competitive labor packers (or unions) buy differentiated labor input, aggregate it through the technology $n_t^{1/\mu_t^w} = \int_0^1 n_{i,t}^{1/\mu_t^w} di$, and supply the effective labor units n_t. Here, the elasticity of substitution between differentiated labor services is given by $\vartheta_t = \frac{\mu_t^w}{\mu_t^w - 1}$, and varies exogenously over time. Specifically we assume that $\vartheta_t > 1$, implying that $0 < \mu_t^w < 1$, where μ_t^w is driven by a wage markup shock. Labor packers (unions) sell effective units of labor to intermediate goods producing firms at price w_t, which denotes the aggregate real wage rate. Profit maximization then leads to the following labor demand[46]

$$n_{i,t} = \left(\frac{w_{i,t}}{w_t}\right)^{-\vartheta_t} n_t. \tag{4.7}$$

The first order condition for the wage rate $w_{i,t}$ is derived from the household's utility maximization problem where the labor demand function (4.7) is taken into account for as an additional constraint. Due to the assumption

[46] See Appendix C.3 for derivation of the labor demand function.

of wage adjustment cost we can derive a wage Phillips curve for the model from the first order condition[47]

$$\begin{aligned}&\left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}}-1\right)\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}}y_t-\frac{n_t}{\left(\mu_t^w-1\right)\omega_w}\left(\mu_t^w mrs_t-w_t\right)\\ =&\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}}-1\right)\frac{\pi_{t+1}^w}{\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}}y_{t+1}\right],\end{aligned}\tag{4.8}$$

where $\pi_t^w=\frac{w_t}{w_{t-1}}$ denotes the wage inflation rate, and mrs_t denotes the representative household's marginal rate of substitution between consumption and leisure, where

$$mrs_t=-\frac{u_{n,t}}{\lambda_t}.\tag{4.9}$$

Here we already took into account that trade in contingent assets implies that the marginal utility of consumption is the same across households, any household who is permitted to optimize chooses the same supply of $n_{i,t}$.

The first order conditions (4.4) to (4.6), and (4.8) can then be summarized as

$$\frac{1}{R_t^D}=\beta E_t\left[\frac{1}{\pi_{t+1}}\frac{\xi_{t+1}u_{c,t+1}}{\xi_t u_{c,t}}\left(1+\frac{u_{d,t+1}}{u_{c,t+1}}\right)\right],\tag{4.10}$$

$$\frac{1}{R_t^E}=\beta E_t\left[\frac{1}{\pi_{t+1}}\frac{\xi_{t+1}u_{c,t+1}}{\xi_t u_{c,t}}\right],\tag{4.11}$$

$$\begin{aligned}&\left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}}-1\right)\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}}y_t-\frac{n_t}{\left(\mu_t^w-1\right)\omega_W}\left(\mu_t^w mrs_t-w_t\right)\\ =&\beta E_t\left[\frac{\xi_{t+1}u_{c,t+1}}{\xi_t u_{c,t}}\left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}}-1\right)\frac{\pi_{t+1}^w}{\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}}y_{t+1}\right],\end{aligned}\tag{4.12}$$

where the risk free consumption Euler equation rate R_t^E is defined in the usual way, $R_t^{Euler}=\frac{1}{E_t\varphi_{t,t+1}}$.

4.2.2 Production

The production sector consists of perfectly competitive intermediate goods producing firms, monopolistically competitive retailers, and perfectly competitive bundlers who supply the final wholesale good.

There is a continuum of perfectly competitive intermediate goods producing *firms*. Firm $j\in[0,1]$ produces intermediate goods $y_{j,t}^m$ with labor,

[47]See Appendix C.4 for derivation of the wage Phillips Curve.

which is hired from labor packers, and with its own stock of capital $k_{j,t}$. The production technology is identical for all firms j and exhibits standard neoclassical properties:

$$y_{j,t}^{m} = a_t n_{j,t}^{\alpha} \left(u_{j,t} k_{j,t-1}\right)^{1-\alpha}, \tag{4.13}$$

where $\alpha \in (0,1)$, and a_t is a random productivity level with mean one, and $u_{j,t} \in (0,1)$ governs the firm j's level of capital utilization. A firm j accumulates physical capital $k_{j,t}$ by investing $x_{j,t}$, subject to adjustment costs $\Gamma_I\left(\frac{x_{j,t}}{x_{j,t-1}}\right)$ associated with changes in the level of investment

$$k_{j,t} - (1-\delta)k_{j,t-1} = \epsilon_t^I \left(1 - \Gamma_I\left(\frac{x_{j,t}}{x_{j,t-1}}\right)\right) x_{j,t},$$

where $\Gamma_I\left(\frac{x_{j,t}}{x_{j,t-1}}\right) = \frac{\gamma_I}{2}\left(\frac{x_{j,t}}{x_{j,t-1}} - 1\right)^2$, with $\gamma_I > 0$, and $\delta \in (0,1)$, denotes the depreciation rate of investment expenditures. Investment-specific technology ϵ_t^I is assumed to evolve as a stationary AR(1) process.

Demand for external funds is introduced in Christoffel and Schabert (2013) by assuming that wages have to be paid on a banking account before goods are sold, such that firms have to borrow in terms of one-period loans $L_{j,t}$ from banks at the price $\frac{1}{R_t^L}$:

$$\frac{L_{j,t}}{R_t^L} \geq P_t w_t n_{j,t}. \tag{4.14}$$

Christoffel and Schabert (2013) abstract from asymmetric information issues and limited commitment, and assume that firms fully repay one unit of currency per unit of loan in the subsequent period, such that R_t^L denotes a risk-free rate of return on loans. The budget constraint of firm j can then be written as

$$P_t v_{j,t}^f - Z_t a_t n_t^{\alpha} \left(u_{j,t} k_{j,t-1}\right)^{1-\alpha} + P_t w_t n_{j,t} - \left(\frac{L_{j,t}}{R_t^L}\right) + P_t x_{j,t} \geq L_{j,t-1}, \tag{4.15}$$

where $P_t v_{j,t}^f$ denotes intermediate goods producing firm $j's$ profits and Z_t denotes the price of the intermediate good. Firm j maximizes the present value of profits subject to (4.14), and (4.15), and a no-Ponzi game condition:

$$\max_{\{n_{j,t}, l_{j,t}, x_{j,t}, k_{j,t}\}} E_t \sum_{k=0}^{\infty} \phi_{t,t+k} v_{j,t+k}^f, \text{ s.t. (4.14) and (4.15)},$$

where $\phi_{t,t+k}$, denotes a stochastic discount factor (see 4.2), and given that $k_{j,-1} > 0$, and $x_{j,-1} > 0$. The first order conditions for labor demand and loan demand are

$$mc_{t+k}\alpha a_{t+k} n_{j,t+k}^{\alpha-1} \left(u_{j,t+k} k_{j,t+k-1}\right)^{1-\alpha} = w_{t+k} \cdot \left(\frac{R_{t+k}^L}{R_{t+k}^E}\right), \tag{4.16}$$

$$\frac{l_{j,t}}{R_t^L} \geq w_t n_{j,t}, \tag{4.17}$$

where we used $\frac{Z_{t+k}}{P_{t+k}} = mc_{t+k}$, and (4.17) is binding if $\chi_{t+k} = \left(\frac{R_{t+k}^L}{R_{t+k}^E}\right) - 1 > 0$, where χ_t is the multiplier on the constraint (4.17). The first order conditions for investment expenditures and physical capital read

$$\begin{aligned}
1 = & q_{j,t+k}\epsilon_{t+k}^I \left(1 - \Gamma_I\left(\frac{x_{j,t+k}}{x_{j,t+k-1}}\right) - \Gamma_I'\left(\frac{x_{j,t+k}}{x_{j,t+k-1}}\right)\frac{x_{j,t+k}}{x_{j,t+k-1}}\right) \\
& + E_{t+k}\left[\frac{\phi_{t+k+1}}{\phi_{t+k}} q_{j,t+k+1}\epsilon_{t+k+1}^I \Gamma_I'\left(\frac{x_{j,t+k+1}}{x_{j,t+k}}\right)\left(\frac{x_{j,t+k+1}}{x_{j,t+k}}\right)^2\right], \qquad (4.18) \\
q_{j,t+k} = & E_{t+k}\frac{\phi_{t+k+1}}{\phi_{t+k}}[q_{j,t+k+1}(1-\delta) + rk_{j,t+k+1}u_{j,t+k+1} \\
& - rk\left(u_{t+k+1} - 1\right) + \frac{\varkappa \cdot rk}{2}\left(u_{t+k+1} - 1\right)^2], \qquad (4.19)
\end{aligned}$$

where q_t denotes the standard Tobin's q, and where $rk_{j,t+k}$, which is the first derivative of (4.13) w.r.t. capacity utilization $u_{j,t+k}$, is given by

$$rk_{j,t+k} = mc_{t+k}(1-\alpha)a_{t+k}n_{j,t+k}^{\alpha}\left(u_{j,t+k}k_{j,t+k-1}\right)^{-\alpha} k_{j,t+k-1}. \tag{4.20}$$

Further rk denotes the steady state value of (4.20) and $\varkappa$ denotes the inverse of the steady-state elasticity of the capital adjustment cost function. Note that the firm's investment decision is distorted if the interest rate on loans differs from the Euler equation rate $R_t^L \neq R_t^E$.

Monopolistically competitive *retailers* buy intermediate goods $y_t^m = \int_0^1 y_{j,t}^m dj$, at the real price mc_t. A retailer $k \in [0,1]$, relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive *bundlers*. Those bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk$, where the elasticity of substitution is defined as $\varepsilon_t = \frac{\mu_t^p}{\mu_t^p - 1}$, with $\varepsilon_t > 1$, implying that $0 < \mu_t^p < 1$. Further, μ_t^p

is driven by a price markup shock. The cost minimizing demand for $y_{k,t}$ is therefore given by[48]

$$y_{k,t} = \left(\frac{P_{k,t}}{P_t} \right)^{-\varepsilon} y_t. \tag{4.21}$$

Further, we introduce price rigidities through price adjustment costs $PAC_{k.t}$ which are given by[49]

$$PAC_{k.t} = \frac{\omega_P}{2} \left(\frac{P_{k,t}}{P_{k,t-1} \left(\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p} \right)} - 1 \right)^2 y_t, \tag{4.22}$$

where $\omega_P > 0$, measures the degree of price stickiness and $\iota_p \in (0, 1)$, measures the degree of price indexation to past inflation. Each retailer has to pay an increasing and convex cost which is zero at the steady state. This cost is measured in terms of aggregate output when the pace of its nominal price changes deviates from an index number of the steady state inflation rate and the past inflation rate. From the optimal price setting behavior of the retailer we derive the price Phillips curve:[50]

$$\begin{aligned} &\left(\frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - 1 \right) \frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - \frac{\mu_t^p}{\omega_p \left(\mu_t^p - 1 \right)} \left(mc_t - \frac{1}{\mu_t^p} \right) \\ &= \beta E_t \left[\left(\frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} \frac{y_{t+1}}{y_t} \right]. \end{aligned} \tag{4.23}$$

4.2.3 Financial intermediaries

The basic role of the financial intermediation sector is to provide loans to firms which are required to pay wages before goods are sold. The model allows for the propagation of monetary policy through a bank balance sheet channel by including a bank's balance sheet constraint and costs associated with the supply of credit. Following Curdia and Woodford (2011) there is a real resource cost related to the supply of credit from banks to firms. These credit costs are increasing in the amount of loans and decreasing in the amount of reserves. As pointed out by Christoffel and Schabert (2013), while

[48] Which is derived analogously to the labor demand function (4.7).

[49] The specification of price adjustment cost we use here is taken from Ireland (2007) which is based on Rotemberg's (1982) quadratic cost of price adjustment.

[50] See Appendix C.5 for derivation of the wage Phillips Curve.

lacking an explicit microfoundation this assumption allows to introduce the relation between balance sheet items and the cost of providing credit.[51] In addition to this, we follow Curdia and Woodford (2011) by assuming that banks face a balance sheet constraint which requires deposits holdings to equal the expected payoff from the assets on the balance sheet.

There is a continuum of perfectly competitive financial intermediaries, which are called banks. They receive deposits from household $D_t = \int D_{i,t}di$, and invest in loans $L_t = \int L_{j,t}dj$, and reserves M_t. Further, there are two types of bonds which are held by the banks. Short-term government bonds B_t^S are issued at the price $\frac{1}{R_t^S}$ in period t and deliver the payoff one in period $t+1$, where R_t^S denotes the short-term bonds' gross return. Long-term government bonds B_t^L are assumed to be perpetuities. We model the stock of long-term bonds in the same way as Chen, Curdia, and Ferrero (2012). The perpetuities' price at time t is p_t^L. Perpetuities pay exponentially decaying coupons ρ_t^s in period $t+s+1$, with $\rho_t \in (0,1]$. We assume that the coupon rate is following an AR(1) process. The price of a long-term bond in period t, issued s periods ago, p_{t-s}^L is a function of the coupon and the current price $p_{t-s}^L = \rho_t^s p_t^L$.[52] The gross yield to maturity YTM_t at time t on the long-term bond perpetuity, is a function of its price and the coupon rate[53]

$$YTM_t = \frac{1}{p_t^L} + \rho_t. \tag{4.24}$$

Banks buy perpetuities at the period-t price p_t^L, which pay off $1 + \rho_t p_{t+1}^L = p_{t+1}^L \left(\frac{1}{p_{t+1}^L} + \rho_t \right) = p_{t+1}^L YTM_{t+1}$, units of currency in period $t+1$.

Following Curdia and Woodford (2011), in each period the bank's balance sheet has to be satisfied. It requires the bank to acquire deposits in the maximum amount that it can repay in the end of each period from the expected payoffs from its assets:

$$D_t = M_t + B_t^S + E_t \left[p_{t+1}^L YTM_{t+1} B_t^L \right] + L_t. \tag{4.25}$$

Following Christoffel and Schabert (2013) banks can use government bonds

[51] Christoffel and Schabert (2013) compare Gertler and Kiyotaki (2010) and Gertler and Karadi (2013) for theoretical approaches with an explicit microfoundation and Kashyap and Stein (2000) and Jimenez et al. (2012) for empirical studies on the relation between credit costs and the bank balance sheet.

[52] See Appendix C.6 for the derivation of this expression.

[53] See Appendix C.7 for the derivation of this expression.

in an asset exchange for additional central bank money I_t:

$$I_t \leq \kappa_t^S \frac{B_{t-1}^S}{R_t^m} + \kappa_t^L \frac{p_t^L YTM_t B_{t-1}^L}{R_t^m} + \varepsilon_{t,Coll}, \tag{4.26}$$

where the value of the bond is discounted at the main refinancing rate R_t^m. The parameters κ_t^S and κ_t^L are additional monetary policy instruments. They allow the central bank to set the fractions of short-term bonds and long-term bonds which are eligible for repo contacts, and therefore to control the amount of extra reserves supplied to banks. Further, $\varepsilon_{t,Coll}$ denotes an i.i.d. exogenous money supply shock with zero mean which eases the collateral constraint. Note that we assume that in the absence of LSAPs, the parameter $\kappa_t^L = 0$, implying that only short-term Treasuries are eligible for repos.

Costs of financial intermediation are specified in an implicit way, which is particularly useful for calibration and estimation purposes. Banks face real resource costs $\Xi_t \geq 0$, when they supply loans to firms. Following Curdia and Woodford (2011), these costs are convex, increasing in the amount of loans, with $\Xi_{l,t} \geq 0$, and decreasing in the amount of real reserves $\frac{M_{t-1}+I_t-\mu_t D_{t-1}}{P_t}$, with $\Xi_{m,t} \leq 0$. Christoffel and Schabert (2013) consider that the bank relies on reserves to manage deposits in accordance to the relation $\mu_t D_{t-1}$. Further, μ_t evolves according to an AR(1) process. For the quantitative analysis we follow Christoffel and Schabert (2013) and employ a specific form for the cost of financial intermediation which is given by

$$\Xi_t = \zeta_t \left(\frac{L_t}{P_t}\right)^{\eta_{rc}} \left(\frac{M_{t-1} + I_t - \mu_t D_{t-1}}{P_t}\right)^{-\phi_{rc}}, \tag{4.27}$$

where $\eta_{rc} \geq 0$, $\phi_{rc} \geq 0$, and ζ_t denotes a shock to the cost of financial intermediation. The parameter η_{rc} denotes the elasticity of the banking cost function with respect to real loan provision and ϕ_{rc} is the elasticity of banking cost with respect to real reserves holdings.

Given that bonds are discounted at the rate R_t^m (see 4.26) acquisition of reserves I_t is associated with costs $I_t\left(R_t^m - 1\right)$. The real profits of a bank υ_t^I are thus given by

$$\begin{aligned} P_t \upsilon_t^I = & \frac{D_t}{R_t^D} - D_{t-1} - \frac{B_t^S}{R_t^S} + B_{t-1}^S - p_t^L B_t^L + p_t^L YTM_t B_{t-1}^L \\ & - \frac{L_t}{R_t^L} + L_{t-1} - M_t + M_{t-1} - I_t\left(R_t^m - 1\right) - P_t \Xi_t. \end{aligned} \tag{4.28}$$

The banks aim at maximizing the present value of profits, subject to (4.25) and (4.26). Equation (4.28) can be reduced to

$$P_t v_t^I = \frac{D_t}{R_t^D} - \frac{B_t^S}{R_t^S} - p_t^L B_t^L - \frac{L_t}{R_t^L} - M_t - I_t \left(R_t^m - 1\right) - P_t \Xi_t,$$

by taking into account the bank balance sheet condition (4.25). Further, a no-Ponzi game condition $\lim_{s\to\infty} E_t \phi_{t,t+s} D_{i,t+s} \geq 0$, as well as $L_t \geq 0$, $B_t^S \geq 0$, $B_t^L \geq 0$, and $M_t \geq 0$ have to be satisfied.[54] The banks' optimization problem then reads

$$\max E_t \sum_{k=0}^{\infty} \phi_{t,t+k} v_{t+k}^I, \ s.t. \ (4.25) \text{ and } (4.26),$$

The first order conditions with regard to deposits, short-term bonds, long-term bonds, loans, money holdings, and reserves I_t are given by

$$\begin{aligned}
\frac{1}{R_{t+k}^D} &= \frac{1}{R_{t+k}^E} E_{t+k}\left[-\mu_t \Xi_{m,t+k+1}\right] - \Theta_{t+k}, \\
\frac{1}{R_{t+k}^S} &= \frac{1}{R_{t+k}^E} E_{t+k}\left[\eta_{t+k+1}\kappa_{t+k+1}^S\right] - \Theta_{t+k}, \\
1 &= \frac{1}{R_{t+k}^E} E_{t+k}\left[R_{t+k+1}^{LB} YTM_{t+k+1}\eta_{t+k+1}\kappa_{t+k+1}^L\right] \\
&\quad -\Theta_{t+k} E_{t+k}\left[R_{t+k+1}^{LB} YTM_{t+k+1}\right], \\
\frac{1}{R_{t+k}^L} &= -\Xi_{l,t+k} - \Theta_{t+k}, \\
1 &= \frac{1}{R_{t+k}^E} E_{t+k}\left[-\Xi_{m,t+k+1}\right] - \Theta_{t+k}, \\
\Xi_{m,t+k} &= 1 - \left(1 + \eta_{t+k}\right) R_{t+k}^m.
\end{aligned}$$

Note that $\varphi_{t+k,t+k+1} = \frac{\phi_{t,t+k+1}}{\phi_{t,t+k}\pi_{t+k+1}}$, and $\frac{1}{R_{t+k}^E} = E_{t+k}\left[\varphi_{t+k,t+k+1}\right]$, and $R_{t+k+1}^{LB} = \frac{p_{t+k+1}^L}{p_{t+k}^L}$, and Θ_{t+k} denotes the multiplier on the balance sheet constraint (4.25), and η_{t+k} denotes the multiplier on the collateral constraint (4.26).

[54] Where the real values of the variables are given by $m_t = \frac{M_t}{P_t}$, $i_t = \frac{I_t}{P_t}$, $d_t = \frac{D_t}{P_t}$, $b_t^S = \frac{B_t^S}{P_t}$, $b_t^L = \frac{B_t^L}{P_t}$, and $l_t = \frac{L_t}{P_t}$.

Note that real cost of financial intermediation are given by
$\Xi_t = \zeta_t (l_t)^{\eta_{rc}} \left(\frac{m_{t-1}}{\pi_t} - \frac{\mu_t d_{t-1}}{\pi_t} + i_t\right)^{-\phi_{rc}}$, with the marginal cost of loan provision $\Xi_{l,t} = \eta_{rc}\frac{\Xi_t}{l_t}$, and the marginal contribution of reserves to the reduction of intermediation costs $\Xi_{m,t} = -\frac{\phi_{rc}\Xi_t}{\left(m_{t-1}\pi_t^{-1} - \mu_t d_{t-1}\pi_t^{-1} + i_t\right)}$.

By eliminating Θ_t the banks' first order conditions can be reduced to

$$\frac{1}{R^D_{t+k}} = 1 + (\mu_t - 1)\frac{1}{R^E_{t+k}} E_{t+k}\left[-\Xi_{m,t+k+1}\right], \tag{4.29}$$

$$\frac{1}{R^S_{t+k}} = 1 + \frac{1}{R^E_{t+k}}\left(E_{t+k}\left[\eta_{t+k+1}\kappa^S_{t+k+1}\right] - E_{t+k}\left[-\Xi_{m,t+k+1}\right]\right), \tag{4.30}$$

$$\begin{aligned} 1 = {} & \frac{1}{R^E_{t+k}}(E_{t+k}\left[R^{LB}_{t+k+1} YTM_{t+k+1}\eta_{t+k+1}\kappa^L_{t+k+1}\right] \\ & - E_{t+k}\left[-\Xi_{m,t+k+1}\right] E_{t+k}\left[R^{LB}_{t+k+1} YTM_{t+k+1}\right]) \\ & + E_{t+k}\left[R^{LB}_{t+k+1} YTM_{t+k+1}\right], \end{aligned} \tag{4.31}$$

$$\frac{1}{R^L_{t+k}} = 1 - \Xi_{l,t+k} - \frac{1}{R^E_{t+k}} E_{t+k}\left[-\Xi_{m,t+k+1}\right], \tag{4.32}$$

$$\Xi_{m,t+k} = 1 - \left(1 + \eta_{t+k}\right) R^m_{t+k}. \tag{4.33}$$

Further, the following complementary slackness conditions have to be satisfied

$$i_{t+k} \leq \kappa^S_{t+k}\frac{b^S_{t+k-1}}{\pi_{t+k}R^m_{t+k}} + \kappa^L_{t+k}\frac{p^L_{t+k} YTM_{t+k} b^L_{t+k-1}}{\pi_{t+k}R^m_{t+k}}, \quad \eta_{t+k} \geq 0,$$

$$\eta_{t+k}\left(\kappa^S_{t+k}\frac{b^S_{t+k-1}}{\pi_{t+k}} + \kappa^L_{t+k}\frac{p^L_{t+k} YTM_{t+k} b^L_{t+k-1}}{\pi_{t+k}} - R^m_{t+k} i_{t+k}\right) = 0.$$

as well as the balance sheet constraint (4.25).

The condition for the optimal choice of deposit holdings (4.29) relates the inverse rate of return on deposits to the payoff and the expected marginal reduction of banking costs implied by the acquisition of the last unit of deposits. Condition (4.30) relates the inverse of the return on short-term government bonds to the payoff adjusted by the expected marginal contribution to the relaxation of the collateral constraint which is induced by holding the last unit of short-term bonds and for the expected marginal reduction of the banking costs. Note that the second term of the right-hand side of (4.30) captures the liquidity premium on the short-term bond rate which is induced by the short-term bond being eligible for an exchange against reserves (see 4.26). The condition for the optimal choice of a long-term bond holdings (4.31) relates the perpetuity's price to the expected payoffs corrected for the expected marginal contribution on the banking costs and the expected marginal relaxation of the collateral constraint. The first term on the right-hand side of (4.31) denotes the long term bond's expected liquidity premium. In absence of LSAP measures, which means that the respective policy parameter is set $\kappa_t^L = 0$, this premium will be equal to zero. Condition (4.32) requires that the inverse of the return on loans equals the expected payoff corrected for the marginal cost of loan provision and the expected future marginal change of the banking cost function induced by the future expected payoff from the last unit of loans. Finally, the optimal central bank money demand is determined by (4.33), which relates the marginal contribution of the last unit of reserves to the reduction of the banking cost $\Xi_{m,t}$, to η_t, the multiplier on the collateral constraint (4.26), and the policy rate R_t^m.

4.2.4 The government

The government raises lump-sum taxes τ_t and purchases goods g_t. It issues short-term bonds, where we follow Schabert and Reynard (2009) by assuming that nominal short-term bond supply grows with a constant rate $B_t^{TS} = \Gamma B_{t-1}^{TS}$, with $\Gamma \geq 1$. It further issues nominal long-term debt securities B_t^{TL} which are modeled as perpetuities with coupon payments that decay exponentially at the rate $\rho_t \in (0, 1]$. Since bonds issued in period $t - s$ are equivalent to ρ_t^s bonds issued in t, we assume – without loss of generality – that all long-term bonds are of one type (which implies that the government redeems all old bonds) in each period.

The government's real goods purchases and the supply of long-term bonds are exogenously determined by AR(1) processes. Government goods purchases follow an autoregressive rule for deviations of g_t from the steady state value $\bar{g}$. Following Chen, Curdia, and Ferrero (2012), we assume that the government controls the supply of long-term bonds by an autoregressive rule for deviations of long-term bonds' market values in real terms $p_t^L b_t^{TL}$, from their steady state level $\bar{p}^L \bar{b}^{TL}$:

$$\left(\frac{g_t}{\bar{g}}\right) = \left(\frac{g_{t-1}}{\bar{g}}\right)^{\rho_g} e^{\varepsilon_{g,t}}, \tag{4.34}$$

$$\left(\frac{p_t^L b_t^{TL}}{\bar{p}^L \bar{b}^{TL}}\right) = \left(\frac{p_{t-1}^L b_{t-1}^{TL}}{\bar{p}^L \bar{b}^{TL}}\right)^{\rho_b} e^{\varepsilon_{b,t}}, \tag{4.35}$$

where ρ_b, $\rho_g \in (0,1)$, and $\varepsilon_{b,t}$, and $\varepsilon_{g,t}$ are i.i.d. innovations.

The treasury's budget constraint reads

$$\frac{B_t^{TS}}{R_t^S} + p_t^L B_t^{TL} = B_{t-1}^{TS} + p_t^L YTM_t B_{t-1}^{TL} + P_t g_t - P_t \tau_t - P_t \tau_t^m. \qquad (4.36)$$

The left-hand side of (4.36) is the market value of the total amount of short-term bonds and long-term bonds issued by the treasury at time t, expressed in nominal terms. The right-hand side is the total deficit at time t, which is the cost of servicing bonds maturing in period t plus government providing net of taxes and central bank transfers τ_t^m. Further it is assumed that the government has access to non-distortionary lump-sum transfers, which can be adjusted to balance the budget. Let B_t^{TS} denote the total stock of newly issued short-term bonds, which is either held by banks B_t^S, or the central bank B_t^{CS}, then: $B_t^{TS} = B_t^S + B_t^{CS}$. Analogously let Let B_t^{TL} denote the total stock of newly issued long-term bonds which is either held by banks B_t^L, or the central bank B_t^{CL}, then: $B_t^{TL} = B_t^L + B_t^{CL}$.

