SFB 823

Evaluating the interplay of term premia, monetary policy, and the economy in the euro area

Fabian Herrmann

Nr. 42/2015

Discussion Paper

Evaluating the Interplay of Term Premia, Monetary Policy, and the Economy in the Euro Area

Fabian Herrmann[1]

TU Dortmund University

Version: October 20, 2015

Abstract

This paper investigates the interplay of term premia, monetary policy, and the economy in the euro zone. For this purpose I use a no-arbitrage macro-finance model of the term structure of government bond yields as in Ireland (2015), where yields are modeled as linear-affine functions of the state vector. Movements in term premia are captured by an unobservable risk variable. Restrictions on the dynamic of the state equation are entailed in order to identify the structural model. The model is estimated using Bayesian estimation techniques. The results highlight a rich dynamic between term premia, monetary policy, and the economy. In line with the "practitioners view" I find that an exogenous rise in premia dampens economic activity. Moreover, during the sample period, the ECB lowered the nominal short-term interest rate in response to a rise in term premia.

JEL: C11, E43, E44, E52, G12.

[1]TU Dortmund University , Vogelpothsweg 87, 44227 Dortmund, Germany. Phone: +49 231 755 3441. Email: fabian.herrmann@tu-dortmund.de. Financial support from German Science Foundation (DFG) through SFB 823 is gratefully acknowledged.

1 Introduction

Standard decomposition of yields separates the yield of a long-term bond into an expectation part and a term premium part. The expectation part consists of the average of the expected sum of short-term interest rates until the bond matures while the term premium part compensates risk-averse investors for the risk of holding longer-dated instruments. In order to affect the economy, manipulating the expectations of the future short rates by the forward guidance of future short-term interest rates is one important tool of central banks, as emphasized by Woodford (2005). This routine is known as the term-structure expectation channel. However, to the extent that aggregate demand depends, among other macroeconomic factors, not only on the short-term interest rate but also on long-term interest rates, by influencing the term structure premium incorporated in long-term bond yields, there is another, less conventional way, how central banks might be able to affect economic activity. This paper analyses the effects of movements in term premia on the economy, the effects of monetary policy on term premia, and whether the ECB responds, in turn, on term premia movements.

The effects of variations in term premia on the economy, and how monetary policy affects these premia, are in the focus of policy makers and researchers, not solely, but especially since the financial crisis. During the crisis, with the short-term nominal interest rate at the zero lower bound, unconventional methods of monetary policy sought to reduce term premia in long-term bond yields in order to ease financial conditions. Specifically, as noted by former Federal Reserve Chairman Ben Bernanke (2013, p.7), "to the extent that Treasury securities and agency-guaranteed securities are not perfect substitutes for other assets, Federal Reserve purchases of these assets should lower their term premiums, putting downward pressure on long term interest rates and easing financial conditions more broadly." But also before the onset of the financial crisis, the effects of changes in term premia on the economy and the response of monetary policy to these fluctuations were considered by researchers and policy makers. As explained, again, by then Federal Reserve Chairman Bernanke (2006), "if spending depends on long-term interest rates, special factors that lower the spread between long-term and short-term interest rates will stimulate aggregate demand. Thus, when the term premium declines, a higher short-term rate is required to obtain the long-term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices". Rudebusch, Sack, and Swanson (2007) discuss and summarize this view under the expression "practitioner view". The practitioner view states two assumptions. Firstly, a drop in the term premium and with it in long-term yields, all else is being equal, works to stimulate aggregate demand and output. Secondly, optimal monetary policy requires the central bank to counteract the drop in the premium in order to balance output and inflation. Though this view is prevalent among practitioner (Rudebusch, Sack, and Swanson, 2007), surprisingly less evidence for it has been found so far. The empirical findings of the effects of changes of the premium on output are rather mixed, ranging from exactly the opposite relationship of what one would expect from the practitioner view to the expected inverse relationship between term premia and output. Since a broad literature focuses on the effects of movements of term premia on output, the next section serves a literature overview of the effects of term premia movements on GDP.

However, not are only the effects of changes in term premia on output unclear, but also how monetary policy should respond to these changes (if it responds at all). The practitioner view advocates that in response to a rise in the term premium, the central bank should lower the policy rate to offset the increase.[2] In contrast, Goodfriend (1993) and McCallum (2005) argue that the central bank should increase the short-term interest rate in response to a rise in the term premium. Both interpret the rise in the term premium as evidence for an increase in inflation scares which the central bank should fight by raising the short-term interest rate. More recently, Ireland (2015) investigates the response of monetary policy to changes in the term premium for the US. He provides evidence that an increase in the premium led the Fed tighten monetary policy.

This paper seeks to evaluate the interplay of monetary policy, term premia and the economy in the Euro Area. My analysis focuses on the euro area before and during the financial crisis in order to investigate if movements in term premia affect output and inflation, whether the ECB responds to these movements, and how term premia in turn responds to conventional monetary policy actions. For this purposes, I apply the macro-finance model of the term structure proposed by Ireland (2015) to the euro area.

The recent period raises questions about a non-negativity constraint or lower-bound constraint on the interest rate processes, usually known as the “zero lower bound”. While

[2] Indeed, Carlstrom, Fuerst and Paustian (2014) demonstrate in a DSGE model with segmented financial markets and imperfect financial intermediation that a negative response coefficient in the monetary policy rule on the term premium increases welfare modestly.

the results of Bauer and Rudebusch (2015) stress the relevance of shadow rate models (a particular class of term structure models that respects a lower bound for the short-term interest rate process) for the US, the need of this kind of models for the Euro area is less clear. Indeed, as argued by Christensen and Krogstrup (2014, 2015) standard Gaussian modelling approaches appear to be fully warranted, since particular bond yields in Europe (in their example: German and Swiss bond yields) "have actually been well below zero for intermediate maturities and for extended periods in recent years." Thus, they do not find it obvious that a lower bound should be enforced. Also Dewachter et al. (2014b), using government bond yields of five European countries, do not enforce a zero lower bound on European bond yields and on the process of the risk-free short-term interest rate (proxied by the OIS rate).

For analyzing the yield curve, and especially term structure premia, macro-finance models bring along several benefits. In contrast to pure finance models, macro-finance models allow bond prices and macroeconomic fundamentals to evolve jointly over time. The short-end of the yield curve, that is, the short-term risk-free interest rate is under the control of the central bank, using information of the state of macroeconomy helps to model the short-term interest rate process. Moreover, evidence shows that term premia are not only time varying, but also different across bond maturities. Exploiting all information available over the entire yield curve helps to identify the term premium and thus to separate term premia from the expectation part of long-term yields. Instead of determining specific channels through which macroeconomic and other shocks affect premia and premia affect the economy, macro-finance models do not specify a particular transmission channel. This is in particular appealing because of the conflicting evidence of the effects of movements in term premia on the economy from previous empirical studies.

Yet, some assumptions to ensure identification and to make yield equations consistent with each other in the cross section and the time series have to be made. In order to model the dynamics of yields consistently over the yield curve, cross-equation restrictions are needed. Based on Duffie and Kan (1996), these cross-equation restrictions arise from the assumption of the absence of arbitrage opportunities in bond markets. The precise specification of the term structure part of the model follows Dewachter and Iania (2012), Dewachter et al. (2014a), and Ireland (2015): In order to evaluate the interplay of term premia movements, monetary policy and the economy, a latent risk variable that captures term premia movements is employed. In the spirit of Cochrane and Piazzesi (2005, 2008), the risk variable is constructed to be the only force that drives the one-period expected excess holding return (the one period-return premium) and is integrated into the state space system. The dynamics of the state variables are modeled as a structural vector autoregressive (VAR) model. The risk variable responds to all state variables and, based on evidence that a large fraction of variations in term premia is not fully spanned by macroeconomic factors (Cochrane and Piazzesi, 2009, and Joslin, Priebsch and Singleton, 2014), also exhibits an autonomous dynamic. Moreover, while Dewachter and Iania (2012) and Dewachter et al. (2014a) does not allow term premia to affect the economy, following Ireland (2015), the model allows for feedbacks from term premia movements to the economy. Identification of the structural shocks of the state equations is achieved by imposing restrictions on the contemporaneous relation among the variables of the state equation. The estimation of the model is carried out by Bayesian estimation techniques. The likelihood function is constructed using the Kalman filter. The posterior is evaluated using an Adaptive Metropolis (AM) algorithm in the lines of Haario, Saksman and Tamminen (2001).

My results reveal a rich dynamic between term premia, monetary policy and the economy. In line with the practitioner view, I find that a rise in term premia is associated with a drop in the output gap and a drop in inflation. The ECB lowers the short-term interest rate in response to an increase in term premia. Thus, during the sample period, the ECB mitigates the effect of a rise in the term premium on the yield curve by lowering the short-end of the yield curve. However, I find only negligible effects of conventional monetary policy on term premia in turn.

The remainder of the paper is organized as follows. The next section serves a literature overview of the effects of term premia movements on output. Section 3 explains the macro-finance model and discusses the decomposition of the yield curve into the expectation part and term premia part. The next Section casts the model into the state space system, describes the data and discusses the estimation procedure and the prior distribution. Section 5 presents and discusses the results of the estimation. The last Section concludes.

2 Literature overview

This section covers a literature overview of the empirical results and the theoretical consideration of the effects of term premia movements on output.

In standard linearized New-Keynesian models, term premia do simply not exist. Log-linearization eliminates higher order terms like term premia by construction. In order to analyze term premia in a DSGE framework, limits-to-arbitrage or non-linear setups are required. Rudebusch, Sack and Swanson (2007) show that a non-linear New-Keynesian model with habit formation produces time-varying term premia which respond to the state of the economy. They emphasize that the relationship between the term premium and the output gap depends on the kind of the underlying distortion. However, their model does not offer a feedback from the term premium to the economy. Andrés, López-Salido and Nelson (2004) use a New-Keynesian model with imperfect substitutability between different financial assets and segmented asset markets to analyze the effect of long-term yields on aggregate demand and supply. They demonstrate that an increase in term premia dampens economic activity. Chen, Cúrdia, and Ferrero (2012) estimate a linearized DSGE model with segmented financial markets and limits to arbitrage. They evaluate the effects of LSAP on the economy where the effects are transmitted by a drop in the term premium of long-term government bonds. Though the decrease in term premia works to stimulate economy activity, their results suggest that the effects are only moderate. Similarly, Kiley (2012) estimates a model with segmented markets and limits-to-arbitrage using not only government long-term bond yields, but also private long-term bond yields. His results also suggest that a decline in the term premium has positive, but moderate effects on aggregate spending.

Using less structural approaches, a broad empirical literature analyzes the effect of changes of term premia on the economy, using either macro-finance models or reduced form regressions. The following passage summarizes their findings.

Hamilton and Kim (2002) use a regression to investigate the effects of the short-long term yield spread on GDP growth. They were the first who decompose the yield spread into an expectation part and a term premium part in order to evaluate the effects of both components of the spread on GDP growth separately. Using ex-post observed short rates as instruments for ex-ante expected rates isolate the expectation part, they find that a decline in premia is associated with slower future GDP growth, contradicting the practitioner view. Also, Favero, Kaminska, and Söderström (2005) find that a lower term premium predicts slower future GDP growth. They decompose the yield spread similar to Hamilton and Kim (2002), but use an estimated real-time VAR to predict the expectations of future short-term rates. Wright (2006) investigates whether the return forecast factor of Cochrane and Piazzesi (2005) - a linear combination of the spot rate and four forward rates which predicts term premia in one- to five-year maturity bonds - helps to forecast recessions. He documents that lower term premia raise the odds of a recession.

In contrast to these results, Ang, Piazzesi, and Wei (2006) find that changes in the term premium do not affect output growth. They run a regression of output growth on the term premium and expected future short rates, where the premium and the expected future short rates are computed from the estimates of a VAR with long-term rates, GDP growth, and the short-term interest rate. Also Rosenberg and Maurer (2007) find that the term premium has no predictive power for future GDP growth. They decompose the yield spread as in Hamilton and Kim (2002) and use both components in a recession forecasting model. In their estimation, the term premium is measured by the Kim-Wright (2005) term premium measure - the estimated term premium from a no-arbitrage dynamic latent 3-factor model. Dewachter et al. (2014a) use a macro-finance model of the term structure where a latent variable captures all movement in the one-period expected excess holding return (the return premium). They also find that movements in the term premium have no predictive power for future output growth.

However, in line with the practitioner view, Rudebusch, Sack, and Swanson (2007) find that a decline in the term premium is associated with higher positive GDP growth. They decompose the term spread in order to perform a regression of GDP growth on changes in the term premium, using the Kim-Wright term premium measure. Also, Jardet, Montfort, and Pegoraro (2013) and Joslin, Priebsch, and Singleton (2014) find both that a rise in the term premium lowers GDP growth in the short run, but has positive effects on GDP growth for longer horizons. While the former use a macro-finance near-cointegrated VAR(p) term structure model, the latter employ macro-finance model with imperfect correlated macro risk to explore the sources of variation in expected excess returns on bonds and the effects of term premium shocks on GDP growth and inflation. Recently, using a vector autoregression macro-finance model of the term structure, Ireland (2015) find that a rise in the term premium leads to a drop in output.

3 The Model

In this section the macro-finance model is presented. It is a joint model of the macroeconomy and the term structure as introduced into the macro-finance literature by Ang and Piazzesi (2003). The structure of the macro part of the model follows closely Ireland (2015). The term structure is modeled by an affine no-arbitrage model of the term structure as developed by Duffie and Kan (1996) and Dai and Singleton (2000). Motivated by the evidence of Cochrane and Piazzesi (2008) that one single factor accounts for most of the movements in expected excess holding returns, a latent variable that captures all movements in the one-period return premium is introduced. By restricting the prices of risk, this variable is constructed to be the only potential source for time variation in the market prices of risk and thus, for movements in term premia. The specification of the term structure model follows Dewachter and Iania (2011), Dewachter et al. (2014a) and Ireland (2015).

The model section is structured as follows. The first part describes the structural macroeconomic dynamics and casts the macro model into its state representation. The state variables are then used as pricing factors in the term structure model. Cross-equation restrictions, based on the assumption of no-arbitrage, are employed to tie the movements of yields closely together. Finally, different notion of the term structure premium - the yield and the return premium - are discussed and related to the latent risk variable.

3.1 The Macro Part

Following Ireland (2015) the macroeconomic dynamics are described by five state variables, three of them are observable - the nominal short-term interest rate r_t, the inflation rate π_t, and the output gap g_t^y - and two variables are unobservable, a risk variable v_t and the central bank's inflation target π_t^*.

In order to simplify the notation, the inflation gap and the interest rate gap are defined. The inflation rate gap is defined as the deviation of the inflation rate from central bank's inflation target,

$$g_t^\pi \equiv \pi_t - \pi_t^*,$$

and the interest rate gap is defined as the deviation of the interest rate from the inflation target,

$$g_t^r \equiv r_t - \pi_t^*.$$

The central bank's policy rule for the short-term nominal interest rate can then be specified in terms of the interest rate gap, the inflation gap and the output gap. Specifically, the central bank sets the interest rate according to the following interest rate rule in the spirit of Taylor (1993),

$$g_t^r - g^r = \rho_r \left(g_{t-1}^r - g^r\right) + (1 - \rho_r) \left[\rho_\pi g_t^\pi + \rho_y \left(g_t^y - g^y\right) + \rho_v v_t\right] + \sigma_r \varepsilon_{rt}, \tag{1}$$

where ρ_r, $\rho_r \in (0, 1)$, is the interest rate smoothing parameter, ρ_π, $\rho_\pi > 0$, is the central bank's response parameters on inflation ρ_y, $\rho_y > 0$, is the response parameters on the output gap, and ρ_v is the response parameter on the variation in term premia variable, σ_r, $\sigma_r > 0$, is a volatility parameter, and g^r and g^y are the steady state values of g_t^r and g_t^y, respectively. The shock ε_{rt} is supposed to be standard normally distributed and represents the interest rate policy shock. The notation of the interest rate rule incorporates the assumption that the steady state value of the inflation gap is zero. Thus, it is assumed that in the steady state the actual inflation rate equals the central bank's target rate. While ρ_π and ρ_y are restricted to be non-negative, the sign of the parameter of the term premium variable, ρ_v, is not constrained. A positive value of ρ_v implies that the central bank tends to tighten monetary policy in response to a rise in term premia. Goodfriend (1993) and McCallum (2005) argue that this should be the case if the central bank regards an increase in premia as an indicator of "inflation scares" or as an indicator of policy laxity.[3] In contrast, Bernanke (2006) argues that, to the extent that aggregate demand depends also on long-term interest rates, a rise in the term premium requires the central bank to lower the short-term interest rate in order to offset the effects of the decline in premia and to retain the economic condition, all else being equal. Thus, the coefficient ρ_v should be negative. This so called practitioner view, as labeled and discussed by Rudebusch, Sack and Swanson (2007), states that optimal monetary policy should account for movements in premia by adjusting the interest rate contrary to the direction of the movements in term premia. Apparently, if ρ_v is zero, the central bank does not react at all on changes in the term structure premium.

[3] To be precise, McCallum (2005) suggests that the central bank should tighten monetary policy if the interest rate spread between long-term bond yields and the short-term rate increases, given that the expectation hypothesis holds and that the premium follows an AR(1) process. A rise in the long-short rate spread might be due to two reasons: an increase in future expected short rates or an increase in the term structure premium. In McCallum's specification of the interest rate rule, the central bank reacts on the long-short spread, and with it, in general, on the fluctuation in the term premium. However, the cause for the rise in the spread is not identified.

