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On the evolution of preferences and delegation in Tullock rent-seeking contests

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Chapter 1

Introduction and Tullock Rent-Seeking Contest

1.1 Introduction

Contests are omnipresent in human society and they are a big issue in the lives of everyone. A lobbyist influences the laws that are made by government and therefore determines what we have to pay for health insurance, for example. An application for a new job can be seen as a contest. The result of a football match or the determination of the football champion can also be seen as the outcome of a rent-seeking contest. Wars or battles are also examples. Even finding a girlfriend or boyfriend can be seen as a contest.

In all cases a prize is at stake and people that are interested in this prize will spend irreversibly effort to win it. Nearly every day it is possible to find examples in the news. The beginning of the crisis in the Ukraine in 2013 is a very good one. It first started with a contest between the European Union (EU) and the Russian Federation. In 2013 the president of the Ukraine, Janukowitsch, had the choice whether to sign a contract with the EU or to cooperate with Russia. Both Russia and the EU tried to convince Janukowitsch. In the end Russia won[1] by offering 15 billion US-Dollars worth of credit (without demanding any reform) and reducing the prize of gas for the Ukraine. Also the EU, the International Monetary Fund and the US Government offered credits, but they demanded the enactment of reforms.

Because of the importance of contests in everyday life, we want to examine contests from a theoretical point of view. We will make use of the way how Tullock modeled rent-seeking contests. A description of the model of Tullock is given at the end of this section. But given that there has been and still is very much interest in contests in the literature, we will concentrate on combining contests with another concept

[1] At least until the coup that overturned the government.

that has only received little attention in that context. This concept is delegation. As we will show subsequently, delegation and contests can fit together. Because of the complexity of international affairs delegation is needed and used.
Shelling (1960) first introduced strategic delegation to contests. He shows that delegating a decision, or only a part of it, to an agent can change the result in favor of the delegating principal. One example he uses is a sadist in a prison. Everyone knows that a sadist will always punish someone for misconduct. This may lead to a reduction in the number of misconducts in the prison. Another famous result is the result of Wärneryd in 2000. He states that delegation is able to reduce investments in rent-seeking contests and therefore reduces waste, if it is assumed that such efforts are not welfare enhancing. But on this point there are (at least) two opinions. On the one hand, producing tanks, machine-guns or paying a lobbyist, a lawyer and so forth increases gross national income and the military sector has always been a driver in innovations. Also a tank has an afterlife. It is shown how a Russian tank is used to produce a hammer in a German commercial for a hardware store for example.
But on the other hand, in a symmetric situation no investment in a contest by both sides would always be pareto optimal. Symmetric means that the prize valuation and the technology of the players do not differ. Technology determines how their investment changes their probability of winning. If no one would invest, both would have a chance to win and they would have no costs. In equilibrium the chance is also one half, but both put in effort. Although a tank, for example, has a positive effect the total effect is negative because the money could have been spent elsewhere where more people could benefit from it.
Therefore the question arises whether wasteful investments in contests can be reduced. Wärneryd shows that delegation could work. But in his study delegation has to be mandatory. This is a serious problem, because every principal would benefit from mutual delegation, but not delegating is a dominant strategy. Accordingly, they are kept in a kind of prisoners' dilemma.
To incorporate the fact that the strategies used by an individual may change over time, evolutionary game theory is introduced into the analysis. The biological concept is mainly used for animal populations. To put it briefly, in finite populations an individual should always be concerned with the payoffs or fitness, as it is called, of all other opponents. That is, because evolutionary forces have a bias towards individuals that behave that way. Only if an individual can ensure that his absolute fitness is highest, he will raise more offspring and therefore his strategy will become more frequent in the population. This fact makes hurting oneself to hurt others even more a good idea. In this context we will use the term individualistic player. This refers to a player that only cares for his utility and is not influenced by the utility of

any other player. In game theory we are most of the time concerned with rational agents. But in biology we are concerned with genes. Both concepts are used in the indirect evolutionary approach as will be shown below. But let us first concentrate on the direct evolutionary approach.

In the direct evolutionary approach, evolution works on the level of strategies. Only strategies that are more successful than other strategies will spread further in the population. This may also lead to a situation where players do not behave rationally, i.e. they do not maximize their utility. This can lead to a deviation from Nash equilibrium. In fact it was shown by Schaffer (1988), that one should act in a non-altruistic way if one is to play a contest in a finite population. That means in finite populations the more successful players are not playing the Nash equilibrium strategy, i.e. they hurt themselves just to hurt the opponents even more. An example that was spread in the news on July 31st, 2013 that sheds some light on that case happened in the potash sector[2]. In this particular sector there are two cartels that determine prices. Uralkali (a Russian potash firm) was part of one cartel. But this particular firm decided to leave the cartel and to increase production and therefore to reduce the price. In the potash sector it is common knowledge that Uralkali has the lowest costs. This announcement destroyed 13 billion Dollars of stock exchange capitalisation in a few hours. The stock of Uralkali lost more than 25 percent. Later Uralkali stated that getting rid of some opponents in the sector is the aim of this action[3]. One can see that Uralkali hurts itself to hurt other firms with higher costs even more, just to gain some market share. If opponents that care for high prices leave the market, then Uralkali is successful. But if Uralkali has to leave the market, then collusion is better.

In the indirect evolutionary approach the individuals are rational in terms of maximizing their utility but their objective functions or preferences are determined by nature (by their genes, for example). Accordingly, evolutionary forces work on the level of preferences. Given the preference function of an individual, the individual actively chooses a strategy that maximizes his utility. That means that the Nash equilibrium strategy is played by the individuals. In human society one can think of altruists, non-altruists and individualists. And only the preferences that lead to the highest fitness become more frequent and therefore the population may, in the end, only consist of individuals with a certain preference function. In the direct evolutionary approach individuals choose strategies that proved to be successful in the past and the individuals that use the most successful strategy form the population in the long run. But in the indirect evolutionary approach natural selection

[2] A summary is given in the following newspaper article: http://www.handelsblatt.com/unternehmen/industrie/ks-unter-druck-russen-treiben-dax-konzern-zur-verzweiflung/8579358.html.

[3] It is supposed that a Belarusian firm was the main target.

leads to a population of individuals that choose the appropriate strategies rationally because they have the genes that make them act this way. In nature, there are many examples for species that care for their offspring but there are also many examples for species that do not do that. Let us consider a species that cares for their offspring. The direct evolutionary approach would argue that individuals that cared were more successful because they had a higher fitness. Although this behavior may have reduced utility. But the indirect evolutionary approach argues that only these individuals had a higher fitness that have preferences that make them rationally choose to care. There have also been individuals that did not care. But their fitness was lower and they became extinct.
In Chapter 2 we are concerned with this kind of problem. We will have a short look at the direct and indirect evolutionary approach in context of a contest, and a second kind of indirect evolutionary approach is introduced. Leininger (2009) used a Tullock rent-seeking contest to find out whether it is advantageous to be an altruist, an individualist or a non-altruist. That means he was concerned with whether or not interdependent preferences can enhance the fitness. He states that negatively interdependent preferences yield an advantage in finite populations.
We will show that also the perception of the value of the contested prize can be used in an indirect evolutionary approach and that this leads to the same result. I.e. an agent is willing to exert more effort in order to win the prize than he should do objectively. A good example is the intrinsic motivation to be better than the opponents or that somebody wants to be the winner. Winning gives an individual an extra utility. That also makes cheating worthwhile. The opponents are hurt just to have a higher probability of being the first. In any case the efforts spent are increased compared to individualistic preferences.

On the one hand, efforts exerted in a contest could be used elsewhere to increase welfare. And on the other hand, evolutionary forces even lead to more efforts exerted in contests. Maybe sending delegates could be a good idea to decrease efforts made. A delegate wants to get paid and also a principle wants to get a share of the contested prize. There has to be some kind of split up of the prize. Accordingly, the prize for a delegate is smaller. Therefore, he will invest less compared to a principal. In a situation with negatively interdependent preferences sending a delegate that is not concerned with the payoff of the enemy of the principal can reduce efforts spent. With negatively interdependent preferences an agent is concerned with his payoff and the wish that the other should lose. But a delegate in this situation may only be concerned with his material payoff. A divorce can be a good example. The term “can” refers to the fact that not all divorces are consensual divorces. If we are in a non-consensual divorce, we are clearly confronted with interdependent preferences

and overvaluation of things that may have an additional personal value to one party. In this situation a neutral lawyer who acts as a delegate could reduce efforts spent, because he is only interested in the objective value of things and has no disutility if the other party gets something.
The question of contracting an agent is the question of how to split up the contested prize. Splitting up in this particular context means that the principal gets the whole prize if the contest is won, but he has to pay the agent. It is assumed that the payment to the agent is made conditional on the value of the contested prize. The share the delegate gets is therefore money that represents a share of the monetary benefit for the principal and not a share of the indivisible prize of the contest. In Chapter 3 we want to have a look at how much to offer in order to contract an agent. So far Baik (2007) answered the question, but only for a limited set of contracts. Baik comes to the conclusion that no-win-no-pay contracts should be used, because in such contracts the difference between winning and losing is greatest and therefore the delegate has the greatest incentive to win. But it is shown that this is not true in all cases. Baik assumes that a principal cannot punish the agent for a defeat. Because of that assumption his set of contracts is limited. Why shouldn't it be possible to punish a delegate? Such contracts are quiet common in non-legal sectors. The Yakuza[4] are a good example: An agent has to amputate a part of his finger if the principal is dissatisfied with his performance. Another example was spread in the news on 21st of december 2014[5]. Some eyewitnesses claim that after having lost a battle, the Islamic State executed 45 of its own fighters for this defeat. The Islamic State shares the resources conquered with his soldiers but also punishes them for a defeat. One example for sharing the resources is the sad destiny of displaced women. Some were sold but some were also given to the fighters[6].
Instead of assuming non-legal sectors, we could also have a look at a situation without enforceable (property) rights. Agents may be forced to act for a principal and they may be forced to sign extreme contracts. Young men may be forced to fight for the Islamic State and to pay with their lifes for a lost battle, for example. Whereas in functioning states those contracts are not allowed under the rule of law.
But there are also examples in legal sectors. One could think of contractual penalties that become effective if determined goals were not reached by the agent. Also soccer is an example, if the outcome of a match is interpreted as a contest and it is assumed that both coaches are delegates of the management. The prize is given by winning the match. Both coaches invest effort to win the match, but in most of

[4] I.e. the japanese mafia.
[5] http://www.n-tv.de/politik/IS-Miliz-toetet-noch-mehr-eigene-Kaempfer-article14199261.html
[6] As stated in a notable German newspaper on 11/10/2014: http://www.faz.net/aktuell/politik/ausland/naher-osten/is-dschihadisten-verkauften-yezidische-frauen-als-sklavinnen-13256836.html.

the cases only one coach succeeds. The loser may be fired if losing happens to often. Therefore, he is unemployed and also his future earnings are decreased because other clubs could refrain from hiring a “loser coach”. The coach of the losing team is therefore punished. Also managers are fired if they are not able to stop competitors from increasing their market share at the expense of his own firm.
When penalties are introduced, the difference in payment for the delegate between winning and losing may be expanded. Accordingly, contracts might incentivize delegates to a greater extent than a no-win-no-pay contract. A reduction in efforts spent might therefore be smaller compared to Wärneryd (2000). We will also be concerned with this topic in Chapter 3. Baik uses principals with individualistic preferences. To compare the results with the literature, we will first use individualistic preferences. The consequences of interdependent preferences are also explained. We will also have a look at situation where individualistic principals contract agents with negatively interdependent preferences.

After the optimal contracts are determined, we will have a look at relative contracts. The relation of this type of contract to the contracts determined in Chapter 3 is explained. But in Chapter 4 we will concentrate on negatively interdependent preferences. With negatively interdependent preferences in a contest, a principal values his payoff and also assesses the payoff the opponent does not get, i.e. they want to have more than the opponent. Principals do not play the contest and therefore the efforts invested do not increase directly. But it is shown in Chapter 4 how these negatively interdependent preferences influence the choice of the contract and therefore increase efforts indirectly. Relative contracts are chosen because they reward the delegate if he was able to do better than the other delegate. Accordingly, they can transfer the preferences of the principal to the delegate. In a divorce battle with lawyers as delegates, for example, incentivizing the lawyer to gain more than his opponent might be a good idea.
Relative contracts are therefore less concerned with absolute payoffs, but with the relative position the delegate achieves. Putting a positive weight on sales in manager-contracts is one example for this type of contract.
In Chapter 2 it is explained that negatively interdependent preferences yield an advantage in finite populations compared to altruistic and individualistic preferences. In Chapter 4 we will examine whether something similar can be observed in the selection of contracts or not, i.e. do negatively interdependent contracts have an advantage or not.
Wärneryd (2000) showed that no-win-no-pay contracts can reduce efforts spent in contests in a setting with mandatory delegation. Whether this also holds with relative contracts or not will also be part of my investigation in this chapter.

It is assumed in Chapters 3 and 4 that the principal uses the contract to influence the effort made by the agent. In Chapter 5 we will introduce training into the delegation setting. Training is modeled as an investment of the principal in order to make the agent more effective. Two approaches are used in this chapter: Training can reduce the unit costs of investing effort for the agent or it can influence the weight the effort has, i.e. whether the agent can achieve more without doing more just because he was trained. The principal pays to increase the skills of his delegate. We can think of soldiers and their basic training when they start. Also researchers go to conferences and employees get training when they start a job. These examples show that principals are willing to invest money to make the agents more effective. As we will see, the design of training makes it also possible to include assistance to the delegate. That means that also investing in technical assistance etc. can be included in this setting. A steel helmet for example does not kill an enemy, but it helps to keep the soldier alive and therefore to make him more effective, in a sense. Another example is water and food for soldiers. To provide sustenance can increase the effectiveness of the soldiers in a battle. We will be concerned with the influence of training on principals and delegates. But we will concentrate on the contest actually played. The effects of training in the future are neglected. Also the effects of changing the costs of training or the form of the Contest-Success-Function are shown.

To judge whether delegation can decrease wasteful investment or not, the results are recapitulated in Chapter 6. Also the implications of the development of contracts are shown. So far, introducing mandatory delegation can be seen as an intervention by government to reduce investments in rent-seeking. We will discuss whether this is enough or not, i.e. should the government also determine the contracts to be used. Such a type of governmental interference is the German "Rechtsanwaltsvergütungsgesetz". We will also have a look at how principals can influence the outcome of the contest even though contracts are prescribed.

So far, contracts were given by no-win-no-pay contracts. We want to emphasize that more competitive contracts may also be used by the principals. Also training may be used to influence the effort put in by the delegate. Additionally, in the section "Future Research" we will be concerned with another type of gaining influence on the behavior of the delegate. As Akerlof and Kranton (2000) show, identity is also an important factor for individuals. And we will try to introduce identity in a contest with delegation.

1.2 Tullock Rent-Seeking Contest

Before we start with the model of Tullock (1980), we will give an intuition for the meaning of the terms economic rent and rent-seeking. According to Tollison (1982) an economic rent is “defined as a return in excess of a resource owner’s opportunity cost,[...]”. An economic rent can be artificial or not, i.e. a shortage that leads to an excess return can be created by a shift in demand, for example, or by the decision of an authority. The difference is that artificial rents may be persistent, i.e. they are not driven to normal levels by market forces. We will give an example to demonstrate what this means. Let us consider the National Football League (NFL) in the USA. Each of the 32 clubs is granted a franchise. Each team has a “Home Territory” and a “Home Marketing Area”. In the Home Territory the team can exclusively host professional football games. In the Home Marketing Area a team has the right to advertise, promote, and host events, i.e. a team can act as a monopolist in the defined territories and therefore earns an economic rent. This rent is artificial because the number of franchises is restricted by the NFL to 32. According to Forbes magazine all 32 NFL clubs rank among the top 50 most valuable sports teams in the world[7].

We can also use this example to demonstrate what is meant by rent-seeking. Assume that the league decided to grant another franchise and that there is more than one candidate. Each of the candidates wants to convince the authorities of the NFL that he is the best. Accordingly, every candidate spents money on promotion, bribery and so on. But only one candidate wins the franchise. This expenditure of scarce resources by all candidates to capture an economic rent is called rent-seeking. According to Congleton et. al (2008) “Incentives for rent seeking are present whenever decisions of others influence personal outcomes or more broadly when resources can be used to affect distributional outcomes.” (p. 1).

In most cases a monopoly leads to a loss in welfare. If this monopoly is artificial and scarce resources are spent to gain this monopoly, we have to add these expenditures to the welfare loss of the monopoly to account for all negative effects. The idea that resources spent attempting to make or prevent transfers should be counted as a cost to society was introduced by Tullock (1967). Compared to the economic rent of a monopoly, how much is invested to gain the monopoly rights? Until Tullock (1980) it was believed that the total amount invested is as high as the monopoly rent or even higher. In “Efficient Rent Seeking” Tullock modeled the actual process of rent-seeking and showed that this is not necessarily true. Tullock uses a simple lottery to explain his model. In this section we will recall his model. Assume that

[7] Kurt Badenhausen (April 18, 2012). ”Manchester United Tops The World’s 50 Most Valuable Sports Teams”. Forbes. Retrieved September 12, 2012.

two risk-neutral players can buy lottery tickets. The amount of ticket bought by the first player is denoted by A and the amount of tickets of the second player is given by B. One ticket is randomly chosen and the player who bought it wins the contest. Accordingly, the probability that A wins the lottery is $P_A = \frac{A}{A+B}$. A function that maps efforts ("ticket purchases") to winning probabilities is called a Contest-Success-Function (CSF).
Tullock assumes that the prize of the lottery is an amount of 100 Dollars and that the price of one lottery ticket is one Dollar. He showes that both players should buy 25 lottery tickets and therefore only one half of the rent is dissipated. The money spent for the tickets is sunk from the point of view of A and B. Accordingly, they spent irreversible effort to win the prize; i.e. both players seek to win the rent (100 Dollars) and in order to do this they buy tickets.
To get the odds of one player in the model, we just have to devide the number of his tickets by the sum of all tickets sold. Tullock extends his model to model a broader class of situations. In his extension the probability that the first player wins is given by

$$P_A = \frac{A^r}{A^r + B^r}.$$

We will stick to this form of modeling rent-seeking. r is an exogenously given parameter and r determines the weight of the effort in the CSF. If $r > 1$, then the impact of an additional unit of effort invested is higher than the impact of the unit spent before. If $r < 1$, the impact of an unit of effort decreases the more effort has already been spent. As soon as $r > 2$, we are confronted with multiple equilibria where only one player invests. That would mean that only one player in the contest is active. We want to avoid this rather unrealistic scenario and therefore do not consider this case. Another interpretation is that r is a measure of how much the relative size of effort counts. If r is zero, it does not matter who invested more. Every individual has the same probability of winning. If r is close to infinity, the player who invested the most will win the contest almost certainly.
We will use this so-called exponential form in our analyses. One reason is that we can model many real world scenarios with this function. Another reason is that according to Skaperdas (1996) this is the standard form in modeling rent-seeking.

Chapter 2

On the Evolution of Preferences

This chapter has benefitted from extensive discussions with Wolfgang Leininger and Burkhard Hehenkamp.

2.1 Introduction

Garfinkel and Skaperdas (2007) point out that individuals not only engage in mutual beneficial activities but also use resources to appropriate wealth of other agents. A vast literature concerning this aspect has evolved in the last decades: Tullock (1980) with his idea of efficient rent-seeking and the use of a Contest-Success-Function, Haavelmo (1954) who first modeled the choice between production and appropriation and Hirshleifer (1995) theorizing about conflict, are just a few.

In recent years evolutionary concepts have also been applied in this context. Mostly this is done in order to gain some insights about how strategies evolve over time. In the direct evolutionary approach, evolution works on the level of strategies. A behavior that yields a higher payoff than any other behavior is imitated by others. In the long run, only the strategy which performs best compared to other available strategies survives, and is therefore evolutionarily stable. Ultimately, only those individuals form the population that were more successful than their opponents, because they have more resources to spend on reproduction, for example. In finite populations this can cause a deviation from Nash equilibrium. But evolution does not only work on the level of strategies. For that reason, an indirect evolutionary approach has been termed by Güth and Yaari (1992). The behavior of an individual depends on his preferences which act as stimuli. The strategy chosen according to the given preferences, compared to the strategies used by other individuals in the population, determines the fitness of an individual. Evolution does not take place in the choice of certain strategies, those are chosen rationally, but in the stimuli appearing over time. In the end only individuals with the most "useful" stimuli form the population. By using this approach, Leininger (2009) develops evolutionarily stable preferences (EST) in a two-player Tullock contest. He shows that pure profit maximization is evolutionarily stable only within an infinite population. But as soon as the population is smaller, the individuals are concerned with a weighted relative payoff. This leads to perfectly negatively interdependent preferences for a population of two individuals. It is worth to notice, that even though individuals hurt themselves by such a behavior, they hurt the opponent even more. Of course, negative interdependent preferences are not rational from the point of view of the whole population. Accordingly, the term spiteful is used in the literature.

This chapter is aiming at the indirect evolutionary approach as well. We state that both, the preferences over the outcome for the enemy and the valuation of the contested prize, can act as a stimulus. Of course, some individuals can use the prize more efficiently than others, but this case is of no importance here. The prize has a unique value, but the individuals may rate it differently. This approach is equivalent to a situation where every individual faces the same costs, but the individuals think that the costs are different. Therefore, underestimation of the own costs can act as an explanation for an overestimation of the contested prize and the other way around. Additionally, if we compute our own costs correctly, we can still overestimate the prize if we overestimate the costs of our opponents. Another explanation is an intrinsic value of winning. Getting the desired object gives individuals an extra boost in utility that cannot be explained solely by the value of the prize. Boudreau and Shunda (2010) also used a Tullock rent-seeking contest and the indirect evolutionary approach to determine evolutionarily stable prize perceptions. But they only consider two-player contests and compare their outcome to the direct evolutionary approach.

We assume that there is an ex-post outcome which is unknown to the players ex-ante. We want to show that overvaluation is evolutionarily stable in contests of arbitrary size if the population is finite. It is also shown that overvaluation can occur in infinite populations. This means that making a "mistake" can be beneficial in evolutionary terms. Because of a higher valuation, the opponents are discouraged on the one hand and the player acts more aggressively and therefore hurts his opponent even more than he hurts himself on the other hand. The latter effect is similar to the result of Leininger (2009). We will show that the material outcome and the invested efforts are the same for both indirect approaches. It is also shown that both indirect evolutionary approaches predict more aggressive behavior and therefore lower material payoffs than the direct evolutionary approach. Only in two-player contests and playing the field contests all investigated approaches are equivalent in the behavior they predict.

The remainder of the chapter proceeds as follows: In Section 2.2 we specify an evolutionary game to explain the direct and the indirect evolutionary approach. In section 2.3 we set up the model and recapitulate some basic definitions and concepts. In Section 2.4 evolutionarily stable prize perceptions are derived within an arbitrary but finite population for two-player Tullock contests and for Tullock contests with more than two opponents. Section 2.5 compares our results with the results of the direct evolutionary approach according to Schaffer (1988) and with the results of the indirect evolutionary approach developed by Leininger (2009); Section 2.6 concludes.

2.2 Direct and Indirect evolutionary approach

2.2.1 Direct evolutionary approach

To demonstrate the direct evolutionary approach, we will concentrate on symmetric two-player games. This presentation draws heavily on Schaffer (1988). Assume that there is a population of N individuals. It holds that $2 \leq N < \infty$. All players engage in contests of size two. Suppose that two individuals are randomly chosen to play a contest. We call the players player 1 and player 2. The strategy space of the first player is S_1. The set of strategies of the second player is S_2. It holds that $S_1 = S_2$ and that $S_1, S_2 \subset \mathbb{R}$. $S = S_1 \times S_2$ is the product of strategy sets. Both players want to maximize their material payoff or fitness. We denote by $f_i : S \rightarrow \mathbb{R}, i \in N$, the material payoff function of individual i. The material payoff of any player in the two-player contest depends on his strategy and the strategy of the opponent.

Suppose that one individual in the population is a mutant, i.e. he plays a strategy that is different from the strategy the other $N-1$ individuals in the population are playing. We will denote his strategy by x^M. We will call the strategy the other individuals are using x^{ESS}. The superscript indicates that this strategy is an Evolutionarily Stable Strategy (ESS). Before the conditions for a strategy to be an ESS are explained, we will give a short description of what an ESS is: "Roughly, an ESS is a strategy such that, if most of the members of a population adopt it, there is no 'mutant' strategy that would give higher reproductive fitness." (Maynard Smith and Price (1973), p.15). Higher reproductive fitness in this context means that once the ESS is used by all members of the population, no individual can get more resources for reproduction by playing a different strategy.

Because there is only one mutant, this mutant will always play against a player that plays x^{ESS}. His material payoff is given by

$$f_M = f_M(x^M, x^{ESS}).$$

The payoff of an ESS-player, when he plays the contest against the mutant, is denoted by $f_{ESS}(x^M, x_{ESS})$. The probability that an ESS-player plays the contest against the mutant is given by $\frac{1}{N-1}$. Accordingly, his expected material payoff is

$$f^{ESS} = \frac{N-2}{N-1} f_{ESS}(x^{ESS}, x^{ESS}) + \frac{1}{N-1} f_{ESS}(x^M, x^{ESS}).$$

According to Schaffer (1988) a strategy is an ESS if and only if two conditions are fulfilled:

a) Equilibrium condition and

b) Stability condition.

The Equilibrium condition requires that the following holds:

$$f^{ESS} \geq f_M,$$

i.e. the expected material payoff of an ESS-player has to be at least as high as the payoff of a mutant. Accordingly, there is no strategy that yields a material payoff that is higher.
To find the ESS, we have a look at

$$\max_{x^M} \quad f_M - f^{ESS}.$$

For $x^M = x^{ESS}$ the maximum of zero is reached. By using the conditions for f_M and f_{ESS} derived above, we get

$$\max_{x^M} \quad f_M(x^M, x^{ESS}) - \frac{1}{N-1} f_{ESS}(x^M, x^{ESS}).$$

In a finite population an ESS does not maximize the material payoff of a player. In fact, in a finite population it is evolutionarily stable to maximize a weighted relative payoff. The weight put on the material payoff of the opponent depends on the size of the population. As $N \to \infty$, the concern for the opponent vanishes and the problem is therefore to maximize the own material payoff. Accordingly, in a finite population the ESS can be different from the Nash equilibrium strategy. In an infinite population it is evolutionarily stable to act according to the preferences and therefore to maximize the material payoff. In such a population the ESS is also the Nash equilibrium strategy. But in a finite population it is evolutionary stable to act as if the aim is to maximize a weighted relative payoff. Accordingly, the Nash equilibrium strategy and the ESS need not be the same.
The intuition is that the evolution of strategies works relatively fast, i.e. the individuals realize easily whether a different strategy is more successful or not. Successful

in this context means that a strategy yields a material payoff, and therefore a reproductive fitness, that is higher than the average material payoff of the opponents. The higher the material payoff, the more offspring can be raised. By deviating from the Nash equilibrium, the material payoff is decreased, but the material payoff of the opponents that use the Nash equilibrium strategy is decreased even more. Accordingly, it is evolutionarily stable to hurt oneself in order to hurt the opponents even more.

We just concentrated on one contest that is played in the population. We have to remember that every individual in the population is part of a two-player contest. We have to consider the payoff of each of the individuals to check whether a strategy is evolutionarily stable or not. We also assumed that there is only one mutant. This assumption must also be relaxed to account for the appearence of more than one mutant. Accordingly, we will now turn to the second condition that an ESS must fulfill. Schaffer (1988) defines stability as follows:
"A strategy x^{ESS} is Y-stable if, in a population with a total of up to Y identical mutant strategists with any mutant strategy $s^M \neq s^{ESS}$, $f_M < f_{ESS}$ for all $2 \leq M \leq Y$. The ESS is globally stable if $Y = N - 1$." (p.473)
The equilibrium condition ensures that there is no strategy that is more successful than the ESS in the two player contest. The stability condition deals with strategies that perform as good as the ESS in the two-player contest. In fact, if there are no more than Y mutants, then the expected material payoff of using the ESS is higher than the material payoff of the mutant strategy, i.e. the expected material payoff of a mutant in a contest with another mutant is lower compared to the expected material payoff of an ESS-player playing a contest with another ESS-player.
We will now turn to the indirect evolutionary approach. It is important to remember that the direct evolutionary approach may predict a deviation from Nash equilibrium in finite populations.

