Essays in Dynamic Macroeconomics: Public Policy, Household Savings, and Lack of Commitment

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Joost Röttger

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Part I

Introduction

Introduction

In macroeconomics, it is common to assume that a decision maker, whether it is a household, a firm or a government, has the ability to commit to future actions. While this assumption often is a reasonable or at least an innocuous one, there are many instances where an agent's lack of commitment is arguably more realistic and crucial to understand certain economic phenomena.

This thesis presents three self-contained essays that are dedicated to applications where lack of commitment matters for the decisions made by public or private agents. Chapters 1 and 2 study monetary-fiscal policy interactions when a government cannot commit to future policies and in particular not to service its outstanding debt payments. Chapter 3 presents a model of a two-person household whose members cannot commit to future actions and might not cooperate.

To see under which conditions lack of commitment matters, consider the dynamic decision problem of an agent. This decision problem could be the consumption-savings problem of a household, the pricing decision of a firm or the borrowing decision of a government. Suppose that at some initial point of time $t = 0$, the decision maker chooses a plan for all of his future actions. When the agent arrives at some later date $t > 0$ and still finds it optimal to stick to the plan made in the initial period, this plan is called time consistent. If however, he finds it optimal to deviate from the actions prescribed by his period-0 plan, this plan is called time inconsistent. The implementation of such a time-inconsistent plan would require the decision-making agent to have a commitment technology and will not generally be carried out in practice if decisions are made from period to period.

There are several examples of time-inconsistency problems in macroeconomics. Some of the most famous ones are the taxation of capital in public finance (see e.g. Benhabib and Rustichini, 1997, Klein et al., 2008, Martin, 2010), the use of surprise inflation in monetary policy (see e.g. Kydland and Prescott, 1977, Barro and Gordon, 1983b, Calvo, 1978) and the consumption-savings problem of a household with self-control problems (see e.g. Laibson, 1997, Krusell et al., 2002). In political economy models, time-inconsistency problems arise when political parties with different objectives alternate in power (see e.g. Persson and Svensson, 1989, Alesina and Tabellini, 1990, Aguiar and Amador, 2011). Firms and labor unions have also been shown to suffer from time-inconsistency problems (see e.g. Nakamura and Steinsson, 2011, Krusell and Rudanko, 2013).

This thesis contributes to the study of time-inconsistency problems in macroeconomics. All three essays contained in this thesis analyze the intertemporal trade-offs faced by decision makers in the absence of commitment and study how these trade-offs depend on the details of the respective model environment. Methodologically, a common denominator of all three chapters is the focus on Markov-perfect equilibria (see Maskin and Tirole, 2001). The Markov-perfect equilibrium concept is a refinement of the subgame-perfect equilibrium concept and hence ensures that agents act in a time consistent way. It requires that players base their strategies only on payoff-relevant states, ruling out the possibility of reputational considerations based on trigger strategies. Recent work has applied this equilibrium concept to study macroeconomic questions (see e.g. Krusell et al., 2002, Klein et al, 2008, Azzimonti, 2011) and highlighted its usefulness for understanding how lack of commitment affects aggregate outcomes.

Chapters 1 and 2 contribute to this literature by investigating how lack of commitment shapes the conduct of monetary and fiscal policy. In the literature on optimal monetary and fiscal policy, the dominant approach has been to consider the policy problem of a Ramsey planner who, before time starts, commits to a time- and state-contingent (Ramsey) policy plan for all future periods, anticipating the response of the private sector (see e.g. Chari and Kehoe, 1999). While the Ramsey approach provides a useful benchmark for how policy should ideally be conducted, its policy recommendations typically are not time consistent. A well known source of time inconsistency is the presence of nominal government debt (Lucas and Stokey, 1983). When a policy maker inherits a positive amount of nominal debt, he will be tempted to reduce the real debt burden via surprise inflation and relax the government budget. While inflation might generate distortions in the present that are internalized by the policy maker, he does not internalize that rational lenders in previous periods anticipate the use of inflationary policies and demand higher nominal interest rates in return. By contrast, a Ramsey planner would internalize this effect as well and tend to refrain from using inflation surprises in a systematic way (see e.g. Chari et al., 1991, Schmitt-Grohé and Uribe, 2004, Siu, 2004).

In practice, public policy typically is chosen under discretion. It is thus interesting to ask what optimal policy looks like in the absence of commitment. Can models that have previously been used for normative purposes even explain polices implemented in practice and therefore offer a positive theory of monetary and fiscal policy? Following the work of Klein et al. (2008), recent studies have shed light on such questions by analyzing optimal monetary and fiscal policy without commitment (see e.g. Martin, 2009, Niemann et al., 2013a), demonstrating that fairly standard macroeconomic models can replicate empirically observed features of public policy. While these papers analyze optimal policy without commitment, they maintain the assumption that the policy maker cannot default on his debt payments. Lack of commitment to debt service thus never is an issue. However, in the past, governments have defaulted on nominal debt (see Reinhart and Rogoff, 2011).

The contribution of Chapter 1 is to study optimal monetary and fiscal policy without commitment when the policy maker can choose whether to default on outstanding debt payments. The studied model economy is a version of the cash-credit good economy introduced by Lucas and Stokey (1983), in which a benevolent policy maker (in the remainder also referred to as the government) levies distortionary taxes to finance exogenous government spending. To model the default decision, Chapter 1 follows the recent literature on sovereign default in emerging economies (see e.g. Aguiar and Gopinath, 2006, Arellano, 2008) and introduces a discrete default choice that involves certain costs for the economy if default is chosen over repayment.

The government prefers to default in particularly bad times, i.e. when output is low and/or public debt levels are high, allowing it to reduce labor taxes and inflation. An economy where the government has the option to default is found to experience lower average inflation rates than an otherwise identical economy where default is ruled out by assumption. While a default allows the government to reduce inflation, it is mostly an indirect mechanism that leads to lower average inflation. When a default might occur in equilibrium, lenders price the risk of default, increasing interest rates. The government responds to the change in borrowing conditions by issuing less debt on average, which in turn reduces the incentive to use inflation. The adverse effect of sovereign risk on the interest rate faced by the government also negatively affects the role of public debt as a shock absorber, leading to more volatile policies and aggregate outcomes.

Chapter 2 is also dedicated to the time-inconsistency problem created by the interaction between nominal public debt and monetary policy but looks at it from a different perspective. Instead of studying how optimal policy is conducted in the absence of commitment, it reconsiders a suggestion made by Rogoff (1985) about how to mitigate the time-inconsistency problem. Rogoff (1985) proposes the delegation of monetary policy to an independent central bank that is ”monetary conservative” and has a greater distaste for inflation than society. Even though this central bank acts under discretion and is not benevolent, it might implement a welfare-enhancing policy relative to a benevolent policy maker who lacks commitment because he is less tempted to use surprise inflation.

It is however important to look at the details of an economy to evaluate the costs and benefits of monetary conservatism. Chapter 2 studies the implications of monetary conservatism for a model economy that faces incomplete financial markets, lack of commitment to debt repayment and political economy distortions. These frictions are relevant for many emerging economies, which increasingly introduce independent central banks with a focus on price stability (see Carstens and Jácome, 2005), and might reduce the desirability of monetary conservatism. When a government issues non-state contingent nominal debt, inflation provides a potentially useful hedge against adverse outcomes since it can be used to adjust the real value of nominal debt payments in response to shocks. Similarly, the flexible use of inflation might also reduce the risk of experiencing a sovereign default since such an event can in principle be avoided if a central bank is willing to lower the real debt burden via inflation. By reducing the flexibility of monetary policy, monetary conservatism could thus hurt an economy. If a government mostly issues debt due to political economy considerations (see e.g. Alesina and Tabellini, 1990), monetary conservatism might also entail welfare costs by reducing the cost of borrowing and encouraging the accumulation of public debt levels that are not in the interest of society.

To evaluate the role of monetary conservatism in the presence of financial and political frictions, Chapter 2 follows a recent literature on monetary-fiscal policy interactions (see e.g. Niemann, 2011, Martin, 2014) and considers a model where a government consists of two independent authorities: a central bank and a fiscal authority. The central bank is in charge of monetary policy, whereas the fiscal authority decides on public spending, debt issuance and whether to repay bond holders. The fiscal authority is not benevolent because of a deficit bias that is due to political economy distortions. By contrast, the central bank shares the preferences of society except that it might be monetary conservative. The policy authorities both cannot commit to future policies. The main finding of Chapter 2 is that an increase in the degree of monetary conservatism leads to a higher average debt burden, lower average inflation and more frequent default episodes. In addition, it is also associated with less volatile inflation but an increase in the volatility of government spending. The success of lower and more stable inflation thus comes at the cost of rising indebtedness and more volatile fiscal policy. In terms of welfare, the benefits of monetary conservatism are however shown to still dominate the potential costs. Even economies that experience several frictions can thus benefit from adopting central bank independence.

As in Chapter 2, Chapter 3 also studies decision making without commitment in an environment where two agents with potentially differing objectives interact. However, these two agents now are the individual members of a household instead of independent policy authorities. In economics in general and in macroeconomics in particular, it is common to model households as single decision units and to abstract from the interaction between different members of a household. Chapter 3 explicitly models the within-household interaction between individual household members. More specifically, similar to Hertzberg (2012), the household studied in Chapter 3 consists of two members that are altruistic towards each other and cannot commit to future actions. The main contribution of Chapter 3 is to analyze how time-consistent household behavior depends on whether the household members cooperate or not. In particular, it studies the consumption-savings problem of a two-person household (a couple) whose individual members choose their own consumption and labor supply. The household's savings position then is determined residually via the joint household budget constraint.

When the household members cooperate, the household behaves as in the collective household model introduced by Chiappori (1988, 1992) and recently extended to a dynamic context (see Chiappori and Mazzocco, 2014). In this case, the outcome is equivalent to that chosen under commitment, i.e. there is no time-inconsistency problem when household members act under full cooperation. This changes when household members do not cooperate. The non-cooperative interaction between the household members is modeled as a dynamic game in which the household members take as given the decisions of the spouse. If the household members are imperfectly altruistic, they are shown to save less relative to the cooperative case. When a household member decides whether to consume a unit of resources today or to save it for tomorrow, it takes into account that, in the next period, its spouse will also consume part of the savings. When imperfectly altruistic, consumption of the spouse is however valued less than the own consumption. As a result, imperfectly altruistic household members have an incentive to save less and consume more in any given period. The non-cooperative household thus exhibits an undersaving (or overborrowing) bias relative to the cooperative one when individual members are imperfectly altruistic. If the household members are perfectly altruistic, i.e. they place the same weight on their own utility and the utility of the spouse, the savings distortion vanishes and the household behaves like the cooperative one, eliminating the time-inconsistency problem. Using a quantitative model version with idiosyncratic labor income risk, Chapter 3 shows that lack of cooperation can lead to sizable reductions in household savings when household members are imperfectly altruistic, resulting in substantial welfare losses.

To summarize, this thesis studies decision making in the absence of commitment and shows how details of a model matter for the way lack of commitment affects outcomes.

More specifically, Chapter 1 highlights how the option to default affects policy outcomes when a government lacks commitment. Chapter 2 shows how delegation of monetary policy to a central bank changes the way fiscal policy is chosen without commitment. Chapter 3 shows that lack of commitment matters for the way a household acts, depending on whether individual household members cooperate or not.

Part II

Chapters

Chapter 1

Monetary and Fiscal Policy with Sovereign Default

1.1 Introduction

Suppose that a government faces high nominal debt payments that can only be refinanced at high interest rates. If it is not willing (or able) to raise primary surpluses to pay bond holders, there are essentially two options left: inflation and sovereign default. While default and inflation both can lower the real debt burden, there are several differences between these two policy options which make them imperfect substitutes. For example, a government can collect seigniorage when engineering inflation by issuing currency while a default does not generate additional tax revenues. Another difference is that default directly affects the return on government bonds whereas inflation impacts on the return on all nominal assets. Being a continuous variable, inflation can also be adjusted rather easily while the discrete default choice does not offer the same degree of flexibility. Given the distinct roles of money and government bonds for the private sector, default and inflation may also distort economic activity through different channels.

The contribution of this chapter is to study the implications of allowing a policy maker not only to use standard instruments of monetary and fiscal policy but also to choose outright sovereign default.[1] In particular, it extends previous studies on optimal monetary and fiscal policy with nominal debt that focus on the case of lack of commitment but still assume that the policy maker is always committed to service debt (see e.g. Diaz-Gimenez et al., 2008, Martin, 2009, Niemann et al., 2013a). In the model studied here, a benevolent government jointly chooses monetary and fiscal policy under discretion to finance exogenous government spending in a representative agent cash-credit economy

[1] This chapter is based on Röttger (2014).

that is subject to productivity shocks.[2] More specifically, it sets a labor income tax rate, chooses the money growth rate, issues nominal non-state contingent bonds and decides on whether to repay its outstanding debt or not. The default decision is modeled as a binary choice (see Eaton and Gersovitz, 1981). Following the quantitative sovereign default literature,[3] a default is costly because it leads to a deadweight loss of resources that takes the form of a reduction in aggregate productivity and exclusion from financial markets for a random number of periods.

As is common in the literature on optimal monetary and fiscal policy, I consider a closed economy. This chapter thus contributes to the study of domestic debt default which, despite being a historically recurring phenomenon with severe economic consequences, has not received a lot of attention in the sovereign default literature (see Reinhart and Rogoff, 2011). In a closed economy, a default does not redistribute resources from foreign lenders to domestic citizens. The government may still choose not to repay its debt to relax its budget constraint and reduce distortionary taxes. The model is calibrated to the Mexican economy which has experienced periods of high inflation and sovereign risk in the recent past.

I study the Markov-perfect equilibrium of the public policy problem (see Klein et al., 2008). The government's decisions hence only depend on the payoff-relevant state of the economy which consists of aggregate productivity, the beginning-of-period public debt position and whether the government is in financial autarky or not. Since the government optimizes sequentially, it cannot commit to future policies and does not internalize how its current decisions affect household expectations in previous periods. However, the government is aware that (expected) future policy will depend on its borrowing decision because it will affect the incentive to reduce the real debt burden via default or inflation in the next period. With lack of commitment, the option to default thus matters for the government's response to adverse shocks by allowing it to adjust the real debt burden as well as by affecting the cost of borrowing and thus the attractiveness of debt as a shock absorber. However, only the first effect is internalized by the government.

Compared to an otherwise identical economy without default option (or equivalently an economy with prohibitively high costs of default) the availability of sovereign default results in lower average inflation. Since inflation does not reduce the real debt burden when a default takes place, it is lower when default is chosen instead of repayment. How-

[2] I assume that there is only one policy maker, referred to as the government, who is in charge of both, monetary and fiscal policy. Niemann (2011), Niemann et al. (2013b) and Martin (2014) study time-consistent public policy without sovereign default in models where a central bank and a fiscal authority interact. Chapter 2 in this thesis will consider a model with independent monetary and fiscal authorities that allows for sovereign default and political frictions.

[3] See e.g. Hamann (2004), Aguiar and Gopinath (2006) and Arellano (2008).

ever, this direct effect of default on inflation is negligible at a plausible default frequency. The key mechanism that leads inflation to be lower when the default option is available is an indirect one. The attractiveness and hence the probability of default increases with public debt and decreases with aggregate productivity. With default risk, bond prices become more debt elastic in recessions than with only inflation risk and the marginal revenue from debt issuance decreases faster.[4] Consequently, the government borrows less which reduces its incentive to implement high inflation rates. Since lower average debt is associated with less inflation, less money is issued and seigniorage revenues decline. The government then has to adjust the primary surplus, leading to a higher labor tax rate in the long run. In the short run, the increased sensitivity of bond prices to productivity shocks and bond issuance that is induced by sovereign risk impedes the government's ability to smooth tax distortions across states. Relative to an economy without default option, tax and inflation rates are thus more volatile, amplifying the impact of productivity shocks on the economy. In times of high sovereign risk, debt issuance is costly and the government tries to avoid a default by reducing the real debt burden via inflation. A sovereign debt crisis thus is inflationary which is consistent with empirical evidence (see Reinhart and Rogoff, 2009).

From a welfare perspective, it is not obvious whether it is desirable to endow the government with the option to default when it cannot commit to future actions.[5] As discussed above, the risk of default affects public policy in the short and the long run. With productivity shocks, the government would like to smooth tax distortions by running a budget deficit (surplus) during bad (good) times, following the logic of Barro (1979). Default risk makes debt issuance more expensive in recessions which leads to welfare losses due to more volatile public policy. The long-run implications of sovereign default might however outweigh these costs. As in Martin (2009) and Diaz-Gimenez et al. (2008), the government chooses positive average debt positions because of its lack of commitment and a household money demand that increases with expected inflation. By increasing the cost of borrowing in recessions, risk of default renders public debt accumulation less attractive, thus avoiding high debt levels and the implementation of high average inflation. A welfare exercise reveals that the counterfactual elimination of sovereign default leads to a negligible welfare loss. From a welfare perspective, lack of commitment to debt service hence is not important.

[4]Even without sovereign risk, higher debt issuance leads to higher interest rates by increasing expected inflation.

[5]The same is true in the context of consumer default where there exists a trade-off when reducing the costs of filing for bankruptcy. On the one hand, indebted consumers receive the ability to make debt payments state contingent. On the other hand, this flexibility comes at the cost of higher borrowing costs that compensate lenders for the increased risk of default (see e.g. Livshits et al., 2007).

This chapter builds on the literature on optimal Markov-perfect monetary and fiscal policy with nominal government debt. Martin (2009, 2011, 2013) extensively studies the short- and long-run properties of public debt and inflation when the government lacks commitment. I will discuss how his findings relate to mine in Section 1.3. Diaz-Gimenez et al. (2008) show how public policy and welfare depend on whether debt is indexed to inflation or not. Among other things, they find that without commitment welfare can be lower when debt is indexed. In a model with nominal rigidities, Niemann et al. (2013a) show that the presence of lack of commitment and nominal government debt leads to persistent inflation. Despite highlighting the role of lack of commitment for public policy, all of these studies maintain the assumption that debt is always repaid and thus abstract from sovereign default. This chapter is also related to recent papers that study domestic debt default. In a model with incomplete markets and idiosyncratic income risk, D'Erasmo and Mendoza (2013) show that a sovereign default can occur in equilibrium as an optimal distributive policy. Pouzo and Presno (2014) extend the incomplete markets model of Aiyagari et al. (2002) by considering a policy maker who cannot commit to debt payments. Sosa-Padilla (2013) studies Markov-perfect fiscal policy in a model where a sovereign default triggers a banking crisis. All of these papers feature real economies and hence do not discuss monetary policy.

This chapter also relates to the quantitative sovereign default literature that studies how risk of default affects business cycles in emerging economies.[6] With this literature, it shares the assumption of the government's lack of commitment and the way sovereign default is modeled. Within this literature, the studies that are closest to this chapter are Cuadra et al. (2010), Hur et al. (2014) and Du and Schreger (2015). Cuadra et al. (2010) study a production economy with endogenous fiscal policy but abstract from monetary policy and - as is common in the sovereign default literature - look at a small open economy that trades real bonds with foreign investors. Hur et al. (2014) consider an endowment economy with nominal debt, exogenous shocks to inflation and risk-averse investors. They find that the cyclicality of inflation matters for public debt dynamics by affecting risk premia and thus the cost of borrowing. Du and Schreger (2015) study a model of a small open economy where the government borrows in local currency from foreign investors and can reduce its real debt burden by using inflation. Since domestic entrepreneurs have liabilities denominated in foreign currency but earn revenues in local currency, inflation hurts firm balance sheets by depreciating the local currency.