The incorporation of an unobservable time-varying inflation target is a common approach in the recent macro-finance term structure literature (as in e.g. Dewachter and Lyrio, 2006, Hördahl et al., 2006, Rudebusch and Wu, 2008, or Hördahl and Tristani, 2012). It allows, on the one hand, for some variation of the conduction of monetary policy, and it helps, on the other hand, to capture movements in long-term nominal government bond yields which arise due to changes in central bank's inflation target. In fact, Barr and Campbell (1997) for the UK and Gürkaynak et al. (2005) for the US find that movements in long-term interest rates occur mainly due to changes in expected inflation. Also Hördahl et al. (2006), using a macro-finance term structure model with German data, find that changes in the perceived inflation target tend to have a stronger impact on long-term yields than policy rate-, inflation-, or output shocks. The inflation target π_t^* is supposed to follow a first-order autoregressive process (AR(1)),

$$\pi_t^* = (1 - \rho_{\pi^*})\, \pi^* + \rho_{\pi^*} \pi_{t-1}^* + \sigma_{\pi^*} \varepsilon_{\pi^* t}, \tag{2}$$

where π^* is the steady state level of the inflation target, $\rho_{\pi^*} \in [0, 1)$, $\sigma_{\pi^*} > 0$ and the shock $\varepsilon_{\pi^* t}$ is standard normally distributed. As in Hördahl et al. (2006), Rudebusch and Wu (2008), Hördahl and Tristani (2012), or Ireland (2015), this restriction is imposed to ensure stationarity of the inflation target process. As noted by Ireland (2015) a non-stationary inflation target leads to non-stationary inflation and non-stationary nominal short-term interest rate. As shown by Campbell, Lo and MacKinley (1997 p. 433) or Spencer (2008) for models with homoscedastic shocks a unit root in the nominal short-term interest rate translates in undefined asymptotic long-term bond yields. Thus, the assumption of the stationarity of the inflation target process ensures that the term structure part of the model is well-behaved.

Similar to Ireland (2015), the dynamics of the remaining three state variables are modeled as in more conventional structural VAR models. The inflation gap, the output gap, and the risk variable are linear functions of their own lags, the lags of all other state variables, their own innovations, and in some cases of the innovations of the other state variables. This specification allows for a fairly high degree of flexibility while restrictions on the contemporaneous relationship of these variables ensure identification of the structural model.

Specifically, the output gap is supposed to depend on own lags, on lags of the interest rate gap, the inflation gap and the risk variable, and on the innovations of the inflation target $\varepsilon_{\pi^\star t}$, and on its own innovations ε_{yt},

$$\begin{aligned} g_t^y - g^y &= \sum_{i=1}^{3} \rho_{yr}^i \left(g_{t-i}^r - g^r\right) + \sum_{i=1}^{3} \rho_{y\pi}^i g_{t-i}^\pi + \sum_{i=1}^{3} \rho_{yy}^i \left(g_{t-i}^y - g^y\right) \\ &\quad + \rho_{yv} v_{t-1} + \sigma_{y\pi^\star} \sigma_{\pi^\star} \varepsilon_{\pi^\star t} + \sigma_y \varepsilon_{yt}, \end{aligned} \tag{3}$$

where the volatility parameter σ_y is non-negative, and ε_{yt} is standard normally distributed. The inflation gap is assumed to depend on own lags, on lags of the interest rate gap, the output gap, and the term premium variable, and on innovations of the inflation target $\varepsilon_{\pi^\star t}$ and on its own innovations $\varepsilon_{\pi t}$, innovations of the output gap ε_{yt}

$$\begin{aligned} g_t^\pi &= \sum_{i=1}^{3} \rho_{\pi r}^i \left(g_{t-i}^r - g^r\right) + \sum_{i=1}^{3} \rho_{\pi\pi}^i g_{t-i}^\pi + \sum_{i=1}^{3} \rho_{\pi y}^i \left(g_{t-i}^y - g^y\right) \\ &\quad + \rho_{\pi v} v_{t-1} + \sigma_{\pi\pi^\star} \sigma_{\pi^\star} \varepsilon_{\pi^\star t} + \sigma_{\pi y} \sigma_y \varepsilon_{yt} + \sigma_\pi \varepsilon_{\pi t}, \end{aligned} \tag{4}$$

where the volatility parameter σ_π is non-negative and $\varepsilon_{\pi t}$ is standard normally distributed. Finally, similar to Bekaert et al. (2013) and Ireland (2015), the risk variable is supposed to respond contemporaneously on all distortions of the economy, as bond prices do. Specifically, the risk variable depends on its own lags and lags of all others state variables and on its own innovations ε_{vt} and additionally all innovations in all other state variables,

$$\begin{aligned} v_t &= \rho_{vr} \left(g_{t-1}^r - g^r\right) + \rho_{v\pi} g_{t-1}^\pi + \rho_{vy} \left(g_{t-1}^y - g^y\right) + \rho_{v\pi^\star} \left(\pi_{t-1}^\star - \pi^\star\right) \\ &\quad + \rho_{vv} v_{t-1} + \sigma_{vr} \sigma_r \varepsilon_{rt} + \sigma_{v\pi} \sigma_\pi \varepsilon_{\pi t} + \sigma_{vy} \sigma_y \varepsilon_{yt} + \sigma_{v\pi^\star} \sigma_{\pi^\star} \varepsilon_{\pi^\star t} + \sigma_v \varepsilon_{vt}, \end{aligned} \tag{5}$$

where the volatility parameter σ_v is non-negative, and ε_{vt} is standard normally distributed.

The chosen structure imposes restrictions in order to identify structural shocks. As in Ireland (2015), shocks to the inflation target $\varepsilon_{\pi^* t}$ affect the interest rate gap, the inflation gap, the output gap and the risk variable only contemporaneously. All further effects of fluctuations in the central bank's inflation target affect the economy only if the change in the inflation gap and interest rate gap are not fully offset by a proportional adjustment of the interest rate and the inflation rate (Ireland, 2015). This specification imposes a form of long-run monetary neutrality. In order to separate the effects of monetary policy on term premia from the effects of the changes in term premia on the short-term interest rate, the effects of the short-term interest rate and term premia movements on output and inflation from the effects of inflation and output on the short-term interest rate and term premia, and the effects of inflation on output from the effects of output on inflation the following restrictions on the contemporaneous relationship of these variables are imposed. Shocks to the risk variable affect the interest rate only through the change in the risk variable while a shock to the interest rate directly affects the term premium variable. The interest rate and the risk variable respond on shocks to the inflation gap and the output gap instantly, but innovations in the risk variable ε_{vt} and in the interest rate ε_{rt} do not affect the output gap and the inflation gap immediately, but rather with one period lag. Finally, as in Christiano et al. (2005), the inflation gap shock $\varepsilon_{\pi t}$ does not affect the output gap contemporaneously.

Define the vectors X_t and ε_t containing the state variables and the innovations by

$$X_t = \begin{bmatrix} g_t^r & g_{t-1}^r & g_{t-2}^r & g_t^\pi & g_{t-1}^\pi & g_{t-2}^\pi & g_t^y & g_{t-1}^y & g_{t-2}^y & \pi_t^\star & v_t \end{bmatrix}',$$

and

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{rt} & 0 & 0 & \varepsilon_{\pi t} & 0 & 0 & \varepsilon_{yt} & 0 & 0 & \varepsilon_{\pi^\star t} & \varepsilon_{vt} \end{bmatrix}',$$

then eq., (1) - (5) can be expressed as

$$P_0 X_t = \mu_0 + P_1 X_{t-1} + \Sigma_0 \varepsilon_t. \tag{6}$$

For the specific form of the matrices P_0, P_1, μ_0, and Σ_0 see Appendix (A.1). Eq. (6) gives the structural form of the model. Multiplying by P_0^{-1} yields the reduced form representation of the state equation,

$$X_t = \mu + P X_{t-1} + \Sigma \varepsilon_t, \tag{7}$$

where

$$\begin{aligned} \mu &= P_0^{-1}\mu_0, \\ P &= P_0^{-1}P_1 \end{aligned}$$

and

$$\Sigma = P_0^{-1}\Sigma_0.$$

3.2 The Term Structure Model

Affine term structure models, as developed by Duffie and Kan (1996) and Dai and Singleton (2000), are a particular class of term structure models[4] where the time t yield $y_t^{(\tau)}$

[4]More precisely, the discrete-time term structure model presented in this section belongs to the class of essentially affine models of the term structure, as categorized by Duffee (2002), and introduced by Gourieroux et al. (2002) in discrete time.

of $\tau-$period zero coupon bond is modeled as an affine function of the state vector X_t,

$$y_t^{(\tau)} = A_\tau + B_\tau' X_t,$$

where both coefficients A_τ and B_τ depend on the maturity τ. Though yields are linear affine in the state vector X_t, A_τ and B_τ' are highly non-linear functions of underlying parameters. The particular functional form of these coefficients is derived from cross-equation restrictions, which in turn stem from the assumption of the absence of arbitrage opportunities. These restrictions tie the movements of yields closely together.

The outlined affine term structure model is similar to the one described in Ang and Piazzesi (2003). However, in contrast to Ang and Piazzesi, restrictions are imposed on parameters contained in the matrix of prices of risk which permit the risk variable v_t to be the only source of fluctuations in the prices of risk and with it in the term premium. This subsection is structured as follows: the first part relates the short end of the yield curve to the state vector. The next part derives the pricing kernel which is used to price bonds. Finally, under the assumption of no-arbitrage, the functional form of the affine yield curve representation is derived and the solution for the coefficients A_τ and B_τ is presented.

3.2.1 Short rate equation

The short-term rate, and thus the short end of the yield curve, is from eq. (1) under the control of the central bank. The short end of the yield curve can be modeled as an affine function of the state vector X_t,

$$r_t = \delta_0 + \delta_1' X_t, \tag{8}$$

where δ_0 is a scalar, and δ_1' is a 1x11 selection vector indicating the position of g_t^r and τ_t in X_t. The coefficients δ_0 and δ_1 are set to ensure consistency between the macro part and the term structure part of the model. This requires δ_0 to be equal to zero, $\delta_0 = 0$, and

$$\delta_1' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

so that eq. (8) corresponds to the definition of the interest rate gap.

3.2.2 Pricing Kernel

The prices of government bonds are supposed to be arbitrage free. As shown in Harrison and Kreps (1979) or in Duffie (2001, pp. 108) the assumption of the absence of arbitrage guarantees for the existence of an "equivalent martingale measure" or "risk-neutral measure" Q.[5] Under the risk-neutral measure Q the price $P_t^{(\tau)}$ of any zero-coupon asset maturing in τ periods satisfies

$$P_t^{(\tau)} = E_t^Q \left(\exp\left(-r_t\right) P_{t+1}^{(\tau-1)} \right).$$

Thus, pricing under the risk-neutral measure implies that the price of an asset is given by the expected discounted future value of the asset, where the discounting takes place with the risk-free short-term interest rate. If market participants are risk-neutral, the risk-neutral probability measure coincides with the data generating measure H. However, in general, the risk-neutral probability measure does not coincide with the data generating process (Piazzesi, 2010, p. 697). The Radon-Nikodym derivative, which is denoted in the following by ξ_t, $\xi_t \equiv dQ/dH$, provides the link between the risk-neutral measure Q and the data generating measure H (see Duffie, 2001, p. 110). It is used to convert one probability measure into an equivalent measure.[6]

The specification of the pricing kernel is in reduced form. Though it is not explicitly derived from underlying preferences and is in particular not expressed in terms of marginal utility, it is widely used in the finance and macro-finance literature since it does match empirical properties fairly well (see Dai and Singleton, 2002). For discrete time models, following Ang and Piazzesi (2003), the nominal pricing kernel m_{t+1} is defined by

$$m_{t+1} \equiv \exp\left(-r_t\right) \frac{\xi_{t+1}}{\xi_t}, \tag{9}$$

and ξ_t is supposed to follow the log-normal process

$$\xi_{t+1} = \xi_t \exp\left(-\frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right), \tag{10}$$

where λ_t is an 11-dimensional vector of time-varying prices of risk. Combining eq. (9) and (10) yields for the pricing kernel,

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right). \tag{11}$$

[5] Moreover, if markets are also complete, then this risk neutral probability measure is also unique (Harrison and Kreps, 1979).

[6] Given the existence of the risk-neutral measure, for any random variable with finite variance the following holds:

$$E_t^Q \left(Z_{t+1}\right) = \frac{E_t \left(\xi_{t+1} Z_{t+1}\right)}{\xi_t},$$

where $E_t^Q(\cdot)$ denotes the time $t-$conditional expectations under Q, $E_t(\cdot)$ the time $t-$conditional expectations under H, and where already is implied that ξ_t is martingale (see Duffie, 2001, p. 168).

The log-normal pricing kernel depends on the short-term interest rate, the structural shocks and the prices of risk. The prices of risk drive the response of the long-term government bond yields to macro, policy and risk shocks. If all elements in λ_t are equal to zero, pricing takes places under the risk-neutral probability measure.

The prices of risk are supposed to be affine functions of the state variables, taking the functional form

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{12}$$

where λ_0 is an 11×1 vector and λ_1 is an 11×11 matrix. For the market prices of risk, I assume that only contemporaneous state variables are priced. The vector of constants λ_0 is given by

$$\lambda_0 = \begin{bmatrix} \lambda_0^r & 0 & 0 & \lambda_0^\pi & 0 & 0 & \lambda_0^y & 0 & 0 & \lambda_0^{\pi^*} & \lambda_0^v \end{bmatrix}'.$$

Note that the coefficients in λ_0 and λ_1 do no vary over time. All fluctuations in the prices of risk λ_t are caused by movements in the state variables in X_t. Evidence by Cochrane and Piazzesi (2005,2008) indicates that one single factor accounts for a large portion of variation in one-period return premia. In the spirit of this factor, the risk variable v_t is constructed to be the single source for time variation in the prices of risk. Following Dewachter and Iania(2012), Dewachter et al. (2014a), and Ireland (2015) the identification of the risk variable is done by setting all elements in λ_1, except the last column, to be equal to zero,

$$\lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^\pi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^{\pi^*} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^v \end{bmatrix}. \tag{13}$$

From eq. (12) together with the restrictions in eq. (13) all movements in the price of risk arise only from changes in the variable that is ordered as the last element in the vector X_t, that is, the risk variable v_t. As discussed in Section (3.3), these restrictions work to attribute movements in term premia to changes in the risk variable v_t.

3.2.3 Bond Prices

Given the pricing kernel, the assumption of the absence of arbitrage opportunities implies that under the data generating probability measure for any gross return R_t of a nominal asset the following equation holds

$$E_t\left(m_{t+1}R_{t+1}\right) = 1. \tag{14}$$

Let P_t^{τ} denote the price of a default-free, zero-coupon bond maturing in τ periods. Then, eq. (14) implies that all zero-coupon bond prices can be computed recursively by the no-arbitrage condition

$$P_t^{(\tau)} = E_t\left(m_{t+1}P_{t+1}^{(\tau-1)}\right). \tag{15}$$

That is, the time t price of a $\tau+1$-period zero-coupon bond equals the expected discounted price of a τ-period discount bond in period $t+1$, where pricing occurs under the data-generating measure using the stochastic discount factor m_{t+1}.

Given this set-up, Ang and Piazzesi (2003) demonstrate that the price of a zero-coupon bond $P_t^{(\tau)}$ maturing at time $t+\tau$ can be written as an exponentially affine function of the state vector X_t. Thus, the price of a bond maturing in τ-periods is

$$P_t^{(\tau)} = \exp\left(\bar{A}_\tau + \bar{B}_\tau' X_t\right), \tag{16}$$

where the coefficients $\bar{A}_t$ and $\bar{B}_\tau$ can be computed recursively by the following ordinary differential equations (see Appendix (A.2))

$$\begin{aligned} \bar{A}_{\tau+1} &= \bar{A}_\tau + \bar{B}_\tau'\left(\mu - \Sigma\lambda_0\right) + \frac{1}{2}\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - \delta_0, & (17)\\ \bar{B}_{\tau+1}' &= \bar{B}_\tau'\left(P - \Sigma\lambda_1\right) - \delta_1'. & (18)\end{aligned}$$

Eq. (11), (15) and $P_{t+1}^{0} = 1$ together imply that the log discount bond price of a bond maturing next period is given by $\log\left(P_t^1\right) = -r_t$. Consistency of eq. (8) and (16) for $\tau = 1$, given $\log\left(P_t^1\right) = -r_t$, requires then that the initial condition for $\bar{A}_\tau$ and $\bar{B}_\tau$ are given by: $\bar{A}_1 = \delta_0 = 0$, and $\bar{B}_1' = -\delta_1'$. The τ-period zero-coupon bond yield $y_t^{(\tau)}$ is related to the bond price by

$$y_t^{(\tau)} = -\frac{\log\left(P_t^{(\tau)}\right)}{\tau}. \tag{19}$$

Substituting eq. (16) into eq. (19), yields the affine yield curve representation with functional form

$$y_t^{(\tau)} = A_\tau + B_\tau' X_t. \tag{20}$$

where $A_\tau \equiv -\bar{A}_\tau/\tau$ and $B_\tau \equiv -\bar{B}_\tau/\tau$.

3.3 Term Structure Premia and the Expectation Hypothesis

Term structure premia can be captured in different forms (see e.g. Cochrane and Piazzesi, 2008, or Joslin et al., 2014). In the following, similar to Dewachter et al. (2014a), I will focus on the yield premium and the return premium. The definition of these premia is based on Cochrane and Piazzesi (2008). The yield premium is the most prominent form of the term premium and the one used by Ireland (2015). It can be composed into the average of expected future return premia of declining maturities. The one-period return premium in turn is only driven by the risk variable v_t. Before discussing both types of term structure premia, their relationship to each other and their relation to the risk variable, I will review some relevant basic relationships between holding period returns, excess holding returns and bond prices (see e.g. Cochrane, 2005, or Cochrane and Piazzesi, 2008). The holding period return $hpr_{t+1}^{(\tau)}$ is the return from buying a bond at time t that matures in $t+\tau$ periods and selling this bond the period after. Formally, it is defined by

$$hpr_{t+1}^{(\tau)} \equiv p_{t+1}^{(\tau-1)} - p_t^{(\tau)}, \tag{21}$$

where $p_t^{(\tau)}$ is the log price of a zero-coupon bond maturing in $t+\tau$ periods, $p_t^{(\tau)} \equiv \log\left(P_t^{(\tau)}\right)$. The excess holding period return (or short excess return) $hprx_{t+1}^{(\tau)}$ is the return from buying a long term bond in period t and selling it in the subsequent period in excess of the return from buying and holding a short term bond maturing next period,

$$hprx_{t+1}^{(\tau)} \equiv hpr_{t+1}^{(\tau)} - y_t^{(1)}. \tag{22}$$

The yield of a τ-period zero-coupon default-free long-term bond $y_t^{(\tau)}$ can be decomposed in an expectation part and a part which is denoted as the *yield* premium $\kappa_t^{(\tau)}$ (see e.g. Cochrane and Piazzesi, 2008):

$$y_t^{(\tau)} = \frac{1}{\tau} E_t\left(\sum_{i=0}^{\tau-1} y_{t+i}^{(1)}\right) + \kappa_t^{(\tau)}. \tag{23}$$

The expectation part consists of the average of expected future short rates over the bond's residual maturity. Rearranging eq. (23) gives the definition of the yield premium. Thus, the yield premium can be interpreted as the average expected return from buying a τ-period bond and holding this bond until maturity financed by a sequence of short-term debt. It is the compensation that a risk-averse investor demands for holding a long-term bond instead of a sequence of short-term bonds. Under the (pure) expectation hypothesis of the term structure, this premium is (zero) constant.