4.2.5 The central bank

The central bank supplies money outright, $M_t^H = \int_0^1 M_{i,t}^H di$, and in open market operations via repurchase agreements against short-term bonds, $M_t^R = \int_0^1 M_{i,t}^R di$. Newly issued money thus sums up to $I_t = M_t^H - M_{t-1}^H + M_t^R$, for which the central bank receives government bonds. Hence, in period t the central bank gets $I_t R_t^m$ units of bonds for I_t units of cash, such that its budget constraint reads

$$\begin{aligned} &\frac{B_t^{CS}}{R_t^S} - B_{t-1}^{CS} + p_t^L B_t^{CL} - p_t^L YTM_t B_{t-1}^{CL} + P_t \tau_t^m \\ &= \left(M_t^H - M_{t-1}^H\right) R_t^m + M_t^R \left(R_t^m - 1\right). \end{aligned}$$

Following Schabert and Reynard (2009), we identify seigniorage revenues as interest earnings from issuing money via repos or from holding interest bearing assets:

$$P_t \tau_t^m = B_t^{CS} - \frac{B_t^{CS}}{R_t^S} + p_t^L YTM_t B_{t-1}^{CL} - p_{t-1}^L B_{t-1}^{CL} + M_t^R \left(R_t^m - 1\right).$$

The central bank is assumed to transfer $P_t \tau_t^m$ directly to the public sector. When substituting out central bank transfers, bond holdings evolve according to

$$B_t^{CS} - B_{t-1}^{CS} + p_t^L B_t^{CL} - p_{t-1}^L B_{t-1}^{CL} = R_t^m (M_t - M_{t-1}). \tag{4.37}$$

For the policy rate R_t^m we follow Smets and Wouters (2007) by applying a simple feedback rule, which describes how the central bank adjusts the policy rate in response to deviations of R_{t-1}^m from its steady state level R^m, to deviations of inflation from its steady state level π, to real output deviation from its steady state y, and to the contemporary output growth:

$$R_t^m = \left(R_{t-1}^m\right)^{\rho_r} \left(R^m \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{y_t}{y}\right)^{\rho_y}\right)^{(1-\rho_r)} \left(\frac{y_t}{y_{t-1}}\right)^{\rho_{dy}} \exp \varepsilon_{r,t}, \tag{4.38}$$

where $R^m > 1$, $\rho_r \geq 0$, $\rho_\pi \geq 0$, $\rho_y \geq 0$, and $\rho_{dy} \geq 0$, and the $\varepsilon_{r,t}$'s are normally and i.i.d. with $E_{t-1}\varepsilon_{r,t} = 0$.

We follow Christoffel and Schabert (2013) by assuming that the central bank further controls reserves injections by deciding on the fraction of eligible assets to be purchased in period t, i.e. it sets κ_t^S and κ_t^L. Both variables affect the size and the composition of the central bank balance sheet. There are many possible ways to set κ_t^S and κ_t^L. We will consider different regimes for κ_t^L, for the quantitative analysis. Finally it has to be specified how money is supplied either outright or via repos. For this purpose, it is assumed that the central bank exogenously sets the fraction of repos

$$M_t = \Lambda M_t^R, \text{ with } \Lambda > 0.$$

4.2.6 Equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear. A complete equilibrium definition can be found in appendix C.1. In the following, we describe some particular properties: We will restrict our attention to equilibria where the liquidity constraint (4.14) and the collateral constraint (4.26) are binding, which requires $R_t^L > R_t^E \Rightarrow \eta_t > 0 \Leftrightarrow R_t^m - 1 < -\Xi_{m,t}$. Both requirements will be ensured in equilibrium, by an appropriate choice of parameters for the cost function of financial intermediation (4.27).

We further use that the total stock of short-term bonds and long-term bonds outstanding is either held by banks or the central bank, $B_t^{TS} =$

$B_t^S + B_t^{CS}$, and $B_t^{TL} = B_t^L + B_t^{CL}$, and (4.37) to identify how banks' bond holdings evolve over time:

$$\begin{aligned} & B_t^S - B_{t-1}^S + p_t^L B_t^L - p_{t-1}^L B_{t-1}^L \\ = & B_t^{TS} - B_{t-1}^{TS} + p_t^L B_t^{TL} - p_{t-1}^L B_{t-1}^{TL} - R_t^m \left(M_t - M_{t-1}\right), \end{aligned} \quad (4.39)$$

where

$$\frac{B_t^S}{R_t^S} = \frac{B_t^{TS}}{R_t^S} - \kappa_t^S B_{t-1}^S, \quad (4.40)$$

$$p_t^L B_t^L = p_t^L B_t^{TL} - \kappa_t^L p_t^L YTM_t B_{t-1}^L. \quad (4.41)$$

Condition (4.39) thus describes how banks' bond holdings increase with bond supply and decrease with money supply $R_t^m \left(M_t - M_{t-1}\right)$, while a higher the price of money R_t^m tends to raise the central bank's bond holdings (see 4.37).

Appendix C.2 provides the steady state of the model for a binding collateral constraint ($\eta > 0$) where variables without time indices denote steady state values.

4.3 Calibration and estimation

4.3.1 Calibration

In order to derive quantitative results, the model is partly calibrated and estimated with Bayesian techniques. Table 4.2 summarizes the calibrated parameters and Table 4.3 summarizes the estimated parameters. Most parameters affecting the steady state are calibrated while key parameters driving the dynamics of the model are estimated. For the study presented in this chapter, we try to use standard parameter values as far as possible. The model's interest rates are calibrated to match the average values of their empirical counterparts.[55] These are calculated for the time period covered by the underlying data sample ranging from 1964:Q3 to 2012:Q3. Hence, we calibrate the steady state value for the quarterly gross policy rate to equal $R^m = 1.0142$, the gross deposit rate $R^D = 1.0145$, the gross loan rate $R^L = 1.0214$, and the gross yield to maturity of long-term bonds $YTM = 1.0163$. Note that we calibrate YTM to match the average yield to maturity of 7-year U.S. Treasuries. As depicted in Table 4.1 the Federal

[55] See Appendix C.8 for a data description.

Table 4.1: Distribution of LSAP 2 Treasury purchases by maturity range

Nominal Coupon Securities by Maturity Range					TIPS
$2^{1/2} - 4$ Years	$4 - 5^{1/2}$ Years	$5^{1/2} - 7$ Years	$7 - 10$ Years	$10 - 30$ Years	$1^{1/2} - 30$ Years
20%	20%	23%	23%	6%	3%

Notes: Beginning with the operations included in the tentative schedule announced on July 13, 2011, the Desk planed to distribute purchases across the following seven maturity sectors based on the approximate weights above. The on-the-run 7-year note will be considered part of the $5^{1/2}$- to 7-year sector, and the on-the-run 10-year note will be considered part of the 7- to 10-year sector.
Source: Federal Reserve Bank of New York.

Open Market Committee's Trading Desk distributed its LSAP 2 purchases across seven maturity sectors covering largely Treasury securities with maturity lengths of $5\frac{1}{2}$ to 10 years.[56] For the household preferences we follow Christiano, Eichenbaum, and Evans (2005) by calibrating the inverse of the intertemporal elasticity of substitution to equal $\sigma = 1$ which indicates a log utility function. Due to the the assumption that deposits directly enter households utility we have to define a value for the scaling parameter ϱ. We calibrate ϱ such that the steady state value of R^D implied by (4.10) matches the mean of its empirical counterpart (see Appendix C.2, equation C.24). The steady state elasticity of substitution between differentiated labor types is calibrated as $\vartheta = 6$, implying a steady state value of $\mu^w = 1.2$.[57] The steady state elasticity of substitution between differentiated goods is calibrated as $\varepsilon = 6$, implying a steady state value of $\mu^p = 1.2$. The quarterly time discount factor β is set to 0.9935 and the quarterly steady state price inflation is 0.93 percent[58], implying by $R^E = \frac{\pi}{\beta}$ a quarterly steady state Euler equation rate of 1.0159 percent. The capital depreciation rate is set to $\delta = 0.03$, the steady state capital utilization rate is $u = 1$. We follow Christoffel and Schabert (2013) by setting the steady state working time to

[56]U.S. Treasuries which mature in more than one to up to ten years are denoted as Treasury Notes, U.S. Treasuries maturing after more than ten years are denoted as Treasury Bonds.

[57]For the estimation we use output data but refrain from a further decomposition in order to keep the model and the number of structural shocks small.

[58]The observed annualized average U.S. inflation rate for the data sample, 1964:Q3 to 2012:Q3, is 3.77 percent.

$n = 1/3$ and the fraction of money held outright to $\Lambda = 0.1$. The average duration measure of a 7 year U.S. Treasury is assumed to be of $5\frac{1}{2}$ years[59] for the time period under consideration, implying a steady state decay factor for the perpetuity's coupon rate of $\rho = \frac{Duration \cdot YTM - YTM}{Duration} = 0.83$.[60] By the assumed functional form of the financial intermediation cost function which is given by equation (4.27), the model includes three additional parameters. We calibrate the steady state scaling parameter $\bar{\zeta}$ in order to match the steady state value of the spread $R^L - R^D$ which is implied by (4.32) minus (4.29) with the mean of its empirical counterpart (see Appendix C.2, equation C.26). The parameters ϕ_{rc} and η_{rc} which are the elasticities of the banking cost function with respect to reserves and loans are estimated. The calibration of the model parameters implies the following ordering of the steady state interest rates: $R^L > YTM > R^{Euler} > R^D > R^m$. Following Smets and Wouters (2007) we calibrate an exogenous government spending-to-GDP ratio G/Y. For the present data sample G/Y is found to be equal to 0.2.

Monetary policy in the model operates via two margins. The standard monetary policy channel operates via the interest rate rule. Note that we estimate the parameters governing this policy reaction function. In addition to the policy interest rate there are further monetary policy variables which are directly or indirectly affecting the cost of financial intermediation (4.27). One of those is the fraction of reserves that the banks have to hold to manage the deposits μ_t. We estimate the parameters of the AR(1) which is driving μ_t. For the estimation we set the fraction of short-term bonds eligible for the asset exchange against central bank money $\bar{\kappa}^S$ to unity implying that money injections vary with the real value of short-term bonds. The fraction of long-term bonds eligible for repo agreements κ_t^L basically captures the model's channel for nonstandard balance sheet policy measures, namely LSAP 2. In absence of such a policy measure the parameter κ_t^L is equal to zero. For the policy simulation experiment we analyze the model's predictions for a specific path of κ_t^L which takes non-zero values as a policy reaction to a severe crisis scenario.

[59] This is calculated as the mean of the average values of the duration measure series for U.S. Treasury securities with 5 to 7 years remaining to maturity (Datastream series USSB57L.R) and for U.S. Treasury securities with 7 to 10 years remaining to maturity (Datastream series USSB70L.R).

[60] For the derivation of the relationship between the bond duration measure and the decay factor of a perpetuity see Appendix C.7.

Table 4.2: Values assigned to the calibrated parameters

Parameter	Value	Definition
β	0.9935	Discount factor
δ	0.03	Steady-state capital depreciation rate
u	1	Steady-state capacity utilization rate
μ^w	1.2	Steady-state wage mark-up
μ^p	1.2	Steady-state price mark-up
G/Y	0.2	Steady-state ratio of gov. consumption to GDP
n	0.33	Steady-state of hours worked
κ^S	1	Share of short-term bonds purchased
κ^L	0	Share of long-term bonds purchased
π	1.0093	Steady state price inflation
R^M	1.0142	Steady state main refinancing rate
R^L	1.0214	Steady state loan rate
YTM	1.0163	Steady state yield to maturity
$Duration$	5.5	Duration of long-term bonds
R^D	1.0145	Steady state deposit rate
Λ	0.1	Fraction of money held outright

Notes: This table shows the values for the calibrated parameters and the steady state ratios.

4.3.2 Data and shocks

For the estimation we use in this chapter quarterly U.S. data ranging from 1964:Q3 to 2012:Q3, including data on the recession which followed the subprime crisis. The macroeconomic time series are taken from the Federal Reserve Economic Database (FRED) which is maintained by the Federal Reserve Bank of St. Louis. We use real per capita GDP which is the empirical counterpart to the model's variables $c_t + i_t + g_t$, real per capita consumption c_t, real per capita investment i_t, hours worked n_t, real wages W_t/P_t, inflation calculated from the GDP Implicit Price Deflator deflator $\frac{P_t}{P_{t-1}}$, real per capita bank reserves $\frac{M_t}{P_t}$, and real per capita deposits $\frac{D_t}{P_t}$. We use interest rate data on the federal funds rate to proxy for R_t^m, the 3-month certificate of deposit rate for R_t^D, the 7-year treasury constant maturity rate for YTM_t, and on Moody's Baa-rated corporate bond rate for R_t^L. All time series are detrended, except for the interest rates, hours worked, and real deposits.[61] We use a linear trend, as our model does not explicitly consider growth.

In order to estimate the model we have to introduce as many shocks as observable variables to the model. We employ eight macroeconomic shocks that are used in comparable studies. A time preference shock ($\varepsilon_{\xi,t}$), a total factor productivity shock ($\varepsilon_{a,t}$), a price markup shock ($\varepsilon_{p,t}$), a markup shock on wages ($\varepsilon_{w,t}$), an interest rate shock ($\varepsilon_{r,t}$), a government spending shock ($\varepsilon_{g,t}$), an investment-specific technology shock ($\varepsilon_{\epsilon,t}$), and a shock to the fraction of reserves required to manage deposits ($\varepsilon_{\mu,t}$). Further, we introduce a decay factor shock ($\varepsilon_{\rho^s,t}$), and a shock to the long-term bond supply ($\varepsilon_{b,t}$). In addition to these shocks we introduce a shock that affects the cost of financial intermediation ($\varepsilon_{\zeta,t}$) and a shock driving a wedge between the real value of short-term bonds and the collateral constraint in the provision of money via repos ($\varepsilon_{t,Coll}$). This can be interpreted as a money supply shock and is required to account for the fact that previously to late 2008 only roughly 50 percent of the observed asset holdings by the Fed were short-term government securities.[62] Furthermore, due to reasons of data availability we set the fraction of long-term bonds purchased under repos κ_t^L over the whole data sample range equal to zero. Therefore, the estimation residual $\hat{\varepsilon}_{t,Coll}$ should capture the extra money injections required to match the share of the observed variation in the data on reserves which can not be explained by (4.26) with $\kappa_t^L = 0$:

$$I_t = \kappa_t^S \frac{B_{t-1}^S}{R_t^m} + \hat{\varepsilon}_{t,Coll}, \text{ with } \hat{\varepsilon}_{t,Coll} = \kappa_t^L \frac{p_t^L YTM_t B_{t-1}^L}{R_t^m} + \varepsilon_{t,Coll}. \tag{4.42}$$

Hence, we expect $\hat{\varepsilon}_{t,Coll}$ to capture the increase in reserves supply which was induced by unconventional balance sheet policies such as LSAP 2. All shocks except the policy rate shock and the money supply shock are modelled as AR(1) processes.[63]

[61] See Appendix C.8 for a complete data description and the description of the mapping of these variables to the states.

[62] From the Federal Reserve Bank of St. Louis FRED databank we use monthly data on the face value of U.S. Treasury securities held by the Fed, maturing in within 15 days (TREAS15), maturing in 16 days to 90 days (TREAS1590), and maturing in 91 days to 1 year (TREAS911Y) which sums up to the current face value of the Fed's holdings of Treasury bills. This is divided by the total face value of U.S. Treasury securities held by the Fed (TREAST).

[63] See Appendix C.1.

4.3.3 Estimation

As pointed out by Coenen and Straub (2005), employing Bayesian inference methods allows formalizing the use of prior information from earlier studies at both the micro and macro level for estimating the parameters of a possibly complex DSGE model. This seems particularly appealing in situations where the sample period of the data is relatively short, as it is the case for the present chapter. It is further pointed out, that Bayesian inference may also help to alleviate the inherent numerical difficulties associated with solving the highly non-linear estimation problem.

The log-linearized DSGE model leads to a rational expectations system (see Sims (2002)) which is given by

$$\Gamma_0(\theta)\, s_t = \Gamma_1(\theta)\, s_{t-1} + \Psi(\theta)\, \epsilon_t + \Pi(\theta)\, \eta_t, \tag{4.43}$$

where s_t is a vector of model variables, ϵ_t is a vector of exogenous shocks, η_t is a vector of rational expectations errors, and θ is a vector of structural parameters. Further Γ_0, Γ_1, Ψ, and Π are matrices that are nonlinear functions of structural parameters of the model. Typically standard perturbation techniques are employed which lead as a solution to the linear state-space representation[64]

$$s_t = \phi_1(\theta)\, s_{t-1} + \phi_\varepsilon(\theta)\, \varepsilon_t, \quad \varepsilon_t \sim NID(0, \Sigma_\varepsilon), \tag{4.44}$$

$$Y_t = \psi_0 + \psi_1(\theta)\, s_t + u_t, \quad u_t \sim NID(0, \Sigma_u). \tag{4.45}$$

The first equation is the state transition equation. In particular, ϕ_1 and ϕ_ε are functions of the matrices Γ_0, Γ_1, and Ψ. The second equation is the observation equation with measurement errors being collected in u_t, where Y_t corresponds to the vector of observables at time t. The matrix ψ_1 which contains zeros and ones, relates the model's definitions with the data. The vector ψ_0 is required to match the means of the observed data.

Let $p(\theta|m)$ denote the prior distribution of the parameter vector $\theta \in \Theta$, for some model $m \in M$, and let $L(Y_T|\theta, m)$ denote the likelihood function for the observed data, $Y_T = \{y_t\}_{t=1}^{T}$, conditional on parameter vector θ and model m. The Kalman filter is employed to construct the likelihood of the model under consideration. The joint posterior distribution $p(\theta|Y_T, m)$, conditional on the sample data Y_T and the model m, equals the model

[64] The following description is based on Schorfheide (2000).

likelihood, multiplied by the priors on the model parameters, up to a factor of proportionality which is known as Bayes rule

$$p(\theta|Y_T, m) \propto L(Y_T|\theta, m)p(\theta|m). \tag{4.46}$$

The Kalman filter generates projections of the state of the linear approximate solution of (4.43) for the model, given an information set of observed macro time series.

The posterior is evaluated by applying the Random Walk Metropolis (RWM) algorithm which belongs to the class of Metropolis-Hastings algorithms (see An and Schorfheide (2007), Schorfheide (2000)).[65] For that purpose first a numerical optimization routine is used to maximize the log posterior.[66] The RWM algorithm constructs a Gaussian approximation around the mode of the posterior kernel $L(Y_T|\theta, m)p(\theta|m)$ and a scaled version of the asymptotic covariance matrix which is the inverse of the negative Hessian computed numerically at the posterior mode. The latter serves as a covariance matrix for a proposal distribution. By applying rejection sampling, the algorithm then generates a sequence of dependent draws from the posterior that can be averaged to approximate posterior moments of interest, such as location measures and measures of dispersion.[67]

[65] This algorithm is used by DYNARE.

[66] The log posterior kernel is maximized using Dynare's Covariance Matrix Adaptation Evolution Strategy (CMA-ES) algorithm. This algorithm is applied in the case if derivative based methods, e.g. quasi-Newton BFGS or conjugate gradient, fail due to a rugged search landscape, i.e. a search landscape with high variability in the evaluation function value between neighbouring search positions. The CMA-ES is a second order approach estimating a positive definite covariance matrix within an iterative procedure. Specifically, on convex-quadratic objective functions the covariance matrix of the search distribution is set to the inverse Hessian matrix. In contrast to quasi-Newton methods, the CMA-ES does not use or approximate gradients and does not require their existence (See Hansen (2006), Hansen and Ostermeier (1996)).

[67] Let $\tilde{\Sigma}_m$ denote the inverse of the (negative) Hessian computed at the posterior mode $\tilde{\theta}$. A starting value $\theta^{(0)}$ is drawn from $N\left(\tilde{\theta}, c_0^2\tilde{\Sigma}_m\right)$. For $s = 1, ..., S$: draw ϑ from the proposal distribution $N\left(\theta^{(s-1)}, c_0^2\tilde{\Sigma}_m\right)$. The jump from $\theta^{(s-1)}$ is accepted $\left(\theta^{(s-1)} = \vartheta\right)$ with probability $\min\left\{1, r\left(\theta^{(s-1)}, \vartheta|Y_T\right)\right\}$ and rejected $\left(\theta^{(s)} = \theta^{(s-1)}\right)$ otherwise. Here

$$r\left(\theta^{(s-1)}, \vartheta|Y_T\right) = \frac{L(Y_T|\tilde{\theta}, m)p(\tilde{\theta}|m)}{L(Y_T\theta^{(s-1)}, m)p(\theta^{(s-1)}|m)}.$$

We use $S = 1000000$ and drop the first 500000 to let the Markov chain produced by the

4.3.4 Parameter prior distributions

In Table 4.3 we summarize the prior distributions for the estimated parameters. We follow Chen, Curdia, and Ferrero (2012) to chose the parameters' prior distributions according to whether their supported intervals are in line with economic theory's implications regarding the parameters value ranges. Specifically, we employ a gamma distribution for the parameters that should be positive to constrain their support on the interval $[0, \infty]$, we employ a beta distribution for those parameters that span on the unit interval, and we use the inverse-gamma distribution for the standard deviations of shock innovations.

We chose the prior distributions for the parameters which are generally found in applications of the canonical DSGE model by closely following Smets and Wouters (2007). The utility function's habit parameter h is assumed to be beta distributed fluctuating around 0.7 with a standard error of 0.1, and the inverse of the Frisch elasticity of labor supply υ, is assumed to be gamma distributed with mean 2.0 and standard error of 0.75. The prior on the labor share in production α is beta distributed with the mean set around 0.7 and with a standard deviation of 0.05. Following Christiano, Eichenbaum, and Evans (2005) we assume that the prior on the adjustment cost parameter for investment γ_I, follows a gamma distribution with a mean of 4.0 and a standard deviation of 1.5, and the inverse of the steady-state elasticity of the capital adjustment cost function is assumed to be gamma distributed with mean of 1.0 and a standard error of 0.25. For the priors of the parameters which govern the indexation of wages and prices to past inflation ι_w, and ι_p, we follow Smets and Wouters (2007) and Justiniano, Primiceri, and Tambalotti (2008) by choosing beta distributions with means set to 0.5 and standard errors of 0.15. We chose the priors for the parameters measuring the degree of price and wage stickiness ω_p and ω_w, similar to Gerali et al. (2010). Both priors are gamma distributed with mean of 60 and a standard error of 20.

Regarding the parameters governing the Taylor rule we follow Justiniano, Primiceri, and Tambalotti (2008) who assume that ρ_r which governs the central bank's adjustments of the policy rate in response to changes in it's own lags, is beta distributed with a mean of 0.6 and a standard devia-

RWM algorithm converge. The scaling factor c_0 is set to 0.3 which is intended to achieve an average acceptance rate per chain of approximately 25% (See An and Schorfheide (2007), Schorfheide (2000)).

tion of 0.15. The parameters capturing the policy rate response to steady state deviations of inflation ρ_π, to steady state deviations of output ρ_y, and to contemporary output growth ρ_{dy}, are assumed to be gamma distributed with means of 1.5, 1.25, and 1.25, where standard deviations are assumed to be 0.15, 0.05, and 0.05.

Following Smets and Wouters (2007) we let the priors of the standard errors of the innovations to the exogenous shock processes be inverse-gamma distributed. Further we try to harmonize the priors on the stochastic processes as much as possible. Specifically, the priors for the standard deviations of the shock processes known from the canonical DSGE model, which are the standard deviation for the process of production technology innovations σ_a, preference innovations σ_ξ, government spending σ_g, investment specific technology σ_I, and the wage markup σ_w, are set at a mean of 0.005 with a standard deviation of 2. For the standard deviation of the price markup innovation process σ_p, the prior mean is set at 0.001 with a standard deviations of 2. We set the prior mean of σ_p lower as the prior mean of σ_w in accordance with the results by Gerali et al. (2010) and Benati (2008).[68]

We set the mean of the standard deviation of the innovation to the policy rate σ_{r_m} at 0.001 with a standard deviation of 2. Note that the innovation to the policy rate does not follow an AR(1) process in logs as it is the case for most of the model's innovations. Hence, to harmonize the ex-ante expected contributions of the shocks to the variability of the observed variables we have to resize the mean of this standard deviation. As there is no previous experience regarding the prior choice for the standard deviations of the innovation processes governing the banking cost σ_ζ, the fraction of reserves required to manage deposits σ_μ, the coupon decay factor σ_{ρ^s}, the money supply σ_{Coll}, and the long-term bond supply σ_b, we mainly adapt the same choices regarding distribution, means, and standard deviations as for the other innovation processes. Here we made an exception for the prior on σ_{Coll}, by setting its mean at 0.01. This is done because the shock is required to contribute to the explanation of the observed significant variation in the

[68] Gerali et al. (2010) introduce a banking sector to a DSGE model with financial frictions to analyze the role of banking intermediation in the transmission of monetary impulses and shocks which are originated in the credit markets to the real economy. Wage stickiness is found to be more important than price stickiness while the degree of price indexation is relatively low. This result is in line with Benati (2008) who documents a reduction in indexation for the U.S. in the post WWII sample.

U.S. monetary base over the sample period. This seems to be an appropriate approach as these priors can be regarded as being loose. Following Smets and Wouters (2007), the priors of the persistence parameters of the AR(1) processes are beta distributed with mean of 0.5 and standard deviation 0.2.

Further, the model is estimated with additional three nonstandard parameters. These are the parameter governing the marginal utility of deposits φ_d, and the elasticities of the banking cost with respect to loan provision η_{rc}, and with respect to reserves and injections ϕ_{rc}. Note that these parameters are at the heart of the study presented in this chapter. The latter two parameters govern the model's mechanism which explains an important channel through which the 2008/2009 financial crisis and later the nonstandard balance sheet policy measures worked. We estimate those parameters to avoid biasing the estimations and simulations in either direction. As there is no guidance about the prior shapes we decided to assume loose gamma distributed priors.

4.3.5 Estimation results

Table 4.3 summarizes the posterior means of the estimated parameters, as well as the 90% probability intervals.