In independent and contemporaneous work, Sunder-Plassmann (2014) also studies time-consistent public policy for a version of Martin (2009)'s cash-credit good economy

[6] A recent summary of this literature can be found in Aguiar and Amador (2014).

that allows for a default decision as in Arellano (2008). However, there are several differences between our studies that make them complementary. First, as in Diaz-Gimenez et al. (2008), the focus of her paper is on comparing the long-run properties of a model economy with nominal government debt and an otherwise identical model economy with real government debt. She finds that real debt leads to higher average debt, more frequent default events and higher inflation than nominal debt, which is shown to be consistent with cross-country evidence for selected emerging economies. By contrast, I focus on how the ability to default changes the conduct of monetary and fiscal policy in the short and the long run relative to an economy without default, using a model that can replicate short- and long-run properties of the Mexican economy. To model business cycle fluctuations, I study a model with productivity shocks, whereas she considers government expenditure shocks. Another difference between our two studies is that her model assumes an exogenous and constant labor tax rate, whereas I allow the government to choose the tax rate.

Recently, Aguiar et al. (2013) have also developed a model to jointly study inflation and sovereign default when a government cannot commit to future policy. However, their analysis differs from mine in several ways. First, their model features an endowment economy that is not subject to fundamental shocks and borrows from abroad. Second, the authors assume that the government experiences an ad-hoc utility cost of inflation. Third, in the spirit of Cole and Kehoe (2000), they exclusively focus on self-fulfilling sovereign defaults and characterize analytically how a government's "inflation credibility" - as measured by the weight on its disutility of inflation - affects an economy's vulnerability to sunspot-driven rollover crises.

The rest of the chapter is organized as follows. Section 1.2 presents the model that is analyzed in Section 1.3. The welfare implications of sovereign default are discussed in Section 1.4. Section 1.5 concludes.

1.2 Model

The model extends the cash-credit economy studied in Martin (2009) by allowing for productivity shocks and sovereign default.[7] Time is discrete, starts in $t = 0$ and goes on forever. The economy is populated by a unit mass continuum of homogeneous infinitely-lived households and a benevolent government. Taking government policies and prices as given, the households optimize in a competitive fashion. They supply labor n_t to produce

[7] The focus on productivity shocks allows me to study how the possibility of sovereign default affects the business cycle properties of a monetary economy.

the marketable good y_t, using a linear technology to be specified below. In addition, they choose consumption of a cash good c_{1t} and a credit good c_{2t}, and decide on money ($\tilde{m}_{t+1}$), nominal government bond ($\tilde{b}_{t+1}$) and nominal risk-free private bond ($\tilde{b}^{rf}_{t+1}$) holdings. The unit price of a government (private) bond is denoted as q_t (q^{rf}_t). The risk-free bonds are only traded by households and will thus be in zero net supply. While all assets are nominal and thus subject to inflation risk, only government bonds are subject to default risk. A role for money is introduced by tying consumption of c_{1t} to beginning-of-period money holdings via a cash-in-advance constraint (see Lucas and Stokey, 1983)

$$\tilde{m}_t \geq \tilde{p}_t c_{1t},$$

with $\tilde{p}_t$ denoting the price of consumption in terms of $\tilde{m}_t$.

To finance constant government spending g and outstanding nominal debt payments $\tilde{B}_t$, the government chooses from a set of policies that includes the money growth rate μ_t, a linear labor income tax rate τ_t, the binary default decision $d_t \in \{0,1\}$, and issuance of nominal non-state contingent one-period bonds $\tilde{B}_{t+1}$. A default occurs when $d_t = 1$ is chosen, while the government fully repays its obligations for $d_t = 0$. In the default case, the government is excluded from financial markets for a random number of periods (see Aguiar and Gopinath, 2006, or Arellano, 2008). It can thus neither borrow from nor lend to households during this time, i.e. $\tilde{B}_{t+1} = 0$.

1.2.1 Private Sector

Before formulating the government decision problem, I will first discuss the behavior of the private sector.

Households

The households have preferences given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \right],$$

with discount factor $\beta \in (0,1)$ and period utility function $u : \mathbb{R}^3_+ \to \mathbb{R}$. The utility function is additively separable in all its arguments and satisfies $u_1, u_2, -u_n > 0$ and $u_{11}, u_{22}, u_{nn} < 0$ with u_x (u_{xx}) denoting the first (second) derivative of $u(\cdot)$ with respect to $x \in \{c_1, c_2, n\}$. Households have initial assets (b_0, b^{rf}_0, m_0) and take as given prices $\{\tilde{p}_t, q_t, q^{rf}_t\}^{\infty}_{t=0}$ and government policies $\{d_t, \mu_t, \tau_t, \tilde{B}_{t+1}\}^{\infty}_{t=0}$. The aggregate money stock evolves according to $\tilde{M}_{t+1} = (1+\mu_t)\tilde{M}_t$. The labor productivity of the households $\{a_t\}_{t=0}^{\infty}$ is subject to random shocks and follows a stationary first-order Markov process with continuous support $\mathbb{A} \subseteq \mathbb{R}_+$ and transition function $f_a(a_{t+1}|a_t)$.

Households maximize their expected lifetime utility subject to the period budget constraint

$$(1-\tau_t)\psi(a_t,d_t)n_t + \frac{\tilde{m}_t + (1-d_t)\tilde{b}_t + \tilde{b}_t^{rf}}{\tilde{p}_t} \geq c_{1t} + c_{2t} + \frac{\tilde{m}_{t+1} + q_t\tilde{b}_{t+1} + q_t^{rf}\tilde{b}_{t+1}^{rf}}{\tilde{p}_t},$$

the cash-in-advance constraint

$$\frac{\tilde{m}_t}{\tilde{p}_t} \geq c_{1t},$$

as well as the No-Ponzi game constraint

$$\lim_{T\to\infty}\mathbb{E}_t\left[\left(q_T^{rf}\tilde{b}_{T+1}^{rf} + q_T\tilde{b}_{T+1}\right)\prod_{s=0}^{T-1} q_{t+s}^{rf}\right] \geq 0.$$

Households use their labor supply n_t to produce a marketable good according to the linear technology $y_t = \psi(a_t,d_t)n_t$. They take as given their effective labor productivity $\psi : \mathbb{R}_+ \times \{0,1\} \to \mathbb{R}_+$ which depends on random productivity a_t and the government's default decision d_t. Effective productivity $\psi(\cdot)$ increases with exogenous productivity $(\partial\psi(a_t,d_t)/\partial a_t \geq 0)$ and is negatively affected by a default $(\psi(a_t,0) \geq \psi(a_t,1))$.[8]

In the model, lack of commitment to public debt repayment will require costs of default that are internalized by the government to sustain positive levels of debt. As is common in the quantitative sovereign default literature (see e.g. Arellano, 2008 and Cuadra et al., 2010), there are two types of default costs. First, the government is excluded from the bond market in the default period. Conditional on being in autarky, the economy regains access to financial markets with constant probability θ in the subsequent period. Second, the economy experiences a direct resource loss governed by $\psi(\cdot)$. As in Cuadra et al. (2010) and Pouzo and Presno (2014), these costs capture in reduced form productivity losses that occur in periods of default (and financial autarky). Despite being arguably ad hoc, such a specification allows me not to take a stand on how exactly a sovereign default is propagated through the economy. While there is evidence for domestic output costs, there is still no consensus on which mechanism is the most relevant one (see Panizza et al., 2009). In addition, two recent papers show that models with endogenous default costs that arise due to private credit disruptions (Mendoza and Yue, 2012) or banking crises (Sosa-

[8]It is straightforward to modify the model to include a representative firm that is owned by households and produces the homogeneous good y_t, using labor supplied by households at a real wage w_t. Due to linearity of the production function, the wage rate will equal effective productivity $\psi(a_t,d_t)$ and profits will be zero, such that the behavior of the economy will not change with such a firm sector.

Padilla, 2013) deliver similar qualitative and quantitative results as those with exogenous default costs.

The household optimality conditions are given by the first-order conditions

$$
\begin{aligned}
-\frac{u_n(n_t)}{u_2(c_{2t})} &= (1-\tau_t)\psi(a_t,d_t), & (1.1)\\
u_2(c_{2t}) &= \beta \mathbb{E}_t\left[u_1(c_{1t+1})\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right], & (1.2)\\
u_2(c_{2t})q_t &= \beta \mathbb{E}_t\left[(1-d_{t+1})\,u_2(c_{2t+1})\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right], & (1.3)\\
u_2(c_{2t})q_t^{rf} &= \beta \mathbb{E}_t\left[u_2(c_{2t+1})\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right], & (1.4)
\end{aligned}
$$

and the complementary slackness conditions

$$
\lambda_t = u_1(c_{1t}) - u_2(c_{2t}) \geq 0, \tilde{m}_t/\tilde{p}_t - c_{1t} \geq 0, \lambda_t\left(\tilde{m}_t/\tilde{p}_t - c_{1t}\right) = 0,
$$

with λ_t denoting the Kuhn-Tucker multiplier on the cash-in-advance constraint.[9]

Intuitively, the cash-in-advance constraint is binding whenever the marginal utility of cash-good consumption exceeds the marginal utility of credit-good consumption. The inequality

$$
u_1(c_{1t}) - u_2(c_{2t}) \geq 0, \tag{1.5}
$$

needs to hold in equilibrium to satisfy $\lambda_t \geq 0$. Equation (1.1) characterizes the optimal household labor supply decision which is distorted for non-zero tax rates $\tau_t \neq 0$. The conditions (1.2)-(1.4) are the Euler equations for money and bonds. Since nominal government bonds are defaultable, they have to compensate households for the risk of default (see condition (1.3)). However, all assets need to compensate for expected (gross) inflation $\tilde{p}_{t+1}/\tilde{p}_t$.

As in Martin (2009), I normalize all nominal variables by the beginning-of-period aggregate money stock $\tilde{M}_t$: $x_t \equiv \tilde{x}_t/\tilde{M}_t$ with $x \in \left\{B, b, b^{rf}, m, p\right\}$. This normalization renders the model stationary. It implies that the inflation rate in period t is given as

$$
\pi_t \equiv \frac{p_t\,(1+\mu_{t-1})}{p_{t-1}} - 1,
$$

such that inflation equals money growth in the long run.

[9] In a household optimum, the household budget constraint and the No-Ponzi game constraint hold with equality.

After normalizing nominal variables, the Euler equations are now given as

$$u_2(c_{2t}) = \beta \mathbb{E}_t \left[u_1(c_{1t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1+\mu_t} \right], \tag{1.6}$$

$$u_2(c_{2t}) q_t = \beta \mathbb{E}_t \left[(1-d_{t+1}) u_2(c_{2t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1+\mu_t} \right], \tag{1.7}$$

$$u_2(c_{2t}) q_t^{rf} = \beta \mathbb{E}_t \left[u_2(c_{2t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1+\mu_t} \right]. \tag{1.8}$$

Market Clearing

In equilibrium, goods and asset markets clear:

$$\begin{aligned} \psi(a_t, d_t) n_t &= c_{1t} + c_{2t} + g, \\ b_{t+1} &= B_{t+1}, \\ b_{t+1}^{rf} &= 0, \\ m_{t+1} &= 1. \end{aligned}$$

If real balances are high enough, i.e. $1/p_t \geq u_1^{-1}(u_2(c_{2t}))$ holds (see condition (1.5)), households equalize marginal utility across cash and credit goods. If not, households are cash constrained and the allocation of consumption is distorted. As in Martin (2009), in a monetary equilibrium, i.e. an equilibrium in which money is valued,

$$c_{1t} = 1/p_t,$$

needs to hold. Note that this still allows for an unconstrained consumption allocation if the cash-in-advance constraint is just binding, i.e. when p_t is such that $\lambda_t = 0$ and $c_{1t} = 1/p_t = u_1^{-1}(u_2(c_{2t}))$.

1.2.2 Government Problem

In this section, I formulate the decision problem of the benevolent government. When the government defaults, the economy suffers a productivity loss governed by $\psi(\cdot)$ and it cannot access the bond market for a random number of periods.[10] Conditional on the

[10] Note that households can still trade risk-free bonds among each other when the government is in financial autarky. However, since they are homogeneous and private bonds are in zero net supply, this is not going to affect the way public policy is conducted.

default decision, the government budget constraint is

$$g-\tau_t\psi(a_t,d_t)n_t=\begin{cases}\frac{\tilde{M}_{t+1}+q_t\tilde{B}_{t+1}}{\tilde{p}_t}-\frac{\tilde{M}_t+\tilde{B}_t}{\tilde{p}_t}, & \text{if} \quad d_t=0\\ \frac{\tilde{M}_{t+1}-\tilde{M}_t}{\tilde{p}_t}, & \text{if} \quad d_t=1\end{cases}$$

It can be written as

$$g-\tau_t\psi(a_t,d_t)n_t=\begin{cases}(1+\mu_t)\frac{1+q_tB_{t+1}}{p_t}-\frac{1+B_t}{p_t}, & \text{if} \quad d_t=0\\ \frac{\mu_t}{p_t}, & \text{if} \quad d_t=1\end{cases}$$

by using $\tilde{M}_{t+1}=(1+\mu_t)\tilde{M}_t$ and applying the normalization of nominal variables used before.[11] In the default (and autarky) case, the government has to finance public spending g with income tax revenues $\tau_t\psi(a_t,1)n_t$ and seigniorage $\tau_t^m\equiv\mu_t/p_t$. When the government repays its debt, it additionally has to make debt payments but can access the bond market and issue debt.

The government is benevolent and sets its policy instruments to maximize the expected life-time utility of the households, anticipating the response of the private sector to its policies. However, it cannot commit itself to a state-contingent (Ramsey) policy plan for all current and future policies but optimizes from period to period instead. In the model, the presence of nominal public debt leads to a time-inconsistency problem since a sequentially optimizing government will be tempted to use default or surprise inflation to relax the government budget. To analyze the public policy problem of the government, I restrict attention to stationary Markov-perfect equilibria (see Klein et al., 2008). In such an equilibrium, the optimal decisions of the government in any period will be characterized by time-invariant functions that only depend on the minimal payoff-relevant state of the economy in that respective period. In the model, this state consists of the beginning-of-period debt-to-money ratio B_t, labor productivity a_t and whether the government is in financial autarky or not. By requiring the government to only condition its decisions on the current payoff-relevant aggregate state, the Markov-perfect equilibrium concept rules out the possibility that the government is able to keep promises made in the past. This is because at the start of a period, the government does not care about the past and only considers its payoff in current and future periods.[12] By construction, the government thus is ensured to act in a time-consistent way.

The Markov-perfect policy problem will be formulated recursively. In the remainder, I will thus adopt the notation of dynamic programming. Time indices are hence dropped

[11]Note that the normalized aggregate money stock is constant and equal to one.

[12]The focus on Markov-perfect strategies also rules out the possibility of reputational considerations based on complex trigger strategies as in Chari and Kehoe (1990, 1993).

and a prime is used to denote next period's variables. Given the aggregate state at the start of a period, the government takes as given the policy function $\mathcal{D}(B',a')$ that determines next period's default decision as well as the policy functions $\mathcal{X}^r(B',a')$ and $\mathcal{X}^d(a')$, with $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{P}\}$, that determine consumption and the price index in the next period for the case of repayment (r) and default (d).[13] Expectations of these variables enter the household optimality conditions (1.6) and (1.7) and thus matter for the allocation in the current period.[14] Despite lacking the ability to commit to future policies, the government fully recognizes today that it affects (expected) future policies via its choice of B', which in turn have an effect on the behavior of the private sector in the current period. In a stationary Markov-perfect equilibrium, the policy functions that govern future decisions then coincide with the policy functions that determine current public policy for all states.

As in Klein et al. (2008), one can interpret the formulation of the public policy problem as a Markov-perfect game played between successive governments. Following this interpretation, in each period, a different government is in charge of choosing public policy, taking as given the policies set by the government in the next period. Each government then chooses its optimal strategies, taking as given the optimal responses of the government in the next period and the private sector in the current period.

In each period, the government anticipates how the private sector responds to its actions in the current period as given by the private sector equilibrium conditions.[15] Using the household optimality conditions (1.1),(1.6)-(1.7), the binding cash-in-advance constraint and the aggregate resource constraint, the government budget constraint can be rewritten as

$$\beta \mathbb{E}_{a'|a} \left[\begin{array}{c} u_1(\mathcal{P}^r(B',a')^{-1}) \frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} \\ +u_1(\mathcal{P}^d(a')^{-1}) \frac{\mathcal{D}(B',a')}{\mathcal{P}^d(a')} \end{array} \right] + \beta \mathbb{E}_{a'|a} \left[u_2(\mathcal{C}_2^r(B',a')) \frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} \right] B' \\ + u_n(n)n + u_2(c_2)(c_2 - B/p) = 0, \tag{1.9}$$

for the repayment case and as

$$\beta \mathbb{E}_{a'|a} \left[\theta \times \left\{ \begin{array}{c} u_1(\mathcal{P}^r(0,a')^{-1}) \frac{1-\mathcal{D}(0,a')}{\mathcal{P}^r(0,a')} \\ +u_1(\mathcal{P}^d(a')^{-1}) \frac{\mathcal{D}(0,a')}{\mathcal{P}^d(a')} \end{array} \right\} + (1-\theta) \times \frac{u_1(\mathcal{C}_1^d(a'))}{\mathcal{P}^d(a')} \right] \\ + u_n(n)n + u_2(c_2)c_2 = 0, \tag{1.10}$$

[13]Remember that cash-good consumption c_1 is directly linked to the price index p via the cash-in-advance constraint.

[14]Households do not have an impact on the future government policies but form rational expectations about them based on the policy functions listed above.

[15]The government thus plays a Stackelberg game against the (passive) private sector in every period.

for the default (and autarky) case. This constraint can be seen as the period implementability constraint for the government.[16]

In addition to this constraint, the government has to respect the following two private sector equilibrium conditions:

$$
\begin{aligned}
0 &= \psi(a,d)n - 1/p - c_2 - g, && (1.11) \\
0 &\leq u_1(1/p) - u_2(c_2). && (1.12)
\end{aligned}
$$

The household budget constraint is satisfied by Walras' Law, given the government budget constraint, the market clearing conditions for bonds and money and the resource constraint. Let $\mathbb{B} \equiv [\underline{B}, \overline{B}]$ be the set of possible aggregate debt values with $-\infty < \underline{B} \leq 0$ and $0 < \overline{B} < \infty$. Conditional on entering a period with access to financial markets, the decision problem of the government is given by the following functional equation:

$$
\mathcal{V}(B,a) = \max_{d \in \{0,1\}} \left\{ (1-d)\mathcal{V}^r(B,a) + d\mathcal{V}^d(a) \right\} \tag{1.13}
$$

with the value of repayment given as

$$
\mathcal{V}^r(B,a) = \max_{c_2,n,p,B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\mathcal{V}(B', a') \right] \text{ s.t. } (1.9), (1.11), (1.12),
$$

and the value of default as

$$
\mathcal{V}^d(a) = \max_{c_2,n,p} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(0, a') + (1-\theta) \mathcal{V}^d(a') \right] \text{ s.t. } (1.10) - (1.12).
$$

The value $\mathcal{V}(\cdot)$ is the option value of default. As is standard in the sovereign default literature, the government is assumed to honor its obligations whenever it is indifferent between default and repayment. If the government starts a period in financial autarky, it solves the same problem as in the default case. In periods of default and autarky, the government will regain access to financial markets in the subsequent period with probability θ. With probability $1-\theta$, it will stay in financial autarky.[17]

1.2.3 Equilibrium

The Markov-perfect equilibrium is defined as follows:

Definition 1. *A stationary Markov-perfect equilibrium is given by a set of value functions*

[16] The derivation of the implementability constraint can be found in Appendix 1.A.1.