The yield premium can be written as the average of expected future return premia of declining maturity (as in Cochrane and Piazzesi, 2008, or Ludvigson and Ng, 2009; for a detailed derivation see Appendix (A.3)), where the respective *return* premium is defined as the expected $i+1$-period excess return, $E_t\left(hprx^{\tau}_{t+i+1}\right)$,

$$\kappa_t^{(\tau)} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} E_t\left(hprx_{t+i+1}^{(\tau-i)}\right), \tag{24}$$

with

$$E_t\left(hprx_{t+i+1}^{(\tau-i)}\right) = E_t\left(hpr_{t+i+1}^{(\tau-i)} - y_{t+i}^{(1)}\right).$$

Under the expectation hypothesis, these premia are constant but maturity specific. Eq. (24) illustrates that the yield- and the return premium (subsumed under the expression "term structure premium") are not the same objects, but both are related and can be derived from the other. While the yield premium reflects the premium in a bond yield over the full lifetime of the bond, the return premium reflects the per-period holding premium. Moreover, if return premia are zero or constant also the yield premium would be zero or constant.

In order to compute the yield and the return premium, the expectations of future short rates and excess returns have to be calculated. Following Ireland (2015), the expected value of the future short-term rate can be written as

$$E_t\left(y_{t+j}^1\right) = E_t\left(r_{t+j}\right) = \delta_1' E_t\left(X_{t+j}\right),$$

given eq. (8). Now define the unconditional expectation of the state vector by $\bar{\mu}$, $\bar{\mu} \equiv E\left(X_t\right)$, then, from eq. (7) one can write $\bar{\mu} = \left(I-P\right)^{-1}\mu$. Subtracting $\bar{\mu}$ from both sides of eq. (7) yields the (demeaned) state equation:

$$X_{t+1} - \bar{\mu} = P\left(X_t - \bar{\mu}\right) + \Sigma\varepsilon_{t+1}.$$

Then, the time-t conditional expected future short rate for period $t+j$, $\forall j > 0$, can be computed by

$$E_t\left(r_{t+j}\right)=\delta_1'\left(I-\delta' P^j\right)\bar{\mu}+\delta_1' P^j X_t$$

By rearranging eq. (23), and using $y_t^{(\tau)}=A_\tau+B_\tau' X_t$ the yield premium is given by

$$\kappa_t^{(\tau)}=A_\tau-\delta_1'\left[I-\frac{1}{\tau}\sum_{j=0}^{\tau-1}P^j\right]\bar{\mu}+\left[B_\tau-\delta_1'\frac{1}{\tau}\sum_{j=0}^{\tau-1}P^j\right]X_t.$$

Using $\Sigma_{j=0}^{\tau-1}P^j=\left(I-P^\tau\right)\left(I-P\right)^{-1}$, the yield premium can be expressed in a computationally more convenient form (as in Ireland, 2015)

$$\begin{aligned}\kappa_t^{(\tau)} &= A_\tau-\delta_1'\left(I-\frac{1}{\tau}\left(I-P^\tau\right)\left(I-P\right)^{-1}\right)\bar{\mu} \\ &\quad +\left(B_\tau'-\delta_1'\frac{1}{\tau}\left(I-P^\tau\right)\left(I-P\right)^{-1}\right)X_t.\end{aligned} \tag{25}$$

The return premium can be calculated by plugging the model implied log prices, $p_t^{(\tau)}=\bar{A}_\tau+\bar{B}_\tau' X_t$, into the definition $i+1$-period return premium and rearranging terms (see Appendix (A.4)),

$$\begin{aligned}E_t\left(hprx_{t+i+1}^{(\tau)}\right) &= \bar{B}_{\tau-1}'\Sigma\left[\lambda_0+\lambda_1\left(I-P^i\right)\bar{\mu}+\lambda_1 P^i X_t\right] \\ &\quad -\frac{1}{2}\bar{B}_{\tau-1}'\Sigma\Sigma'\bar{B}_{\tau-1}\end{aligned} \tag{26}$$

If $i=0$, then eq. (26) is the one-period return premium. From the restrictions on the elements in λ_1 in eq. (13) the risk variable v_t is identified as the driving force of the one-period return premium. Precisely, the one-period return premium of a bond with maturity τ is given by

$$E_t\left(hprx_{t+1}^{(\tau)}\right)=\bar{B}_{\tau-1}'\Sigma\left(\lambda_0+\lambda_1 X_t\right)-\frac{1}{2}\bar{B}_{\tau-1}'\Sigma\Sigma'\bar{B}_{\tau-1}. \tag{27}$$

Eq. (27) reveals that all variation over time in one-period return premia arises solely from fluctuations in v_t for all bond maturities. In contrast, to the extent that the risk variable is not zero over time, the yield premium is affected by all state variables if $\tau>1$. To see this, recall that the yield premium can be written as the average of expected future return premia of declining maturity. Since the i-period return premium, in general, depends on all state variables, from eq. (24) also the yield premium depends on all state variables if $\tau>1$.

Finally, if all elements in the matrix λ_1 are equal to zero, then the one-period return premium and the yield premium are constant. In this case, in eq. (27) the term $\lambda_1 X_t$

disappears, eliminating all time variation in the one-period return premium. Similar, as shown by Ireland (2015) taking $\lambda_1 = 0_{11x11}$ into account in eq. eq. (18) leads to

$$B'_\tau = \delta'_1 \frac{1}{\tau} \left(I - P^\tau\right) \left(I - P\right)^{-1}.$$

Plugging B'_τ in eq. (25) confirms that $\kappa_t^{(\tau)}$ is constant, if all elements in the matrix λ_1 are equal to zero. The discussion of the different types of term premia completes the model section.

4 Estimation

The first part of this section presents the state space system. The next part summarizes the data set that is used for the estimation of the model. Then the estimation method is discussed. The last part presents and discusses the choice of the prior distribution for the parameters.

4.1 The State Space System

The macro part and the affine term structure model form a state-space system. The state equation, given by eq. (7), describes the dynamic of the state vector, while the observables - output gap, inflation, the short-term interest and the long-term government bond yields - are linked to the state vector by measurement equations.

For the estimation, a version of the state-state space model without constant terms is employed. By dropping the constant terms appearing in eq. (7) and (20) , and using demeaned data the estimation is simplified. Precisely, under the assumption that the central bank is able - on average - to implement its target inflation rate, so that the average of the actual inflation rate equals the average target inflation rate, the steady state values of g^r, τ and g^y can be calibrated to match the data averages of the short-term interest rate, the output gap and inflation. Moreover, as demonstrated in Ireland (2015), the values of the elements in λ_0 can be calibrated so that the steady state values of yields match the average yields. Thus, the state-space system is given by

$$X_t = P X_{t-1} + \Sigma \varepsilon_t, \tag{28}$$

$$Z_t = U X_t + V \eta_t, \tag{29}$$

where the vector Z_t containing the eight observables is defined by

$$Z_t \equiv \begin{bmatrix} r_t & \pi_t & g_t^y & y_t^{12} & y_t^{24} & y_t^{36} & y_t^{48} & y_t^{60} \end{bmatrix}',$$

the matrix U is specified by

$$U = \begin{bmatrix} U_r \\ U_\pi \\ U_y \\ B'_{12} \\ B'_{24} \\ B'_{36} \\ B'_{48} \\ B'_{60} \end{bmatrix},$$

with

$$\begin{aligned} U_r &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\ U_\pi &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\ U_y &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

in order to connect the observable macro variables to the state vector X_t, the vector B'_n, $n = \{12,\ 24,\ 36,\ 48,\ 60\}$, is determined by eq. (18), given the definition $B_\tau \equiv -\bar{B}_\tau/\tau$ and for given starting values $\bar{B}'_1 = -\delta'_1$, and the matrix V contains the volatility parameters of the measurement errors η_t. These errors are attached in order to avoid stochastic singularity. The problem of stochastic singularity arises in this type of models because numerous yield data are observed, but only a few structural shocks of potentially also observable state variables are used, so that the number of observable variables exceeds the number of shocks. Noise or measurement errors are added in order to give the model the ability to fit the high dimensional data vector with a lower dimensional state vector. Two different assumptions on the nature of these measurement errors are commonly drawn: Either only some yields are measured with errors (as e.g. in Ang and Piazzesi, 2003, or Ireland, 2015) or all yields are measured with errors (as e.g. in Ang et al., 2007, or Chib and Ergashev, 2009). Following, among others, Chib and Ergashev (2009), I will treat all yields (except the policy rate) as measured with errors.[7] Specifically, the matrix V is

[7]As discussed in Piazzesi (2007, p. 726), supposing that only a certain number of yields - that is, the

given by

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sigma_{12} & 0 & 0 & 0 & 0 \\ 0 & \sigma_{24} & 0 & 0 & 0 \\ 0 & 0 & \sigma_{36} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{48} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{60} \end{bmatrix}$$

with σ_{12}, σ_{24}, σ_{36}, σ_{48} and $\sigma_{60} > 0$ and the vector of the corresponding measurement errors η_t is given by

$$\eta_t = \begin{bmatrix} \eta_t^{12} & \eta_t^{24} & \eta_t^{36} & \eta_t^{48} & \eta_t^{60} \end{bmatrix}'.$$

These zero-mean measurement errors are supposed to be standard normally distributed.

4.2 Data

I include euro area data from September 2004 to April 2014 in my sample. The data set contains macro data and yield data. The data is taken from the Bundesbank and the ECB. The macroeconomic variables are the inflation rate, the output gap, and the nominal short-term interest rate. The financial variables are the yields from an index of risk-free zero-coupon treasury bonds of European countries with maturities of 12, 24, 36, 48 and 60 months. The yield data is only available from the ECB since Fall 2004, restricting effectively the size of the available sample. Due to the short sample size of the dataset - roughly ten years - I use monthly data. This compromises between the high-frequency yield data and the lower frequency macro data. The sample space covers 116 observations per time series. Moreover, data for the risk-free short-term interest rate - the OIS rate for the Eurozone - is only available since mid-2005. During the estimation, the yield data from Fall 2004 until June 2005 are treated as missing observations. The time path of the missing observations is constructed by the Kalman filter.

required number of shocks that needs to be added in order to avoid stochastic singularity - is observed with errors seems to be arbitrary, especially which particular yield should be observed with an error and which particular yield not. Data entry mistakes and interpolation methods for construction the zero-coupon yield date might lead to errors that should potentially affect all yields. Thus, if some yields are measured with errors the assumption that possibly all yields are observed with errors seems to be plausible. See Piazzesi (2007, pp. 726) for a more detailed discussion of noise- or measurement errors in the context of affine term structure models.

The output gap variable is defined as the percentage (logarithmic) deviation of actual output from trend output. Since GDP data is only available on a quarterly frequency, I use the seasonally adjusted industrial production index of the Euro area (Euro area 18, fixed composition) as a proxy for output (as e.g. Clarida, Galí and Gertler, 1998, Ang and Piazzesi, 2003, or Favero, 2006). Trend output is constructed using the HP filter with a smoothing parameter equal to 14.400. The inflation rate is measured by the annual rate of change of the seasonally adjusted HICP of the Euro area in percentage. For the risk-free zero-coupon yield data, an index of government bonds of countries from the euro area is used. The government bond index consists of all countries of the euro area that are AAA rated. All yields are continuously compounded. The yield data is taken from the ECB. The yield index of risk-free zero-coupon treasury bonds is not available for bonds with one-month residual maturity. To overcome this shortcoming, the risk-free nominal short-term interest rate is proxied by the Overnight Indexed Swap (OIS) rate. The OIS rate is an interest rate swap with a floating rate indexed on an overnight interbank rate. In the case of the Euro area, this overnight interbank rate is the EONIA. It had become, in particular for the euro area, a lately widely used measure for the risk-free rate (among others by Borgy et al. 2012, Dewachter et al., 2014b, Dubeq et al., 2013, Finlay and Chambers, 2009, Filipović and Trolle, 2013, or Joyce et al., 2011), rather than inter-bank rates like the EONIA.[8] The OIS rate date is taken from the Bundesbank.

Table (1) provides some summary statistics of the data for the macroeconomic variables and the yield data. The sample average of inflation is around the ECB's announced inflation target of 2 percent. By construction, the mean of the output gap is equal to zero. All macroeconomic variables are persistent, reflected by high first to third order autocorrelation. The summary statistics of the yields confirm that the employed yield data are line with stylized facts of yield curves[9] (though the sample space covers the financial crisis): First, the average yield curve is upward slopping. Thus, the longer the residual maturity of a government bond, the higher are yields. Second, the term structure of volatility of yields is downward slopping. The standard deviation of yields declines with maturity. Third, yields are highly autocorrelated. The first to third-order sample autocorrelations

[8] Euro area inter-bank rates, which are on unsecured interbank lending, are quite likely to compromise a certain amount of premia for credit risks, in particular, since the onset of the financial crisis in 2007. In contrast, netting and credit enhancement mechanisms of in swap contracts seem to work, also in times of financial turmoil, to mitigate counterparty risk (see Bomfim, 2003).

[9] See for example Campbell, 1995, Ang and Piazzesi, 2003, or Hördahl, Tristani and Vestin, 2008.

Table 1: Descriptive summary statistics

Obs.:	Moments				Autocorrelation		
116*	Mean	S.d.	Skewness	Kurtosis	1. Lag	2. Lag	3. Lag
π_t	1.9619	0.9031	−0.5073	3.4419	0.9591	0.8970	0.8101
g_t^y	0	3.6550	−0.8297	4.4141	0.9548	0.8963	0.8086
r_t	1.5976	1.5223	0.5976	1.7124	0.9938	0.9791	0.9579
y_t^{12}	1.6825	1.4942	0.4331	1.6909	0.9918	0.9761	0.9547
y_t^{24}	1.8873	1.4262	0.2816	1.7316	0.9892	0.9722	0.9511
y_t^{36}	2.1010	1.3350	0.1095	1.7670	0.9877	0.9705	0.9497
y_t^{48}	2.3142	1.2431	−0.0487	1.8122	0.9862	0.9690	0.9487
y_t^{60}	2.5185	1.1566	−0.1835	1.8681	0.9843	0.9666	0.9468

Source: yield data, industrial production and inflation: ECB; OIS rate: Bundesbank. The 12 - 60 month yields are annual zero coupon bond yields. Inflation is calculated as the percentage year-to-year change of the HICP of the Eurozone. Output is measured by industrial production and the output gap is defined as the deviation of actual output from its trend.
* For the OIS rate, the sample period is 2005:07 to 2014:04, covering 106 observations in total.

are not below 0.94. Fourth, yields move closely together. The correlation between yields of treasury bonds with maturity of 12 and yield of treasuries with maturity of 36 months is equal to 0.9769 (not displayed in the table) and the correlation between yields of treasury bonds with maturity of 60 months and yields of treasuries with maturity of 60 months is equal to 0.9880.

4.3 Method

To estimate the state space model, I apply Bayesian estimation techniques. As often noted in the literature, even the estimation of pure affine term structure model is computationally challenging and time-consuming (see e.g. Christensen et al, 2011, or Chib and Ergashev, 2009). Adding the macro-dynamics enhances these difficulties due to the complexity of the macroeconomic interactions with the term structure and vice versa (Rudebusch and Wu, 2008). The parameters in the $B_{(\tau)}$ matrices of the observation equations are highly non-linear functions of the underlying parameters of the state equations and the prices of risk. This non-linearity, as demonstrated by Chib and Ergashev (2009), can produce multimodal likelihood functions. Applying Bayesian estimation techniques allow to employ a priori information which help to down-weight regions of the parameter space which are not economically reasonable and help to rule out economically implausible parameter values. As a result, the posterior distribution can be smoother than the likelihood function (see Chib and Ergashev, 2009). Moreover, the usage of prior information is helpful when dealing with short data sets.

4.3.1 Posterior and Likelihood function

Formally, let Z denotes the data set, $Z = (Z_1, ..., Z_T)'$, where T is the number of total observations, and let θ denotes the vector of all parameters contained in the matrices P, Σ, Λ and V, then from Bayes rule, the joint posterior distribution of θ, $\pi(\theta|X)$, is obtained by combining the likelihood function of the observables, the prior distribution of the parameter vector and a norming constant, thus,

$$\pi(\theta|Z) \propto L(Z|\theta)\, p(\theta),$$

where $L(Z|\theta)$ is the likelihood function, and $p(\theta)$ is the prior distribution. Denote by $\mathcal{Z}_{t-1}$ all available information of the observable variables at time $t-1$, $\mathcal{Z}_{t-1} \equiv (Z_1, ..., Z_{t-1})'$. If the initial state X_0 and the innovations $\{\varepsilon_t, \eta_t\}_{t=1}^{T}$ are multivariate Gaussians, then the conditional distribution of the observables Z_t on $\mathcal{Z}_{t-1}$ is also Gaussian (see Hamilton, 1994, p. 385)

$$Z_t|\mathcal{Z}_{t-1} \sim N\left(UX_{t|t-1}, R_{t|t-1}\right),$$

where $X_{t|t-1}$ denotes the one step ahead forecast, $X_{t|t-1} \equiv E[X_t|\mathcal{Z}_{t-1}, \theta]$, and $R_{t|t-1}$ denotes the conditional variance, $R_{t|t-1} \equiv Var(Z_t|\mathcal{Z}_{t-1}, \theta)$.[10] Since two of the state variables are latent, the likelihood $L(Z|\theta)$ is constructed using the standard Kalman filter recursions (see Harvey, 1991). Hence, the joint density of the date set Z given θ can be written as

$$\begin{aligned} L(Z|\theta) &= \prod_{t=1}^{T} (2\pi)^{-\frac{T}{2}} \left[\det\left(R_{t|t-1}\right)\right]^{-\frac{1}{2}} \\ &\quad \times \exp\left(-\frac{1}{2}\left(Z_t - UX_{t|t-1}\right)'\left(R_{t|r-1}\right)^{-1}\left(Z_t - UX_{t|t-1}\right)\right). \end{aligned}$$

At the start of the recursions, the initial matrix of the variance of the forecast errors is set equal to the unconditional variance of the state variables.