2.2.2 Indirect evolutionary approach

We will concentrate on symmetric two-player games to explain the indirect evolutionary approach. This subsection follows the idea of Güth and Peleg (2001) and Leininger (2009). The notation and the depiction follow Leininger (2009).
Assume that a game is played by two players. We call these players player 1 and player 2. The set of strategies of player 1 is denoted by S_1. Because this game is symmetric, it holds that $S_1 = S_2$, where S_2 is the strategy set of player 2. Accordingly, $S = S_1 \times S_2$ is the product of strategy sets. The preference of any player determines how he will act in the game. But the preferences of any player can change

in this game. M_i denotes the set of all kinds of preferences player $i = 1, 2$ can be stimulated by. Accordingly $M = M_1 \times M_2$ is the "'mutation space' of stimuli", as Leininger (2009) called it.
The utility Π_i of player i is given by

$$\Pi_i : S \times M \to \mathbb{R}.$$

We can see that the payoff of any player is determined by his preference and the preference of the other player, but also by his strategy and the strategy of the other player. The question arises how strategies are chosen and how preferences evolve. The preference of any player is the result of an evolutionary process. For any combination of preferences, the strategy chosen by the players is always the result of (Nash-)equilibrium behavior. Evolution determines the utility functions of the players. Both players are totally rational and play the unique Nash equilibrium of the game that is specified by the utility functions. The intuition is as follows: Preferences evolve slowly. We can think of a father and his son. The preference of the father is determined when he is born and cannot change. But the prference of the son can differ from the preference of the father. Because of this slow process, the individuals have time to find the optimal strategy.
Figuratively speaking, the evolutionary game consists of two games. One game is played by the players, who choose the best strategy in order to maximize their utility. The second game is played by nature. The strategies of nature are the preferences. Only these preferences that make the players choose the correct strategies will survive. To judge whether a preference function is better fit than another preference function or not, we have to introduce the evolutionary fitness function $f_i : S \to \mathbb{R}$ of player $i \in N$. N is the number of individuals in the evolutionary game and again N is at least two. The fitness function of player i gives us the material payoff of player i for any combination of strategies. The players perceive a utility, but normally this says little about his material payoff. Consider a player that goes base jumping. His utility is positive, but he risks his life and spents money. Accordingly, his material payoff may be negative.
Note that the fitness of player i is only dependent on the chosen strategies. The preferences determine the strategies that are chosen, but they do not influence the fitness of a player directly. The higher is the fitness of an individual, the more offspring he can raise and therefore the higher is the probability that his preference will prevail in the population.
To formalize the evolutionary game, assume that two players are randomly chosen from the population to play the game. The preference of the first player is given by m_1 and the preference of the second player is m_2. The preferences are determined,

therefore we denote the game the players are playing as

$$E(m_1, m_2) \;=\; (\{1,2\}, S, \Pi_1(x_1, x_2, m_1, m_2), \Pi_2(x_1, x_2, m_1, m_2)).$$

For simplicity, we assume that E is well defined and has a unique Nash Equilibrium for each $m \in M$. The Nash Equilibrium strategies are given by $x_1^*(m_1, m_2)$ and $x_2^*(m_1, m_2)$. As mentioned above, the strategies chosen by the players only depend on the preferences. The utilities of the players in equilibrium are dependent on the strategies and the preferences. Accordingly, they are given by $\Pi_1^*(x_1^*(m_1, m_2), x_2^*(m_1, m_2), m_1, m_2)$ and $\Pi_2^*(x_1^*(m_1, m_2), x_2^*(m_1, m_2), m_1, m_2)$. But we can also compute the evolutionary fitness of the players:

$$f_1(x_1^*(m_1, m_2), x_2^*(m_1, m_2)) \quad \text{and} \quad f_2(x_1^*(m_1, m_2), x_2^*(m_1, m_2)).$$

"This gives rise to *indirect* fitness functions

$$F_i(m_1, m_2) \;=\; f_i(x_1^*(m_1, m_2), x_2^*(m_1, m_2)), \qquad i = 1, 2$$

for the players, which can be regarded as the payoff functions of an evolutionary game that is played over types."(Leininger (2009), p.347)
This is the second game we mentioned above. Following Leininger, we denote this game

$$\bar{E} \;=\; (\{1,2\}, M, F_1(m_1, m_2), F_2(m_1, m_2)).$$

The preferences determine the Nash equilibrium and therefore the payoffs of the first game. But utility normally tells us nothing about the material payoff. Therefore we use the fitness functions to calculate the material payoffs. Using F_i we can judge whether player $i = 1, 2$ has a higher material payoff or not. If his material payoff is higher, then his preference yield an advantage in the evolutionary game. But by using these indirect fitness functions, we can judge for any combination of preferences, which one is better. The solution concept applied to this problem is ESS (evolutionarily Stable Strategies). But since we are confronted with preferences, we will call it ESP (Evolutionarily Stable Preferences), i.e. we are searching for the preference that, once spread in the whole population, cannot be invaded by a sufficiently small number of mutants with another preference. Assume that $m^* = (m_1^*, m_2^*)$ is the unique ESP of $\bar{E}$. Hence, the Nash equilibrium of E is given by $x^*(m^*) = (x_1^*(m_1^*, m_2^*), x_2^*(m_1^*, m_2^*))$.
Accordingly, the solution of the indirect evolutionary game is given by a vector of preferences and strategies. Note that the strategies are induced by these preferences.

2.3 The Model

As Alchian (1950) points out, it might to some degree be the case that environment adapts individuals through natural selection instead of individuals adapting to environment. Güth and Yaari (1992) had this in mind when they wrote about an indirect evolutionary approach. They allow for an indirect dependence of behavior on genetically determined “stimuli”. These stimuli define the game played directly by determining the preferences of the players. Leininger (2009) argued that the preferences for the opponent can act as such a stimulus. In this article we show that this need not necessarily be the only possibility. The perception of the prize can act as a stimulus as well. The strategies and therefore the actions depend on the valuations the players have.

Time and time again, rational players are involved in Tullock contests with $r \leq 1$ and with an arbitrary number of opponents $n \geq 2$ within a given population of arbitrary size N. It holds that N is at least as high as n. As will be shown below, we have to consider r to be smaller or equal to one because the material payoff of the players is in expectation negative otherwise. The consequences of a negative material payoff are given in the first chapter. The contest is for the prize V. We assume that this is also the objective value of the prize. The perception of player $i \in \{1, ..., n\}$ is given by $\sigma_i V$, with $\sigma_i \in [0, \infty)$. Note that we have overvaluation if $\sigma_i > 1$, undervaluation if $\sigma_i < 1$ and a correct prize perception if $\sigma_i = 1$.
According to his valuation, player i invests effort denoted by x_i. The sum of all efforts by all players, except player i, is denoted by x_{-i}.
Accordingly, the probability that player i wins the contest is given by

$$p_i(x_i, x_{-i}) \quad = \quad \begin{cases} \frac{x_i^r}{x_i^r + x_{-i}^r} & \text{for } x_i + x_{-i} > 0 \\ \frac{1}{n} & \text{for } x_i + x_{-i} = 0. \end{cases}$$

After all players have exerted effort, the winner is chosen by nature according to the achieved probabilities and the prize is handed over.
To determine the amount of effort a player is willing to invest, we have to take a look at his utility function. The utility function of an arbitrary player i is given by

$$\Pi_i \quad = \quad \frac{x_i^r}{x_i^r + x_{-i}^r} \sigma_i V - x_i.$$

With probabiliy $\frac{x_i^r}{x_i^r + x_{-i}^r}$ player i wins the contest. If he is successful, he gets the prize which is worth $\sigma_i V$ to him. If he loses, he gets nothing. But in both cases player i has to bear the costs for the invested effort.

We denote by $x_i^*(\sigma_i, \sigma_{-i})$ the (optimal) effort player i is willing to exert in the Nash equilibrium of this contest. Because the invested efforts are determined by the valuation parameters, the utility in the Nash equilibrium of the contest $\Pi_i^*(\sigma_i, \sigma_{-i})$ of any player i is also dependent on these parameters. This gives rise to the indirect evolutionary approach. Note that we get a second game that is played by nature with preference parameters acting as "strategies". The valuations evolve through an evolutionary process. The most successful valuation can spread faster and therefore increase its share of the population. Evolution "determines" the valuations, but the choice of strategies is the result of (Nash) equilibrium behavior of the players.

From time to time, mutations with a different prize perception than the prevailing one appear. If the new valuation is fitter, then it will become the dominant prize perception through natural selection. In order to judge whether a prize perception is fitter or not, we take a look at material payoffs. Only if an individual is relatively more successful in material terms than his opponent, he can raise more offspring and the perception reproduces faster.
To determine the evolutionarily stable valuation, we have to have a look at the evolutionary success function, as Leininger (2009) termed it.
The material payoff of any player i is given by

$$f_i = \frac{(x_i^*(\sigma_i, \sigma_{-i}))^r}{(x_i^*(\sigma_i, \sigma_{-i}))^r + (x_{-i}^*(\sigma_i, \sigma_{-i}))^r} V - x_i^*(\sigma_i, \sigma_{-i}).$$

Note that f_i is defined with regard to the true, not perceived value of the prize. But as stated above, absolute payoff is not all that matters. Absolute payoff has to be compared to the payoff the opponents get. Only if we have a higher absolute payoff than our opponents, we are more successful in evolutionary terms. To determine the fitness of any player, it is necessary to subtract the weighted material payoffs of the opponents from the material payoff. The weight on the material payoff of the opponents is determined by the number of opponents and by the share of the population these opponents represent. If the opponents in the contest represent a large fraction of the population, then the weight is high because beating the opponents leads to an advantage over a large fraction of the population. The indirect fitness function of player i is given by

$$F_i = f_i(\sigma_i, \sigma_{-i}) - \frac{1}{(N-1)} \sum_{\substack{j=1 \\ j \neq i}}^{n} f_j(\sigma_j, \sigma_{-j}).$$

The evolutionarily stable valuation parameter for the contested prize is given by σ^*. Given that any player has a valuation of $\sigma^* V$, any deviation leads to a fitness that is lower or equal compared to the fitness with σ^*. Additionally, $x^*(\sigma^*)$ is the Nash equilibrium of the contest played by the individuals.

2.4 Evolutionarily stable prize perception

As mentioned in Section 2.1, we first concentrate on contests between two-players with $r \leq 1$, before we turn to contests between an arbitrary number of players with $r = 1$. In the last subsection we also have a look at n-player contests and $r \leq 1$.

2.4.1 Two-player contests

Any two players[1] out of an arbitrary but finite population N are chosen to play a Tullock contest. Suppose that $N-1$ individuals rate the contested prize according to the evolutionarily stable prize perception (ESV) σ_{ESV}. The remaining individual is a mutant with a different valuation. This valuation is denoted by σ_M. The effort invested by the mutant is given by x_M, and the effort exerted by a random player with the evolutionarily stable prize perception is denoted by x_{ESV}. If the mutant was to play the contest, he would maximize

$$\Pi_M \quad = \quad \frac{x_M^r}{x_M^r + x_{ESV}^r}\sigma_M V - x_M.$$

An ESV-player is concerned with

$$\Pi_{ESV} \quad = \quad \frac{x_{ESV}^r}{x_M^r + x_{ESV}^r}\sigma_{ESV} V - x_{ESV}.$$

Deriving both first order conditions and setting them equal yields

$$x_{ESV} \quad = \quad \frac{\sigma_{ESV}}{\sigma_M} x_M.$$

The equilibrium efforts are

$$\begin{aligned} x_M &= \frac{\sigma_M^{r+1}\sigma_{ESV}^r}{(\sigma_M^r + \sigma_{ESV}^r)^2} rV, \\ x_{ESV} &= \frac{\sigma_M^r \sigma_{ESV}^{r+1}}{(\sigma_M^r + \sigma_{ESV}^r)^2} rV. \end{aligned}$$

The probability that the mutant wins the contest is given by

$$p_M \quad = \quad \frac{\sigma_M^r}{\sigma_M^r + \sigma_{ESV}^r}.$$

[1] see Boudreau and Shunda (2010) for the case of $r = 1$.

Whether the probability of winning the contest is higher for the mutant or for the ESV-player depends on the relative size of the prize perceptions of both players. If the mutant values the prize higher than the ESV-player, he will invest more and will therefore win with a higher probability.
Given the probability and the efforts, we can determine the material payoffs of the players. The indirect fitness function is used to decide whether a prize perception is evolutionarily stable or not. The indirect fitness function of the mutant reads

$$\begin{aligned} F_M &= \frac{\sigma_M^r}{\sigma_M^r + \sigma_{ESV}^r} V - \frac{\sigma_M^{r+1}\sigma_{ESV}^r}{(\sigma_M^r + \sigma_{ESV}^r)^2} rV \\ &- \frac{1}{(N-1)} \frac{\sigma_{ESV}^r}{\sigma_M^r + \sigma_{ESV}^r} V + \frac{1}{(N-1)} \frac{\sigma_M^r \sigma_{ESV}^{r+1}}{(\sigma_M^r + \sigma_{ESV}^r)^2} rV. \end{aligned}$$

F_M is composed of four terms. The first is the expected prize the mutant gets. The second term is what he spends in the contest. The last term is the investment of the opponent. The third term is the share of the prize the ESV-player gets in expectation. The last two terms are weighted by the share of the remaining population the ESV-player represents.
The first order condition with respect to σ_M is used to determine the influence of the prize perception on the evolutionary fitness. Since σ_{ESV} is the evolutionarily stable prize perception, the maximum fitness of the mutant is reached when his prize perception is equal to σ_{ESV}. Therefore, we can set $\sigma_M = \sigma_{ESV} = \sigma$. The first order condition reduces to

$$\begin{aligned} 0 &= (2r\sigma^{2r-1} - r\sigma^{2r-1})2\sigma^r - 2(r+1)r\sigma^{3r} + 2r^2\sigma^{3r} \\ &+ \frac{2r}{(N-1)}\sigma^{3r-1} + \frac{r}{(N-1)}(2r\sigma^{3r} - 2r\sigma^{3r}) \\ 0 &= 2r\sigma^{3r-1} - 2r\sigma^{3r} + 2r\frac{1}{N-1}\sigma^{3r-1}. \end{aligned}$$

Searching for the evolutionarily stable prize perception yields

Theorem 2.1: *For a finite population of size N and any $r \leq 1$, the unique symmetric evolutionarily stable prize perception in a two-player contest is given by $\sigma^* = \frac{N}{N-1}$; i.e. the evolutionary stable preference function of player i=1,2 is determined as*

$$\Pi_i = \frac{x_i^r}{x_1^r + x_2^r} \frac{N}{N-1} V - x_i.$$

Accordingly, we can state:

Lemma 2.1: *In a two-player Tullock contest the material payoff of a player is*

$$\pi_M = \pi_{ESV} = \pi_1 = \frac{2N-2-rN}{4(N-1)}V.$$

And the utility is given by

$$U_M = U_{ESV} = U_1 = \frac{N}{(N-1)}\frac{(2-r)}{4}V.$$

The effort invested is given by

$$x_M = x_{ESV} = x_1 = \frac{N}{(N-1)}\frac{rV}{4}.$$

Note that the utility is always positive because $r \leq 1$. The utility is also always greater than the material payoff because the contested prize is overvalued, at least in finite populations. Therefore the individuals feel better since they believe that the value of the prize is greater than it actually is. In contrast, the material payoff is zero for $r = 1$ and $N = 2$. The evolutionary fitness of every player is positive as long as $N > 2$.
Because winning has an intrinsic value, full dissipation can occur. This happens for $N = 2$ and $r = 1$. Although the individuals spent everything the prize is worth, they still have a positive utility that is created by the overvaluation of the prize. We may think of a collector of coins. Although he spent everything the coin is worth objectively in the contest for this coin (on Ebay, for example), he still is happy because he got the desired coin, which might have been the final piece to complete his collection.
Boudreau and Shunda (2010) also find that the evolutionarily stable prize perception is equal to σ^*. But because they only used $r = 1$, they do not find that this prize perception is independent of r and therefore holds for a much broader set of Contest-Success-Functions.

2.4.2 n-player contests

We now assume that $2 \leq n \leq N$. Individuals are chosen to play a contest for V. Only one of the chosen players is a mutant and has a prize perception σ_M^A. The prize perceptions of the other players are given by the ESV-prize perception σ_{ESV}^A. In contrast to Section 2.4.1, we assume $r = 1$.

Accordingly, if the mutant is chosen to play the contest, his utility is given by

$$\Pi_M^A = \frac{x_M}{x_M + (n-1)x_{ESV}} \sigma_M^A V - x_M.$$

And the utility of an arbitrary opponent is

$$\Pi_{E\hat{S}V}^A = \frac{\hat{x}_{ESV}}{x_M + \hat{x}_{ESV} + (n-2)x_{ESV}} \sigma_{ESV}^A V - \hat{x}_{ESV},$$

where $\hat{x}_{ESV}$ denotes the effort the chosen ESV-player exerts.
Accordingly, the first order condition of any of the ESV-players is given by

$$\frac{\partial \Pi_{E\hat{S}V}^A}{\partial \hat{x}_{ESV}} = \frac{x_M + (n-2)x_{ESV}}{(x_M + (n-1)x_{ESV})^2} \sigma_{ESV}^A V - 1 \overset{!}{=} 0.$$

Setting this equal to the first order condition of the mutant yields

$$x_M = \frac{(n-1)\sigma_M^A - (n-2)\sigma_{ESV}^A}{\sigma_{ESV}^A} x_{ESV}.$$

It is worthwhile to have a look at the resulting equilibrium efforts invested by the players:

$$\begin{aligned} x_M &= \frac{(n-1)\sigma_M^A \sigma_{ESV}^A V((n-1)\sigma_M^A - (n-2)\sigma_{ESV}^A)}{((n-1)\sigma_M^A + \sigma_{ESV}^A)^2}, \\ x_{ESV} &= \frac{(n-1)\sigma_M^A (\sigma_{ESV}^A)^2 V}{((n-1)\sigma_M^A + \sigma_{ESV}^A)^2}. \end{aligned}$$

By using the first derivatives with respect to the prize perceptions, it is possible to identify two effects of an increase in the prize perception. The first order conditions are the following:

$$\begin{aligned} \frac{\partial x_M^A}{\partial \sigma_M^A} &= \frac{(n-1)(\sigma_{ESV}^A)^2 V}{((n-1)\sigma_M^A + \sigma_{ESV}^A)^3} \left(n(n-1)\sigma_M^A - (n-2)\sigma_{ESV}^A \right), \\ \frac{\partial x_M^A}{\partial \sigma_{ESV}^A} &= \frac{(n-1)^2 (\sigma_M^A)^2 V}{((n-1)\sigma_M^A + \sigma_{ESV}^A)^3} \left((n-1)\sigma_M^A - (2n-3)\sigma_{ESV}^A \right), \\ \frac{\partial x_{ESV}^A}{\partial \sigma_{ESV}^A} &= \frac{2(n-1)^2 (\sigma_M^A)^2 \sigma_{ESV}^A V}{((n-1)\sigma_M^A + \sigma_{ESV}^A)^3}, \\ \frac{\partial x_{ESV}^A}{\partial \sigma_M^A} &= \frac{(n-1)(\sigma_{ESV}^A)^2 V}{((n-1)\sigma_M^A + \sigma_{ESV}^A)^3} \left(\sigma_{ESV}^A - (n-1)\sigma_M^A \right). \end{aligned}$$

If the value the mutant assigned to the prize increases, for example, his invested effort would rise if $n(n-1)\sigma_M^A > (n-2)\sigma_{ESV}^A$. Whereas, if $\sigma_{ESV}^A < (n-1)\sigma_M^A$, the ESV-players would be discouraged and would therefore invest less. The chance of winning the contest for an arbitrary ESV-player is rather low. If a mutant invests more, the contest becomes unattractive and therefore the invested effort decreases. We term the first effect incentive-effect, because a higher valuation for the contested prize incentivizes the player to act more aggressively. But if a player faces an opponent with a higher prize perception, then he is discouraged and invests less. Accordingly, we term this effect discouragement-effect. These two effects are also present if the prize perception of the ESV-players increases: The first derivative of the efforts invested by the ESV-players with respect to σ^A_{ESV} is always positive. But the discouragement-effect on the mutant is only present if $(2n-3)\sigma^A_{ESV} > (n-1)\sigma^A_M$. If $(n-1)\sigma^A_M < \sigma^A_{ESV}$, the mutant is encouraged to react with a higher investment against opponents with an increasing prize perception. The mutant is alone and wants to survive. If the pressure on him increases from all sides, he can only have success in evolutionary terms if he can hold the ground. This means, that he has to invest more effort.

Note that the discouragement-effect vanishes in a symmetric equilibrium in two-player contests since they cancel out. The mutant is discouraged by the ESV-player to the same degree he discourages the ESV-player. This is an explanation why this effect is not present in Section 2.4.1.

We can compute the winning probabilities of the mutant and an arbitrary ESV-player by using the equilibrium efforts:

$$\begin{aligned} p^A_{ESV} &= \frac{\sigma^A_{ESV}}{(n-1)\sigma^A_M + \sigma^A_{ESV}}, \\ p^A_M &= \frac{(n-1)\sigma^A_M - (n-2)\sigma^A_{ESV}}{(n-1)\sigma^A_M + \sigma^A_{ESV}}. \end{aligned}$$

Accordingly, the indirect evolutionary fitness of the mutant is given by

$$\begin{aligned} F^A_M &= \frac{(n-1)\sigma^A_M - (n-2)\sigma^A_{ESV}}{(n-1)\sigma^A_M + \sigma^A_{ESV}}V - \frac{(n-1)\sigma^A_M\sigma^A_{ESV}V((n-1)\sigma^A_M - (n-2)\sigma^A_{ESV})}{((n-1)\sigma^A_M + \sigma^A_{ESV})^2} \\ &- \frac{(n-1)}{(N-1)}\frac{\sigma^A_{ESV}}{(n-1)\sigma^A_M + \sigma^A_{ESV}}V + \frac{(n-1)}{(N-1)}\frac{(n-1)\sigma^A_M(\sigma^A_{ESV})^2V}{((n-1)\sigma^A_M + \sigma^A_{ESV})^2}. \end{aligned}$$

By deriving the first order condition with respect to σ^A_M and setting $\sigma^A_M = \sigma^A_{ESV} = \sigma^*$ afterwards, we can calculate the evolutionarily stable prize perception:

Theorem 2.2: *For a finite population of size N and r = 1, the unique symmetric evolutionarily stable prize perception in a n-player contest is given by* $\sigma^* = \frac{n(n-1)N}{N(n^2-2n+2)-n}$*; i.e. the evolutionary stable preference function of player i=1,...,n is determined as*

$$\Pi_i \quad = \quad \frac{x_i^r}{x_i^r + x_{-i}^r}\left(\frac{n(n-1)N}{N(n^2-2n+2)-n}\right)V - x_i.$$

Note that $\sigma^* = \frac{n(n-1)N}{N(n^2-2n+2)-n}$ reduces to $\frac{N}{N-1}$ if $n = 2$ (Theorem 2.1).
σ^* is maximal for a population of N individuals if $n_{\max} = \frac{2N}{N+1} + \sqrt{\frac{2N(N-1)}{(N+1)^2}}$ players are active in the contest. The latter expression converges to $2+\sqrt{2}$ for N approaching infinity. Accordingly, the evolutionarily stable preference for the contested price is highest for two-and three-player contests, even within a very large population. To have three players in the contest already leads to the highest valuation for populations with more than three individuals. It can be shown that even for an infinite population the evolutionarily stable valuation is greater than one for three-and four-player contests. In fact, for N approaching infinity and $n = 3$ or $n = 4$, σ^* converges to $\frac{6}{5}$. Even if there is an infinite number of individuals, an overvaluation of the prize occurs, except for playing the field contests. A reason for this is that a player with a higher prize perception acts more aggressively in the single contest. This behavior yields an advantage in the contest. And in evolutionary terms, the player can have an edge over these competitors if he is successful.
Another striking result is that $\sigma^* = \frac{N}{N-1}$ for playing the field contests, i.e. $N = n$ and for contests with only two players. By comparison with σ^* from Section 2.4.1, one can also see that the equilibrium prize perception in contests between more than two-players is greater than the equilibrium prize perception in two-player contests as long as $N > n$. In two-player contests no discouragement-effect exists. Accordingly, a valuation that is higher than $\sigma = \frac{N}{N-1}$ is not beneficial for a player in a two-player contest. But as soon as n is higher than 2, the discouragement-effect occurs and a further increase of σ may be beneficial. With a higher prize perception an individual has a higher chance winning the prize. One reason is that he invests more and another reason is that the opponents are discouraged and therefore invest less.
To see why it is not beneficial for any player to have a prize perception higher than $\frac{N}{N-1}$ in a playing the field contest, we have a look at the first derivative of the fitness function of the mutant with respect to σ_M^A after we applied the symmetry

assumption:

$$\begin{aligned}0 &= n(n-1)\sigma_1^2 \\ &- (n^2-2(n-1))\sigma_1^3 \\ &+ \frac{n-1}{N-1}n\sigma_1^2 \\ &- \frac{n-1}{N-1}(n-2)\sigma_1^3.\end{aligned}$$

The first line gives the effect of a higher valuation on the winning probability. Of course, this effect is positive. In the second line the effect on the invested effort is given. Because more exerted effort leads to higher costs, this effect is negative. The third line states that a higher prize perception decreases the probability of winning the contest of an opponent, and is therefore beneficial. The reason for the relatively low prize perception in playing the field contests is shown in the fourth line. It is true that a higher valuation discourages the opponents. But this effect is not only beneficial as can be seen in the fitness function. By discouraging the opponents, their investments are reduced. This means that their costs are lower and therefore they have more resources left. Accordingly, the effect in the fourth line is negative. Because this effect is weighted with $\frac{n-1}{N-1}$, the loss in fitness due to discouraging the opponents is the highest in playing the field contests.

Lemma 2.2: *The material payoff for a player with the evolutionarily stable prize perception σ^* in the equilibrium of a contest of n players with $r = 1$ is*

$$\pi_M^A = \pi_{ESV}^A = \pi_2 = \frac{N-n}{(N(n^2-2n+2)-n)}\frac{V}{n}.$$

The utility in equilibrium is given by

$$U_M^A = U_{ESV}^A = U_2 = \frac{(n-1)}{n}\frac{N}{N(n^2-2n+2)-n}V.$$

The equilibrium effort invested is

$$x_M^A = x_{ESV}^A = x_2 = \frac{(n-1)^2N}{nN(n^2-2n+2)-n^2}V.$$

Note that the material payoff is zero for playing the field contests and positive otherwise. Again, the intrinsic value makes the individuals spend as much as the prize is worth objectively. The reason why it is not beneficial to spend even more is given by the fact that this leads to a negative fitness. The utility is always positive

and greater than the material payoff. The reason for this is the evolutionarily stable prize perception, i.e. even if the fitness is negative, the utility can be positive. An individual may feel happy even if the opponents are more successful. In a finite population, σ^* is strictly greater than one. Accordingly, it is beneficial to overvalue the contested prize within a finite population. On the one hand it makes an agent "happy" because he believes that he has more in expectation and on the other hand he acts more aggressively and hurts the opponents more than he hurts himself.
Another important result is that the efforts invested in equilibrium are falling in n and N. This holds for all but one case. The effort invested by an arbitrary player in a three-player contest is higher than in a two-player contest. Individuals in a contest are concerned with the size of the population and the number of opponents. Holding the number of opponents in the contest constant and increasing the number of individuals in the population always reduces the invested effort. The smaller is the number of players in the contest compared to the number of individuals in the population, the smaller is the share of the population you can beat in this particular contest. This makes saving resources compared to a smaller population worthwhile. If we add a player to a given contest within a population with a determined number of individuals, there are two effects. On the one hand an additional player can be beaten and therefore a victory becomes more important from an evolutionary point of view. This increases the investment by a player. On the other hand an additional player invests and therefore winning becomes less likely. This effect decreases the investment of a player. In equilibrium the latter effect dominates the first one for contests with at least three players; i.e. only in two player contests the equilibrium investment per player increases, if an additional player is added.

2.5 n-player contests with $r \leq 1$

The assumption of $r = 1$ is relaxed now. Accordingly, the utility of the mutant is given by

$$\Pi_M^C = \frac{(x_M^C)^r}{(x_M^C)^r + (n-1)(x_{ESV}^C)^r}\sigma_M^C V - x_M^C.$$

It is rather difficult to find the evolutionarily stable prize perception analytically. But it is possible to state the following:

Theorem 2.3: *The evolutionarily stable prize perception without discouragement-effect is given by*

$$\sigma^C = \frac{N}{N-1}.$$

Proof: Suppose that we are in a symmetric situation with n arbitrary players, out of a population of size N, playing a given contest. The players do not differ with respect to their prize perception. This common prize perception is denoted by σ.
The utility of player h, $h = 1, ..., n$, is given by

$$\Pi_h = \frac{x_h^r}{\sum_{i=1}^{n} x_i^r}\sigma V - x_h.$$

Because of the symmetric situation, the first order condition with respect to x_h reduces to

$$n^2 x^{2r} = (n-1) r x^{2r-1} \sigma V.$$

Accordingly, the equilibrium effort is given by $x^* = \frac{n-1}{n^2} r\sigma V$.

Now suppose that we have two types of players. The type is determined by the prize perception. A player of type 1 has a valuation of $\tilde{\sigma}$ for the prize, and a player of type 2 has a valuation of $\hat{\sigma}$. We have one player of type 1 and $(n-1)$ players of type 2. The player of type 1 believes that he is in a symmetric situation, i.e. that the other players are also of type 1. Accordingly, his invested effort is given by $x_1 = \frac{(n-1)}{n^2} r\tilde{\sigma} V$. A type 2 player also believes that he is only facing type 2 players. His invested effort is given by $x_2 = \frac{(n-1)}{n^2} r\hat{\sigma} V$.
By employing this assumption, we can exclude the discouragement-effect because the investment of any player depends solely on his prize perception. An increase in valuation only increases the effort invested by this particular player, but it does not alter the efforts spent by any other player because they still believe that they are in a symmetric situation.
Subsequently, we can calculate the material payoffs

$$f_1 = \frac{(\tilde{\sigma})^r}{(n-1)(\hat{\sigma})^r + (\tilde{\sigma})^r} V - \frac{(n-1)}{n^2} r\tilde{\sigma} V,$$
$$f_2 = \frac{(\hat{\sigma})^r}{(n-1)(\hat{\sigma})^r + (\tilde{\sigma})^r} V - \frac{(n-1)}{n^2} r\hat{\sigma} V.$$

Accordingly, the indirect fitness function of the type 1 player is

$$\begin{aligned} F_1 &= \frac{(\tilde{\sigma})^r}{(n-1)(\hat{\sigma})^r + (\tilde{\sigma})^r} V - \frac{(n-1)}{n^2} r\tilde{\sigma} V \\ &- \frac{(n-1)}{(N-1)} \left(\frac{(\hat{\sigma})^r}{(n-1)(\hat{\sigma})^r + (\tilde{\sigma})^r} V - \frac{(n-1)}{n^2} r\hat{\sigma} V \right). \end{aligned}$$

Since we are searching for a symmetric solution, we substitute $\tilde{\sigma}$ and $\hat{\sigma}$ by $\bar{\sigma}$ in the first order condition:

$$\frac{1}{n^2} = \frac{N}{N-1} \frac{\bar{\sigma}^{2r-1}}{n^2 \bar{\sigma}^{2r}}.$$

Therefore we can state that the evolutionarily stable prize perception without the discouragement-effect is given by $\bar{\sigma} = \frac{N}{N-1}$.