[17] On average, the government thus spends $1/\theta$ periods in autarky after a default.

$\mathcal{V}(B,a)$, $\mathcal{V}^r(B,a)$, $\mathcal{V}^d(a)$, and policy functions $\mathcal{D}(B,a)$, $\mathcal{B}^r(B,a)$, $\mathcal{X}^r(B,a)$, $\mathcal{X}^d(a)$, with $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, such that for all $(B,a) \in \mathbb{B} \times \mathbb{A}$:

$$\mathcal{D}(B,a) = \underset{d \in \{0,1\}}{\arg\max} \left\{ (1-d)\mathcal{V}^r(B,a) + d\mathcal{V}^d(a) \right\},$$

$$\begin{aligned} \{\mathcal{X}^r(B,a)\}_{\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P},\mathcal{B}\}} &= \underset{c_2,n,p,B' \in \mathbb{B}}{\arg\max}\ u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\mathcal{V}(B',a') \right] \\ & \quad s.t.\ (1.9), (1.11), (1.12), \end{aligned}$$

and

$$\begin{aligned} \left\{\mathcal{X}^d(a)\right\}_{\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P}\}} &= \underset{c_2,n,p}{\arg\max}\ u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(0,a') + (1-\theta)\mathcal{V}^d(a') \right] \\ & \quad s.t.\ (1.10) - (1.12), \end{aligned}$$

as well as

$$\mathcal{V}(B,a) = (1 - \mathcal{D}(B,a)) \times \mathcal{V}^r(B,a) + \mathcal{D}(B,a) \times \mathcal{V}^d(a),$$

$$\mathcal{V}^r(B,a) = u(\mathcal{P}^r(B,a)^{-1}, \mathcal{C}_2^r(B,a), \mathcal{N}^r(B,a)) + \beta \mathbb{E}_{a'|a} \left[\mathcal{V}(\mathcal{B}^r(B,a), a') \right],$$

and

$$\mathcal{V}^d(a) = u(\mathcal{P}^d(a)^{-1}, \mathcal{C}_2^d(a), \mathcal{N}^d(a)) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(0,a') + (1-\theta)\mathcal{V}^d(a') \right].$$

This definition highlights the stationarity of the policy problem since the functions that solve the decision problem of the government in a given period coincide with the policy functions that govern the optimal decisions of the government in future periods.[18]

1.3 Model Analysis

In this section, the role of sovereign default for public policy is investigated. Because the model cannot be evaluated analytically due to the discrete default option, numerical methods are applied. Appendix 1.A.3 contains details regarding the numerical computation of the equilibrium. The next section presents the model specification. A discussion of the public policy choices can be found in Section 1.3.2. Simulation results are presented in Section 1.3.3.

[18] The definition of the equilibrium is formulated following Martin (2009).

Parameter	Description	Value
β	Discount factor	0.9900
g	Government spending	0.0379
γ_1	Cash-good weight	0.0030
γ_2	Credit-good weight	0.3370
σ_1	Cash-good curvature	2.4300
σ_2	Credit-good curvature	1.0000
σ_n	Leisure curvature	2.0000
$\tilde{a}$	Default cost parameter	0.9900
θ	Probability of reentry	0.2000
σ	Std. dev. productivity shock	0.0169
ρ	Persistence of productivity	0.9000

Table 1.1: Parameter values

1.3.1 Model Specification

To explore the model properties by computational means, functional forms and parameters need to be chosen.

Functional Forms

Productivity follows a log-normal AR(1)-process,

$$a_t = a_{t-1}^{\rho} \exp(\sigma\varepsilon_t), \; \varepsilon_t \overset{i.i.d.}{\sim} N(0,1).$$

The household utility function is specified as

$$u(c_1,c_2,n) = \gamma_1 \frac{c_1^{1-\sigma_1}-1}{1-\sigma_1} + \gamma_2 \frac{c_2^{1-\sigma_2}-1}{1-\sigma_2} + (1-\gamma_1-\gamma_2)\frac{(1-n)^{1-\sigma_n}-1}{1-\sigma_n},$$

with $\gamma_1, \gamma_2, \sigma_i > 0$, $i \in \{1,2,n\}$ and $\gamma_1+\gamma_2 < 1$.[19]

The resource costs of default are specified as in Cuadra et al. (2010):

$$\psi(a,d) = a - d \times \max\{0, a-\tilde{a}\}.$$

If a default takes place, effective productivity equals $\tilde{a}$ when a exceeds $\tilde{a}$ while there are no costs of default when productivity a is below the threshold $\tilde{a}$. This default cost specification implies that a default is more costly in booms than in recessions. In the quantitative sovereign default literature, it is well known that this feature is crucial for

[19]For $\sigma_i = 1$, $i \in \{1,2,n\}$, household utility is logarithmic for the respective variable.

default to mostly take place in bad states and hence for countercyclical sovereign risk to emerge (see e.g. Aguiar and Amador, 2014). This property is consistent with empirical evidence (see Tomz and Wright, 2007) and also present in models with endogenous costs of default (see Mendoza and Yue, 2012, Sosa-Padilla, 2013).[20]

Parameters

A model period corresponds to one quarter. The selected model parameters are listed in Table 1.1. They are either set to standard values or chosen to replicate certain short- or long-run properties of the Mexican economy.[21] The productivity parameter ρ is set to 0.9 while σ is chosen to match the standard deviation of HP-filtered Mexican log real GDP. As is common in business cycle models, a discount factor of $\beta = 0.99$ is selected, implying an annual real risk-free rate of 4%. Based on World Bank data for 1980-2008, g is set to 0.0379 to match an average ratio of public spending to GDP of around 11%.[22] The credit-good preference parameter σ_2 is normalized to 1. Targeting a cash-credit good ratio and an average working time of one third each, γ_1 is set to 0.003 and γ_2 to 0.337. For the inverse of the elasticity of leisure σ_l, a rather standard value of 2 is selected. The probability of reentry θ is set to 0.2, implying that financial autarky lasts for 5 quarters on average. This parameter value is in line with values considered in the quantitative sovereign default literature which range from 0.0385 (Chatterjee and Eyigungor, 2012) to 0.282 (Arellano, 2008).

The size of σ_1 is crucial for long-run debt and inflation as will be explained in the next section. To match the Mexican average annual inflation rate of 29.69%, σ_1 is set to 2.43.[23] The incentive to default critically depends on $\tilde{a}$. For Mexico, Reinhart (2010) documents that there have been domestic defaults in 1982 and between 1929 and 1938. Based on this observation, I set the default cost parameter to match an annual default frequency of 2%. The model is also solved and simulated with prohibitively high productivity costs of default which rule out equilibrium default. This benchmark economy yields the same results as a model without default option and will be referred to as ”no-default economy”. The model with default option will be referred to as ”baseline economy”.

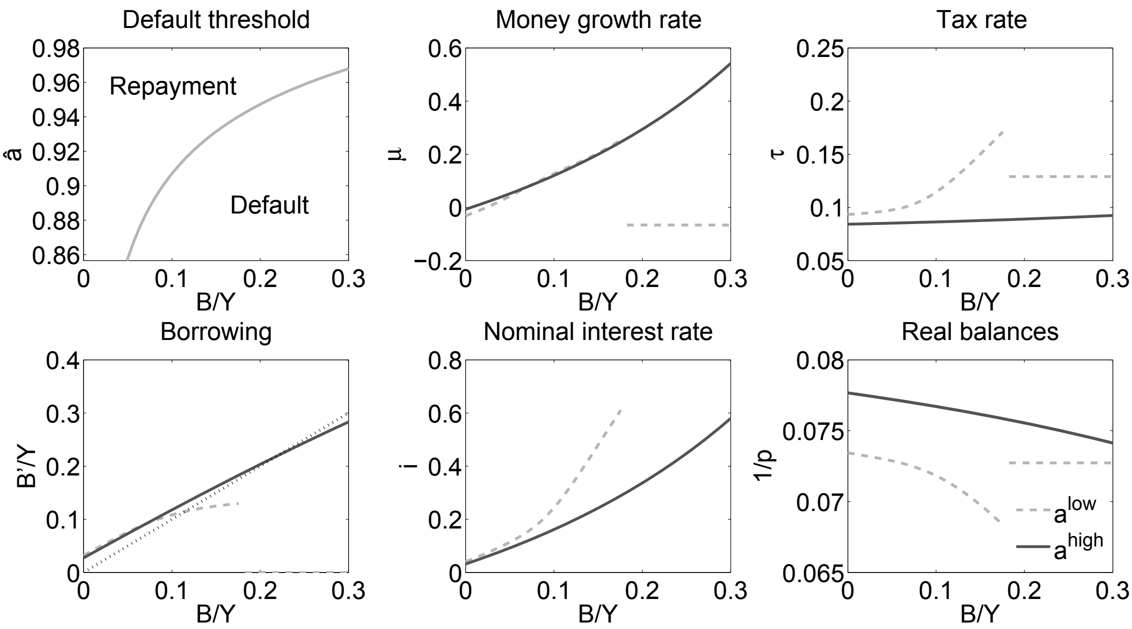
[20]Allowing for default costs that enter the the aggregate resource constraint (or the government budget constraint) in a lump-sum way does not change the results of this chapter as long as these losses are also relatively higher in good than in bad states, preserving countercyclical default incentives.

[21]The time series for real GDP and the GDP deflator are taken from Cuadra et al. (2010) and cover the time period from 1980:I to 2007:I. They are seasonally adjusted via EViews' multiplicative X-12 routine.

[22]More specifically, I use annual data from the World Bank's World Development Indicators for general final government consumption as a share of GDP.

[23]The inflation rate is calculated based on the quarterly GDP deflator time series for Mexico provided by Cuadra et al. (2010). Using alternative measures such as the CPI also yields average inflation rates of around 30%.

Figure 1.1: Default threshold and selected policy functions (baseline economy)



Notes: The policy functions are depicted for productivity values 1.5 standard deviations below (a^{low}) and above (a^{high}) the unconditional mean of productivity. The money growth rate and the nominal interest rate are displayed in annual terms.

1.3.2 Public Policy Decisions

The optimal policies for the economy with default can be seen in Figure 1.1. Debt is normalized by nominal output $Y \equiv py$, evaluated at the unconditional mean of productivity. The default decision is visualized using the default threshold $\hat{a}(B)$ which is the lowest productivity level that leads to repayment for given debt B: $\mathcal{V}^r(B,\hat{a}(B)) = \mathcal{V}^d(\hat{a}(B))$. The threshold separates the state space (B,a) into two areas: the default region ($a < \hat{a}(B)$, i.e. below the line) and the repayment region ($a \geq \hat{a}(B)$, i.e. on and above the line). As in the quantitative sovereign default literature, default becomes more attractive with higher debt and lower productivity (see Arellano, 2008).

The remaining policies are presented for productivity levels 1.5 standard deviations below (dashed line) and above (solid line) the unconditional mean of productivity. The nominal interest rate is defined as $i = 1/q - 1$. Since the continuous policy decisions depend on the default decision, the objects displayed in Figure 1.1 exhibit kinks at states where default is optimally chosen.[24] In the default case, the policies also do not change

[24]The policy functions displayed in Figure 1.1 are given as $\mathcal{X}(B,a) = (1-\mathcal{D}(B,a))\mathcal{X}^r(B,a) + \mathcal{D}(B,a)\mathcal{X}^d(a)$, with $\mathcal{X} \in \{\mathcal{B},\mathcal{C}_2,\mathcal{N},\mathcal{P}\}$. The remaining variables (μ,τ,i) are calculated by using these policy functions and the private sector equilibrium conditions (1.1),(1.6)-(1.7).

with B anymore.

The optimal labor and inflation tax distortions reflect the government's financing needs. By relaxing the government's budget, a sovereign default allows to reduce labor taxation and increase real balances relative to full debt repayment. The income tax rate and the price index p both increase with B. An inflationary monetary policy becomes particularly more attractive with higher debt because it lowers the real debt burden. This implies that default and inflation are substitutes since inflation as "partial default" becomes useless for $d = 1$. However, they are only imperfect substitutes due to the discrete nature of default.

The intertemporal policy trade-off can be illustrated via the generalized Euler equation

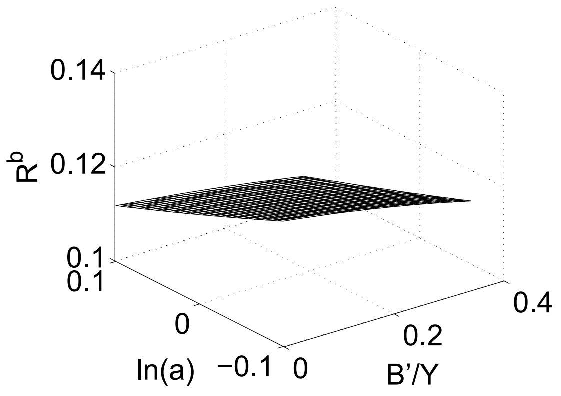
$$\beta \int_{\hat{a}(B')}^{\infty} (\xi' - \xi) \frac{u_2(c_2')}{p'} f_a(a'|a) da' = \xi \left(\frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right), \tag{1.14}$$

where ξ denotes the multiplier on the implementability constraint (1.9), $R^b = ((1+\mu)\, q u_2)/p$ average revenues from bond issuance and $R^m = ((1+\mu)\, u_2)/p$ (gross) revenues from money creation.[25] With the model specification of Section 1.3.1, these revenues are given by the following functions of productivity a and borrowing B':

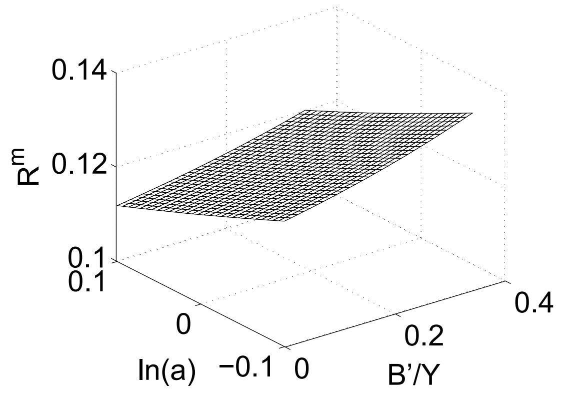
$$\begin{aligned} R^b(B',a) &= \beta \int_{\hat{a}(B')}^{\infty} \frac{\gamma_2 \mathcal{C}_2^r(B',a')^{-\sigma_2}}{\mathcal{P}^r(B',a')} f_a(a'|a) da', \\ R^m(B',a) &= \beta \times \left\{ \begin{array}{c} \int_0^{\hat{a}(B')} \gamma_1 \mathcal{P}^d(a')^{\sigma_1 - 1} f_a(a'|a) da' \\ + \int_{\hat{a}(B')}^{\infty} \gamma_1 \mathcal{P}^r(B',a')^{\sigma_1 - 1} f_a(a'|a) da' \end{array} \right\}. \end{aligned}$$

Households dislike volatile consumption and leisure. In the presence of productivity shocks, the government can issue debt to accommodate these preferences and smooth tax distortions as measured by ξ across states (see the LHS of (1.14)). Its ability to do so is constrained by financial market incompleteness and lack of commitment. Since only nominal non-state contingent bonds are available, the government has an incentive to make real debt payments state contingent via inflation or default. However, because it cannot commit to a state-contingent repayment plan for the next period, public financing conditions will depend on the chosen debt position B' since it affects the risk of inflation and default. The derivatives on the RHS of (1.14) reflect this channel. The optimal debt

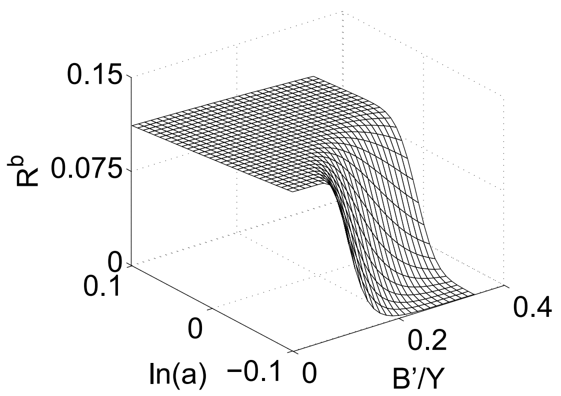
[25] The derivation of the Euler equation assumes differentiability of $\mathcal{V}^r$, R^b and R^m with respect to debt (see Appendix 1.A.2 for details). As is common in the sovereign default literature (see e.g. Cuadra and Sapriza, 2008, or Hatchondo et al., 2015), the generalized Euler equation is only presented here to highlight the intertemporal public policy trade-off in an intuitive way. The numerical algorithm that is used to solve the model is not based on this Euler equation and does not require differentiability to hold (see Appendix 1.A.3 for details).



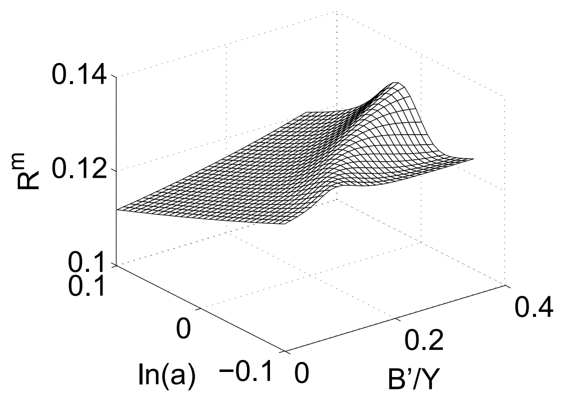
(a) $R^b(B', a)$ (no default)



(b) $R^m(B', a)$ (no default)



(c) $R^b(B', a)$ (baseline)



(d) $R^m(B', a)$ (baseline)

Figure 1.2: Average bond revenues $R^b(B', a)$ and money revenues $R^m(B', a)$ for the no-default economy and the baseline economy

policy then trades off the tax smoothing motive against the time-inconsistency problem, i.e. how current debt issuance affects revenues $R^b(B', a) \times B'$ and $R^m(B', a)$ by changing household expectations of inflation and default, which matter for household bond and money demand.[26]

The impact of debt issuance on public revenues is visualized in Figure 1.2. It depicts the functions R^b and R^m for the no-default economy (first row) and the baseline economy with equilibrium default (second row). The no-default case has previously been studied by Martin (2009, 2011, 2013) and Diaz-Gimenez et al. (2008).