Since the posterior density is, in general, not known in closed form, I apply Markov Chain Monte Carlo (MCMC) methods (the Adaptive-Metropolis algorithm) in order to simulate draws from the joint posterior distribution.

[10] See Appendix (A.5) for the explicit expressions of the prediction and updating equations of the mean and the variance.

4.3.2 MCMC Method

The choice of the proposal density of the Metropolis-Hastings algorithm is crucial for the speed of the convergence of the chain (Rosenthal, 2010). The scaling of the posterior distribution is often done by trial and error. But not only is the scaling of the proposal density "by hand" in general time-consuming, improving the proposal distribution manually also becomes very difficult, if not infeasible, in high-dimensional problems. Therefore, I employ the Adaptive Metropolis (AM) algorithm as introduced by Haario et al. (2001) to evaluate the posterior. The main idea of the AM algorithm is to run a chain that alters its own proposal distribution by using all information about the posterior cumulated so far. Thus, the algorithm improves on the fly. Precisely, the covariance of the proposal distribution is updated each step using all available information. Apart from the updating scheme, the algorithm is identical to the standard random walk Metropolis-Hastings algorithm. Due to the adaptive nature of the algorithm it is non-Markovian, but Haario et al. (2001) show that it still has the correct ergodic properties.

Let θ_0, ..., θ_{j-1}, denote the sampled parameters until $j-1$ iterations, where θ_0 is the initial set of parameters. I follow Haario et al. (2001) and let the proposal distribution, denoted by $q(\cdot|\theta_0, ..., \theta_{j-1})$, be a multivariate Gaussian distribution with mean at the current value of the parameter vector θ_{j-1} and a covariance matrix C_t. The algorithm starts with a pre-specified strictly positive proposal distribution covariance C_0. After an initial period n_0 the adaption takes place by updating the covariance of the proposal distribution according to $C_j = s_d Cov(\theta_0, ...\theta_j) + s_d \varepsilon I_d$, where s_d is a parameter that depends only on the dimension d of the parameter vector θ and $\varepsilon > 0$ is a (very small) constant employed to prevent C_j from becoming singular. In practice, the calculation of the covariance C_j is simplified using the following recursion formula (see Haario et al., 2001):

$$C_{j+1} = \frac{j-1}{j} C_j + \frac{s_d}{j} \left(\bar{\theta}_{j-1} \bar{\theta}'_{j-1} - (j+1) \bar{\theta}_j \bar{\theta}'_j + \theta_j \theta'_j + \varepsilon I_d \right).$$

Precisely, the AM algorithm is given by the following steps:

1. Set the number of total iterations n and specify the initial period n_0 $(n_0 < n)$ after which the adaption starts. Chose an (arbitrary) positive definite initial covariance matrix C_0 and specify the initial parameter vector θ_0. Set $C_j = C_0$ and $\theta_{j-1} = \theta_0$.

2. Draw a candidate θ_j^* from $q(\cdot|\theta_{j-1}, C_j)$

3. Compute $\alpha\left(\theta_j^*, \theta_{j-1}\right) = \min\left[1, \frac{\pi\left(\theta_j^*|\cdot\right)}{\pi\left(\theta_{j-1}|\cdot\right)}\right]$.

4. Set $\theta_j = \theta_j^*$ with probability $\alpha\left(\theta_j^*, \theta_{j-1}\right)$

   and set $\theta_j = \theta_{j-1}$ with probability $1 - \alpha\left(\theta_j^*, \theta_{j-1}\right)$.

5. Update $C_{j+1} = \begin{cases} C_0, & j \leq n_0 \\ s_d Cov\left(\theta_0, ...\theta_j\right) + s_d \varepsilon I, & j > n_0 \end{cases}$.

6. Repeat step 2-5 until $j = n$.

Haario et al. (2001) note that the choice of an appropriate initial covariance C_0 helps to speed up the algorithm and thus to increase efficiency. Therefore, I use a scaled down version of the inverse of the Hessian matrix computed at the posterior mode for the initial covariance matrix. The initial parameter vector is set to the parameter values at the mode. For the choice of the scaling parameter s_d I follow Haario et al. (2001) (whose choice in turn is based on Gelman et al. (1996)) and set $s_d = (2.4)^2/d$. The initial period is set to $n_0 = 20,000$ and the number of draws is set to $n = 1,000,000$.

As noted by Chib and Ergashev (2009), the mode of the posterior can in general not be found using Newton-like optimization methods. Therefore, I employ the Covariance Matrix Adaption Evolution Strategy (CMA-ES) algorithm. The CMA-ES is a stochastic method for numerical parameter optimization of non-linear, non-convex functions with many local optima. It belongs to the class of evolutionary optimization algorithms (Hansen and Ostermeier, 2001). The computation of the mode is conducted by the software package Dynare (Adjemian et al., 2011).

4.4 Parameter Restrictions and Prior Distributions

4.4.1 Parameter Restrictions

During the estimation the following restrictions, in addition to restrictions on the interest rate rule parameters and on the parameter of the inflation target process (the non-negativity restrictions of ρ_y and ρ_π, and the restriction that ρ_r and $\rho_\tau \in [0,1)$, are imposed.

To ensure stationarity of the VAR part the eigenvalues of P are constrained to be less than unity in absolute value, $eig\left(P\right) < |1|$. Likewise, a similar eigenvalue restrictions need to be imposed in order to ensure stability of the no-arbitrage recursions (see Dai and

Singleton, 2000). Specifically, the eigenvalues of $P - \Sigma\lambda_1$ are constraint to be less than unity in absolute value, $eig\left(P - \Sigma\lambda_1\right) < |1|$. For identification, the parameter σ_v of the latent variable needs to be normalized. As well known in the literature of latent factor models (e.g. Dai and Singleton, 2000), multiplicative transformations of the latent factor lead to observationally equivalent systems. In order to fix the scale of the latent variable, the constraint $\sigma_v = 0.01$ is imposed. Additionally, the direction in which an increase in the risk variable v_t moves term structure premia needs to be pinned down. Following Ireland (2015), without loss of generality the constraint $\Lambda^\pi \leq 0$ is imposed during the estimation. Finally, similar to Dewachter et al. (2014a) and Ireland (2015), to impose that v_t only moves the prices of risk, which are associated to the other four state variables, the constraint $\Lambda^v = 0$ is imposed.

After imposing these restrictions, there are 50 parameters left to estimate in eq. (28) - (29). The next sub-section presents the prior distribution of these parameters.

4.4.2 Prior Distributions

Using prior information from previous studies and restricting parameters to lie in an economically reasonable region helps to reduce the complexity of the maximization problem by down-weighting economically non-meaningful regions of the parameter space (see Chib and Ergashev, 2009, for a deeper discussion). The first part of table (2) displays the prior distributions of the coefficients in the monetary policy rule. I follow closely Smets and Wouters (2003) for the choice of these priors. Since the parameter capturing the degree of interest rate smoothing ρ_r is supposed to be in the interval between 0 and 1, it is assumed that ρ_r is Beta distributed. I set the prior mean equal to 0.8 and the standard deviation equal to 0.05, assuming a high degree of interest rate inertia. The parameter governing central bank's reaction on deviation of the actual inflation rate from its target rate is assumed to be Gamma distributed with a mean of 1.5 and a standard deviation of 0.25. I employ the Gamma distribution to ensure that the parameter ρ_π cannot be negative. The prior mean satisfies the Taylor principle. Likewise, I also suppose that the prior for the parameter of central bank's reaction on deviation from the output gap is Gamma distributed. The prior mean is chosen to correspond roughly to the Taylor coefficient of 0.5. Finally, the coefficient of central bank's response on movements in term premia ρ_v is assumed to be Normal distributed with a mean of 0 and a standard deviation of 0.5, so that the interval $[-1.96; 1.96]$ covers 95% of the probability mass. The choice of the prior

Table 2: Summary of the prior distribution

Taylor Rule							
Parameter	type	mean	std. dev.	Parameter	type	mean	std. dev.
ρ_r	B	0.80	0.05	ρ_π	G	1.500	0.250
ρ_v	N	0.00	0.250	ρ_y	G	0.045	0.025
Macro Part							
Parameter	type	mean	s.d.	Parameter	type	mean	std. dev.
$\rho^1_{\pi r}$	N	-0.20	0.150	$\rho^2_{y\pi}$	N	0.00	0.075
$\rho^2_{\pi r}$	N	0.00	0.075	$\rho^3_{y\pi}$	N	0.00	0.050
$\rho^3_{\pi r}$	N	0.00	0.050	ρ^1_{yy}	N	0.90	0.150
$\rho^1_{\pi\pi}$	N	0.90	0.150	ρ^2_{yy}	N	0.00	0.075
$\rho^2_{\pi\pi}$	N	0.00	0.075	ρ^3_{yy}	N	0.00	0.050
$\rho^3_{\pi\pi}$	N	0.00	0.050	$\rho_{v\pi}$	N	0.00	0.150
$\rho^1_{\pi y}$	N	0.00	0.150	ρ_{vv}	N	0.90	0.150
$\rho^2_{\pi y}$	N	0.00	0.075	ρ_{vy}	N	0.00	0.150
$\rho^3_{\pi y}$	N	0.00	0.050	ρ_{vr}	N	0.00	0.150
ρ^1_{yr}	N	-0.20	0.150	$\rho_{v\tau}$	N	0.00	0.150
ρ^2_{yr}	N	0.00	0.075	ρ_τ	B	0.90	0.100
ρ^3_{yr}	N	0.00	0.050	ρ_{yv}	N	0.00	0.250
$\rho^1_{y\pi}$	N	0.00	0.150	$\rho_{\pi v}$	N	0.00	0.250
Volatility and co-movement parameters							
Parameter	type	mean	std. dev.	Parameter	type	mean	std. dev.
σ_{vr}	N	0.00	2.00	σ_π	IG	0.01	0.200
$\sigma_{v\pi}$	N	0.00	2.00	σ_y	IG	0.01	0.200
σ_{vy}	N	0.00	2.00	σ_τ	IG	0.01	0.200
$\sigma_{v\tau}$	N	0.00	2.00	σ_{12}	IG	0.0001	0.001
$\sigma_{y\pi}$	N	0.00	2.00	σ_{24}	IG	0.0001	0.001
$\sigma_{y\tau}$	N	0.00	2.00	σ_{36}	IG	0.0001	0.001
$\sigma_{\pi\tau}$	N	0.00	2.00	σ_{48}	IG	0.0001	0.001
σ_r	IG	0.01	0.20	σ_{60}	IG	0.0001	0.001
Prices of Risk							
Parameter	type	mean	std. dev.	Parameter	type	mean	std. dev.
Λ^r	N	0.00	25.00	Λ^π	N	0.00	25.00
Λ^τ	N	0.00	25.00	Λ^y	N	0.00	25.00

means implies that monetary policy is, a priori, characterized by a standard Taylor rule. Given the normalization of σ_v the choice of the standard deviation implies a relatively uninformative prior.

The choice of the priors of the parameters describing the dynamics of the macroeconomy is displayed in the second part of table (2). As described in Section (2.1), these dynamics are modeled as in a structural VAR model. The priors for the VAR part (eq. 1 - 5) are chosen in the spirit of Minnesota (see Litterman, 1986) by assuming that almost all coefficients are normal distributed and by setting the prior means of most of the coefficients equal to zero except for these coefficients corresponding to the first own lags of the dependent variables. These coefficients are set equal to 0.9 as suggested by Koop and Korobilis (2010). The choice of the prior means reflects the assumption that these variables exhibit a high degree of persistence, but do not follow a unit root process. The standard deviation of the prior distribution of the parameters is weighted by the lag length, implying that with increasing lag length the coefficients are shrunk towards zero. As in Dewachter et al. (2014a), I set the standard deviations for the coefficients on the first lags equal to 0.15. Departing from Minnesota and following Dewachter and Iania (2011) and Dewachter et al. (2014a), I choose a negative prior mean for the parameters ρ_{yr}^1 and $\rho_{\pi r}^1$. These choices capture beliefs that an increase in the interest rate dampens economic activity. For the parameters ρ_{yv} and $\rho_{\pi v}$ I choose a relatively uninformative prior. Precisely, I set the prior mean equal to zero and the standard deviation equal to 0.25, assuming that movements in the term premium do not affect output and inflation a priori. The coefficient of the inflation target process is Beta distributed with a mean of 0.9 and a standard deviation of 0.1. Employing the Beta distribution guarantees that the process of the inflation target is stationary while avoiding that the central bank's inflation target jumps erratically. The overall choice of these priors satisfies the stationarity of the macro dynamics.

The third part of table (2) presents the prior distributions of the volatility parameters associated with the structural shocks and the measurement errors, and the prior distributions of the co-movement parameters. The prior distributions of the volatility parameters corresponding to the structural shocks and the measurement errors follow, similar to Dewachter (2008), the Inverse Gamma distribution with a mean of 0.01 and 0.0001, respectively, and a standard deviation of 0.2 and 0.001, respectively, corresponding to a mean of 1 percentage of the structural shocks and a mean of 0.01 percentage of the measurement errors. This specification captures the beliefs that measurement errors should be rather small. I employ the Inverse Gamma distribution in order to prevent the volatility parameter to be negative or equal to 0. The prior distributions for the co-movement parameters follow a Normal distribution with a mean of 0 and standard deviation of 2. Noteworthy, the choice of the priors satisfies the stationarity condition and the stability condition of the no-arbitrage recursions. Hence, under the chosen prior specification $eig\,(P) < |1|$ and $eig\,(P - \Sigma\lambda_1) < |1|$ hold.

Finally, for the choice of the prior distributions of the coefficients Λ^{π}, Λ^{y}, Λ^{r}, and Λ^{τ} (the elements in the prices of risk), I follow Dewachter and Iania (2011) and Dewachter et al. (2014a). The last part of table (2) presents the priors for the prices of risk. I use relatively uninformative priors, reflected by the choice of large standard deviations. More precisely, each element in the prices of risk is assumed to be Normal distributed with a mean of 0 and a standard deviation of 25.

5 Results

Table (3) and (4) list the results of the estimation. They report the posterior modes of the parameters, the posterior means, and the 90% highest posterior density (HPD) interval. While the posterior mode is obtained by maximizing the (log-) posterior distribution, the latter results are obtained by using the Adaptive Metropolis algorithm outlined in Section (4.3.2). First, the estimated values of the interest rate rule parameters are discussed. Then, I will evaluate the estimated mode by plotting impulse response functions (IRF) and decomposing the error forecast variance.

5.1 Policy coefficients

Focusing on the four estimated parameters of the interest rate rule displayed in the first four rows in the table (3), I find that all four parameters are significantly different from zero, including the ECB's response parameter to movements in term structure premia ρ_v. The posterior mean of ρ_v is significantly different from zero and negative, $\rho_v = -0.4419$, implying that the ECB lowered the interest rate in response to a rise in term premia. Thus, in line with the practitioner view, this indicates that the central bank counteracted changes in term premia, presumably to retain the overall mix of financial conditions, balancing output and inflation. Carlstrom, Fuerst, and Paustian (2015) demonstrate that in a DSGE model with imperfect financial markets a negative response coefficient on term premia in the monetary policy rule improves welfare. In contrast, Ireland (2015), who estimated the same parameter, but for the Fed with US data from the 1950th until 2007 (and for an extended sample until the end of 2014), using a restricted maximum likelihood approach, finds a positive and significant coefficient.

The estimated values of the other three parameters of the interest rate rule are similar to those from studies using a more standard interest rate rules specification for the Euro Area (e.g. Andrés et al., 2006, or Smets and Wouters, 2003). The estimate of the interest rate inertia $\rho_r = 0.8894$ reflects a high degree of interest rate smoothing. The estimate of the coefficient measuring central bank's response to changes in the output gap is $\rho_y = 0.0335$. The estimated coefficient of the central bank's response to a change in inflation is larger than one, $\rho_\pi = 1.2220$, satisfying the Taylor principle.

5.2 Model's Dynamic

The estimation results for the remaining parameters are summarized in table (3) and table (4). Rather than interpreting each coefficient separately I will describe the results of the parameter estimates jointly by computing impulse response functions (IRFs) of the model's variables to the fundamental shocks of the economy and by decomposing the forecast error variance. Both methods help to examine the dynamic of the estimated model and to describe the propagation and the relevance of different shocks.