For most structural parameters the mean of the posterior distribution is found to be relatively close to the mean of the prior assumptions. Exceptions are ω_w and ι_p. The mean of the parameter measuring the degree of wage stickiness ω_w is estimated to be far higher than 60. The mean of the degree of price indexation to inflation ι_p is estimated to be much less than 0.5. Together with the posterior estimates for the parameters measuring the degree of price stickiness ω_p, and indexation of wages to inflation ι_w, results imply that the labor market nominal rigidity is more important than the goods market nominal rigidity. These results are in line with the estimations of Gerali et al. (2010). The posterior means of the investment adjustment cost γ_I, and the labor share in production α, are close to the estimates which Smets and Wouters (2007) find for their canonical medium scale DSGE model. As in Smets and Wouters (2007) the estimate of the investment adjustment cost is notably higher than assumed a priori, suggesting a response of investment to the changes in the value of capital which is even slower than expected. Further, the posterior mean of the consumption habit parameter h is higher than assumed for the prior distribution, indicating a significantly higher desire to smooth consumption. However, h is higher than in Smets and Wouters (2007) and with a mean value of

Table 4.3: Parameter estimates

Parameters		Prior			Posterior	
		Type	Mean	STD	Mean	[5% − 95%]
Structural parameters						
Labor share in production	α	B	0.7	0.05	0.71	[0.70 − 0.73]
Investment adj. cost	γ_I	G	4.0	1.5	5.67	[4.13 − 7.11]
Banking cost elast. loans	η_{bc}	G	6.0	3.46	0.38	[0.32 − 0.43]
Banking cost elast. reserves	ϕ_{bc}	G	6.0	3.46	0.25	[0.19 − 0.31]
Consumption habit	h	B	0.7	0.1	0.91	[0.89 − 0.93]
Inverse of Frisch elast.	υ	G	2.0	0.75	0.89	[0.41 − 1.32]
Deposit utility	φ_d	G	6.0	3.46	3.36	[0.56 − 6.04]
Inv. elast. of capital util. cost	$\varkappa$	G	1.0	0.25	1.02	[0.62 − 1.38]
Degree of price stickiness	ω_p	G	60	20	87	[69 − 104]
Degree of wage stickiness	ω_w	G	60	20	153	[105 − 198]
Price indexation to past infl.	ι_p	B	0.5	0.15	0.04	[0.02 − 0.07]
Wage indexation to past infl.	ι_w	B	0.5	0.15	0.63	[0.43 − 0.82]
Taylor Rule Parameters						
Interest rate smoothing	ρ_r	B	0.6	0.15	0.76	[0.72 − 0.79]
Resp. to inflation	ρ_π	G	1.5	0.15	1.80	[1.62 − 1.98]
Resp. to output	ρ_y	G	0.125	0.05	0.06	[0.04 − 0.09]
Resp. to output growth	ρ_{dy}	G	0.125	0.05	0.18	[0.14 − 0.21]
Autocorrelation of shocks						
Technology	ρ_a	B	0.5	0.2	0.91	[0.88 − 0.95]
Preference	ρ_ξ	B	0.5	0.2	0.50	[0.42 − 0.59]
Gov. spending	ρ_g	B	0.5	0.2	0.93	[0.89 − 0.96]
Invest. spec. tech.	ρ_ϵ	B	0.5	0.2	0.23	[0.13 − 0.33]
Price mark-up	ρ_p	B	0.5	0.2	0.03	[0.01 − 0.05]
Wage mark-up	ρ_w	B	0.5	0.2	0.39	[0.27 − 0.52]
Banking cost	ρ_ζ	B	0.5	0.2	0.04	[0.01 − 0.09]
Deposit management	ρ_μ	B	0.5	0.2	0.85	[0.82 − 0.87]
Coupon decay factor	ρ_{ρ^s}	B	0.5	0.2	0.92	[0.89 − 0.96]
Long term bond supply	ρ_b	B	0.5	0.2	0.98	[0.97 − 0.99]
Standard deviation of innovations						
Technology	σ_a	IG	0.005	2	0.007	[0.006 − 0.008]
Preference	σ_ξ	IG	0.005	2	0.059	[0.047 − 0.071]
Gov. spending	σ_g	IG	0.005	2	0.017	[0.015 − 0.018]
Invest. spec. tech.	σ_I	IG	0.005	2	0.164	[0.126 − 0.203]
Price mark-up	σ_p	IG	0.001	2	0.059	[0.047 − 0.071]
Wage mark-up	σ_w	IG	0.005	2	0.319	[0.214 − 0.423]
Banking cost	σ_ζ	IG	0.005	2	1.554	[1.404 − 1.699]
Deposit management	σ_μ	IG	0.005	2	0.078	[0.058 − 0.086]
Coupon decay factor	σ_{ρ^s}	IG	0.005	2	0.006	[0.004 − 0.008]
Long term bond supply	σ_b	IG	0.005	2	0.018	[0.017 − 0.019]
Interest rate shock	σ_{r_m}	IG	0.001	2	0.002	[0.002 − 0.003]
Money supply shock	σ_{Coll}	IG	0.01	2	0.140	[0.122 − 0.157]

Note: $\mathcal{B}, \mathcal{G}$, and $\mathcal{IG}$ correspond to Beta, Gamma, and inverse Gamma distributions. Posterior densities were computed by creating a sample of 1'000'000 draws with initial burning sample of 500'000 draws. Average acceptance rate of the chain was roughly 25%. The estimation sample is 1964:Q3 - 2012:Q3.

0.91 even out of the range of the estimates of Justiniano, Primiceri, and Tambalotti (2008). The inverse of the Frisch elasticity of labor supply υ, and the inverse elasticity of capacity utilization are estimated to be close to unity which is for both cases lower compared to the estimates by Smets and Wouters (2007). In general, the data are found to be informative on the structural parameters.

Regarding most of the exogenous disturbances the data appear to be informative as well. This is however, not the case for the standard deviations of the innovation processes for the price markup and wage mark-up, σ_p and σ_w, as well as for the AR(1) coefficient of the wage markup process ρ_w. The AR(1) coefficients for the processes governing technology ρ_a, government spending ρ_g, fraction of reserves required for deposit management ρ_μ, coupon decay factor ρ_{ρ^s}, and long-term bond supply ρ_b, are estimated to be highly persistent. In contrast to that, the persistence parameter for the AR(1) process governing the banking cost innovations is found to be close to zero.

Regarding the parameters describing the central bank's policy rate rule, we estimate a relatively high value for the posterior mean of the coefficient governing the reaction to steady state deviations of inflation ρ_π (1.81). Further, the estimate of ρ_{r_m} implies a considerable degree of interest rate smoothing (0.76). The estimates of ρ_y (0.06), and ρ_{dy} (0.17), imply that interest rate policy reacts weaker than expected to deviations from the output steady state, but stronger than expected to changes in output growth. These results are in the range of the estimates of Justiniano, Primiceri, and Tambalotti (2008).

For the estimates of the parameters governing the cost of financial intermediation, namely the loan provision elasticity η_{rc}, and the elasticity with respect to reserves and injections ϕ_{rc}, the data appear to be informative. The posterior means for these parameters are estimated to be 0.38 and 0.25. This provides evidence that the model's mechanism for financial intermediation contributes to explain variation in real macroeconomic variables. Note that data do not seem to be informative about the utility's parameter φ_d.

4.3.6 Business cycle moments

To assess the empirical fit of the model we calculate in this section the model-implied moments at the posterior mean conditional on the structural shocks and compare them to the observed moments of the data sample.

Table 4.4: Selected moments of observed data and model-implied moments

	Standard deviation		Std. deviation rel. to output		Correlation with output		Autocorrelation of order 1		Autocorrelation of order 4	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Output	3.39	5.19	1.00	1.00	1.00	1.00	0.97	0.98	0.79	0.84
Consumption	3.48	4.92	1.03	0.95	0.93	0.84	0.98	0.98	0.87	0.89
Investment	11.08	19.50	3.26	3.75	0.78	0.90	0.95	0.97	0.68	0.78
Inflation	0.59	1.18	-	-	-0.17	0.73	0.87	0.94	0.76	0.67
Hours worked	3.47	6.03	1.01	1.16	0.82	0.80	0.97	0.89	0.79	0.66
Wages	2.62	3.61	0.77	0.69	0.13	0.77	0.96	0.97	0.84	0.81
Reserves	67.35	61.43	19.86	11.83	-0.35	-0.01	0.94	0.09	0.75	0.01
Deposits	9.29	8.07	2.74	1.55	0.25	0.21	0.97	0.98	0.84	0.93
Policy rate	0.84	2.26	-	-	-0.13	0.85	0.95	0.97	0.78	0.78
Loan rate	0.62	1.22	-	-	-0.46	0.09	0.97	0.35	0.86	0.24
Deposit rate	0.81	0.72	-	-	-0.13	0.18	0.95	0.72	0.79	0.52
7yr. Treas. yld.	0.66	0.71	-	-	-0.32	0.14	0.95	0.92	0.84	0.75

Notes: The model-implied moments are computed from the solution of the model at the posterior mean. The standard deviations of inflation and the interest rates are expressed in annual terms.

Here we focus on the model's business cycle implications in terms of selected moments.[69] Table 4.4 compares the standard deviations, correlations with output, and autocorrelations of the observed data with the corresponding model-implied moments. Comparing the model-implied standard deviations with the observed data's standard deviations shows that output, consumption, investment, inflation, hours worked, wages, the policy rate, the loan rate, and the 7 year Treasury yield are more volatile in the model, whereas deposits, reserves, the deposit rate are less volatile in the model. Note that the high standard deviation of the reserves time series is mainly a result of the large unconventional reserves provision induced by the LSAP programs.[70] Output and consumption volatility in the model exceed the observed volatilities in the data by the factor 1.5. The implied volatility of investment, inflation, and hours worked exceed the observed volatilities by roughly the factor 2. These implications are except for inflation volatility close to the results by Justiniano, Primiceri, and Tambalotti (2008). However, the implied volatility of the policy rate exceeds the observed volatility by almost the factor 3. Note that the model-implied standard deviations calculated relative to the output standard deviation are very close to the respective moments calculated for the data sample. This however, is not the case for the model-implied volatility of reserves and the volatility of deposits relative to output.

We find some significant discrepancies for the comparison of the model-implied variables' correlations with output versus the observed variables' correlations with output. While the model-implied correlations of consumption, investment, hours worked, and deposits, match the observed correlations with output rather well, implied correlations of the models interest rates with output have positive signs compared to their empirical counterparts which have negative signs.[71] Furthermore, the observed correlation between inflation and output is negative while the model-implied correlation coefficient is positive. However, the models of Smets and Wouters (2007) and Justiniano, Primiceri, and Tambalotti (2008) are able to repli-

[69]The evaluation of a model's empirical fit by such a moment comparison is widely applied in the literature on DSGE models (see Prescott (1986), King and Rebelo (1999), and Gali and Rabanal (2004)).

[70]For the data sample ranging from 1964:Q3 to 2007:Q4 the standard deviation of reserves is 14.12. Compared to that the model estimated for the same data sample implies a standard deviation of 12.63.

[71]Note that it is particularly problematic to match the observed second moments of interest rates and inflation.

cate this negative correlation. The model-implied autocorrelation patterns match the observed autocorrelation patterns rather well although there are again some exceptions. Notably, the implied autocorrelation coefficients for reserves and the loan rate are significantly different from their observed empirical counterparts.

4.3.7 Variance decomposition

To assess the relative contributions of the model's structural shocks to the variation in the observed data we conduct an unconditional posterior variance decomposition. Results reported in Table 4.5 imply that in terms of driving forces we can distinguish between variables which are usually analyzed within the canonical macroeconomic model framework and variables newly introduced for the purpose of the present study. The former group is comprised of output, consumption, investment, hours worked, inflation, and the policy rate. Results show that these variables are mainly driven by shocks which are as well commonly employed in the canonical approach. In this regard the shocks are namely on preferences ε_ξ, technology ε_a, price markup ε_p, wage markup ε_w, investment specific technology ε_ϵ, the monetary policy rate ε_r, and government spending ε_g. Overall, the decomposition of driving factors for the variation of these macroeconomic variables yields results which are in line with previous evidence (see Smets and Wouters (2007)).

Regarding the variables which are not part of the canonical model framework, we find that the observed variation of reserves is almost completely explained by the money supply shock ε_{Coll}. Note that for the present model estimation, where we assume that the parameter governing the share of short-term bonds eligible for open market operations κ_t^S is unity, and that the fraction of long-term bonds eligible for repos κ_t^L is equal to zero, an unconventional central bank money supply can only be explained by innovations to ε_{Coll} (see 4.42). The variation of the observed series on deposit holdings is found to be mainly driven by the shock on the supply of long-term government bonds ε_b. This is due to the close relationship between financial intermediaries's assets and liabilities captured by the bank balance sheet constraint (4.25). Further, results show that shocks to technology, the price markup, and the wage markup, explain roughly 82 percent of the variation in the loans series. This reflects that the model-implied loan demand mechanism is closely connected to key variables of the model's firm sector. The banking cost shock ε_ζ contributes about 7.5 percent to the variation in

Table 4.5: Variance decomposition of observed variables

Variable			**Shocks**			
	preference	technology	interest rate	mark-up prices	mark-up wages	money supply
Output	2.04	28.59	3.74	42.57	17.26	0.12
Consumption	10.00	27.87	3.22	36.51	14.72	0.13
Investment	8.13	23.65	3.30	37.68	15.93	0.07
Hours worked	1.30	7.36	4.57	52.13	19.84	0.57
Wages	3.71	38.62	2.59	33.23	17.56	0.06
Inflation	2.85	6.04	6.32	65.34	5.93	0.10
Reserves	0.00	0.01	0.00	0.05	0.01	99.91
Deposits	0.12	1.09	0.31	3.92	0.72	0.07
Loans	1.80	16.30	4.52	54.51	10.60	1.36
Policy rate	3.31	9.93	3.47	63.18	7.99	0.10
Deposit rate	0.16	1.12	0.30	4.67	0.77	21.81
Loan rate	0.06	0.40	0.11	1.70	0.28	23.53
7yr. Treas. yld.	0.10	0.88	0.18	2.63	0.55	0.56
	banking cost	invest. spec. tech.	gov. spending	deposit man.	decay factor	long-term bond sup.
Output	0.99	2.83	0.52	1.03	0.00	0.31
Consumption	0.85	3.12	2.36	0.98	0.00	0.26
Investment	0.87	7.81	1.27	1.00	0.00	0.29
Hours worked	6.79	4.88	0.85	1.33	0.00	0.33
Wages	0.66	1.92	0.78	0.58	0.00	0.78
Inflation	1.33	3.56	0.69	2.29	0.00	5.47
Reserves	0.00	0.00	0.00	0.00	0.00	0.00
Deposits	0.43	0.12	0.02	0.25	0.00	92.93
Loans	7.37	2.05	0.41	0.68	0.00	0.40
Policy rate	1.37	2.72	0.70	2.26	0.00	4.95
Deposit rate	0.27	0.14	0.03	47.65	0.00	23.08
Loan rate	38.82	0.05	0.01	24.36	0.00	24.36
7yr. Treas. yld.	1.36	0.06	0.01	17.45	53.26	24.26

Notes: Values refer to the unconditional variance in percentage terms at the posterior mean.

loan supply and almost 40 percent to the variation in the loan rate. This implies that loan supply and the loan rate are to a significant extent driven by the present model's proposed banking cost channel.[72] Further ε_ζ contributes about 7 percent to the variation in hours worked which reflects the model's link between loan supply and labor demand (see 4.14). The shock to the fraction of reserves required for deposit management ε_μ explains more than 45 percent of the variation in the deposit rate, about 25 percent of the variation in the loan rate, and about 18 percent of the variation of the 7-year Treasury notes yield. The relative importance of the shock to the fraction of reserves required for deposit management is due to μ_t which directly governs the cost of financial intermediation in (4.27). The variation in the observed 7-year Treasury Notes yield is to roughly 45% driven by the decay factor shock ε_{ρ^s} which is due to the relationship (4.24).

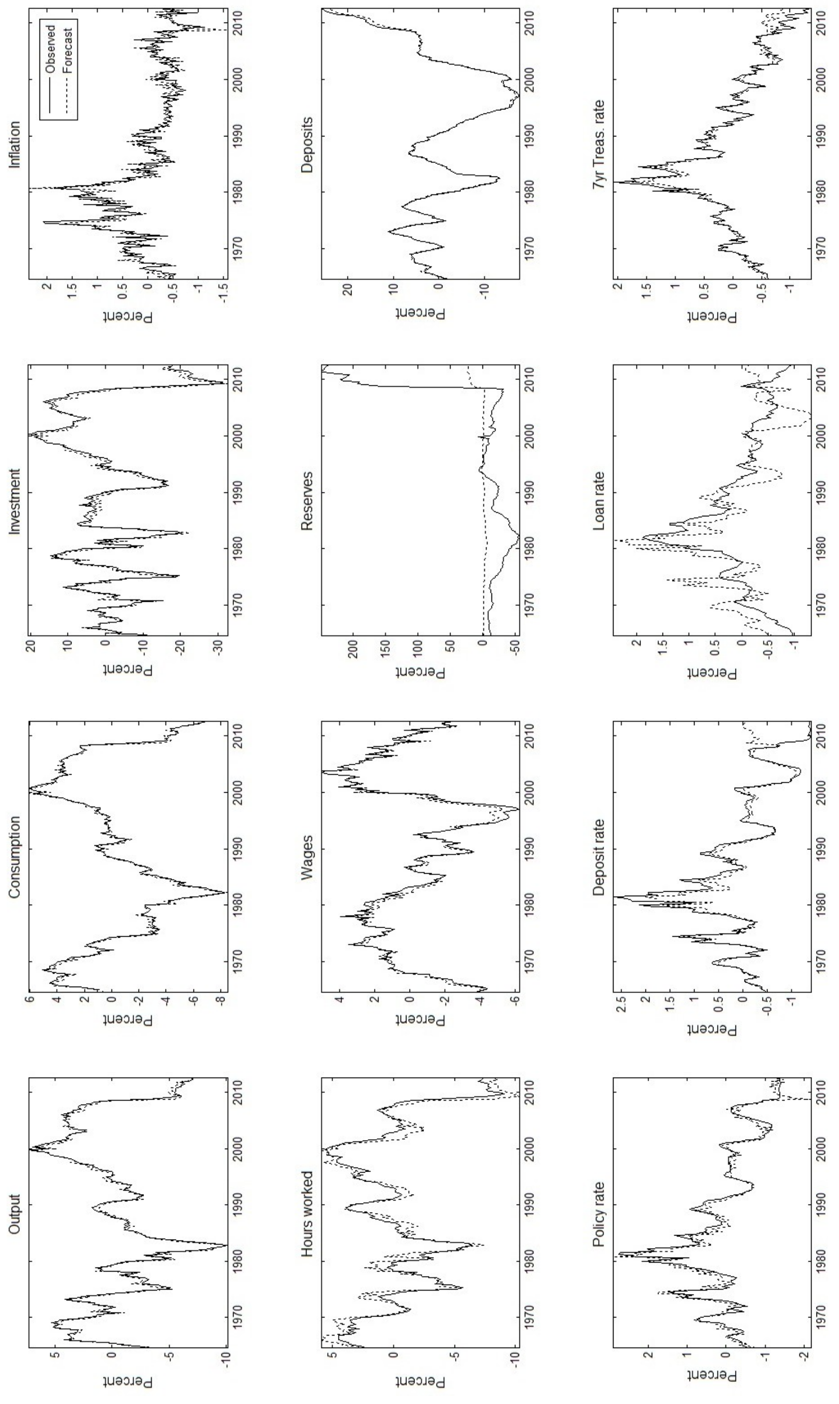
The analysis of the unconditional posterior variance decomposition implies that the nonstandard shocks except for the banking cost shock yield no significant contribution to the variation of the model variables which are commonly found in the canonical approach. The shocks known from the canonical model are found to only contribute a sizeable share to the variation of loans.

4.3.8 Forecasting performance

In Figure 4.2 the time series of the observed macroeconomic variables are plotted together with their respective one-step ahead forecasts implied by the model. The computation of the posterior distribution of one-step ahead forecasts, i.e. $E_t y_{t+1}$, is done by applying the Kalman filter at the posterior mean. By visual inspection one can easily assert that the model provides relatively small one-step ahead forecast errors for output, consumption, investment, inflation, hours worked, wages, deposits, the policy rate, deposit rate, and the 7yr Treasury securities rate. In contrast to that the model-implied one-step ahead forecasts for the expected deviations of the loan rate from the long-run mean considerably overpredict and underpredict the actual deviations for most periods. Furthermore, expected model-implied deviations of reserves injections from their linear trend hardly match the actual dynamics of reserve injections at any period. This is due to the fact that the model has to generate highly autocorrelated sequences of positive

[72] Note that we use data from 1964:Q3 to 2012:Q3 to estimate the model. We expect the contribution of the credit cost shock to the variation of loan supply and the loan rate to increase during the financial crises relative to the pre-crisis period.

Figure 4.2: Observed data and one-step ahead forecasts



Notes: The one-step ahead forecasts for the period 1964:Q3 - 2012:Q3. Real variables are measured in percentage deviations from their linear trend. Inflation and interest rates are measured in annualized percentage deviations from their sample mean.

and negative money supply shocks ε_{Coll}, in order to match the dynamics of observed reserve injections. This however, comes at the expense of bad forecast performance for reserves. We further compute the mean forecast errors (MFEs) as a measure of how accurate the model forecasts the observed percentage deviations of key macroeconomic variables from their trends and means. Further, the MFE is a means to assess whether a variable is systematically overpredicted or underpredicted. Results which are calculated based on the one-step ahead forecasts are reported in Table 4.6.[73] They indicate that the present model tends to considerably underpredict investment. Further, it is implied that output and hours worked are slightly underpredicted. Most of the mean forecast errors are however, close to zero implying generally an accurate forecasting performance of the model. Note that the zero value of the MFE for reserves holdings is due to the construction of the measure. Forecast errors may sum up to zero despite of the fact that forecast errors might be large and show a highly autocorrelated pattern (see Figure 4.2).

4.3.9 Model dynamics

In this section of the present chapter we examine the short-run dynamics of the estimated model. In particular, we compute impulse responses to unexpected innovations to the model's stochastic processes. We analyze the impact of changes in policy variables on the equilibrium allocation and prices. For that purpose the model is solved by using a first-order local approximation of the model's equilibrium conditions around the deterministic steady state.[74] The percent deviation of each of the model's real variables z_t from its steady state value z is given by $\hat{z}_t = 100 \cdot [\log(z_t) - \log(z)]$. Nominal interest rates and inflation are calculated as annualized absolute deviations $\hat{R}_t^* = 400 \cdot \left(\hat{R}_t^* - \hat{R}^*\right)$, implying that $\hat{R}_t^* = 1$ is an increase of the respective gross interest rate by 100 basis points. Note that we assume for the subsequent analysis that $\kappa_t^L = 0$.

Response to a monetary policy shock Figure 4.3 shows estimated mean impulse response functions to a monetary policy shock, measured by an innovation to the policy rate (see 4.38). An increase in the policy rate

[73] Note that zero MFE values are due to rounding error.

[74] The full set of (non-linearized) equilibrium conditions can be found in Appendix C.1.

Table 4.6: One-step ahead forecast errors

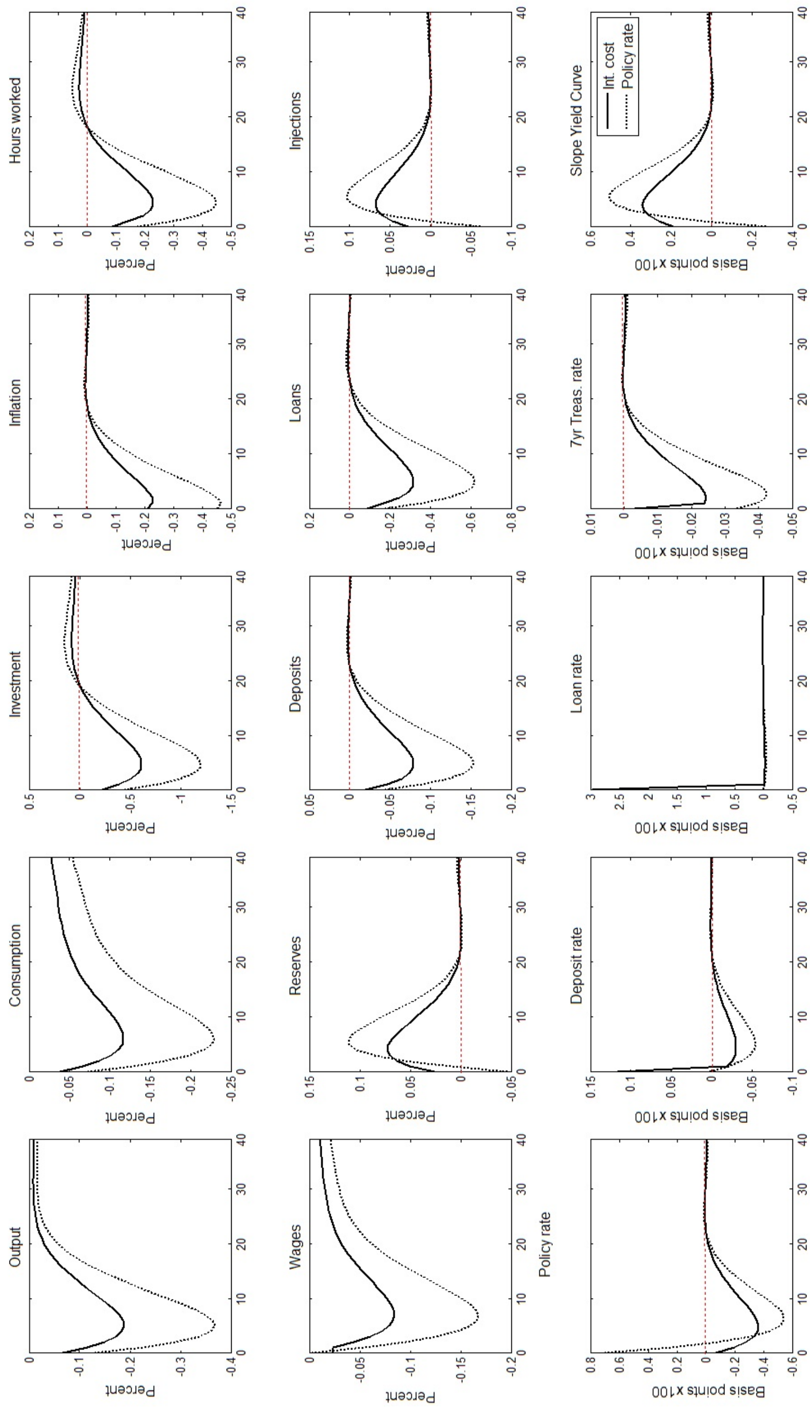
	Mean forecast error MFE
Output	0.04
Consumption	0.02
Investment	0.16
Inflation	0.00
Hours worked	0.08
Wages	0.00
Reserves	0.00
Deposits	0.02
Policy rate	0.01
Loan rate	0.00
Deposit rate	0.00
7yr. Treas. yld.	0.00

Notes: The forecast errors F_t are computed as the difference between the observed variable x_t and its one-step ahead forecast x_t^f as $F_t = x_t - x_t^f$. The MFE is calculated by $MFE = T^{-1} \sum_{t=1}^{T} F_t$.

(i.e. the main financing rate) reduces the amount of real central bank money that can be acquired by banks in open market operations (see 4.26). Accordingly the monetary tightening increases the real cost of central bank money acquisition. This induces the banks to gradually decrease loan provision and to decrease their deposits holding. Therefore, the hump-shaped responses of deposits and loans to a monetary policy shock are due to the reduced amount of reserves acquirable in equilibrium. The decreased loan provision induces a reduction of the wages which have to be paid up front by the firms. This leads in turn to a decline in hours worked, and thus to a decline in output and consumption. Hence, the monetary policy shock exerts a contractionary effect on the allocation of resources. Furthermore, the slope of the yield curve which is defined as the difference between the long-term bond rate and the short-term bond rate declines on impact of the shock. This is caused by a positive response of the short-term bond rate and a decline in the long-term bond rate. As short-term bonds are eligible for repos in open market operations their price carries a liquidity premium. Due to the assumption that $\kappa_t^L = 0$, the price of long-term bonds does not carry a liquidity premium. The liquidity premium declines as the policy rate shock lowers the short-term bonds' value as a means for the real reserves acquisition. The long-term bond rate declines as the monetary tightening increases the expected marginal contribution of reserves to a reduction in the cost of loan creation $E_t[\Xi_{m,t+1}]$ (see 4.31). This reflects the notion that banks as investors value the fact that future expected payoffs from holding long-term bonds might be used to acquire reserves. This implies an increased demand for long-term bonds which drives up their prices and decreases their yields. This reflects a shift in investors' portfolios from short-term bonds to long-term bonds. Note that after the impact of the innovation, the policy rate decreases with a fast pace which is leading after a few periods to a negative deviation of the policy rate from its steady state value. This is due to the strong decline in inflation and output which is induced by the monetary policy rate shock. This forces the Taylor rule type of policy reaction function to gradually lower the policy rate. This in turn increases the amount of reserves that can be acquired in open market operations which leads to a quick recovery of output towards the steady state.