It is useful to first look at the case without default to understand how sovereign risk

[26]Note that revenues are weighted by the marginal utility of credit-good consumption $u_2(c_2)$ and expressed in real terms.

affects the debt policy.[27] The shape of R^b reflects the relation between inflation and beginning-of-period debt (see panel (a)). Due to its lack of commitment, the government optimizes from period to period and therefore does not internalize how its current policy choices affect outcomes in previous periods. More specifically, it does not internalize that its current actions have an impact on households' demand for money and bonds in the last period. Failing to recognize this impact, the government decides to erode the real value of beginning-of-period debt via inflation to relax its budget. Since the temptation to use inflation in this way increases with B, expected inflation becomes an increasing function of end-of-period debt and the price of nominal government bonds responds to B' in a negative way, causing borrowing to become more expensive when more debt is issued ($\partial R^b/\partial B' < 0$). The shape of R^m reflects the way public debt, inflation and household money demand are related (see panel (b)). Given that real balances $1/p$ are a decreasing function of B, the real payoff of money is expected to decrease with borrowing B', reducing household money demand today. However, lower real balances $1/p$ also increase the marginal utility of cash-good consumption $u_1(1/p)$, such that the demand for money increases with B' since households expect to be more cash-constrained in the subsequent period. Whether higher borrowing B' increases net household money demand depends on the size of the parameter σ_1. For $\sigma_1 > 1$, it does and an increase in B' leads to higher money revenues ($\partial R^m/\partial B' > 0$). Since a household's valuation of money increases with B', the government can simply issue more currency to implement a particular price index and thus obtain more revenues from money issuance.

The long-run debt position is determined by the two effects described above. A positive sign for $\partial R^m/\partial B'$ is crucial for non-zero long-run debt.[28] For $\sigma_1 = 1$, money revenues do not respond to borrowing ($\partial R^m/\partial B' = 0$) which eliminates the incentive to borrow in the long run. With $\sigma_1 \in (0,1)$, the government even has an incentive to accumulate assets ($B' < 0$) due to $\partial R^m/\partial B' < 0$.[29] These two cases are not further discussed here because they make default a redundant policy option. When there are productivity shocks, a positive response of money revenues to borrowing also matters for the government's ability to smooth tax distortions across states since, without this effect, only the negative bond price effect would be operative and make it more expensive to issue debt in low productivity states.

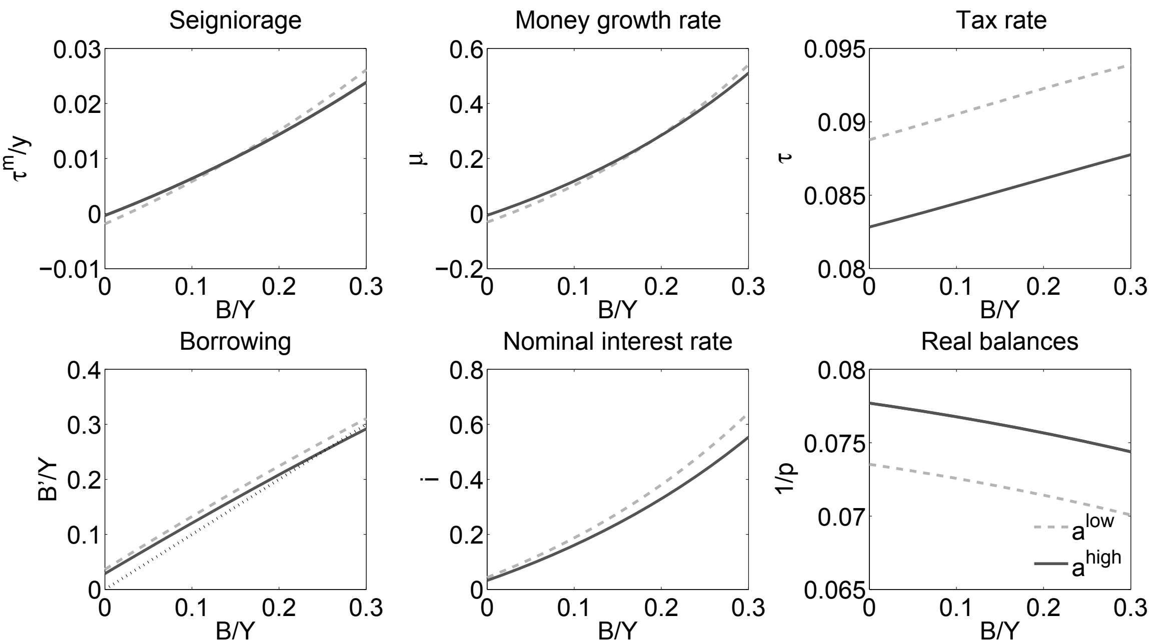
When the government can default on its debt, sovereign risk changes the impact of

[27] In this case, the default threshold is given as $\hat{a}(B) = 0$, i.e. default is prohibitively costly.

[28] Looking at Markov-perfect public policy in a real economy setting with endogenous government spending and without default, Debortoli and Nunes (2013) show - for analytical and quantitative examples - that long-run debt only deviates from zero for a small range of parameter values.

[29] For more details see Proposition 5 in Martin (2009) who proves these properties for a deterministic model without default option.

Figure 1.3: Selected policy functions (no-default economy)



Notes: The policy functions are depicted for productivity values 1.5 standard deviations below (a^{low}) and above (a^{high}) the unconditional mean of productivity. The money growth rate and the nominal interest rate are displayed in annual terms.

borrowing on public revenues. Panel (c) displays R^b for the baseline economy with sovereign default. Because a default is more likely for higher amounts of debt, the bond price would respond to B' in a negative way even in the absence of inflation risk. As in Arellano (2008), the debt elasticity of interest rates is higher in bad (low productivity) than in good (high productivity) states, reflecting the default incentives of the government. Panel (d) shows how money revenues respond to B' when there is sovereign risk. The non-monotonic shape results from the optimal mix of default and inflation. Since default and inflation are substitutes, higher borrowing can lower expected inflation by increasing the probability of default. While making money as a store of value more valuable, this interaction also lowers the expected marginal utility of cash-good consumption and hence R^m. Due to its adverse effect on bond and money revenues, sovereign risk thus ultimately makes debt issuance less attractive. The consequences for the long-run debt position can be illustrated via Figure 1.1 and Figure 1.3. By looking at the intersection between the (dotted) 45-degree line and the borrowing policies, one can already see without having simulated the model that average debt is going to be lower in the baseline model with default. The quantitative dimension of the model properties discussed so far is explored in the next section.

	Baseline	No default
Mean		
Default probability (annual)	0.0206	0
Debt-to-GDP	0.1866	0.3340
Tax rate	0.1043	0.0912
Seigniorage-to-GDP	0.0155	0.0296
Inflation rate (annual)	0.3011	0.6503
Nominal interest rate (annual)	0.3958	0.7175
Standard deviation		
Output	0.0236	0.0192
Tax rate	0.0124	0.0012
Inflation rate (annual)	0.1077	0.0544
Nominal interest rate (annual)	0.0782	0.0252
Correlation with output		
Tax rate	-0.8597	-0.9991
Inflation rate (annual)	-0.2990	-0.5599
Nominal interest rate (annual)	-0.4725	-0.9093

Table 1.2: Selected model statistics

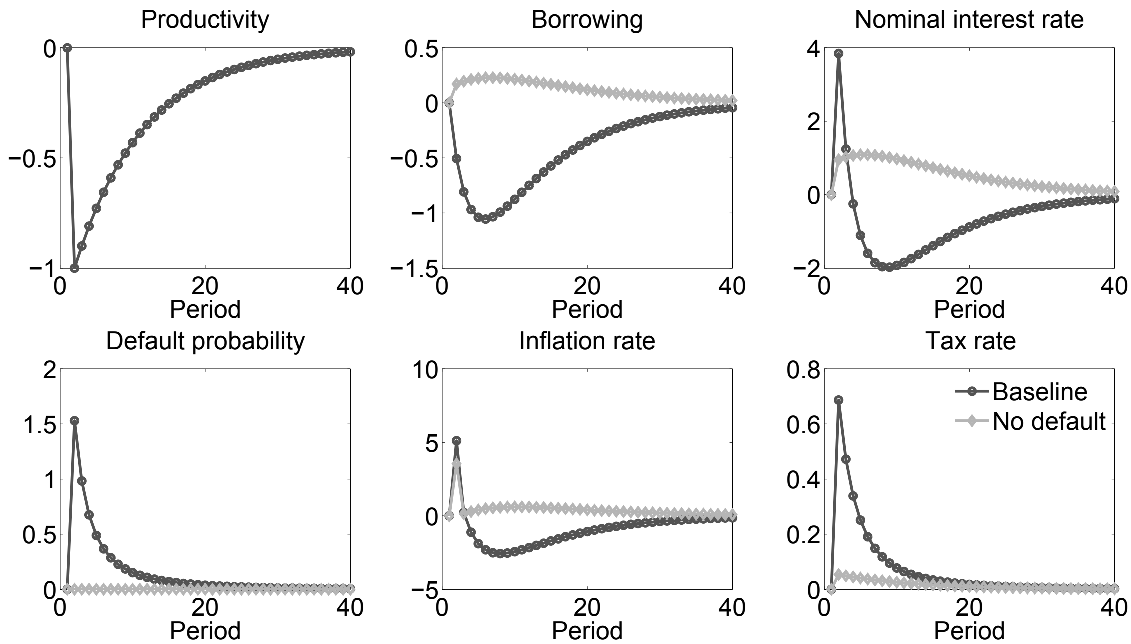
1.3.3 Simulation Results

Table 1.2 presents the averages of statistics calculated for 2500 simulated economies with 2000 periods each. The time series are filtered using the Hodrick-Prescott filter with a smoothing parameter of 1600. Output is in logs. All simulations are initialized with $S_0 = (0, \mathbb{E}[a])$ and the first 500 observations of each sample are discarded.

Average debt and inflation are lower with default option. More specifically, the average inflation rate in the no-default economy is more than twice as large as in the baseline economy with default. The possibility of default reduces average inflation through a direct and an indirect effect. When the government chooses to default, there is no incentive to use inflation to reduce the real debt burden anymore. As a result, inflation is lower in default (and autarky) periods than in periods of repayment. The role of this direct effect is however limited by the frequency of default and does not contribute much to the average inflation rate.[30] The indirect effect of default on inflation is related to how the risk of default affects the government's borrowing behavior. Default risk raises the cost of rolling over even low amounts of debt in recessions. This mechanism restricts the build up of large public debt positions which would make higher inflation more attractive. Less debt also implies that the tax base of the income tax increases relative to that of infla-

[30]The average inflation rate for periods of repayment only is 31.08% and thus only slightly higher than the overall average inflation rate of 30.11% which includes periods of default and autarky as well.

Figure 1.4: Impulse responses of selected model variables to a negative one-time productivity shock



Notes: All variables are expressed as absolute deviations (multiplied by 100) from their values at the stationary state to which the respective economy converges when productivity is kept fixed at its unconditional mean.

tion. Hence, the benefit of raising inflation is lower, leading to a higher average labor tax rate in the baseline economy.[31] While the accumulation of debt crucially depends on the government's ability to collect seigniorage (see the discussion in the previous section), the average seigniorage-to-GDP ratio is moderate and of plausible size.[32] When the government has the option to default, borrowing is more expensive in recessions due to the increased risk of default. The average nominal interest rate however is higher in the no-default economy since it experiences more inflation on average.

The short-run implications of sovereign risk for public policy can be illustrated via Figure 1.4. For the baseline and the no-default economy, it displays impulse responses of selected model variables to a negative one-time productivity shock. The variables are expressed as absolute deviations from their values at the stationary state to which the economies converge when productivity is kept fixed at its long-run mean.[33] Since pro-

[31]For Mexico, Ilzetzki (2011) calculates an average marginal income tax rate of 12.1% which is close to the average tax rate in the baseline model (10.43%).

[32]Using the same definition of seigniorage as in the model, Aisen and Veiga (2008) calculate that average seigniorage is 2.2% of GDP for Mexico.

[33]Of course, the two economies do not exhibit the same stationary state. The variables in such a stationary state are close to the average values listed in Table 1.2.

	Mexico (1980-2007)	Baseline	No default (recalibrated)
Standard deviation	0.2423	0.1077	0.0703
Correlation with output	-0.2734	-0.2990	-0.4585

Table 1.3: Cyclical properties of inflation

ductivity is persistent ($\rho > 0$), the negative shock immediately raises the risk of default as the incentive to default is more likely to be strong in the subsequent period. The high sensitivity of interest rates to changes in debt issuance forces the government to reduce its debt position in order to avoid an even larger decline of the bond price. As a result, the government has to resort to large increases in inflation and taxes to finance debt payments and government spending.[34] Consistent with empirical evidence (see Reinhart and Rogoff, 2009), a sovereign debt crisis thus is inflationary. As productivity reverts back to its mean and debt is reduced even further, expected inflation and sovereign risk both decline, leading the government to take advantage of the improved borrowing conditions and accumulate debt again. In the no-default economy, borrowing conditions do not deteriorate very much in response to the negative productivity shock. This property allows the government to effectively smooth tax distortions across states by issuing debt which avoids large increases in taxes and inflation. Because debt cannot be easily rolled over in the baseline model, the impact of productivity shocks on the economy is more pronounced and output volatility is 23% higher relative to the model without default.

Since one of the main contributions of this chapter is to offer a joint analysis of inflation and sovereign default, it is interesting to compare the cyclical properties of inflation generated by the model with and without default to those observed in the data. Table 1.3 shows the results. In Mexico, inflation is very volatile and countercyclical. The baseline model with default can replicate these findings. To give the model without default a fair chance, it is recalibrated to match the average inflation rate and the volatility of output in Mexico.[35] While the recalibrated no-default economy yields countercyclical inflation, the baseline model predicts more volatile and less countercyclical inflation than the no-default model which is closer to what is observed empirically.

[34]This mechanism is related to the one studied by Cuadra et al. (2010) in a model of a small open economy with real government debt. The authors show that countercyclical default risk can rationalize the procyclical consumption taxation observed in emerging economies.

[35]The changed model parameter values are $\gamma_1 = 0.015$, $\gamma_2 = 0.325$, $\sigma_1 = 1.73$ and $\sigma = 0.02$.

1.4 The Welfare Implications of Sovereign Default

This section discusses the welfare implications of sovereign default. With commitment, the option to default will not decrease welfare since the government would otherwise refrain from using it.[36] Without commitment, this is not necessarily the case anymore. Section 1.3 has shown that the default option has implications for public policy in the short and the long run. On the one hand, by increasing the sensitivity of bond prices with respect to debt and productivity, countercyclical risk of default entails short-run costs because the government loses some of its ability to smooth tax distortions across states. On the other hand, default risk might lead to welfare gains due to its impact on long-run debt. The model features a long-run borrowing motive that stems from the presence of two frictions, lack of commitment and a liquidity constraint (see the discussion in Section 1.3.2). The government acts in a time-consistent way and does not internalize the effect of its current choice of inflation on the borrowing behavior in previous periods. When household money demand and thus the value of money are increasing in the amount of issued debt, the government persistently chooses positive debt positions which then lead to high average inflation. By limiting public debt accumulation via more sensitive interest rates, the default option reduces average inflation and the misallocation of consumption compared to the no-default setting.

To evaluate whether the addition of the default option to the set of policy instruments is welfare enhancing, welfare measure Δ is calculated. It measures the percentage increase in credit-good consumption that households in the no-default economy need to be given in each period to achieve the same expected lifetime utility as in the baseline economy with default:

$$\mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t u(c_{1t}^D, c_{2t}^D, n_t^D)\right] = \mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t u(c_{1t}^N, c_{2t}^N(1+\Delta), n_t^N)\right].$$

The sequences of consumption and labor supply in the economy with ($j = D$) and without default option ($j = N$) are denoted as $\{c_{1t}^j, c_{2t}^j, n_t^j\}_{t=0}^T$. Expected lifetime utility is calculated for both types of economies by averaging realized lifetime utility of 2500 samples with simulated time series of effective length $T = 1500$ each.

The calculated welfare measure is $\Delta = 0.00006$. For the no-default economy, credit-good consumption thus needs to be increased by only 0.006% in each period to equalize household welfare in both types of economies. Since these gains are of negligible size,

[36] For a real small open economy with incomplete markets and costly sovereign default, Adam and Grill (2012) show that welfare can be increased when the Ramsey planner can commit to a state-contingent default plan.

one can argue that, from a welfare perspective, lack of commitment to repayment is not important.

1.5 Conclusion

To understand the implications of the option to default on debt payments for public policy, this chapter has studied optimal monetary and fiscal policy without commitment in a cash-credit economy with nominal debt and endogenous government default. While a default allows the government to reduce inflation and distortionary labor taxation by relaxing its budget constraint, the default option mainly induces lower rates of inflation by constraining debt issuance via endogenous default risk premia. This mechanism reduces the average debt position and the government's incentive to implement high inflation in the long-run. Less debt also implies that the income tax becomes more attractive relative to inflation, resulting in a higher average labor tax rate. Taxes and inflation are more volatile when the default option is available because the government's ability to smooth tax distortions across states is reduced by the presence of default risk. For the case of Mexico, a counterfactual exercise has demonstrated that the consequences of the option to default for welfare are negligible.