Each of the following figures shows the impulse response of the model's variables to a particular shock. Each shock is of a size of one-standard-deviation. The first column of each figure displays the impulse responses of the macroeconomic variables (the nominal short-term interest rate r_t, the inflation rate π_t, the output gap g_t^y, and central bank's inflation target $\pi_t^\star$). The second column contains the impulse responses of the yield rates (from the 12-month rate to the 60-month rate). The third and fourth column display the IRFs of the one-period return premium $E_t\left(hprx_{t+1}^{(\tau)}\right)$ and the yield premium κ_t, respectively, incorporated in yield rates with corresponding maturities. By construction, the one-period return premium is driven only by the risk variable υ_t, while the yield premium, which captures the premium in yields over the full lifetime of the bond, is affected by all state variables. The light gray shaded areas cover the 90 percentage HPD

Table 3: Results: Posterior Distribution

Parameter	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
ρ_r	0.8000	0.8894	0.8880	0.8623	0.9156	$\mathcal{B}$
ρ_π	1.5000	1.2151	1.2220	0.9557	1.4898	$\mathcal{G}$
ρ_y	0.0450	0.0387	0.0335	0.0066	0.0565	$\mathcal{G}$
ρ_v	0.0000	-0.3734	-0.4419	-0.5631	-0.3152	$\mathcal{N}$
ρ_{vv}	0.9000	0.9121	0.8810	0.8318	0.9287	$\mathcal{N}$
ρ_{vr}	0.0000	-0.3025	-0.3268	-0.4596	-0.2015	$\mathcal{N}$
$\rho_{v\pi}$	0.0000	-0.1544	-0.0781	-0.2496	0.0977	$\mathcal{N}$
ρ_{vy}	0.0000	0.2206	0.1852	0.1339	0.2386	$\mathcal{N}$
$\rho_{v\pi^*}$	0.0000	-0.3309	-0.2981	-0.3694	-0.2326	$\mathcal{N}$
ρ_{π^*}	0.9000	0.9911	0.9914	0.9848	0.9983	$\mathcal{B}$
$\rho_{\pi v}$	0.0000	-0.0287	-0.0304	-0.0480	-0.0127	$\mathcal{N}$
ρ_{yv}	0.0000	-0.0672	-0.0513	-0.0989	-0.0067	$\mathcal{N}$
$\rho_{\pi r}^1$	-0.2000	-0.0817	-0.0759	-0.1943	0.0457	$\mathcal{N}$
$\rho_{\pi r}^2$	0.0000	0.0009	0.0070	-0.0816	0.1006	$\mathcal{N}$
$\rho_{\pi r}^3$	0.0000	-0.0308	-0.0221	-0.0898	0.0434	$\mathcal{N}$
$\rho_{\pi\pi}^1$	0.9000	1.0409	0.9550	0.8510	1.0510	$\mathcal{N}$
$\rho_{\pi\pi}^2$	0.0000	-0.0613	0.0226	-0.0654	0.1068	$\mathcal{N}$
$\rho_{\pi\pi}^3$	0.0000	-0.0266	-0.0561	-0.1170	0.0040	$\mathcal{N}$
$\rho_{\pi y}^1$	0.0000	-0.0104	-0.0089	-0.0441	0.0272	$\mathcal{N}$
$\rho_{\pi y}^2$	0.0000	0.0165	0.0230	-0.0214	0.0690	$\mathcal{N}$
$\rho_{\pi y}^3$	0.0000	0.0341	0.0262	-0.0022	0.0527	$\mathcal{N}$
ρ_{yr}^1	-0.2000	-0.0805	0.0045	-0.1536	0.1737	$\mathcal{N}$
ρ_{yr}^2	0.0000	-0.0004	0.0185	-0.0955	0.1259	$\mathcal{N}$
ρ_{yr}^3	0.0000	-0.0536	-0.0169	-0.0952	0.0576	$\mathcal{N}$
$\rho_{y\pi}^1$	0.0000	0.1303	0.0966	-0.0667	0.2539	$\mathcal{N}$
$\rho_{y\pi}^2$	0.0000	0.0698	-0.0099	-0.1153	0.1032	$\mathcal{N}$
$\rho_{y\pi}^3$	0.0000	-0.0173	-0.0060	-0.0766	0.0650	$\mathcal{N}$

Parameter	Prior Mean	Post. Mode	Post. Mean	90% HPD Interval		Prior
ρ_{yy}^1	0.9000	1.0774	1.0535	0.9724	1.1343	$\mathcal{N}$
ρ_{yy}^2	0.0000	-0.0295	-0.0025	-0.0890	0.0867	$\mathcal{N}$
ρ_{yy}^3	0.0000	-0.1568	-0.1580	-0.2121	-0.0997	$\mathcal{N}$
σ_r	0.0100	0.0015	0.0015	0.0013	0.0017	$\mathcal{IG}$
σ_π	0.0100	0.0024	0.0024	0.0021	0.0027	$\mathcal{IG}$
σ_y	0.0100	0.0100	0.0101	0.0090	0.0112	$\mathcal{IG}$
σ_{π^*}	0.0100	0.0012	0.0013	0.0012	0.0014	$\mathcal{IG}$
$\sigma_{\pi\pi^*}$	0.0000	-0.9346	-0.8115	-1.1483	-0.4841	$\mathcal{N}$
$\sigma_{\pi y}$	0.0000	0.0343	0.0400	0.0066	0.0764	$\mathcal{N}$
$\sigma_{y\pi^*}$	0.0000	0.3129	0.1885	-1.1469	1.4834	$\mathcal{N}$
σ_{vr}	0.0000	-0.0498	0.9940	-0.4997	2.5729	$\mathcal{N}$
$\sigma_{v\pi}$	0.0000	2.6076	1.9368	1.1157	2.7783	$\mathcal{N}$
σ_{vy}	0.0000	-0.5391	-0.4342	-0.7126	-0.1729	$\mathcal{N}$
$\sigma_{v\pi^*}$	0.0000	-0.6049	0.6033	-1.5026	2.4979	$\mathcal{N}$
Λ^r	0.0000	1.1078	2.2702	-1.4728	6.0125	$\mathcal{N}$
Λ^π	0.0000	-2.4393	-5.0232	-9.9820	-0.0001	$\mathcal{N}$
Λ^y	0.0000	0.1887	-1.4573	-5.4025	2.6230	$\mathcal{N}$
Λ^{π^*}	0.0000	-0.5709	-0.8612	-1.3745	-0.3219	$\mathcal{N}$
σ_{12}	0.0001	0.0005	0.0005	0.0005	0.0006	$\mathcal{IG}$
σ_{24}	0.0001	0.0001	0.0001	0.0001	0.0001	$\mathcal{IG}$
σ_{36}	0.0001	0.0000	0.0000	0.0000	0.0000	$\mathcal{IG}$
σ_{48}	0.0001	0.0000	0.0000	0.0000	0.0000	$\mathcal{IG}$
σ_{60}	0.0001	0.0001	0.0001	0.0001	0.0001	$\mathcal{IG}$

Table 4: Results: Posterior Distribution

interval while the dark gray shaded areas cover the 68 HPD interval. The IRF (displayed by the blue line) is computed as the mean impulse response. The output gap is depicted in percentage deviation of the steady state, and the inflation- and the yield rate are shown in annualized percentage points. One period corresponds to one month.

Figure (1) shows the response to a term premium shock. The increase in the risk variable causes one-period return premia and yield premia for bond yields of all maturities to rise. Similar to a negative demand shock, output and inflation drop in response to a term premium shock. In line with the findings of Ireland (2015) for the US, the plots show that an exogenous rise in term premia works to dampen economic activity. According to the interest rate rule, the rise in the risk variable causes the central bank to ease monetary policy and to drop short-term rate. The one-period return premia and the yield premia follow the risk variable closely. The effects of a term premium shock are more pronounced for premia of bonds with longer maturities. Following the short-term interest rate, long-term yields decline. Notably, the decline in long-term yields is mitigated by the increase

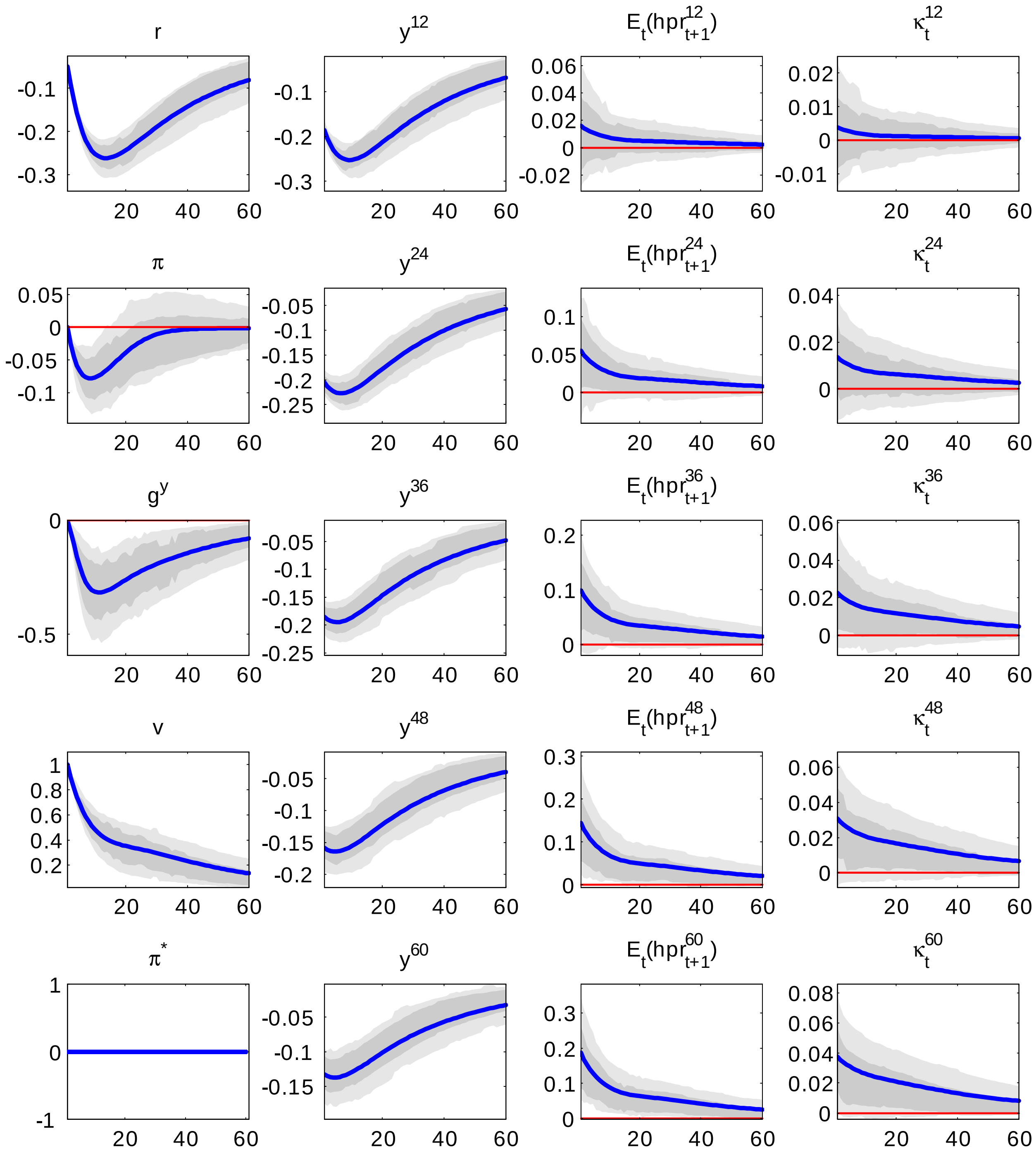

Figure 1: Impulse responses of the model's variables to a one-standard-deviation term premia variable shock ε_{vt}.

in term premia. Since the inflation target is given by a univariate autoregressive process, the inflation target is not affected by shocks to the other state variables.

Figure (2) displays the response of the economy to a positive interest rate rule shock. The interest rate rises on impact and stays above its steady state level for more than 12 months, converging back to its steady state. The response of the output gap and the response of the inflation rate to the interest rate shock are in line with previous study and economic theory. The tightening of monetary policy dampens economic activity, leading to a drop in output and inflation though the response of the output gap is not statistically significant from zero on the 90 percent level. The mean IRFs of the risk variable and of the term premia variables confirm their close relationship though the response of the risk variable and the term premia variables to the interest rate shock is not significantly different from zero.

The impulse responses to the output shock ε_{yt} are displayed in figure (3). The output gap rises sharply on impact and decreases slowly over the next 16 months back to its steady state. Inflation rises slowly with its peak after 8 months and remains positive for another 12 months. The increase in the output gap and inflation causes monetary policy to tighten. Following the rise in the short-term interest rate, all yields move upward. Overall, the effects of this shock work similar to an aggregate demand shock. The IRF of the risk variable reveals an interesting dynamic of term premia over time in response to an output shock. On impact, the term premia variable drops and converges back to its steady state value for the next 3 months. After recovering, v_t remains significantly above zero for more than 18 months.

The impulse responses to an innovation in the inflation rate are shown in figure (4). Inflation rises sharply and converges back to its steady state in less than 16 months. According to the interest rate rule, the short-term interest rate is raised in response to the jump in inflation. Yields follow the short-term interest rate. The response of the output gap is not significantly different from zero. The response of the economy to the inflation shock affect mainly inflation and the risk variable. In response to the inflation shock, the risk variable rises on impact and stays significantly different from zero for more than 12 months. Following the risk variable, the one-period return premium and the yield premium both rise.

Finally, figure (5) presents the impulse responses to a shock to the inflation target π_t^*.

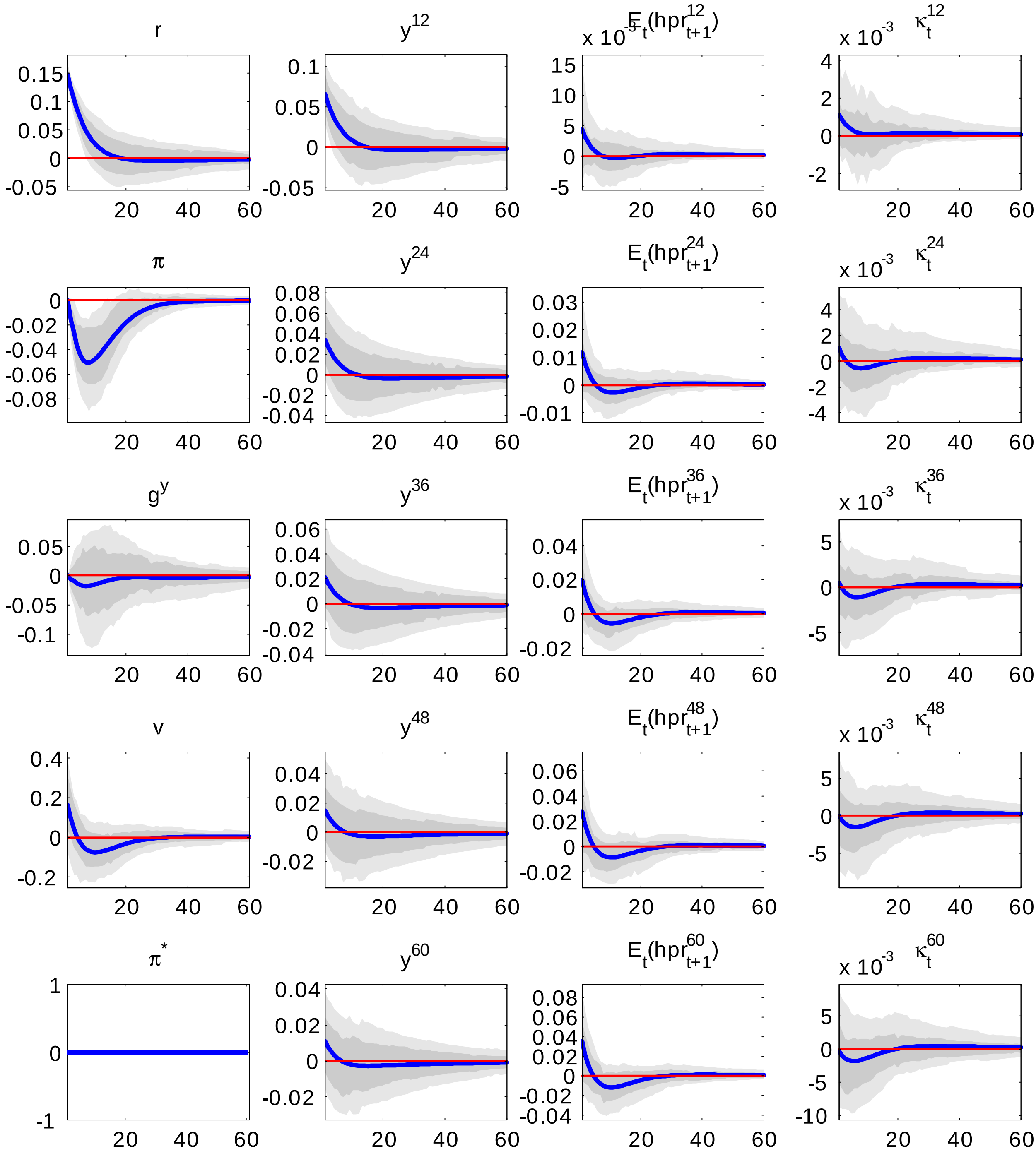

Figure 2: Impulse responses of the model's variable to a one-standard-deviation interest rate shock ε_{rt}.

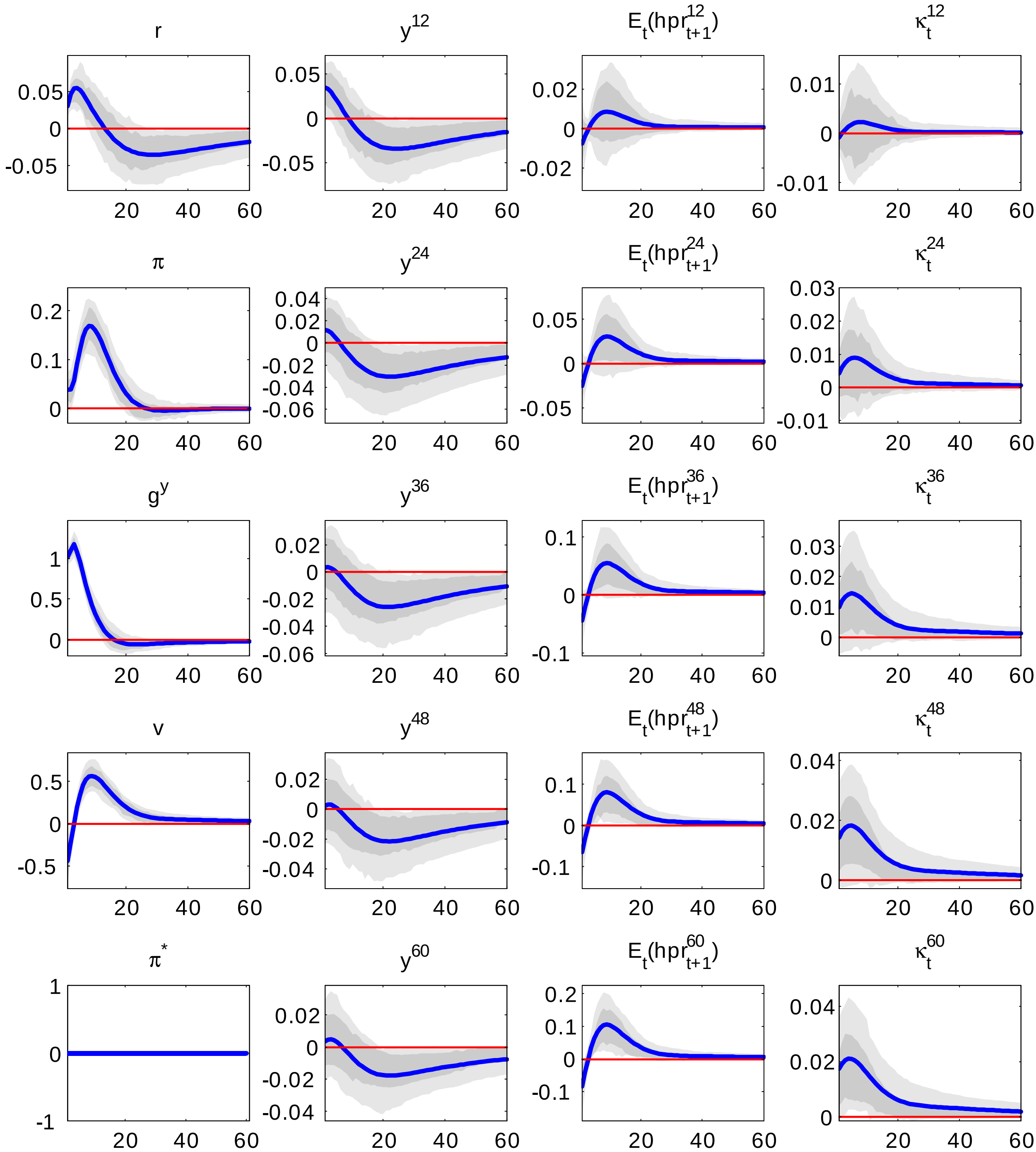

Figure 3: Impulse responses of the model's variables to a one-standard-deviation output gap shock ε_{yt}.