Response to a banking cost shock Figure 4.3 shows impulse responses estimated for a shock to the cost of financial intermediation, measured by an innovation to the scaling factor ζ_t which governs the cost of loan creation Ξ_t (see 4.27). The increase in the banks' cost of financial intermediation induces a decline in loan provision and deposit holdings. Both variables respond in a hump-shaped way similar to the pattern described in the paragraph above on the model's response to a negative monetary policy shock. The notable difference between the impact of a banking cost shock and the impact of a monetary policy shock on the model's variables is that the former does not exert a negative effect on the value of short-term bonds for the asset exchange against real central bank money. Therefore, immediately after the shock to ζ_t occurs, the financial intermediary can acquire additional real reserves to alleviate the increase in the cost of financial intermediation. Specifically, the banking cost shock induces reserves and injections to respond positively on impact while these variables respond negatively on impact in the case of a policy rate shock. This implies that in absence of the asset exchange mechanism of short-term bonds against reserves, the banking cost shock would have a way stronger negative impact on the real macroeconomic variables. This seems to be even more likely when turning

Figure 4.3: Estimated impulse responses to contractionary shocks



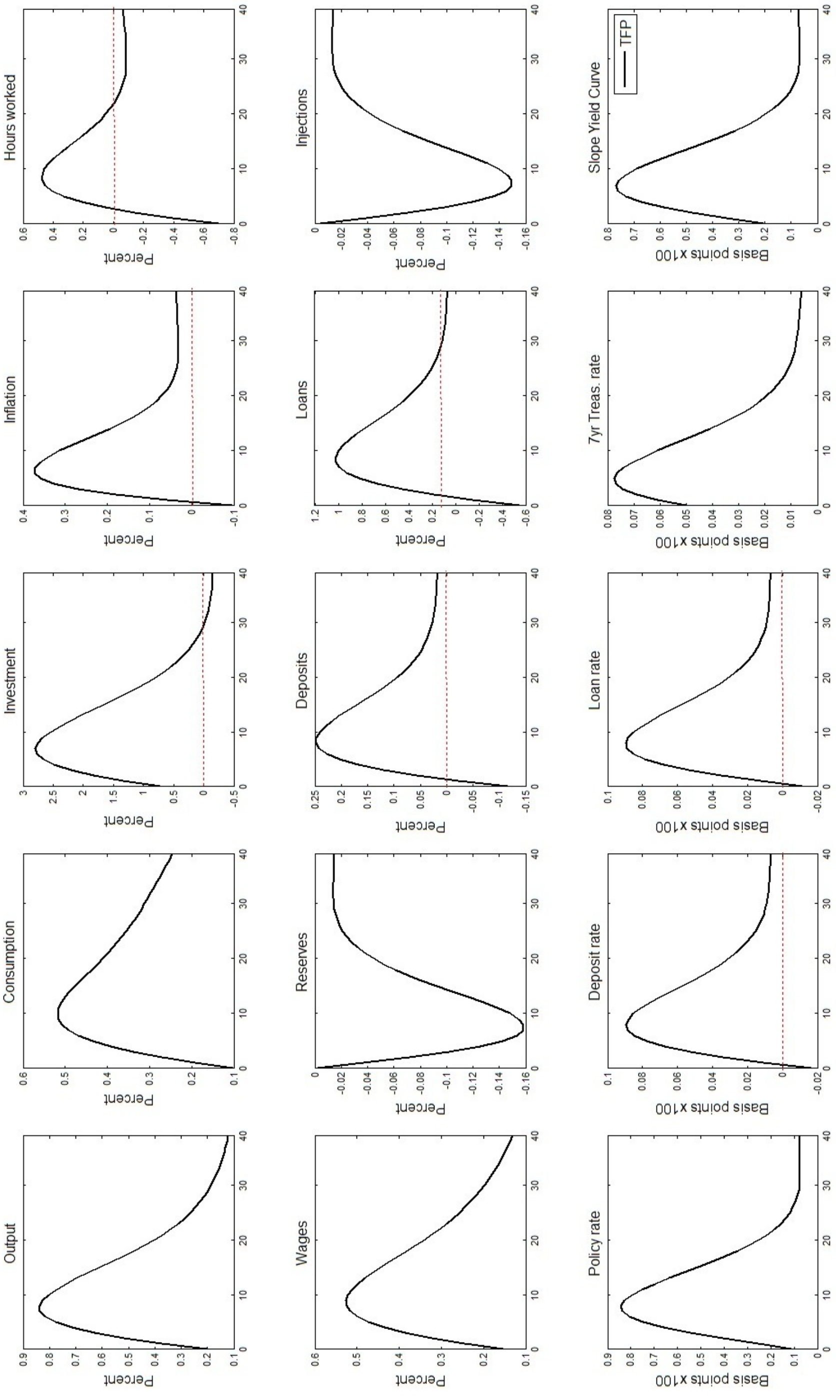
Notes: Estimated impulse responses are calculated at the posterior mean. Real economic variables are in percentage deviations from the steady state. Inflation and interest rates are in percentage point deviations from the steady state.

to the impulse response functions for the interest rates. Accordingly, the banking cost shock induces on impact a rise of the loan rate by 297 basis points which would have been even stronger in the absence of the model's asset exchange mechanism. The implied loan rate returns back to its steady state value after only two periods. In addition to the correspondent increase in real reserves one important reason for this observation is a low estimated persistence parameter ρ_ζ for the process governing ζ_t. Further, the banking cost shock induces the slope of the yield curve to increase. The long-term bond rate decreases at maximum by less than 3 basis points induced by the shock. Hence, the rise in the slope of the yield curve is mainly due to a large negative response of the short-term bond rate. As for the present analysis only short-term bonds are eligible for open market asset exchange operations, the increased demand for reserves will increase the demand for short-term bonds. This drives up short-term bond prices and therefore, increases their liquidity premium.

Response to a total factor productivity shock Figure 4.4 shows impulse responses for a shock to total factor productivity. The unexpected increase in total factor productivity induces a positive response of production, consumption, investment, hours worked, and wages. The increased wage rate and the increase in hours worked require the firms to demand more loans (see 4.14). Further, an increase in expected output growth leads to a positive response of inflation (see 4.23). By the central bank's monetary policy reaction function an increasing inflation induces a rise in the policy rate. This in turn reduces the amount of real reserves that can be acquired by banks in open market operations. A higher provision of loans together with a reduced reserve acquisition rises the cost of financial intermediation and induces the loan rate to increase gradually. By this mechanism the loan provision will be dampened which forces production and the other macroeconomic variables back to their steady state values. This further explains the hump-shaped paths of impulse responses. The positive response of the slope of the yield curve basically follows the long-term bond rate which outweighs the positive response of the short-term bond rate. The latter is driven by a diminishing liquidity premium which is due to the decreased real value of short-term bonds for reserves acquisition.

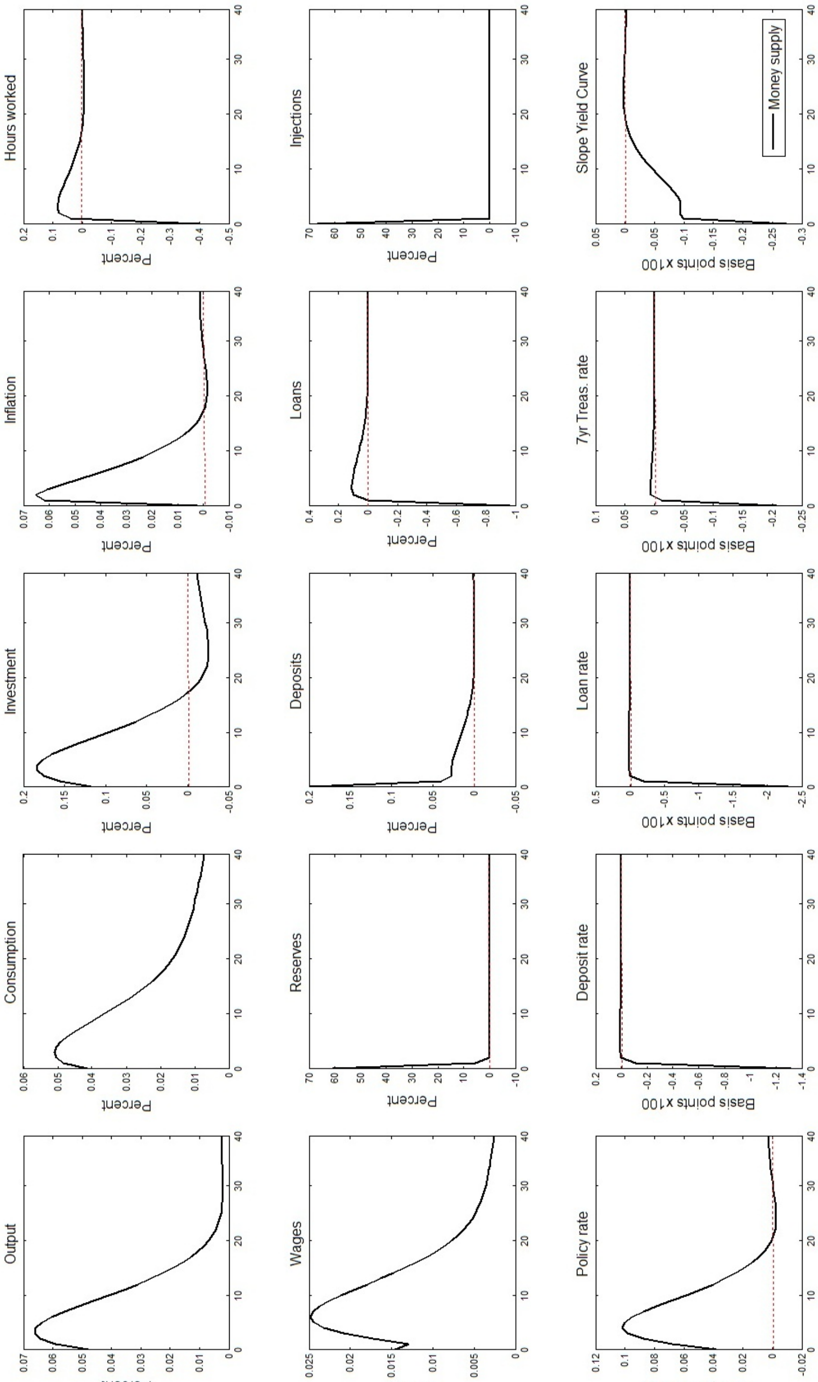
Response to a money supply shock Figure 4.5 shows impulse responses for a money supply shock, which is measured by an innovation

Figure 4.4: Estimated impulse responses to expansionary shock I



Notes: Estimated impulse responses are calculated at the posterior mean. Real economic variables are in percentage deviations from the steady state. Inflation and interest rates are in percentage point deviations from the steady state.

Figure 4.5: Estimated impulse responses to expansionary shock II



Notes: Estimated impulse responses are calculated at the posterior mean. Real economic variables are in percentage deviations from the steady state. Inflation and interest rates are in percentage point deviations from the steady state.

to ε_{Coll} (4.26). The money supply shock affects the banks' cost of loan provision by a direct injection of additional central bank money. This is done at no cost for the banks and without requiring short-term bonds in exchange. Therefore, the shock induces a negative response of the loan rate and the deposit rate which leads to an increase of loan provision. By this, output, consumption, investment, hours worked and wages respond positively. In turn, inflation increases which induces a monetary tightening in terms of an increasing policy rate. Further, the slope of the yield curve decreases on impact as the open market operation shock decreases the liquidity premium on short-term bonds.

4.3.10 Observed variables decomposition

Figure 4.6 shows the model-implied contributions of the macroeconomic shocks to the observed historical variations of real per capita output around its linear trend. The black line depicts the deviations of the value of the corresponding endogenous model variable from its steady state at the posterior mode. The colored bars correspond to the contribution of the respective smoothed shocks to the deviation of the endogenous variable from its steady state. The length of the bars refers to the absolute value of the shock contributions. These values are expressed in percentage deviations from the steady state. They are derived by using the Kalman smoother to compute the most likely shock realizations that lead to the observed values of the data.

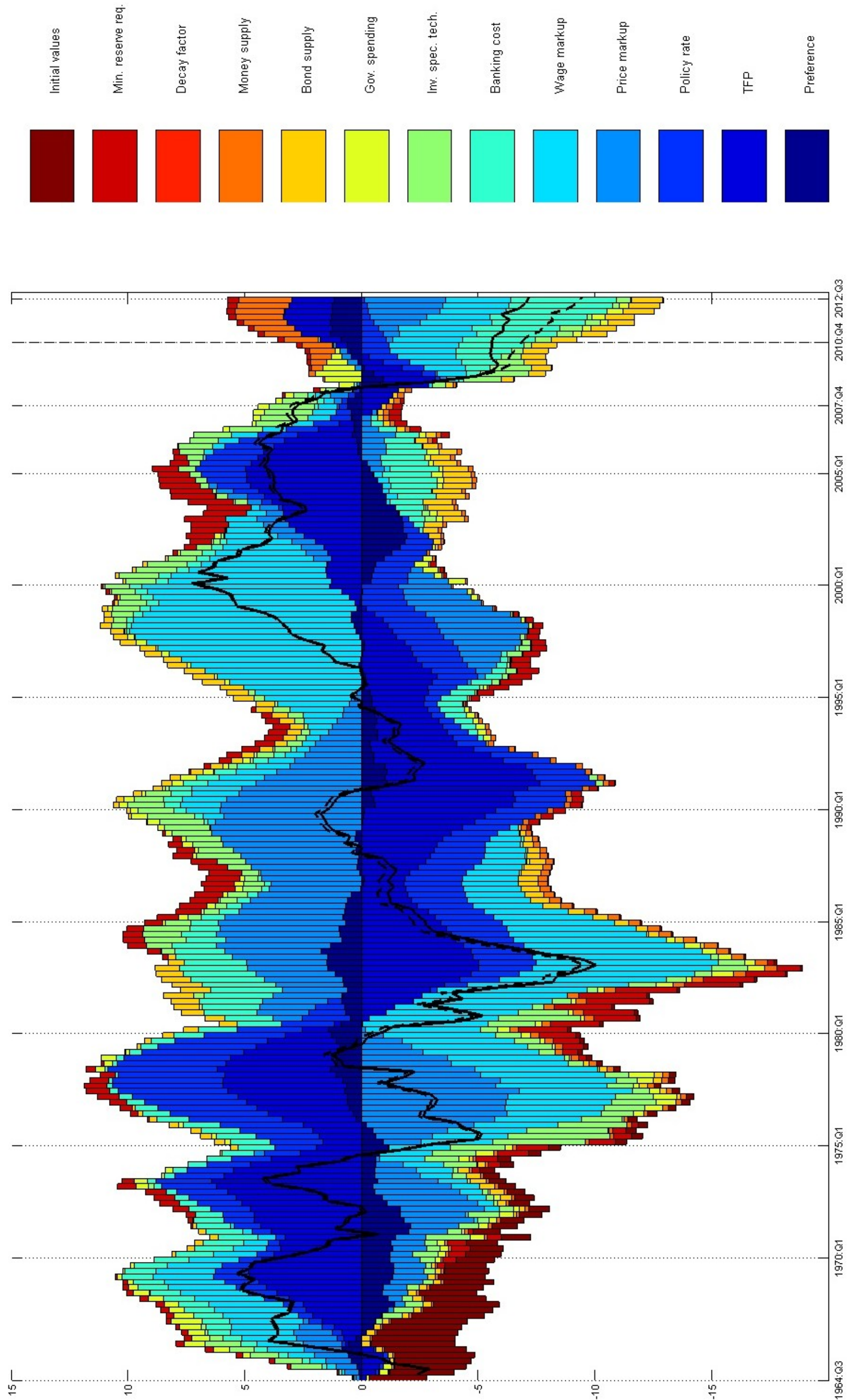
Analyzing the contributions of the model's macroeconomic shocks for the pre-crisis period from 1964:Q3 to 2007:Q4, implies that total factor productivity, policy rate, and the markup shocks are the main drivers of output deviations from its steady state. This is in line with the results by Smets and Wouters (2007). These shocks are common for the canonical DSGE model analysis framework. According to the historical variable decomposition presented in Figure 4.6 the model explains the recession which is observed following the emergence of the 2008/2009 crisis, by a different set of shocks. Specifically, there is a simultaneous contribution of shocks known from the canonical model and shocks which are introduced for the purposes of the present study. In detail, the main model-implied contributors to the negative trend deviation of real per capita output which followed the 2008:Q3 collapse of Lehman Brothers are the total factor productivity shock, the investment-specific technology shock, the markup shocks, and the shock to the cost of financial intermediation.[75] At the same time there are several implied sources of positive contributions to output. The most important among them are the government spending shock, a shock to preference for contemporary consumption, a money supply shock, and with some delay, a positive total factor productivity shock. With the beginning of 2010:Q4, which was the quarter where LSAP 2 was initiated, the observed trend deviation of real per capita output was -5.60 percent. The model-implied contributions of the single shocks to that observed value are for the combined markup shocks -3.26 percent, for the investment-specific technology shock -0.68 percent, for the policy rate shock -1.05 percent, for the banking cost shock -3.12 percent, for the preference shock 0.87 percent, for the total factor productivity shock 0.92 percent, and the money supply shock 1.43 percent. Note that the implied contributions of the money supply shock and the banking cost shock to output variation become even larger following 2010:Q4 while those shocks do not significantly contribute to output deviation prior to 2008:Q3.

There are two conclusions we draw from this evidence. *First*, shocks to financial intermediation increase the model's ability to explain the drop of real per capita output following the 2008/2009 crisis. These shocks significantly contribute to the recession but at the same time they are not the single source of it. *Second*, for the quarters following 2010:Q4, which is the quarter when LSAP 2 was launched, the model implies a significant positive contribution of the money supply shock to the variation in real per capita output. The collateral constraint (4.26) governs the acquisition of extra central bank money and constitutes the model's mechanism for the central bank's non-conventional balance sheet policies. As we set the share of long-term bonds purchased κ_t^L equal to zero so far, we interpret the model-implied contributions of the money supply shock to output variation as the effects of LSAPs.

Figure 4.7 shows the evolution of U.S. real per capita GDP, real per capita consumption, real per capita investment, inflation, hours worked, real wages, the long-term treasury rate, and the loan rate from 2007:Q4 to 2012:Q3. We follow Del Negro et al. (2013) and calculate the deviations

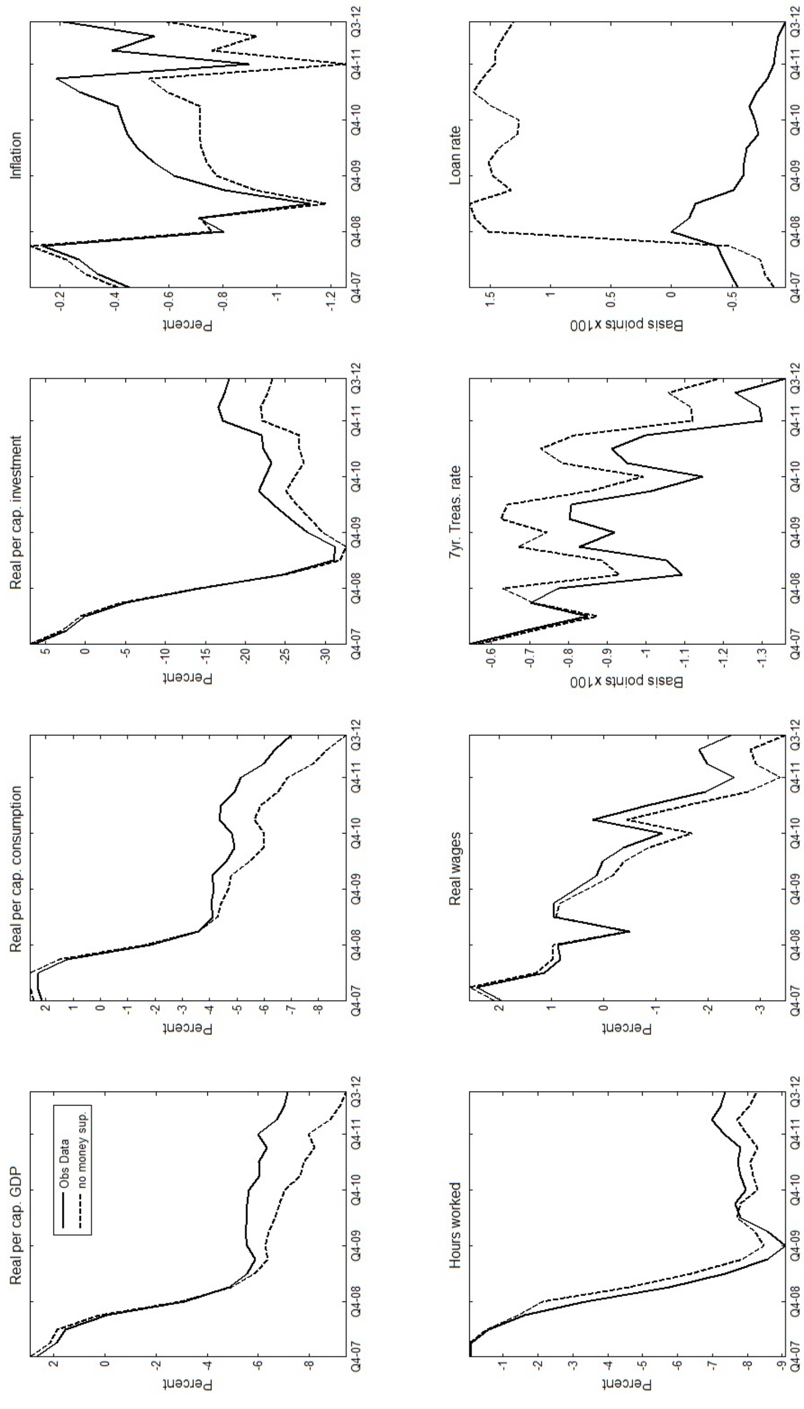
[75] Furthermore the model implies that the policy rate shock is a source of negative output deviation. Note that the observed data on the policy rate which are included in the estimation reach the zero lower bound by 2008:Q3. At the same time the endogenous model mechanisms would require the policy rate to drop even stronger. Therefore, the policy rate shock has to absorb the gap which leads to a negative contribution to output.

Figure 4.6: Observed variable decomposition



Notes: Observed deviations of real per capita output from its linear trend. Variations are explained by the contributions of the twelve included shocks.

Figure 4.7: Dynamics of key macro variables following the crisis



Notes: Evolution of real GDP, real consumption, real investment, inflation, hours worked, real wages, long-term Treasury rate, and loan rate following the 2008/2009 crisis. This is compared to the implied variables' paths excluding the money supply shock.

of the variables from their long-run trend which is extracted from the pre-crisis data subsample ranging from 1964:Q3 to 2007:Q4. This is done except for hours worked, inflation, and the interest rates, for which we calculate deviations from their respective means. In 2010:Q3 which was the quarter before LSAP 2 was launched the observed trend deviation of real GDP was −5.56 percent, of real consumption −4.93 percent, of real investment −27.71 percent, of hours worked −7.66 percent, and of real wages −0.38 percent. By 2010:Q3 the decrease of inflation relative to its long-run mean was −0.43 percentage points. The long-term treasury rate decreased by 45 basis points relative to 2007:Q4 to a −102 basis points deviation from the long-run mean. Further the loan rate decreased by 17 basis points relative to 2007:Q4 to a −72 basis points deviation from the long-run mean.[76]

The dashed lines in Figure 4.7 represent the model-implied paths of the endogenous variables which are calculated from the linear state-space model where the residual of the reserves injections $\hat{\varepsilon}_{Coll}$ is set equal to zero. Note that the model-implied negative deviation of output from the steady state then exceeds the trend deviation of the observed variable by −1.28 percentage points. By the end of the last observed quarter 2012:Q3, the model-implied trend deviation of output exceeds the observed by −2.26 percentage points. This evaluation implies that the recession following the 2008/2009 financial crisis would have been more severe without central bank's unconventional balance sheet policy interventions. Further for the quarter 2012:Q3 we find that the model-implied deviation of real per capita consumption exceeds the observed by −1.99 percentage points, the drop in real per capita investment exceeds the observed by −5.47 percentage points, hours worked decline exceeds the observed decline by −0.89 percentage points, and the drop in real wages exceeds the observed drop by −1.03 percentage points. Further in the absence of the money supply shock the model-implied inflation rate is lower than the observed. For 2012:Q3 the difference between implied inflation rate and the observed is at −0.39 percentage points. Hence, the model implies inflationary effects of non-conventional balance sheet policies. Further, in the absence of the money supply shock the implied long-term Treasury bond rate and the implied loan

[76] Over the period 2007:Q4 to 2012:Q4 the maximum peak effect for real GDP is −7.16 percent, for real consumption −6.99 percent, for real investment −31.21 percent, for inflation −1.24 percentage points, for hours worked −9.08 percent, and for real wages −2.34 percent. The long-term treasury rate falls at maximum by 82 basis points relative to 2007:Q4 and the loan rate increases at maximum by 55 basis points relative to 2007:Q4.

rate are higher than the observed rates. For 2012:Q3 the implied long-term bond rate exceeds the observed by 17 basis points and the implied loan rate exceeds the observed by 225 basis points. Note that unconventional monetary policy measures such as LSAP 2 were intended to lower market interest rates by large-scale asset purchases. This should in turn have been transmitted to private borrowing rates e.g. the loan rate. Therefore, the difference between the observed interest rates and the ex-post calculated implied rates captures the model-implied effect of unconventional monetary policies.

4.4 Simulating LSAP II

The main purpose of the study presented in this chapter is to identify the macroeconomic effects of LSAP 2 on the U.S. economy. As pointed out in Section 4.3.10 from an ex-post point of view the model explains the variation of real per capita output following 2010:Q4, which is the quarter when LSAP 2 was launched, to a significant extent by a shock to money supply. In this section of the chapter we investigate whether this model-impled effect of the money supply shock captures the impact of unconventional balance sheet policies such as LSAP 2. Therefore, in a next step the present study intends to analyze the effects of balance sheet policies as a structural feature of the model while focussing on large-scale purchases of long-term Treasury bonds. For this purpose we employ the model to simulate the recession and the Fed's unconventional balance sheet policies following 2010:Q4. Specifically, we use information on the macroeconomic shocks which was derived in Section 4.3.10, to make the model's simulated endogenous variables match the observed trend deviations of key macroeconomic variables in 2010:Q4.[77] At the same time the model simulation accounts for the LSAP 2 unconventional monetary policy intervention which is comprised of large-scale purchases of long-term Treasuries. In particular, we calibrate the path of κ_t^L, which is the share of long-term bonds purchased under repo contracts, so as to match the observed evolution of the U.S. Fed's long-term treasuries holdings relative to long-term treasuries outstanding. To assess the effectiveness of the large-scale Treasury purchases we follow Del Negro et al. (2013) by analyzing the counterfactual responses of key macroeconomic variables. In particular, responses of the model's variables are simulated

[77] Note that the contribution of the money supply shock to the output deviation from the steady state has to be set equal to zero here.

in the absence of the policy intervention and compared to the variables' simulated responses under inclusion of the policy intervention.

4.4.1 Recession scenario

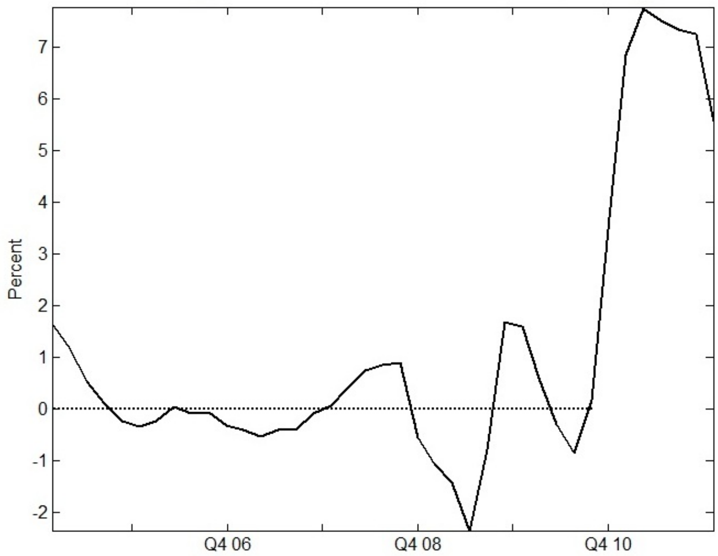
We simulate a recession scenario where the implied dynamics of the endogenous model variables match the observed trend deviations of key macroeconomic variables in 2010:Q4 as presented in Figure 4.7. For this purpose we use information on the model-implied contributions of the shocks to real output trend deviation from the historical variable decomposition which was performed in chapter 4.3.10. Specifically, this information is used for the simulation to determine size and duration of the preference shock, the total factor productivity shock, the markup shocks, the banking cost shock, and the investment-specific technology shock. We simulate LSAP 2 as a series of non-zero values assigned to κ_t^L. Specifically, as κ_t^L is the share of long-term bonds purchased in open market trades, we track the share of the U.S. Fed's holdings of U.S. Treasury notes and bonds relative to total notes and bonds outstanding.[78] We calculate this share starting from the first available data on the Fed's U.S. Treasury notes and bonds holdings in 2003:Q1 to 2012:Q3. As the observed share of Fed holdings of Treasury notes and bonds relative to notes and bonds outstanding is positive over the whole data sample we demean the time series and assume that $\kappa_t^L = 0$ before 2010:Q4 (see Figure 4.8).