1.A Appendix

1.A.1 Derivation of the Implementability Constraint

I will only derive the implementability constraint for the repayment case. The constraint for the default case is derived similarly. First, take the household optimality conditions (1.1),(1.6)-(1.7) and rewrite them (in recursive notation) as

$$\begin{aligned}
\tau &= 1+\frac{u_n(n)}{u_2(c_2)}\frac{1}{\psi(a,0)}, \\
\frac{1+\mu}{p} &= \beta\mathbb{E}_{a'|a}\left[\frac{u_1(1/p')}{u_2(c_2)}\frac{1}{p'}\right], \\
\frac{(1+\mu)q}{p} &= \beta\mathbb{E}_{a'|a}\left[\frac{u_2(c_2')}{u_2(c_2)}\frac{1-d'}{p'}\right].
\end{aligned}$$

After using these expressions to eliminate the terms on the LHS of these equations in the government budget constraint

$$g-\tau\psi(a,0)n+\frac{1+B}{p}=(1+\mu)\,\frac{1+qB'}{p},$$

one obtains

$$\begin{aligned}
& g-\left(1+\frac{u_n(n)}{u_2(c_2)}\frac{1}{\psi(a,0)}\right)\psi(a,0)n+1/p+B/p \\
= \; & \beta\mathbb{E}_{a'|a}\left[\frac{u_1(1/p')}{u_2(c_2)}\frac{1}{p'}\right]+\beta\mathbb{E}_{a'|a}\left[\frac{u_2(c_2')}{u_2(c_2)}\frac{1-d'}{p'}\right]B',
\end{aligned}$$

or

$$\begin{aligned}
& g-\psi(a,0)n-\frac{u_n(n)}{u_2(c_2)}n+1/p+B/p \\
= \; & \beta\mathbb{E}_{a'|a}\left[\frac{u_1(1/p')}{u_2(c_2)}\frac{1}{p'}\right]+\beta\mathbb{E}_{a'|a}\left[\frac{u_2(c_2')}{u_2(c_2)}\frac{1-d'}{p'}\right]B'.
\end{aligned}$$

Now, eliminate $\psi(a,0)n$ via the resource constraint $\psi(a,0)n=1/p+c_2+g$,

$$\begin{aligned}
& g-(1/p+c_2+g)-\frac{u_n(n)}{u_2(c_2)}n+1/p+B/p \\
= \; & \beta\mathbb{E}_{a'|a}\left[\frac{u_1(1/p')}{u_2(c_2)}\frac{1}{p'}\right]+\beta\mathbb{E}_{a'|a}\left[\frac{u_2(c_2')}{u_2(c_2)}\frac{1-d'}{p'}\right]B'.
\end{aligned}$$

After multiplying both sides of the equation with $u_2(c_2)$ and using the policy functions to replace next period's variables, one arrives at the implementability constraint

$$\begin{aligned} & -u_n(n)n - u_2(c_2)(c_2 - B/p) \\ = \;& \beta\mathbb{E}_{a'|a}\left[\begin{array}{c} u_1(\mathcal{P}^r(B',a')^{-1})\frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')} \\ +u_1(\mathcal{P}^d(a')^{-1})\frac{\mathcal{D}(B',a')}{\mathcal{P}^d(a')} \end{array}\right] + \beta\mathbb{E}_{a'|a}\left[u_2(\mathcal{C}_2^r(B',a'))\frac{1-\mathcal{D}(B',a')}{\mathcal{P}^r(B',a')}\right]B'. \end{aligned}$$

1.A.2 First-Order Conditions for the Policy Problem

Conditional on repayment, the necessary first-order condition for an interior debt choice B' is

$$0 = \xi\left(R^b + \frac{\partial R^b}{\partial B'}B' + \frac{\partial R^m}{\partial B'}\right) + \beta\int_{\hat{a}(B')}^{\infty}\frac{\partial \mathcal{V}^r(B',a')}{\partial B'}f_a(a'|a)da',$$

with ξ denoting the multiplier on the implementability constraint (1.9).[37]

When combined with definition

$$R^b = \beta\int_{\hat{a}(B')}^{\infty}\frac{u_2(c_2')}{p'}f_a(a'|a)da',$$

and envelope condition

$$\frac{\partial \mathcal{V}^r(B,a)}{\partial B} = -\xi\frac{u_2(c_2)}{p},$$

the first-order condition yields the generalized Euler equation

$$\beta\int_{\hat{a}(B')}^{\infty}(\xi' - \xi)\frac{u_2(c_2')}{p'}f_a(a'|a)da' = \xi\left(\frac{\partial R^b}{\partial B'}B' + \frac{\partial R^m}{\partial B'}\right).$$

The first-order conditions for the price index p, credit consumption c_2, and labor supply n are

$$0 = -p^{-2}\left[u_1(1/p) - \xi u_2(c_2)B - \phi + \vartheta u_{11}(1/p)\right], \quad (1.15)$$

$$0 = u_2(c_2) + \xi\left[u_{22}(c_2)(c_2 - B/p) + u_2(c_2)\right] - \phi - \vartheta u_{22}(c_2), \quad (1.16)$$

$$0 = u_n(n) + \xi\left[u_{nn}(n)n + u_n(n)\right] + \phi\psi(a,0), \quad (1.17)$$

where ξ, ϕ and ϑ are the multipliers related to the constraints (1.9), (1.11) and (1.12). In addition to these three conditions, the complementary slackness conditions

$$\vartheta \geq 0, u_1(1/p) - u_2(c_2) \geq 0, \vartheta \times [u_1(1/p) - u_2(c_2)] = 0, \quad (1.18)$$

[37] The derivation of the generalized Euler equation follows Martin (2009) and Arellano and Ramanarayanan (2012).

need to be satisfied as well.

1.A.3 Numerical Solution

The task of the numerical solution algorithm is to find the policy and value functions $\mathcal{X}^r(B,a)$, $\mathcal{X} \in \{\mathcal{B},\mathcal{C}_2,\mathcal{N},\mathcal{P},\mathcal{V}\}$, and $\mathcal{X}^d(a)$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P},\mathcal{V}\}$. Following Hatchondo et al. (2010), I approximate these functions on discrete grids for debt and productivity, and use cubic spline interpolation to allow for off-grid values of B and a. The solution algorithm involves the following steps:

1. Construct discrete grids for debt $[\underline{B},\overline{B}]$ and productivity $[\underline{a},\overline{a}]$.

2. Choose initial values for the policy and value functions $\mathcal{X}^r_{start}(B,a)$ and $\mathcal{X}^d_{start}(a)$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P},\mathcal{V}\}$, at all grid point combinations.

3. Set $\mathcal{X}^j_{next} = \mathcal{X}^j_{start}$, $j \in \{r,d\}$ and fix an error tolerance ε.

4. For each discrete grid point combination $(B,a) \in [\underline{B},\overline{B}] \times [\underline{a},\overline{a}]$, find the optimal policies $\mathcal{X}^r_{new}(B,a)$, $\mathcal{X} \in \{\mathcal{B},\mathcal{C}_2,\mathcal{N},\mathcal{P}\}$, and the associated value of repayment $\mathcal{V}^r_{new}(B,a)$. For each productivity value $a \in [\underline{a},\overline{a}]$, compute the optimal policies $\mathcal{X}^d_{new}(a)$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P}\}$, and the value of default $\mathcal{V}^d_{new}(a)$.

5. If $|\mathcal{X}^r_{new}(B,a) - \mathcal{X}^r_{next}(B,a)| < \varepsilon$ and $\left|\mathcal{X}^d_{new}(a) - \mathcal{X}^d_{next}(a)\right| < \varepsilon$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P},\mathcal{V}\}$, for all grid point combinations, go to step 6, else set $\mathcal{X}^j_{next} = \mathcal{X}^j_{new}$, $j \in \{r,d\}$ and repeat step 4.

6. Use $\mathcal{X}^j_{new}(\cdot)$, $j \in \{r,d\}$, as approximations of the respective equilibrium objects in the infinite-horizon economy.

The grid points are distributed evenly. Since the asymmetric default cost specification leads to a kink at $a = \tilde{a}$ in $\mathcal{X}^d(a)$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P},\mathcal{V}\}$, I partition the productivity grid for the default case as in Hatchondo et al. (2010) to account for this discontinuity.

As is known in the literature (see e.g. Krusell and Smith, 2003, or Martin, 2009), there might be multiple Markov-perfect equilibria in models with infinitely-lived agents. In particular, there could be equilibria with discontinuous policy functions which do not arise in the infinite-horizon limit of a finite-horizon model version. To avoid such equilibria, I follow Hatchondo et al. (2010) and solve for the infinite-horizon limit of a finite-horizon model version.[38] In practice, this means that I compute the value and policy functions for

[38] Martin (2009) also solves for the infinite-horizon limit. As pointed out by him, using a Svensson (1985)-type beginning-of-period cash-in-advance constraint in a finite-horizon model requires a terminal

the final period problem where no borrowing takes place and use these objects as initial values $\mathcal{X}^j_{start}$, $j \in \{r,d\}$, for step 2.

For a given state $(B,a) \in [\underline{B},\overline{B}] \times [\underline{a},\overline{a}]$, the objective function of the government is the sum of two parts, the period utility function $u(1/p,c_2,n)$ and (in the repayment case) the continuation value $\beta\mathbb{E}_{a'|a}[\mathcal{V}_{next}(B',a')]$, with $\mathcal{V}_{next}(B,a) = \max\left\{\mathcal{V}^r_{next}(B,a), \mathcal{V}^d_{next}(a)\right\}$. The optimal policies for step 4 are then computed as follows. I use a sub-routine that calculates the optimal static policies c_2, n, and p for given debt and productivity values $(B,a) \in [\underline{B},\overline{B}] \times [\underline{a},\overline{a}]$ and an arbitrary, i.e. possibly off-grid, borrowing value $\hat{B}'$. More specifically, I use a non-linear equation solver to find the variables that satisfy the static optimality conditions (1.9),(1.11),(1.15)-(1.18).[39] Since these conditions involve the complementary slackness conditions (1.18), I follow Brumm and Grill (2014) and use the trick by Garcia and Zangwill (1981) to transform the set of optimality conditions into a system of equations.[40] Using the static policy sub-routine, (c_2,n,p) and thus period utility $u(1/p,c_2,n)$ can be expressed as functions of $(B,a,\hat{B}')$.[41] As a result, given $(B,a) \in [\underline{B},\overline{B}] \times [\underline{a},\overline{a}]$, the government objective can be expressed as a function of $\hat{B}'$ as well: $u(1/p,c_2,n) + \beta\mathbb{E}_{a'|a}\left[\mathcal{V}_{next}(\hat{B}',a')\right]$.

For each discrete grid point combination $(B,a) \in [\underline{B},\overline{B}] \times [\underline{a},\overline{a}]$, the optimal debt policy $\mathcal{B}^r_{new}(B,a)$ then is computed via a global non-linear optimizer, calling the static policy routine to calculate the objective function for each candidate debt value $\hat{B}'$. More specifically, for each (B,a), I first perform a grid search over a pre-defined grid for $\hat{B}'$ and then use the solution as an initial guess for the Nelder-Mead algorithm.[42] The optimal policies $\mathcal{X}^r(B,a)$, $\mathcal{X} \in \{\mathcal{C}_2,\mathcal{N},\mathcal{P}\}$ then are computed by using the static policy routine for the optimal borrowing value $\mathcal{B}^r_{new}(B,a)$. The algorithm iterates on the policy and value functions until the maximum absolute difference between value and policy functions obtained in two subsequent iterations is below $\varepsilon = 10^{-5}$ for each combination $(B,a) \in [\underline{B},\overline{B}] \times [\underline{a},\overline{a}]$.[43]

money value for a monetary equilibrium to exist. Otherwise, households will not be willing to invest in money in the final period and by backward induction not in any of the previous periods. The impact of the final-period value of money vanishes over time and does not affect the final results.

[39] In the default case, condition (1.9) is replaced by condition (1.10)

[40] Alternatively, one can also use a non-linear constrained optimizer to compute the optimal static policies for each combination $(B,a,\hat{B}')$. Using a sequential quadratic programming algorithm (see e.g. Nocedal and Wright, 1999 for details), I found this approach to be both slower and less accurate.

[41] The routine is also used to obtain the optimal policies in the final period, where $\hat{B}' = 0$ holds.

[42] Instead of the Nelder-Mead algorithm, I also solved for the optimal debt policy using Golden section search and a mesh adaptive direct search algorithm (as implemented by the optimizing routine NOMAD provided by the OPTI Toolbox), which did not affect the results.

[43] Using a tighter convergence criterion did not affect the results.

To evaluate value and policy functions at debt and productivity states that are off-grid, cubic spline interpolation is used.[44] To approximate expected values in an accurate way, one needs to account for the default threshold. This can be seen by looking at the expected option value of default:

$$\mathbb{E}_{a'|a}\left[\mathcal{V}_{next}(B',a')\right] = \int_0^{\hat{a}(B')} \mathcal{V}^d_{next}(a') f_a(a'|a) da' + \int_{\hat{a}(B')}^{\infty} \mathcal{V}^r_{next}(B',a') f_a(a'|a) da'.$$

Gauss-Legendre quadrature nodes and weights are used to approximate the integrals above. The default threshold $\hat{a}(B)$ satisfies $\mathcal{V}^r_{next}(B,\hat{a}(B)) - \mathcal{V}^d_{next}(\hat{a}(B)) = 0$ and is computed via bisection method.

[44] Hatchondo et al. (2010) show that allowing for a continuous state space is crucial for accurate solutions of models with equilibrium default.

Chapter 2

Monetary Conservatism and Sovereign Default

2.1 Introduction

At least since the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983b), it is known that often times, the optimal monetary policy under commitment is time inconsistent and hence not going to be implemented by a benevolent policy maker who - quite realistically - is unable to commit to future policy. More specifically, when monetary policy is set under discretion, it tends to result in an inflation bias, i.e. an inflation rate that is persistently higher than the optimal one under commitment. This result holds for model environments where monetary policy is tempted to use surprise inflation to stimulate the economy (see e.g. Barro and Gordon, 1983b, or Clarida et al., 1999) or relax the government budget by reducing the real value of outstanding nominal public debt payments (see e.g. Lucas and Stokey, 1983). In doing so, the policy maker does not internalize that rational agents anticipate the use of such inflationary policies in earlier periods which (partially) offsets the potentially positive effects of inflation surprises. To avoid (or at least reduce) this inflation bias, Rogoff (1985) has suggested the delegation of monetary policy to a monetary conservative central banker who views inflation as more costly than society. Although he is not benevolent, his appointment makes monetary policy less tempted to resort to inflationary policies and might thus increase welfare relative to a scenario with a benevolent policy maker.

In practice, most developed economies have delegated monetary policy to independent central banks that emphasize low and stable inflation. These reforms have shielded monetary policy from the sequential nature of policy making in democratic societies and have usually been accompanied by lower inflation rates. Recently, many emerging economies have also introduced central bank independence in an attempt to bring down their persistently high inflation rates (see e.g. Carstens and Jácome, 2005). However, a lot of these countries are subject to frictions that might undermine the success of such reforms.

This chapter studies the effectiveness and desirability of monetary conservatism in a model which accounts for three frictions that matter for many emerging economies: (i) incomplete financial markets, (ii) risk of default, and (iii) political distortions. I study a model of a small open economy where fiscal and monetary policy is chosen without commitment. With lack of commitment, the presence of nominal non-state contingent government debt introduces an incentive to reduce the real debt burden by using surprise inflation or default and relax the government budget. Fiscal policy is set by a fiscal authority that exhibits a deficit bias due to political economy frictions whereas monetary policy is controlled by an independent central bank. Reflecting its independence, the central bank's objective might differ from that of the fiscal authority and society. In particular, the central bank might not be subject to political economy constraints and place a higher value on price stability (see Rogoff, 1985, or Adam and Billi, 2008). The interaction between the fiscal authority and the central bank is modeled as a stationary Markov-perfect game. Because this equilibrium concept is a refinement of the subgame-perfect equilibrium concept, monetary and fiscal policies are ensured to be set in a time-consistent way, reflecting the policy makers' lack of commitment. In a Markov-perfect equilibrium, the strategies chosen by the two authorities are only conditioned on the minimal payoff-relevant state of the economy, which includes the public debt position. Since the policy makers re-optimize from period to period, they do not internalize the impact of their decisions on expectations and outcomes in previous periods, and take as given next period's monetary and fiscal policies as well as the policies set by the respective other authority in the current period. Although the fiscal authority and the central bank take future policies as given, they recognize that they can affect these policies via the amount of debt carried into the next period.

The frictions (i)-(iii) matter for the implications of monetary conservatism for the following reasons. When the central bank places a higher (utility) weight on inflation than the fiscal authority and society, it is less tempted to use inflation to reduce the real debt burden. However, when financial markets are incomplete and only non-state contingent bonds can be issued, this also implies that the central bank is less willing to use inflation as a shock absorber and make real debt payments state contingent. As a result, even if monetary conservatism can bring down inflation, it is not clear that this is welfare enhancing. The central bank's willingness to use inflation might also affect the economy's vulnerability to sovereign debt crises (see e.g. Kocherlakota, 2012). The more conservative the central bank is, the more attractive the default option might become for the fiscal authority to relax the government budget, potentially increasing the likelihood of a debt crisis. Finally, political frictions might render a higher credibility for low inflation costly as well. In the model, the fiscal authority exhibits a deficit bias due to political disagreement and turnover risk (see Cuadra and Sapriza, 2008, or Aguiar and Amador, 2011). It therefore has a long-run borrowing motive that does not reflect the preferences of society, leading to debt levels that are higher relative to those accumulated by a benevolent fiscal authority. The nominal interest rate reflects expected inflation which tends to increase with the level of borrowing when there is no commitment. Importantly, the elasticity of the nominal interest rate with respect to future debt affects the fiscal authority's incentive to issue nominal bonds (see also Aguiar et al., 2014). When the central bank is less tempted to raise inflation, this elasticity decreases which tends to encourage the fiscal authority to borrow even more and might reduce household welfare.

The main finding of this chapter is that an economy with a more conservative central bank ends up with more debt, more frequent default events and lower inflation. Monetary conservatism can thus successfully reduce the inflation bias. This success comes however at a cost. By lowering expected inflation and hence nominal interest rates, it makes debt accumulation more attractive for the fiscal authority and thereby exposes the economy more often to sovereign debt crises since the incentive to default increases with debt. By reducing the time-inconsistency problem related to inflation, it aggravates the time-inconsistency problem related to sovereign default. The resulting increase in sovereign risk leads to more sensitive interest rates which in turn make it more costly to smooth government spending in response to bad fiscal shocks. As a result, fiscal policy becomes more volatile when the degree of monetary conservatism is increased. A welfare comparison reveals that the benefits of lower and more stable inflation outweigh the welfare costs of experiencing higher average debt, more frequent debt crises and more volatile fiscal policy.

This chapter is related to three strands of literature. First, it is related to recent papers that study central bank independence in the presence of nominal government debt and lack of commitment. In particular, it relates to Niemann (2011) who studies a Markov-perfect policy game between a monetary conservative central bank and a myopic fiscal authority, using the cash-in-advance model of Nicolini (1998). In his model, the fiscal authority has a lower discount factor than society and does not internalize the effect of its borrowing decision on future policy. When nominal debt is the only source of the time-inconsistency problem, he shows that monetary conservatism backfires. While it lowers average inflation when the degree of monetary conservatism is sufficiently high, it encourages the fiscal authority to borrow more in the long run, decreasing welfare. Other related papers are Niemann et al. (2013b) and Martin (2014) who also investigate central bank independence in models with nominal debt and lack of commitment but abstract from monetary conservatism.[1] All of these papers do not consider sovereign default, micro-founded political distortions and uncertainty.

Second, this chapter is related to the recent literature on sovereign default and incomplete markets (see e.g. Aguiar and Amador, 2014, for details). Within this growing literature, the studies that are closest to this chapter are Cuadra and Sapriza (2008), Du and Schreger (2015) and Nuño and Thomas (2015). The former paper introduces political polarization and turnover risk into the sovereign default model of Arellano (2008), showing that such political frictions make policy makers act in a more impatient manner. In this chapter, the economy faces similar political distortions. Du and Schreger (2015) develop a quantitative sovereign default model in which a government can reduce the real debt burden by raising inflation (and thereby depreciate the domestic currency) at the cost of hurting the balance sheet of domestic firms which issue debt denominated in foreign currency and earn revenues in local currency. Nuño and Thomas (2015) study a continuous-time model in which a policy maker borrows from abroad and monetary policy is either chosen under discretion or always following a zero-inflation policy that is not responsive to the state of the economy. In contrast to this chapter, they consider a benevolent policy maker. As in this chapter, they also find that the economy is better off when the government is not tempted to reduce the real debt burden via inflation. The authors also briefly consider the case of delegating monetary policy to a monetary conservative central bank but do not allow for political distortions and do not discuss how disagreement between the fiscal and the monetary authority might affect outcomes.

Third, this chapter is related to Aguiar et al. (2013, 2015). Aguiar et al. (2013) study a continuous-time model of discretionary monetary and fiscal policy where default events are self-fulfilling in the spirit of Cole and Kehoe (2000). Building on this paper, Aguiar et al. (2015) consider a model of a monetary union with a continuum of countries which independently choose fiscal policy and a common central bank that is in charge of monetary policy. They show the existence of a fiscal externality that encourages countries to overborrow. In contrast to this chapter, the authors focus on benevolent policy makers and thus do not allow for political frictions and varying degrees of central bank independence. In addition, they abstract from fundamental uncertainty and only consider sunspot-driven default events.[2]

[1] Adam and Billi (2008) study the role of monetary conservatism in a sticky price model with endogenous fiscal policy but without public debt.