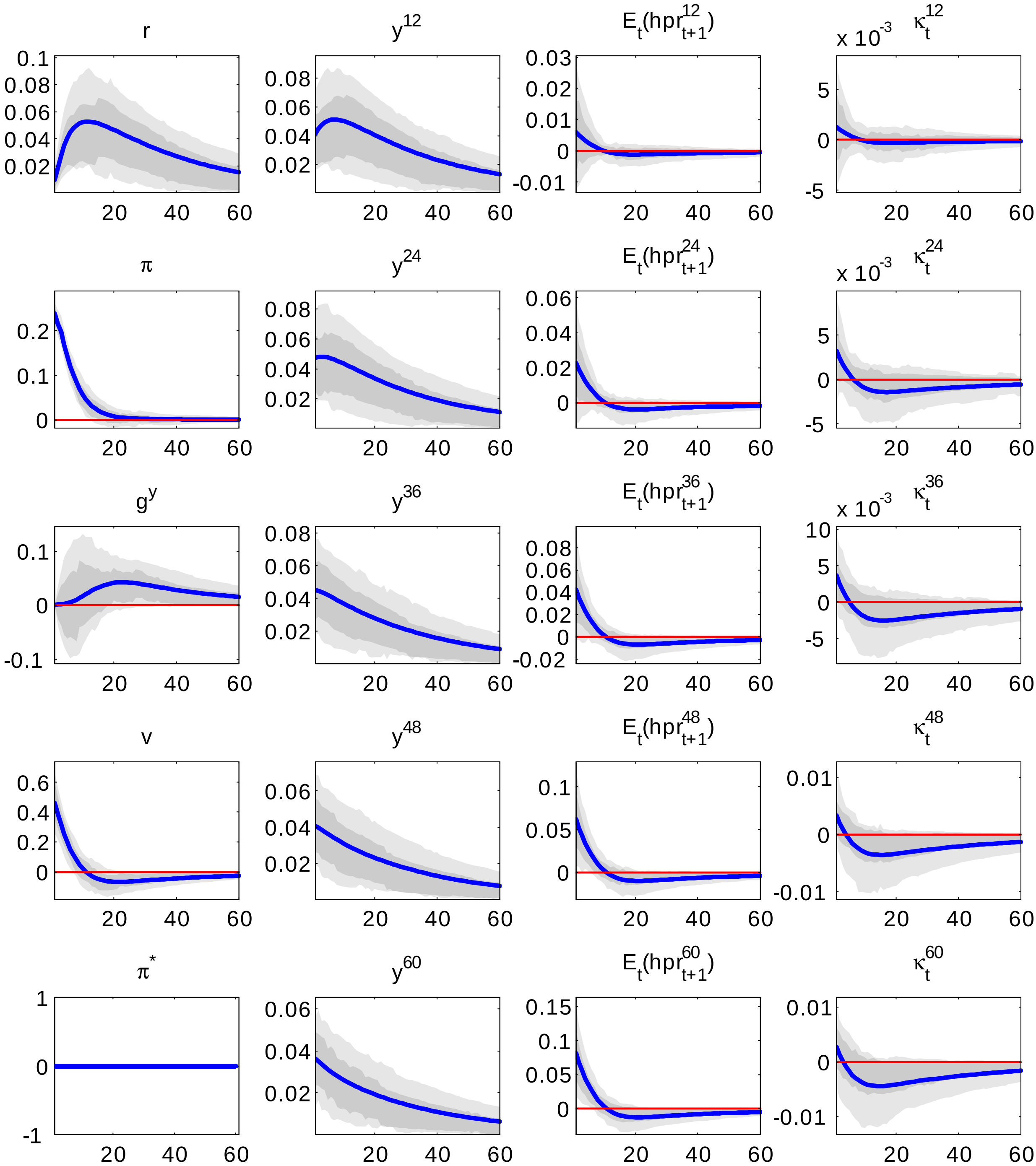

Figure 4: Impulse responses of the model's variables to a one-standard-deviation inflation shock $\varepsilon_{\pi t}$.

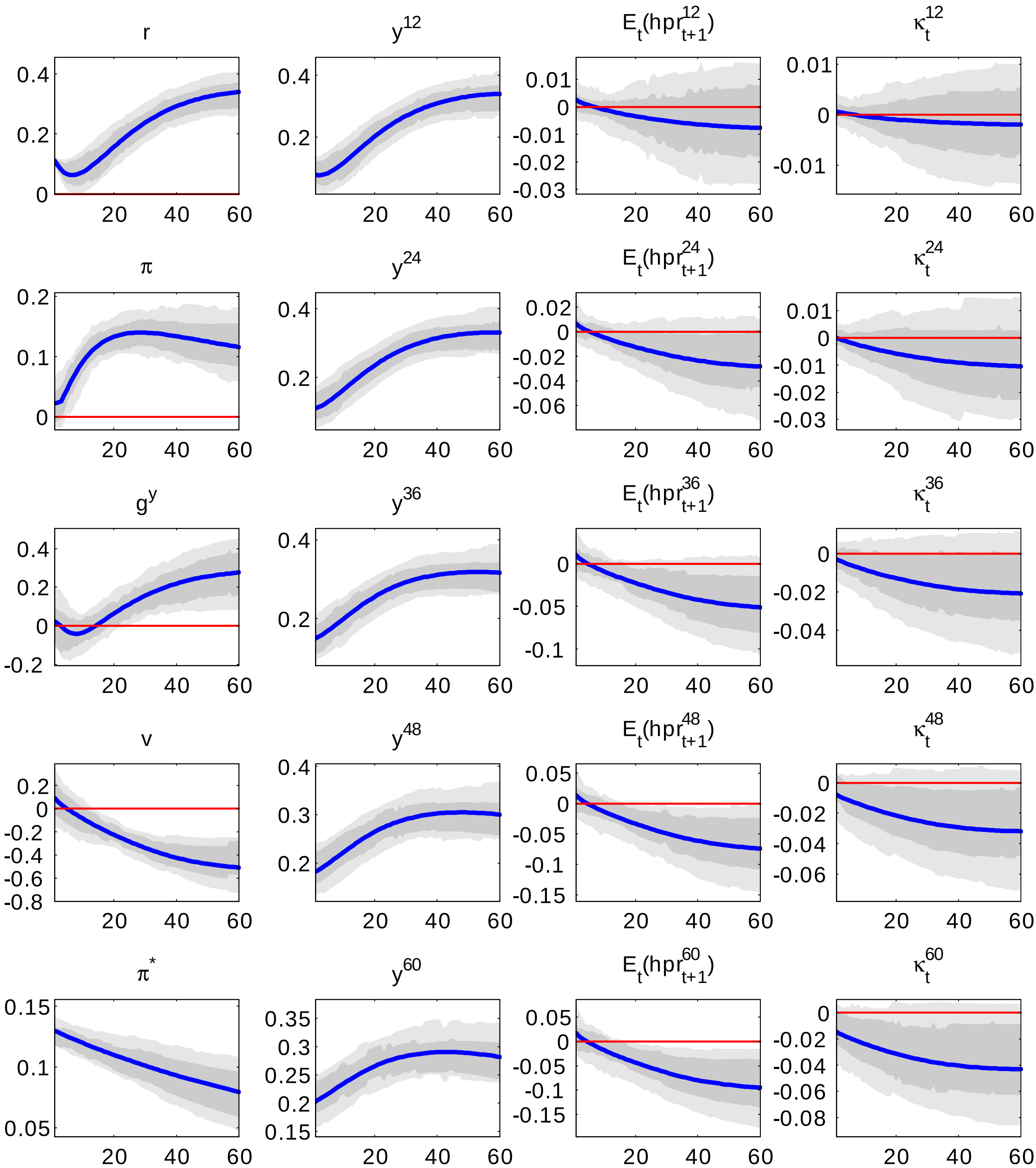

Figure 5: Impulse responses of the model's variables to a one-standard-deviation output gap shock $\varepsilon_{\pi^* t}$.

From the parameter estimates of the inflation target process $\rho_{\pi^*} = 0.9939$ the inflation target process is highly persistent. Actual inflation rises along with the new inflation target rate. Also, the nominal short-term interest rate and bond yields rise. The inflation target shock works similar to the level factor shock in finance term structure models.[11] It moves bond yields simultaneously and persistently upward, resulting in a higher level of the yield curve. The output gap does not respond on impact but starts to rise slowly after two years. Similarly to the findings of Ireland (2015), term premia drop in response to the shock to the inflation target.

The displayed results highlight a multidirectional interaction of term premia, monetary policy, and the macroeconomy. Previous empirical works indicate that the bond term premium varies over the business cycle and that this variation is countercyclical (Cochrane and Piazzesi, 2005; Piazzesi and Swanson, 2008; or Ludvigson and Ng, 2009). My results correspond to these findings, but emphasizes that the kind of underlying disturbance is crucial for the sign of the correlation between output gap and term premia, as theory suggests (see e.g. Hördahl et al., 2008, Rudebusch, Sack and Swanson, 2007, or Rudebusch and Swanson, 2012). Shocks to the risk variable move output gap and term premia in opposite directions, leading to a countercyclical relationship. Shocks to the inflation target do not move output gap and term premia on impact, but with a delay. In contrast, the relation between output gap and term premia dynamics in response to output shocks is more complicated. A positive innovation to the output gap results into a countercyclical movement of term premia on impact and for the next periods. However, after around 18 months the relationship is eventually reversed. The results indicate, thus, whether term premia are countercyclical over the business cycle or not depends on the source of the fluctuations.

Next, in order to assess the relative importance of different shocks for the variability of a variable, I compute the forecast error variance decomposition (FEVD). The FEVD helps to quantify the contribution of each of the five structural shocks to the forecast error variance of the model's variables. Formally, the fraction of the forecast error variance of variable i due shock j for horizon h, denoted by $\phi_{i,,j}(h)$, is defined by

$$\phi_{i,,j}(h) = \frac{\omega_{i,,j}(h)}{\Omega_i(h)},$$

[11]The first three latent factors studied in affine term structure models in finance are commonly denoted by "level", "slope", and "curvature" factor, referring to the effect the factors have on the yield curve.

Table 5: FEVD of macroeconomic variables

Short-Term Interest Rate					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	59.35	0.12	2.64	4.53	33.26
12	15.19	2.86	2.98	64.91	14.06
36	3.87	1.83	1.34	51.70	41.26
60	2.01	1.06	0.90	31.40	64.99
∞	0.70	0.38	0.33	11.26	87.33
Inflation					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	0.00	97.96	1.93	0.00	0.11
12	4.44	45.07	34.09	9.25	7.15
36	4.74	13.79	22.55	8.42	40.49
60	4.00	20.04	18.97	10.32	46.67
∞	3.86	19.30	18.29	10.97	47.59
Output Gap					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	0.00	0.00	99.85	0.00	0.15
12	0.49	0.07	90.91	8.46	0.10
36	0.62	0.08	84.29	10.71	4.30
60	0.59	0.08	80.25	10.24	8.84
∞	0.53	0.07	71.49	9.14	18.76
Term Premium					
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}
1	0	23.87	17.07	58.73	0.32
12	2.82	11.31	31.43	52.98	1.47
36	3.51	6.47	22.67	53.31	14.04
60	2.20	4.16	14.01	40.74	38.88
∞	0.76	1.45	4.83	14.92	78.04

where $\omega_{i,,j}(h)$ is the forecast error variance of variable i due to shock j at horizon h and $\Omega_i(h)$ is the total error forecast variance of variable i at horizon h. Table (5) and (6) present the FEVD of the model's variables for different horizons to the five structural disturbances. The FEVD of the macroeconomic variables are displayed in table (5). Since the inflation target does only react on own innovations, over all horizons 100 percent of the forecast error variance is simply explained by inflation target shocks. Therefore it is omitted from table (5).

In the short run, more than half of the variability of the short-term interest rate is due to interest rate shocks. Term premium shocks ε_{vt} account for between 30 to 65 percent of the error forecast variance of the interest rate on a two to five-year horizon. In the long run, inflation target shocks account for more than 87 percent of movements in the interest rate.

Term premia shocks do not only move the short term interest rate, but do also account for sizeable variations in inflation, output gap and the risk variable itself, revealing a non-negligible influence of term premia shocks on the economy. In line with the "practitioners view", risk shocks play an important role for economic activity. They account for between 8 - 10 percent of the forecast error variance in the output gap, and also for between 8 - 10 percent in the inflation rate, both at horizons between one and five years. The forecast error variance of the risk variable in turn is driven by different disturbances, each differently important on different horizons. On a one to five-year horizon, term premium shocks account for the bulk of movements in term premia (between 40 and 53 percent). This corresponds to the findings of Dewachter et al. (2015a) and Ireland (2015) who find that a large fraction of movements in term premia is not driven by macroeconomic shocks, but by exogenous term premia shocks. In the short run, in addition to term premia shocks, inflation shocks account for a large fraction of the forecast error variance in term premia, while in the long run inflation target shocks account for more than 78 percent of the forecast error variance in term premia. At the horizon between three and five years, output gap shocks ε_{yt} account for between 14 and 22 percent of variations in term premia. The results indicate a bidirectional linkage, running from the macroeconomic to term premia and vice versa. According to the estimated model, interest rate shocks did not account for much variance of the other variables over the sample period. Movements in the output gap are mainly driven by own shocks. Variations in the inflation rate are due to inflation shocks and output gap shocks in the short run and inflation target shocks in the long run. Term premia and output shocks account for a sizeable fraction of the forecast error variance in the inflation rate on a one to three years horizon.

The FEVD of bond yields is presented in table (6). In addition to the five fundamental disturbances, also the components of the forecast error variance stemming from measurement errors are reported. Term premium shocks account for sizeable variation in bond yields, in particular, for bonds with shorter terms to maturity and for short forecast horizons. In line with evidence of Barr and Campbell (1997), Gürkaynak et al. (2005) or Hördahl et al. (2006), most of the variation in bond yields is caused by inflation target shocks. The contribution of inflation target shocks to the forecast error variance in bond yields is even more pronounced for long-term bonds and increasing in the forecast horizon. Inflation target shocks account for between 51 and 92 percent of movements in yields of

Table 6: FEVD of Bond yields

One Year Yield Rate										
Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	13.98	2.68	2.53	60.48	14.74	5.60	0	0	0	0
12	3.98	2.94	0.59	74.17	17.98	0.35	0	0	0	0
36	1.25	1.53	0.82	45.11	51.20	0.08	0	0	0	0
60	0.69	0.90	0.58	27.12	70.67	0.05	0	0	0	0
∞	0.26	0.35	0.24	10.54	88.60	0.02	0	0	0	0
Two Year Yield Rate										
Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_π	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	4.79	3.03	0.18	67.16	24.65	0	0.20	0	0	0
12	1.58	2.36	0.19	62.12	33.73	0	0.01	0	0	0
36	0.55	1.13	0.59	34.11	63.62	0	0	0	0	0
60	0.32	0.68	0.42	20.80	77.78	0	0	0	0	0
∞	0.13	0.35	0.24	10.54	98.60	0	0	0	0	0
Three Year Yield Rate										
Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	2.31	2.54	0.00	53.30	41.83	0	0	0	0	0
12	0.82	1.77	0.17	45.97	51.27	0	0	0	0	0
36	0.30	0.82	0.44	24.46	73.98	0	0	0	0	0
60	0.18	0.50	0.32	15.27	83.72	0	0	0	0	0
∞	0.08	0.22	0.14	6.74	92.82	0	0	0	0	0
Four Year Yield Rate										
Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	1.29	2.00	0.00	54.30	41.83	0	0	0	0	0
12	0.46	1.30	0.11	32.61	65.52	0	0	0	0	0
36	0,18	0.60	0.31	17.42	81.49	0	0	0	0	0
60	0.12	0.38	0.23	11.16	88.12	0	0	0	0	0
∞	0.05	0.17	0.11	5.20	94.46	0	0	0	0	0
Five Year Yield Rate										
Horizon	Structural Shock					Measurement errors				
h	ε_r	ε_π	ε_y	ε_v	ε_{π^*}	η_t^{12}	η_t^{24}	η_t^{36}	η_t^{48}	η_t^{60}
1	0.77	1.56	0.00	27.04	70.56	0	0	0	0	0.07
12	0.37	0.97	0.06	23.04	75.65	0	0	0	0	0.01
36	0.11	0.45	0.21	12.52	86.70	0	0	0	0	0
60	0.07	0.29	0.16	8.22	91.25	0	0	0	0	0
∞	0.04	0.14	0.08	4.02	95.72	0	0	0	0	0

bonds of 3- to 5-year residual maturity at forecast horizons between one and five years. But also for bonds with shorter terms to maturity (two years and less), inflation target shocks are important determinants of the forecast error variance. These findings confirm the earlier observation that inflation target shocks work similar to a level shock, moving the entire yield curve upward. Notably, measurement error shocks do not contribute to much movement in bond yields, confirming a good fit of the model. They account for around 5 percent of the one-month ahead forecast error variance in the one-year rate, less than 0.23 percent of the one-month ahead forecast error variance in the two-year rate and even less for rates of bonds with longer terms to maturity. The contribution of measurement errors to the variance of bond yields declines considerably with the forecast horizon.

6 Conclusion

In this work, I evaluate the interplay of term premia, monetary policy and the economy in the euro area. Using a macro-finance model of the term structure, which explicitly allows term premia to affect the economy, my findings reveal a broad interaction among term premia, monetary policy, and the economy. Movements in term premia are captured by an unobservable risk variable, which responds to all other state variables, but also exhibits an autonomous dynamic. By restricting the prices of risk in the pricing kernel (as in Dewachter et al., 2015a, and Ireland, 2015) this variable is identified to account for all variations in the one-period return premium. Furthermore, restrictions, similar to those from more conventional VAR models, on the state process of macroeconomic variables are entailed to disentangle the effects of fundamental shocks to the endogenous variables. In line with earlier studies of the term structure and term premia, I find that the term premium is time-varying and that it responds to the state of the economy, contradicting the expectation hypotheses.