The share of long-term Treasuries purchased in open market operations κ_t^L is in the present framework set to zero in the absence of the unconventional policy intervention. Our experiment is to simulate the model under the recession scenario together with the policy intervention specified by nonzero values of κ_t^L. This is compared to a model simulation of the recession scenario without the policy intervention. The aim of this counterfactual policy analysis is to identify the effects of LSAP 2 on the U.S. economy. Note that LSAP 1 started in late 2008 and lasted until 2010 while LSAP 2 was launched in 2010:Q4 and ended in 2011:Q2. Thus, our recession scenario takes the potential impact of LSAP 1 on the U.S. economy

[78] We use quarterly time series data on U.S. Treasuries outstanding and on U.S. Treasuries held by the Fed. The series on the amount of Treasury Notes outstanding (Code 440109711) and on Treasury Bonds outstanding (Code 440109712) are from Datastream. From the Federal Reserve Bank of St. Louis FRED databank we take the series on U.S. Treasury securities held by the Fed, maturing in 1 to 5 years (TREAS1T5), maturing in 5 to 10 years (TREAS5T10), and maturing in over 10 years (TREAS10Y).

as given without explicitly specifying this policy measure within the model framework.

Figure 4.8: Fed holdings of Treasury notes and bonds rel. to total outst.



Notes: Observed share of Fed holdings of Treasury notes and bonds relative to Treasury notes and bonds outstanding.

Note that we use Dynare to compute a simulation of the model. As explained above we specify deterministic paths for exogenous shocks. Hence, we employ Dynare's *forecast* command for the simulation. In this case, the trajectories of the endogenous variables are computed conditionally on the agents knowing the future values of the deterministic exogenous variables. Therefore, we would have to presume that agents know the entire path of the share of long-term Treasuries purchased in open market operations κ_t^L for the whole duration of the LSAP program. However, there is no clear information about the future path of κ_t^L following the last observed value in 2012:Q3. Therefore, we assume that after the last observed value, the share of U.S. Treasury notes and bonds held by the Fed relative to total notes and bonds outstanding is in each following quarter reduced by roughly 25 percent. This value matches the decrease of the market share of long-term bonds held by the Fed from the second last observed quarter (7.25 percent) to the last observed quarter (5.54 percent).

4.4.2 Simulation results

In this section we simulate and compare the effects of the crisis scenario on the model's endogenous variables for the case of the balance sheet policy intervention which is interpreted as LSAP2 program versus the case of no policy intervention. This is done by calculating the trajectories of the stochastic model's variables starting from the steady state. Figure 4.9 shows the trajectories of the endogenous variables and compares them to the model's implied variables' paths in the absence of the LSAP 2 program. The deviation of each of the model's real variables from the steady state value is denoted in percentage terms. Nominal interest rates and inflation are given in annualized absolute deviations.

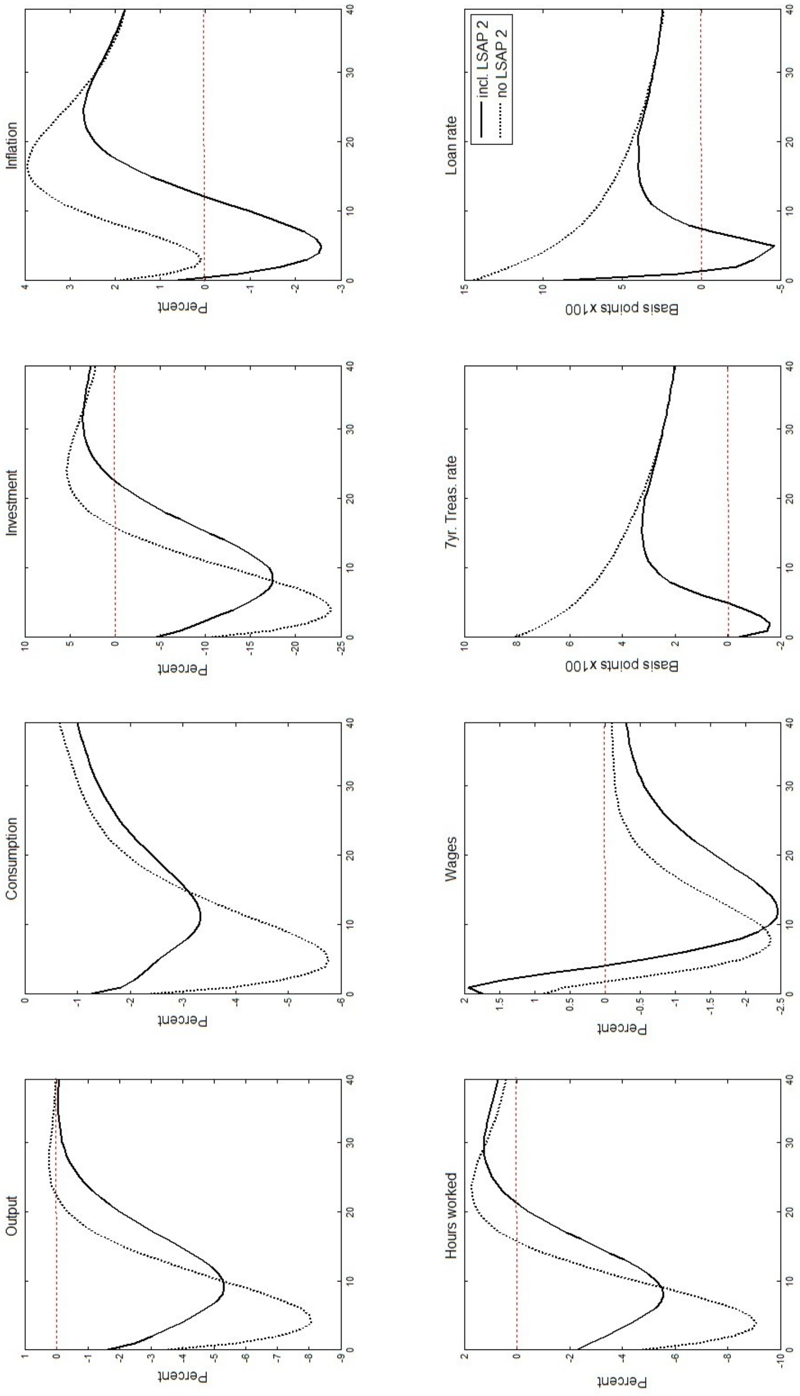
The solid lines in Figure 4.9 show the response of output, consumption, investment, inflation, hours worked, real wages, the long-term treasury rate, and the loan rate to the crisis scenario together with the policy intervention.[79] The model predicts a simultaneous drop in output by -5.31 percent (versus -5.61 percent observed in 2010:Q4), real consumption by -3.33 percent (-4.83 percent), real investment by -17.49 percent (-22.43 percent), hours worked by -5.56 percent (-7.87 percent),and real wages by -2.74 percent (-1.12 percent). Comparing the movements of the macroeconomic variables simulated by the model with their observed empirical counterparts, shows that the model is capable to reproduce the effects of the recession on the real economy.[80]

The dashed lines in Figure 4.9 depict the responses of the endogenous model variables to the crisis scenario without the policy intervention. In general the model's equilibrium conditions imply that an increase in the share of long-term Treasuries purchased in open market operations κ_t^L increases the banks' amount of acquirable central bank money. This directly

[79] Same as impulse response functions the trajectories show an expected path of a given variable from an ex ante perspective, not a particular realized path in the model. Impulse response functions plot the expected path of variables conditional on the shock hitting at the beginning of the first period. Specifically, all possible state contingent paths induced by the Markov process are averaged, while using the associated probabilities as weights. Therefore it should not be expected that implied variables' trajectories have to completely match the observed variables' paths. Hence we try to match the observed variables' trend deviations in 2010:Q4 by the simulation of the recession scenario.

[80] Del Negro et al. (2013) note that the disparity between model and data regarding investment may be due to the fact that the model does not account for an explicit residential sector. Note that by following Smets and Wouters (2007) we exclude durable goods from our consumption measure and instead treat it as a part of investment.

Figure 4.9: Simulated impulse responses to recession scenario



Notes: Simulated trajectories of output, consumption, investment, inflation, hours worked, wages, long-term Treasury rate, and loan rate.

affects the banks' cost of financial intermediation which induces a negative response of the loan rate and the deposit rate, and therefore increases the amount of loan provision. By this, output, consumption, investment, hours worked and wages respond positively. Further, declaring long-term Treasuries as eligible for open market operations induces a liquidity premium on their prices which should c.p. decrease the spread between the long-term bond rate and the short-term bond rate. Or put differently, this operation would c.p. decrease the slope of the yield curve. Note that the positive reaction of real output would increase the monetary policy rate which is assumed to follow a Taylor-rule type of policy reaction function (see 4.38). However, we assume that the central bank holds the policy rate close to the zero lower bound during the simulation experiment.

We expect the simulation which is excluding the balance sheet policy intervention to imply a deeper recession for the real macroeconomic variables and to imply a sudden and strong increase in the private sector borrowing rate. In fact, in the absence of the policy intervention the output contraction following the simulated crisis event is up to -8.06 percent. This exceeds the drop in output calculated for the simulation including the policy intervention by 2.75 percentage points. We find that compared to the simulation including the policy intervention in the absence of the policy intervention the drop in consumption is increased by 2.45 percentage points, the drop in investment is increased by 6.46 percentage points, the drop in hours worked is increased by 3.52 percentage points, and the drop in the real wage rate is decreased by 0.11 percentage points. These effects on the model's real variables mainly result from the fact that in case of the asset purchasing program the loan provision does not decrease as strong as in the absence of this program. It is noteworthy that under the policy intervention scenario the model predicts a strong decrease of inflation. The model's equilibrium conditions imply a strong positive relation between inflation and output growth (see 4.23). As the pace of output recovery is in the case without policy intervention faster than in the case with the policy intervention, inflation therefore will increase stronger in the former case.

The policy intervention significantly affects interest rates. Specifically, in absence of the intervention the model simulation implies an increase of the loan rate by 14.40 percentage points on impact and an increase in the long-term Treasury rate by 8.06 percentage points on impact. Simulating the model with the policy intervention implies that the first period's impact of the crisis scenario on the loan rate is dampened by 560 basis points relative to the case of an absence of the program. In the second period there is a further decrease by 722 basis points driving down the loan rate to a 154 basis points deviation from the steady state value. The impact of the asset purchase programs on the long-term bond rate is sizably stronger. The policy intervention reduces the long-term bond rate on impact by 851 basis points compared to the simulation without the bond purchases. This number can be interpreted as the model-implied liquidity premium on long-term bonds, measured in the first simulation period. Note that on impact, the model predicts a drop in the long-term bond rate by 44 basis points in case of the policy intervention. This value lies between the estimates of Krishnamurthy and Vissing-Jorgensen (2011) (33 basis points) and D'Amico and King (2013) (55 basis points) for the impact of LSAP 2 on the ten-year Treasury Note yield. The response of the loan rate does obviously not match the observed data in Figure 4.7. This is due to the fact that the simulated model requires a relatively strong increase of the banking cost shock and therefore of the loan rate to be able to match the observed evolution of the key macroeconomic variables. However, it is to note that we actually do not know how the loan rate would have evolved without any emergency and short-term policy responses[81] as well as without the (announcement-)effects of the first round of LSAP. Those measures were launched previous to LSAP 2 and we de not identify the effects of these measures on the loan rate in the present chapter. Specifically, we do not model any of the previous emergency policy interventions and focus on explaining the impact of LSAP 2 on real macroeconomic variables and on the long-term bond rate.

4.5 Conclusion

In this chapter we applied a monetary DSGE model with financial frictions and an explicitly specified set of central bank balance sheet policy options to identify the macroeconomic effects of LSAP 2. The model contains a banking sector which requires central bank reserves to facilitate financial intermediation. Reserves are supplied by the central bank in exchange for eligible assets. Multiple assets, such as short-term bonds and long-term bonds, are considered which differ with respect to their ability to serve as a mean for reserves acquisition. By employing Bayesian estimation tech-

[81] See measures like the Term Auction Facility (TAF) and Term Asset-Backed Securities Loan Facility (TALF).

niques we find that the model provides a considerably good fit to the data. We show that the model generates responses to commonly studied macroeconomic shocks which are qualitatively consistent with existing empirical evidence. Beyond that, we introduce shocks which directly affect financial intermediation. We find that the impact on financial intermediation is particularly relevant for the size and the persistence of shock responses. Further, we calculated the model-implied contributions of macroeconomic shocks to the observed historical variations of the model's key variables. We find that the shocks to the costs of financial intermediation and to central bank money supply explain a significant share of the deviations of real per capita output from its trend which is observed following the emergence of the 2008/2009 crisis.

In a next step we have investigated whether the model-impled effect of the central bank money supply shock on output captures the impact of unconventional balance sheet policies such as LSAP 2. In particular, we analyzed the effects of balance sheet policies as a structural feature of the model while focussing on large-scale purchases of long-term Treasury bonds. For that purpose, we employed the model to simulate the recession and the central bank's balance sheet policy intervention following 2010:Q4. From a counterfactual simulation experiment we conclude that LSAP 2 was likely to yield a considerable contribution to dampen the adverse impact of the crisis on the macroeconomy. We find that output contraction following the simulated crisis event is dampened by 2.75 percentage points. This poses evidence in favour of the notion that unconventional balance sheet policy operations such as large-scale asset purchase programs are effective at stimulating the economy. Further, results support the model's proposed transmission channel of unconventional balance sheet policy operations. Specifically, LSAP 2 successfully lowers the cost of financial intermediation and interest rates of longer-termed Treasury securities which is transmitted to private sector borrowing rates.

Chapter 5

Testing Uncovered Interest Parity under the Assumption of Liquidity Premia

5.1 Introduction

Uncovered interest parity (UIP) implies that, under the assumption that covered interest parity holds, the differential between two countries' risk-free interest rates is an estimate of future changes in the spot exchange rate. If expectations are rational then the interest differential should be an unbiased predictor of future bilateral exchange rate changes.[82] There is a large branch of empirical works in the literature on testing UIP which employs forward premium regressions. These studies regress realized exchange rate changes on the interest differential or forward premium resp. between two countries.[83] Under rational expectations and risk neutrality, UIP predicts this regression to yield a positive coefficient of unity on the forward premium. The empirical failure of UIP has been documented by various evidence from forward premium regressions.[84] The widely quoted result by

[82]See Bilson (1981), Fama (1984), Froot and Thaler (1990).

[83]As pointed out by Frankel (1982), employing regression analysis will test the joint hypothesis of the UIP's implications regarding expected exchange rate changes together with unbiased expectations both to hold. This is denoted as the "unbiasedness hypothesis".

[84]For seminal survey articles about the empirical work on testing UIP see Hodrick (1987), Froot (1990), and Engel (1996). See Burnside et al. (2006) and Chinn (2006) for

Froot (1990) finds that the average estimate of the mentioned coefficient across 75 published studies is -0.88. Furthermore, only a few estimates are positive but none is equal or greater than unity. This result is known as the forward premium puzzle. It implies that the forward premium predicts future changes in the spot exchange rate which are inconsistent with UIP, in terms magnitude and in terms of the direction of the movement.

The present chapter investigates whether liquidity premia can explain deviations from UIP. Specifically, in this chapter I examine the impact of liquidity premia on international interest rate differentials, namely the U.S.-U.K. Treasury yield spread.

UIP is a key no-arbitrage condition in international bond markets. Canzoneri et al. (2013) explain deviations from UIP by relaxing the assumption that risk-free bonds which are denominated in different currencies are perfect substitutes. Specifically, home and foreign bonds are imperfect substitutes for money in each countries' transaction technology. Canzoneri et al. (2013) argue that the U.S. dollar's role as a key currency in the international monetary system is the reason for the relatively low U.S. Treasuries' interest rates. It is pointed out that U.S. Treasuries facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, individuals hold them in money market accounts that offer checking services, and importers and exporters hold them as transaction balances. Therefore, the liquidity of U.S. Treasury bonds is interpreted as a non-pecuniary return to investors which poses the rationale for why U.S. Treasuries will be held at a discount. Hence, the key currency feature of the U.S. dollar can contribute to the explanation of deviations from UIP.

A recent study by Krishnamurthy and Vissing-Jorgensen (2012) provides evidence that the corporate-Treasury bond yield spread is to a significant extent driven by the total amount of U.S. Treasuries outstanding which is proxied by the government Debt-to-GDP ratio (i.e. the market value of publicly held U.S. government debt to U.S. GDP). They argue that investors value certain features of U.S. Treasuries, namely their liquidity and their "absolute security of nominal return". This affects prices of Treasuries and drives down their yields compared to assets that do not to the same extent share these features. As a theoretical rationale for this observation Krishnamurthy and Vissing-Jorgensen (2012) assume that the holder of a U.S. Treasury security obtains some services and gains to the subjective level

recent empirical studies and Engel (2013) for a recent survey.

of well-being. Those benefits are summarized as "convenience yield" which directly contribute to investors' utility and lead U.S. Treasuries to have a lower yield than they would have in a standard asset-pricing framework.

In this chapter I follow Krishnamurthy and Vissing-Jorgensen (2012) by modifying a standard representative agent asset-pricing model by allowing agents to derive utility directly from holdings of U.S. Treasuries. In a next step I derive no-arbitrage conditions for the international bond market which are implied by the modified model. Here I follow Canzoneri et al. (2013) by relaxing the assumption that U.S. domestic bonds and foreign bonds are perfect substitutes. Specifically, it is assumed that U.S. Treasuries provide unique liquidity services. Therefore, the model-implied no-arbitrage conditions allow for liquidity premia induced by the U.S. dollar's postulated key currency feature. The no-arbitrage conditions are further derived for the cases of explicitly accounting for foreign exchange risk and price risk, and for neglecting these risk premia. I employ regression analysis to empirically test whether the model-implied no-arbitrage conditions for U.S. data and U.K. data can explain deviations from UIP. In this context I follow Fuhrer (2000) by assuming that the households' expectations regarding the dynamics of consumption and the depreciation rate of the domestic currency can be described by an unconstrained vector autoregression.

In this chapter I find that investors' valuation for U.S. Treasuries' liquidity contributes to explain deviations from UIP. Further, estimation results imply a positive association between the expected depreciation rate of the U.S. currency relative to the U.K. currency and the U.S.-U.K. Treasury yield spread or forward premium. However, the point estimate of the coefficient still is below unity.

There have been many attempts to account for departures from UIP. One of the most influential of these is Fama (1984) who attributes deviations of realized exchange rate changes from UIP to a time-varying risk premium.[85] However, studies like Backus, Foresi, and Telmer (2001) and Brandt, Cochrane and Santa-Clara (2006) confirm the result that risk premia cannot resolve the forward premium puzzle while assuming standard preferences. There are recent studies in this field which are able to account for deviations from UIP by assuming utility maximizing representative agents in home and foreign countries with non-standard preferences. E.g. Verdelhan (2010) employs a utility specification of external habit pref-

[85] Specifically, a risk premium arises in such models due to the degree to which the exchange rate return covaries with consumption growth.

erences, and Bansal and Shaliastovich (2012) use Epstein-Zin preferences. However, these studies rely on simulations of models which are calibrated to match a set of macroeconomic and financial data features. Further, small open economy models commonly include a UIP condition (see Gali and Monacelli (2005)). Authors like McCallum and Nelson (2000) and Kollmann (2005) add an exogenous UIP shock which is calibrated to align the model-implied volatility of exchange rates with the observed interest rate differentials. Justiniano and Preston (2010) conduct a Bayesian estimation of a small open economy model with an exogenous UIP shock. They find that the volatility in the real exchange rate is almost completely explained by a risk premium shock. This is interpreted as an extreme version of exchange rate disconnect. The generally bad empirical performance is attributed to the failure of the estimated model to link movements in the exchange rate with macroeconomic fundamentals. Therefore, addressing the issue of exchange rate disconnect is regarded as a key to improve the model's quantitative performance.

This chapter is organized as follows. In section 5.2 I derive the modified asset pricing model, no-arbitrage conditions for the international bond market, and specify regression models which are estimated to test the no-arbitrage condition's implications. Section 5.3 presents estimation results. Section 5.4 concludes.

5.2 Yield spread model

In the following section I modify an asset pricing model under the assumption that U.S. Treasuries' liquidity services are valued by investors. This is done along the lines of Krishnamurthy and Vissing-Jorgensen (2012). Spreads between the yields of U.S. Treasuries and foreign Treasury securities which do not provide liquidity services are then explained by a no-arbitrage condition. The goal is to obtain a model of spread determinants which can be empirically tested for its ability to explain observed international Treasury yield spreads.

5.2.1 Household's problem

A representative household is assumed to maximize the expected sum of a discounted stream of utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t, \nu \left(b_t, GDP_t, \xi_t\right)\right), \tag{5.1}$$

subject to the budget constraint

$$P_t c_t + P_t^T b_t + X_t P_t^{T*} b_t^* \leq P_t y_t + P_t^T b_{t-1} + X_t P_t^{T*} b_{t-1}^*, \tag{5.2}$$

where E_0 is the expectation operator conditional on the information set in the initial period, and $\beta \in (0,1)$ is the subjective discount factor. The utility function is specified by $u_t = \frac{1}{1-\sigma} c_t^{1-\sigma} + \nu \left(b_t, GDP_t, \xi_t\right)$, with $\sigma \geq 1$, where c_t denotes consumption and $\nu(\cdot)$ represents the agent's gained convenience yield. I follow Krishnamurthy and Vissing-Jorgensen (2012) by assuming that convenience yields are a function of the U.S. gross domestic product (GDP) and the investor's holdings of real U.S. Treasuries b_t.[86] The latter further captures the assumption by Canzoneri et al. (2013), that the U.S. dollar's role as a key currency in the international monetary system induces holdings of U.S. Treasuries to yield unique non-pecuniary returns to the investors. The term ξ_t is a preference shock. Following Krishnamurthy and Vissing-Jorgensen (2012) the convenience yield function $\nu(\cdot)$ is concave with $\nu'(\cdot) > 0$, and $\nu''(\cdot) < 0$. Further, $\nu(\cdot)$ shall be homogenous of degree one in GDP_t and b_t.[87] The household earns a real endowment income y_t, and carries wealth into the next period by investing into nominal holdings of U.S. Treasuries $P_t^T b_t$, and by investing into nominal holdings of the foreign country's Treasuries $P_t^{T*} b_t^*$. In order to measure the purchasing power of a foreign currency pay-off in a particular period t, the nominal exchange rate X_t is introduced. The exchange rate is measured as the price of foreign currency in units of domestic currency at time t. Assume for simplicity that the agent buys zero coupon discount bonds which pay out one unit of currency when being held to maturity. The aggregate price level at date t is denoted by P_t. The nominal prices for one-period investments into U.S. Treasuries, and into the foreign country's Treasuries are denoted as P_t^T, and P_t^{T*}. Real holdings of foreign Treasuries are denoted as b_t^*.

[86] Specifically, Krishnamurthy and Vissing-Jorgensen (2012) assume that convenience yields are driven by a set of macroeconomic factors which will influence the household's level of well-being. The U.S. GDP acts as a shortcut to capture these factors.

[87] Hence $\nu(\cdot)$ can be transformed in the following manner: $\nu \left(b_t, GDP_t, \xi_t\right) \equiv \nu \left(\frac{b_t}{GDP_t}, \xi_t\right) GDP_t$.

Maximizing the objective function (5.1) subject to the budget constraint (5.2) leads for given initial values and non-negativity constraints for b_t, and b_t^* to the following first order conditions for consumption c_t, and investments into U.S. Treasuries b_t, and foreign Treasuries b_t^*:

$$c_t^{-\sigma} = \lambda_t, \tag{5.3}$$

$$\nu'(\cdot)\frac{P_t^T}{P_t} + \beta E_t\left[\lambda_{t+1}\frac{P_{t+1}^T}{P_{t+1}}\right] = \lambda_t\frac{P_t^T}{P_t}, \tag{5.4}$$

$$\beta E_t\left[\lambda_{t+1}\frac{X_{t+1}P_{t+1}^{T*}}{P_{t+1}}\right] = \lambda_t\frac{X_t P_t^{T*}}{P_t}, \tag{5.5}$$

and (5.2) holding with equality, and the transversality conditions $\lim_{j\to\infty}\beta^j E_t\left(\lambda_{t+j}P_{t+j}^T b_{t+j}\right) = 0$, and $\lim_{j\to\infty}\beta^j E_t\left(\lambda_{t+j}X_{t+j}P_{t+j}^{T*}b_{t+j}^*\right) = 0$.

The stochastic discount factor for nominal payoffs is denoted as $M_{t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}$, such that (5.4), and (5.5) combined with (5.3) imply

$$P_t^T = \frac{E_t\left[M_{t+1}P_{t+1}^T\right]}{1-\nu'(\cdot)/c_t^{-\sigma}}, \tag{5.6}$$

$$P_t^{T*} = E_t\left[M_{t+1}\left(1+q_{t+1}\right)P_{t+1}^{T*}\right], \tag{5.7}$$

where I denote $(1+q_{t+1}) = X_{t+1}/X_t$, as the gross return on holding one unit of foreign currency. Equation (5.6) requires that under the assumption that U.S. Treasuries provide liquidity services as an argument of the investor's utility function, increasing the amount of U.S. Treasuries held, will decrease their price P_t^T. Specifically, increasing the stocks of U.S. Treasuries will lower the investor's willingness to pay for another unit of such assets. This is due to the assumption of $\nu(\cdot)$ being a concave function of b_t. Note that foreign Treasuries do not provide liquidity services.

5.2.2 No-arbitrage condition without risk premium

In this section I derive the no-arbitrage condition for the international bond market under the assumption that financial markets are complete.[88] Note that I consider zero coupon discount bonds. Further, I assume that there is no price risk or default risk for the two Treasury bonds under consideration.

[88] In a similar way Gali and Monacelli (2005) derive the UIP condition for a small open economy model.

Therefore, $P_{t+1}^{T} = P_{t+1}^{T*} = 1$. Hence, $P_t^T = \frac{1}{R_t^T}$, and $P_t^{T*} = \frac{1}{R_t^{T*}}$, where R_t^T and R_t^{T*} are the risk-free gross returns. Equations (5.4) and (5.5) can now be written as

$$\frac{\nu'(\cdot)}{\lambda_t}\frac{1}{R_t^T} + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right] = \frac{1}{R_t^T}, \tag{5.8}$$

$$\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{X_{t+1}}{X_t}\frac{1}{\pi_{t+1}}\right] = \frac{1}{R_t^{T*}}, \tag{5.9}$$

where inflation is given by $\pi_{t+1} = P_{t+1}/P_t$. Now denote the left-hand side of (5.9) as stochastic discount factor in terms of purchasing power in the foreign currency:

$$M_{t+1}^* = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\frac{X_{t+1}}{X_t} = M_{t+1}\frac{X_{t+1}}{X_t}.$$

Substituting out in (5.8) yields

$$\frac{\nu'(\cdot)}{c_t^{-\sigma}}\frac{1}{R_t^T} + E_t\left[M_{t+1}^*\frac{X_t}{X_{t+1}}\right] = \frac{1}{R_t^T}.$$

Given that $1/R_t^{T*} = E_t\left[M_{t+1}^*\right]$, is the foreign currency rate of return on a nominally risk-free Treasury, the no-arbitrage condition can be derived

$$\frac{R_t^T}{R_t^{T*}}E_t\left[\frac{X_t}{X_{t+1}}\right] = 1 - \frac{\nu'(\cdot)}{c_t^{-\sigma}}.$$

Taking the logarithm of the former expression yields then

$$r_t^T - r_t^{T*} = E_t\left[q_{t+1}\right] - \nu'(\cdot)/c_t^{-\sigma}. \tag{5.10}$$

This approximation uses that $\ln(1-y) \approx y$, for small y. Note that $E_t\left[x_{t+1}\right] - x_t = E_t\left[q_{t+1}\right]$.[89] Equation (5.10) implies that the investors' marginal valuation for the U.S. Treasuries' liquidity induces a deviation from UIP. Specifically, this equation implies a positive relation between the holdings of U.S. Treasuries and the interest rate differential $r_t^T - r_t^{T*}$. Increasing the holdings of U.S. Treasuries decreases the investor's marginal valuation for any further unit of U.S. Treasuries $\nu'(\cdot)$. This in turn reduces the U.S. Treasuries' prices and will therefore drive up the expected returns.