■

We have seen in Section 2.4.2 that an increase in the prize perception of one player increases the effort invested by that particular player and also decreases the efforts invested by any other player, except in two-player contests. Without the second effect the evolutionarily stable valuation in a Tullock contest is given by $\frac{N}{(N-1)}$. But by incorporating the second effect, even higher prize perceptions can be evolutionarily stable. Accordingly, it is purely the discouragement-effect which varies and determines the evolutionarily stable prize perception. Making a mistake changes the own behavior, but it also has an effect on the opponents. This can help to make the mistake to be persistent, to become more severe, or make an even bigger mistake evolutionarily stable. In this model a prize perception of more than one leads to an advantage in the contest. On the one hand because it induces more aggressive behavior, but on the other hand because the opponents are deterred. By natural selection, this may lead to even higher prize perceptions.

2.6 Comparison

2.6.1 Direct evolutionary approach

Schaffer (1988) showed that it is not evolutionarily stable to be an absolute payoff maximizer in a finite population. Only if the evolutionarily stable strategy (ESS) - as proposed by Maynard Smith (1973)- is employed by a player, then he is able to compete with the other individuals in the population. Evolution takes place on the level of strategies. If a player uses a new strategy and is more successful than the other players in the population, then this strategy is imitated and becomes the predominant strategy. This movement stops if the ESS is used by any player. To derive the ESS in the given contest, suppose that we have one mutant. We also have $(n-1)$ players that already use the ESS. ESS-players and the mutant are playing the Tullock-contest with $r \leq 1$. The mutant's material payoff is given by

$$f_M^D = \frac{x_M^r}{x_M^r + (n-1)x_{ESS}^r}V - x_M^D.$$

As opposed to this, the payoff of an arbitrary ESS-player is given by

$$f_{ESS}^D = \frac{x_{ESS}^r}{x_M^r + (n-1)x_{ESS}^r}V - x_{ESS}.$$

The equilibrium condition for x_{ESS} being an evolutionarily stable strategy according to Schaffer (1988) is

$$f_M^D \leq f_{ESS}^D, \qquad \text{for any strategy } x_M^D.$$

Accordingly, the problem for the mutant's strategy is $\max\limits_{x_M^D}(f_M^D - f_{ESS}^D)$. Which can be written as

$$\max_{x_M^D} \quad \left\{ \frac{x_M^r}{x_M^r + (n-1)x_{ESS}^r}V - x_M - \frac{(n-1)}{(N-1)}\left(\frac{x_{ESS}^r}{x_M^r + (n-1)x_{ESS}^r}V - x_{ESS}\right)\right\}.$$

The solution for this problem is given by $x_M = x_{ESS}$ and the corresponding maximum is zero. It is evolutionarily stable to act as if the preferences are given by a weighted relative payoff. A striking result is that in finite populations the ESS is a deviation from the Nash equilibrium strategy.
Note that the stability condition used by Schaffer (1988) is omitted here due to simplicity.
The probability that the mutant does not win the contest
is given by $\left(1 - \frac{x_M^r}{x_M^r + (n-1)x_{ESS}^r}\right)$. Accordingly, the objective function of the mutant can be rewritten as

$$\begin{aligned}\Pi_M^D &= \frac{x_M^r}{x_M^r + (n-1)x_{ESS}^r}V - x_M \\ &- \frac{(n-1)}{(N-1)}\left(\frac{1}{(n-1)}\left(1 - \frac{x_M^r}{x_M^r + (n-1)x_{ESS}^r}\right)\right)V + \frac{(n-1)}{(N-1)}x_{ESS}.\end{aligned}$$

Which reduces to

$$\Pi_M^D = \frac{x_M^r}{x_M^r + (n-1)x_{ESS}^r}\frac{N}{(N-1)}V - x_M + \frac{1}{(N-1)}((n-1)x_{ESS} - V).$$

We can immediately state the following:

Theorem 2.4: *The direct evolutionary approach and the indirect evolutionary approach that is concerned with prize perceptions are only equivalent in the behavior they predict for two-player and playing the field contests in finite populations.*

The evolutionarily stable prize perception for two-player and playing the field contests is given by $\frac{N}{N-1}$. This is exactly the weight the direct evolutionary approach "puts" on the prize (implicitly) as can be seen by having a look at Π_M^D. "Puts" is put in quotation marks here, because the direct evolutionary approach just predicts that it is evolutionarily stable to weight the contested prize with this factor.
$\frac{1}{(N-1)}((n-1)x_{ESS} - V)$ in Π_M^D is independent of the effort invested by the mutant. This term therefore has no influence on the investment choice of the M-player. That leads to the same first order condition the indirect evolutionary approach would predict. And therefore the effort chosen in the Nash equilibrium of the indirect evolutionary approach and the ESS-strategy are the same.
The reason why the direct evolutionary approach can be transformed is that any player accounts for the part of the prize he is supposed to win and additionally the share of the prize the opponents do not get. This relation is described by Leininger (2003). He compared Nash equilibrium behavior in a transfer contest to finite population ESS-behavior in a contest. He found that both are identical. In a transfer contest individuals care for the expected payoff of the opponents because they have to pay them in expectation. And in an evolutionary equilibrium the players care for their opponents' payoff because they have to beat them to have a higher fitness. It is also obvious why this case creates the same behavior in the evolutionary equilibrium as the indirect evolutionary approach concerning prize perceptions without the discouragement-effect suggests. The prize perception is determined to be $\frac{N}{N-1}$, the opponent cannot be discouraged by a high valuating opponent because they all have the same prize perception. Accordingly, the material payoffs in the two-player and the playing the field case are identical in equilibrium for both approaches. However, when the discouragement-effect is at work (for $2 < n < N$), the two approaches predict different behavior: For $r = 1$ and $2 \leq n \leq N$, the material payoff of an arbitrary player is $f^D = \left(\frac{(N-n)}{n^2(N-1)}V\right)$. This material payoff is greater than the material payoff in the indirect evolutionary approach using prize perceptions for $2 < n < N$, because the higher valuation makes the players act more aggressively. Nevertheless, the "utility" is smaller in the direct evolutionary approach unless overdissipation occurs, since $\frac{1}{(N-1)}((n-1)x_{ESS} - V)$ is negative otherwise. The reason for that is that the prize perception is smaller.

2.6.2 Indirect evolutionary approach according to Leininger (2009)

As stated above, Leininger (2009) argued that the preferences about the opponent's expected payoff can act as stimuli. Only the preferences about the opponent's expected payoff that make the player choose the most successful strategy will survive in the long run. Suppose that n players, drawn out of a population of N individuals, are chosen to play a contest with $r \leq 1$ for the prize V. Note that Leininger (2009) only considers two-player contests with $r = 1$. We assume that $n-1$ players are concerned with the evolutionarily stable preferences (ESP) and that only one mutant has a different preference function. Because the non-mutants are all alike, we assume that the preferences of the mutant do not depend on the non-mutant he is facing in the contest, i.e. he treats them all alike. We also assume that the preferences of any ESP-player stay the same for any opponent he is facing in the contest. Accordingly, if the mutant was chosen to play the contest, his utility is given by

$$\Pi_M^L = \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r} V - x_M - (n-1)\delta_M \left(\frac{x_{ESP}^r}{x_M^r + (n-1)x_{ESP}^r} V - x_{ESP} \right).$$

Where $\delta_M \in [-1, 1]$ denotes the preferences of the mutant about the expected payoff of an arbitrary ESP-player. If $\delta_M < 0$, he is an altruistic player, he is an individualistic player for $\delta_M = 0$ and a spiteful player otherwise. The evolutionary process works on the level of preferences about the opponents' expected payoff and operates in the interval $[-1, 1]$. The ESP is given by δ^*.
It is possible to state the following:

Lemma 2.3: *Concerning the evolutionarily stable preferences and the evolutionarily stable prize perception σ^*, it always holds that*

$$\sigma^* = (1 + \delta^*).$$

Proof: The probability that the contest is won by an ESP-player is given by $1 - \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r}$. Because they are all alike, the probability for one ESP-player is $\frac{1}{n-1}$ times the latter expression. Accordingly, the utility function of the mutant can be rewritten as

$$\begin{aligned} \Pi_M^L &= \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r} V - x_M \\ &- (n-1)\delta \left(\frac{1}{(n-1)} \left(1 - \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r} \right) V - x_{ESP} \right). \end{aligned}$$

After solving the brackets we get

$$\Pi_M^L = \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r} V - x_M - \delta V + \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r} \delta V + (n-1)\delta x_{ESP},$$

which reduces to

$$U_M^L = \frac{x_M^r}{x_M^r + (n-1)x_{ESP}^r}(1+\delta)V - x_M + \delta((n-1)x_{ESP} - V).$$

Substitute $(1+\delta)$ by σ_M^C. The only differences to the utility function Π_M^C from section 2.4.3 are $(n-1)\delta x_{ESP}$ and $-\delta V$. These two terms are independent of the efforts invested by the mutant, and have therefore no influence on the decision of the mutant.[2] Accordingly, both indirect evolutionary approaches lead to identical first order conditions and therefore to no difference in behavior by the players in the Nash equilibrium of the contest.

■

It is straightforward to show that

Theorem 2.5: *Both indirect evolutionary approaches are equivalent in the behavior they predict in the Nash equilibrium of the contest.*

This theorem shows that the indirect evolutionary approach concerning preferences about the opponents' payoffs, which works on the interval $[-1, 1]$ can be transformed into the indirect evolutionary approach that is concerned with price perceptions and that works on the interval $[0, 2]$. The intuition behind the fact that interdependent preferences can be transformed is that any player does not only take account of the share of the prize he might get but also of the prize the opponents do not get. This means the contest is played for a quasi-prize that is greater than the objective value. In the indirect evolutionary approach concerning prize perceptions this effect is internalized by the higher prize perception.
That theorem also suggests, for example, that a perfectly altruistic player $(\delta = -1)$ in the approach of Leininger (2009) is equivalent to a player with a prize perception of zero in the second indirect approach. Both players will invest nothing. The player in the first approach will do so because he does not want to harm the opponent, and the player in the second approach does not want to pay anything for the prize.

Calculating the evolutionarily stable values for the two-player and for the n-player case with $r = 1$ yields the following:

[2]Note that we can employ the same type of analysis for any of the ESP-players.

Lemma 2.4: *In the evolutionary equilibrium of a two-player contest with* $r \leq 1$ *the payoff of the opponent is weighted by*

$$\delta_1 = \sigma^* - 1 = \frac{1}{N-1}.$$

Accordingly, the material payoff and the utility of any player is given by

$$\begin{aligned} f_1^L &= \frac{2N-2-rN}{4(N-1)}V, \\ \text{and} \quad \Pi_1^L &= \frac{(N-2)(2N-2-rN)}{4(N-1)^2}V, \text{ respectively.} \end{aligned}$$

Contrary to this, in the evolutionary equilibrium of a contest between n *players and* $r = 1$ *the payoff of the opponent is weighted by*

$$\delta_2 = \sigma_1^* - 1 = \frac{nN+n-2N}{N(n^2-2n+2)-n}.$$

Accordingly, the material payoff and the utility of any player in equilibrium is given by

$$\begin{aligned} f_2^L &= \frac{(N-n)}{(N(n^2-2n+2)-n)}\frac{V}{n}, \\ \text{and} \quad \Pi_2^L &= \frac{(N-n)^2}{(N(n^2-2n+2)-n)}V, \text{ respectively.} \end{aligned}$$

As induced by Theorem 2.5, both approaches yield the same material payoff for the players because their behavior does not change[3]. The only difference is that the utilities in both approaches differ. It can be shown that the utility for any player is always higher in the indirect evolutionary approach used in section 2.4, for both cases. One reason is that they believe that the value of the prize is higher than it actually is. Another reason is that the material payoff of the opponent is not included in their preference function.

In both approaches the individuals invest the same effort, but they are supposed to be happier in the approach that assigns an additional value to the prize. Pointing to this result, a soccer-trainer in Germany recently said, "Ich glaube nicht daran, dass die Angst vor dem Verlieren dich eher zu einem Sieger macht als die Lust auf Gewinnen." (Jürgen Klopp)

That means "I don't believe that the fear of losing makes you become a winner more

[3]The equilibrium efforts have already been computed in sections 2.4.1 and 2.4.2.

easily than the desire to win."
The fear of losing is the approach of Leininger (2009). The fear is incorporated by the negative weight on the payoff of the opponents. The desire to win is given by the higher prize perception. You do not care for the payoff of the opponents. The only thing that matters is to win.

2.7 Conclusion

We asked whether it is beneficial in evolutionary terms to overvalue the prize in a Tullock contest or not. We find that this is indeed the case. Because of a higher valuation for the contested prize, the players invest more and the opponents are discouraged and exert less effort. Accordingly, we named these two effects incentive-effect and discouragement-effect. Both effects make overvaluation evolutionarily stable. The discouragement-effect is only at work in contests between more than two players but not in playing the field contests. Even without the discouragement-effect overvaluation does occur in finite populations. Without the discouragement-effect overvaluation is not evolutionarily stable in an infinite population. But if this effect is at work, then overvaluation can be evolutionarily stable even in an infinite population. This is especially true for three- and four-player contests.

Subsequently, we compared the indirect evolutionary approach concerning prize perceptions to the direct evolutionary approach and to the indirect evolutionary approach according to Leininger (2009). We find that all three approaches predict the same behavior for two-player contests and for playing the field contests. The direct evolutionary approach and the indirect evolutionary approach that introduces preferences about the opponents' payoffs implicitly put a weight on the contested prize. In the direct evolutionary approach the implicit prize valuation is constant. This constant prize perception is equal to the prize perception in the indirect evolutionary approach concerning prize valuations without the discouragement-effect. Accordingly, both approaches do not differ in the behavior they predict in the equilibrium of two-player and playing the field contests. In a two player contest both players get discouraged by a high prize perception to the same amount they discourage the opponent. In playing the field contests the discouragement-effect raises the costs of a higher prize perception to such an amount that the players only act as aggressively as in the direct evolutionary approach. All other opponents exert less effort if the prize perception is high. But because there are many players, the probability of winning changes only slightly. Since the opponents save effort and therefore have more resources to spend, the evolutionary costs for the high valuating player rises. The material payoff in the direct evolutionary approach is higher

because of the less aggressive behavior.
We also show that for contests that have more than two players and that are not playing the field the behavior predicted by the indirect approaches is more aggressive. This is due to the discouragement-effect.

The indirect evolutionary approach according to Leininger (2009) predicts exactly the same aggressive behavior in the Nash equilibrium of the contest as the indirect evolutionary approach using prize perceptions because the weights on the prizes are identical. The implicit weight is not fixed. Therefore this weight is allowed to evolve evolutionarily and in equilibrium amounts to the same amount as in the new indirect evolutionary approach. Accordingly, the material payoffs are the same. Only the utility of the players differs. In the indirect evolutionary approach according to Leininger (2009) the utility is smaller as long as we do not face overdissipation. The reason for this is that the material payoff of the opponent is not subtracted and that the prize is overvalued. The individuals feel better because of their desire to win and they are not depressed by their fear of losing.

We are able to calculate the result for more than two players and for decreasing marginal efficiency of effort only if we control for the discouragement-effect. It would shed some light on the relations in perceived utility between direct and indirect evolutionary approach if we could find an equilibrium for that case. It also might be interesting whether the result changes if the non-mutants treat the mutants differently from the way they treat a player that has the same preferences he has.

Chapter 3

Equilibrium contracts in two-player Tullock contests

3.1 Introduction

Since Shelling wrote "The Strategy of Conflict"(1960), strategic delegation is an important topic in economic research. By using a contract a delegate is hired to act on behalf of a principal. The behavior of the agent is influenced directly by the chosen terms of the contract. Accordingly, the principal can influence the outcome of a game in his favor, if he chooses an appropriate contract. Delegation is also an important issue in human society. Firm owners hire managers, lawyers are hired to act in lawsuits or employees are sent to negotiations to act on behalf of their employer. There are many reasons for delegation. The agent may be more skilled, as stated by Baik and Kim (1997), or the agent may have more instruments that he can use compared to the principal, as explained by Schoonbeek (2007). Opportunity costs or better information of the agent also act as explanations. Delegation may even be required by law. Another interesting point compared to Chapter 2 is that delegation is not present in animal populations. Delegation is therefore a pure invention of humans. This Chapter will show whether delegation can decrease wasteful expenditures. This is one explanation for why delegation has been invented.

In economics, delegation is often used in contests. In contests, delegation is able to reduce the effort made, as is shown by Wärneryd (2000) for two-player Tullock rent-seeking contests by using no-win-no-pay contracts. The agent has to be paid and the principal also wants to get a part of the desired prize. Accordingly, the contested prize has to be split up and therefore the incentives for both are smaller compared to the incentives of a principal playing the contest on his own behalf. But if principal and agent do not differ, for example in their abilities, then no principal would voluntarily choose to delegate. The principal would always have a higher probability of winning and therefore a higher expected payoff in a contest due to the higher incentives. Accordingly, mandatory delegation is assumed by Wärneryd (2000).

We also use a two-player Tullock contest for a given and indivisible prize in this chapter. The prize has the same value to both players. It is assumed that agents and principals are risk neutral. The effort made by the agent is not revealed to the principal. This gives rise to moral hazard. But it will be shown that moral hazard can be neutralized by choosing an appropriate contract. Mandatory delegation is assumed, but it is shown that it is possible to relax this assumption. By choosing an appropriate contract, the incentives for the agent can be increased to the same level a principal, acting on his own, is incentivized with. Konrad (2009) assumed that the effort choice of the agent is revealed to the principal. Using this assumption, he derives that the principal can also incentivize his agent to the extent he is incentivized with. But, as we will show, it is not even necessary to know the investment of the agent. Also, selling the right to participate is no preferred option in this article. Baik (2007) showed that no-win-no-pay contracts are optimal for hiring an agent if the effort made by the agent is unobservable for the principal. In this paper it is shown that a no-win-no-pay contract is only one representative of a broader class of contracts and that Baik (2007) has not examined all possible types of contracts. Baik (2007) showed that principals only use a part of the prize to pay the agent and do not use fixed payments. In this chapter, we assume that principals offer a payment contingent on the prize in case of victory, but they also implement a fine that is also contingent on the prize if the agent loses the contest. A game structure similar to the prisoners' dilemma prevents moral hazard on part of the agents. The fine is included to meet the idea of contractual penalties. Penalties are often used in contracts in everyday life to ensure a desired behavior by the contract partners. But in the theory of contests, this type of contract parameter was excluded so far. In a contest, the contestants fight against each other to gain the desired prize. A defeat in this contest is an unwanted result because you have costs and your opponent a gain. In a delegation scenario a fine represents the wish of the principal not to lose. This wish can be transferred to the agent through a fine. The reward and the fine may be bounded, in order to represent contractual law or common rules in contracting.

If no fine is used, the setting of Baik (2007) results. To the best of my knowledge, these kind of contracts have been introduced by Harris and Raviv (1979). The contracts are called dichotomous contracts and they are used in a monitoring model. The contract consists of two branches. If the action of the agent is acceptable for the principal, then the agent is paid according to a predetermined schedule. But if the action is found to be unacceptable, the agent receives a less preferred payment or he may be dismissed. A signal that may be correlated to the effort invested by the agent is used to judge whether the action is acceptable or not. In this chapter the signal is the outcome of the contest, i.e. who is the winner. If the contest is won, the agent is rewarded; otherwise the agent is punished. Introducing a fine is quite common in real life, either directly or indirectly. Yubitsume in Japan is a very good example. This ritual is mainly performed by the Yakuza. To apologize for disappointing or offending one's principal, a member amputates parts of his fingers. This weakens the members ability to fight. Because of his weakness, the ties between principal and agent become stronger. We also mentioned the "contracts" the Islamic State uses to contract his soldiers. A situation without enforceable property rights may also lead to such contracts.

But also in legal sectors such dichotomous contracts may be used. To show this, we will refer back to contractual penalties. There is a well-known example for such a penalty in Germany. The government delegated the introduction of the toll system in Germany to a group of firms (Toll Collect). The contract also defined a fine for Toll Collect. And indeed Toll Collect had to pay this fine, because the firms were not able to get the system started on time. The fine stood for the forgone tolls for the government's budget.

Another example is a lawyer who loses an important lawsuit. He may not be hired again and even other individuals may refrain from hiring him. The forgone future earnings act as an implicit fine.

By using a fine, the incentives for the agent can be increased. Accordingly, by choosing an appropriate contract, a principal can induce more invested effort on part of the agent and therefore more aggressive behavior than by using a no-win-no-pay contract.

It is shown that no-win-no-pay contracts are not chosen in equilibrium and that it is possible to incentivize an agent by contracting to the same extent the principal himself is incentivized with. Accordingly, a reduction in effort by sending a delegate cannot be observed if the set of contracts is not limited. By prescribing the contracts to use, i.e. by forbidding Yubitsume, the legislator can reduce wasteful expenditures. Concerning Yubitsume, one has to admit that it is still used although it is illegal to force an agent to do this. One explanation for the usage is that it proved to be an effective mean to encourage the agents, even if the use is sentenced. Additionally, other contractual penalties are quiet common in real life, which is another hint that they proved to be useful.

The influence of a limited fine and reward is explained. Selling the right to participate is excluded in this setting. But it is described how this assumption changes

the results.
Leininger (2009) showes that in finite populations interdependent preferences yield an advantage. In such populations hurting oneself in order to hurt the opponent even more may be a good idea. This is also a topic in economics, as the Uralkali example shows. Also Uralkali hurts itself to get rid of some of the competitors. Another example is given by a divorce battle. Both parties want to get more than the opponent. In the latter example mandatory delegation is prescribed in Germany. It is not immediately clear whether interdependent preferences change the chosen contract by the principal or not. On the one hand, the principal wants to transfer his preferences on the delegate, but on the other hand, the agent is only interested in his material payoff. If an evolutionary setting is employed, an important question can be asked: Will the delegates also have interdependent preferences? This topic is also addressed. In this chapter, the influence of interdependent preferences of the agents on the chosen contracts by principals with independent preferences is also considered. Whether the results of this model differ or not is also explained.

This chapter is organized as follows. In section 3.2 we set up the model. Afterwards the game is solved using Baik's (2007) no-win-no-pay contracts and the contracts using a fine. In section 3.4 the model is solved for a limited fine and a limited reward. Section 3.5.1 deals with the influence of interdependent preference on the side of the principals on the chosen contract and 3.5.2 is concerned with agents with interdependent preferences. The conclusion is given in section 3.6.

3.2 The Model

Two risk neutral players are chosen to play a contest for the exogenously given, indivisible, and strictly positive prize V. The players are named principal one and principal two. Each player has to choose an agent that plays the contest for him. We assume that there is perfect competition on the labor market. The reservation wage is normalized to zero. Accordingly, their participation constraint is met as long as their wage is in expectation non-negative. This also means that the contested prize is always greater than the reservation wage, otherwise delegation would be excluded by the principals. The agents and the principals do not differ in any detail except that the principals can hire the delegates. Principals and agents are only interested in their material payoff.
The game has two stages. At stage one both principals hire a delegate. Katz (1991) showed that it does not matter whether the terms of the contract are observable or not. At stage two the agents put in irreversible effort to win the prize V. The unit costs of effort are constant and are given by one for any player. To hire a delegate at stage one, both principals determine the share they want to pay to the delegate

if he wins and the share the agent is fined with if he is defeated (the contractual penalty). The penalty and the offered share may be limited. This extension is discussed in Section 3.4. The principal does not have the possibility to sell the right of participating.
The share of the contested prize, paid by principal one if his agent wins, is given by $\alpha_1 \in [0,1]$. The share demanded, if his agent is not successful, is given by $\alpha_2 \in [0,1]$. For simplicity it is assumed that both shares are not greater than one. Nevertheless, if both shares are used, it is possible to incentivize the delegate with an amount greater than the contested prize. The corresponding shares used by principal two are $\beta_i \in [0,1], i = 1,2$. Note that we make no assumption concerning the relative size of the shares, i.e. whether for principal one it holds that $\alpha_1 > \alpha_2$ or not, for example.

Both principals are aware of the fact that their agent's expected payoff must be non-negative, i.e. that the reward for a victory is high enough to make the agents willing to sign the contract, even if they have to pay a contractual penalty and the costs of the effort made when they are not successful.
The model of Baik results, if $\beta_2 = \alpha_2 = 0$.

The effort invested by the agent of principal one is given by d_1. The corresponding effort of the second delegate is given by d_2. The winner of the contest is determined by using the well-known Tullock contest success function with $r \leq 2$. Accordingly, the probability that the delegate of player one wins the contest is given by

$$p_1 = \begin{cases} \frac{1}{2} & \text{for } d_1 + d_2 = 0, \\ \frac{d_1^r}{(d_1^r + d_2^r)} & \text{for } d_1 + d_2 > 0. \end{cases}$$

After both agents put in their efforts, the winner is determined and the prize is handed out. In Chapter 2 r has to be smaller or equal to one because a higher r could lead to negative material payoffs. The reason is the more aggressive behavior induced by the negatively interdependent preferences. But in this chapter the individuals act not as aggressively as they do in Chapter 2. Accordingly, r can rise to values smaller or equal to two.

The game is solved by using backward induction. Both principals think about how the behavior of the agents at stage two is affected by the parameters of the contract. If they know the relation between contract and behavior, both principals can choose their preferred contract, contingent on the contract of the opponent, at stage one. The equilibrium is reached if no principal wants to change a parameter of the contract, for given parameters of the opponent.

We start at the second stage. The agent of principal one maximizes

$$\pi_{D1} = \frac{d_1^r}{(d_1^r + d_2^r)}\alpha_1 V - \left(1 - \frac{d_1^r}{(d_1^r + d_2^r)}\right)\alpha_2 V - d_1.$$

By calculating the first order condition with respect to d_1 and setting it equal to zero, it results

$$(d_1^r + d_2^r)^2 = (\alpha_1 + \alpha_2)d_1^{r-1}d_2^r rV. \tag{3.1}$$

Calculating the first order condition of the second delegate, and setting both conditions equal afterwards, yields

$$d_2 = \frac{\beta_1 + \beta_2}{\alpha_1 + \alpha_2}d_1.$$

The effort invested by an agent in equilibrium depends on the sum of both shares. The relation of the sum of both shares of both principals determines which delegate invests more; i.e. the agent with the higher total incentive also invests more.
To find the equilibrium effort levels, this condition is plugged into (3.1), it results

Lemma 3.1: *The equilibrium efforts of the agents contingent on the offered shares are given by*

$$d_1 = \frac{(\alpha_1 + \alpha_2)^{r+1}(\beta_1 + \beta_2)^r}{((\alpha_1 + \alpha_2)^r + (\beta_1 + \beta_2)^r)^2}rV,$$
$$d_2 = \frac{(\alpha_1 + \alpha_2)^r(\beta_1 + \beta_2)^{r+1}}{((\alpha_1 + \alpha_2)^r + (\beta_1 + \beta_2)^r)^2}rV.$$

Accordingly, the probability that the agent of principal one wins the prize is given by

$$p_1 = \frac{(\alpha_1 + \alpha_2)^r}{((\alpha_1 + \alpha_2)^r + (\beta_1 + \beta_2)^r)}$$

and analogously for the agent of principal two

$$p_2 = \frac{(\beta_1 + \beta_2)^r}{((\alpha_1 + \alpha_2)^r + (\beta_1 + \beta_2)^r)}.$$

It can be seen from the effort made that any agent is incentivized by the sum of both shares. A delegate considers the amount he gets if he wins, and he also pays attention to the share he has to pay if he loses. By implementing a fine, a principal can induce more aggressive behavior. If the contest is lost, then the agent has a negative payoff because he has to pay the fine and has to bear the costs of the invested effort. It is assumed that the stake of any agent is sufficient to pay the fine. If the contest is won, then the payoff for the agent is positive. Baik (2007) explained that no-win-no-pay contracts are optimal because the agent is incentivized by the difference in payment between winning and losing. This difference is maximized by no-win-no-pay contracts if no payment takes place if the agent is defeated. By introducing a fine, the difference can be increased even further. An employee, for example, acts more committed if his job is on the line.

3.3 Equilibrium contracts

To analyze the implications of all parts of the contract, the analysis is split up. In the first subsection no fine is allowed. That means $\alpha_2 = \beta_2 = 0$. Since this connotes that only no-win-no-pay contracts are used, the subsection is named that way.
Afterwards, a fine is allowed for but the participation constraint is not considered. In the last subsection the principal uses both parts of the contract and designs contracts that the agent is willing to sign. This subsection contains the equilibrium contract for the game specified above. Without loss of generality, we concentrate on principal one.

3.3.1 No-win-no-pay contracts

If an agent wins, then his principal gets the contested prize. But the principal has to pay the share specified in the contract to his delegate. If the agent loses, he gets paid nothing. Accordingly, the expected payoff for principal one is given by

$$\pi_{P1} \;=\; p_1(1-\alpha_1)V \;=\; \frac{\alpha_1^r}{(\alpha_1^r+\beta_1^r)}(1-\alpha_1)V.$$

Calculating the first order condition and setting $\alpha_1 = \beta_1$ afterwards, because we are looking for a symmetric solution, yields the contract used in Nash equilibrium:

$$\alpha_1 \;=\; \beta_1 \;=\; \frac{r}{2+r}.$$

It can be established

Lemma 3.2: *If only no-win-no-pay contracts are allowed, the efforts invested and payoffs achieved in equilibrium are given by*

$$\begin{aligned} d_1 = d_2 &= \frac{r^2}{(2+r)}\frac{V}{4}, \\ \pi_{P1} = \pi_{P2} &= \frac{V}{(2+r)}, \\ \pi_{D1} = \pi_{D2} &= \frac{(2-r)}{(2+r)}\frac{rV}{4}. \end{aligned}$$

By using no-win-no-pay contracts, the principal offers a share of the prize to the delegate that is sufficient to endow him with an expected payoff that is non negative. The expected payoff is strictly positive as long as $r < 2$. The offered share is always smaller than $\frac{1}{2}$. Note that the efforts made are smaller than in the case without mandatory delegation. Accordingly, the payoffs for the principals are higher compared to not hiring a delegate[1]. Another result is that the expected payoff of the principal is always higher than the expected payoff of his delegate. The principal has the initial right to participate in the contest and therefore he benefits the most. This is ensured formally by the fact that he knows how the agent reacts to the share offered. He can use the share most useful to him.