I emphasize two aspects of my findings. First, a rise in the term premium does affect the economy. Precisely, proving evidence for the practitioner view, a pure exogenous term premium shock dampens output and inflation, similar to an aggregate demand shock. Second, the analysis reveals that the ECB reacted on movements in the term premium during the sample period. Indeed, in order to counteract the change in the premia, the central bank shifts the policy rate contrary to the change in the premium. Furthermore, this paper does not find evidence for strong effects of conventional monetary policy on term premia. Nevertheless, overall, the results indicate a broad dynamic between term premia, monetary policy- and the economy.

Examining how term premia movements affect the economy and the effects of conventional monetary policy on term premia is one first step. A natural question arising from these finding is how unconventional monetary policy actions, in particular, "quantitative easing" (QE), affects the term premium. QE intends to stimulate the economy through aggregate demand channels not only by reducing long-term yields, the so-called signaling channel but also by reducing the term premium part in long-term yields, the so-called "portfolio-balance" channel (International Monetary Fund report, 2013). Recent studies[12] find that QE worked to reduce long-term yields though the magnitude of these effects differs greatly and the channel through which large-scale asset purchases affects long-term yields is not clear. If changes in term premia work to affect the economy, what are the qualitative and quantitative effects of QE on term premia? However, the analysis of these effects is beyond the scope of this paper.

[12] Among many others, Bauer and Rudebusch (2015), Carlstrom, Fuerst, and Paustian (2014), Chen, Cúrdia, and Ferrero (2012), Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), and Woodford (2012).

A Appendix

A.1 Parameter Vectors and Matrices

The vectors and matrices P_0, P_1, μ_0, and Σ_0 in eq. (7) are defined as

$$P_0 \equiv \begin{bmatrix} 1 & 0 & 0 & -(1-\rho_r)\,\rho_\pi & 0 & 0 & -(1-\rho_r)\,\rho_y & 0 & 0 & 0 & -(1-\rho_r)\,\rho_v \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$P_1 \equiv \begin{bmatrix} \rho_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho^1_{\pi r} & \rho^2_{\pi r} & \rho^3_{\pi r} & \rho^1_{\pi\pi} & \rho^2_{\pi\pi} & \rho^3_{\pi\pi} & \rho^1_{\pi y} & \rho^2_{\pi y} & \rho^3_{\pi y} & 0 & \rho_{\pi v} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho^1_{yr} & \rho^2_{yr} & \rho^3_{yr} & \rho^1_{y\pi} & \rho^2_{y\pi} & \rho^3_{y\pi} & \rho^1_{yy} & \rho^2_{yy} & \rho^3_{yy} & 0 & \rho_{yv} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\pi^\star} & 0 \\ \rho_{vr} & 0 & 0 & \rho_{v\pi} & 0 & 0 & \rho_{vy} & 0 & 0 & \rho_{v\pi^\star} & \rho_{vv} \end{bmatrix},$$

$$\mu_0 \equiv \begin{bmatrix} (1-\rho_r)\left(g^r - \rho_y g^y\right) \\ 0 \\ 0 \\ -\left(\rho^1_{\pi r} + \rho^2_{\pi r} + \rho^3_{\pi r}\right) g^r - \left(\rho^1_{\pi y} + \rho^2_{\pi y} + \rho^3_{\pi y}\right) g^y \\ 0 \\ 0 \\ \left(1 - \left[\rho^1_{yy} + \rho^2_{yy} + \rho^3_{yy}\right]\right) g^y - \left(\rho^1_{yr} + \rho^2_{yr} + \rho^3_{yr}\right) g^r \\ 0 \\ 0 \\ (1-\rho_{\pi^\star})\,\pi^\star \\ -\rho_{vr} g^r - \rho_{vy} g^y - \rho_{v\pi^\star}\pi^\star \end{bmatrix}$$

and

$$\Sigma_0 \equiv \begin{bmatrix} \sigma_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\pi & 0 & 0 & \sigma_{\pi y}\sigma_y & 0 & 0 & \sigma_{\pi\pi^\star}\sigma_{\pi^\star} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_y & 0 & 0 & \sigma_{y\pi^\star}\sigma_{\pi^\star} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\pi^\star} & 0 \\ \sigma_{vr}\sigma_r & 0 & 0 & \sigma_{v\pi}\sigma_\pi & 0 & 0 & \sigma_{vy}\sigma_y & 0 & 0 & \sigma_{v\pi^\star}\sigma_{\pi^\star} & \sigma_v \end{bmatrix}.$$

A.2 Recursive bond prices

Following Ang and Piazzesi (2003), the difference equations are derived by induction using eq. (15). Start with $\tau = 0$, then, from $P^0_{t+1} = 1$, eq. (15) implies

$$\begin{aligned} P^1_t &= E_t\left(m_{t+1}\right) \\ &= E_t\left(\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\right) \\ &= \exp\left(-r_t\right) \end{aligned}$$

where I used that ε_t is standard normally distributed so that m_{t+1} log-normal distributed with mean $\mu = -r_t - \frac{1}{2}\lambda_t'\lambda_t$ and variance $\sigma^2 = \lambda_t'\lambda_t$. Now suppose that

$P_t^1 = \exp\left(\bar{A}_1 + \bar{B}_1 X_t\right)$ holds, then substituting eq. (8) for r_t leads to

$$\exp\left(\bar{A}_\tau + \bar{B}_\tau' X_t\right) = \exp\left(-\delta_1' X\right).$$

Matching coefficients leads to the initial conditions $\bar{A}_1 = 0$ and $\bar{B}_1' = -\delta_1'$. Next, in order to show that the recursions in eq. (17) and (18) hold for arbitrary values of $\tau > 1$ suppose that $P_t^\tau = \exp\left(\bar{A}_\tau + \bar{B}_\tau X_t\right)$. Substitute eq. (7), eq. (12), (11) and (16) into eq. (15) yields

$$\begin{aligned}
P_t^{\tau+1} &= E_t\left(\exp\left(-\delta_1' X_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\exp\left(\bar{A}_\tau + \bar{B}_\tau' X_{t+1}\right)\right) \\
&= \exp\left(-\delta_1' X_t - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau\right) E_t\left(\exp\left(\bar{B}_\tau' X_{t+1} - \lambda_t'\varepsilon_{t+1}\right)\right) \\
&= \exp\left(-\delta_1' X_t - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau\right) E_t\left(\exp\left(\bar{B}_\tau'\left[\mu + P X_t + \Sigma\varepsilon_{t+1}\right] - \lambda_t'\varepsilon_{t+1}\right)\right) \\
&= \exp\left(-\frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau + \bar{B}_\tau'\mu + \left[\bar{B}_\tau' P - \delta_1'\right] X_t\right) E_t\left(\exp\left(\bar{B}_\tau'\Sigma\varepsilon_{t+1} - \lambda_t'\varepsilon_{t+1}\right)\right) \\
&= \exp\left(-\frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau + \bar{B}_\tau'\mu + \left[\bar{B}_\tau' P - \delta_1'\right] X_t\right) E_t\left(\exp\left(\left[\bar{B}_\tau'\Sigma - \lambda_t'\right]\varepsilon_{t+1}\right)\right) \\
&= \exp\left(-\frac{1}{2}\lambda_t'\lambda_t + \bar{A}_\tau + \bar{B}_\tau'\mu + \left[\bar{B}_\tau' P - \delta_1'\right] X_t + \frac{1}{2}\left[\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - 2\bar{B}_\tau'\Sigma\lambda_t + \lambda_t'\lambda_t\right]\right) \\
&= \exp\left(\bar{A}_\tau + \bar{B}_\tau'\mu + \frac{1}{2}\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - \bar{B}_\tau'\Sigma\lambda_t + \left[\bar{B}_\tau' P - \delta_1'\right] X_t\right) \\
&= \exp\left(\bar{A}_\tau + \bar{B}_\tau'\mu + \frac{1}{2}\bar{B}_\tau'\Sigma\Sigma'\bar{B}_\tau - \bar{B}_\tau'\Sigma\lambda_0 + \left[\bar{B}_\tau' P - \bar{B}_\tau'\Sigma\lambda_1 - \delta_1'\right] X_t\right),
\end{aligned}$$

where the sixth equality is obtained by computing the expectation of the exponential function using the normality of ε_{t+1} and

$$E_t\left(\exp\left(\left[\bar{B}_\tau'\Sigma - \lambda_t'\right]\varepsilon_{t+1}\right)\right) = \exp\left(\bar{\mu} + \frac{1}{2}\bar{\sigma}^2\right)$$

with $\bar{\mu} = 0$ and $\bar{\sigma}^2 = \bar{B}_\tau\Sigma\Sigma'\bar{B}_\tau' - 2\bar{B}_\tau\Sigma\lambda_t + \lambda_t'\lambda_t$ Matching coefficients yields the recursive relations in eq. (17) and (18).

A.3 Yield premium and return premium

This part of the appendix demonstrates that the yield premium can be written as the average of expected future return premia of declining maturity. The yield premium of

τ-period bond $\kappa_t^{(\tau)}$ is given by

$$\begin{aligned}
\kappa_t^{(\tau)} &= y_t^{(\tau)} - \frac{1}{\tau} E_t \left[\sum_{i=0}^{\tau-1} y_{t+i}^{(1)} \right] = \frac{1}{\tau} \left[\tau y_t^{(\tau)} - E_t \sum_{i=0}^{\tau-1} y_{t+i}^{(1)} \right]. \\
&= \frac{1}{\tau} \left[-p_t^{(\tau)} - E_t \sum_{i=0}^{\tau-1} y_{t+i}^{(1)} \right]
\end{aligned}$$

where the last equality uses the relation $y_t^{\tau} = -p_t^{\tau}/\tau$. Now add $E_t \sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1} - E_t \sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1}$, rearrange terms, and use the definition $E_t \left(hprx_{t+i+1}^{\tau-i} \right) = p_{t+i+1}^{(\tau-i-1)} - p_{t+i}^{(\tau-i)} - y_{t+i}^{(1)}$ to obtain

$$\begin{aligned}
\kappa_t^{(\tau)} &= \frac{1}{\tau} \left[-p_t^{(\tau)} - E_t \sum_{i=0}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[\sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=0}^{\tau-1} p_{t+i+1}^{\tau-i-1} - p_t^{(\tau)} - \sum_{i=0}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[p_{t+1}^{\tau-1} - p_t^{(\tau)} - y_t^{(1)} - p_{t+1}^{\tau-1} + \sum_{i=1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=1}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[hprx_{t+1}^{\tau} - p_{t+1}^{\tau-1} + \sum_{i=1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=1}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[hprx_{t+1}^{\tau} + p_{t+2}^{\tau-2} - p_{t+1}^{\tau-1} - y_{t+1}^{(1)} - p_{t+2}^{\tau-2} + \sum_{i=2}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=2}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=2}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[hprx_{t+1}^{\tau} + hprx_{t+2}^{\tau-1} - p_{t+2}^{\tau-2} + \sum_{i=2}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=2}^{\tau-1} p_{t+i+1}^{\tau-i-1} - E_t \sum_{i=2}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \ldots \\
&= \frac{1}{\tau} E_t \left[\sum_{i=0}^{\tau-2} hprx_{t+i+1}^{\tau-i} - p_{t+\tau-1}^{\tau-(\tau-2)-1} + \sum_{i=\tau-1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=\tau-1}^{\tau-1} p_{t+i+1}^{\tau-i-1} - \sum_{i=\tau-1}^{\tau-1} y_{t+i}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[\sum_{i=0}^{\tau-2} hprx_{t+i+1}^{\tau-i} - p_{t+\tau-1}^{\tau-(\tau-2)-1} + p_{t+\tau-1+1}^{\tau-(\tau-1)-1} - p_{t+\tau-1+1}^{\tau-(\tau-1)-1} - y_{t+\tau-1}^{(1)} \right] \\
&= \frac{1}{\tau} E_t \left[\sum_{i=0}^{\tau-2} hprx_{t+i+1}^{\tau-i} + p_{t+\tau}^{(0)} - p_{t+\tau-1}^{(1)} - y_{t+\tau-1}^{(1)} - p_{t+\tau}^{(0)} \right]
\end{aligned}$$

Finally, note that $p_{t+\tau}^{(0)} = 0$ (since $P_{t+\tau}^{0} = \exp\left(p_{t+\tau}^{(0)}\right) = 1$) and $E_t \left(hprx_{t+\tau}^{1} \right) = p_{t+\tau}^{(0)} - p_{t+\tau-1}^{(1)} - y_{t+\tau-1}^{(1)}$. hence,

$$\kappa_t^{(\tau)} = \frac{1}{\tau} \sum_{i=0}^{\tau-1} E_t \left(hprx_{t+i+1}^{\tau-i} \right)$$

A.4 Computation of the $i+1$-period return premium

The return premium is given by (for $\tau > i$)

$$
\begin{aligned}
E_t\left(hprx_{t+i+1}^{(\tau)}\right) &= E_t\left(hpr_{t+i+1}^{(\tau)}\right) - E_t\left(y_{t+i}^{(1)}.\right). \\
&= E_t\left(p_{t+i+1}^{(\tau-1)} - p_{t+i}^{(\tau)}\right) - E_t\left(y_{t+i}^{(1)}.\right)
\end{aligned}
$$

Plugging the log prices and the expected short rate into the equation above yields

$$
E_t\left(hprx_{t+i+1}^{(\tau)}\right) = \bar{A}_{(\tau-1)} + \bar{B}'_{(\tau-1)}E_tX_{t+i+1} - \bar{A}_{(\tau)} - \bar{B}'_{(\tau)}E_tX_{t+i} - \delta'\bar{\mu} - \delta'P^i\left(X_t - \bar{\mu}\right)
$$

Using $E_tX_{t+j} = \bar{\mu} + P^j\left(X_t - \bar{\mu}\right)$, $\mu = (I-P)\,\bar{\mu}$, eq. (17), rearranging, and collecting terms yields

$$
\begin{aligned}
E_t\left(hprx_{t+i+1}^{(\tau)}\right) &= -\bar{B}'_{(\tau-1)}\left(\mu - \Sigma\lambda_0\right) - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{\tau-1} + \bar{B}'_{(\tau-1)}E_tX_{t+i+1} \\
&\quad -\bar{B}'_{(\tau)}E_tX_{t+i} - \delta'\bar{\mu} - \delta'P^i\left(X_t - \bar{\mu}\right) \\
&= -\bar{B}'_{(\tau-1)}\left(\mu - \Sigma\lambda_0\right) - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{\tau-1} + \bar{B}'_{(\tau-1)}\bar{\mu} \\
&\quad -\bar{B}'_{(\tau-1)}P^{i+1}\bar{\mu} - \bar{B}'_{(\tau)}\bar{\mu} + \bar{B}'_{(\tau)}P^i\bar{\mu} - \delta'\bar{\mu} + \delta'P^i\bar{\mu} \\
&\quad +\bar{B}'_{(\tau-1)}P^{i+1}X_t - \bar{B}'_{(\tau)}P^iX_t - \delta'P^iX_t \\
&= c + \left[\bar{B}'_{(\tau-1)}P^{i+1} - \bar{B}'_{(\tau)}P^i - \delta'_1P^i\right]X_t
\end{aligned}
$$

where c is defined by

$$
\begin{aligned}
c &\equiv -\bar{B}'_{(\tau-1)}\left(\mu - \Sigma\lambda_0\right) - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{\tau-1} - \bar{B}'_{(\tau-1)}P^{i+1}\bar{\mu} \\
&\quad +\bar{B}'_{(\tau-1)}\bar{\mu} - \bar{B}'_{(\tau)}\bar{\mu} - \delta'\bar{\mu} + \delta'P^i\bar{\mu} + \bar{B}'_{(\tau)}P^i\bar{\mu} \\
&= \bar{B}'_{(\tau-1)}\Sigma\lambda_0 - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}'_{(\tau-1)} + \left[\bar{B}'_{(\tau-1)}\left(P - P^{i+1}\right) - \delta'_1 + \delta'_1P^i - \bar{B}'_{(\tau)} + \bar{B}'_{(\tau)}P^i\right]\bar{\mu}
\end{aligned}
$$

Now use $\bar{B}'_{(\tau)} = \bar{B}'_{(\tau-1)}\left(P - \Sigma\lambda_1\right) - \delta'_1$ to see that

$$
\begin{aligned}
c &= \bar{B}'_{(\tau-1)}\Sigma\lambda_0 - \frac{1}{2}\bar{B}'_{\tau-1}\Sigma\Sigma'\bar{B}_{(\tau-1)} + \left[\bar{B}'_{(\tau-1)}\left(P - P^{i+1}\right) - \delta'_1 + \delta'_1P^i\right]\bar{\mu} \\
&\quad -\left[\left[\bar{B}'_{(\tau-1)}\left(P - \Sigma\lambda_1\right) - \delta'_1\right] + \left[\bar{B}'_{(\tau-1)}\left(P - \Sigma\lambda_1\right) - \delta'_1\right]P\right]^i\bar{\mu} \\
&= \bar{B}'_{(\tau-1)}\Sigma\left[\lambda_0 + \lambda_1\left(I - P^i\right)\bar{\mu}\right] - \frac{1}{2}\bar{B}'_{(\tau-1)}\Sigma\Sigma'\bar{B}_{(\tau-1)}.
\end{aligned}
$$

and

$$
E_t\left(hprx_{t+i+1}^{(\tau)}\right) = c + \bar{B}_{\tau-1}\Sigma\lambda_1P^iX_t.
$$

Hence,

$$E_t\left(hprx_{t+i+1}^{(\tau)}\right)=\bar{B}'_{\tau-1}\Sigma\left[\lambda_0+\lambda_1\left[\left(I-P^i\right)\bar{\mu}+P^iX_t\right]\right]-\frac{1}{2}\bar{B}'_{\tau-1}\Sigma\Sigma'\bar{B}_{\tau-1}$$

Note that the $i+1$-period return premium depends on the state of the economy only due to the term $\lambda_1 P^i X_t$. As long as not only the elements in the last columns of P^i but also other elements in the columns in P^i are different from zero and $P^i \neq I$, all variation in the variables in X_t affect $E_t\left(hprx_{t+i+1}^{(\tau)}\right)$. For $i=0$ follows $P^i=I$ so that the 1-period return premium reads

$$\begin{aligned}E_t\left(hprx_{t+1}^{(\tau)}\right) &= \bar{B}'_{\tau-1}\Sigma\lambda_0-\frac{1}{2}\bar{B}'_{\tau-1}\Sigma\Sigma'\bar{B}'_{\tau-1}+\bar{B}_{\tau-1}\Sigma\lambda_1X_t\\ &= \bar{B}'_{\tau-1}\Sigma\left[\lambda_0+\lambda_1X_t\right]-\frac{1}{2}\bar{B}'_{\tau-1}\Sigma\Sigma'\bar{B}'_{\tau-1}.\end{aligned}$$

Due to the restricted form of λ_1 the only source of variation in $E_t\left(hprx_{t+1}^{(\tau)}\right)$ is the variable that is ordered at the last position in X_t.