[2] Other recent papers that study the interaction between monetary policy and self-fulfilling sovereign

The rest of this chapter is organized as follows. Section 2.2 describes the model and Section 2.3 discusses the main policy trade-offs. Section 2.4 presents the quantitative model analysis. Section 2.5 concludes.

2.2 Model

In the model, there is a small open economy and a continuum of risk-neutral foreign investors. The small open economy is inhabited by households and a government. The government consists of two independent authorities: a central bank and a fiscal authority. In the economy, there are two political parties that might be in charge of the fiscal authority. These parties randomly enter and leave office, i.e. only one party chooses fiscal policy in a given period.

2.2.1 Small Open Economy

Consider a small open economy that is inhabited by a unit-mass continuum of households. Time is discrete, indexed with $t = 0, 1, 2, ...$ and goes on forever. Households have preferences over private consumption c_t and a public good g_t, given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \tilde{U}(c_t, g_t) \right], \; 0 < \beta < 1,$$

where the period utility function is given by

$$\tilde{U}(c_t, g_t) = c_t + u(g_t),$$

with $u_g(\cdot), -u_{gg}(\cdot) > 0$.[3] Households face the period budget constraint

$$y = c_t + \tau_t + \psi(\pi_t),$$

where y is a constant endowment that they receive in each period, τ_t are exogenous tax payments and $\psi(\pi_t)$ are resource losses of inflation as in Calvo and Guidotti (1992) that satisfy $\psi_\pi(\cdot), \psi_{\pi\pi}(\cdot) > 0$.[4] The endowment y is in terms of a tradable good that will be the numeraire in the model. Its international price is normalized to one.

debt crises are Araujo et al. (2013), Da-Rocha et al. (2013), Corsetti and Dedola (2014) and Bachetta et al. (2015).

[3]The same quasi-linear household utility function is also used in Cole and Kehoe (2000).

[4]These resource losses could, for instance, be interpreted as price adjustment costs in the spirit of Rotemberg (1982).

Using the period budget constraint to eliminate private household consumption and dropping policy-invariant terms, welfare of the average citizen can be written as

$$\mathcal{U} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(g_t, \pi_t) \right],$$

with

$$U(g_t, \pi_t) = u(g_t) - \psi(\pi_t).$$

This welfare measure reflects the preferences of society and will be used to evaluate the welfare properties of public policy.[5]

In the economy, a government is in charge of setting monetary and fiscal policy. This government consists of two separate entities: a fiscal authority and a monetary authority (from now on referred to as central bank). Both authorities re-optimize in each period and cannot commit to future policies. Similar to Cuadra and Sapriza (2008), the fiscal authority is controlled by either one of two political parties. These parties have symmetric objectives and randomly enter and leave office. The objective of political party $i \in \mathbb{I} \equiv \{1,2\}$ is given by

$$\mathcal{F}_i = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U_i^{\mathcal{F}}(g_t, \pi_t) \right],$$

where

$$U_i^{\mathcal{F}}(g_t, \pi_t) = \tilde{\theta}_{it} u(g_t) - \psi(\pi_t),$$

with $\tilde{\theta}_{it} = \theta > 1$ if party i is in office and $\tilde{\theta}_{it} = 1$ if it is not.

In each period, the fiscal authority chooses the supply of the public good, trades one-period bonds with foreign investors and decides on whether to repay outstanding debt or not. As in Aguiar and Amador (2011), both political parties place a higher weight θ on the utility derived from the public good when they are in office.[6] While the political parties disagree about the value of the public good, there is no disagreement about the cost of inflation $\psi(\pi_t)$. However, $\tilde{\theta}_{it}$ leads to disagreement about the optimal inflation rate between the two parties since the incumbent party places a lower relative weight on the cost of inflation $\psi(\pi_t)$.[7]

As argued by Aguiar and Amador (2011), the weight $\tilde{\theta}_{it}$ can be interpreted in sev-

[5] Aguiar et al. (2013, 2015) and Nuño and Thomas (2015) directly assume a utility cost of inflation.

[6] Cuadra and Sapriza (2008) consider a model with two population groups where each group is favored by one of two potentially ruling parties.

[7] Aisen and Veiga (2005) document a positive relationship between political instability and average inflation. The fiscal authority's lower relative emphasis on price stability compared to society is consistent with this pattern.

eral ways. For instance, it can represent disagreement between the two political parties about the implementation of public policy, leading to a higher marginal utility of public consumption for the incumbent political party since it can carry out its desired policy. Alternatively, the assumption $\theta > 1$ can be viewed as a shortcut for the incumbent's ability to divert public funds into its own pocket via pork-barrel spending (see e.g. Battaglini and Coate, 2008) or corruption.

For simplicity, I assume that the political parties have completely symmetric objectives. In addition, once in office, the probability of being in office in the next period μ is the same for both parties.[8] These assumptions imply that - for the recursive model formulation below - there is no need to keep track of which particular party is in office since they will choose the same policies in a symmetric equilibrium. To smooth public spending across states, the government can trade nominal and real one-period bonds with risk-neutral foreign investors. These bonds are non-state contingent and defaultable, i.e. the fiscal authority can refuse to repay bondholders. Following the recent sovereign default literature, a default is costly because of direct resource costs and a temporary loss of access to international financial markets (see e.g. Aguiar and Gopinath, 2006, or Arellano, 2008).

The presence of political disagreement and turnover risk leads the fiscal authority to exhibit a present bias that makes it behave similarly to a decision maker who discounts in a quasi-geometric fashion (see Laibson, 1997, Krusell et al., 2002).[9] As a result, it is effectively less patient than a policy maker who does not face the risk of leaving office. In the context of the model, increased impatience implies that the fiscal authority has an incentive to front-load public spending by either borrowing more or defaulting on debt payments. In any period, the costs associated with these policies are (partly) borne in the future, either through increases in the primary surplus or temporary financial autarky. When less patient, these costs are discounted more by the fiscal authority, making borrowing and default more attractive policy options. It is important to note that the strength of the present bias varies with the state of the economy. The present bias of the fiscal authority in this chapter thus is different from that of a policy maker who simply has a low discount factor β relative to the lenders (see Niemann, 2011, and Aguiar et al., 2014).

If the fiscal authority repays its debt, the period government budget constraint is

$$P_t \tau_t + q_{Nt} B_{Nt+1} + P_t q_{Rt} b_{Rt+1} \geq P_t g_t + B_{Nt} + P_t b_{Rt},$$

[8] On average, a newly appointed incumbent thus spends $1/(1-\mu)$ subsequent periods in office.

[9] Persson and Svensson (1989) and Alesina and Tabellini (1990) were the first to recognize that political polarization and turnover risk lead to a present bias. Aguiar and Amador (2011) and Chatterjee and Eyigungor (2014) show in detail how quasi-geometric discounting can result in such a political economy model when when $\mu = 0.5$.

where P_t is the price level, q_{Nt} the price of nominal bonds B_{Nt+1} and q_{Rt} the price of real bonds b_{Rt+1}. Tax revenues τ_t are random and follow a first-order Markov process with continuous support $\mathbb{T} \subseteq \mathbb{R}_+$.[10]

I consider exogenous tax revenues for three reasons.[11] First, for many countries it is difficult, if not virtually impossible, to quickly change the tax code in the short run. By contrast, sudden adjustments of public spending tend to be easier to carry out in practice. Second, since the sovereign default literature mostly considers endowment economies (see e.g. Aguiar and Gopinath, 2006, or Arellano, 2008), a setting that models public resources also as an endowment makes it easier to relate the model to this literature. Third, the numerical solution of the model is quite difficult as it involves solving the decision problems of two distinct authorities. Abstracting from the tax rate as a decision variable for the fiscal authority substantially reduces the computational burden.

In real terms, the budget constraint is

$$\tau_t + q_{Nt} b_{Nt+1} + q_{Rt} b_{Rt+1} \geq g_t + \pi_t^{-1} b_{Nt} + b_{Rt},$$

with (gross) inflation $\pi_t = P_t / P_{t-1}$ and normalized nominal debt $b_{Nt} = B_{Nt} / P_{t-1}$. The initial price level $P_{-1} \in (0, \infty)$ is taken as given. For tractability reasons, I assume that nominal debt issuance b_{Nt+1} always accounts for a fixed share $\lambda \in [0, 1]$ of total debt $b_{t+1} = b_{Nt+1} + b_{Rt+1}$.[12] In the repayment case, the government budget constraint then becomes

$$\tau_t + (\lambda q_{Nt} + (1 - \lambda) q_{Rt}) b_{t+1} \geq g_t + \left(\lambda \pi_t^{-1} + 1 - \lambda\right) b_t,$$

while in the default case, it is

$$\tau_t - \phi(\tau_t) \geq g_t,$$

where $\phi(\tau_t) \geq 0$ are (public) resource costs of default. In the sovereign default literature, such resource costs are standard but modeled in terms of aggregate output and not in terms of public funds (see e.g. Arellano, 2008). One interpretation for public resource costs is that they result from the abandonment of public projects which leads to net losses for the government. Another interpretation is that in the default case, the country experiences

[10] I will occasionally refer to shocks to tax revenues as fiscal shocks.

[11] In the context of a real economy without default but with private government information, Halac and Yared (2014) also look at a fiscal policy maker who exhibits a present bias and finances the supply of a public good with exogenous tax revenues and borrowing.

[12] Chatterjee and Eyigungor (2012) make a similar assumption in a model with long-term debt. They keep the debt maturity structure fixed and let the sovereign choose the level of debt. In this chapter, it is the currency composition of debt that is kept constant. I allow for a nominal debt share that is not equal to one to compare the effects of changing the currency composition of debt with the effects of changing the monetary policy stance.

a decline in tax morale which makes it more difficult for the government to collect tax payments. As a result, it has to spend additional resources on tax enforcement to raise a given amount of revenues τ_t.

Monetary policy is controlled by the central bank. I assume that the central bank can directly choose the inflation rate by setting its policy instruments in an appropriate way. Reflecting its independence, the central bank's objective may differ from that of the fiscal authority:

$$\mathcal{M} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U^{\mathcal{M}} (g_t, \pi_t) \right],$$

where

$$U^{\mathcal{M}} (g_t, \pi_t) = u(g_t) - \alpha \psi(\pi_t),$$

with $\alpha \geq 0$.

Following the literature (see Rogoff, 1985, Adam and Billi, 2008, and Niemann, 2011), monetary policy is delegated to a monetary conservative central banker who has the same preferences as the average citizen, except that he has an inherent distaste for inflation: $U^{\mathcal{M}} (g_t, \pi_t) = U(g_t, \pi_t) - (\alpha - 1) \psi(\pi_t)$. The parameter α is the central banker's degree of monetary conservatism.[13] For $\alpha > 1$ ($\alpha < 1$), the central banker values price stability more (less) than society. Since the economy (partly) borrows in its own currency, the central bank can reduce the real debt burden and relax the government budget by raising inflation. The temptation to do so strongly depends on α. In contrast to the fiscal policy maker, the central banker is not subject to political risk and remains in power forever. Importantly, the central banker also does not derive additional utility from the public good like the incumbent political party. In the economy, the degree of central bank independence thus is characterized by the central bank's monetary conservatism and its independence from political economy considerations. For $\alpha = \alpha_\theta \equiv 1/\theta$, the central bank puts the same relative weights on $u(g)$ and $\psi(\pi)$ than the fiscal authority. This case will be a useful benchmark since it implies that the main source of disagreement between the fiscal and the monetary authority is the fiscal authority's deficit bias.

The interaction between the political parties, which determines the actions of the fiscal authority, and the interaction between the fiscal authority and the central bank is modeled as a Markov-perfect game (see e.g. Niemann et al., 2013b). As is common in the literature, I restrict attention to stationary equilibria. In a stationary Markov-perfect equilibrium, the policy functions that characterize the optimal decisions of the two authorities only depend on the minimal payoff-relevant state, which includes the beginning-of-period

[13] Aguiar et al. (2013) refer to this parameter as "inflation credibility" in a model without delegated monetary policy.

debt position b_t.[14] As in Cuadra and Sapriza (2008), I only study symmetric equilibria in which the two political parties choose the same policies when in power, given the aggregate state. This way, fiscal policy does not depend on which party is in office. Because the two authorities optimize under discretion, they do not internalize the effect of their actions on previous periods and have no incentive to honor promises made by policy makers in the past. As a result, they cannot credibly commit to carry out specific actions in the future and take the policies set in the subsequent period as given. However, since these policies will depend on the future aggregate state, the authorities can influence the way public policy is conducted in the future via the debt position b_{t+1}.

Conditional on entering a period with debt b_t, the within-period timing is as follows. First, the revenue shock τ_t is realized and the office-holder is determined. Then, the fiscal authority chooses whether to repay its debt. After this, the central bank sets the inflation rate, followed by the fiscal authority's spending and borrowing decisions. Conditional on the default decision, the two authorities thus play a Stackelberg game with the central bank acting as the Stackelberg leader. This particular timing is chosen for two reasons. First, it implies that the central bank is not powerless and can influence the decisions of the fiscal authority via the inflation rate. If the fiscal authority were the Stackelberg leader and made all its decisions before the central bank acts, it could effectively also control the inflation rate since the central bank would have no other choice than to set the inflation rate that satisfies the budget constraint.[15] Second, the value and policy functions are not generally differentiable due to the discrete default option, which implies that the intertemporal decisions might not be characterized via Euler equations as in Niemann et al. (2013b) or Martin (2014). The Stackelberg leadership timing allows to solve the model numerically by sequentially solving the decision problems of the two authorities in any period, given the respective aggregate state at the beginning of the period (see Appendix 2.A.2 for details).

2.2.2 International Investors

The small open economy can trade non-state contingent nominal and real bonds with a continuum of homogeneous risk-neutral foreign investors who can borrow and save

[14]Since the optimal strategies are only conditioned on the current payoff-relevant (fundamental) state of the economy, the Markov-perfect equilibrium concept rules out reputational considerations as discussed by Barro and Gordon (1983a) that rely on trigger strategies which require strategies to exhibit complex history dependence.

[15]Alternatively, one could follow Niemann et al. (2013b) and assume that the fiscal authority chooses public spending but not borrowing. The end-of-period debt position then is determined residually to satisfy the budget constraint, given the spending and inflation decisions of the fiscal authority and the central bank.

on international financial markets at the real risk-free rate r_f. Although the small open economy may refuse to repay its debt, investors always honor their obligations. Risk neutrality and expected profit maximization imply the bond pricing conditions

$$q_N\left(b',\tau\right) = \frac{1}{1+r_f}\mathbb{E}_{\tau'|\tau}\left[\frac{1-\mathcal{D}(b',\tau')}{\Pi^r\left(b',\tau'\right)}\right], \tag{2.1}$$

$$q_R\left(b',\tau\right) = \frac{1}{1+r_f}\mathbb{E}_{\tau'|\tau}\left[1-\mathcal{D}(b',\tau')\right]. \tag{2.2}$$

The bond price schedules $q_N\left(b',\tau\right)$ and $q_R\left(b',\tau\right)$ reflect rational expectations of future default and inflation. Given the focus on Markov-perfect public policy, next period's default and inflation policies $\mathcal{D}(\cdot)$ and $\Pi^r(\cdot)$ depend on end-of-period debt b' as well as future tax revenues τ'. Following the sovereign default literature (see e.g. Arellano, 2008), I assume that the investors act after all public policies have been determined. As a result, the central bank and the fiscal authority anticipate how their decisions affect bond prices.

2.2.3 Public Policy

Conditional on having access to financial markets, the beginning-of-period value of the central bank is denoted as $\mathcal{M}(s)$, that of an incumbent as $\mathcal{F}(s)$ and that of a party not in office as $\mathcal{F}^*(s)$, where $s \equiv (b,\tau)$.[16]

The default decision of the fiscal authority solves

$$\mathcal{F}(b,\tau) = \max_{d\in\{0,1\}}\left\{(1-d)\,\mathcal{F}^r(b,\tau)+d\mathcal{F}^d(\tau)\right\}, \tag{2.3}$$

where $\mathcal{F}^r(b,\tau)$ is the value of repayment and $\mathcal{F}^d(\tau)$ the value of default.

The beginning-of-period values of the central bank and the political party currently not in office satisfy

$$\mathcal{M}(b,\tau) = (1-\mathcal{D}(b,\tau))\,\mathcal{M}^r(b,\tau)+\mathcal{D}(b,\tau)\mathcal{M}^d(\tau), \tag{2.4}$$

$$\mathcal{F}^*(b,\tau) = (1-\mathcal{D}(b,\tau))\,\mathcal{F}^{*r}(b,\tau)+\mathcal{D}(b,\tau)\mathcal{F}^{*d}(\tau), \tag{2.5}$$

where $\mathcal{D}(b,\tau)$ characterizes the optimal default decision of the fiscal authority.

[16] In addition to $s=(b,\tau)$, whether the economy is in financial autarky or not also counts as a state variable in the model. The model formulation below accounts for this by formulating the public policy problem conditional on the economy's default/autarky status.

After the default decision has been made, the central bank acts, solving

$$\mathcal{M}^r(b,\tau) = \max_{\pi \geq \pi_{min}} \left\{ \hat{\mathcal{M}}^r(\pi,b,\tau) \right\}, \tag{2.6}$$

if the fiscal authority repays and

$$\mathcal{M}^d(\tau) = \max_{\pi \geq \pi_{min}} \left\{ \hat{\mathcal{M}}^d(\pi,\tau) \right\}, \tag{2.7}$$

if it defaults. The lower bound on the inflation rate $\pi_{min} = 1/(1+r_f)$ ensures non-negative nominal interest rates ($q_N \leq 1$). The value functions $\hat{\mathcal{M}}^r(\pi,b,\tau)$ and $\hat{\mathcal{M}}^d(\pi,\tau)$ are the intra-period continuation values for the central bank. They are determined below and depend on how the fiscal authority sets its policies, given the inflation rate π.

For the political parties, the repayment and default values satisfy

$$\begin{aligned}
\mathcal{F}^r(b,\tau) &= \hat{\mathcal{F}}^r(\Pi^r(b,\tau),b,\tau), && (2.8)\\
\mathcal{F}^d(\tau) &= \hat{\mathcal{F}}^d(\Pi^d(\tau),\tau), && (2.9)\\
\mathcal{F}^{*r}(b,\tau) &= \hat{\mathcal{F}}^{*r}(\Pi^r(b,\tau),b,\tau), && (2.10)\\
\mathcal{F}^{*d}(\tau) &= \hat{\mathcal{F}}^{*d}(\Pi^d(\tau),\tau), && (2.11)
\end{aligned}$$

where $\Pi^r(b,\tau)$ and $\Pi^d(\tau)$ denote the policy functions for inflation that solve the central bank's decision problem, $\hat{\mathcal{F}}^r(\pi,b,\tau)$ and $\hat{\mathcal{F}}^d(\pi,\tau)$ the intra-period continuation values for the incumbent party, and $\hat{\mathcal{F}}^{*r}(\pi,b,\tau)$ and $\hat{\mathcal{F}}^{*d}(\pi,\tau)$ the intra-period continuation values for the party not in office. When choosing whether to default or repay, the fiscal authority thus internalizes how its default decision affects the inflation rate.