3.3.2 Equilibrium contracts without participation constraint

Now suppose that the principal can use a fine. This fine has to be paid by the agent if the contest is lost. This can be seen as an insurance against losing. The participation constraint is not taken into account here. That means that the expected payoff of the delegate may be negative. The purpose of this section is to show which tool is preferred by the principal if he does not have to ensure that the participation constraint is fulfilled.
The payoff of principal one is given by

$$\pi_{P1} = \frac{(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}(1-\alpha_1)V + \frac{(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}\alpha_2 V.$$

By using the first derivatives, it is possible to show

[1] For a detailed interpretation, see Baik (2007) and Wärneryd (2000).

Lemma 3.3: *The Nash equilibrium in the contract game without participation constraint is given by*

$$
\begin{aligned}
\alpha_1 &= \beta_1 = 0 \quad \textit{and} \\
\alpha_2 &= \beta_2 = 1.
\end{aligned}
$$

Proof:
The payoff of principal one is given by

$$
\pi_{P1} = \frac{(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}(1-\alpha_1)V + \frac{(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}\alpha_2 V.
$$

First of all, we have a look at α_1 and under what circumstances this tool is used.

$$
\begin{aligned}
\frac{\partial \pi_{P1}}{\partial \alpha_1} &= \frac{r(\alpha_1+\alpha_2)^{r-1}((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}V - \frac{r(\alpha_1+\alpha_2)^{r-1}(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}V \\
&- \frac{((\alpha_1+\alpha_2)^r+r\alpha_1(\alpha_1+\alpha_2)^{r-1})((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}V \\
&+ \frac{r(\alpha_1+\alpha_2)^{r-1}\alpha_1(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}V - \frac{r(\alpha_1+\alpha_2)^{r-1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}\alpha_2 V > 0.
\end{aligned}
$$

Substituting $(\alpha_1+\alpha_2)$ by a, $(\beta_1+\beta_2)$ by b and simplifying the expression yields

$$
b^r(ra^{r-1}-(1+r)a^r) > a^{2r}.
$$

The term in brackets on the left hand side can be positive or negative. Assume for the moment that $r \neq (1+r)a$. Accordingly, we have to consider two cases. For $\frac{r}{1+r} > a$, we get

$$
b^r > \frac{a^{r+1}}{r-(1+r)a}.
$$

If the total incentive of the opponent is high enough, then this inequality is fulfilled, although the total incentive of principal one is strictly smaller than one.
The second case is given for $\frac{r}{1+r} < a$.

$$
b^r < \frac{a^{r+1}}{r-(1+r)a}.
$$

This case is ruled out because the right hand side is negative. The total incentive cannot be negative.
Accordingly, principal one will only use α_1 if the total incentive he wishes to use is

smaller than $\frac{r}{1+r}$.
Now we have a look at α_2.
This tool is only used as long as the first derivative is positive.

$$\frac{\partial \pi_{P1}}{\partial \alpha_2} \quad > \quad 0.$$

Computing this expression and simplifying it, yields

$$r(\alpha_1+\alpha_2)^{r-1} + (1-r)(\alpha_1+\alpha_2)^r + (\beta_1+\beta_2)^r \quad > \quad 0.$$

This condition is fulfilled because r as well as $(\alpha_1+\alpha_2)$ are not greater than two. That means that increasing the fine is always beneficial for the principal. Accordingly, the total incentive will be equal to one because as soon as α_2 is greater than $\frac{r}{1+r}$, the principal reduces the reward to $\alpha_1 = 0$. But α_2 is still increased. This movement stops if the upper bound is reached. The same kind of analysis can be applied to the second principal. And also $r \neq (1+r)a$ holds.

■

Lemma 3.3 states that if the agent works for the principal anyway[2], then there will only be punishment for defeats. If the participation constraint is disregarded, also the effects of both tools on the payoff of the agent are ignored. If a principal increases the share of the prize offered to the delegate to reward a victory, then the willingness of the agent to sign the contract increases. Suppose that only a fine is used. No rational agent will sign the contract because the payoff is negative for sure. The agent has to pay something if he loses and gets paid nothing if he wins, and the agent has to pay for the effort invested anyway. But if a reward is used, then the agent gets a share of the prize. Accordingly, it is possible that his participation constraint is met. Nevertheless, using a reward that is too high lowers the payoff for the principal. And a fine that is too high attracts no agent. These effects are ignored if the participation constraint is neglected.

3.3.3 Equilibrium contracts

In this subsection any principal can reward his delegate for a victory, punish his agent for a defeat, and has to ensure that the payoff for the delegate is non-negative. Hence, principal one is confronted with

$$\max_{\alpha_1,\alpha_2} \pi_{P1} \quad \text{subject to} \quad \pi_{D1} \geq 0.$$

[2]We would call this slavery.

The expected payoff for the delegate in equilibrium of stage two is given by

$$\begin{aligned}\pi_{D1} &= \frac{(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}\alpha_1 V - \frac{(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}\alpha_2 V \\ &- \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV.\end{aligned}$$

The first part is the share of the prize the agent gets weighted by the winning probability. The second part represents the fine and the last part are the costs of effort invested by the agent.
We conclude that principal one does not want to pay more than necessary to his agent. Accordingly, he only ensures that the payoff of the delegate is zero, no more, no less. Thus,

$$\begin{aligned}\frac{(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}\alpha_2 V &= +\frac{(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)}\alpha_1 V \\ &- \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV.\end{aligned}$$

Applying this to the payoff function of principal one yields

$$\pi_{P1} = \frac{(\alpha_1+\alpha_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)} V - \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV.$$

Note, that any principal cannot observe the effort invested by any delegate directly. But by assuming rational behavior of the delegate and knowledge of the fact that all individuals maximize their material payoff, any principal can conclude how the delegates react to the terms of the contract. Accordingly, the design of the contract is used to provide the incentives most useful to the principal. The principal knows the fine he gets paid and the reward that he has to pay. The delegate has to be compensated for the fine, but the reward reduces the compensation. Accordingly, both cancel out if they are applied to the problem of the principal. In expectation the principal only has to compensate the delegate for the effort invested.
By introducing a fine, it is possible for the principal to offset the reduction in incentive due to the split up of the prize.
By substituting $(\alpha_1+\alpha_2)$ by a and $(\beta_1+\beta_2)$ by b, the problem of principal one reduces to

$$\pi_{P1} = \frac{a^r}{a^r+b^r} V - \frac{a^{r+1}b^r}{(a^r+b^r)^2} rV.$$

By calculating the first derivative and simplifying the first order condition, it can be shown that

$$a^r + b^r - (r+1)a^{r+1} - (r+1)ab^r + 2ra^{r+1} \;=\; 0.$$

Rearranging yields

$$\frac{a^r}{b^r} \;=\; \frac{a(r+1)-1}{a(r-1)+1}. \tag{3.2}$$

Doing the same for the second principal yields

$$\frac{a^r}{b^r} \;=\; \frac{b(1-r)-1}{1-b(r+1)}. \tag{3.3}$$

Setting (3.2) and (3.3) equal yields

$$b \;=\; \frac{a}{2a-1}. \tag{3.4}$$

And a is implicitly defined by

$$(2a-1)^r \;=\; \frac{a(r+1)-1}{a(r-1)+1}. \tag{3.5}$$

Using (3.4) and (3.5), it is possible to state

Lemma 3.4: *The Nash equilibria in the contract game are given by* $(a^*, b^*) = \left(a^*, \frac{a}{2a^*-1}\right)$*. Where* a^* *is determined by (3.5).*

According to Lemma 3.4 we can state that there is a symmetric equilibrium that is independent of r. In fact, $a = 1$ always solves equation (3.5) for any r. Another result is that for $r = 1$ any a solves the equation. We will be concerned with this case in Section 3.3.4 Whether there are asymmetric equilibria or not, depends on r. For the symmetric equilibrium we can state

Theorem 3.1: *In the symmetric Nash equilibrium any principal incentivizes his agent with the whole prize.*

The proof is straightforward and therefore omitted[3]. In essence, it is optimal for any principal to use a contract that incentivizes the agent with the whole prize. The principal tries to put the agent in the same situation he would be in. That means

[3]Using the fact that we are searching for a symmetric equilibrium, we can set (3.2) equal to (3.3). This leads to $a = b = 1$ because of symmetry.

that winning the prize is as important for the agent as it would be for the principal if he were fighting himself. Selling the right to participate is no preferred option in this setting. Whether the right is sold or not, has no influence on the efforts invested by the delegate because in both cases his incentives do not differ.
Konrad (2009) shows that for observable effort choices the principal can incentivize the agent to invest as much as he would. Theorem 3.1 shows that this is also the case for unobservable effort choices. By using an appropriate contract, the principal can completely overcome the problem of moral hazard.
In the symmetric equilibrium any agent invests $\frac{rV}{4}$ to win the contest. In a contest with no-win-no-pay contracts a delegate invests $\frac{r^2}{(2+r)}\frac{V}{4}$. Accordingly, an agent is more aggressive and therefore invests more when there is a fine for a defeat.

To determine the values for the fine and the reward that are chosen by the principal, we use the participation constraint.

$$\begin{aligned}\frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_1 V &= \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_2 V \\ &+ \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV.\end{aligned}$$

In expectation the payment to the delegate is as high as the fine that the agent has to pay and the costs of effort he makes.

Because this is the symmetric equilibrium and $\alpha_1+\alpha_2=1$, we get

Theorem 3.2: *The shares used by the principals in the symmetric equilibrium contracts are*

$$\begin{aligned}\alpha_1 = \beta_1 &= \frac{(2+r)}{4} \text{ and} \\ \alpha_2 = \beta_2 &= \frac{(2-r)}{4}.\end{aligned}$$

It is obvious that the reward is always greater than the fine. If the reward is greater than $\frac{(2+r)}{4}$, the delegate has a positive payoff in expectation. But if the reward is smaller, then the agent has a negative payoff. α_1 and β_1 are greater than $\frac{1}{2}$ for every r and they increase in r. α_2 and β_2 are always smaller than $\frac{1}{2}$ and they decrease in r. The reason for this are the incentives for the principal. If r is very low, investing more effort than the opponent has only a small effect on the winning probability. For very low values of r, the determination of the winner is comparable to a coin flip. Accordingly, the principal wants to be prepared for both cases. He offers only a small share as a reward, because the positive effect of more effort is only small and luck plays an important role. The fine is high, because the principal wants to insure himself against losing. If r is high, then investing effort leads to a higher winning probability and the agent has a high incentive to invest effort. And because he invests much, the principal cannot use a high fine because this would lead to a negative expected payoff.

Concerning the payoffs, it can be established

Corollary 3.1: *The material payoff of any principal is given by* $\pi_{P1} = \pi_{P2} = \frac{(2-r)}{4}V$ *in equilibrium. The material payoff of the agent is always zero.*

The principal only ensures the participation of the agent. Accordingly, the payoff for any delegate in equilibrium is zero. Note that the payoff for the principal is as high as his payoff would be in an equilibrium without mandatory delegation. Suppose that mandatory delegation is not assumed anymore. Whether a delegate is hired or not, has no effect on the expected payoff of the principal[4]. Accordingly, any principal is indifferent between delegating or playing the contest himself. This may lead to a situation where only one principal delegates or no principal delegates, i.e. the assumption of mandatory delegation can be relaxed.

Note that if the contract space is unlimited, agents act as aggressive as principals would do and it is therefore not possible to observe a reduction in wasteful investments by introducing mandatory delegation. The efforts invested by the players can only be decreased if the number of possible contracts is decreased. This would lead to higher payoffs for the principals and can also lead to higher payoffs for the delegates. How these restrictions affect the equilibrium is investigated in the next section.

Concerning the asymmetric equilibria, if they exist, we can state that a principal sets the total incentive greater than 1 if the other principal sets the total incentive smaller than 1. One property of these equilibria is that the sum of both total incentives of the principals is greater than 2. In the asymmetric equilibria they act overall even more aggressive. If a principal is confronted with an opponent with a rather low incentive scheme, he wants to use this weakness by setting an even

[4]If an aditional stage is introduced to account for the decision on delegating or not, we have four subgames: no principal delegates, both principals delegate and only one principal delegates. It is straightforward to show that setting a total incentive of one is an equilibrium even if the opponent does not delegate. The material payoff for both principals does not differ compared to the no delegation scenario. On the one hand the non-delegation principal is confronted with an agent that is as incentivized as the other principal is. And on the other hand the delegating principal made the agent act as he would do himself.

higher incentive for his agent. An interesting case where it is possible to see such a behavior is the case of $r = 1$.

3.3.4 The case of $r = 1$

Setting $r = 1$ in (3.2) and (3.3) yields for both conditions

$$b = \frac{a}{2a-1}. \tag{3.6}$$

Accordingly, the best response curves are completely identical[5]. Therefore, there is one symmetric equilibrium and there are multiple asymmetric equilibria.
In the symmetric equilibria a=b=1 holds and $\pi_{P1} = \pi_{P2} = \frac{V}{4}$. The asymmetric equilibria are characterized by $\left(a, \frac{a}{2a-1}\right)$.
Now assume, without loss of generality, that $a > b$. According to (3.4) this is true for $a > 1$. It also holds that

$$a + b > 2.$$

The sum of incentives in an asymmetric equilibrium is always higher than in the symmetric equilibrium.
The payoffs for both principals are given by

$$\pi_{P1} = \frac{2a-1}{4a}V$$
$$\pi_{P2} = \frac{1}{4a}V.$$

It is easy to check that

$$\pi_{P1} > \frac{V}{4} > \pi_{P2}.$$

Therefore, the more aggressive principal has a higher payoff than the principal with the weaker incentive scheme.
Suppose that one principal uses a low incentive for his agent. In this case the other principal increases the incentives for his principal to make profit out of this weak incentive scheme.

[5]Leininger (2009) searching for evolutionarily stable preferences derived a similar result in his model. He concentrates on the relevant symmetric equilibrium.

3.4 Limited reward and/or fine

Principal 1 has to solve

$$\begin{aligned}
\max_{\alpha_1,\alpha_2} \quad & \pi_{P1} \\
\text{wrt} \quad & \pi_{D1} \geq 0 \\
& \bar{\alpha} - \alpha_1 \geq 0 \\
& \underline{\alpha} - \alpha_2 \geq 0.
\end{aligned}$$

$\bar{\alpha}$ represents the upper bound for the reward and $\underline{\alpha}$ the upper bound for the fine. All constraints can be binding or not. That means we have to consider the following cases:

Case	c_1	c_2	c_3
1	+	+	+
2	+	+	0
3	+	0	+
4	+	0	0
5	0	+	+
6	0	+	0
7	0	0	+
8	0	0	0

c_1 stands for the participation constraint, c_2 for the upper bound constraint for the reward and c_3 for the upper bound constraint for the fine. A "+" indicates that the constraint is binding. We will concentrate on the symmetric equilibrium.
Case 6 can be excluded because the principal has an incentive to increase the fine. An increase in the fine increases the incentives for the agent and increases the expected payoff for the principal. This upward shift continues until the participation constraint or the upper bound of the fine (or both) are met. Using this argument Case 8 can also be excluded. Case 4 is examined in Section 3.3.3. We can also exclude Case 3. This case is equivalent to Case 4. The principal does not want to increase the reward. An increase would increase the payoff for the agent and therefore the participation constraint would not be binding anymore. The only case where the upper bound of the fine is reached, the principal does not want to increase the reward and the participation constraint is binding is when the chosen fine in Section 3.3.3 is equal to the upper bound.
The participation constraint is binding if

$$\alpha_1 = \frac{2+r}{2-r}\alpha_2.$$

Accordingly, this condition is fulfilled in cases 1 to 4. In the other cases α_1 is greater than the right-hand side. There will be positive profits for the agent in these cases. Of course, for the upper bound constraint for the fine and for the reward to be binding, the shares have to be lower than the values given in Theorem 3.2. The second condition can only be binding as long as $\alpha_1 + \alpha_2 \leq \frac{r}{2+r}$.

In the following, we will concentrate on symmetric equilibria. The results for the maximization problem are given in the following table

Case	α_1	α_2	π_{P1}
1	$\bar{\alpha}$	$\underline{\alpha}$	$(1-\bar{\alpha}+\underline{\alpha})\frac{V}{2}$
2	$\bar{\alpha}$	$\frac{2-r}{2+r}\underline{\alpha}$	$\frac{2+r-2r\bar{\alpha}}{2+r}\frac{V}{2}$
4	$\frac{2+r}{4}$	$\frac{2-r}{4}$	$\frac{2-r}{4}V$
5	$\bar{\alpha}$	$\underline{\alpha}$	$(1-\bar{\alpha}+\underline{\alpha})\frac{V}{2}$
7	$\frac{r}{2+r}-\underline{\alpha}$	$\underline{\alpha}$	$(\frac{1}{2+r}+\underline{\alpha})V$

If the use of the fine is forbidden, the problem given by Case 7 characterizes a no-win-no-pay contract.

An important result of this table is that limiting the set of contracts is beneficial for the principal (and it can also be beneficial for the agent)[6]. The payoff of the principal is lowest in Case 4, i.e. when the restrictions do not affect his contract choice. The payoff of the agent is positive in Cases 5 and 7. In any other case the payoff is zero. Note that in Case 5 and in Case 7 the fine is limited.

Another result is that limiting the reward leads to a higher payoff than limiting the fine does, i.e. the payoff of the principal in Case 2 is higher than in Section 3.3.3. This is due to the participation constraint. The principal has to compensate the agent with a higher reward if he uses a fine. If the reward is limited, the fine is also limited.

Cases 5 and 1 are not equal. For a given fine, the reward in Case 5 is higher than in Case 1.

3.5 Interdependent Preferences

We are concerned with interdependent preferences and their influence on the chosen contract in this section. Common sense tells us that negatively interdependent preferences would also lead to more aggressive behavior. Whether this is true or not is explained below. But before, we have to say a few words about negatively interdependent preferences. In a divorce battle, for example, the negatively interdependent preferences result from the relationship between two individuals. If one of these two has to play a contest with a foreigner, he may not have this kind of

[6]The upper bounds must be strictly smaller than the desired shares given in Theorem 3.2.

preferences anymore. But in Chapter 2 we described the work of Leininger (2009). He derives the result that negatively interdependent preferences are evolutionarily stable in finite populations. Hence, it seems sensible to examine the effects of such preferences in our framework as well. But this opens another problem: Given that negatively interdependent preferences are evolutionarily stable, why don't the delegates, or principals in the other model respectively, also have negatively interdependent preferences?
We will address two possible explanations here. The decision about the plausibility of either one is left to the reader.
One answer is that we have two populations. In only one population evolution led to negatively interdependent preferences. And the same government that introduced mandatory delegation also obliged that one has to choose a delegate from the population with individualistic preferences.
Another answer could be that the delegates are only hired and have no personal concern for the prize at stake. A soldier that has to defend his hometown is more incentivized than a mercenary fighting on the same battlefield, for example. This could also be an explanation why a delegate may or may not share the principal's negatively interdependent preferences. For a principal it may be a good decision to hire an agent that hates the agent of the opponent. Common sense tells us that an agent with negatively interdependent preferences will invest more.

3.5.1 Principals with negatively Interdependent Preferences

For simplicity, let us assume that both principals have negatively interdependent preferences. δ, with $0 \leq \delta \leq 1$, represents the degree of negatively interdependent preferences. δ is the same for both principals. $\delta = 0$ holds for an individualistic player. The higher δ is, the higher are the concerns for the payoff of the opponent. For $\delta = 1$, an individual is interested in the difference between his payoff and the payoff of his opponent. Accordingly, he has relative preferences. If an evolutionary setting is employed, $\delta = \frac{1}{N-1}$ holds. N stands for the number of individuals in the population.
The preferences of the agents and the set of possible contracts are not changed.

Accordingly, the results at the last stage stay the same:

$$\begin{aligned}
\pi_{P1} &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}(1-\alpha_1)V + \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_2 V,\\
\pi_{P2} &= \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}(1-\beta_1)V + \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\beta_2 V,\\
\pi_{D1} &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_1 V - \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_2 V\\
&- \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV.
\end{aligned}$$

As stated above π_{Pi} ($i = 1, 2$) gives the material payoff for principal i and π_{D1} determines the payoff for the delegate of principal one.
Principal one does not maximize his material payoff given by π_{P1}. Because of the interdependent preferences he maximizes:

$$U_{P1} = \pi_{P1} - \delta\pi_{P2}.$$

And principal one has to meet the participation constraint of his agent, i.e. π_{D1} has to be non-negative in equilibrium.

Theorem 3.3: *The total incentive a principal with negatively interdependent preferences uses in the unique symmetric equilibrium is also one, i.e.* $\alpha_1+\alpha_2 = \beta_1+\beta_2 = 1$.
Proof:

$$\begin{aligned}
U_1 &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}(1-\alpha_1)V + \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_2 V\\
&- \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\delta(1-\beta_1)V - \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\delta\beta_2 V.
\end{aligned}$$

The first order condition with respect to α_2 is given by

$$\begin{aligned}
\frac{\partial U_1}{\partial \alpha_2} &= \frac{r(\alpha_1+\alpha_2)^{r-1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}(1-\alpha_1)V + \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r} V\\
&- \frac{r(\alpha_1+\alpha_2)^{r-1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}\alpha_2 V + \frac{r(\alpha_1+\alpha_2)^{r-1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}\delta(1-\beta_1)V\\
&- \frac{r(\alpha_1+\alpha_2)^{r-1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}\delta\beta_2 V.
\end{aligned}$$

This first derivative is greater or equal to zero, for

$$(\beta_1+\beta_2)^r + (\alpha_1+\alpha_2)^r(1-r) + r(\alpha_1+\alpha_2)^{r-1}(1+\delta-\delta(\beta_1+\beta_2)) \geq 0.$$

This condition is always fulfilled, because $r \leq 2$, the total incentive is smaller or equal to two and $\delta \leq 1$.
Accordingly, principal one wants to increase the penalty, as long as his delegate is willing to participate. To ensure that the payoff of the delegate is non-negative, the principal will use the fine until $\pi_{D1} = 0$.
Using this condition yields

$$\begin{aligned}\frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_2 V &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_1 V \\ &- \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV.\end{aligned}$$

Applying this to U_1 yields

$$\begin{aligned}U_1 &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}V - \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV \\ &- \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\delta(1-\beta_1)V - \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\delta\beta_2 V.\end{aligned}$$

$(\alpha_1+\alpha_2)$ represent the total incentive principal one uses to incentivize his agent. To facilitate our further analysis, we will set $t = (\alpha_1+\alpha_2)$:

$$\begin{aligned}\frac{\partial U_1}{\partial t} &= \frac{rt^{r-1}(\beta_1+\beta_2)^r}{(t^r+(\beta_1+\beta_2)^r)^2}V \\ &- \frac{(r+1)t^r(\beta_1+\beta_2)^r(t^r+(\beta_1+\beta_2)^r)^2 - 2(t^r+(\beta_1+\beta_2)^r)rt^{2r}(\beta_1+\beta_2)^r}{(t^r+(\beta_1+\beta_2)^r)^4} rV \\ &+ \frac{rt^{r-1}(\beta_1+\beta_2)^r}{(t^r+(\beta_1+\beta_2)^r)^2}(1-\beta_1)\delta V - \frac{rt^{r-1}(\beta_1+\beta_2)^r}{(t^r+(\beta_1+\beta_2)^r)^2}\beta_2\delta V \\ &= 0.\end{aligned}$$

That expression reduces to

$$\begin{aligned}&(\beta_1+\beta_2)^r(1+\delta-\delta(\beta_1+\beta_2)) \\ =\ &\frac{(r+1)t(\beta_1+\beta_2)^r(t^r+(\beta_1+\beta_2)^r) - 2rt^{r+1}(\beta_1+\beta_2)^r}{t^r+(\beta_1+\beta_2)^r}.\end{aligned}$$

Setting $t = (\beta_1+\beta_2)$ (symmetry assumption) yields

$$2t^r(1+\delta-t\delta) = t(rt^r+t^r+t^r-rt^r).$$

And therefore $t = 1$.
We excluded altruism ($\delta < 1$) and therefore $t = 1$ is the unique symmetric solution.

■

The negatively interdependent preferences are supposed to increase spiteful behavior. And indeed they increase the incentive to win the contest for the principal. But the participation constraint is a barrier that keeps the total incentive from rising. In the setting with individualistic preferences, the payoff of the delegate was already zero. That means the delegate already invested everything the contract allowed him to. Accordingly, increasing α_2 is not possible for principal one.
An increase of α_1 also does not pay. A higher α_1 decreases the share he gets if the delegate wins and this case happens with a higher probability. This increase would also allow a higher fine, but the case in which he gets paid the fine occurs with a lower probability. On the other hand, a higher α_1 and α_2 decreases the probability of winning for the opponent. But because of the fine, the effect on the material payoff of the opponent is mitigated. In the symmetric equilibrium all these effects cancel each other out. Accordingly, it is still optimal to use a total incentive of one in equilibrium.
As can be seen, delegation can indeed be an adequate mean to decrease wasteful investments in contests. Negatively interdependent preferences would make the principals act more aggressively in the contest if they had to play it. But the participation constraint in the delegation setting prevents the principal from increasing the incentives by using the terms of the contract. With individualistic preferences, the principals are able to put the agent in the same situation they would be in when playing themselves. But they are not able to transfer the negatively interdependent preferences to their agents. Note that the principals also benefit from this because their material payoff stays the same compared to the case without negatively interdependent preferences. But without delegation it would decrease, and the utility would be the same as in the delegation setting.
The principal has a positive material payoff. By using a fixed payment, the principal could increase the total incentive and compensate the delegate for a possible loss. We will introduce fixed payment F_1 for principal one and F_2 for principal two, in order to show that this is no option. A fixed payment does not change the behavior of the agent directly, but it enlarges the contract space for the principal. Due to changed contract parameters the behavior of the delegate can be changed. With the fixed payment the payoff of the delegate of principal one is

$$\begin{aligned}\tilde{\pi}_{D1} &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_1 V - \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\alpha_2 V \\ &- \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2} rV \\ &+ F_1.\end{aligned}$$

Increasing the fine until the participation constraint is met is still beneficial for the principal. Setting $\tilde{\pi}_{D1} = 0$ and using this condition yields

$$\begin{aligned}\tilde{U}_1 &= \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}V - \frac{(\alpha_1+\alpha_2)^{r+1}(\beta_1+\beta_2)^r}{((\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r)^2}rV \\ &- \frac{(\beta_1+\beta_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\delta(1-\beta_1)V - \frac{(\alpha_1+\alpha_2)^r}{(\alpha_1+\alpha_2)^r+(\beta_1+\beta_2)^r}\delta\beta_2 V.\end{aligned}$$

This is the same function as in the setting without a fixed payment. Accordingly, the result remains unchanged and the principal is still not able to increase the total incentive to more than one. That also means that he is still not able to put the delegate in the same situation he would be in.

3.5.2 Agents with negatively Interdependent Preferences

In this subsection the principals have independent preferences, but the agents have interdependent preferences. To keep the analysis simple, it is assumed that an agent is concerned with the payoff of the other agent. Accordingly, we don't consider the effects of an agent that is concerned with the payoff of the other principal, of his principal[7] or even both. To avoid the strategic choice of an agent's type[8], δ, with $0 \leq \delta \leq 1$, represents the degree of negatively interdependent preferences. δ is the same for both and to ensure comparison, it is the same as in section 3.5.1. There are two effects in this model. The agent will invest more compared to a situation with independent preferences. This may decrease the incentives offered by the principals, because the agents want to beat the opponent. But by using a high incentive, the principal can increase the probability of winning.
The model has two stages. At the first stage the principals choose the contract. The contract chosen determines the effort invested by the agent at the second stage. The agent of principal one maximizes his utility u^I_{D1}. His utility is given by

$$U^I_{D1} = \pi^I_{D1} - \delta\pi^I_{D2}.$$

π^I_{D1} is his material payoff and π^I_{D2} is the material payoff of the other agent. α^I_1 is the share used by principal one as a reward and α^I_2 is the share of the prize that is used as a fine. β^I_1 and β^I_2 are the shares used by principal two. The efforts invested by the agents are denoted by d_1 and d_2. Accordingly, the agent of principal one

[7] Such a kind of model is considered in Chapter 6.