A.5 The Likelihood Function

The likelihood function reads

$$\begin{aligned}L\left(Z|\theta\right) &= \prod_{t=1}^{T}\left(2\pi\right)^{-\frac{T}{2}}\left[\det\left(R_{t|t-1}\right)\right]^{-\frac{1}{2}}\\ &\quad\times\exp\left(-\frac{1}{2}\left(Z_t-UX_{t|t-1}\right)'\left(R_{t|r-1}\right)^{-1}\left(Z_t-UX_{t|t-1}\right)\right).\end{aligned}$$

where$R_{t|t-1}$ denotes the conditional variance,

$$R_{t|t-1}\equiv Var\left(Z_t|\mathcal{Z}_{t-1},\theta\right)=U\Xi_{t|t-1}U'+VV'$$

$X_{t|t-1}$ denotes the one step ahead forecast,

$$X_{t|t-1}\equiv E\left[X_t|\mathcal{Z}_{t-1},\theta\right]=PX_{t-1|t-1}$$

with

$$X_{t|t}\equiv X_{t|t-1}+\Xi_{t|t-1}U\left(U'\Xi_{t|t-1}U+VV'\right)^{-1}\left(Z_t-UX_{t|t-1}\right),$$

and $\Xi_{t+1|t}$ denotes the mean squared error of the forecasts

$$\begin{aligned}\Xi_{t+1|t} &\equiv E\left[\left(X_{t+1}-X_{t|t}\right)\left(X_{t+1}-X_{t+1|t}\right)'\right]\\ &= P\left(\Xi_{t|t-1}-\Xi_{t|t-1}U\left(U'\Xi_{t|t-1}U+VV'\right)^{-1}U'\Xi_{t|t-1}\right)P'+\Sigma\Sigma'.\end{aligned}$$

The Kalman filter is implemented by iterating on $X_{t|t-1}$ and$\Xi_{t|t-1}$for given initial values $\Xi_{1|0}$ and $X_{1|t}$.

References

[1] Adjemian, Stéphane, Bastani, Houtan, Juillard, Michel, Karamé, Frédéric, Mihoubi, Ferhat, Perendia, George, Pfeifer, Johannes, Ratto Marco, and Sébastien Villemot, 2011, "Dynare: Reference Manual, Version 4", *Dynare Working Papers*, 1, CEPREMAP

[2] Andrés, Javier, López-Salido, J. David, and Edward Nelson, 2004, "Tobin's imperfect asset substitution in optimizing general equilibrium", *Journal of Money, Credit and Banking*, vol. 36, no. 4, pp. 665-690.

[3] Ang, Andrew, and Monika Piazzesi, 2003, "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables", *Journal of Monetary Economics*, vol. 50, no. 4, pp. 745-787.

[4] Ang, Andrew, Dong, Sen, and Monika Piazzesi, 2007, "No-arbitrage Taylor rules",. No. w13448, National Bureau of Economic Research.

[5] Ang, Andrew, Piazzesi, Monika, and Min Wei, 2006, "What does the yield curve tell us about GDP growth?", *Journal of Econometrics*, vol. 131, no. 1, pp. 359-403.

[6] Barr, David G., John Y. Campbell, 1997, "Inflation, real interest rates, and the bond market: a study of UK nominal and index-linked government bond prices", *Journal of Monetary Economics*, vol. 39 pp. 361-383.

[7] Bauer, Michael D. and Glenn D. Rudebusch, 2015, "Monetary Policy Expectations at the Zero Lower Bound", Federal Reserve Bank of San Francisco Working Paper 2013-18.

[8] Bekaert, Geert, Hoerova, Marie, and Marco Lo Duca, 2013, "Risk, uncertainty and monetary policy", *Journal of Monetary Economics*, vol. 60, no. 7 , pp. 771-788.

[9] Bernanke, Ben S., 2006, "Reflections on the Yield Curve and Monetary Policy", presented at the Economic Club of New York, New York, March 20, 2006.

[10] Bernanke, Ben S., 2013, "Long-Term Interest Rates", Speech Given at the Annual Macroeconomics Conference: the Past and Future Monetarry Policy", San Francisco: Federal Reserve Bank of San Francisco, 1 March 2013.

[11] Bomfim, Antulio Neves, 2003, "Counterparty credit risk in interest rate swaps during times of market stress", Federal Reserve Board.

[12] Borgy, Vladimir, Laubach, Thomas., Mesonnier, Jean-Stéphane, and Jean-Paul Renne, 2011, "Fiscal sustainability, default risk and euro area sovereign bond spreads" markets "*Banque de France*", No. 350.

[13] Campbell, John Y.,1995, "Some Lessons from the Yield Curve", *Journal of Economic Perspectives*, vol. 9.3, pp. 129-152.

[14] Campbell, John Y., Lo, Andrew W., and A. Craig MacKinlay, 1997, *The Econometrics of Financial Markets*, Princeton: Princeton University Press,.

[15] Carlstrom, Charles T., Fuerst, Timothy S., and Matthias Paustian, 2014, "Targeting Long Rates in a Model with Segmented Markets", *Federal Reserve Bank of Cleveland*, working paper no. 14-19.

[16] Chen, Han, Cúrdia, Vasco, and Andrea Ferrero, 2012, "The Macroeconomic Effects of Large-scale Asset Purchase Programmes", The Economic Journal, vol. 122, no. 564, pp F289-F315.

[17] Christensen, Jens H.E., Diebold,Francis X., and Glenn D. Rudebusch, 2011, "The affine arbitrage-free class of Nelson–Siegel term structure models", *Journal of Econometrics*, vol. 164, no.1, pp. 4-20.

[18] Christensen, Jens H.E., and Signe Krogstrup, 2014, "Swiss unconventional monetary policy: lessons for the transmission of quantitative easing", Federal Reserve Bank of San Francisco.

[19] Christensen, Jens H.E., and Signe Krogstrup, 2015, "Transmission of Quantitative Easing: The Role of Central Bank Reserves", Federal Reserve Bank of San Francisco Working Paper 2014-18.

[20] Christiano, Lawrence J., Eichenbaum, Martin, and Charles L. Evans, 2005, "Nominal rigidities and the dynamic effects of a shock to monetary policy", *Journal of Political Economy*, vol. 113, no. 1, pp. 1-45.

[21] Cochrane, John H., and Monika Piazzesi, 2005, "Bond Risk Premia", *The American Economic Review*, vol. 95.1, pp. 138-160.

[22] Cochrane, John H., and Monika Piazzesi, 2008, "Decomposing the Yield Curve", Manuscript, Chicago: University of Chicago.

[23] Clarida, Richard, Galí, Jordi, and Mark Gertler, 1998,. "Monetary policy rules in practice: some international evidence", *European Economic Review*, vol. 42.6, pp.1033-1067.

[24] Dai, Qiang, and Kenneth J. Singleton, 2000, "Specification analysis of affine term structure models", *The Journal of Finance*, vol. 55.5, pp. 1943-1978.

[25] Dai, Qiang and Kenneth Singleton, 2002, "Expectation puzzles, time-varying risk premia, and affine models of the term structure.", *Journal of Financial Economics* 63

[26] Dewachter, Hans, 2008, "Imperfect Information, Macroeconomic Dynamics and the Yield Curve: An Encompassing Macro-Financial Model", National Bank of Belgium Working Paper 144.

[27] Dewachter, Hans and Marco Lyrio, 2006, "Macro Factors and the Term Structure of Interest Rates", *Journal of Money, Credit and Banking*, vol. 38.1, pp. 119-140.

[28] Dewachter, Hans, and Leonardo Iania, 2012, "An extended macro-finance model with financial factors", *Journal of Financial and Quantitative Analysis*, vol. 46.06, pp. 1893-1916.

[29] Dewachter, Hans, Iania, Leonardo, and Marco Lyrio, 2014a, "Information in the yield curve: A macro finance approach", *Journal of Applied Econometrics*, vol. 29, pp. 42-64.

[30] Dewachter, Hans, Iania, Leonardo, Lyrio, Marco, and Maite de Sola Perea, 2014b, "A macro-financial analysis of the euro area sovereign bond market", *Journal of Banking & Finance*, forthcoming.

[31] Diebold, Francis X., Rudebusch Glenn D., and S. Boragan Aruoba, 2006, "The macroeconomy and the yield curve: a dynamic latent factor approach", *Journal of Econometrics*, vol. 131 no. 1-2, pp. 309-338.

[32] Dubeq, Simon, Monfort, Alain, Renne, Jean-Paul, and Guillaume Roussellet, 2013, "Credit and liquidity in interbank rates: a quadratic approach", *Banque de France.*

[33] Duffee, Gregory, R., 2002 "Term Premia and Interest Rate Forecasts in Affine Models." *Journal of Finance* 57, pp. 405-443.

[34] Duffie, Darrell, 2001, "*Dynamic Asset pricing Theory.*", Third ed. Princeton: Princeton University Press.

[35] Duffie, Darrell and Rui Kan, 1996, "A yield factor model of interest rates.", *Mathematical Finance*, vol 51, pp. 379–406.

[36] Favero, Carlo A., 2006, "Taylor rules and the term structure.", *Journal of Monetary Economics*, vol 53, pp. 1377–1393.

[37] Favero, Carlo A., Kaminska, Iryna, and Ulf Söderström, 2005, "The predictive power of the yield spread: further evidence and a structural interpretation", No. 4910, CEPR Discussion Papers.

[38] Filipović, Damir, and Anders B. Trolle, 2013, "The term structure of interbank risk", *Journal of Financial Economics*, vol. 109.3, pp. 707-733.

[39] Finlay, Richard, and Mark Chambers, 2009, "A Term Structure Decomposition of the Australian Yield Curve", *Economic Record*, vol. 85.271, pp. 383-400.

[40] Gagnon, Joseph, Raskin, Matthew, Remache, Julie, and Brian Sack, 2011, "The financial market effects of the Federal Reserve's large-scale asset purchases", *International Journal of Central Banking*, vol. 7, no. 1, pp. 3-43.

[41] Gelman, Andrew, G. Roberts, and W. Gilks, 1996, "Efficient metropolis jumping rules", *Bayesian statistics,* no. 42, pp. 599-608.

[42] Goodfriend, Marvin, 1993, "Interest rate policy and the inflation scare problem: 1979-1992", *Federal Reserve Bank of Richmond Economic Quarterly*, vol. 79, no. 1, pp. 1-24.

[43] Gourieroux, Christian, Monfort, Allan., and Vassilis Polimenis, 2002, "Affine Term Structure Models.", Working Paper, CREST, Paris.

[44] Gürkaynak, Refet, Sack, Brian and Eric Swanson, 2005, "The sensitivity of long term interest rates to economic news: evidence and implications for macroeconomic models", *American Economic Review*, vol. 95, pp. 425-436.

[45] Haario, Heikki, Saksman, Eero, and Johanna Tamminen, 2001, "An adaptive Metropolis algorithm", *Bernoulli*, pp. 223-242.

[46] Hamilton, James D.,1994, *Time series analysis,* Vol. 2, Princeton: Princeton university press.

[47] Hamilton, James D., and Dong H. Kim, 2002, "A re-examination of the predictability of the yield spread for real economic activity", *Journal of Money, Credit, and Banking*, vol. 34.2, pp. 340-360.

[48] Hamilton, James D., and Jing Cynthia Wu, 2012, "The effectiveness of alternative monetary policy tools in a zero lower bound environment", *Journal of Money, Credit and Banking*, vol. 44, no. 1, pp. 3-46.

[49] Hansen, Nikolaus, and Andreas Ostermeier, 2001, "Completely derandomized self-adaptation in evolution strategies", *Evolutionary computation*, vol. 9, no. 2, pp. 159-195.

[50] Harrison, J. Michael, and David M. Kreps, 1979, "Martingales and arbitrage in multiperiod securities markets", *Journal of Economic theory*, vol. 20, no. 3, pp. 381-408.

[51] Harvey, A. C., 1991, *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge, UK: Cambridge University Press.

[52] Hördahl, Peter, Tristani, Oreste, and David Vestin, 2006, "A joint econometric model of macroeconomic and term-structure dynamics", *Journal of Econometrics*, vol. 131, no. 1-2, pp. 405-444.

[53] Hördahl, Peter, Tristani, Oreste, and David Vestin, 2008, "The yield curve and macroeconomic dynamics", *The Economic Journal*, vol 118, pp. 1937-1970.

[54] Hördahl, Peter and Oreste Tristani, 2012, "Inflation Risk Premia in the Term Structure of Interest Rates", Journal of the European Economic Association, vol. 10, pp. 634-647.

[55] Ireland, Peter N., 2015, “Monetary Policy, Bond Risk Premia, and the Economy”, No. 852. Boston College Department of Economics.

[56] Jardet, Caroline, Monfort, Alain, and Fulvio Pegoraro, 2013, “No-Arbitrage Near-Cointegrated VAR(p) Term Structure Models, Term Premia and GDP Growth”, *Journal of Banking and Finance*, vol. 37.2, pp. 389-402.

[57] Joslin, Scott, Priebsch, Marcel and Kenneth J. Singleton, 2014, “Risk premiums in dynamic term structure models with unspanned macro risks”, *The Journal of Finance*, vol. 69.3, pp. 1197-1233.

[58] Joyce, Michael A. S., Lasaosa, Anna, Stevens, Ibramhim, and Matthew Tong, 2011, ”The Financial Market Impact of Quantitative Easing in the United Kingdom”, *International Journal of Central Banking*, vol. 7.3, pp. 113-161.

[59] Kiley, Michael, 2012, “The aggregate demand effects of short-and long-term interest rates”, *Finance and Economics Discussion Series*, no. 54.

[60] Knez, Peter J., Litterman, Robert, and Jose Scheinkman, 1994, “Explorations into factors explaining money market returns”, *The Journal of Finance*, vol. 49, no. 5, pp. 1861-1882.

[61] Koop, Gary, and Dimitris Korobilis, 2010, “Bayesian Multivariate Time Series Methods for Empirical Macroeconomics”, *Foundations and Trends in Econometrics*, vol. 3.4, pp. 267-358.

[62] Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2011, “The effects of quantitative easing on interest rates: channels and implications for policy”, No. w17555, National Bureau of Economic Research.

[63] Litterman, Robert B., 1986, “Forecasting with Bayesian vector autoregressions—five years of experience”, *Journal of Business & Economic Statistics*, vol. 4, no. 1, pp. 25-38.

[64] Ludvigson, Sydney C., and Serena Ng, 2009, “Macro Factors in Bond Risk Premia”, Review of Financial Studies, vol. 22(12), pp. 5027-5067.

[65] McCallum, Bennett, 2005, "Monetary Policy and the Term Structure of Interest Rates", Federal Reserve Bank of Richmond *Economic Quarterly*, vol. 91, pp. 1-21.

[66] Piazzesi, Monika, and Eric T. Swanson, 2008, "Futures prices as risk-adjusted forecasts of monetary policy", *Journal of Monetary Economics*, vol. 55, no. 4, pp. 677-691.

[67] Piazzesi, Monika., 2010, "Affine Term Structure Models", In Aït-Sahalia, Yacine, and Lars Peter Hansen (eds.), *Handbook of Financial Econometrics: Volume 1: Tools and Techniques*, Amsterdam: Elsevier, pp. 691-766.

[68] Rosenberg, Joashua V. and Samuel Maurer, 2008, "Signal or noise? Implications of the term premium for recession forecasting", *Economic Policy Review*, nr. July, pp. 1-11.

[69] Rosenthal, Jeffrey S, 2011, "Optimal proposal distributions and adaptive MCMC", in Brooks, S., A. Gelman, G. Jonesand X. L. Meng (eds.), *Handbook of Markov Chain Monte Carlo*, pp. 93-112.

[70] Rudebusch, Glenn D., Sack, Brian P., and Eric T. Swanson, 2007, "Macroeconomic implications of changes in the term premium", Federal Reserve Bank of St. Louis Review, vol. 89, Nr. July/August 2007, pp. 241-269.

[71] Rudebusch, Glenn D. and Eric T. Swanson, 2012, "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks", *American Economic Journal: Macroeconomic*, vol. 4, pp. 241-269.

[72] Rudebusch, Glenn D., Swanson, Eric T, and Tao Wu, 2006, "The bond yield "conundrum" from a macro-finance perspective", Institute for Monetary and Economic Studies, Bank of Japan.

[73] Rudebusch, Glenn, D. and Tao Wu, 2008, "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy", *The Economic Journal*, vol. 118, pp. 906 - 926.

[74] Spencer, Peter D, 2008, "Stochastic Volatility in a Macro-Finance Model of the US Term Structure of Interest Rates 1961–2004", *Journal of Money, Credit and Banking*, vol. 40, no. 6, pp. 1177-1215.

[75] Smets, Frank, and Raf Wouters, 2003, "An estimated dynamic stochastic general equilibrium model of the euro area", *Journal of the European economic association*, vol. 1.5, pp. 1123-1175.

[76] Wallace, Neil., 1981,"A Modigliani-Miller theorem for open-market operations", *The American Economic Review*, pp. 267-274.

[77] Woodford, Michael, 2005, "Central bank communication and policy effectiveness", *Proceedings - Economic Policy Symposium - Jackson Hole*, Federal Reserve Bank of Kansas City, Issue Aug, pages 399-474.

[78] Woodford, Michael, 2012, "Methods of policy accommodation at the interest-rate lower bound", The Changing Policy Landscape: 2012 Jackson Hole Symposium, Federal Reserve Bank of Kansas City.

[79] Wright, Jonathan H, 2006, "The yield curve and predicting recessions", Finance and Economic Discussion Series No. 2006-07, Board of Governors of the Federal Reserve System.