After the central bank has set the inflation rate, the fiscal authority makes its spending and borrowing decisions. Its decision problem is given by

$$\begin{aligned}
\hat{\mathcal{F}}^r(\pi,b,\tau) &= \max_{g,b'} \left\{ \begin{array}{c} \theta u(g) - \psi(\pi) \\ +\beta \mathbb{E}_{\tau'|\tau} \left[\begin{array}{c} \mu \mathcal{F}(b',\tau') \\ +(1-\mu)\mathcal{F}^*(b',\tau') \end{array} \right] \end{array} \right\} && (2.12)\\
\text{subject to } 0 &\leq \tau - g - \left(\lambda\pi^{-1} + 1 - \lambda\right) b + \left[\begin{array}{c} \lambda q_N(b',\tau) \\ +(1-\lambda) q_R(b',\tau) \end{array} \right] b',
\end{aligned}$$

in the repayment case and by

$$\hat{\mathcal{F}}^d(\pi,\tau) \quad = \quad \max_g \left\{ \begin{array}{c} \theta u(g) - \psi(\pi) \\ +\delta\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c} \mu\mathcal{F}(0,\tau') \\ +(1-\mu)\,\mathcal{F}^*(0,\tau') \end{array} \right] \\ +(1-\delta)\,\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c} \mu\mathcal{F}^d(\tau') \\ +(1-\mu)\,\mathcal{F}^{*d}(\tau') \end{array} \right] \end{array} \right\} \tag{2.13}$$

$$\text{subject to } 0 \quad \leq \quad \tau - g - \phi(\tau),$$

in the default case.

If the fiscal authority reneges on debt payments, it is excluded from international financial markets for the current period. Conditional on being in autarky, the economy regains access to international financial markets with probability δ in the following period. The average duration of financial autarky hence is $1/\delta$ periods. Regardless of whether the party currently in charge of fiscal policy defaults or repays, it remains in office in the subsequent period with probability μ and is replaced by the opposite party with the counter-probability $1-\mu$.

For the central bank, the intra-period continuation values $\hat{\mathcal{M}}^r(\pi,b,\tau)$ and $\hat{\mathcal{M}}^d(\pi,\tau)$ satisfy

$$\hat{\mathcal{M}}^r(\pi,b,\tau) = \left\{ \begin{array}{c} u(\hat{\mathcal{G}}^r(\pi,b,\tau)) - \alpha\psi(\pi) \\ +\beta\mathbb{E}_{\tau'|\tau}\left[\mathcal{M}(\hat{\mathcal{B}}^r(\pi,b,\tau),\tau')\right] \end{array} \right\}, \tag{2.14}$$

and

$$\hat{\mathcal{M}}^d(\pi,\tau) = \left\{ \begin{array}{c} u(\hat{\mathcal{G}}^d(\pi,\tau)) - \alpha\psi(\pi) \\ +\beta\mathbb{E}_{\tau'|\tau}\left[\delta\mathcal{M}(0,\tau') + (1-\delta)\,\mathcal{M}^d(\tau')\right] \end{array} \right\}, \tag{2.15}$$

and for the party not in office, the continuation values $\hat{\mathcal{F}}^{*r}(\pi,b,\tau)$ and $\hat{\mathcal{F}}^{*d}(\pi,\tau)$ satsify

$$\hat{\mathcal{F}}^{*r}(\pi,b,\tau) = \left\{ \begin{array}{c} u(\hat{\mathcal{G}}^r(\pi,b,\tau)) - \psi(\pi) \\ +\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c} \mu\mathcal{F}^*(\hat{\mathcal{B}}^r(\pi,b,\tau),\tau') \\ +(1-\mu)\,\mathcal{F}(\hat{\mathcal{B}}^r(\pi,b,\tau),\tau') \end{array} \right] \end{array} \right\}, \tag{2.16}$$

and

$$\hat{\mathcal{F}}^{*d}(\pi,\tau) = \left\{ \begin{array}{c} u(\hat{\mathcal{G}}^d(\pi,\tau)) - \psi(\pi) \\ +\delta\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c} \mu\mathcal{F}^*(0,\tau') \\ (1-\mu)\,\mathcal{F}(0,\tau') \end{array} \right] \\ +(1-\delta)\,\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c} \mu\mathcal{F}^{*d}(\tau') \\ (1-\mu)\,\mathcal{F}^d(\tau') \end{array} \right] \end{array} \right\}, \tag{2.17}$$

where $\hat{\mathcal{B}}^r(\pi,b,\tau)$, $\hat{\mathcal{G}}^r(\pi,b,\tau)$ and $\hat{\mathcal{G}}^d(\pi,\tau)$ denote the policy functions for borrowing and government spending that solve the fiscal authority's decision problems (2.12) and (2.13). These functions characterize the fiscal authority's optimal response to the inflation rate π set by the central bank. The probabilities μ and $1-\mu$ do not enter the continuation values of the central bank $\hat{\mathcal{M}}^r$ and $\hat{\mathcal{M}}^d$ since future fiscal policy does not depend on which of the political parties will be in office. The objective of the central bank does not vary with the office-holder of the fiscal authority either.

Equations (2.14) and (2.15) illustrate that inflation affects the objective of the central bank in two ways. First, there is a direct effect of π on the cost of inflation $\psi(\pi)$. Second, there is an indirect effect that operates through the optimal response functions of the fiscal authority. When solving the decision problems (2.6) and (2.7), the central bank internalizes both of these effects.

The policy functions for inflation $\Pi^r(b,\tau)$ and $\Pi^d(\tau)$ then determine

$$\mathcal{B}^r(b,\tau) = \hat{\mathcal{B}}^r(\Pi^r(b,\tau),b,\tau), \tag{2.18}$$
$$\mathcal{G}^r(b,\tau) = \hat{\mathcal{G}}^r(\Pi^r(b,\tau),b,\tau), \tag{2.19}$$
$$\mathcal{G}^d(\tau) = \hat{\mathcal{G}}^d(\Pi^d(\tau),\tau), \tag{2.20}$$

such that in the repayment case, the equilibrium policies will only depend on (b,τ) and in the default/autarky case on τ, since the inflation choices are conditioned on these states as well.

Conditional on having access to financial markets, the equilibrium policies are ultimately pinned down by the fiscal authority's default decision, such that

$$\Pi(b,\tau) = (1-\mathcal{D}(b,\tau))\,\Pi^r(b,\tau) + \mathcal{D}(b,\tau)\Pi^d(\tau), \tag{2.21}$$
$$\mathcal{B}(b,\tau) = (1-\mathcal{D}(b,\tau))\,\mathcal{B}^r(b,\tau), \tag{2.22}$$
$$\mathcal{G}(b,\tau) = (1-\mathcal{D}(b,\tau))\,\mathcal{G}^r(b,\tau) + \mathcal{D}(b,\tau)\mathcal{G}^d(\tau). \tag{2.23}$$

The Markov-perfect equilibrium for the model is then defined as follows:

Definition 2. *A stationary Markov-perfect equilibrium is given by bond pricing functions q_N and q_R that satisfy the zero-expected profit conditions (2.1)-(2.2), value functions $\{\mathcal{F},\mathcal{F}^r,\mathcal{F}^d,\hat{\mathcal{F}}^r,\hat{\mathcal{F}}^d,\mathcal{F}^*,\mathcal{F}^{*r},\mathcal{F}^{*d},\hat{\mathcal{F}}^{*r},\hat{\mathcal{F}}^{*d},\mathcal{M},\mathcal{M}^r,\mathcal{M}^d,\hat{\mathcal{M}}^r,\hat{\mathcal{M}}^d\}$ that satisfy the equations (2.3)-(2.17) and policy functions $\{\Pi,\Pi^r,\Pi^d,\mathcal{B},\mathcal{B}^r,\hat{\mathcal{B}}^r,\mathcal{D},\mathcal{G},\mathcal{G}^r,\mathcal{G}^d,\hat{\mathcal{G}}^r,\hat{\mathcal{G}}^d\}$ that satisfy the conditions (2.1)-(2.2), (2.4)-(2.5), (2.8)-(2.11), (2.14)-(2.17) and (2.18)-(2.23). The functions $\{\Pi^r,\Pi^d\}$ furthermore solve the policy problems of the central bank (2.6)-(2.7) and the functions $\{\hat{\mathcal{B}}^r,\mathcal{D},\hat{\mathcal{G}}^r,\hat{\mathcal{G}}^d\}$ solve the policy problems of the fiscal authority*

(2.3), (2.12)-(2.13).

2.3 Policy Trade-Offs

Before moving to the quantitative analysis, it is helpful to first take a look at the first-order conditions for the fiscal authority and the central bank to understand the forces that drive policy making in the model. In this section, I will abstract from default and consider a model version where the fiscal authority always repays. The quantitative evaluation in Section 2.4 will then consider the full model of the previous section with default. Without default, the policy and value functions do not need to carry an index d or r since the fiscal authority always repays

Interior solutions to the public policy problems then satisfy the generalized Euler equations (GEEs)

$$
\begin{aligned}
0 \;=\; & \theta u_g(g)\Delta_q \\
& -\mu\beta\mathbb{E}_{\tau'|\tau}\left[\theta u_g(g')\Delta'_\lambda - \Delta'_\theta \frac{\partial\Pi(b',\tau')}{\partial b'}\right] \\
& +(1-\mu)\beta\mathbb{E}_{\tau'|\tau}\left[u_g(g')\frac{\partial\mathcal{G}(b',\tau')}{\partial b'} - \psi_\pi(\pi')\frac{\partial\Pi(b',\tau')}{\partial b'}\right] \\
& -\mu\beta\mathbb{E}_{\tau'|\tau}\left[\theta u_g(g')\Delta'_q \frac{\partial\mathcal{B}(b',\tau')}{\partial b'}\right] \\
& +(2\mu-1)\beta\mathbb{E}_{\tau'|\tau}\left[\beta\mathbb{E}_{\tau''|\tau'}\left[\theta u_g(g'')\Delta''_\lambda - \Delta''_\theta\frac{\partial\Pi(b'',\tau'')}{\partial b''}\right]\frac{\partial\mathcal{B}(b',\tau')}{\partial b'}\right],
\end{aligned}
\tag{2.24}
$$

and

$$
\begin{aligned}
0 \;=\; & u_g(g)\Delta_q\frac{\partial\hat{\mathcal{B}}(\pi,b,\tau)}{\partial\pi} + \Delta_\alpha \\
& -\beta\mathbb{E}_{\tau'|\tau}\left[u_g(g')\Delta'_\lambda - \Delta'_\alpha\left(\frac{\partial\Pi(b',\tau')}{\partial b'} - \frac{\frac{\partial\mathcal{B}(b',\tau')}{\partial b'}}{\frac{\partial\hat{\mathcal{B}}(\pi',b',\tau')}{\partial\pi'}}\right)\right]\frac{\partial\hat{\mathcal{B}}(\pi,b,\tau)}{\partial\pi},
\end{aligned}
\tag{2.25}
$$

with

$$
\begin{aligned}
\Delta_\lambda &\equiv \lambda\pi^{-1}+1-\lambda, \\
\Delta_q &\equiv \lambda q_N(b',\tau)+(1-\lambda)q_R(b',\tau)+\left(\lambda\frac{\partial q_N(b',\tau)}{\partial b'}+(1-\lambda)\frac{\partial q_R(b',\tau)}{\partial b'}\right)b', \\
\Delta_\theta &\equiv \theta u_g(g)\lambda\pi^{-2}b-\psi_\pi(\pi), \\
\Delta_\alpha &\equiv u_g(g)\lambda\pi^{-2}b-\alpha\psi_\pi(\pi).
\end{aligned}
$$

Condition (2.24) characterizes the optimal borrowing decision of the fiscal authority, whereas (2.25) is the optimality condition for the inflation rate set by the central bank.[17]

As a benchmark, it is useful to first look at the optimality conditions

$$u_g(g)\Delta_q = \beta \mathbb{E}_{\tau'|\tau}\left[u_g(g')\Delta'_\lambda\right], \quad (2.26)$$

$$u_g(g)\lambda\pi^{-2}b = \psi_\pi(\pi), \quad (2.27)$$

which characterize the optimal borrowing and inflation decisions for a benevolent government that also lacks commitment and is in charge of setting monetary and fiscal policy. These conditions also apply when the fiscal authority and the central bank both are benevolent and jointly choose fiscal and monetary policy.

The government wants to trade non-state contingent bonds to smooth the impact of fiscal shocks on public consumption (see condition (2.26)). The marginal revenues obtained by borrowing more today are given by Δ_q. Due to lack of commitment, they do not equal average revenues $\lambda q_N(b',\tau)+(1-\lambda)q_R(b',\tau)$. The reason for this is that bond prices respond to the amount of borrowing because expected inflation depends on next period's debt position b'. This effect is captured by the derivatives $\partial q_N(b',\tau)/\partial b'$ and $\partial q_R(b',\tau)/\partial b'$ and is internalized by the fiscal authority when choosing end-of-period debt b'. Note that without default, $\partial q_R(b',\tau)/\partial b' = 0$ holds.

In a stationary Markov-perfect equilibrium, current and future inflation rates are governed by the same policy functions, reflecting that, in each period, inflation is chosen in the same way, given the aggregate state. For the current period, condition (2.27) depicts the trade-off involved when setting the optimal inflation rate without commitment. When the government inherits positive nominal debt λb, it wants to reduce real debt payments to free resources for public spending (LHS). The optimal inflation rate equates these marginal benefits of inflation to the marginal costs of inflation $\psi_\pi(\pi)$. Since the government optimizes sequentially, it does not internalize that an increase in π additionally affects the nominal bond price in the previous period in an adverse way. The failure to internalize this effect is the source of the time-inconsistency problem of monetary policy in the model. As the temptation to raise inflation increases with the nominal debt position λb, expected inflation is an increasing function of end-of-period debt b'. This implies that the elasticity of the nominal bond price schedule with respect to b' is negative, which tends to discourage the government from borrowing and impedes its ability to respond to (adverse) fiscal shocks by issuing bonds.

While nominal debt introduces a time-inconsistency problem that can increase the cost

[17]The derivation of these conditions can be found in Appendix 2.A.1.

of borrowing, it also has potential benefits. When only non-state contingent bonds can be issued, the debt contract does not specify future debt payments conditional on future fiscal shocks. Inflation offers a very flexible way of adjusting payments in response to fluctuating tax revenues, making nominal debt a potentially useful hedge against bad fiscal shocks.[18] This hedging property of nominal government debt is captured by the RHS of the Euler equation (2.26). When only real debt is issued ($\lambda = 0$), the (marginal) debt payment that the government will have to make in the next period Δ'_λ does not change with the realization of τ'. By contrast, when the public debt portfolio involves nominal debt ($\lambda > 0$), real payments (negatively) depend on future inflation π'. Since the government will tend to increase inflation in response to adverse fiscal shocks, i.e. when τ is low and $u_g(g)$ is high, the effective debt payment will decline exactly when public resources are scarce. Of course, this state-contingency of real debt payments will be anticipated by rational investors, who demand to be compensated for this inflation risk, and therefore comes at a cost.

Now consider the case without political frictions ($\theta = \mu = 1$) but with disagreement between the fiscal authority and the central bank ($\alpha \neq 1$). In this case, the first-order condition for the fiscal authority is given by

$$u_g(g)\Delta_q = \beta \mathbb{E}_{\tau'|\tau}\left[u_g(g')\Delta'_\lambda - \Delta'_\theta \frac{\partial \Pi(b',\tau')}{\partial b'}\right], \tag{2.28}$$

whereas the optimality condition for the central bank is given by (2.25).

The expressions Δ_α and Δ_θ measure the net marginal gains of inflation from the perspective of the central bank and the fiscal authority, respectively. If the fiscal authority and the central bank agree on the optimal inflation rate ($\alpha = 1/\theta = 1$), $\Delta_\alpha = \Delta_\theta = 0$ as well as (2.27) hold. If there is however disagreement about the optimal inflation rate ($\alpha \neq 1/\theta$), $\Delta_\alpha \neq \Delta_\theta$ holds and the two authorities use their policy instruments to strategically manipulate the policies chosen by the other authority. By comparing (2.28) to (2.26), one can see that disagreement about future inflation - as measured by Δ'_θ - introduces a wedge into the first-order condition (2.26), distorting public consumption smoothing.[19] The size of this wedge depends on $\partial\Pi(b',\tau')/\partial b'$, i.e. on the response of future inflation to an increase in borrowing. As argued above, this derivative tends to be positive which implies that if, from the perspective of the fiscal authority, the expected marginal benefits of inflation exceed the respective marginal costs ($\Delta'_\theta > 0$), the fiscal authority has an incentive to

[18] The hedging benefit of nominal government debt is discussed in detail by Bohn (1988). The hedging benefit of long-term debt relative to short-term debt is highlighted by Arellano and Ramanarayanan (2012) for a sovereign default model with real debt only.

[19] Similar wedges can be found in Niemann (2011) and Martin (2014).

increase borrowing to reduce the gap Δ'_θ. Similarly, the central bank has an incentive to use inflation to distort the borrowing decision of the fiscal authority (see condition (2.25)). In contrast to (2.27), the inflation choice now also involves intertemporal considerations because the central bank has an incentive to influence the borrowing decision of the fiscal authority in the current period via the inflation rate.

If, in addition to disagreement between the fiscal authority and the central bank ($\alpha \neq 1/\theta$), there are also political frictions ($\theta > 1$, $\mu < 1$), the first-order condition for the fiscal authority changes from (2.28) to (2.24). It can be thought of as a version of the GEE derived in Cuadra and Sapriza (2008) for the case with default, extended to incorporate monetary-fiscal policy interactions as in Niemann (2011) or Martin (2014). As in Cuadra and Sapriza (2008), the existence of political disagreement ($\theta > 1$) and turnover risk ($\mu < 1$) affects the borrowing decision of the fiscal authority via three effects.[20] The first effect is captured by the second term on the RHS of (2.24) and is referred to as "impatience effect" by Cuadra and Sapriza (2008). Because the incumbent party only stays in office with probability μ, it discounts the expected marginal costs of debt repayment more than without turnover risk. As a result, it is encouraged to front-load public consumption by borrowing more in the current period.

The third term on the RHS of (2.24) displays what Cuadra and Sapriza (2008) call the "disagreement effect". With probability $1-\mu$, the opposite party takes over office in the subsequent period. In this case, the implemented fiscal policy will be different from what the party currently in office would prefer since it will have a lower marginal valuation of the public good when it is not in power anymore. In the current period, the incumbent party then uses borrowing as a strategic device to manipulate future fiscal policy set by the opposite political party in case there is a change in power. More specifically, the incumbent party increases borrowing (or reduces savings) to leave less financial resources for the other party to spend on public spending in the next period.

The last two terms on the RHS of (2.24) capture the third effect by which political frictions affect the fiscal authority's borrowing decision. It shows that there is not only disagreement about future public spending - as captured by the "disagreement effect" above - but also about future borrowing. While the role of this effect for the borrowing decision of today's incumbent party is not clear ex ante, the two other effects tend to lead the fiscal authority to borrow more relative to a scenario without political frictions.