[8] Of course, choosing the agent with the more negatively interdependent preferences may be advantagous for the principal.

maximizes

$$U_{D1}^I = \frac{d_1^r}{d_1^r + d_2^r}\alpha_1^I V - \frac{d_2^r}{d_1^r + d_2^r}\alpha_2^I V - d_1 - \delta\left(\frac{d_2^r}{d_1^r + d_2^r}\beta_1^I V - \frac{d_1^r}{d_1^r + d_2^r}\beta_2^I V - d_2\right).$$

Deriving the first order condition and rearranging it yields

$$(d_1^r + d_2^r)^2 = d_1^{r-1} d_2^r (\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I) rV.$$

Doing the same for the second agent yields

$$(d_1^r + d_2^r)^2 = d_1^r d_2^{r-1} (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I) rV. \tag{3.7}$$

Setting both conditions equal yields

$$d_2 = \frac{(\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)}{(\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)} d_1. \tag{3.8}$$

In the models so far the total incentive of an agent was given by the sum of both shares, i.e. the determined fine and reward. But in this model the total incentive is given by the sum of the own shares and the sum of the shares of the opponent weighted by δ. That means, as long as $\delta > 0$, the incentives for an agent to invest effort are higher in this model compared to the model with agents with independent preferences for any value of the shares used by the principal. But it still holds that the relation of the sum of both shares of both principals determines which delegate invests more.
Using (3.7) and (3.8) yields the equilibrium efforts of the second stage. We can state

Lemma 3.5: *The equilibrium efforts of the agents contingent on the offered shares are given by*

$$d_1 = \frac{(\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^{r+1} (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^r}{((\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^r + (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^r)^2} rV,$$
$$d_2 = \frac{(\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^r (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^{r+1}}{((\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^r + (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^r)^2} rV.$$

Accordingly, the probabilities of winning are

$$
\begin{aligned}
p_1^I &= \frac{(\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^r}{(\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^r + (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^r}, \\
p_2^I &= \frac{(\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^r}{(\alpha_1^I + \alpha_2^I + \delta\beta_1^I + \delta\beta_2^I)^r + (\delta\alpha_1^I + \delta\alpha_2^I + \beta_1^I + \beta_2^I)^r}.
\end{aligned}
$$

Let us turn to the first stage of the game. Any principal maximizes his expected material payoff. The payoff function of principal one is given by

$$\pi_{P1}^I = (1 - \alpha_1^I)p_1^I V + p_2^I \alpha_2^I V.$$

But the principal must still ensure participation of the agent. Without the participation constraint, the principal will only punish his agent. To prevent this, we will assume that any agent will sign the contract only if the expected utility is at least zero. Note that we don't only consider the material payoff. Accordingly, the material payoff can also be negative. The agent with interdependent preferences is, for δ sufficiently high, willing to suffer a loss but only if the opponent suffers a loss that is even greater. A zero material payoff assumption would include situations with negative utility[9]. Of course, no agent that maximizes his utility will sign such a contract. But we have to mention that a negative material payoff for an agent is problamatic, because he loses money by signing such a contract. We will see that this problem can be neglected in equilibrium.
The utility of the agent of principal one is

$$U_{D1}^I = p_1^I \alpha_1^I V - p_2^I \alpha_2^I V - d_1 - \delta p_2^I \beta_1^I V + \delta p_1^I \beta_2^I V + \delta d_2.$$

Setting this expression equal to zero and rearranging it afterwards, yields

$$p_2^I \alpha_2^I V = p_1^I \alpha_1^I V - d_1 - \delta p_2^I \beta_1^I V + \delta p_1^I \beta_2^I V + \delta d_2.$$

Accordingly, the expected material payoff of principal one is

$$\pi_{P1}^I = p_1^I V - d_1 - \delta p_2^I \beta_1^I V + \delta p_1^I \beta_2^I V + \delta d_2.$$

[9]Assume that $\delta = 1$, the material payoff of the first agent to be zero and the material payoff of the second agent to be greater zero.

Using the expressions derived at the second stage, leads to

$$
\begin{aligned}
\pi_{P1}^{I} &= \frac{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r}{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r+(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r}V \\
&- \frac{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^{r+1}(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r}{((\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r+(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r)^2}rV \\
&- \frac{(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r}{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r+(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r}\delta\beta_1^I V \\
&+ \frac{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r}{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r+(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r}\delta\beta_2^I V \\
&+ \frac{(\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^{r+1}}{((\alpha_1^I+\alpha_2^I+\delta\beta_1^I+\delta\beta_2^I)^r+(\delta\alpha_1^I+\delta\alpha_2^I+\beta_1^I+\beta_2^I)^r)^2}\delta rV.
\end{aligned}
$$

The first two terms represent the benefit created by the contest for principal and agent. The last three terms represent the concern of the agent for the material payoff of his opponent. We can see that the principal cares for the material payoff of the other agent, although he has independent preferences. The reason for this is that these preferences influence the behavior of his agent and therefore he has to take these preferences into account.

Note that for principal one only the sum of both shares used by him matters. Accordingly, we will set $(\alpha_1^I+\alpha_2^I)=a^I$. When possible, we will set $(\beta_1^I+\beta_2^I)=b^I$.

The first order condition with respect to a is given by

$$
\begin{aligned}
0 &= \frac{r(a^I+\delta b^I)^{r-1}\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^2}V \\
&- \frac{\left(r(a^I+\delta b^I)^{r-1}+r\delta(\delta a^I+b^I)^{r-1}\right)(a^I+\delta b^I)^r}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^2}V \\
&- \frac{((r+1)(a^I+\delta b^I)^r(\delta a^I+b^I)^r)\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)^2}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}rV \\
&- \frac{(r\delta(\delta a^I+b^I)^{r-1}(a^I+\delta b^I)^{r+1})\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)^2}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}rV \\
&+ \frac{2\left((a^I+\delta b^I)^r\right)\left(r(a^I+\delta b^I)^{r-1}+r\delta(\delta a^I+b^I)^{r-1}\right)(a^I+\delta b^I)^{r+1}(\delta a^I+b^I)^r}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}rV \\
&+ \frac{2(\delta a^I+b^I)^r\left(r(a^I+\delta b^I)^{r-1}+r\delta(\delta a^I+b^I)^{r-1}\right)(a^I+\delta b^I)^{r+1}(\delta a^I+b^I)^r}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}rV \\
&- \frac{r\delta(\delta a^I+b^I)^{r-1}\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^2}\delta\beta_1^I V \\
&- \frac{\left(r(a^I+\delta b^I)^{r-1}+r\delta(\delta a^I+b^I)^{r-1}\right)(\delta a^I+b^I)^r}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^2}\delta\beta_1^I V \\
&+ \frac{r(a^I+\delta b^I)^{r-1}\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^2}\delta\beta_2^I V \\
&- \frac{\left(r(a^I+\delta b^I)^{r-1}+r\delta(\delta a^I+b^I)^{r-1}\right)(a^I+\delta b^I)^r}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^2}\delta\beta_2^I V \\
&+ \frac{(r(a^I+\delta b^I)^{r-1}(\delta a^I+b^I)^{r+1})\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)^2}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}\delta rV \\
&+ \frac{((r+1)\delta(a^I+\delta b^I)^r(\delta a^I+b^I)^r)\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)^2}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}\delta rV \\
&- \frac{2(a^I+\delta b^I)^r(\delta a^I+b^I)^{r+1}\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)\left(r(a^I+\delta b^I)^{r-1}\right)}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}\delta rV \\
&- \frac{2(a^I+\delta b^I)^r(\delta a^I+b^I)^{r+1}\left((a^I+\delta b^I)^r+(\delta a^I+b^I)^r\right)r\delta(\delta a^I+b^I)^{r-1}}{((a^I+\delta b^I)^r+(\delta a^I+b^I)^r)^4}\delta rV.
\end{aligned}
$$

We are searching for a symmetric equilibrium. Accordingly, we set $a = b = c$. It results

$$
\begin{aligned}
0 &= (1-\delta) - (r+1)(c+\delta c) + r(c+\delta c) - 2\delta^2\beta_1 + (1+\delta)\delta\beta_1 + 2\delta\beta_2 \\
&- (1+\delta)\delta\beta_2 + \delta(c+\delta c)(r+\delta(r+1)) - \delta(c+\delta c)r(1+\delta).
\end{aligned}
$$

Which reduces to

$$
0 = (1-\delta) - (c+\delta c) + (1-\delta)\delta c + \delta^2(c+\delta c).
$$

Accordingly, we can state

Lemma 3.6: *The symmetric Nash equilibrium in the contract game is given by* $a^* = b^* = \frac{1}{\delta^2+\delta+1}$.

We can see that as long as $0 \leq \delta \leq 1$ holds, $\frac{1}{\delta^2+\delta+1}$ is falling in δ. Accordingly, the total incentive set by the principals is highest for $\delta = 0$. We have seen that in this case the total incentive is given by one. That means that the principal benefits from the negatively interdependent preferences of his agent. The principal pays less to an agent with negatively interdependent preferences. Because he is only interested in his monetary payoff, he does not want to defeat the other principal at any cost. He therefore pays less, because the agent is intrinsically motivated by the wish to defeat the other agent. Note that this result does not depend on r.
Another striking result is that the total incentive is $\frac{1}{3}$ when δ is equal to one. Note that $\frac{1}{3}$ is the share of the prize used by a principal, if $r = 1$, only no-win-no-pay contracts are allowed and we are not confronted with negatively interdependent preferences. That means the total incentive in a model with the new kind of contracts and negatively interdependent preferences is always higher. Although the principals benefit from the negatively interdependent preferences, they still use a relatively high share of the prize to incentivize their agent.
For the effort invested by any agent, it holds

$$d_1^* = d_2^* = \frac{(1+\delta)rV}{4(\delta^2+\delta+1)}.$$

In the symmetric equilibrium of the model without negatively interdependent preferences the effort invested by an agent is $\frac{rV}{4}$. Accordingly, the effort invested by an agent is smaller if he has negatively interdependent preferences. We have two effects. On the one side, the agent invests more because of the wish to defeat the opponent, but, on the other hand, the principal reduces the incentive for the agent and therefore investment falls. The latter effect dominates the first effect. The prize has to be split up between principal and agent. But the share offered to the agent is so small, that, although he hates the opponent, invests less.
Using the zero-utility condition and the result in Lemma 3.6, we can state

Theorem 3.4: *The shares used by a principal in the symmetric equilibrium contract are*

$$\begin{aligned} \alpha_1^{I*} &= \beta_1^{I*} = \frac{2+(1+\delta)r}{4(\delta^2+\delta+1)} \quad \textit{and} \\ \alpha_2^{I*} &= \beta_2^{I*} = \frac{2-(1+\delta)r}{4(\delta^2+\delta+1)}. \end{aligned}$$

We can see that both shares decrease if δ increases. The reduction in total incentive used by the principal is therefore due to a reduction in the fine and a reduction in the reward.
Concerning the payoffs, we can state

Corollary 3.2: *The material payoff of any principal in equilibrium is given by* $\pi_{P1}^I = \pi_{P2}^I = \frac{(2\delta^2+2\delta+2-r-r\delta)}{(\delta^2+\delta+1)}\frac{V}{4}$. *The material payoff and the utility of any agent in equilibrium are both zero.*

Compared to the result of the contract choice game without negatively interdependent preferences, we can state that if $\delta > 0$, i.e. the agents do indeed have negatively interdependent preferences, the material payoff of the principal is higher. The principal uses the intrinsic motivation of the agent to increase his share of the pie. You have to pay more to a mercenary to fight for you, than you have to pay to a person that hates your opponent (or his soldiers) for some reason.
The material payoff of the agent is zero. Accordingly, we are not confronted with the problem of a negative material payoff. Although the agent has negatively interdependent preferences, he has no disadvantage: His material payoff does not decrease.

3.6 Conclusion

Wärneryd (2000) showed that mandatory delegation may decrease efforts invested in a two-player Tullock rent-seeking contest. Because the contested prize is split up between principal and agent, the agent does not act as aggressively as the principal would do. Accordingly, efforts made are reduced and expected payoffs for the principals are increased. This holds true for no-win-no-pay contracts. In this paper it is shown that no-win-no-pay contracts are only a subset of a broader class of contracts. In fact, a fine and a reward, both depending on the contested prize, are introduced. Any principal can punish his delegate for a defeat. An example is found in Yubitsume by the Yakuza, contractual penalties, the hiring of lawyers etc. The agent acts more aggressive if he is in danger of losing the invested effort and

some extra money, compared to only losing the effort put in. This fine also prevents moral hazard by the agent. The probability of shirking is reduced if a defeat is very bitter.
In a no-win-no-pay contract only a reward is used.

In the symmetric equilibrium with risk neutral agents and principals both principals put their delegate in the same situation they would be in without mandatory delegation. In fact, the share offered as a reward and the share used as a fine must add up to one. The sum of both shares determines the total incentive of the agent because this is the difference between winning and losing. For any principal this is also one because either he wins the contest, or he loses it. The expected payoff for the agents in equilibrium is always zero, but the payoff of the principal is positive and is as high as in a situation without delegation. In fact, whether a principal delegates or not does not alter the payoff in equilibrium. Accordingly, the principals are indifferent between delegating or not. In the asymmetric equilibria, if they exist, one agent is incentivized with more than the complete prize and the other one with less. In total the sum of incentives is greater than two times the prize. If a principal is confronted with a principal using a weak incentive scheme, he wishes to overrun the enemy by incentivizing his agent over proportionally compared to the prize at stake.
The efforts invested in the contest only decrease, as Wärneryd stated, as long as the contract space is limited. That means, upper bounds for penalties and for rewards have to be introduced. By introducing a fine, it is possible for the principals to contract a delegate by using more aggressive contracts. An agent contracted with the new type acts as a principal would do and, as Wärneryd (2000) showed, has therefore an advantage compared to a delegate with a no-win-no-pay contract. The positive effects of delegation are mitigated by more aggressive contracts. If only no-win-no-pay contracts are allowed, then a principal using a fine to contract an agent has an advantage. But he has to fear detection. Introducing mandatory delegation alone is not beneficial, but introducing mandatory delegation and restricting the possible contracts combined with law enforcement is. If the contract space is limited, for example by law, then wasteful investments in contests can be reduced. An example is the lawyers' compensation act (Rechtsanwaltsvergütungsgesetz) in Germany. By introducing upper bounds, even a positive payoff for the agent is possible.
Another important result is the influence of interdependent preferences on the side of the principal on the chosen contract. The incentives of a principal with negatively interdependent preferences to win the contest are higher than for a principal with individualistic preferences. But the participation constraint keeps the spiteful

principal from transferring the increased incentive to the agent. Accordingly, the negative effect of interdependent preferences is mitigated and delegation can indeed decrease efforts invested, even with the increased contract set. In a divorce battle both parties want their lawyer to feel the pain they do and to act as he would be in the same situation. But the lawyer is only interested in his material payoff, and he has no advantage from hurting himself, just to hurt the opponent even more.
Delegation can also decrease the effort invested if agents have negatively interdependent preferences. Because the agent is intrinsically motivated to invest efforts, the principal decreases the total incentive. He uses a lower fine and a lower reward. This reduction is high enough to make the agents invest less compared to a situation without agents with negatively interdependent preferences. The principal benefits from this reduction, because his material payoff increases. But the agent does not benefit. He still has a material payoff of zero.
Delegation leads to a split up of the contested prize between agent and principal. If the agent has negatively interdependent preferences, the principal can increase his share, because the agent is intrinsically motivated. But this motivation makes the agent spent his share in the contest.

It is an interesting endeavor to investigate empirically whether individuals choose contracts that include punishments or not, and which implications this has on the behavior of the agents. Also the influence of risk aversion on the delegation decision and on the chosen contracts seems to be an interesting topic.

Chapter 4

In defense of lawyers II: Delegation as an Aid against too aggressive behavior.

4.1 Introduction

In this chapter we want again show that delegation is beneficial. In order to show the benefits of delegation, it is assumed that the players in the contest have negatively interdependent preferences. This assumption can also be justified by the fact that individuals are to some extent concerned with their relative payoff. As Riechman (2007) states, this might be the case due to the imitation of successful opponents or a payment scheme used by firm owners to pay the manager in the rent-seeking contest. Riechmann also states that experimental evidence supports the significance of relative payoffs. We also examined evolutionary reasons in Chapter 2.
In this model, two players will compete for a single indivisible prize, and they are each obliged to hire a delegate who has to compete for them. We use the well-known Tullock Contest-Success-Function (CSF) to determine the winner. But we will use a different way of modelling the contest than we used in Chapter 3. In the previous chapter we assumed that the principals can choose the fine and the reward without any restriction. But in this chapter we will use a model where the principals have to choose between two types of contracts. Accordingly, the contest has three stages. At the first stage, the principals simultaneously choose the contracts they wish to use. They then contract a delegate and truthfully announce the terms of the contract to the other principal and to both delegates. As Katz (1991) showed, it is possible to assume unobservability of contracts without changing the results. At the third stage, the actual contest between the delegates takes place. A principal has the choice of two kinds of contract. The first offers a share of the prize

to incentivize the delegate. Such contracts have been used previously in literature, by Baik and Kim (1997) or Baik (2007) for example. As before, we will call these types of contracts no-win-no-pay contracts. The second type consists of a payment that is made conditionally upon the agent's success in the contest. Neither delegate is involved in the conflict before he is contracted and maximizes his individualistic payoff. Suppose there are two groups of agents. One of these groups consists of players that maximize weighted relative payoff, irrespectively of the reason which may have caused this behavior. In the other group the individuals maximize their absolute payoff. The delegates are hired from the latter group to act in a conflict between two individuals from the first group. In Germany, for example, a lawyer has to be hired in divorce proceedings. The spouses may be driven by the maximization of their relative payoff but normally the lawyers are not.
We will show that both contracts make the principals at least as successful as players that maximize their absolute payoff in a setting without delegation. If the agent is paid according to relative success, the principal has to pay a fixed sum to hire an agent. But a prisoners' dilemma-like game-structure will prevent moral hazard in the relationship between the agent and the principal. Accordingly, the second contract used here differs in to important aspects from the contract in the third chpater. On the one hand, we introduce fixed payments into the analysis and will therefore be able to explain there existence. Note that in Chapter 3 they had no effect. But in this model we will show that they are necessary. On the other hand, we will be confronted with contracts that reward the agent according to his relative success, i.e. whether the agent is in expectation more successful than his opponent.

The chapter is structured as follows. First of all, we will recall the equilibrium outcomes without delegation within a population of absolute and relative payoff maximizing players as a benchmark in Section 4.2. In Section 4.3, we will establish the model with both kinds of contracts. The contract choice game is solved in Section 4.4 before the conclusion in Section 4.5.

4.2 Rent-seeking without delegation

This paper examines Tullock's (1980) contest with two opponents. Both opponents make irreversible effort to win the indivisible prize V. The valuation is the same for both. Since no delegation takes place, there is only one stage. The winner is determined using the common Tullock Contest-Success-Function (CSF) where $r = 1$ (constant marginal efficiency of effort as it is called by Guse and Hehenkamp (2006)). Accordingly, the probability of player i, $i \in \{1, 2\}$, winning the prize is given by

$$p_i = \begin{cases} \frac{x_i}{x_i+x_{-i}} & \text{for } x_i + x_{-i} > 0 \\ \frac{1}{2} & \text{for } x_i + x_{-i} = 0. \end{cases}$$

4.2.1 Maximizing absolute payoff

Every principal strikes for his own benefit, irrespectively of group size and outcomes for the other players. Accordingly, the expected utility of player i is given by

$$\pi_i = \frac{x_i}{x_i + x_{-i}} V - x_i.$$

Deriving the first order condition for player i yields

$$\frac{\partial \pi_i}{\partial x_i} = \frac{x_{-i}}{(x_i + x_{-i})^2} V - 1 \overset{!}{=} 0.$$

In equilibrium we get

$$\begin{aligned} x_1 &= x_2 = \frac{V}{4} \\ \pi_1 &= \pi_2 = \frac{V}{4} = \pi. \end{aligned}$$

4.2.2 Maximizing weighted relative payoff

An arbitrary player j, $j \in \{1, 2\}$, is now concerned with a weighted relative payoff. The term relative is used because the utility of a player not only depends on his own material payoff but also on the material payoff of the opponent weighted by δ, with $\delta \in [0, 1]$. Here, δ states whether a player is an individualistic player or not. If $\delta = 0$, a player is only concerned with his individualistic payoff. But $\delta = 1$ indicates that a player is maximizing his relative payoff. Note that in Leininger's (2009) derivation $\delta = \frac{1}{N-1}$, if preference evolution takes place in a population of size N, i.e. $\delta = 1$ for $N = 2$. The effort invested by player j is given by y_j. Player j maximizes:

$$u_j = \frac{y_j}{y_j + y_{-j}} V - y_j - \delta \frac{y_{-j}}{y_j + y_{-j}} V + \delta y_{-j}.$$

By deriving the first-order conditions, it results that

$$y_j = y_{-j} = (1 + \delta) \frac{V}{4}.$$

Since the winning probability is $\frac{1}{2}$ for all players, the material payoff of any player is

$$\pi_R = (1-\delta)\frac{V}{4},$$

the corresponding weighted relative payoff is

$$u_R = (1-\delta)^2\frac{V}{4}.$$

Note that there is full dissipation if $\delta = 1$, as both players then maximize relative payoffs. But if δ comes close to zero, the utility function converges to the individualistic payoff function. As δ decreases, the concern regarding the other player's payoffs becomes smaller and smaller. Accordingly, as can be seen from the above formulas $u_R = \pi_R = \pi = \frac{V}{4}$ from section 4.2.1 for $\delta = 0$. But for $\delta > 0$ it holds that $\frac{V}{4} > \pi_R$. If $0 < \delta < 1$, it holds that $\pi_R > u_R$. Because of the negatively interdependent preferences the utility is smaller than the material payoff. The perceived state of the world is inferior to the actual state of the world. This does not hold for relative payoff maximization, i.e. $\delta = 1$. In that case utility and material payoff is zero. This is due to the fact that the payoff of the opponent is also zero.

4.3 The Model

We will now address contests with mandatory delegation. Suppose there is a group with negatively interdependent preferences. It is common knowledge that this preference structure induces aggressive and therefore spiteful behavior in contests. Since more aggressive behavior yields an advantage compared to absolute payoff maximization, no player has an incentive to change his behavior. But to compensate for the disadvantages, mandatory delegation is introduced by law, for example. For the purpose of simplicity, it is assumed that only two-player contests are allowed to take place. The reason why the principals are assumed to hire a delegate is that delegation by both principals is not an equilibrium. The contested prize has to be split up between the delegate and the principal. The incentives for the principal acting for himself are therefore higher than the incentives for a delegate would be. The principal invests more and is therefore more successful. In equilibrium, all principals prefer not to delegate and have a lower material payoff, although the utility does not change. There are many real life examples for mandatory delegation. In Germany it is mandatory to hire a lawyer for legal proceedings that take place in the so-called Landesgericht (regional court) and higher courts. In other words, if the contested "rent" is sufficiently high, delegation to a lawyer is mandatory (in Germany).

Both principals have negatively interdependent preferences. It is worth mentioning that this also ensures participation on the part of the principals: The individuals experience negative utility if they do not invest, since the opponent then wins for sure. As before, δ (with $\delta \in [0, 1]$) denotes the player's concern for the material payoff of his opponent. For the purpose of simplicity, it is assumed that both players have the same concern for their opponent, i.e. δ is constant and the same for both principals.

The game has three stages. At the first stage, both principals choose a contract simultaneously. At stage II, the principals contract a delegate. At stage III, both delegates then engage in the contest and exert efforts. Finally, the winner is determined, and the prize is handed over. Note that the principals cannot observe the effort of both delegates. They find out whether their delegate won or not, and they infer the objective functions of their opponent and of the agents. Both principals can choose between two types of contracts: No-win-no-pay contracts and so-called relative contracts defined below.

First of all, the model is examined with both principals using no-win-no-pay contracts. The delegate obtains part of the prize but only if he wins. After that the model is defined with a payment to delegates in case of success and a forfeit in case of a defeat. This results in relative contracts. The idea behind these contracts is that the other-regarding preferences of a principal might induce the corresponding delegate to act according to the other-regarding preferences via these contracts. These contracts also act as "defeat insurance" for the principal. We will then examine the asymmetric case, namely where one principal uses a no-win-no-pay contract and the other uses the alternative contract.

In all cases the efforts invested by a delegate are denoted by d. A subscript indicates the principal the delegate is working for. A superscript refers to the case under examination.

4.3.1 No-win-no-pay contracts

Baik (2007) states that it is optimal to use no-win-no-pay contracts for absolute payoff maximizing principals if they have to hire a delegate. Note that no payment is made if the contest is lost, neither positive nor negative. Only if the contest is won, the agent will get a share of the contested prize. The share principal l is using as an incentive is denoted with α_l, where $\alpha_l \in [0, 1]$ and $l \in \{1, 2\}$. We are ruling out the possibility of selling the right to participate to another agent. If the right is sold, no delegation will take place but the principal alone will be substituted by the buyer. It is shown that we can rule out this case without loss of generality. Negative amounts as a fixed part of compensation are excluded here because a fixed

payment does not change the behavior of the delegate and therefore the equilibrium strategies. In Chapter 3 this was shown for a special case. Another result of that chapter is that delegation with no-win-no-pay contracts is beneficial for a principal even without a negative fixed payment for the delegate.
The utility function of principal l is given by

$$u_{Pl}^{A} = \frac{d_l^A}{d_l^A + d_{-l}^A}(1-\alpha_l)V - \delta\left(\frac{d_{-l}^A}{d_l^A + d_{-l}^A}\right)(1-\alpha_{-l})V.$$

The first term represents the expected material payoff received by the principal. He does not have to bear any expenses directly. On account of this, he only gets the remaining share of the contested prize in expectation. The second term reflects the principal's concern for his relative position. It is the expected material payoff of his opponent (after delegation) weighted by δ.
The delegate is rewarded according to the following payoff function:

$$\pi_{Dl}^{A} = \frac{d_l^A}{d_l^A + d_{-l}^A}\alpha_l V - d_l^A.$$

The reservation wage of the delegate is normalized to zero, so as long as the expected payoff for the delegate is negative, no rational agent will sign this contract.
The model is solved by using backward induction, starting at stage III. In order to determine the optimal effort, we have to derive the first-order conditions

$$\begin{aligned}\frac{\partial \pi_{D1}^A}{\partial d_1^A} &= \frac{d_2^A}{(d_1^A + d_2^A)^2}\alpha_1 V - 1 \overset{!}{=} 0,\\ \frac{\partial \pi_{D2}^A}{\partial d_2^A} &= \frac{d_1^A}{(d_1^A + d_2^A)^2}\alpha_2 V - 1 \overset{!}{=} 0.\end{aligned}$$

By solving these equations for d_1^A and d_2^A, it is possible to calculate the winning probability for any principal l:

$$p_l^A = \frac{\alpha_l}{\alpha_l + \alpha_{-l}}.$$

Accordingly, the probability is given by the ratio of the offered share and the sum of all shares, i.e. if a principal used a greater part of the prize to incentivize his agent than his opponent did, he will also win the contest with a higher probability.

We will now continue by analyzing the second stage. Principal l chooses the offered share α_l such that he maximizes his utility

$$\max_{\alpha_l} \quad \left\{ u_{Pl}^A = \frac{\alpha_l - \alpha_l^2}{\alpha_l + \alpha_{-l}} V - \delta \frac{\alpha_{-l} - \alpha_{-l}^2}{\alpha_l + \alpha_{-l}} V \right\}.$$

Focusing on a symmetric equilibrium, let $\alpha_1 = \alpha_2 = \alpha$. The first order condition of any of the principals can be rewritten as

$$\alpha - 3\alpha^2 + \delta(\alpha - \alpha^2) \;\; = \;\; 0.$$

By solving this for α, we obtain

$$\alpha \;\; = \;\; \frac{(1+\delta)}{(3+\delta)}.$$

$\alpha = 0$ is discarded here as a solution. The chosen α of a player with other-regarding preferences is $\frac{1}{2}$ for $\delta = 1$ and $\frac{1}{3}$ for $\delta = 0$. An absolute payoff maximizing player would use one third of the contested prize to incentivize his delegate. We can therefore state that a player with other-regarding preferences will incentivize his delegate more than an absolute payoff regarding principal. He therefore acts spitefully.
It is established:

Lemma 4.1: *The utility of the principals in equilibrium is given by*

$$u_{P1}^A = u_{P2}^A \;\; = \;\; \frac{(1-\delta)}{(3+\delta)} V \;\; = \;\; u^A.$$

The underlying material payoff that determines the fitness of the principals in evolutionary terms is

$$\pi_{P1}^A = \pi_{P2}^A \;\; = \;\; \frac{V}{(3+\delta)} \;\; = \pi_P^A.$$

A delegate in this conflict has an equilibrium payoff of

$$\pi_D^A \;\; = \;\; \frac{(1+\delta)}{(3+\delta)} \frac{V}{4} \;\; = \;\; \frac{\alpha V}{4} \;\; > \; 0.$$

The material payoff of both principals is positive in equilibrium for any value of δ, though they experience zero utility for $\delta = 1$.
Note that $\pi_P^A > \pi_R$ for $\delta < 1$. Delegation is therefore beneficial for the principals in material terms because the expenditures in the contest are reduced and the winning

probability remains unchanged since we are in a symmetric equilibrium. Through introducing delegation with no-win-no-pay contracts, the prize has to be split up between the agent and the principal. The incentives for a delegate to make effort and for a principal to incentivize the agent are therefore always lower than without delegation. Exerting effort also becomes more expensive because the principal has to pay the delegate to put in more effort, yet only a fraction of this extra incentive is expended. The reason for this is, that the expected payoff for a delegate is strictly positive because α is assumed to be non-negative. This result is in line with the writings of Baik (2007), who predicted positive profits in the "delegate industry" for contests between absolute payoff maximizing principals. As in the article by Baik, positive profits arise from strategic decisions by the principals. The principals try to put their agents into a situation similar to that which they would be in themselves without delegation. This is achieved by using αV as the prize in a new contest between the delegates. According to Section 4.2.1, the absolute payoff maximizing behavior of the delegates leads to positive profits for them. Note that the principal and the agent are assumed to have the same abilities in the contest and the positive profits in the delegation industry are therefore not due to a skill advantage of the agent.

4.3.2 Relative contracts

We will now address the case of relative contracts. Note that Baik (2007) ruled out a punishment for the delegate if the contest is lost. This assumption is relaxed here. Both principals can observe whether their delegate has won or not. If the contest is won, the corresponding principal l will pay a share of γ_l (where $\gamma_l \geq 0$ and $l \in \{1, 2\}$) of the prize. In case of a defeat the agent will have to pay a penalty. He will have to pay a share of γ_l of the prize to his principal. The penalty is like insurance for the principal. If the contest is lost, the principal will experience a negative utility. But the contest will not be lost completely since he will get compensation from the agent.
It is still assumed that all delegates and principals are risk-neutral. In addition, it is possible to pay a fixed amount F_l (where $F_l \geq 0$ and $l \in \{1, 2\}$) to the delegate to meet his participation constraint. This means an agent gets a fixed amount even if the contest is lost. In Chapter 3, fixed payments were ruled out, but they have to be used in this case. The reason why is given by the specific contract. In the latter Chapter the fine is always smaller than the reward. But in this case both are equal. The fixed payment compensates the agent for the higher fine.
It is also forbidden to sell the right of participation in this case. We will begin by

examining stage III. Given the terms of the contract, each delegate chooses how much effort to put in. Knowing that the delegates are risk-neutral and that they maximize their absolute payoffs, principal l chooses his offered share γ_l to maximize his weighted relative payoff at stage II.