[89] Note that x_t and x_{t+1} are the logarithms of the period t and period $t+1$ exchange rates, and that the net returns r_t^T and r_t^{T*} are the logarithms of the gross returns R_t^T and R_t^{T*}.

5.2.3 Yield spread model with risk premium

In a next step this section employs the modified asset pricing model to explain international Treasury bond yield spreads while accounting for price risk and foreign exchange risk. I follow Krishnamurthy and Vissing-Jorgensen (2012)[90] by computing the τ-period yields for U.S. Treasury debt securities $i^T_{t,\tau}$, and for foreign Treasury debt securities $i^{T*}_{t,\tau}$:

$$i^T_{t,\tau} = -\frac{1}{\tau} lnP^T_t, \text{ and } i^{T*}_{t,\tau} = -\frac{1}{\tau} lnP^{T*}_t,$$

where τ is the number of periods to maturity. By this, the price of a zero coupon bond is converted into a continuously compounded zero coupon bond yield. Therefore, for discount bonds with $P^T_\tau = P^{T*}_\tau = 1$, the yield spread for securities with any number of periods to maturity τ, can be expressed as:

$$i^T_{t,\tau} - i^{T*}_{t,\tau} = \frac{1}{\tau}\left(lnP^{T*}_t - lnP^T_t\right).$$

Now plug in (5.6) for P^T_t and (5.7) for P^{T*}_t

$$= \frac{1}{\tau}\left(ln\left(E_t\left[M_{t+\tau}\left(1+q_{t+\tau}\right)\right]\right) - ln\left(\frac{E_t\left[M_{t+\tau}\right]}{1-\nu'\left(\cdot\right)/c_t^{-\sigma}}\right)\right),$$
$$\approx \frac{1}{\tau}\left(E_t\left[M_{t+\tau}\left(1+q_{t+\tau}\right)\right] - E_t\left[M_{t+\tau}\right] - \nu'\left(\cdot\right)/c_t^{-\sigma}\right).$$

This approximation uses that $\ln(1+y) \approx y$, for small y. Denote the yield spread as $\Delta i_{t,\tau} = i^T_{t,\tau} - i^{T*}_{t,\tau}$, and rearrange

$$\Delta i_{t,\tau} = \frac{1}{\tau}E_t\left[M_{t+\tau}\right]E_t\left[q_{t+\tau}\right] + \frac{1}{\tau}cov_t\left(M_{t+\tau}, q_{t+\tau}\right) - \frac{1}{\tau}\nu'\left(\cdot\right)/c_t^{-\sigma}. \quad (5.11)$$

Equation (5.11) implies that the τ-period spread between the yield of a U.S. Treasury security and the yield of a foreign Treasury security with remaining term to maturity τ, is determined by the product of the expected $t+\tau$-periods-ahead stochastic discount factor times the expected $t+\tau$-periods ahead exchange rate growth rate, the covariance between the

[90] This is applied by Backus, Foresi, and Telmer (2001) to calculate prices of bonds with different maturities in the context of affine models of the term structure of interest rates.

$t + \tau$-periods-ahead stochastic discount factor and the $t + \tau$-periods-ahead exchange rate growth rate, and the period t marginal convenience yield of U.S. Treasuries divided by $c_t^{-\sigma}$. The former two terms reflect the foreign exchange risk premium which arises due to the comovement of the future expected spot exchange rate growth rate with the expected change in the stochastic discount factor. Note that the third term on the right-hand side of (5.11) reflects the modification of the standard asset pricing model by the assumption that investors value features of U.S. Treasuries which are unique to them. By the assumption of the U.S. dollar to be the key currency, these features are not shared with any other Treasury debt security issued by any other country.

5.2.4 Estimation strategy

The purpose of the present chapter is to test the hypothesis that investors value the unique liquidity of U.S. Treasuries which leads to deviations from UIP. This is done by investigating whether the marginal convenience yield terms in (5.10) and (5.11) significantly contribute to the explanation of the observed yield spreads for U.S. Treasuries compared to U.K. Treasury debt securities.[91] For that purpose I specify the following regression models:

$$\Delta i_t^{US,UK} = \alpha^{np} + \beta_1^{np} \log\left(\frac{Debt_t}{GDP_t}\right) + \beta_2^{np} \hat{q}_{t+1} + \varepsilon_t^{np}, \tag{5.12}$$

$$\Delta i_t^{US,UK} = \alpha^{rp} + \beta_1^{rp} \log\left(\frac{Debt_t}{GDP_t}\right) + \beta_2^{rp} \left(\begin{array}{c} \hat{M}_{t+1}\hat{q}_{t+1} \\ +cov\left(M,q\right) \end{array}\right) + \varepsilon_t^{rp}, \tag{5.13}$$

where $\Delta i_t^{US,UK}$ denotes the spread between the yields of a U.S. Treasury and a U.K. Treasury with same maturity length, and ε_t^{np} and ε_t^{rp} denote error terms. The dependent variable is a quarterly yield spread measured in percentage points. Specifically, I use quarterly data and 3-month Treasury bill yields.[92] The superscript np denotes the estimation model for the no-arbitrage condition (5.10), and the superscript rp denotes the estimation model for the yield spread model (5.11) which accounts for risk premia.

[91] For the present study I focus on U.S. and U.K. data because Treasury debt securities issued in both countries can be regarded as close substitutes apart from the postulated key currency feature of the U.S. dollar. Specifically, financial market integration between both countries is intense and trading volumes of U.S. Treasuries and U.K. Treasuries are large.

[92] See Appendix D.1 for a description of the data.

The proxy for the marginal convenience yield which is divided by marginal utility of consumption in the equations (5.10) and (5.11) is the logarithm of the face value of the outstanding stock of U.S. Treasuries, scaled by U.S. GDP. This proxy is denoted as $\log\left(Debt_t/GDP_t\right)$. A log functional form is used because it provides a good fit and requires estimation of only one parameter. Further, the interpretation of a regression coefficient for a log independent variable on a dependent variables denoted in percentage points is more convenient.

I follow Fuhrer (2000) by assuming that the dynamics of the stochastic discount factor and the growth rate of the exchange rate can be described by an unconstrained vector autoregression.[93] In particular, the vector autoregression is used to generate the households' forecasts of the future changes in consumption and inflation, which are required to calculate the expected changes in the stochastic discount factor $\hat{M}_{t+1}$, and forecasts of the exchange rate $\hat{q}_{t+1}$. These variables enter the right-hand sides of the yield spread regression models (5.12) and (5.13).[94] Note that I regard the covariance between the stochastic discount factor and the growth rate of the exchange rate $cov\left(M,q\right)$ which enters the right hand side of (5.13), as being constant.[95]

[93] This approach is similar to the one applied in Campbell and Shiller (1987) for present value models, such as consumption functions that relate consumption, income, and interest rates. Campbell and Shiller (1987) point out that employing an unconstrained vector autoregression to generate forecasts implies the choice of the information set which includes all relevant information of market participants at the time when expectations are formed.

[94] Canzoneri et al. (2007) use this approach to generate the forecasts of the future changes in consumption and inflation. Their aim is to compute implied consumption Euler equation rates under a number of different preference specifications and compare them to observed nominal and real market rates.

[95] The conditional moments are obtained form a VAR(p) model with k endogenous variables which are elements of the vector $\vec{Y_t} = (Y_{1,t}, ..., Y_{k,t})$:

$$\begin{aligned} &\vec{Y_t} = A_0 + A_1\vec{Y}_{t-1} + ... + A_p\vec{Y}_{t-p} + \vec{e}_t, \\ &\vec{e}_t \sim IID\left(0, \Sigma_e\right), \end{aligned}$$

where A_0 and $\vec{e}_t$ are $k \times 1$ are vectors of the constant terms and the independent and identically distributed random error terms. The $k \times k$ matrices A_1 and A_p contain the regressors' parameters. The conditional expectations for the h-periods-ahead consumption and exchange rate are derived by computing

$$E_t\vec{Y}_{t+h} = \hat{A}_0 + \hat{A}_1\vec{Y}_{t+h-1} + ... + \hat{A}_p\vec{Y}_{t+h-p},$$

The VAR is estimated from 1985:Q3 to 2008:Q1 on quarterly U.S. data. Following Fuhrer (2000) the VAR is estimated for log per capita real nondurable goods and services consumption, the log per capita real disposable income, the effective federal funds rate, the log per capita real nonconsumption GDP, the log change in the price index for nondurables and service consumption, and a commodity price index. Further, I follow Eichenbaum and Evans (1995) by additionally considering the U.S. dollar relative to U.K. pound exchange rate for the VAR model estimation.[96]

Note that by employing this VAR model to generate households' forecasts, the exogeneity assumption for the variables $\hat{q}_{t+1}$ and $\hat{M}_{t+1}$ in the regression models (5.12) and (5.13) might be violated. In this case OLS estimates would be invalid. To justify the use of OLS for the purpose of the present study it is assumed that the forecasts of $\hat{q}_{t+1}$ and $\hat{M}_{t+1}$ are contemporaneously uncorrelated with the disturbances ε_t^{np} and ε_t^{rp}.[97]

By estimating (5.12) and (5.13) the present chapter intends to test the following hypotheses:

Hypothesis 1 *The yield spread models (5.10) and (5.11) require that an increase in the U.S. Debt to GDP ratio which is a proxy for the holdings of liquid U.S. Treasuries, increases the observed U.S.-U.K. Treasury yield spreads. Hence, the regression analysis would provide support in favor of the assumption that investors value unique liquidity features of U.S. Treasuries if point estimates for the regression coefficients would imply that* $\beta_1^{np} > 0$, *and* $\beta_1^{rp} > 0$.

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where $\hat{A}_0$, is the vector of the regression intercepts, and $\hat{A}_1$, and $\hat{A}_p$ contain the estimated regression coefficients. The conditional second moments are given by the elements of the estimated covariance matrix

$$\hat{\Sigma} = \frac{T}{T - kp - 1}\hat{U}\hat{U}'$$

where $\hat{U}$ is the $k \times T$ matrix of the regression residuals.

[96] See Appendix D.1 for a detailed description of the data.

[97] Following Mehra and Minton (2007), OLS has been employed in contributions like Orphanides (2001) and Boivin (2006), to estimate forward-looking Taylor rules using the Federal Reserve Board of Governors' Greenbook forecasts. As further pointed out, the use of OLS requires the assumption that the Greenbook forecasts are uncorrelated with the regression error which is interpreted as a monetary policy rate shock. Boivin (2006) argues that this exogeneity assumption can not be directly verified but is implicitly made by studies like Orphanides (2001) when using OLS to estimate forward-looking Taylor rules with forecast data. As the present study is conceptually similar I follow these authors and assume that the VAR forecasts are uncorrelated with the disturbances ε_t^{np} and ε_t^{rp}.

This would imply that liquidity premia can contribute to explain deviations from UIP. Further, I investigate whether foreign exchange risk provides in this context a significant contribution to the explanation of the observed variation in the U.S.-U.K. Treasury yield spread.

Hypothesis 2 *The yield spread model (5.13) provides a better empirical fit. Employing the regression model (5.12) to explain the spread between U.S. Treasury yields and U.K. Treasury yields neglects important information.*

5.3 Empirical results

Equations (5.12) and (5.13) are estimated on quarterly data ranging from 1985:Q3 to 2008:Q1. This data sample is chosen for the present analysis as it covers roughly the period on which recent empirical work testing the UIP condition is estimated.[98] The dependent variable is the spread between the 3-month U.S. Treasury bill yield and the U.K. Treasury bill yield with the same maturity length.

Estimation results are summarized in Table I. The first column of Panel A presents coefficient estimates for the regression of the U.S.-U.K. Treasury yield spread on the measure for U.S. Treasury holdings $\log(Debt_t/GDP_t)$, the expected next quarter's growth rate of the exchange rate $\hat{q}_{t+1}$, and a constant term. The mean value of the U.S.-U.K. Treasury bill yield spread is -266 basis points (bp) for the sample period 1985:Q3 to 2008:Q1. The coefficient of 11.18 on the $\log(Debt_t/GDP_t)$ variable implies that a one percentage point increase of the average U.S. Debt-to-GDP ratio, increases the U.S.-U.K. Treasury bill yield spread by 21 bp. Note that a one standard deviation increase in the U.S. Debt-to-GDP ratio, from its mean value of 0.52 to 0.65, increases the U.S.-U.K. Treasury bill yield spread by 249 bp. From the perspective of the no-arbitrage condition (5.10) one would argue that such an increase in the holdings of Treasuries which are denominated in the key currency, decreases the investors' valuation and willingness to pay for an other unit of such Treasuries. This in turn drives up the yield of a U.S. Treasury bill compared to the yield of a U.K. Treasury bill. This finding is consistent with Hypothesis 1 and statistically significant. Further, it implies that U.S. Treasury supply is an important determinant of the spread. The covariate $\hat{q}_{t+1}$ is in this setting estimated to be significantly related to the spread. The point estimate for β_2^{np} is 0.34 which implies that an expected

[98] See Burnside et al. (2006) and Chinn (2006).

depreciation of the U.S. currency relative to the U.K. currency is positively related to an increase in the U.S.-U.K. Treasury yield spread. Under the standard specification of the forward premium regression model one would test the hypothesis whether the estimated coefficient on $\hat{q}_{t+1}$ is unity. This however, is not found in the present study, but in contrast to most empirical studies on forward premium regressions, the point estimate of the coefficient in the present study is significantly larger zero. In the second column of Panel A the estimated coefficients are presented for a regression where the measure for U.S. Treasury holdings is not included. Results imply that $\hat{q}_{t+1}$ has in this case no significant impact on the spread. Therefore, the positive association of $\hat{q}_{t+1}$ with the spread found for the regression presented in the first column of Panel A, depends on the inclusion of $\log(Debt_t/GDP_t)$ as covariate to the estimation model. Further, including the $\log(Debt_t/GDP_t)$ regressor sharply increases the R^2 measure.

In the first column of Panel B results are shown for estimating the regression model (5.13). In this case the U.S.-U.K. Treasury yield spread is regressed on the proxy for the expected foreign exchange risk $\hat{M}_{t+1}\hat{q}_{t+1} + cov(M, q)$, instead of the expected next quarter's growth rate of the exchange rate $\hat{q}_{t+1}$. Further, a constant and the measure for U.S. Treasury holdings $\log(Debt_t/GDP_t)$ are included. Again, estimating the model with the $\log(Debt_t/GDP_t)$ regressor increases the R^2 measure. Further, by comparison with the results presented in the second column of Panel B, it appears that by inclusion of the U.S. Treasury holdings proxy the coefficient on the proxy for the foreign exchange risk becomes significant. However, the size of the estimated coefficient implies a small effect of foreign exchange risk on the spread. Further, comparing the results across the first columns of Panel A and Panel B shows that the values of the point estimates for the coefficients on $\log(Debt_t/GDP_t)$, the regression constants, and the values of the R^2 measures lie very close together. Hence, the proxy for foreign exchange risk does not seem to contain important information for the U.S.-U.K. Treasury yield spread regression. Hence, I consider Hypothesis 2 to be rejected by this result.

5.4 Conclusion

For the present chapter I modified a representative agent asset-pricing model by assuming that investors value liquidity services which are unique features of U.S. Treasuries. Further, the assumption that U.S. domestic bonds and

Table 5.1: Impact of US Debt/GDP on U.S.-U.K. Treasury bills yield spread

Period	1985:Q3 - 2008:Q1			
	(A)		(B)	
$log(Debt/GDP)$	11.180		11.122	
	[5.491]		[5.354]	
$\hat{q}_{t+1}$	0.339	0.091		
	[2.526]	[0.520]		
$\hat{M}_{t+1}\hat{q}_{t+1}$			0.002	0.001
			[2.200]	[0.223]
Intercept	3.168	-2.665	3.122	-2.665
	[2.930]	[-11.419]	[2.838]	[-11.387]
R^2	0.263	0.004	0.250	0.001
N	89	89	89	89

Notes: The sample period is 1985:Q3 - 2008:Q1. t-statistics are reported in brackets.

foreign bonds are perfect substitutes was relaxed. In a next step model-implied no-arbitrage conditions for the international bond market were derived. These are interpreted as UIP conditions which are adjusted for liquidity premia. Estimation results provide support for the hypothesis that investors value the liquidity of U.S. Treasuries which yields a significant contribution to the explanation of the U.S.-U.K. 3-month Treasury bill yield spread. This implies that investors' valuation for U.S. Treasuries' liquidity contributes to explain deviations from UIP. Estimation results however, imply that foreign exchange risk can only explain a very low share of the observed variation in the U.S.-U.K. 3-month Treasury bill yield spread. In contrast to most forward premium regression estimations I find a positive association between the expected depreciation rate of the U.S. currency relative to the U.K. currency and the U.S.-U.K. Treasury yield spread. However, the point estimate of the coefficient is below unity.

Chapter 6

Concluding Remarks

This thesis has presented several essays on the problem of modelling the notion of aggregate liquidity as a potential driver of asset returns and of macroeconomic evolutions.

Chapter 2 has demonstrated that an asset pricing model which is modified to allow liquidity services to enter investors' utility can contribute to explain observed corporate-U.S. Treasury bond yield spreads. Chapter 3 has shown that observed data on investors' consumption and liquid asset demand are consistent with revealed preference conditions for utility maximization and weak separability. However, estimation results have either implied rejection of proposed specifications of household preferences or parameter values that indicate misspecification. Chapter 4 has addressed the identification of the effects of the LSAP 2 longer-term Treasury purchase program on the U.S. economy. This chapter has found that shocks to financial intermediation have significantly contributed to the evolution of U.S. key macroeconomic variables following the 2008/2009 crisis. A counterfactual policy simulation has estimated that in the absence of LSAP 2 the U.S. real per capita GDP would have dropped by additional 2.75 percentage points. Chapter 5 has estimated forward premium regression models which are modified by assuming that investors value U.S. Treasuries' liquidity services induced by the U.S. dollar's role as a key currency. Estimation results have indicated that implied liquidity premia can explain deviations from UIP, and a positive association between the expected depreciation rate of the U.S. currency relative to the U.K. currency and the forward premium.

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Appendix A

Appendix to Chapter 2

A.1 Regression variables

Aaa-Treasury yield spread: This variable is constructed as the monthly spread between Moody's Aaa-rated long maturity corporate bond yield and the average yield on long term Treasury bonds measured in percentage points. The Moody's Aaa index is constructed from a sample of long maturity ($\geq$ 20 years) industrial and utility bonds (industrial only from 2002 onward). The yield on long maturity Treasury bonds is the average yield on long-term government bonds. The Treasury bonds included are due or callable after 10 years for the period 1971-1999. For 2000-2008 the yields on 20-year maturity Treasuries are used. All three data series are from the Federal Reserve's FRED database (series AAA, LTGOVTBD, and GS20). Monthly data for April 1971 up to September 2008 is used leaving out the sub prime crisis market turmoil and fiscal and GDP response.

CP-Bills yield spread: The yield spread between commercial paper and Treasury bills measured in percentage points. For the whole period 1971-2008 the commercial paper yield is from the FRED database. The period 1971-1996 is covered by the series CP3M (the average of offering rates on 3-month commercial paper placed by several leading dealers for firms whose bond rating is AA or equivalent) and for 1997-2008 by the series CPN3M (the 3-month AA nonfinancial commercial paper rate). The Treasury bill yield is for 3-month Treasury bills from 1971-2008 (FRED series TB3MS).

Debt/GDP: This variable is intended to proxy the holdings of Treasuries scaled by GDP. This variable is calculated from April 1971 until September 2008. I use time series data on the total amount of Treasury securities outstanding from Datastream (series USSECMNSA). Quarterly GDP data is from Federal Reserve's FRED database (series GDP). To derive monthly GDP data I used a cubic spline interpolation on the time series of quarterly U.S. GDP. Unlike KVJ I do not calculate Debt/GDP at market value. However, KVJ show that over the period 1949-2008 the correlation between Debt/GDP at face value and Debt/GDP at market value is 0.992.

MB/GDP: This variable is intended to proxy for the holdings of money and close to money substitutes scaled by GDP. From FRED I use the series BOGAMBSL, "Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements". Therefore, notes and coins (currency) in circulation (outside Federal Reserve Banks, and the vaults of depository institutions), currency in bank vaults, and Federal Reserve Bank credit (minimum reserves and excess reserves) are included which is widely interpreted as base money or total currency. MB/GDP hence, is derived from the most liquid measure of money supply actually leaving out close to money assets like demand deposits and savings deposits.

(M3-M2)/GDP: This variable is intended to proxy for the holdings of long-term close to money substitutes, and long-term assets with the highest possible degree of liquidity, scaled by GDP. M3-M2 covers the positions of large time deposits, institutional money market funds, repurchase agreements and other larger liquid assets. Data on the two empirical measures for aggregate money supply M3, and M2 are from FRED (series M3SL and M2SL). Data for M3 is only available until February 2006. Hence, this variable is calculated for the period April 1971 until February 2006.

CD/GDP: This variable is intended to proxy for the holdings of corporate debt securities scaled by GDP. I use the FRED series CPLBSNNCB, "Commercial Paper - Liabilities - Balance Sheet of Nonfarm Nonfinancial Corporate Business", for the face value of outstanding commercial paper and the series CBLBSNNCB, "Corporate Bonds - Liabilities - Balance Sheet of Nonfarm Nonfinancial Corporate Business", for the face value of outstanding corporate bonds. The sum of both series is assumed to measure the total holdings of corporate debt securities.

Volatility: This measure is based on standard deviations of weekly log stock returns on the S&P 500 index. Weekly returns are calculated on the value-weighted S&P 500 index based on daily returns obtained from Federal Reserve's FRED database (series SP500). As a volatility measure for a given month, the standard deviation of the weekly log returns are calculated up to the end of the month. The standard deviation of weekly log returns is then multiplied by the square root of 4.

Slope: The slope of the Treasury yield curve measured as the spread between the 10-year Treasury yield and the 3-months Treasury bill yield. The interest rate on Treasuries with 10 year maturity is from FRED (series GS10). The interest rate on Treasuries with 3 month maturity is from FRED as well (series TB3MS).

ASW: The measure for the difference in asset-swap spreads between corporate debt securities and Treasury securities. From Datastream the time series ICUSS2Y is used which captures the asset-swap rate of benchmark securities over the 2-year Treasury rate.

Agency: This is the measure for the spread between yields of Refcorp and Treasury securities. Time series data on yields of Freddie Mac securities due after one year and yields on Treasuries with the same maturity length are from Datastream (series USMIA1 and FRTCM1Y).

Appendix B

Appendix to Chapter 3

B.1 Data sources

CnDUR: This variable is constructed as the monthly real per capita consumption expenditures on nondurable goods. Monthly data on aggregate expenditures on nondurable consumption goods are from the Federal Reserve's FRED database (series PCEND). Monthly real values are obtained by a using a deflator calculated from a chain-type price index for personal consumption expenditures (FRED series PCEPI) with $2005 = 100$. Further, per capita data are derived by dividing through monthly total population (FRED series POP).

M: Measures monthly real per capita money balances. This is proxied by the data series on the currency component of M1 measure plus demand deposits from the FRED database (series CURRDD). As for CnDUR, real per capita balances are derived by using the same price index deflator and by dividing through total U.S. population.

TrBi: This variable is intended to proxy for the real per capita holdings of Treasury bills (4-week to 52-week maturity). Here, data on the face value of outstanding marketable U.S. Treasury bills is taken from Datastream (series name: U.S. Federal Debt - Marketable Securities Treasury Bills Curn, Id: USSECTRBA). Note that these data do not contain non-marketable Treasuries i.e. as held in a TreasuryDirect account. Unfortunately I do not have data on the distribution of maturities among non-marketables. So I can not quantify the shares of bills, notes, and bonds. Therefore, data on the face value of non-marketable bills is left out. The time series is as well transformed to real per capita values in the same way as CnDUR. Prices are calculated from the 3-Month Treasury Bill Secondary Market Rate (FRED series TB3MS) which is assumed to be a holding period return of a zero coupon bill. Further, from the raw data returns are calculated on monthly basis and deflated by the gross growth rate of the price index PCEPI. Quantities held are then derived by dividing correspondent real per-capita values by the implied prices. This is done for all groups of assets (See **TrBo**, **CP**, and **CB**).

TrBo: This variable is intended to proxy for the real per capita holdings of Treasury bonds (20 to 30 years maturity). Data is taken from Thomson Reuters Datastream (series name: U.S. Federal Debt - Marketable Securities Treasury Bonds Curn, Id: USSECTRDA). The time series is transformed to real per capita values in the same way as CnDUR. Prices and monthly net returns are calculated in the same way as for the variable TrBi. Correspondent data is taken from the yields on Long-Term U.S. Government Securities (FRED series LTGOVTBD for the period 1969 - 2000 and series GS20 from 2000 - 2008).

CP: Proxies the per capita holdings of commercial paper. Here data on the face value of outstanding commercial paper issued by nonfarm and nonfinancial corporate business is taken from FRED (series CPLBSNNCB). As before the time series is converted to real per capita values. Prices monthly net returns are derived from commercial paper yields in the same way as for the variable TrBi. Prior to 1971 I use the commercial paper yields series for prime commercial paper, 4-6 month maturity, from Banking and Monetary Statistics (Table 12.5 for 1941-1970). For 1971-1996 it is the 3-Month Commercial Paper Rate from FRED (series CP3M) and for 1997-2008 the 3-Month AA Nonfinancial Commercial Paper Rate from FRED (series CPN3M).

CB: Proxies the per capita holdings of corporate bonds. Here data on the face value of outstanding corporate bonds issued by nonfarm and nonfinancial corporate business is taken from FRED (series CBLBSNNCB). As before the time series is converted to real per capita values. Prices monthly net returns for corporate bonds are calculated in the same way as for the variable TrBi. Data are taken from Moody's Seasoned Aaa Corporate Bond Yield Index (FRED series AAA).

Further, for the GMM estimation model monthly returns on the numeraire asset r_t^d are proxied by returns calculated on the S&P 500 Stock Price Index (FRED series SP500). Further returns are deflated by the gross growth rate of the price index PCEPI.