Accordingly, the expected payoff of the delegate of player l at stage III is given by

$$\pi_{Dl}^B = \gamma_l \frac{d_l^B}{d_l^B + d_{-l}^B} V - \gamma_l (1 - \frac{d_l^B}{d_l^B + d_{-l}^B}) V - d_l^B + F_l^B.$$

Which can be rewritten as

$$\pi_{Dl}^B = \gamma_l \left(\frac{d_l^B}{d_l^B + d_{-l}^B} - \frac{d_{-l}^B}{d_l^B + d_{-l}^B} \right) V - d_l^B + F_l^B.$$

We call this type of contract a relative contract, because the payment for any delegate depends on the relative success compared to his opponent. This means if any delegate succeeds in achieving a higher winning probability than the other delegate by exerting effort, then he will be rewarded. Note that a delegate will be penalized if his opponent outperforms him. Payment according to the difference in the winning probabilities is called a relative payment.
It was already explained in Chapter 3, why introducing a fine may be a good idea. We also stated some examples where this happens. In this chapter, however, we examine a special case that was excluded in Chapter 3[1]. But they are related in some sense. Using relative contracts is quite common in everyday life. One may think of payments for managers that are made contingent on market share or on sales. An increased market share is clearly due to the fact that the competitors did not as well as the firm of the manager.

Returning to the analysis by deriving both delegates' first-order condition and by setting them equal, we get

$$d_1^B \gamma_2 = d_2^B \gamma_1.$$

Using this relationship for determining the optimal efforts and probabilities yields

$$d_l^B = \frac{2\gamma_l^2 \gamma_{-l}}{(\gamma_l + \gamma_{-l})^2} V,$$
$$p_l^B = \frac{\gamma_l}{\gamma_l + \gamma_{-l}}.$$

[1]This is excluded in Chapter 3 because the fixed payment was excluded.

At stage II, any principal l maximizes

$$u_{Pl}^B = p_l^B(1-\gamma_l)V + (1-p_l^B)\gamma_l V - F_l^B - \delta\left(p_{-l}^B(1-\gamma_{-l})V + (1-p_{-l}^B)\gamma_{-l}V - F_{-l}^B\right).$$

The first term represents the share of the contested prize the principal gets, if the contest is won. The second term represents the payment by the agent if the contest is lost and the third part is the fixed payment. These three terms are the material payoff of principal l. The fourth term stands for the concern of principal l for the monetary payoff of the other principal.
Rearranging this expression yields

$$u_{Pl}^B = p_l^B V - \gamma_l V(p_l^B - (1-p_l^B)) - F_l^B - \delta\left(p_{-l}^B V - \gamma_{-l}V(p_{-l}^B - (1-p_{-l}^B)) - F_{-l}^B\right).$$

Using the fact that $(1-p_l^B) = p_{-l}^B$ and $p_l^B = \frac{\gamma_l}{\gamma_l+\gamma_{-l}}$ leads to

$$\begin{aligned} u_{Pl}^B &= \frac{\gamma_l}{\gamma_l+\gamma_{-l}}V - \gamma_l\left(\frac{\gamma_l}{\gamma_l+\gamma_{-l}} - \frac{\gamma_{-l}}{\gamma_l+\gamma_{-l}}\right)V - F_l^B \\ &- \delta\left(\frac{\gamma_{-l}}{\gamma_l+\gamma_{-l}}V - \gamma_{-l}\left(\frac{\gamma_{-l}}{\gamma_l+\gamma_{-l}} - \frac{\gamma_l}{\gamma_l+\gamma_{-l}}\right)V - F_{-l}^B\right). \end{aligned}$$

As before, the payoff function of the principal consists of his own material payoff and the material payoff of the opponent weighted with the concern the individual has. The material payoff comprises the fixed and the relative payment to the delegate as well as the contested prize if the contest is won.
The first order condition for player 1 is given by

$$\begin{aligned} \frac{\partial u_{P1}^B}{\partial \gamma_1} &= \frac{\gamma_2}{(\gamma_1+\gamma_2)^2}V - \frac{2\gamma_1(\gamma_1+\gamma_2)-\gamma_1^2}{(\gamma_1+\gamma_2)^2}V \\ &+ \frac{\gamma_2(\gamma_1+\gamma_2)-\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}V + \delta\frac{\gamma_2}{(\gamma_1+\gamma_2)^2}V \\ &- \delta\frac{\gamma_2^2}{(\gamma_1+\gamma_2)^2}V - \delta\frac{\gamma_2(\gamma_1+\gamma_2)-\gamma_2\gamma_1}{(\gamma_1+\gamma_2)^2}V \overset{!}{=} 0. \end{aligned}$$

Using symmetry, we can solve this condition for γ^*:

$$\gamma^* = \frac{1}{2}.$$

$\gamma^* = 0$ is omitted. We also don't consider altruism. In equilibrium, the difference in payment for a delegate between winning and losing is V. By introducing a penalty, a principal is able to incentivize his delegate with the same amount the delegate would be incentivized with if he was a principal. As long as $\delta < 1$, it holds that $\alpha < \gamma^*$. Because of the payment in the event of a defeat, a principal is able to offer a greater share of the contested prize. The delegate therefore acts more aggressively. Note that, since we are in a symmetric equilibrium, the expected relative payment to the agent is zero because no delegate is more successful than the other one. His payoff would be negative and no rational agent will ever sign this contract without a fixed amount as compensation. Therefore the fixed amount is used to meet the participation constraint of an agent. The only sensible amount is $F = d$. As long as $F > d$, other individuals will offer to act as a delegate because positive profits are possible. This trend stops if the expected payoff is zero and therefore $F = d$. This means the principal pays the equilibrium expenses. But why should a delegate invest anything when the fixed amount is paid in any case? This problem is solved by a kind of prisoners' dilemma that both agents are in. Suppose that neither invests anything. In expectation they will get their compensation

$$F_1 = F_2 = \frac{V}{4}.$$

But what happens if the agent of player l deviates and invests an infinitely small, positive amount ε and the other invests nothing? The deviating agent would not only get the fixed payment but also half of the prize because he would be more successful than the "lazy" delegate. He would therefore get

$$\pi_{Dl} = \frac{3}{4}V - \varepsilon, \text{ which is strictly greater than } F = \frac{1}{4}V.$$

The other agent would lose and be penalized by his principal, thus receiving strictly less than the fixed amount F. We can therefore see why an agent would want to deviate by making effort. Unlike no-win-no-pay contracts, both agents earn an expected material payoff of zero in equilibrium.

We have seen that a principal pays indirectly for the effort made. He offers a contract and raises the fixed amount until an agent's participation constraint is met. By using relative contracts in a contest with two players and mandatory delegation, we get

Lemma 4.2: *The utility of the principals in equilibrium with relative contracts is given by*

$$u^B_{P1} = u^B_{P2} = (1-\delta)\frac{V}{4} = u^B_P.$$

In material terms each principal has a payoff of

$$\pi^B_{P1} = \pi^B_{P2} = \frac{V}{4} = \pi^B_P.$$

We can see that the material payoff is positive, although a principal might experience a utility of zero. And the result from Chapter 3 is also valid here: By using appropriate contracts, the principal can prevent moral hazard even if the invested efforts are unobservable. As in the model with observable effort choice used by Konrad (2009), an agent being incentivized with relative contracts and a principal playing the contest make the same effort.

Note that $\pi_P^B < \pi_P^A$ for $\delta < 1$, and $\pi_P^B = \pi_P^A$ for $\delta = 1$. What makes the result more striking is that we excluded negative fixed payments in the no-win-no-pay contract case. We therefore have positive profits in the delegation industry. But in the case with relative contracts we assumed that the fixed payment were only used to endow the delegate with zero utility and therefore to meet his participation constraint.

Due to the introduction of relative contracts, some of the effects of delegation with no-win-no-pay contracts are reversed and any principal as well as his relevant delegate act more aggressively. The reason for this is that the only way for a delegate to obtain a positive profit is to get ahead of the other delegate. Hence, any agent is incentivized to exert more effort. This is similar to the situation the principals are in since they are also concerned with maximizing a weighted relative payoff, making them more aggressive. But since $\gamma^* = \frac{1}{2}$, the prize for the delegate is smaller than for the principal. He therefore does not act as aggressively as a principal would in a situation without delegation. Thus, delegation with relative contracts is beneficial compared to a situation without delegation.

One may think of the following example: Two lawyers get a premium if they are successful and are not hired again if they are unsuccessful. But they still invest less than their principals would invest, because defeating the opponent is not as important to them as it is to their principals.

4.3.3 Asymmetric case

In this case, one principal uses a no-win-no-pay contract and the other rewards his delegate according to the relative contracts specified above. Without loss of generality, it is assumed that principal one uses a no-win-no-pay contract and principal two uses a relative contract. Let $\alpha^C \in [0,1]$ denote the share of the contested prize that principal one offers to his agent. And let $\gamma^C \geq 0$ be the corresponding factor the second principal uses. F^C is the fixed payment used by the second principal.

Starting at stage III, the payoff functions of the delegates are

$$\begin{aligned}\pi^C_{D1} &= \frac{d^C_1}{d^C_1+d^C_2}\alpha^C V - d^C_1\\ \pi^C_{D2} &= F^C + \left(\frac{d^C_2-d^C_1}{d^C_1+d^C_2}\right)\gamma^C V - d^C_2.\end{aligned}$$

Using both first-order conditions to determine the winning probabilities in equilibrium, yields

$$\begin{aligned}p^C_1 &= \frac{\alpha^C}{\alpha^C+2\gamma^C},\\ p^C_2 &= \frac{2\gamma^C}{\alpha^C+2\gamma^C}.\end{aligned}$$

Principal one then maximizes

$$u^C_{P1} = p^C_1(1-\alpha^C)V - \delta\left(p^C_2(1-\gamma^C)V + (1-p^C_2\gamma^C V - F^C)\right).$$

Again, the first part is the monetary payoff of principal one and the second part is his concern for the monetary payoff of the other principal.
Using p^C_1 and rearranging leads to

$$u^C_{P1} = \frac{(\alpha^C-(\alpha^C)^2)}{(\alpha^C+2\gamma^C)}V - \frac{2\gamma^C}{(\alpha^C+2\gamma^C)}\delta V + \left(\frac{(2\gamma^C-\alpha^C)}{(\alpha^C+2\gamma^C)}\right)\gamma^C\delta V + \delta F^C.$$

Using his first-order condition to calculate the reaction function of principal one, yields

$$\alpha^C = -2\gamma^C + \sqrt{4(\gamma^C)^2(1-\delta)+2\gamma^C(1+\delta)}. \tag{4.1}$$

Since $\alpha^C \in [0,1]$, we excluded $-2\gamma^C - \sqrt{4(\gamma^C)^2(1-\delta)+2\gamma^C(1+\delta)}$ as a solution. The reaction function of principal 2 is given by

$$\gamma^C = -\frac{\alpha^C}{2} + \sqrt{\frac{(\alpha^C)^2(1-\delta)+\alpha^C(1+\delta)}{2}}. \tag{4.2}$$

Apparently, $\alpha^C = \gamma^C = 0$ is an intersection point of both reaction curves but with mandatory delegation this is not an equilibrium since both principals have an incentive to deviate.
It is rather difficult to find the equilibrium analytically. But it is not necessary to know the exact answer for our further analysis. It is possible to state the following:

Lemma 4.3: *The upper bound for the share of the contested prize player one chooses in equilibrium is* $\frac{1}{2}$ *and the lower bound for the chosen share in equilibrium is given by* 0.3854, *i.e.* $\alpha^C \in [0.3854, \frac{1}{2}]$. *The corresponding upper bound for the share player two chooses in equilibrium is* $\sqrt{\frac{1}{2}} - \frac{1}{4}$ *and the lower bound is* 0.324, *i.e.* $\gamma^C \in [0.324, \sqrt{\frac{1}{2}} - \frac{1}{4}]$.

Proof:
The superscripts are omitted here to simplify the examination.

(i) Lower bounds:
Suppose that $\delta = 0$. Accordingly, the reaction functions are given by

$$\begin{aligned} \alpha &= -2\gamma + \sqrt{4\gamma^2 + 2\gamma}, \\ \gamma &= -\frac{1}{2}\alpha + \sqrt{\frac{(\alpha^2 + \alpha)}{2}}. \end{aligned}$$

The intersection of both reaction curves is given for $\alpha = 0.3854$ and $\gamma = 0.324$. These values are the candidates for the lower bounds of α and γ. By using the first derivatives with respect to δ and the share offered by the opponent, the lower bound for γ is verified because γ is increasing in δ and α. But for α, we can see that the first derivative with respect to δ is positive. The first derivative with respect to γ increases for low values of δ but decreases for high values.
To check whether 0.3854 is the lower bound, it is necessary to replace α by using (4.1). It can be obtained:

$$-2\gamma + \sqrt{4\gamma^2(1-\delta) + 2\gamma(1+\delta)} \geq 0.3854.$$

This inequality holds whenever the following inequality holds

$$\delta \geq \frac{(-0.4584\gamma + 0.1485)}{(2\gamma - 4\gamma^2)}.$$

This condition is fulfilled for any δ if $\gamma \geq 0.324$, which is indeed the case.
It is therefore possible to state that 0.3854 and 0.324 are the lower bounds for the shares α and γ respectively.

(ii) Upper bounds:
The principal using a no-win-no-pay contract will choose an $\alpha \leq \frac{1}{2}$ in equilibrium if he has to deal with a principal using a relative contract, whatever value γ and δ are.

$$\frac{1}{2} \geq \alpha = -2\gamma + \sqrt{4\gamma^2(1-\delta) + 2\gamma(1+\delta)}.$$

After some rearrangement, it results

$$16\gamma^2\delta + 1 \geq 8\gamma\delta.$$

For $\delta < 1$, the left side is greater than the right side for any value of γ since $\delta \in [0, 1]$. Additionally, for $\delta = 1$ both sides are equal if $\gamma = 0.25$ but for any other value of γ the left side is also greater. We can therefore state that the inequality holds true, whatever values γ and δ take.
To get the upper bound of γ, it is useful to use the fact that γ increases in δ and α. Using (4.2) and setting $\delta = 1$ and $\alpha = \frac{1}{2}$ yields the upper bound $\sqrt{\frac{1}{2} - \frac{1}{4}}$. Note that the chosen γ for $\delta = 1$ is $\frac{4}{9}$. Accordingly, the upper bound is not a chosen value but the highest possible value.
The upper bound for α is $\frac{1}{2}$ and the upper bound for γ is $\sqrt{\frac{1}{2} - \frac{1}{4}}$.

■

In addition, as can be seen from Lemma 4.3, the following inequality holds:

$$\alpha^C < 2\gamma^C.$$

The amount the agent of principal two is incentivized with is $2\gamma^C$ since this is the difference between winning the contest and losing it. Since this is always greater than α^C, it is possible to conclude that the incentives for the agent of player one are always lower and that the winning probability of the agent of principal two is higher. Using relative contracts therefore makes the agent act more aggressively. To increase his probability of winning, principal one offers a greater share of the prize compared to 4.3.1, since 0.3854 is greater than $\frac{1}{3}$. This holds for low values of δ. But if the concern regarding the opponent is high, principal one reduces his offer from 4.3.1 because it is too expensive to compete with the offer of principal two.

It is also worth having a look at the equilibrium payoff of the delegate of principal two:

$$\pi_{D2}^C = \left(\frac{(2\gamma^C - \alpha^C)}{(\alpha^C + 2\gamma^C)}\right)\gamma^C V - \left(\frac{4\alpha^C(\gamma^C)^2}{(\alpha^C + 2\gamma^C)^2}\right) V + F^C.$$

Using the reaction functions derived above, it can be shown

Lemma 4.4: *Principal two always has to use a fixed payment to attract a delegate and therefore* $F^C > 0$.

Proof: Principal two has to pay a fixed amount if the payoff for his agent would be negative without a transfer. We have to show the following

$$\frac{(2\gamma^C - \alpha^C)}{(\alpha^C + 2\gamma^C)}\gamma^C V - \frac{4\alpha^C(\gamma^C)^2}{(\alpha^C + 2\gamma^C)^2}V \;<\; 0.$$

After some rearrangement, it can be obtained that

$$(2\gamma^C - \alpha^C)^2 - 2(\alpha^C)^2 \;<\; 0.$$

Applying (4.2) yields

$$2\sqrt{\frac{(\alpha^C)^2(1-\delta) + \alpha^C(1+\delta)}{2}} \;<\; (\sqrt{2}+2)\alpha^C.$$

After some rearrangement, it is possible to conclude that the material payoff is negative whenever the following inequality holds:

$$\frac{(1+\delta)}{(2+2\sqrt{2}+\delta)} \;<\; \alpha^C.$$

The left hand side is strictly increasing in δ. Accordingly, the condition is fulfilled whenever the inequality holds for $\delta = 0$ and $\delta = 1$, which is indeed the case (Lemma 4.3). The expected payoff for the delegate of principal two is therefore negative without a fixed payment.

■

Although the winning probability for the second delegate is higher than that for his opponent, it does not pay for him in expectation. The costs of exerting the effort to achieve this advantage are too high. Accordingly, principal two has to compensate him for the expected loss.

4.4 Equilibrium in the contract choice game

Summing up the results derived in Section 4.3, we get the following normal form game.

		Principal two	
		NW	RC
Principal one	NW	u^A, u^A	u^C_{P1}, u^C_{P2}
	RC	u^C_{P2}, u^C_{P1}	u^B, u^B

Where NW stands for no-win-no-pay contracts and RC for relative contracts. We will now show

Theorem 4.1: *If two players are involved in a contest with mandatory delegation, the unique Nash Equilibrium is given by (RC, RC).*

Proof:
It has to be shown that using a relative contract is a dominant strategy. The following must hold

(i) $u^C_{P2} > u^A$ and

(ii) $u^B > u^C_{P1}$.

For simplicity of examination, we have omitted the superscript.

(i) Claim: Using a relative contract is the best response to a no-win-no-pay contract.
This means

$$\frac{2\gamma}{(\alpha+2\gamma)}V - \frac{(2\gamma-\alpha)}{(2\gamma+\alpha)}\gamma V - \frac{(\alpha-\alpha^2)}{(\alpha+2\gamma)}\delta V - F \quad > \quad \frac{(1-\delta)}{(3+\delta)}V.$$

Since F accounts for the difference between relative payment and effort made, we get

$$\frac{2\gamma}{(\alpha+2\gamma)} - \frac{(\alpha-\alpha^2)}{(\alpha+2\gamma)}\delta - \frac{4\alpha\gamma^2}{(\alpha+2\gamma)^2} \quad > \quad \frac{(1-\delta)}{(3+\delta)}.$$

Rearranging the terms leads to

$$\begin{aligned} & 6\alpha\gamma + 12\gamma^2 + 2\alpha\gamma\delta + 4\gamma^2\delta + \alpha^2\delta(3+\delta)(\alpha+2\gamma) + \delta(\alpha+2\gamma)^2 \\ > \quad & 4\alpha\gamma^2(3+\delta) + (\alpha+2\gamma)^2 + \alpha\delta(3+\delta)(\alpha+2\gamma). \end{aligned}$$

Using Lemma 4.3, it can be easily shown that the first four terms on the left hand side are always greater than the first two terms on the right hand side. Looking at the remaining parts of the inequality, it is clear to see that the terms on the left-hand side are greater than the term on the right-hand side. The inequality is therefore fulfilled. It is beneficial for a principal to react to a no-win-no-pay contract with a relative contract.

(ii) Claim: For any principal, it has to be beneficial to react with a relative contract to an opponent using a relative contract. Therefore

$$\frac{\alpha - \alpha^2}{(\alpha + 2\gamma)} V - \frac{2\gamma}{(\alpha + 2\gamma)} \delta V + \frac{4\alpha\gamma^2}{(\alpha + 2\gamma)^2} \delta V - (1 - \delta) \frac{V}{4} \quad < \quad 0. \tag{4.3}$$

Using Lemma 4.3, it can be shown that this inequality is fulfilled for $\delta = 1$ and $\delta = 0$. To see what happens in between, suppose that both sides are equal. We obtain

$$\frac{(\alpha - \alpha^2)}{(\alpha + 2\gamma)} - \frac{1}{4} = \frac{2\gamma}{(\alpha + 2\gamma)} \delta - \frac{4\alpha\gamma^2}{(\alpha + 2\gamma)^2} \delta - \frac{\delta}{4}.$$

The solution for δ yields

$$\delta \quad = \quad \frac{(3\alpha^2 + 4\alpha\gamma - 4\alpha^3 - 8\alpha^2\gamma - 4\gamma^2)}{(4\alpha\gamma + 12\gamma^2 - 16\alpha\gamma^2 - \alpha^2)}.$$

According to Lemma 4.3, the denominator is positive but the numerator is not. This is ruled out since $\delta \in [0, 1]$. Accordingly, equality does not hold in (4.3) for any meaningful value of δ, α and γ. Since all terms are continuous in the domain given by Lemma 4.3 and by assumption, we can conclude that u^B is greater than u_{P1}. Using a relative contract to counter a relative contract is beneficial for both principals.

Conditions (i) and (ii) are both fulfilled. Using a relative contract is therefore the dominant strategy in the given 2x2 game.

■

Note that this theorem also holds for players that maximize their individualistic payoff. The agent's more aggressive behavior induced by relative contracts is beneficial for the principal. Even without concern regarding the opponent.

Now that the equilibrium in the contract choice game has been revealed, we will now address the respective material payoffs. Assuming that no delegation has been introduced, a player would receive a material payoff of $\pi = \frac{V}{4}$ if the contest took place between absolute payoff maximizers. In contrast, if there were only relative payoff maximization, the material payoffs would be $\pi^R = \frac{(1-\delta)}{4} V$. For the purpose of comparison, we will refer to the first case. The reason is, that it is necessary to demonstrate that delegation can offset any negative effect of other-regarding preferences. Thus, we will show that delegation can provide the principals with as much material payoff as the simple payoff maximization by all players would.

It can be seen that

$$\pi^B = \pi = \frac{V}{4}.$$

We can therefore immediately state:

Theorem 4.2: *Contracting delegates by using relative contracts neutralizes (in equilibrium) the effect of negatively interdependent preferences completely. The material payoffs to principals with negatively interdependent preferences, who use equilibrium relative contracts, is the same as the one of principals with independent preferences and no delegates.*[2]

The reason for this is the symmetry of the opponents. In expectation no delegate is more successful than his opponent. The material payoff function therefore decreases to

$$\pi^B_{Pl} = \frac{d^B_l}{d^B_l + d^B_{-l}} - F^B_l.$$

We have seen that a principal pays for the effort made indirectly, i.e. $F^B_l = d^B_l$. All that remains is the well-known payoff function for a contest used by Tullock (1980) involving absolute payoff maximizing players. The only difference is that the efforts are made by the delegates. The material payoff for any delegate is zero because no principal is willing to pay more in equilibrium, since this is the lowest value for which the participation constraint is met. The reason for that result is that individualistic players are contracted. These agents are only interested in their monetary payoff. They do not care for the payoff of any other player. Accordingly, a principal can only incentivize them with the monetary value of the contested prize. To use the whole prize to incentivize the agent is beneficial for the principal, and therefore agents act as a player without interdependent preferences would.

It is also worth noting that $\frac{V}{4}$ is the maximum amount an absolute payoff maximizing player is willing to pay for the right to participate in the contest. The amount decreases if the expected opponent or the buyer has a negatively interdependent utility function. In the equilibrium of the contract choice game any principal is expected to earn the same amount by participating in the contest. It therefore does not pay to sell the right of participation and it is therefore possible to exclude this case without loss of generality.

[2] This theorem is also true for two-player contests with $r \leq 1$. The proof is straightforward and therefore omitted.

4.5 Conclusion

The question we tried to answer is: Are there mechanisms to reduce competition and thus wasteful investments in rent-seeking contests, when preferences induce spiteful behavior?
To this end, this paper has explored the effects of delegation in a Tullock rent-seeking contest where principals maximize a weighted relative payoff. Maximizing the relative position leads to higher efforts and reduces the material outcome for each player.

It is shown that delegation makes the principals better off in a two-player Tullock contest. We assume that the agents have no particular concern regarding the opponent. An example is a lawyer acting for a woman in a divorcement process that was cheated on by her husband. With prescribed delegation, each principal can do at least as well as if he were in a group consisting only of individualistic payoff maximizing players, i.e. "spiteful" preferences are "neutralized" in the contract choice game. No-win-no-pay contracts can even overcompensate the negative effects of weighted relative payoff maximizing behavior. But since interdependent preferences yield an advantage in equilibrium in rent-seeking contests, contracts that reward the delegate according to his relative success, these are developed and used by the principals. Relative contracts mean that the delegate has something to lose. He is therefore incentivized more and has an advantage if his opponent is not penalized for a loss but only paid in the case of a win. This reduces the efficiency gained by introducing delegation with no-win-no-pay contracts. However, compared to a group of individualistic players, the efficiency is fully restored. In essence, the economic institution of contracting delegates can completely offset the inefficiency caused by negatively interdependent preferences. In theory, delegation has an even better effect, but is held back by the same competitive forces at the contracting level. More aggressive contracts drive out more moderate ones. Another result is that delegation with relative contracts among individualistic principals does not give any advantage over the equilibrium without delegation. This is due to the fact that relative contracts incentivize the agent with the whole prize. Accordingly, the agent behaves just like a principal with independent preferences would behave if he were to play the contest. By delegating, a principal with negatively interdependent preferences can be replaced by an individualistic player. And therefore there is no welfare gain for individualistic principals compared to a situation without delegation. This is also a drawback of delegation. Wasteful investments can only be reduced if the agent does not care as much as the principals for the material payoff of the opponent. This is especially true for lawyers. Lawyers are only interested in their income.
It is shown that there is a fixed amount the principal has to pay to hire a delegate

in the equilibrium of the contract choice game. The fixed amount does not alter the invested effort but is necessary in order to make an agent willing to sign a contract. A game-structure like in a prisoners' dilemma prevents the delegate from acting as a free rider.
One policy implication of the above analysis is, that mandatory delegation combined with a limited contract space should be introduced. In Germany, for example, a related system is at work: as stated previously, a lawyer has to be hired in divorce proceedings and if the contested "rent" is high enough. The payment of lawyers is also fixed in the lawyers' compensation act (Rechtsanwaltsvergütungsgesetz).

Mandatory delegation has been assumed. The consequences of relative payoff maximizing players on the terms of contract have been shown. But it is interesting to find out whether prescribed delegation leads to changes for a larger group of contests. Also empirical findings on this topic seem interesting.

Chapter 5

Training in a Tullock Rent-Seeking contest with delegation

5.1 Introduction

In Chapter 3 and Chapter 4 the principal used the parameters of the contract to influence the effort put in by his delegate. To pay the agent, the prize at stake is split up. Accordingly, the incentives for the delegate are decreased and therefore investments are decreased. In this chapter the principal can also use training. The principal cannot reverse the split up, but he can increase the weight of the effort put in by the agent. On the one hand, the principal uses a contract to hire an agent, and, on the other hand, he determines the amount of training to employ. Training increases the skills of the agent and therefore makes his invested effort count more. Accordingly, he is more effective. Because of his increased skills, he can reach his desired impact in the Contest-Success-Function with a lower investment than without training. This can also be interpreted as technical assistance the principal pays for and that does not "count" in the Contest-Success-Function directly because it only influences the weight of the effort made by the agent. There are many examples where a principal either invests in training or gives support to his agent. The basic training in the army is one example for better skills. But also support plays an important role in an army. An idiom that expresses that a principal may support his delegate is: An army marches on its stomach. A soldier that is not hungry is a better fighter than a soldier that nearly starves. One can also think of researchers going to conferences paid for by their employer. An increased knowledge may increase the possibility of winning a patent race.

To the best of our knowledge, this is the first concern of training in a rent-seeking contest with delegation. Baik and Kim (1997) use an approach to include different skill-levels in the analysis. They assumed that the delegate has an advantage in the contest compared to the principal. One example is a lawyer that knows the laws and the verdicts so far. But a principal has to invest more effort because he has to acquire all the information and skills that can be helpful. Accordingly, the lawyer can have the same impact in the lawsuit at lower costs. Schoonbeek (2007) introduces another kind of difference in skills between agent and principal. He assumes that a delegate can use two instruments and the principal can only use one. We have to say that the second instrument increases the impact of the first one. We can think of a traffic violation in Germany. If a driver is accused of a traffic violation in Germany, he has got the possibility to enter a caveat. A lawyer can do the same but he can also have a look at the documents of the police which can make his caveat more powerful. A variation of this approach is used here. Schoonbeek assumes that the delegate can choose whether to use both instruments or only one. We will assume that the agent only decides how much effort to put in. The principal can decide whether to increase the weight of the effort put in or not. He also has to bear the costs of the support given to the agent. We will also have a look at a situation where training decreases the unit costs of investing effort. The principal decides whether to decrease the unit costs of investing effort or not. On the one hand, that increases the costs for the principal, but this can also make the agent invest more. One may think of a worker that is more effective because of training and therefore produces more units per hour. The results of this case are compared to the first approach of including training. By using these two approaches, we can compare a reduction in unit costs to an increase in the weight of effort in the Contest-Success-Function. Accordingly, we introduce two additional ways of how a principal can influence the outcome of the contest. If the unit costs are reduced, the principal influences the effort choice of the agent. But if the weight is increased, he gains influence directly, because he can choose how much the effort that was invested counts.

As the examples above indicate, we assume mandatory delegation. The reason for this assumption can be seen in the assumed contract. We assume that no-win-no-pay contracts are used. This ensures comparison with the delegation literature so far. It also shows that training does not need additional assumptions than already made in the literature. The game has two stages. At the first stage the delegate is hired and the amount of training is determined. At the second stage the contest is played by the agents. After the effort was put in by both delegates, the winner is determined, the prize is handed over, and payments take place. We will use the Tullock Contest-Success-Function to determine the winner. It is shown under which circumstances training is used. It is also shown how the parameter of the contract and the amount of training change with the parameters of the model.

As stated above, Wärneryd (2000) showed that splitting the contested prize up leads to an increased payoff for the principal. Whether this holds true if training is introduced or not, is not immidiately clear. On the one hand, the prize is still split up and therefore the incentives to put in effort are decreased. On the other hand, the principal can also influence the outcome of the contest by his investment. Accordingly, his payoff may be decreased. The answer to this question is also given in this chapter. Also the consequences for the delegate are given. Because of the training used by the principals, the delegate can have a bigger impact in the contest, without investing more. Common sense tells us that a delegate may decrease his investment to increase his payoff. Whether this is true or not is stated below.

In section 5.2 the model is explained. Section 5.3 gives the equilibrium of the game. Accordingly, the contract-parameters and the amount of training is determined. Section 5.4 explains how the equilibrium changes if the parameters of the model are changed and compares the result to the scenarios without delegation and without training. Section 5.5 concludes.

5.2 The Model

We will consider two models here. In the first model, training increases the weight of the effort invested by the agent. In the second model, training reduces the unit costs of investing effort. But before we will turn to the first model, we give the assumptions used in both models:
Two risk-neutral principals are involved in a contest for a single, indivisible prize. The value of the prize is given by V and is the same for all players. This is also the objective value of the prize. Both principals have to hire a delegate. A delegate puts in effort for his employer. The game has got two stages. At the first stage the principal contracts the delegate and determines the amount of training the agent gets. At the second stage the contest is played by the delegates. The Tullock Contest-Success-Function is used to determine the winner. r is assumed to be smaller or equal to two. The delegates are also risk-neutral. The reservation wage of both agents is normalized to zero. The delegates are contracted with no-win-no-pay contracts. $\alpha_i \in [0, 1]$ determines the share of the prize the delegate of principal i, $i = 1, 2$, gets if the contest is won. If the contest is lost, the principal pays nothing to the agent. The agent has to bear the costs of the effort he made in both cases. If the expected payoff is positive, then the agent will sign the contract. It is assumed that all individuals only care for their absolute payoff.

5.2.1 Increased weight of invested effort

The amount of training offered by principal i is given by $\tau_i \geq 0$. The costs of one unit of training is given by $\frac{V}{c}$. c is an exogenously given parameter and is always greater than zero. If c is too low, then the principal will only use the contract to influence the outcome of the contest. If c is very high, then the principal will train the agent a lot. Note that the training costs are forgone. As we will show later, the agent has no positive effect from training, except that he has better skills that he can use in the future. But the future effects of training are not considered here. The principal may not use training, but he will always offer a positive share of the prize to the agent. If no share of the prize is offered, then no delegate will sign the contract. Only a positive α_i ensures a positive effort made by the delegate of principal i. The offered share will always be smaller than one. The principal would suffer a loss otherwise. The effort a delegate puts in is given by d_i. The unit costs of effort are constant and equal to one. They are the same for any delegate. To sum up, the probability that the agent of principal i wins is

$$p_i = \begin{cases} \frac{(1+\tau_i)^r d_i^r}{(1+\tau_1)^r d_1^r + (1+\tau_2)^r d_2^r}, & \text{for } d_1 + d_2 > 0 \\ \frac{1}{2}, & \text{for } d_1 + d_2 = 0. \end{cases}$$

Of course, if no effort is put in, then training has no effect on the probability. If both principals decide to use no training, then the standard Tullock contest with delegation is played, as used in Baik and Kim (1997). We assume a linear costs function for training, i.e. all units of training cost the same amount. But in reality improving the skills of an individual is harder the more skilled he already is. This concept can also be observed in our model. The first derivative of the winning probability of i with respect to his training decision is positive. But the second derivative is negative for $r \leq 1$ and for a situation where the effort choices do not differ very much. But in equilibrium this derivative will also be negative. That means that the more training was used, the smaller is the effect on the winning probability. The same incremental amount invested in training has therefore a smaller impact the more money was already invested.

The model is again solved by backward induction. The principals first think about the effect of their decision on the actions of the delegates but they also think of the effect on the actions on the other principal. Afterwards, they choose the action that maximizes their payoff. At the second stage, the agent of principal i decides on his effort d_i to put in to maximize

$$\pi_{Di} = \frac{(1+\tau_i)^r d_i^r}{(1+\tau_1)^r d_1^r + (1+\tau_2)^r d_2^r} \alpha_i V - d_i.$$

The equilibrium is reached if the following holds

$$d_2 = \frac{\alpha_2}{\alpha_1} d_1.$$

Both agents cannot influence the amount invested in training. Accordingly, training is viewed as a constant term and does not influence the equilibrium condition at this stage. But training does have an influence on the effort put in and therefore on the winning probability:

Lemma 5.1: *At the second stage, the effort put in by the delegates in equilibrium is*

$$\begin{aligned} d_1 &= \frac{\alpha_1^{r+1}\alpha_2^r(1+\tau_1)^r(1+\tau_2)^r}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2} rV \\ d_2 &= \frac{\alpha_1^r\alpha_2^{r+1}(1+\tau_1)^r(1+\tau_2)^r}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2} rV. \end{aligned}$$

Accordingly, the probability that agent i wins the contest is

$$p_i = \frac{\alpha_i^r(1+\tau_i)^r}{\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r}.$$

As can be seen from the formula, the probability is influenced by both variables the principal can use. Before we turn to the solution of the model, we will recapitulate some results to ensure comparison.

On the one hand, we want to compare the model to a situation without training. But on the other hand, we want to compare it to a situation without delegation. The first situation was analyzed in section 3.3.1 (Lemma 3.2). The result was

$$\begin{aligned} \alpha_1 &= \alpha_2 = \frac{r}{r+2}, \\ d_1 &= d_2 = \frac{r^2}{(2+r)}\frac{V}{4}, \\ \pi_{P1} &= \pi_{P2} = \frac{V}{(2+r)}, \\ \pi_{D1} &= \pi_{D2} = \frac{(2-r)}{(2+r)}\frac{rV}{4}. \end{aligned}$$

The latter situation was analyzed in section 4.2.1. But it was assumed that $r = 1$. Accordingly, the model has to be adjusted. It is assumed that both principals play the contest. x_i stands for the effort principal i puts in. The probability that he wins is

$$q_i = \begin{cases} \frac{x_i^r}{x_1^r+x_2^r}, & \text{for } x_1+x_2>0 \\ \frac{1}{2}, & \text{for } x_1+x_2=0. \end{cases}$$

Principal i maximizes $\pi_i = q_i V - x_i$.
The equilibrium of this contest is

$$\begin{aligned} x_1 &= x_2 = \frac{rV}{4}, \\ \pi_1 &= \pi_2 = (2-r)\frac{V}{4}. \end{aligned}$$

As can be seen from the results, mandatory delegation is always beneficial for the principal. This is the result of Wärneryd (2000).

5.2.2 Training decreases the unit costs

In this model the amount of training used by the principal has no influence on the weight of the invested effort in the Contest-Success-Function. The amount of effort that is invested by the delegate of principal i is given by $\bar{d}_i$. Accordingly, the probability that the agent of principal i wins the contest is:

$$\bar{p}_i = \begin{cases} \frac{\bar{d}_i^r}{\bar{d}_1^r+\bar{d}_2^r}, & \text{for } \bar{d}_1+\bar{d}_2>0 \\ \frac{1}{2}, & \text{for } \bar{d}_1+\bar{d}_2=0. \end{cases}$$

Training has an influence on the unit costs of effort and therefore on the amount invested by the agent. In this model the unit costs of effort for the delegate of principal i are given by $\frac{1}{1+\bar{\tau}_i}$. $\bar{\tau}_i$ denotes the amount of training used by principal i. If no training is used, the unit costs of effort are constant and equal to one for all agents. But as soon as the amount of training differs, also the unit costs differ. The more training is used by principal i (the higher $\bar{\tau}_i$), the lower are the unit costs for his agent. Because of the chosen functional form of the unit costs, the effect of one additional unit of training is the lower the more units have been already used. If there are two soldiers, one soldier is untrained and the other soldier goes jogging in his leisure time, then the same amount of physical training will have a greater effect on the first soldier. To represent the costs of training, we will use an approach similar to the first model. The costs of one unit of training are given by $\frac{V}{\bar{c}}$. $\bar{c}$ is exogenously given, fixed, and determines whether a lot of training is used or no training takes place at all. For very high values of $\bar{c}$ there will also be a lot of training.

The share used by principal i is given by $\bar{\alpha}_i \in [0,1]$ in this subsection. Hence, the payoff of the agent of principal i is

$$\bar{\pi}_{Di} = \frac{\bar{d}_i^r}{\bar{d}_1^r + \bar{d}_2^r}\bar{\alpha}_i V - \frac{\bar{d}_i}{(1+\bar{\tau}_i)}.$$

The equilibrium condition at the second stage is given by

$$\bar{d}_2 = \frac{(1+\bar{\tau}_2)\bar{\alpha}_2}{(1+\bar{\tau}_1)\bar{\alpha}_1}\bar{d}_1.$$

In the first model the intersection of the reaction curves of both agents was determined by the chosen effort. But in this model we can see that the amount of training chosen by the principal influences the point at which the reaction curves intersect.

Lemma 5.2: *At the second stage, the effort put in by the delegates in equilibrium is*

$$\begin{aligned}
\bar{d}_1 &= \frac{\bar{\alpha}_1^{r+1}\bar{\alpha}_2^r(1+\bar{\tau}_1)^{r+1}(1+\bar{\tau}_2)^r}{(\bar{\alpha}_1^r(1+\bar{\tau}_1)^r + \bar{\alpha}_2^r(1+\bar{\tau}_2)^r)^2} rV \\
\bar{d}_2 &= \frac{\bar{\alpha}_1^r\bar{\alpha}_2^{r+1}(1+\bar{\tau}_1)^r(1+\bar{\tau}_2)^{r+1}}{(\bar{\alpha}_1^r(1+\bar{\tau}_1)^r + \bar{\alpha}_2^r(1+\bar{\tau}_2)^r)^2} rV.
\end{aligned}$$

Note that Lemma 5.2 predicts more invested efforts than Lemma 5.1 does if the amounts of training and the offered shares do not differ. By increasing the weight of effort, the effort actually invested counts more. By decreasing the costs of effort, more effort is invested by the agents.
In this case the winning probability for the agent of principal i is

$$\bar{p}_i = \frac{\bar{\alpha}_i^r(1+\bar{\tau}_i)^r}{\bar{\alpha}_1^r(1+\bar{\tau}_1)^r + \bar{\alpha}_2^r(1+\bar{\tau}_2)^r}.$$

We can see that the share offered and the amount of training used both influence the probability of winning. Note that $\bar{p}_i$ is equal to p_i if the offered shares and the amount of training used do not differ. Decreasing the unit costs of effort makes the agent act more aggressively than increasing the weight does. But if $\tau_i = \bar{\tau}_i$ and $\alpha_i = \bar{\alpha}_i$, then the increased weight compensates the principal for the lower investment.

5.3 Equilibrium Contract and amount of Training

We will turn to the first stage, now. First, we will consider the model with an increased weight. The principals know how their decisions influence the outcome at the second stage. The objective of principal one is to maximize

$$\pi_{P1} = \frac{\alpha_1^r(1+\tau_1)^r}{\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r}(1-\alpha_1)V - \tau_1\frac{V}{c}.$$

An increase in α_1 increases the winning probability, but it also decreases the share the principal gets if the contest is won. Training also increases the probability of winning, but he has to pay for every unit. The principal has to ensure that the payoff for the delegate is not negative. Because of the no-win-no-pay contracts, we can ignore this for the moment. The principal can only apply a non-negative amount of training.

Before we solve the model, we will have a look at the model with decreased unit costs. In this model, any principal i maximizes

$$\bar{\pi}_{Pi} = \frac{\bar{\alpha}_i^r(1+\bar{\tau}_i)^r}{\bar{\alpha}_1^r(1+\bar{\tau}_1)^r+\bar{\alpha}_2^r(1+\bar{\tau}_2)^r}(1-\bar{\alpha}_i)V - \bar{\tau}_i\frac{V}{\bar{c}}.$$

We can see that the problem for the principal is similar in both models. Accordingly, we will concentrate on the first model. The analysis for the second model can be done in the same manner. We will just give the results in the corresponding Lemma and Theorems. But when it comes to the analysis of the efforts invested by the delegates, we will analyze both models separately.

Coming back to the problem of the principals in the model with an increased weight, we can derive the first order conditions. The first order conditions for both principals are

$$\begin{aligned}
\frac{\partial \pi_{P1}}{\partial \alpha_1} &= \frac{(r\alpha_1^{r-1}-(r+1)\alpha_1^r)(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2}(1+\tau_1)^rV \\
&- \frac{r\alpha_1^{r-1}(1+\tau_1)^r(\alpha_1^r-\alpha_1^{r+1})}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2}(1+\tau_1)^rV \overset{!}{=} 0, \\
\frac{\partial \pi_{P1}}{\partial \tau_1} &= \frac{r(1+\tau_1)^{r-1}(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)-r\alpha_1^r(1+\tau_1)^{2r-1}}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2}\alpha_1^r(1-\alpha_1)V \\
&- \frac{V}{c} \overset{!}{=} 0, \\
\frac{\partial \pi_{P2}}{\partial \alpha_2} &= \frac{(r\alpha_2^{r-1}-(r+1)\alpha_2^r)(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2}(1+\tau_2)^rV \\
&- \frac{r\alpha_2^{r-1}(1+\tau_2)^r(\alpha_2^r-\alpha_2^{r+1})}{(\alpha_1^r(1+\tau_1)^r+\alpha_2^r(1+\tau_2)^r)^2}(1+\tau_1)^rV \overset{!}{=} 0,
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \pi_{P2}}{\partial \tau_2} &= \frac{r(1+\tau_2)^{r-1}(\alpha_1^r(1+\tau_1)^r + \alpha_2^r(1+\tau_2)^r) - r\alpha_2^r(1+\tau_2)^{2r-1}}{(\alpha_1^r(1+\tau_1)^r + \alpha_2^r(1+\tau_2)^r)^2}\alpha_2^r(1-\alpha_2)V \\
&- \frac{V}{c} \overset{!}{=} 0.
\end{aligned}
$$

The first condition can be reduced to

$$
\frac{\alpha_2^r(1+\tau_2)^r}{\alpha_1^r(1+\tau_1)^r} = \frac{\alpha_1}{r-(r+1)\alpha_1}. \tag{5.1}
$$

Rearranging the third condition yields

$$
\frac{\alpha_2^r(1+\tau_2)^r}{\alpha_1^r(1+\tau_1)^r} = \frac{r-(r+1)\alpha_2}{\alpha_2}. \tag{5.2}
$$

Setting the r.h.s's of (5.1) and (5.2) equal leads to

$$
\alpha_1 = \frac{r-(r+1)\alpha_2}{(r+1)-(r+2)\alpha_2}. \tag{5.3}
$$

Let us now turn to the second and fourth condition. The aim is to get an equation that tells us how the amount of training depends on the offered shares. With such an equation and (5.3) we can get an equation of how α_2 depends on r. Rearranging $\frac{\partial \pi_{P1}}{\partial \tau_1} = 0$ yields

$$
\frac{1}{c}(\alpha_1^r(1+\tau_1)^r + \alpha_2^r(1+\tau_2)^2) = r\alpha_1^r(1-\alpha_1)\alpha_2^r(1+\tau_1)^{r-1}(1+\tau_2)^r. \tag{5.4}
$$

Rearranging $\frac{\partial \pi_{P2}}{\partial \tau_2} = 0$ in the same way and setting it equal to (5.4) yields

$$
(1+\tau_2) = \frac{(1-\alpha_2)}{(1-\alpha_1)}(1+\tau_1). \tag{5.5}
$$

Using (5.4) and (5.5) leads to

$$
(1+\tau_1) = \frac{\alpha_1^r\alpha_2^r(1-\alpha_1)^{r+1}(1-\alpha_2)^r}{(\alpha_1^r(1-\alpha_1)^r + \alpha_2^r(1-\alpha_2)^r)^2}rc, \tag{5.6}
$$

$$
(1+\tau_2) = \frac{\alpha_1^r\alpha_2^r(1-\alpha_1)^r(1-\alpha_2)^{r+1}}{(\alpha_1^r(1-\alpha_1)^r + \alpha_2^r(1-\alpha_2)^r)^2}rc. \tag{5.7}
$$

Combining (5.2), (5.3), (5.6), and (5.7) leads to

Lemma 5.3: *The share that principal two uses in the equilibrium at the first stage of the game is determined by*

$$\begin{aligned} \alpha_2^{r+1}((r+1)-(r+2)\alpha_2)^{2r} &= (r-(r+1)\alpha_2)^{r+1}, \quad \textit{and} \\ \bar{\alpha}_2^{r+1}((r+1)-(r+2)\bar{\alpha}_2)^{2r} &= (r-(r+1)\bar{\alpha}_2)^{r+1} \quad \textit{respectively.} \end{aligned}$$

Accordingly, we get

Theorem 5.1: *There are no asymmetric equilibria at the first stage of both models.*

Proof: Assume, without loss of generality, that in an asymmetric equilibrium $\alpha_1 > \alpha_2$ holds. Using (5.3) leads to

$$\frac{r-(r+2)\alpha_2}{(r+1)-(r+2)\alpha_2} > \alpha_2. \tag{5.8}$$

The denominator of the left hand side of the inequality is negative for $\alpha_2 > \frac{r+1}{r+2}$. The numerator of the same side is negative for $\alpha_2 > \frac{r}{r+2}$. A principal will always use a share that is strictly positive. Accordingly, the right hand side is always positive. For $\alpha_2 \in (\frac{r}{r+2}, \frac{r+1}{r+2}]$ the inequality is wrong because the left hand side is negative. To proof Theorem 5.1, we have to show that the inequality is wrong for $\alpha_2 < \frac{r}{r+2}$ and for $\alpha_2 > \frac{r+1}{r+2}$. The latter can be shown by having a look at the choice of principal one. Principal one will only increase his offered share if this increases his expected payoff. That means he will increase α_1 as long as $\frac{\partial \pi_{P1}}{\partial \alpha_1} > 0$. This first order condition is negative if

$$\frac{\alpha_1^r(1+\tau_1)^r}{\alpha_2^r(1+\tau_2)^r} > \frac{r-(r+1)\alpha_1}{\alpha_1}.$$

Using (5.6) and (5.7) leads to

$$\frac{\alpha_1^r(1-\alpha_1)^r}{\alpha_2^r(1-\alpha_2)^r} > \frac{r-(r+1)\alpha_1}{\alpha_1}.$$

The left hand side is always positive. But the right hand side is negative for $\alpha_1 \geq \frac{r}{r+1}$. If $\alpha_1 \geq \frac{r}{r+1}$, then the first derivative with respect to α_1 is negative, i.e. principal one will never use a share that is that high. Because $\frac{r+1}{r+2} > \frac{r}{r+1}$ and $\alpha_1 > \alpha_2$, (5.8) is not true for $\alpha_2 > \frac{r+1}{r+2}$.
The only case that is left is $\alpha_2 < \frac{r}{r+2}$. It is straightforward to use $\frac{\partial \pi_{P2}}{\partial \alpha_2}$ to show that it is beneficial for principal two to increase his share if α_2 is that low. Accordingly,

(5.8) is not true and therefore there is no asymmetric equilibrium.

■

Both principals are facing a symmetric situation. The desire for the contested prize is the same for both and therefore they incentivize their delegate to the same extent. By applying the symmetry assumption to (5.3), it results

Theorem 5.2: *The share of the contested prize offered by the principal to his delegate in equilibrium is given by*

$$\begin{aligned} \alpha_1 &= \alpha_2 = \frac{r}{r+2}, \qquad \textit{and} \\ \bar{\alpha}_1 &= \bar{\alpha}_2 = \frac{r}{r+2} \qquad \textit{respectively.} \end{aligned}$$

It is straightforward to show that this indeed maximizes the expected payoff for the principals. $\alpha_1 = \alpha_2 = 1$ is also a solution of (5.3), but this is no equilibrium because both principals can increase their expected payoff by reducing their offer. Note that $\frac{r}{r+2}$ is exactly the same offer as in the situation without training. Because the principal pays for the training and therefore for the increasing effectiveness, the higher skill-level does not pay for the delegate. Figuratively speaking, the principal pays the delegate for the skills he had before the training. Note that this might only hold for this particular contest. Whether the delegate gets higher wages because of his increased skills in the future or not is not considered in this chapter.
In equilibrium, the amount of training that principal i=1,2 pays for is given by

$$\tau_i = \begin{cases} 0 & \text{for } c \leq \frac{2(r+2)}{r} \quad \Leftrightarrow \frac{V}{c} \geq \frac{r}{2(r+2)}V \\ \frac{rc-2(r+2)}{2(r+2)} & \text{for } c > \frac{2(r+2)}{r} \quad \Leftrightarrow \frac{V}{c} < \frac{r}{2(r+2)}V. \end{cases}$$

If one unit of training is too expensive, then the principal prefers not to train the agent. But if training is relatively cheap, then the principal will invest in the increase of skills of his delegate. Battles in the last century are a good example. Firearms can improve the effectiveness of a soldier. But there are some battles in the 20th century where not all soldiers had firearms because it was too expensive. Examples are the war between Germany and Poland in 1939 and the Korean War 1950-1953. In the equilibrium of the model with decreased unit costs of effort, the equilibrium amount of training is given by

$$\bar{\tau}_i = \begin{cases} 0 & \text{for } \bar{c} \le \frac{2(r+2)}{r} \quad \Leftrightarrow \frac{V}{\bar{c}} \ \ge \ \frac{r}{2(r+2)}V \\ \frac{r\bar{c}-2(r+2)}{2(r+2)} & \text{for } \bar{c} > \frac{2(r+2)}{r} \quad \Leftrightarrow \frac{V}{\bar{c}} \ < \ \frac{r}{2(r+2)}V. \end{cases}$$

A principal will only invest in decreasing the unit costs of effort for his delegate if the costs of one unit of training are low enough. If the costs for decreasing the unit costs of effort are too high, then the additional effort invested by the agent in the contest does not compensate the principal for his investment. Accordingly, no training is used. One may think of an unskilled worker that only has to operate a machine in a factory. If anyone could handle the machine, then the factory owner will not invest in training. But if a manufactory is considered, training may be used if the output of the agent can be increased by a sufficiently large amount.
The effort made by any delegate in the symmetric equilibrium of the model with an increased weight of effort is given by

$$d_1 = d_2 = \frac{r^2}{(r+2)}\frac{V}{4}.$$

Surprisingly, the effort put in is exactly the same as given in Lemma 3.2. That means the amount of training has no effect on the effort choice of the delegate. Even if the principal trains the agent, the delegate invests as much as in the delegation scenario that only allowed for no-win-no-pay contracts. The reason for this result can be seen in Lemma 5.1. The first derivative of the effort choice of delegate one with respect to training is given by

$$\frac{\partial d_1}{\partial \tau_1} = \frac{r(1+\tau_1)^{r-1}(\alpha_2^r(1+\tau_2)^r - \alpha_1^r(1+\tau_1)^r)}{(\alpha_1^r(1+\tau_1)^r + \alpha_2^r(1+\tau_2)^r)^3}\alpha_1^{r+1}\alpha_2^r(1+\tau_2)^r rV,$$

which can be rewritten as

$$\frac{\partial d_1}{\partial \tau_1} = (p_1 - p_2)\frac{r\alpha_1^{r+1}\alpha_2^r(1+\tau_1)^{r-1}(1+\tau_2)^r rV}{(\alpha_1^r(1+\tau_1)^r + \alpha_2^r(1+\tau_2)^r)^2}.$$

Accordingly, training incentivizes the agent to invest more if he is left behind. This movement stops if both go head to head. If the winning probability of agent one is higher, then training reduces the investment because the same impact is achieved with less effort put in. With relative contracts, that are used in the chapter before, there would be an additional inventive to have a higher probability of winning. But the specific effect on this setting is left to future research.

In the second model the effort invested in the symmetric equilibrium is given by

$$\bar{d}_1 \;=\; \bar{d}_2 = \begin{cases} \frac{r^2}{(r+2)}\frac{V}{4}, & \text{if no training is used, and} \\ \frac{r^3 \bar{c}}{2(r+2)^2}\frac{V}{4}, & \text{if training is used.} \end{cases}$$

As stated above, training is used if $\frac{V}{\bar{c}} < \frac{r}{2(r+2)}V$. We can see that, if training is actually used, then the effort invested by the agent is always higher compared to the situation without training, and compared to the situation where training increases the weight of effort. Using training to decrease the unit costs of effort is therefore an effective mean to increase investment by the agents.

5.4 Comparison

We will first turn to the payoff of the delegate. The probability of winning in the symmetric equilibrium is $\frac{1}{2}$ for both agents in both models. In the model with an increased weight, the effort put in and the share of the contested prize offered by the principal have not changed compared to the situation in Chapter 3. Accordingly, the payoff of the agent also does not change and is given by

$$\pi_{D1} \;=\; \pi_{D2} = \frac{(2-r)}{(2+r)}\frac{rV}{4}.$$

Note that this holds with and without training. On the one hand, the principal contracts the agent and has to ensure participation. Moreover, the principal can determine the weight of the effort put in. The effort made by the delegate is determined by the contract. Because the amount invested is determined, the principal can increase the impact. But the payoff of the agent is not affected because this is solely a decision of the principal. A soldier, for example, acts according to the contract he signed and he also has a certain capability. But the weapon he handles crucially determines the impact he has on the battlefield.

We will now turn to the model with a reduction in unit costs of effort. If no training is used, then, of course, the same result holds as in Chapter 3. But if training is used, then more effort is put in. On the one hand, this reduces the payoff of the agent, on the other hand, the costs are reduced, which is beneficial for the agent. To see which effect dominates, we have a look at the payoff in the symmetric equilibrium:

$$\bar{\pi}_{D1} \;=\; \bar{\pi}_{D2} = \frac{1}{2}\bar{\alpha}V - \bar{\alpha}(1+\bar{\tau})\frac{rV}{4}\frac{1}{1+\bar{\tau}},$$

where $\bar{\alpha}$ represents the share offered in equilibrium by both principals and $\bar{\tau}$ gives the amount of training used in the symmetric equilibrium. Using the result from

Section 5.3, we get

$$\begin{aligned}\bar{\pi}_{D1} &= \bar{\pi}_{D2} = \frac{r}{r+2}\frac{V}{2} - \frac{r^2}{2(r+2)^2}\frac{rV}{4}\frac{2(r+2)}{rc} \\ &= \frac{(2-r)}{(r+2)}\frac{rV}{4}.\end{aligned}$$

We can state that both effects cancel each other out. The reduction in unit costs compensates the agent for the increased effort. Accordingly, in both models the payoff of the delegate is not affected. Whether there is an effect in the future is not considered here.

We will now turn to the decision of the principals. To shorten the analysis, we will concentrate on the first model. To get the results for the second model, we just have to replace c by $\bar{c}$.

If c is rather low, training is expensive and therefore the principal may not use training. In such a situation we are in the model described in Chapter 3 and therefore the result of Wärneryd (2000) still holds: Delegation is able to reduce the investments and therefore increases the payoff of the principals. If c is high, i.e. the unit costs of training are low, then training may be used. The payoff of principal i, i=1,2 with respect to c is

$$\pi_{Pi} = \begin{cases} \frac{V}{(r+2)} & \text{if } c \leq \frac{2(r+2)}{r} \\ \frac{c(2-r)+2(r+2)}{2c(r+2)}V & \text{if } c > \frac{2(r+2)}{r}. \end{cases} \tag{5.9}$$

The critical value of c is therefore $\hat{c} = \frac{2(r+2)}{r}$. As can be seen from the formula, $\hat{c}$ decreases if r increases. If r is high and the impact of the effort in the Contest-Success-Function is high, then the principal will use training even if it is expensive. If r is rather low, then the principal will use training only if it is cheap. For very low values of r, investing effort has only a very limited influence on the probability of winning. If r is close to zero, then the probability of winning is close to $\frac{1}{2}$, irrespectively of how much effort was invested. In such a situation training is not used very much.

If an additional unit of effort put in may improve the chances a lot, then the principal will use everything he has to incentivize the delegate. Accordingly, r influences the decision whether training is used or not. But it has also influence on the amount of training that is used. Once c is greater than the critical value $\hat{c}$, the amount of training used by principal i is $\tau_i = \frac{rc-2(r+2)}{2(r+2)}$. The first derivative with respect to r is $\frac{\partial \tau_i}{\partial r} = \frac{4c}{2r+4}$ and therefore strictly positive. If the impact of effort put in and therefore also the impact of training is large, then the principal will train a lot. This result is quite intuitive. For example, we can think of a soccer team. Although all players

do their best, the result may be bad because they did not train to play together or they did not train the moves.
Considering the payoff of the principal, assume that training is used. The payoff of principal i is given by the lower part of (5.9). Of course, it always holds that $\pi_{Pi} = \frac{V}{(r+2)}$ is greater. It also holds that π_{Pi} is greater than π_i. Using training is beneficial for the principal if the following holds:

$$\frac{c(2-r)+2(r+2)}{2c(r+2)} \quad > \quad (2-r)\frac{V}{4}.$$

This only holds if $c < \frac{4(r+2)}{r(2-r)}$. This leads to:

Theorem 5.3: *The introduction of mandatory delegation and the possibility of training the delegate is beneficial for the principal if and only if*

$$\begin{aligned} c &< \frac{4(r+2)}{r(2-r)}, \quad \textit{and} \\ \bar{c} &< \frac{4(r+2)}{r(2-r)} \quad \textit{respectively.} \end{aligned}$$

On the one hand, if $c \in (\frac{2(r+2)}{r}, \frac{4(r+2)}{r(2-r)})$, then the introduction of delegation is beneficial for the principal, although he trains his delegate. But even if the introduction of training may not be beneficial, the payoff of any principal is always positive. On the one hand, by delegation the prize is split up and therefore incentives to invest are reduced. But on the other hand, training gives the principal the possibility to engage in the contest indirectly and therefore investments increase. The latter effect may even lead to a situation where delegation is not beneficial anymore. The reason for this result is that the principals are kept in a kind of prisoners' dilemma. Both principals would benefit if training is abandoned, but once it is introduced and the costs are not to high, they will use training. If the opponent does not use training, it is beneficial to use it because training leads to an advantage and therefore to a higher probability of winning. Assume, for example, that soldiers have the same capabilities. But the soldier using better equipment may have a higher probability of defeating the enemy.