B.2 Pricing equations

i. Assume that the aggregator function $\nu(\cdot)$ is CES, and is further nested in a CRRA utility function (this is similar to Poterba and Rotemberg (1986)):

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right\}^{\sigma},$$

$$L_t = \left[\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma} + \delta_B\left(\frac{B_t}{P_t}\right)^{\gamma} + (1-\delta_M-\delta_B)\left(\frac{S_t}{P_t}\right)^{\gamma}\right]^{\frac{1}{\gamma}},$$

where $\sigma < 0$, $0 < \gamma < 1$, and $0 < \delta_M$, $\delta_B < 1$. Then from equations (3.16) - (3.18) it follows that

$$\begin{aligned}
\frac{c_t}{L_t}\delta_M\left(\frac{M_t}{P_t}\right)^{\gamma-1} + E_t\left[M_{t+1}\right] &= 1,\\
\frac{c_t}{L_t}\delta_B\left(\frac{B_t}{P_t}\right)^{\gamma-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^b},\\
\frac{c_t}{L_t}(1-\delta_M-\delta_B)\left(\frac{S_t}{P_t}\right)^{\gamma-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^s}.
\end{aligned}$$

For the case of a nested CES aggregator each liquidity services premium on the asset's prices is not only determined by the level of holdings of the respective asset but also determined by the total holdings of liquidity services providing assets, aggregated by L_t. Hence, prices decrease with additional asset holdings as well as in L_t.

ii. Now assume that the aggregator function $\nu(\cdot)$ is Cobb-Douglas, which is nested in a CRRA utility function:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left\{c_t L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right\}^{\sigma},$$

$$L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)},$$

with $\sigma < 0$, and δ_M, δ_B, $(1 - \delta_M - \delta_B)$ summing up to 1. Then from equations (3.16) - (3.18)

$$\begin{aligned} c_t \delta_M \left(\frac{M_t}{P_t}\right)^{-1} + E_t\left[M_{t+1}\right] &= 1, \\ c_t \delta_B \left(\frac{B_t}{P_t}\right)^{-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^b}, \\ c_t \left(1 - \delta_M - \delta_B\right) \left(\frac{S_t}{P_t}\right)^{-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^s}. \end{aligned}$$

For the case of the nested Cobb-Douglas aggregator each asset's liquidity services premium is driven by the current level of holdings of the respective asset but not by the total holdings of liquid assets. This implication is not in line with KVJ and NIE.

iii. Next assume again that the utility function is CRRA with a nested CES aggregator. However, utility is in this case additively separable in its arguments:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left(c_t + L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right)^{\sigma},$$

$$L_t = \left[\delta_M \left(\frac{M_t}{P_t}\right)^{\gamma} + \delta_B \left(\frac{B_t}{P_t}\right)^{\gamma} + \left(1 - \delta_M - \delta_B\right)\left(\frac{S_t}{P_t}\right)^{\gamma}\right]^{\frac{1}{\gamma}},$$

where $\sigma < 0$, $0 < \gamma < 1$, and $0 < \delta_M$, $\delta_B < 1$. Then from equations (3.16) - (3.18)

$$\begin{aligned} L_t^{\frac{1}{\gamma}-1} \delta_M \left(\frac{M_t}{P_t}\right)^{\gamma-1} + E_t\left[M_{t+1}\right] &= 1, \\ L_t^{\frac{1}{\gamma}-1} \delta_B \left(\frac{B_t}{P_t}\right)^{\gamma-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^b}, \\ L_t^{\frac{1}{\gamma}-1} \left(1 - \delta_M - \delta_B\right) \left(\frac{S_t}{P_t}\right)^{\gamma-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^s}. \end{aligned}$$

As for a CES function it is assumed that $0 < \gamma < 1$, it implies for this model specification that increasing the holdings of one of the three assets under consideration increases asset prices. This implication is not in line with the assumptions by KVJ and NIE.

iv. Assume that the utility function is CRRA with a nested Cobb-Douglas aggregator and that it is additively separable in its arguments:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma}\left(c_t + L_t\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right)^{\sigma},$$

$$L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M}\left(\frac{B_t}{P_t}\right)^{\delta_B}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)},$$

with $\sigma < 0$, and δ_M, δ_B, $(1-\delta_M-\delta_B)$ summing up to 1. Then from equations (3.16) - (3.18)

$$\begin{aligned}
\delta_M\left(\frac{M_t}{P_t}\right)^{\delta_M-1}\left(\frac{B_t}{P_t}\right)^{\delta_B}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)} + E_t\left[M_{t+1}\right] &= 1,\\
\delta_B\left(\frac{M_t}{P_t}\right)^{\delta_M}\left(\frac{B_t}{P_t}\right)^{\delta_B-1}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^b},\\
(1-\delta_M-\delta_B)\left(\frac{M_t}{P_t}\right)^{\delta_M}\left(\frac{B_t}{P_t}\right)^{\delta_B}\left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)-1} + E_t\left[M_{t+1}\right] &= \frac{1}{R_t^s}.
\end{aligned}$$

As $0 < \delta_M < 1$, and $0 < \delta_B < 1$, the premium on each asset's price decreases with the level of the respective asset's holdings. However, the premium on each asset's price increases with additional holdings of each of the respective other two assets. This implication is not in line with KVJ and NIE.

Appendix C

Appendix to Chapter 4

C.1 Rational expectations equilibrium

Definition 1 *A RE equilibrium is given by a set of sequences* $\{c_t,\, n_t,\, d_t,\, \pi_t,\, \pi_t^w,\, w_t,\, mc_t,\, mrs_t,\, k_t,\, u_t,\, rk_t,\, x_t,\, q_t,\, m_t,\, m_t^R,\, p_t^L,\, b_t^S,\, b_t^{TS},\, b_t^L,\, b_t^{TL},\, \tau_t,\, \tau_t^m,\, g_t,\, l_t,\, i_t,\, \eta_t,\, y_t,\, R_t^m,\, R_t^L,\, R_t^D,\, R_t^S,\, R_t^L,\, R_t^{LB},\, R_t^{Euler},\, YTM_t\}_{t=0}^{\infty}$ *satisfying the following conditions summarizing the optimal behavior of households*

$$mrs_t = -\frac{u_{n,t}}{u_{c,t}} = -\frac{\nu n_{i,t}^{\upsilon}}{(c_{i,t} - hc_{t-1})^{-\sigma}}, \tag{C.1}$$

$$\frac{1}{R_t^D} = \beta E_t \left[\frac{1}{\pi_{t+1}} \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \left(1 + \frac{u_{d,t+1}}{u_{c,t+1}} \right) \right], \tag{C.2}$$

$$\frac{1}{R_t^E} = \beta E_t \left[\frac{1}{\pi_{t+1}} \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \right], \tag{C.3}$$

$$n_{i,t} = \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_t^w}{\mu_t^w - 1}} n_t,$$

$$w_t = w_{t-1} \frac{\pi_t^w}{\pi_t},$$

$$\left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}} - 1 \right) \frac{\pi_t^w}{\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}} y_t - \frac{n_t}{\omega_w (\mu_t^w - 1)} (\mu_t^w mrs_t - w_t)$$

$$= \beta E_t \left[\frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w} \pi_t^{\iota_w}} - 1 \right) \frac{\pi_{t+1}^w}{\bar{\pi}^{1-\iota_w} \pi_t^{\iota_w}} y_{t+1} \right], \tag{C.4}$$

with

$$u_{c,t} = [c_t - hc_{t-1}]^{-\sigma}, \; u_{d,t} = \varrho d_t^{-\varphi_d}, \; u_{n,t} = -\nu n_t^{\upsilon},$$

of firms

$$w_t = mc_t \alpha a_t n_t^{\alpha-1} (u_t k_{t-1})^{1-\alpha} \left(\frac{R_t^L}{R_t^E} \right)^{-1}, \tag{C.5}$$

$$rk_t = mc_t (1-\alpha) a_t n_t^{\alpha} (u_t k_{t-1})^{-\alpha} k_{t-1}, \tag{C.6}$$

$$\frac{l_t}{R_t^L} = w_t n_t, \tag{C.7}$$

$$\begin{aligned} 1 &= q_t \epsilon_t^I \left(1 - \frac{\gamma_I}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 - \gamma_I \left(\frac{x_t}{x_{t-1}} - 1 \right) \frac{x_t}{x_{t-1}} \right) \\ &+ \beta E_t \left[\frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} q_{t+1} \epsilon_{t+1}^I \gamma_I \left(\frac{x_{t+1}}{x_t} - 1 \right) \left(\frac{x_{t+1}}{x_t} \right)^2 \right], \end{aligned}$$

$$q_t = \beta E_t \frac{\xi_{t+1} u_{c,t+1}}{\xi_t u_{c,t}} \left[\begin{array}{c} q_{t+1}(1-\delta) + rk_{t+1} u_{t+1} \\ -rk \left(u_{t+1} - 1 \right) + \frac{\varkappa \cdot rk}{2} \left(u_{t+1} - 1 \right)^2 \end{array} \right], \tag{C.8}$$

$$\begin{aligned} &\left(\frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - 1 \right) \frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - \frac{\mu_t^p}{\omega_p \left(\mu_t^p - 1 \right)} \left(mc_t - \frac{1}{\mu_t^p} \right) \\ &= \beta E_t \left[\left(\frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} \frac{y_{t+1}}{y_t} \right], \end{aligned} \tag{C.9}$$

of banks

$$YTM_t = \frac{1}{p_t^L} + \rho_t^s, \tag{C.10}$$

$$\frac{1}{R_t^D} = 1 + (\mu_t - 1)\frac{1}{R_t^E}E_t\left[-\Xi_{m,t+1}\right], \tag{C.11}$$

$$\frac{1}{R_t^S} = 1 + \frac{1}{R_t^E}\left(E_t\left[\eta_{t+1}\kappa_{t+1}^S\right] - E_t\left[-\Xi_{m,t+1}\right]\right), \tag{C.12}$$

$$\begin{aligned} 1 = \frac{1}{R_t^E}(&E_t\left[R_{t+1}^{LB}YTM_{t+1}\eta_{t+1}\kappa_{t+1}^L\right] \\ &-E_t\left[-\Xi_{m,t+1}\right]E_t\left[R_{t+1}^{LB}YTM_{t+1}\right]) + E_t\left[R_{t+1}^{LB}YTM_{t+1}\right], \end{aligned} \tag{C.13}$$

$$\frac{1}{R_t^L} = 1 - \Xi_{l,t} - \frac{1}{R_t^E}E_t\left[-\Xi_{m,t+1}\right], \tag{C.14}$$

$$R_t^m = 1 - \Xi_{m,t} - \eta_t R_t^m, \tag{C.15}$$

$$d_t = m_t + b_t^S + E_t\left[p_{t+1}^L YTM_{t+1}b_t^L\right] + l_t,$$

$$\frac{b_t^S}{R_t^S} = \frac{b_t^{TS}}{R_t^S} - \kappa_t^S\frac{b_{t-1}^S}{\pi_t},$$

$$p_t^L b_t^L = p_t^L b_t^{TL} - \kappa_t^L p_t^L YTM_t b_{t-1}^L/\pi_t,$$

with

$$\Xi_t(l_t, i_t) = \zeta_t\left(l_t\right)^{\eta_{rc}}\left(m_{t-1}\pi_t^{-1} - \mu_t d_{t-1}\pi_t^{-1} + i_t\right)^{-\phi_{rc}},$$

$$\Xi_{l,t} = \eta_{rc}\frac{\Xi_t}{l_t},$$

$$\Xi_{m,t}(l_t, i_t) = -\phi_{rc}\Xi_t\left(m_{t-1}\pi_t^{-1} - \mu_t d_{t-1}\pi_t^{-1} + i_t\right)^{-1},$$

of the central bank

$$i_t = \kappa_t^S\frac{b_{t-1}^S}{\pi_t R_t^m} + \kappa_t^L\frac{p_t^L YTM_t b_{t-1}^L}{\pi_t R_t^m} + \varepsilon_{t,Coll}, \quad \varepsilon_{t,Coll} \sim N\left(0, \sigma_{Coll}\right), \tag{C.16}$$

$$i_t = m_t - m_{t-1}\pi_t^{-1} + m_t^R,$$

$$m_t = \Lambda m_t^R,$$

$$R_t^m = \left(R_{t-1}^m\right)^{\rho_r}\left(R^m\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{y_t}{y}\right)^{\rho_y}\right)^{(1-\rho_r)}\left(\frac{y_t}{y_{t-1}}\right)^{\rho_{dy}}\exp\varepsilon_{r,t},$$

and of the government

$$b_t^{TS} = \Gamma \frac{b_{t-1}^{TS}}{\pi_t},$$

$$\left(\frac{p_t^L b_t^{TL}}{\bar{p}^L \bar{b}^{TL}} \right) = \left(\frac{p_{t-1}^L b_{t-1}^{TL}}{\bar{p}^L \bar{b}^{TL}} \right)^{\rho_b} e^{\varepsilon_{b,t}}, \quad \varepsilon_{b,t} \sim N\left(0, \sigma_b\right),$$

and aggregate resources

$$\begin{aligned} y_j &= a_t n_t^{\alpha} \left(u_t k_{t-1}\right)^{1-\alpha}, \qquad \text{(C.17)} \\ y_t &= c_t + x_t + g_t + \Xi_t + \left(rk\left(u_t - 1\right) + \frac{\varkappa \cdot rk}{2} \left(u_t - 1\right)^2 \right) k_{t-1} \\ &+ \frac{\omega_p}{2} \left(\frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - 1 \right)^2 y_t + \frac{\omega_w}{2} \left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}} - 1 \right)^2 y_t, \\ k_t &= (1-\delta) k_{t-1} + \epsilon_t^I \left(1 - \frac{\gamma_I}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 \right) x_t, \end{aligned}$$

where $\frac{1}{R_t^E} = E_t \varphi_{t,t+1}$, $R_{t+1}^{LB} = \frac{p_{t+1}^L}{p_t^L}$, *as well as the transversality conditions, a monetary policy setting* $\{R_t^m \geq 1\}_{t=0}^{\infty}$, *and* $\pi \geq \beta$, *and* κ_t^S, *and* κ_t^L, *and a fiscal policy* $\{g_t, \tau_t\}_{t=0}^{\infty}$ *satisfying*

$$\tau_t = b_{t-1}^{TS} - \frac{b_t^{TS}}{R_t^S} + p_t^L YTM_t b_{t-1}^{TL} - p_t^L b_t^{TL} + g_t - \tau_t^m \;, \qquad \text{(C.18)}$$

$$\left(\frac{g_t}{\bar{g}} \right) = \left(\frac{g_{t-1}}{\bar{g}} \right)^{\rho_g} e^{\varepsilon_{g,t}}, \quad \varepsilon_{g,t} \sim N\left(0, \sigma_g\right),$$

and the stochastic processes for the shocks

$$\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}, \quad \varepsilon_{\xi,t} \sim N(0, \sigma_\xi),$$
$$\log\left(\frac{\mu_t^w}{\bar{\mu}^w}\right) = \rho_w \log\left(\frac{\mu_{t-1}^w}{\bar{\mu}^w}\right) + \varepsilon_{w,t}, \quad \varepsilon_{w,t} \sim N(0, \sigma_w),$$
$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim N(0, \sigma_a),$$
$$\log(\epsilon_t^I) = \rho_\epsilon \log(\epsilon_{t-1}^I) + \varepsilon_{\epsilon,t}, \quad \varepsilon_{\epsilon,t} \sim N(0, \sigma_I),$$
$$\log\left(\frac{\mu_t^p}{\bar{\mu}^p}\right) = \rho_p \log\left(\frac{\mu_{t-1}^p}{\bar{\mu}^p}\right) + \varepsilon_{p,t}, \quad \varepsilon_{p,t} \sim N(0, \sigma_p),$$
$$\log\left(\frac{\rho_t^s}{\bar{\rho}^s}\right) = \rho_{\rho^s} \log\left(\frac{\rho_{t-1}^s}{\bar{\rho}^s}\right) + \varepsilon_{\rho^s,t}, \quad \varepsilon_{\rho^s,t} \sim N(0, \sigma_{\rho^s}),$$
$$\log\left(\frac{\mu_t}{\bar{\mu}}\right) = \rho_\mu \log\left(\frac{\mu_{t-1}}{\bar{\mu}}\right) + \varepsilon_{\mu,t}, \quad \varepsilon_{\mu,t} \sim N(0, \sigma_\mu),$$
$$\log\left(\frac{\zeta_t}{\bar{\zeta}}\right) = \rho_\zeta \log\left(\frac{\zeta_{t-1}}{\bar{\zeta}}\right) + \varepsilon_{\zeta,t}, \quad \varepsilon_{\zeta,t} \sim N(0, \sigma_\zeta),$$

and a given initial values $m_{-1} > 0$, $l_{-1} > 0$, $b_{-1}^{LT} > 0$, $b_{-1}^{TS} > 0$, $k_{-1} > 0$, $x_{-1} > 0$, $\pi_{-1} > 0$.

C.2 Steady state

In this appendix, we examine the steady state of the economy. For a given value of π, the following conditions determine the steady state values of $\{R^{Euler}, k/n, x/n, mc, q\}$:

Equations (C.4) and (C.9) imply for the steady state marginal cost and the steady state marginal rate of substitution

$$mc = \frac{1}{\mu^p}, \; mrs = \frac{w}{\mu^w}.$$

Combining (C.11) and (C.13), dropping time indices, and assuming for the steady state an absence of LSAPs with $\kappa^L = 0$, yields for the steady state minimum fraction of reserves required for deposit management

$$\mu = \left(\frac{1}{R^D} - \frac{1}{YTM}\right)\frac{YTM}{YTM - 1}.$$

This, as well as calibrating the scaling factors ζ and ϱ which will be done below, is required to make the steady state model calibration of interest rates match the historically observed means of the interest rates. Further, the steady state Euler rate is determined by the steady state inflation π and the discount factor β, $R^E = \frac{\pi}{\beta}$. The steady state Tobin's q is equal to one, the steady state investment-to-labor ratio equals the capital-to-labor ratio times the capital depreciation ratio, $\frac{x}{n} = \delta \frac{k}{n}$, and the steady state long-term bond price is, implied by (C.10), equal to $p^L = \frac{1}{YTM - \rho^S}$. Combining equations (C.8) and (C.6) implies for the steady state capital-to-labor ratio

$$\frac{k}{n} = \left(\frac{\beta \cdot mc(1-\alpha)}{1 - \beta(1-\delta)} \right)^{1/\alpha},$$

and hence (C.17) implies for the steady state level of output

$$y = n \cdot \left(\frac{k}{n} \right)^{(1-\alpha)}.$$

Combining (C.5) and (C.7) implies for the steady state amount of loans

$$l = mc \cdot \alpha \cdot n \frac{k}{n}^{(1-\alpha)} R^{Euler}. \tag{C.19}$$

Further, we calibrate the steady state deposit holdings as

$$d = DL \cdot l, \tag{C.20}$$

where DL is the historically measured deposits-to-loans ratio. From (C.6) and (C.7) it is now implied that the steady state capital rental rate and the steady state wage rate are

$$\begin{aligned} rk &= mc(1-\alpha) n^{\alpha} (uk)^{-\alpha}, \\ w &= \frac{l}{n} \cdot R^L. \end{aligned}$$

Now take into account the banking cost function without time indices

$$\Xi = \zeta \frac{l^{\eta_{rc}}}{\left(m\left(1+\Lambda^{-1}\right) - \mu d \pi^{-1}\right)^{\phi_{rc}}}, \tag{C.21}$$

$$\Xi_l = \eta_{rc} \frac{\Xi}{l}, \tag{C.22}$$

$$\Xi_m = -\phi_{rc} \frac{\Xi}{\left(m\left(1+\Lambda^{-1}\right) - \mu d \pi^{-1}\right)}, \tag{C.23}$$

where we used that steady state central bank money injections are given by

$$i = m\left(1 - \pi^{-1} + \Lambda^{-1}\right),$$

which follows from assuming that money supplied outright is a constant fraction of money reserves $m = \Lambda m^R$. Now drop time indices in (C.14) and use (C.22) to substitute out Ξ_l. Then substitute out $\Xi_m = -\left(1 - YTM^{-1}\right) R^E$ which is implied by (C.13), to get the steady state banking cost

$$\Xi = \left(\frac{1}{YTM} - \frac{1}{R^L}\right)\frac{l}{\eta_{rc}}.$$

Then we can derive the steady state value for the marginal contribution of loans to banking cost (C.22). Further we define $gdp = y - \Xi$ as steady state GDP. Where g is calibrated with the historical government spending-to-GDP ratio, GY, implying $g = GY \cdot y$. Now we can determine the steady state value of consumption from the steady state aggregate resource constraint

$$c = y - g - x - \Xi.$$

Further, dropping time indices for marginal utility of consumption yields $u_c = \left(\left(1 - h\right) c\right)^{-\sigma}$. Plug the former into (C.1) and combine with the steady state marginal rate of substitution, $mrs = \frac{w}{\mu^w}$, we can calibrate the Frisch elasticity ν, and determine the steady state value for marginal utility of labor u_n

$$\nu = \frac{u_c w}{\mu^w n^{\upsilon}},$$
$$u_n = -\nu n^{\upsilon}.$$

Next is to calibrate the scaling factor ϱ which governs marginal utility of deposits. For that reason plug (C.3) into (C.2) and set this equal to (C.11). Now substitute out Ξ_m from (C.13)

$$\varrho = \left(\frac{d}{\pi}\right)^{\varphi}\left(\frac{\left(\left(1 - h\right) c\right)^{-\sigma}}{R^E\left(\mu\left(1 - YTM^{-1}\right) + YTM^{-1}\right) - 1}\right)^{-1}. \tag{C.24}$$

Hence, we can determine the steady state value of marginal utility of deposits $u_d = \varrho\left(\frac{d}{\pi}\right)^{-\varphi}$. To determine the steady state money holdings, solve (C.23) for m and substitute out Ξ_m by (C.13) which yields

$$m = \frac{1}{1 + \Lambda^{-1}}\left(\mu\frac{d}{\pi} + \frac{\phi_{rc}\Xi}{\left(1 - YTM^{-1}\right) R^E}\right). \tag{C.25}$$

Now we can determine steady state money reserves $m^R = \frac{m}{\Lambda}$. Further we can now calibrate the steady state value of the scaling parameter within the banking cost function ζ from (C.21)

$$\zeta = \frac{\Xi}{l^{\eta_{rc}}} \left(m \left(1 + \Lambda^{-1} \right) - \mu d \pi^{-1} \right)^{\phi_{rc}}. \tag{C.26}$$

From (C.15) the steady state multiplier on the collateral constraint is implied by

$$\eta = \frac{1 - \Xi_m}{R_t^m} - 1.$$

This can be used to determine the value of the steady state injections i

$$i = m \left(1 - \pi^{-1} + \Lambda^{-1} \right).$$

Next we can solve (C.12) for R^S. By substituting out Ξ_m we get the steady state short-term bond rate

$$R^S = \left(\frac{1}{R^E} \eta \kappa^S + \frac{1}{YTM} \right)^{-1}.$$

Banks' steady state short-term bond holdings are implied by (C.16)

$$b^S = \frac{i \pi R m}{\kappa^S}.$$

Banks' steady state long-term bond holdings are derived from the steady state bank balance sheet condition $d = m + b^S + p^L YTM b^L + l$,

$$b^L = \frac{d - m - b^S - l}{p^L YTM}.$$

For the government's steady state long-term bond supply we assume that $b^{TL} = b^L$, as $\kappa^L = 0$. For the short-term steady state bond supply we sum up the financial intermediaries' steady state holdings and the central bank's steady state holdings

$$b^{TS} = b^S + R^S R^m \frac{\kappa^S b^S}{\pi R^m}.$$

Further the steady state lump-sum taxes are implied by (C.18)

$$\tau_t = b^{TS} \left(\frac{1}{\pi} - \frac{1}{R^S} \right) + p^L b^{TL} \left(\frac{YTM}{\pi} - 1 \right) + g.$$

C.3 Derivation of labor demand

Perfectly competitive labor packers (or unions) buy differentiated labor input, aggregate it through the technology $n_t^{1/\mu_t^w} = \int_0^1 n_{i,t}^{1/\mu_t^w} di$, and supply the effective labor units n_t. They sell effective units of labor to intermediate goods producing firms at price w_t. Profit maximization of labor packers

$$\max_{n_{i,t}} \left[w_t \left(\int_0^1 n_{i,t}^{1/\mu_t^w} di \right)^{\mu_t^w} - \int_0^1 w_{i,t} n_{i,t} di \right],$$

yields the FOC

$$w_{i,t} = w_t \mu_t^w \left(\int_0^1 n_{i,t}^{1/\mu_t^w} \right)^{\mu_t^w - 1} \frac{1}{\mu_w} n_{i,t}^{1/\mu_t^w - 1},$$

and the demand function

$$n_{i,t} = \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_t^w}{\mu_t^w - 1}} n_t.$$

C.4 Wage Phillips curve

To derive the first order condition for the wage rate $w_{i,t}$, let the household i maximize the objective function (4.1) subject the budget constraints (4.2) and the labor demand function (4.7)

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t u \left(c_{i,t}, \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_t^w}{\mu_t^w - 1}} n_t, \frac{d_{i,t-1}}{\pi_t} \right),$$

$$\begin{aligned} ... + P_t c_{i,t} \leq P_t w_{i,t} & \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_t^w}{\mu_t^w - 1}} n_t \\ & - P_t \frac{\omega_W}{2} \left(\frac{P_t w_{i,t}}{P_{t-1} w_{i,t-1} \left(\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w} \right)} - 1 \right)^2 y_t + ... \end{aligned}$$

Here we plugged in the labor demand function for $n_{i,t}$ and the wage adjustment cost (4.3) for WAC_t.