On a final note, we turn to the cost-parameter of training. If $c = \bar{c}$ then both approaches predict the same behavior by the principals, i.e. the share offered and the amount of training used do not differ. But if training is used, then the predicted effort will always be higher in the model with decreased unit costs.

There are four cases: no training, training in both models and only training predicted by one model. If c and $\bar{c}$ are smaller or equal $\frac{2(r+2)}{r}$, then no training is used by the principals. Training the agent is too costly for the principal in both approaches.
Training is used by both principals if c and $\bar{c}$ are greater than $\frac{2(r+2)}{r}$. The costs of training are sufficiently low, and therefore both approaches predict the principals to train the agent. In this case more efforts are invested in the model with decreased unit costs. And the model with a higher costs-parameter also predicts a higher amount of training that is used in equilibrium.
If only c or $\bar{c}$ are greater than $\frac{2(r+2)}{r}$, then only the model with the smaller costs-parameter will predict no training. If training is used to decrease the unit costs of effort, then more effort is invested.

5.5 Conclusion

The introduction of mandatory delegation with no-win-no-pay contracts into a Tullock rent-seeking contest decreases the effort made and increases the payoff of the principal. The contested prize is split up and therefore the incentives to invest are lower for the agent compared to the incentives a principal has without delegation. In this chapter this model is also applied and it is considered that the principal may influence the weight of the effort made by the agent in the contest. It is also considered that the principal can decrease the agent's unit costs of investing effort. This is used to model that the principal can train his delegate. This also includes the possibility that the principal can support his delegate. The principal pays for the training. There are no additional costs for the agent. The costs of training are dependent on the contested prize. The marginal costs are constant. Because of the specification of the Contest-Success-Function, the effect of one additional unit of training decreases in the amount of training used. Normally, a machine gun has a fixed price. The more machine guns are on the battlefield, the smaller is the effect of an additional machine gun. And therefore, more machine guns are needed to get the same impact the first machine gun had. The model has two stages. At the first stage the principal contracts the agent and determines either to what extent the skills of the delegate are improved, or by how much the unit costs are decreased. At the second stage the contest is played by the delegates and after they invested, the winner is determined and payments take place. An army is a good example to demonstrate this. The strength of an army crucially depends on the weapons it uses and how well trained the soldiers are. The soldiers are contracted to serve in the army, but the impact of the effort they put in depends on the support the army gives to them. Firms also constantly invest in the skills of their employees. They also invest in technical support. The skills of an agent have been modeled before by Baik and Kim (1997) and Schoonbeek (2007). Schoonbeek assumes that an agent can use two instruments. One instrument increases the effect of the other instrument. A similar approach is used here. Training acts as the instrument that increases the impact of the other instrument, i.e. the effort the agent puts in. It is also considered here that training does not have an effect on the weight of the invested effort directly. Training can be used to decrease the unit costs of effort and therefore increases the investment of the agent.

In both models the principal can influence the outcome of the contest. He can use the terms of the contract, and, additionally, he can decide on how to improve the skills of the agent. In the first model the contract determines the effort the agent puts in. But the impact of the effort is influenced by the principal. In the second model training influences the investment of the agent. Training is therefore a way to bypass mandatory delegation. By investing in training, the principal intervenes in the contest. Whether the principal uses training or not depends on the costs of training and on the marginal efficiency of effort. If the costs of training are high, then the principal may not use training. Accordingly, the beneficial effect of mandatory delegation is still intact. But if the costs are low, then the principal uses training and therefore increases the weight the effort of his delegate has or decreases the unit costs, respectively. If modern weapons are cheap, then an army may be modernized with a higher probability than when new weapons are expensive.

The marginal efficiency of effort influences the decision on whether to use training or not, and it influences the amount of training that is used. If the effect of effort on the probability of winning is only small and the decision on the winner is more like a coin flip, then the principal will use training only if the costs a very low. He will also use a small amount of training. If the marginal efficiency is high, i.e. the effect of effort is very high, then training is used even if the costs are high. He will also use a lot of money to improve the skills of the agent. If a new weapon can ensure victory, then this weapon is worth much to the army and it may also be used very often.

Because of the way to bypass mandatory delegation, the effect on the payoff of the principal is negative. A situation without training and with mandatory delegation is best for the principal. If the costs of training are moderate, then the situation with training leads to a higher payoff compared to a situation without delegation. But if training is very cheap, then abolishing mandatory delegation is beneficial for the principal. The principals are kept in a kind of prisoners' dilemma. Not using training gives a disadvantage if the opponent uses training. This decreases the probability of winning for the principal that does not use training.

An example is the use of poison gas in the First World War. It was used by the Germans to end positional warfare and to help the soldiers win. In the end, the gas was used by both sides, increased the death toll, and the positional warfare held on. The payoff of the agents does not change. In the model with increased weight the agent's decision on how much effort to put in is determined by the contract. The contract is the same as in the situation without training and therefore they do not invest more. They also do not pay for training. Accordingly, the payoff in the symmetric equilibrium is the same as without training. In the second model, training increases the investment of the agent. But because of the decreased unit costs, the payoff is held constant.

Overall, the effect of training is therefore negative. The principal loses and the delegate does not gain anything. Only the investments in the contest are increased. One policy implication is therefore to prevent the costs of training from being too low if the principals are in a contest. Taxation may be used to reach this goal.

The question of whether the results also hold if interdependent preferences are included or if the contract space is extended or not, is a topic that is postponed to the future. It is an interesting approach to combine both models, i.e. training increases the weight of effort *and* decreases the unit costs. A machine gun is one example. Using a machine gun improves the impact of a soldier. This is the direct effect on the Contest-Success-Function. Additionally training can be used to improve handling. This makes fighting with the gun more efficient and therefore can be seen as a reduction in unit costs of effort.

Chapter 6

Summary and Future Research

6.1 Summary

Contests are omnipresent in human society. Everywhere people are investing irreversible effort to win an indivisible prize. The effort put in the contest does not increase the welfare of the society. These investments belong to the dark side of the force, as Hirshleifer (2001) called it. Instead of creating wealth, people try to appropriate the wealth that was created by other people. Contests are used here to model this process of appropriation. In the second chapter, we had a look at the effect of evolution on the effort exerted in contests in finite populations. In fact, we asked whether it is beneficial in evolutionary terms to overvalue the prize in a Tullock contest or not. We found that overvaluation can indeed be beneficial. Because of a higher valuation for the contested prize, the players invest more and the opponents may be discouraged and therefore exert less effort. We found two effects: an incentive-effect and a discouragement-effect. The discouragement-effect is only operant in contests with more than two individuals, but not in playing the field contests. Even without the discouragement-effect overvaluation occurs.

A comparison is drawn between the indirect evolutionary approach concerning prize perceptions, the direct evolutionary approach and the indirect evolutionary approach according to Leininger (2009). All three approaches predict the same behavior in equilibrium for two-player contests and for playing the field contests. The direct evolutionary approach and the indirect evolutionary approach that introduces preferences for the opponents put a weight on the contested prize implicitly. In the direct evolutionary approach this weight is constant. The indirect evolutionary approach according to Leininger (2009) produces exactly the same aggressive behavior as the indirect evolutionary approach using prize perceptions because the weights on the prize are identical. The implicit weight is not fixed. This weight is therefore allowed to evolve evolutionarily and in equilibrium amounts to the same amount as in the new indirect evolutionary approach. This weight depends crucially on the size of the population. Accordingly, the material payoffs are the same. Only the utility of the players differs. In the indirect evolutionary approach according to Leininger (2009) the utility is smaller as long as we do not face overdissipation of the contested prize. The reason for this is that the material payoff of the opponent is not subtracted and that the prize is overvalued. The individuals feel better because of their desire to win and they are not depressed by their fear of losing. Although, they invest more compared to a contest where all value the prize equally and correctly. In finite populations this yields an evolutionary advantage. Accordingly, it is shown that all three approaches lead to more effort put in to appropriate wealth that other people created. This makes the problem of unwanted investments even more severe. It was our aim to show that delegation can be a tool to decrease such wasteful investments. Wärneryd (2000) showed that mandatory delegation may decrease efforts invested in a two-player Tullock rent-seeking contest. Because the contested prize is split up between principal and agent, the agent acts not as aggressively as the principal would do. Accordingly, efforts made are reduced and expected payoffs for the principals are increased. But this is not the end of the story. On the one hand, the contract space is limited, and on the other hand, the principal has only the contract to gain influence on the contest.

In fact, the result of Wärneryd only holds true for no-win-no-pay contracts. In this book it is shown that no-win-no-pay contracts are only one example of a broader class of contracts. In fact, a fine and a reward, both depending on the contested prize, are introduced. Any principal can punish his delegate for a defeat. In equilibrium the share offered as a reward and the share used as a fine must add up to one. By using appropriate contracts, the principal can eradicate the effect of the split up of the prize on the incentives of the agent. The expected payoff for the agents in equilibrium is always zero, but the payoff for any principal is positive and is as high as in a situation without delegation. Accordingly, the principals are indifferent between delegating or not and therefore delegation does not need to be mandatory. There may also be asymmetric equilibria.

Introducing mandatory delegation alone is not beneficial, but introducing mandatory delegation and restricting the possible contracts combined with law enforcement is. If the contract space is limited, for example by law, then wasteful investments in contests can be reduced. An example is the lawyers' compensation act (Rechtsanwaltsvergütungsgesetz) in Germany. By introducing upper bounds, even a positive payoff for the agent is possible.

Another important result is the influence of negatively interdependent preferences on the side of the principal on the chosen contract. These preferences are created by evolution in finite populations. The incentives of a principal with negatively interdependent preferences to win the contest are higher than for a principal with individualistic preferences. But the participation constraint keeps the first principal from transferring the increased incentive to the agent. Accordingly, the negative effect of interdependent preferences is mitigated and delegation can indeed decrease efforts invested, even with the broader contract set.

In Chapter 3, we also consider individualistic principals that contract agents with interdependent preferences. the agents are concerned with the outcome for the agent, but not for their principal or the principal of the other agent. We show that delegation can also decrease the effort invested if agents have negatively interdependent preferences. Because of his wish of defeating the other agent, the agent is intrinsically motivated to invest. The principal therefore reduces the total incentives. In fact, he reduces the fine and the reward. This reduction is high enough to make the agent invest less compared to a situation without agents with negatively interdependent preferences. The material payoff of any agent is zero, but the material payoff of the principal increases.

If the agent is intrinsically motivated by his negatively interdependent preferences, the principal can increase his share of the prize.

In Chapter 4 we have look at a special form of contracts that are also used in the literature: relative contracts. Interdependent preferences are assumed in this chapter. It is shown that by delegating the players are better off in a two-player Tullock contest in this particular situation. We assume that the delegates do not have a particular concern regarding the opponent. Accordingly, the result from Chapter 3 also holds for relative contracts. With prescribed delegation, each player can do at least as well as if he was in a group consisting only of individualistic payoff maximizing players, i.e. "spiteful" preferences are "neutralized" in the contract game. No-win-no-pay contracts can even overcompensate the negative effects of weighted relative payoff maximizing behavior. But since interdependent preferences yield an advantage in equilibrium in rent-seeking contests with contracts that reward the delegate according to his relative success, these are developed and used by the principals. In essence, the economic institution of contracting delegates can completely offset the inefficiency caused by negatively interdependent preferences. This result also holds for relative contracts. In theory, delegation may have an even better effect, but is held back by the same competitive forces at contracting level. More aggressive contracts drive out more moderate ones.

It is shown for relative contracts that there is a fixed amount the principal has to pay to hire a delegate in the equilibrium of the contract choice game. The fixed amount does not alter the invested effort but is necessary in order to make an agent willing to sign the contract. A game-structure like in a prisoners' dilemma prevents the delegate from acting as a free rider.

Again, one policy implication of the above analysis is that mandatory delegation combined with a limited contract space should be introduced.
The possibility that a principal can intervene into the contest is addressed in Chapter 5. It is considered that the principal may influence the weight the effort exerted by the agent has in the contest. This approach is used to model that the principal can train his delegate. This also includes the possibility of supporting the delegate. The principal pays for the training. The costs of training are dependent on the contested prize. At the first stage the principal contracts the agent and determines to what extent the skills of the delegate are improved. At the second stage, the contest is played by the delegate and after they invested, the winner is determined and payments take place.
This means that the principal can influence the outcome of the contest in two ways: He can use the terms of the contract, and, additionally, he can decide on how to improve the skills of the agent. We considered two approaches of how training affects the outcome. In the first approach the terms of the contract determine the effort the agent puts in. But the impact of the effort is influenced by the principal. In the second approach the unit costs of effort are decreased by using training. Accordingly, the effort exerted by the agent is determined by the amount of training and the terms of the contract. Whether the principal uses training or not depends on the costs of training and on the marginal efficiency of effort. If the costs of training are high, then the principal may not use training. Accordingly, the beneficial effect of mandatory delegation is still intact. But if the costs are low, then the principal uses training and therefore increases the weight of his delegate's effort.
The marginal efficiency of effort influences the decision on the use of training and it influences the amount of training that is used. If the decision on the winner is more like a coin flip (r is close to zero), then the principal will use training only if the costs are very low. He will also use only a small amount of training. If the marginal efficiency is high, then training is used even if the costs are high. The principal will also use a large amount of money to improve the skills of the agent.
But the use of training influences the payoff of the principal negatively. It is also possible that the principal is worse off compared to a situation without delegation. Accordingly, giving the principal the opportunity to gain influence on the decision leads to an increase in total investment. And therefore the desired effect of delegation is only present if the principal cannot intervene into the contest. The principals are kept in a kind of prisoners' dilemma. Not using training is disadvantageous if the opponent uses training and therefore increases his probability of winning. On the other hand, the payoffs of the agents do not change. In the approach that considers an increased weight, his decision on effort to put in is determined by the contract. The contract is the same as in the situation without training and therefore they

do not invest more. They also do not pay for training. Accordingly, the payoff in the symmetric equilibrium is the same as without training. In the second approach, training increases the effort exerted by the agents because the unit costs are decreased. The reduction in unit costs is positive for the agent. In equilibrium both effects cancel each other out and the payoffs of the agents do not change compared to a situation without training. Note that we only considered one single contest. We do not consider the effect of training on contests played in the future.
In essence, it is shown that delegation can indeed reduce the wasteful investments. The negative effect of negatively interdependent preferences is neutralized by delegation. Because the delegate is only interested in his material outcome, he does not act as aggressively as the principal would do. This holds true despite the fact, that the principal can incentivize the agent with the full objective value of the contested prize by using an appropriate contract. The positive effect of delegation may also be mitigated if the principal has the opportunity to train his delegate.

6.2 Future Research

In the last decades, there has been an increasing interest in introducing concepts of various sciences into economic literature. In Chapter 2, we explained how findings from biology were introduced. Another finding that enriches economics, is also of interest for delegation and therefore for research. In an article, written in 2000, Akerlof and Kranton introduced identity into the economic analysis. They define identity as a person's sense of self. Akerlof and Kranton state that "Identity can account for many phenomena that current economics cannot well explain." (p.715). Examples are the economics of poverty and exclusion. A person thinks in social categories. The categories the individual is in, or sometimes chooses to be in, define the identity of this person. A social category is gender, for example. For the individual there are ways the people in each social category should behave. An individual has a disutility if he is not able to behave like his social category tells him to do. He also has a disutility if other individuals do not behave like they should do. A woman, for example, feels a discomfort if another woman next to her dresses and talks like a male construction worker. Or a manager does not like it when another manager collects bottles on the street. And it is also not right to the manager if he collects bottles because such a behavior is attributed to poor and unemployed people.
In 2005 Akerlof and Kranton extended their model to organizations. In their introduction they use the drill at West Point as an example. They write about the army: "They wish to inculcate non-economic motives in the cadets so that they have the same goals as the U.S. Army."(p.9)
That means, they introduce another way how the behavior of individuals can be

influenced. Namely by changing the experienced identity.
It is an interesting endeavor to transfer this topic to a contest with delegation. So far, a principal can use the contract to influence the behavior of the delegate, but now he can also influence the behavior by changing the way the delegate sees himself. That means, the principal might convince the agent that they have a common goal. Akerlof and Kranton (2005) state this for the Army. In Chapter 5 we saw how training can affect the weight the effort invested by the agent has. But we neglected the effect on identity. To incorporate this additional effect in the future, we want to use a three stage model to show the consequences of the introduction of identity. We will use a two-player Tullock-contest. At the first stage, both principals choose to delegate or not to delegate. If at least one principal delegates, the delegate is contracted at the second stage. The contract consists of a monetary component and training. The monetary component is a share of the contested prize the delegate gets if the contest is won. Training increases the ability of the delegate, on the one hand, on the other hand, it makes the delegate also regard the payoff of the principal. Training is paid for by the principal. At the last stage the contest is played and payments take place.
In the game there are four subgames: No one delegates, both delegate or only one principal delegates. We want to explain shortly the mutual delegation subgame to demonstrate the model. The model is again solved by backward induction. $r = 1$ is assumed to ease calculus. The payoff of the delegate of principal i ($i = 1, 2$) is given by

$$\pi_{Di} = \frac{(1+\theta_i)d_i}{(1+\theta_i)d_i + (1+\theta_{-i})d_{-i}}\alpha_i V - d_i + \theta_i \frac{(1+\theta_i)d_i}{(1+\theta_i)d_i + (1+\theta_{-i})d_{-i}}(1-\alpha_i)V.$$

θ_i ($\theta_i > 0$) represents, on the one hand, to what extend the delegate identifies himself with the principal (shown by his principal's payoffs). On the other hand the change of identity is due to training. As in West Point, this training may also increase abilities. Accordingly, θ_i increases the weight the effort of the delegate d_i has in the Contest-Success-Function. This is the way Baik and Kim (1997) modeled abilities of delegates. Like in the chapters above, α_i represents the share of the contested prize the principal pays to the delegate if the contest is won.
Without identity the prize is split up. Principal and delegate only cared for their share. The introduction of training reversed this split up. The equilibrium amount of training and the according degree of reversal is determined in the second stage of the game and crucially depends on the costs of training.
The equilibrium at the third stage is given by the winning probability

$$p_i = \frac{(1+\theta_i)(\alpha_i + \theta_i(1-\alpha_i))}{(1+\theta_i)(\alpha_i + \theta_i(1-\alpha_i)) + (1+\theta_{-i})(\alpha_{-i} + \theta_{-i}(1-\alpha_{-i}))}.$$

$(1+\theta_i)$ gives the effect of the increased ability. $\alpha_i + \theta_i(1-\alpha_i)$ gives the total incentive of the agent. As stated above, a positive θ_i mitigates the split up of the prize. For $\theta_i = 1$, a delegate is incentivized with the whole prize and the share α_i does not matter anymore. We can also think of a delegate that thinks that he and his principal share the same goals.
At the second stage, principal i determines the contract. His problem is

$$\begin{aligned} \max_{\alpha_i,\theta_i} \quad & \frac{(1+\theta_i)(\alpha_i+\theta_i(1-\alpha_i))}{(1+\theta_i)(\alpha_i+\theta_i(1-\alpha_i))+(1+\theta_{-i})(\alpha_{-i}+\theta_{-i}(1-\alpha_{-i}))}(1-\alpha_i)V \\ - \quad & \theta_i(1-\alpha_i)\frac{V}{c}, \\ \text{wrt} \quad & \pi_{Di} \geq 0. \end{aligned}$$

$\theta_i(1-\alpha_i)\frac{V}{c}$ stands for the costs of training. $(1-\alpha_i)$ was included to express a so called "real-world" phenomenon: It is easier to convince someone that your motives are good when you are not greedy. A commander that eats the same things his soldiers eat and sleeps in the same tents has more motivated soldiers than a commander that sleeps in the hotel far away from the enemy. Also soldiers tend to follow commanders more glowingly that share the glory and the money they get with their soldiers. And a commander that gives away everything he earns can convince soldiers at low costs.
c was introduced to ensure that there are no extreme solutions. If the costs of training are too high, then the standard model without identity results. If the costs of training are too low, then the soldier may believe in the aims of the principal more than the principal does. This case is given for $\theta_i > 1$. Religious leaders whose followers are willing to die for their beliefs, but the leader is not willing to die and prefers negotiations, are an example. $c = 1.5$ is an example for a situation with a θ between one and zero. We want to solve the given model for all four subgames and determine under which circumstances this kind of training is used.

Another shortcoming of this research at hand so far is the assumption that there is an objective value of the prize. Individuals have different valuations for the same thing. Bajari and Hortacsu (2003) show for coin auctions on ebay that even if there is a book value for the coin, the paid price can differ considerably from this value. Accordingly, it is worthwhile to introduce heterogeneity to the valuation of the prize.

Chapter 7

References

Akerlof, G. A. and Kranton, R. E. (2000). Economics and Identity. *The Quarterly Journal of Economics*, Vol. 115, Issue 3, 715-753.

Akerlof, G. A. and Kranton, R. E. (2005). Identity and the Economics of Organizations. *Journal of Economic Perspectives*, Volume 19, Number 1, 9-32.

Alchian, A. (1950). Uncertainty, evolution and economic theory. *Journal of Political Economy*, 58, 211-221.

Alos-Ferrer, C. and Ania, A. B. (2005). The evolutionary stability of perfectly competitive behavior. *Economic Theory*, 26, 497-516.

Appelbaum, E. and Katz, E. (1987). Seeking Rents by Setting Rents: The Political Economy of Rent-Seeking. *The Economic Journal*, 1987, 685-699.

Baik, K. H. (2007). Equilibrium Contingent Compensation in Contests with Delegation. *Southern Economic Journal*, 73, 986-1002.

Baik, K. H. and Kim, I.-G. (1997). Delegation in Contests. *European Journal of Political Economy*, Vol. 13 1997, 281-298.

Bajari, P. and Hortacsu, A. (2003). The Winner's Curse. Reserve Prices, and Endogenous Entry: Empirical Insights from ebay Auctions. *The Rand Journal of Economics*, Vol.34 No.2, 329-355.

Boudreau, J. and Shunda, N., On the Evolution of Prize Perceptions in Contests (August 25, 2010). Available at SSRN:
http://ssrn.com/abstract=1666394 or http://dx.doi.org/10.2139/ssrn.1666394

Chowdhury, S. M. and Sheremeta, R. M. (2010). A generalized Tullock Contest. *Public Choice*, 147, 413-420.

Congleton, R., Hillman, A. and K. Konrad (eds.) (2008).*40 Years of Research on Rent*

Seeking, Volumes 1 and 2, Springer Verlag, Heidelberg

Davis, J. B. (2007). Akerlof and Kranton on identity in economics: inverting the analysis. *Cambridge Journal of Economics*, 31, 349-362.

Dixit, A. (1987). Strategic behavior in contests. *American Economic Review*, 77, 891-898.

Dufwenberg, M. and Güth, W. (1999). Indirect evolution vs. strategic delegation: a comparison of two approaches to explaining economic institutions. *European Journal of Political Economy*, Vol. 15, 281-295.

Dufwenberg, M. and Güth, W. (2000). Why do you hate me? On the survival of spite. *Economics Letters*, 67, 147-152.

Eaton, B. C. and Eswaran, M. (2003). The evolution of preferences and competition: a rationalization of Veblen's theory of invidious comparisons. *Canadian Journal of Economics*, 36, 832-859.

Fershtman, C. and Judd, K. L. (1986). Strategic Incentive Manipulation in Rivalrous Agency. *Hoover Institution Working Papers in Economics*, E-87-11 1987.

Fershtman, C. and Judd, K. L. (1987). Equilibrium Incentives in Oligopoly. *The American Economic Review*, 77, 927-940.

Fershtman, C., Judd, K. L. and Kalai, E. (1991). Observable Contracts: Strategic Delegation and Cooperation. *International Economic Review*, 32, 551-559.

Fershtman, C. and Kalai, E. (1997). Unobserved Delegation. *International Economic Review*, 38, 763-774.

Garfinkel, M. R. and Skaperdas, S. (2007). Economics of Conflict. An Overview. In T. Sandler and K. Hartley (eds.), *Handbook of Defense Economics, Vol. II* (pp. 649-709), New York: North-Holland.

Guse, T. and Hehenkamp, B. (2006). The strategic advantage of interdependent preferences in rent-seeking contests. *Public Choice*, 129, 323-352.

Güth, W. and Peleg, B. (2001). When will payoff maximization survive? An indirect evolutionary analysis. *Journal of Evolutionary Economics*, 11, 479-499.

Güth, W. and Yaari, M. (1992). An evolutionary approach to explaining reciprocal behavior in a simple strategic game. In U. Witt (Ed.), *Explaining process and change - approaches to evolutionary economics* (pp.23-34). Ann Arbor: University of Michigan Press.

Haavelmo, T. (1954). A Study in the Theory of Economic Evolution. North Holland Publishing Company.

Hamilton, W. (1970). Selfish and spiteful behavior in an evolutionary model. *Nature*, 228, 1218-1220.

Harris, M. and Raviv, A. (1979). Optimal Incentive Contracts with Imperfect Information. *Journal of Economic Theory*, 20, 231-259.

Hehenkamp, B., Leininger, W. and Possajennikov, A. (2004). Evolutionary equilibrium in Tullock contests: spite and overdissipation. *European Journal of Political Economy*, 20, 1045-1105.

Hirshleifer, J. (1988). The Analytics of Continuing Conflict. *Synthese*, 76, 201-233.

Hirshleifer, J. (1991). The Technology of Conflict as an Economic Activity. *American Economic Review*, 81, 130-134.

Hirshleifer, J. (2001). The Dark Side of the Force: Economic Foundations of Conflict Theory. *Cambridge University Press* ISBN:0521009170

Katz, M. L. (1991). Game-playing agents: unobservable contracts as precommitments. *RAND Journal of Economics*, 22, 307-328.

Kockesen, L. (2007). Unobservable contracts as precommitments. *Economic Theory*, 31, 539-552.

Kockesen, L. and Ok, E. A. (2004). Strategic Delegation by Unobservable Incentive Contracts. *Review of Economic Studies*, 71, 397-424.

Kockesen, L., Ok, E. and Sethi, R.(2000). The strategic advantage of negatively interdependent preferences. *Journal of Economic Theory*, 92, 274-299.

Konrad, K. (2009). Strategy and Dynamics in Contests. *Oxford University Press*, ISBN 978-0-19-954960-3.

Lockard, A. and Tullock, G. (eds.) (2001).*Efficient Rent-Seeking. Chronicle of an Intellectual Quagmire*, Kluwer Academic Publish- ers, Boston.

Leininger, W. (2003). On evolutionarily stable behavior in contests. *Economics of Governance*, 4, 177-186.

Leininger, W. (2009). Evolutionarily stable preferences in contests . *Public Choice*, 140, 341-356.

Lockard, A. and Tullock, G. (eds.) (2001).*Efficient Rent-Seeking. Chronicle of an Intellectual Quagmire*, Kluwer Academic Publish- ers, Boston.

Maynard Smith, J. (1974). The Theory of Games and the Evolution of Animal Conflicts. *Journal of Theoretical Biology*, 47, 209-221.

Pratt, J. W. (1964). Risk Aversion in the Small and in the Large. *Econometrica*, Vol. 32, No.1/2, pp. 122-136.

Riechmann, T. (2007). An analysis of rent-seeking games with relative-payoff maximizers. *Public Choice*, 133, 147-155.

Rosen, S. (1986). Prices and Incentives in Elimination Tournaments. *American Economic Review*, 76, 701-14.

Schaffer, M. (1988). Evolutionarily stable strategies for a finite population and a variable contest size. *Journal of Theoretical Biology*, 132, 469-478.

Schelling, T. C.(1960). *The Strategy of Conflict*. 1980th Edition,Harvard University Press, Cambridge and London 1980.

Schoonbeek, L. (2002). A delegated agent in a winner-take-all contest. *Applied Economics Letters, Taylor and Francis Journals*, 9, 21-23.

Schoonbeek, L. (2004). Delegation in a group-contest. *European Journal of political Economy*, 20, 263-272.

Schoonbeek, L. (2007). Delegation with multiple instruments in a rent-seeking contest. *Public Choice*, 131, 453-464.

Skaperdas, S. (1996). Contest success functions. *Economic Theory*, Nr. 7 (1996), 283-290.

Sklivas, S. D. (1987). The strategic choice of managerial incentives. *RAND Journal of Economics*, Vol. 18, No. 3, Autumn 1987, 452-458.

Tollison, R. D. (1982). Rent Seeking: A Survey. *Kyklos*, Vol. 35 -1982-, 575-602.

Tullock, G. (1967). The Welfare Costs of Tariffs, Monopolies, and Theft. *Western Economic Journal*, Vol. 5, June 1967, 224-232.

Tullock, G. (1980). Efficient Rent Seeking. In Buchanan, Tollision and Tullock (Eds.), *Toward a theory of the rent-seeking society* (pp. 3-15). College station: Texas A&M University Press.

Vega-Redondo, F. (1996). *Evolution, Games and Economic Behavior*. Oxford University Press.

Vega-Redondo, F. (1997). The Evolution of Walrasian Behavior. *Econometrica*, 65, 375-384.

Vickers, J. (1985). Delegation and the Theory of the Firm. *The Economic Journal*, 95, 138-147.

Wärneryd, K.(2000). In Defense of Lawyers: Moral hazard as an Aid to cooperation. *Games and Economic Behavior*, 33, 145-158.