The FOC w.r.t. $w_{i,t}$ reads

$$
\begin{aligned}
0 = &\beta^t \lambda_{i,t} \left(-\frac{1}{\mu_t^w - 1} w_{i,t}^{-\frac{1}{\mu_t^w-1}-1} w_t^{\frac{\mu_t^w}{\mu_t^w-1}} n_t \right) \\
&-\beta^t \lambda_{i,t} \left(\omega_W \left(\frac{P_t w_{i,t}}{P_{t-1} w_{i,t-1} \left(\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}\right)} - 1 \right) y_t \frac{P_t}{P_{t-1} w_{i,t-1} \left(\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}\right)} \right) \\
&+\beta^{t+1} \lambda_{i,t+1} \omega_W \left(\frac{P_{t+1} w_{i,t+1}}{P_t w_{i,t} \left(\bar{\pi}^{1-\iota_w} \pi_t^{\iota_w}\right)} - 1 \right) y_{t+1} \frac{P_{t+1} w_{i,t+1}}{P_t w_{i,t}^2 \left(\bar{\pi}^{1-\iota_w} \pi_t^{\iota_w}\right)} \\
&+\beta^t u_{n,i,t} \left(-\frac{\mu_t^w}{\mu_t^w - 1} \right) w_{i,t}^{-\frac{\mu_t^w}{\mu_t^w-1}-1} w_t^{\frac{\mu_t^w}{\mu_t^w-1}} n_t.
\end{aligned}
$$

Rearranging gives

$$
\begin{aligned}
0 = &-\frac{1}{\mu_t^w - 1} \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_t^w}{\mu_t^w-1}} w_{i,t} \\
&-\omega_W \left(\frac{P_t w_{i,t}}{P_{t-1} w_{i,t-1} \left(\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}\right)} - 1 \right) \frac{y_t}{n_t} \frac{P_t w_{i,t}}{P_{t-1} w_{i,t-1} \left(\bar{\pi}^{1-\iota_w} \pi_{t-1}^{\iota_w}\right)} \\
&+\beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \omega_W \left(\frac{P_{t+1} w_{i,t+1}}{P_t w_{i,t} \left(\bar{\pi}^{1-\iota_w} \pi_t^{\iota_w}\right)} - 1 \right) \frac{y_{t+1}}{n_t} \frac{P_{t+1} w_{i,t+1}}{P_t w_{i,t} \left(\bar{\pi}^{1-\iota_w} \pi_t^{\iota_w}\right)} \\
&-\frac{\mu_t^w}{\mu_t^w - 1} \frac{u_{n,i,t}}{\lambda_{i,t}} \left(\frac{w_{i,t}}{w_t} \right)^{-\frac{\mu_t^w}{\mu_t^w-1}}.
\end{aligned}
$$

For $\omega_W = 0$ we get

$$
\begin{aligned}
\frac{1}{\mu_t^w - 1} w_{i,t} &= \frac{\mu_t^w}{\mu_t^w - 1} \left(-\frac{u_{n,i,t}}{\lambda_{i,t}} \right), \\
w_{i,t} &= \mu_t^w \left(\underbrace{-\frac{u_{n,i,t}}{\lambda_{i,t}}}_{MRS} \right).
\end{aligned}
$$

For $\omega_W \neq 0$, define $\pi_t^w = \frac{P_t w_t}{P_{t-1} w_{t-1}} = \pi_t \frac{w_t}{w_{t-1}}$. Note that we assume trade in contingent claims here. Hence, the MRS is the same across households and each household will supply labor for the same wage rate w_t. Therefore, we get

$$
\begin{aligned}
&\omega_W\left(\frac{P_t w_t}{P_{t-1}w_{t-1}\left(\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}\right)}-1\right)\frac{y_t}{n_t}\frac{P_t w_t}{P_{t-1}w_{t-1}\left(\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}\right)}\\
&=\beta\frac{\lambda_{t+1}}{\lambda_t}\omega_W\left(\frac{P_{t+1}w_{t+1}}{P_t w_t\left(\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}\right)}-1\right)\frac{y_{t+1}}{n_t}\frac{P_{t+1}w_{t+1}}{P_t w_t\left(\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}\right)}\\
&-\frac{1}{\mu_t^w-1}w_t-\frac{\mu_t^w}{\mu_t^w-1}\frac{u_{n,t}}{\lambda_t},
\end{aligned}
$$

after rearranging

$$
\begin{aligned}
&\left(\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}}-1\right)\frac{\pi_t^w}{\bar{\pi}^{1-\iota_w}\pi_{t-1}^{\iota_w}}y_t-\frac{n_t}{\left(\mu_t^w-1\right)\omega_W}\left(\mu_t^w\left(-\frac{u_{n,t}}{\lambda_t}\right)-w_t\right)\\
&=\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_{t+1}^w}{\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}}-1\right)\frac{\pi_{t+1}^w}{\bar{\pi}^{1-\iota_w}\pi_t^{\iota_w}}y_{t+1}.
\end{aligned}
$$

C.5 Price Phillips curve

The retailer maximizes profits by optimally setting the intermediate good's price $P_{k,t}$, taking into account cost for acquisition mc_t, price adjustment cost (4.22), and the market's demand function for $y_{k,t}$ (4.21). The retailer's problem therefore reads

$$
\max_{\{P_{k,t}\}} E_t\sum_{k=0}^{\infty}\beta^t\left[\frac{P_{k,t}}{P_t}y_{k,t}-mc_t y_{k,t}-\frac{\omega_P}{2}\left(\frac{P_{k,t}}{P_{k,t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)}-1\right)^2 y_t\right],
$$

$$
\text{s.t. } y_{k,t}=\left(\frac{P_{k,t}}{P_t}\right)^{-\varepsilon}y_t.
$$

Plugging in for $y_{k,t}$ yields

$$
\sum_{k=0}^{\infty}\beta^t\left[\begin{array}{c}\frac{P_{k,t}}{P_t}\left(\frac{P_{k,t}}{P_t}\right)^{-\frac{\mu_t^p}{\mu_t^p-1}}y_t-mc_t\left(\frac{P_{k,t}}{P_t}\right)^{-\frac{\mu_t^p}{\mu_t^p-1}}y_t\\ -\frac{\omega_P}{2}\left(\frac{P_{k,t}}{P_{k,t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)}-1\right)^2 y_t\end{array}\right].
$$

The first order condition with respect to $P_{k,t}$ reads

$$
\begin{aligned}
0 = &-\frac{1}{\mu_t^p - 1} P_{k,t}{}^{-\frac{1}{\mu_t^p-1}-1} P_t^{\frac{1}{\mu_t^p-1}} y_t \\
&+ mc_t \frac{\mu_t^p}{\mu_t^p - 1} P_{k,t}^{-\frac{\mu_t^p}{\mu_t^p-1}} P_t^{\frac{\mu_t^p}{\mu_t^p-1}} y_t \\
&- \omega_P \left(\frac{P_{k,t}}{P_{k,t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)} - 1 \right) y_t \frac{1}{P_{k,t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)} \\
&+ \beta\omega_P \left(\frac{P_{k,t+1}}{P_{k,t}\left(\bar{\pi}^{1-\iota_p}\pi_t^{\iota_p}\right)} - 1 \right) y_{t+1} \frac{P_{k,t+1}}{P_{k,t}^2\left(\bar{\pi}^{1-\iota_p}\pi_t^{\iota_p}\right)},
\end{aligned}
$$

where we obtain after some rearranging

$$
\begin{aligned}
0 = &-\frac{1}{\mu_t^p - 1} \left(\frac{P_{k,t}}{P_t}\right)^{-\frac{1}{\mu_t^p-1}} y_t \frac{1}{P_{k,t}} \\
&+ mc_t \frac{\mu_t^p}{\mu_t^p - 1} \left(\frac{P_{k,t}}{P_t}\right)^{-\frac{\mu_t^p}{\mu_t^p-1}} y_t \frac{1}{P_{k,t}} \\
&- \omega_P \left(\frac{P_{k,t}}{P_{k,t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)} - 1 \right) y_t \frac{1}{P_{k,t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)} \\
&+ \beta\omega_P \left(\frac{P_{k,t+1}}{P_{k,t}\left(\bar{\pi}^{1-\iota_p}\pi_t^{\iota_p}\right)} - 1 \right) y_{t+1} \frac{P_{k,t+1}}{P_{k,t}^2\left(\bar{\pi}^{1-\iota_p}\pi_t^{\iota_p}\right)}.
\end{aligned}
$$

Now aggregate over all retailers prices $P_t = \int_0^1 P_{k,t} dk$

$$
\begin{aligned}
0 = &-\frac{1}{\mu_t^p - 1} \left(\frac{P_t}{P_t}\right)^{-\frac{1}{\mu_t^p-1}} y_t \\
&+ mc_t \frac{\mu_t^p}{\mu_t^p - 1} \left(\frac{P_t}{P_t}\right)^{-\frac{\mu_t^p}{\mu_t^p-1}} y_t \\
&- \omega_P \left(\frac{P_t}{P_{t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)} - 1 \right) y_t \frac{P_t}{P_{t-1}\left(\bar{\pi}^{1-\iota_p}\pi_{t-1}^{\iota_p}\right)} \\
&+ \beta\omega_P \left(\frac{P_{k,t+1}}{P_t\left(\bar{\pi}^{1-\iota_p}\pi_t^{\iota_p}\right)} - 1 \right) y_{t+1} \frac{1}{P_t^1\left(\bar{\pi}^{1-\iota_p}\pi_t^{\iota_p}\right)},
\end{aligned}
$$

then define $\pi_t = \frac{P_t}{P_{t-1}}$

$$0 = mc_t \frac{\mu_t^p}{\mu_t^p - 1} + \frac{1}{\mu_t^p - 1}$$
$$-\omega_P \left(\frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - 1 \right) \frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} + \beta \omega_P \left(\frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} \frac{y_{t+1}}{y_t},$$

and rearrange

$$\left(\frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - 1 \right) \frac{\pi_t}{\bar{\pi}^{1-\iota_p} \pi_{t-1}^{\iota_p}} - \frac{\mu_t^p}{\omega_p \left(\mu_t^p - 1 \right)} \left(mc_t - \frac{1}{\mu_t^p} \right)$$
$$= \beta E_t \left[\left(\frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}^{1-\iota_p} \pi_t^{\iota_p}} \frac{y_{t+1}}{y_t} \right].$$

C.6 Long-term bond prices

Consider the following investment problem. Following Woodford (2001) we assume that agents can invest in perpetuities B_{t-s}^L issued in period $t-s$, which pay ρ^s units of currency in each period t with $\rho \in [0,1]$. Hence, coupons decay exponentially, such that ρ governs the duration of a bond. Newly issued perpetuities exhibit the price p_t^L

$$... + p_t^L B_t^L + P_t c_t + ... \leq ... + \sum_{s=1}^{\infty} \rho^{s-1} B_{t-s}^L + P_t w_t n_t + ...$$

Define $\mathcal{B}_{t-1}^L$ as the sum of all payments from past bond issuances in period t, like in Arellano and Ramanarayanan (2012)

$$\mathcal{B}_{t-1}^L = \sum_{s=1}^{\infty} \rho^{s-1} B_{t-s}^L.$$

Given that

$$\mathcal{B}_t^L = \sum_{s=1}^{\infty} \rho^{s-1} B_{t+1-s}^L = B_t^L + \sum_{s=2}^{\infty} \rho^{s-1} B_{t+1-s}^L = B_t^L + \rho \sum_{s=1}^{\infty} \rho^{s-1} B_{t-s}^L,$$

we get the following relation between $\mathcal{B}_t^L$ and $\mathcal{B}_{t-1}^L$ such that

$$\mathcal{B}_t^L = \rho \mathcal{B}_{t-1}^L + B_t^L.$$

Substituting out B_t^L, the budget constraint can be rewritten as
$p_t^L\left(\mathcal{B}_t^L-\rho\mathcal{B}_{t-1}^L\right)\leq\mathcal{B}_{t-1}^L+P_t\left(w_tn_t-c_t\right)+...\Leftrightarrow$

$$...+p_t^L\mathcal{B}_t^L+P_tc_t+...\leq...+\left(1+\rho p_t^L\right)\mathcal{B}_{t-1}^L+P_tw_tn_t+...$$

Consider now a secondary market for long-term bonds. The period t price of a bond issued in $t-s$ is $p_{t,t-s}^L$

$$p_{t,t}^LB_{t,t}^L+\sum_{s=1}^{\infty}p_{t,t-s}^LB_{t,t-s}^L\leq\sum_{s=1}^{\infty}\left(p_{t,t-s}^L+\rho^{s-1}\right)B_{t-1,t-s}^L+...,$$

where $B_{t-1,t-s}^L$ denotes the beginning-of-period t holdings of long-term bonds issued in $t-s$, and $B_{t,t-s}^L$ its end-of-period t holdings. Compare two bonds issued in t and $t-s$. Period t investments in bonds issued in $t-s$, $B_{t,t-s}^L$, satisfy

$$p_{t,t-s}^L=\beta E_t\left[\frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}\left(p_{t+1,t-s}^L+\rho^{s-1+1}\right)\right],$$

and in period $t+1$, $B_{t+1,t-s}^L$

$$p_{t+1,t-s}^L=\beta E_{t+1}\left[\frac{u_{c,t+2}}{u_{c,t+1}\pi_{t+2}}\left(p_{t+2,t-s}^L+\rho^{s-1+2}\right)\right].$$

Iterating forward we get,

$$\begin{aligned}
p_{t,t-s}^L &= \beta E_t\left[\frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}\left(\beta E_{t+1}\left[\frac{u_{c,t+2}}{u_{c,t+1}\pi_{t+2}}\left(p_{t+2,t-s}^L+\rho^{s-1+2}\right)\right]+\rho^{s-1+1}\right)\right]\\
&= E_t\left[\begin{array}{c}\beta\frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}\rho^{s-1+1}+\beta^2\frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}\frac{u_{c,t+2}}{u_{c,t+1}\pi_{t+2}}\rho^{s-1+2}\\ +\beta^2\frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}\frac{u_{c,t+2}}{u_{c,t+1}\pi_{t+2}}p_{t+2,t-s}^L\end{array}\right]\\
&\vdots\\
&= E_t\sum_{k=1}^{\infty}\beta^k\frac{u_{c,t+k}}{u_{c,t}\pi_{t+k}}\rho^{s-1+k}. \qquad \text{(C.27)}
\end{aligned}$$

Applying the same procedure, for a bond issued in t, leads to the first order condition $p_{t,t}^L=\beta E_t\left[\frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}\left(1+p_{t+1,t}^L\right)\right]$, implying that its price $p_{t,t}^L$ satisfies

$$p_{t,t}^L=E_t\sum_{k=1}^{\infty}\beta^k\frac{u_{c,t+k}}{u_{c,t}\pi_{t+1}}\rho^{k-1}.$$

Hence, that secondary market prices satisfy

$$p^L_{t,t-s} = \rho^s p^L_{t,t}. \tag{C.28}$$

Using this price relation, we get

$$\begin{aligned}
p^L_{t,t} B^L_{t,t} + \sum_{s=1}^{\infty} p^L_{t,t-s} B^L_{t,t-s} &\leq \sum_{s=1}^{\infty} \left(p^L_{t,t-s} + \rho^{s-1}\right) B^L_{t-1,t-s} + ... \\
\Leftrightarrow p^L_{t,t} \left(B^L_{t,t} + \sum_{s=1}^{\infty} \rho^s B^L_{t,t-s} \right) &\leq \sum_{s=1}^{\infty} \left(\rho^s p^L_{t,t} + \rho^{s-1}\right) B^L_{t-1,t-s} + ... \\
\Leftrightarrow p^L_{t,t} \left[\sum_{s=0}^{\infty} \rho^s B^L_{t,t-s} \right] &\leq \left(1 + \rho p^L_{t,t}\right) \left[\sum_{s=1}^{\infty} \rho^{s-1} B^L_{t-1,t-s} \right] + ...
\end{aligned}$$

Note that in period t, the sum of all payments from past bond issuances is given by $\mathcal{B}^L_{t-1} = \sum_{s=1}^{\infty} \rho^{s-1} B^L_{t-1,t-s}$, and in period $t+1$ by

$$\mathcal{B}^L_t = \sum_{s=1}^{\infty} \rho^{s-1} B^L_{t,t+1-s} = \sum_{s=0}^{\infty} \rho^{s+1-1} B^L_{t,t+1-(s+1)} = \sum_{s=0}^{\infty} \rho^s B^L_{t,t-s},$$

where $\mathcal{B}^L_t = \rho \mathcal{B}^L_{t-1} + B^L_{t,t}$. We can write the budget constraint again as above

$$... + p^L_{t,t} \mathcal{B}^L_t + ... \leq ... + \left(1 + \rho p^L_{t,t}\right) \mathcal{B}^L_{t-1} + ...$$

We now follow Woodford (2001) and apply a simplification, which hugely reduces the dimensionality of the investor's problem. Consider the issuer of long-term debt, who satisfies the budget constraint

$$... + p^L_{t,t} B^L_{t,t} + \sum_{s=1}^{\infty} p^L_{t,t-s} B^L_{t,t-s} + ... \leq ... + \sum_{s=1}^{\infty} \left(p^L_{t,t-s} + \rho^{s-1}\right) B^L_{t-1,t-s} + ...$$

Recall that bonds issued in period $t-s$ are equivalent as ρ^s bonds issued in t (see C.28). We assume that in each period the issuer redeems all previously issued debt by issuing new debt $B^L_{t,t}$.

Assumption 1 *Suppose that debt is issued* $\forall t \geq 0$ *according to* $\sum_{s=1}^{\infty} p^L_{t,t-s} B^L_{t,t-s} = 0$.

Assumption 1 implies for period $t-1$: $\sum_{s=1}^{\infty} p_{t-1,t-(s+1)}^{L} B_{t-1,t-(s+1)}^{L} = 0$. Then, the term $\sum_{s=1}^{\infty} \left(p_{t,t-s}^{L} + \rho^{s-1}\right) B_{t-1,t-s}^{L}$, can be rewritten as

$$
\begin{aligned}
&\sum_{s=1}^{\infty} \left(p_{t,t-s}^{L} + \rho^{s-1}\right) B_{t-1,t-s}^{L} \\
&= \left(p_{t,t-1}^{L} + 1\right) B_{t-1,t-1}^{L} + \sum_{s=2}^{\infty} \left(p_{t,t-s}^{L} + \rho^{s-1}\right) B_{t-1,t-s}^{L}, \\
&= \left(p_{t,t-1}^{L} + 1\right) B_{t-1,t-1}^{L} + \sum_{s=1}^{\infty} \left(p_{t,t-(s+1)}^{L} + \rho^{s+1-1}\right) B_{t-1,t-(s+1)}^{L}, \\
&= \left(p_{t,t-1}^{L} + 1\right) B_{t-1,t-1}^{L} + \sum_{s=1}^{\infty} p_{t-1,t-(s+1)}^{L} \begin{pmatrix} \frac{p_{t,t-(s+1)}^{L}}{p_{t-1,t-(s+1)}^{L}} \\ + \frac{\rho^{s+1-1}}{p_{t-1,t-(s+1)}^{L}} \end{pmatrix} B_{t-1,t-(s+1)}^{L}, \\
&= \left(p_{t,t-1}^{L} + 1\right) B_{t-1,t-1}^{L} + \frac{1+\rho p_{t,t}^{L}}{p_{t-1,t-1}^{L}} \cdot \sum_{s=1}^{\infty} p_{t-1,t-(s+1)}^{L} B_{t-1,t-(s+1)}^{L}, \\
&= \left(1 + p_{t,t-1}^{L}\right) B_{t-1,t-1}^{L}, \\
&= \left(1 + \rho p_{t,t}^{L}\right) B_{t-1,t-1}^{L},
\end{aligned}
$$

where we used $p_{t-1,t-(s+1)}^{L} = \rho^{s} p_{t-1,t-1}^{L}$, $p_{t,t-(s+1)}^{L} = \rho^{s+1} p_{t,t}^{L}$, and $p_{t,t-1}^{L} = \rho p_{t,t}^{L}$. Hence, we end up with

$$
... + p_{t,t}^{L} B_{t,t}^{L} + ... \leq ... + \left(1 + \rho p_{t,t}^{L}\right) B_{t-1,t-1}^{L} + ...,
$$

where we can simplify notation by dropping the double time index $B_{t}^{L} = B_{t,t}^{L}$, etc. Hence, the investor's problem can be written as

$$
... + p_{t}^{L} B_{t}^{L} + P_{t} c_{t} ... \leq ... + \left(1 + \rho p_{t}^{L}\right) B_{t-1}^{L} + P_{t} w_{t} n_{t} + ...
$$

The first order condition for holdings of long-term debt B_{t}^{L} is thus given by

$$
1 = \beta E_{t} \left[\frac{u_{c\,t+1}}{u_{c\,t}} \frac{R_{t+1}^{L}}{\pi_{t+1}} \right],
$$

where the one period rate of return on long-term bonds is given by

$$
R_{t+1}^{L} = \frac{1 + \rho p_{t+1}^{L}}{p_{t}^{L}},
$$

which is obviously state contingent. Suppose for a moment that prices are stable ($\pi_t = 1$) and agents are risk neutral ($u_{c,t} = const.$). Then, the pricing condition (C.27) implies $p^L_{t,t} = \sum_{k=1}^{\infty} \beta^k \rho^{k-1} = \rho^{-1} \sum_{k=1}^{\infty} \beta^k \rho^k = \frac{\beta}{1-\beta\rho}$, and $p^L_{t+1,t} = \sum_{k=1}^{\infty} \beta^k \rho^{k-1+1} = \frac{\beta\rho}{1-\beta\rho}$, such that the one period rate of return $\left(1 + \rho p^L_{t+1}\right) / p^L_{t,t} = \left(1 + p^L_{t+1,t}\right) / p^L_{t,t}$, is given by $1/\beta$, which satisfies arbitrage freeness.

C.7 Duration and yield to maturity

The **Yield to Maturity** is the internal rate of return of an investment, taking into consideration all incomes and expenses and their timing. Hence, the Yield to Maturity makes all future payments of a perpetuity bond equal to the current market value. For a perpetuity bought in t an investor would gain the following stream of incomes $\sum_{s=1}^{\infty} \rho^{s-1}$ discounted by the current Yield to Maturity YTM_t. Hence, the Yield to Maturity YTM_t is simply given by

$$\begin{aligned}
p^L_t &= \sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(YTM_t)^s}, \\
p^L_t &= \frac{1}{\rho} \sum_{s=1}^{\infty} \frac{\rho^s}{(YTM_t)^s} = \frac{1}{\rho} \frac{\rho}{YTM_t} \frac{1}{1 - \frac{\rho}{YTM_t}}, \\
p^L_t \left(1 - \frac{\rho}{YTM_t}\right) &= \frac{1}{YTM_t}, \\
YTM_t &= \left(\frac{1}{p^L_t}\right) + \rho.
\end{aligned}$$

The problem of an investor then reads

$$... + P_t c_t + p^L_t B^L_t + ... \leq ... + p^L_t YTM_t B^L_{t-1} + P_t w_t n_t + ...,$$

leading to the first order condition for holdings of long-term debt

$$1 = \beta E_t \left[\frac{u_{c\,t+1}}{u_{c\,t}} \frac{1}{\pi_{t+1}} \frac{p^L_{t+1} YTM_t}{p^L_t} \right].$$

The concept of **Duration** measures the number of periods it takes for the price of a bond to be repaid by its internal cash flows. A bond's duration is calculated as a weighted average of the time horizons at which the cash flows from a bond are received. Each time horizon's weight is the percentage of the total present value of the bond (bond price) paid at that time. For that purpose, the bond's yield to maturity is used to calculate the present values

$$D_t = \frac{\sum_{s=1}^{\infty} s \frac{\rho^{s-1}}{(YTM_t)^s}}{\sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(YTM_t)^s}} = \frac{\sum_{s=1}^{\infty} s \frac{\rho^{s-1}}{(YTM_t)^s}}{p_t^L} = \frac{\frac{1}{YTM_t} + 2\frac{\rho}{(YTM_t)^2} + 3\frac{\rho 2}{(YTM_t)^3} + ...}{\frac{1}{YTM_t} + \frac{\rho}{(YTM_t)^2} + \frac{\rho 2}{(YTM_t)^3} + ...},$$

where $p_t^L = \sum_{s=1}^{\infty} \frac{\rho^{s-1}}{(YTM_t)^s} = (YTM_t - \rho)^{-1}$, is the bond's present value in t. Note that

$$\sum_{s=1}^{\infty} s \frac{\rho^{s-1}}{(YTM_t)^s} = \frac{1}{\rho} \sum_{s=1}^{\infty} s \frac{\rho^s}{(YTM_t)^s} = \frac{1}{\rho} \frac{\rho}{YTM_t} \frac{1}{\left(1 - \frac{\rho}{YTM_t}\right)^2},$$

where we used that $\sum_{s=1}^{\infty} s x^s = \frac{x}{(x-1)^2}$, for $x \in (0,1)$. Dividing by $p_t^L = (YTM_t - \rho)^{-1}$ gives

$$\begin{aligned} D_t &= \frac{1}{\rho} \frac{\rho}{YTM_t} \frac{1}{\left(1 - \frac{\rho}{YTM_t}\right)^2} \left(\frac{1}{YTM_t - \rho}\right)^{-1}, \\ &= \frac{YTM_t}{YTM_t - \rho}. \end{aligned}$$

Alternatively it can be shown that the duration is the elasticity of the bond's present value with respect to the discount factor YTM_t : $D_t = -\frac{dP_t}{dYTM_t} \frac{YTM_t}{P_t}$, implying $D_t = \frac{1}{(YTM_t - \rho)^2} \frac{YTM_t}{\frac{1}{YTM_t - \rho}} = \frac{YTM_t}{YTM_t - \rho}$.

C.8 Data

We use quarterly U.S. data ranging from 1964:Q3 to 2012:Q3. Time series are taken from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. We construct real per capita GDP by dividing the nominal GDP (GDP) series by population (CNP160V) and the GDP deflator (GDPDEF). Consumption is measured by the sum of private sector nondurable goods consumption (PCND) and private sector services consumption (PCESV). We measure investment by the sum of the series "Gross Private Domestic Investment" (GDPI) and durable goods consumption (PCDG). Bank reserves are proxied by the Federal Reserve

Bank of St. Louis' measure for monthly Adjusted Reserves (ADJRESSL). We proxy for deposits by the sum of the series "Total Checkable Deposits" (TCDSL), "Small Time Deposits" (STDSL), and "Savings Deposits" (SAVINGSL) (as part of M2). Real per capita values for the consumption, investment, reserves, and deposits series are calculated by dividing through population (CNP160V) and the GDP deflator (GDPDEF). Inflation is calculated as the gross growth rate of the GDP implicit price deflator (GDPDEF). We define hours worked by multiplying average weekly hours worked in the nonfarm business sector (PRS85006023) with the series "Civilian Employment" (CE16OV) and dividing by total population (CNP16OV). Real wages are derived by dividing the time series "Nonfarm Business Sector: Compensation Per Hour" (COMPNFB) by the GDP deflator. We use the effective Federal Funds rate (FEDFUNDS) as our measure for the model's money market rate, the 3-month certificate of deposit secondary market rate (CD3M) as measure for the model's deposit rate, Moody's Baa corporate bond yield index (BAA) as a measure for the loan rate, and the 7-Year Treasury constant maturity rate (GS7) as a measure for the yield to maturity on long-term treasuries. All time series are detrended by using a linear trend, except for the interest rates, hours worked, and real deposits. The interest rates are all demeaned. The mapping of the variables to the states is

$$\hat{y}_t^{obs} = 100 \cdot \log\left(\frac{gdp_t}{gdp}\right),$$

$$\hat{c}_t^{obs} = 100 \cdot \log\left(\frac{c_t}{c}\right),$$

$$\hat{i}_t^{obs} = 100 \cdot \log\left(\frac{i_t}{i}\right),$$

$$\hat{n}_t^{obs} = 100 \cdot \log\left(\frac{n_t}{n}\right),$$

$$\hat{w}_t^{obs} = 100 \cdot \log\left(\frac{w_t}{w}\right),$$

$$\hat{m}_t^{obs} = 100 \cdot \log\left(\frac{m_t}{m}\right),$$

$$\hat{d}_t^{obs} = 100 \cdot \log\left(\frac{d_t}{d}\right),$$

$$R_t^{m,obs} = 100 \cdot \left(R_t^m - R^m\right),$$

$$R_t^{D,obs} = 100 \cdot \left(R_t^D - R^D\right),$$
$$R_t^{L,obs} = 100 \cdot \left(R_t^L - R^L\right),$$
$$YTM_t^{obs} = 100 \cdot (YTM_t - YTM),$$
$$\pi_t^{obs} = 100 \cdot (\pi_t - \pi),$$

where all state variables are in deviations from their steady-state values. The detrended observable $\hat{y}_t^{obs}$ corresponds to the first difference in the detrended log real per capita GDP series, multiplied by 100. Analogously we determine $\hat{c}_t^{obs}$, $\hat{\imath}_t^{obs}$, $\hat{n}_t^{obs}$, $\hat{w}_t^{obs}$, $\hat{m}_t^{obs}$, $\hat{d}_t^{obs}$. The demeaned observable $R_t^{m,obs}$ corresponds to the contemporary deviation of the effective Federal Funds rate from its sample mean. Analogously we calculate $R_t^{D,obs}$, $R_t^{L,obs}$, YTM_t^{obs}, π_t^{obs}.

Appendix D

Appendix to Chapter 5

D.1 Data

U.S.-U.K. Treasury yield spread: This variable is constructed as the percentage spread between the U.S. Treasury bill yield for 3-month Treasuries extracted from the Federal Reserve of St. Louis' FRED database (series TB3MS), and the U.K. Treasury bill yield with the same maturity length from Datastream (series UKTBTND).

Debt/GDP: This variable is intended to proxy for the holdings of U.S. Treasuries scaled by U.S. GDP. Here I use time series Data on the total amount of Treasury securities outstanding from Datastream (series USSECMNSA). U.S. GDP data is extracted from the FRED database (series GDP).

VAR model: The vector of the VAR model's endogenous variables is given by

$$\vec{Y}_t = \left(c_t, \pi_t, y_t^{Dis}, i_t^{FED}, p_t^{Ind}, (y_t - c_t), X_t\right).$$

The endogenous variables are calculated using FRED data:

per capita real nondurable goods and services consumption:

$$c_t = \frac{PCNDGC96_t + PCESVC96_t}{POP_t},$$

inflation, measured by the log change in the price index for nondurables and service consumption:

$$\pi_t = \log\left(\frac{PCND_t + PCESV_t}{PCNDGC96_t + PCESVC96_t}\right) - \log\left(\frac{PCND_{t-1} + PCESV_{t-1}}{PCNDGC96_{t-1} + PCESVC96_{t-1}}\right),$$

per capita real disposable income:

$$y_t^{Dis} = \frac{DPIC96_t}{POP_t},$$

the effective federal funds rate, $i_t^{FED} = FEDFUNDS_t$,

a commodity price index, $p_t^{Ind} = PPIIDC_t$,

the nominal U.S. Dollar to British Pound exchange rate $X_t = EXUSUK_t$,

per capita real nonconsumption GDP:

$$(y_t - c_t) = \frac{GDP96_t}{POP_t} - \frac{PCECC96_t}{POP_t}.$$

Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Arbeit selbständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus fremden Quellen direkt oder indirekt entnommenen Gedanken sind als solche kenntlich gemacht.

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Dortmund, Januar 2014