Having nominal and real debt in the model allows me to highlight the different implications that a more conservative central bank and a lower nominal debt share have for public policy. Although both of these changes tend to reduce the temptation to lower the

[20]The description of these effects follows Cuadra and Sapriza (2008, p. 84).

real debt burden via inflation, they do so in different ways. Whereas λ affects the gains of inflation, α impacts on the costs of inflation as perceived by the central bank. Obviously, setting $\alpha \to \infty$ or $\lambda = 0$ delivers the same allocation with full price stability since monetary policy does not respond to the debt position at all. For the remaining cases $\lambda \in (0,1]$ and $\alpha \in [0,\infty)$ however, public policy is affected differently.

To see this, consider the optimality conditions (2.26) and (2.27) associated with the benevolent government as the starting point. If monetary policy is delegated and $\alpha > 1$ holds, the temptation to use inflation for a given debt position declines. This effect comes however at the expense of disagreement between the (benevolent) fiscal authority and the monetary conservative central bank about the cost of inflation and thus about the optimal inflation rate, i.e. $\Delta_\theta > \Delta_\alpha$. This disagreement distorts the borrowing decision of the fiscal authority (see condition (2.28)) which then feeds back into decision of the central bank (see condition (2.25)). By contrast, a reduction in the nominal debt share λ reduces the incentive to resort to inflation by reducing the gains of inflation as perceived by both authorities, $u_g(g)\lambda\pi^{-2}b$. As a result, a reduction in λ does not have the negative side effects associated with an increase in α. With political frictions ($\theta > 1$), there always is disagreement between the fiscal authority and the central bank about the inflation rate for $\alpha \neq \alpha_\theta$ such that the differences between an increase in α and a decrease in λ are not as clear.

2.4 Quantitative Analysis

After having discussed the main forces of the model in the previous section, this section presents a quantitative analysis of the model's properties when the fiscal authority may default on its debt. Section 2.4.1 is concerned with model specification. Section 2.4.2 presents simulation results for different model versions. Section 2.4.3 evaluates the welfare properties of different monetary policy regimes. Appendix 2.A.2 provides computational details about the numerical solution of the model.

2.4.1 Model Specification

This section discusses how the model is specified.

Functional Forms

For the objective functions, an iso-elastic utility function

$$u(g) = \begin{cases} \frac{g^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln g & \text{if } \gamma = 1 \end{cases}$$

and quadratic inflation costs

$$\psi(\pi) = \frac{\chi}{2} (\pi - 1)^2, \ \chi > 0,$$

are used.[21]

Following Arellano (2008), I adopt an asymmetric specification for the resource costs of default:

$$\phi(\tau) = \max\{0, \tau - \tilde{\tau}\}.$$

This default cost specification implies that the resource costs of default increase overproportionally with tax revenues. As a result, default is particularly attractive in bad states, i.e. when tax revenues are low, which is a feature that is both intuitive and empirically plausible (see Tomz and Wright, 2007).

Finally, tax revenues follow a log-normal AR(1)-process:

$$\tau_t = \tau_{t-1}^{\rho} \exp(\sigma \varepsilon_t), \ \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1).$$

Parameters

The model is calibrated under the assumption that there is no central bank independence and monetary policy is directly set by the party currently in office, which is not benevolent due to $\theta > 1$ and $\mu < 1$. Section 2.4.2 will then look at how different monetary policy regimes affect public policy relative to this scenario. In particular, I will consider α-values relative to $\bar{\alpha} = 1/\theta$. If $\alpha = \alpha_\theta$, the central bank and the fiscal authority put the same relative weights on $u(g)$ and $\psi(\pi)$, such that the main source of disagreement between the two authorities is the present bias of the fiscal authority.

One model period corresponds to one quarter. The parameters are set as follows. For γ, a standard value of 2 is used. The value for the real risk-free rate $r_f = 0.017$ is taken from Arellano (2008). The probability of reentry δ is set to 0.1 as in Aguiar and Gopinath (2006). Following Cuadra and Sapriza (2008), the probability of remaining in power μ

[21] This inflation cost function implies that positive ($\pi > 1$) and negative ($\pi < 1$) deviations from full price stability are costly.

is set to 0.9. For the inflation cost parameter χ, the default cost parameter $\tilde{\tau}$ and the disagreement parameter θ, I use values of 1.53, 0.988 and 2.75 to match an annual default probability of roughly 1% and an average annual inflation rate of 20.68%. Loungani and Swagel (2001) list average annual inflation rates for 53 developing economies for the time period 1964-1998, documenting an average inflation rate of 16.4%. The focus of this chapter is on economies that experience persistently high inflation rates. When only economies that experienced inflation rates above 10 percent and below 50 percent are considered, the average inflation rate goes up to 20.68% for the sample.[22] An average annual default probability of 1% means that the economy defaults once in 100 years. This value implies that the government is not a notorious serial defaulter but that it defaults frequently enough for sovereign risk to matter for the public borrowing conditions. For the nominal debt share, I use an empirically plausible value of $\lambda = 0.58$.[23] For the tax revenue process, I set the persistence parameter ρ to 0.9 and calibrate the shock variance to match the volatility of log-government spending for Mexico, which is a typical emerging economy, resulting in the parameter value $\sigma = 0.022$.[24]

The household discount factor β is set to the investor discount factor $1/(1+r_f)$, implying that there is no long-run borrowing motive for the economy that is driven by its impatience relative to foreign investors. Most quantitative sovereign default models use discount factors that are much lower than $1/(1+r_f)$.[25] In these models, a high degree of impatience is needed to make the government accumulate debt levels that are sufficiently high to render default an attractive policy option. Such low discount factors can be motivated by referring to political economy distortions as modeled by Cuadra and Sapriza (2008). For a strictly positive analysis, it might not be of first-order importance to explicitly model the source of the government's impatience. A welfare analysis as performed in this chapter should however consider the possibility that a government borrows due to political frictions and not simply because its citizens are more impatient than foreign investors.

[22]There are 21 countries left in their sample that fit these criteria. Only Argentina (78.4%), Brazil (142.2%) and Peru (60.4%) experienced inflation rates above 50%.

[23]Du and Schreger (2015) list the share of external government debt that is issued in local currency for 14 emerging economies. In 2012, the average share for these economies was 57.99%.

[24]I use the quarterly time series for log real government expenditure provided Cuadra et al. (2010). The time series have been seasonally adjusted via EViews' multiplicative X-12 routine and filtered via the Hodrick-Prescott filter with a smoothing parameter of 1600. The calculated standard deviation for government expenditure is 0.03.

[25]For instance, Aguiar and Gopinath (2006) consider a quarterly discount factor of 0.8.

2.4.2 Simulation Results

The simulation results are shown in Table 2.1. It presents average statistics calculated for a panel of 2500 simulated economies with 2000 periods each, where the first 500 observations of each sample were discarded to eliminate the impact of initial conditions. The time series are filtered using the Hodrick-Prescott filter and a smoothing parameter of 1600. The baseline scenario corresponds to the model version without central bank independence. The main observations are that a higher degree of monetary conservatism α is associated with an increase in average debt and the default probability as well as a decline in inflation.

In a Markov-perfect equilibrium, the borrowing decision of the fiscal authority crucially depends on how elastic the bond price schedules $q_N(b', \tau)$ and $q_R(b', \tau)$ respond to changes in the level of future debt b' (see Figure 2.1). This bond price elasticity reflects the incentive to use inflation or default to reduce the real debt burden. As in Arellano (2008), a default is more attractive in adverse states, i.e. when tax revenues are low and/or debt is high. As a result, for such combinations the bond price schedules are lower and more responsive to debt issuance, reflecting an increase in the probability of default. Expected inflation is also higher in this case but default risk is the dominant force for bond pricing and therefore the borrowing decision. This changes when tax revenues are high and sovereign risk is low. Now, the nominal bond price schedule mostly reflects inflation risk while the real bond price hardly responds to b' at all (for low and intermediate borrowing levels). As debt increases, monetary policy will increase inflation to reduce the real debt burden, not internalizing how this choice affects borrowing costs in the previous period. When the degree of monetary conservatism is increased, the central bank is less tempted to use inflation to adjust debt payments which translates into a nominal bond price schedule that is less responsive to the level of borrowing. This in turn encourages the fiscal authority to borrow more in good times, leading to higher average debt, a decline in average inflation and an increase in the default frequency since the incentive to default increases with debt.[26] As discussed in Section 2.3, disagreement between the fiscal authority and the central bank about the marginal gains and costs of inflation might also matter for the fiscal authority's borrowing behavior. However, the contribution of this channel is difficult to assess quantitatively in the presence of lack of commitment to

[26] Aguiar et al. (2014) make a related argument in a model of a small open (endowment) economy without policy interaction between a fiscal and a monetary authority and (equilibrium) default. They also highlight the link between the incentive to use inflation, the elasticity of the nominal interest rate and the evolution of debt. Niemann (2011) also finds that increased monetary conservatism leads to increased debt accumulation in a model where the fiscal authority is myopic and does not internalize its effect on future policies. He also abstracts from sovereign default

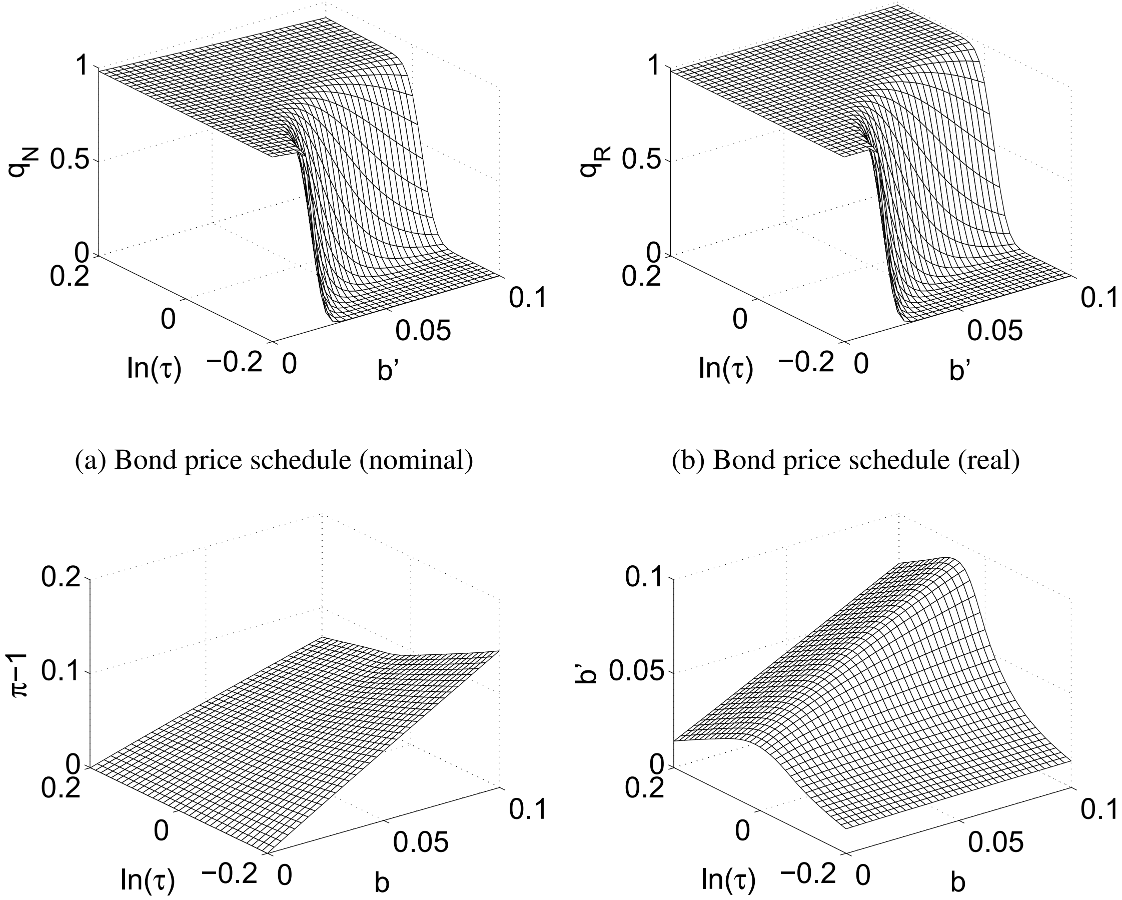
	Baseline	$\alpha = \alpha_\theta$	$\alpha = 2\alpha_\theta$	$\alpha = 3\alpha_\theta$	$\alpha = 10\alpha_\theta$
Avg. default prob. (annual)	0.0087	0.0085	0.0155	0.0175	0.0192
Mean(b/τ)	0.0531	0.0519	0.0654	0.0684	0.0708
Mean($\pi - 1$) (annual)	0.2095	0.2116	0.1322	0.0930	0.0291
Std(g)/Std(τ)	1.1041	1.0957	1.1956	1.2262	1.2614
Std(π)/Std(τ)	1.3399	1.3118	1.2152	0.9516	0.3263
Welfare measure ω (in %)	-	0.0005	0.0523	0.0789	0.1051

Table 2.1: Selected model statistics

debt repayment.

How does monetary conservatism affect the economy's vulnerability to a sovereign debt crisis? When the central bank is more conservative, one might expect that - for a given state (b, τ) - it will be more attractive for the fiscal authority to default since the central bank is less willing to reduce the real debt burden via inflation. However, this reasoning ignores that the fiscal authority will also face lower nominal interest rates for a given amount of debt issuance because the central bank's tougher monetary policy stance reduces expected inflation. These improved borrowing conditions might then encourage the fiscal authority not to default, reducing the likelihood of such an event for a given amount of debt. Figure 2.2 shows that the improvement in borrowing conditions indeed reduces the attractiveness of default. It depicts the bond price schedules as well as the inflation and borrowing policy functions for different degrees of monetary conservatism, given that tax revenues are at their unconditional mean. The changes in the probability of default can be observed by looking at the real bond price schedule which increases when the degree of monetary conservatism goes up. While monetary conservatism reduces the incentive to default for a given amount of debt, the economy still experiences more frequent default events because of the bond price elasticity channel described in the previous paragraph, which increases the average debt burden and as a result the fiscal authority's incentive to default.

When the central bank places the same relative weights on $u(g)$ and $\psi(\pi)$ as the fiscal authority ($\alpha = \alpha_\theta$), its main incentive to deviate from the policy chosen in the baseline scenario without central bank independence is to correct the deficit bias of the fiscal authority. To do so, it chooses a higher inflation rate for a given debt and tax revenue combination relative to the baseline scenario. This policy disciplines the fiscal authority's deficit bias in two ways. First, by reducing the real value of outstanding debt, the central bank reduces the fiscal authority's incentive to borrow by relaxing the government budget constraint. Second, since this policy implies a tighter link between the debt position and the inflation rate, the nominal bond price becomes more responsive to the debt position



(a) Bond price schedule (nominal)

(b) Bond price schedule (real)

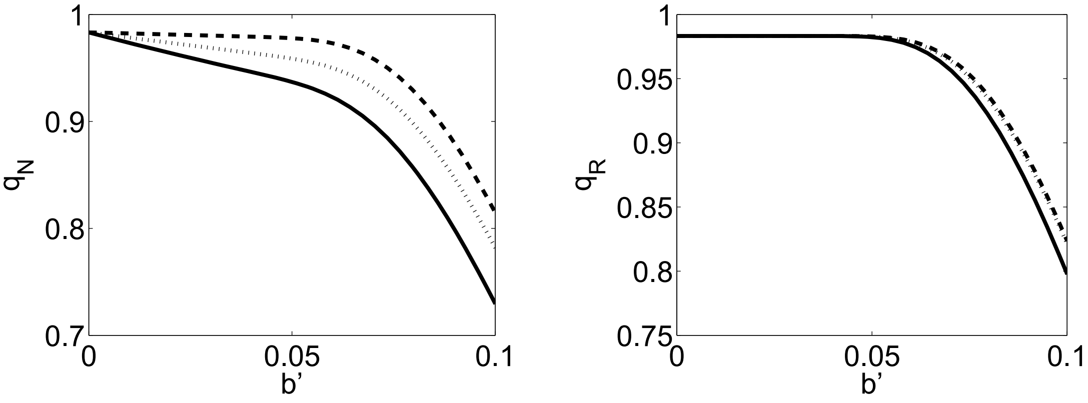
(c) Inflation rate

(d) Borrowing

Figure 2.1: Bond price schedules and policy functions for inflation and borrowing in the repayment case ($\alpha = \alpha_\theta$)

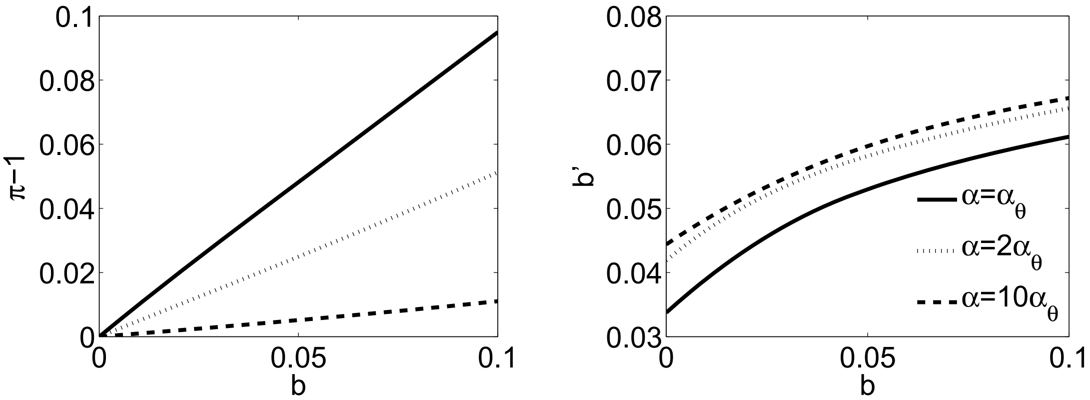
which additionally discourages the fiscal authority from issuing debt. Note that only the first effect is internalized by the central bank, whereas the second effect is unintended. As shown by Table 2.1, for $\alpha = \alpha_\theta$, the policy outcomes hardly differ compared to the baseline scenario. Inflation slightly increases and debt accumulation declines a little bit, leading to a small drop in the average default probability.

The degree of monetary conservatism also has important implications for the fiscal authority's ability to smooth government spending across states. When α is higher, a less responsive nominal bond price leads to a higher average debt burden, which in turn increases the likelihood of a debt crisis by making default more attractive. The increase in sovereign risk raises the volatility of fiscal policy since borrowing becomes more expensive in response to adverse shocks. The volatility of inflation can however successfully be reduced.



(a) Bond price schedule (nominal)

(b) Bond price schedule (real)



(c) Inflation rate

(d) Borrowing

Figure 2.2: Bond price schedules and policy functions for inflation and borrowing in the repayment case for different degrees of monetary conservatism ($\tau = \mathbb{E}[\tau]$)

Are the model implications presented in this section consistent with real world experiences? An extreme but useful example which suggests that it might be is offered by the recent Greek sovereign debt crisis that started in late 2009 (see Lane, 2012, for details). By joining the European Monetary Union, Greece has adopted an extreme version of central bank independence by delegating the control of monetary policy to the European Central Bank. In doing so, it has gained access to improved borrowing conditions since investors did not need to worry about inflation risk as much as before (see Aguiar et al., 2014). The Greek government then has taken advantage of the low nominal interest rate environment and experienced an increase in borrowing.[27] Eventually, this development

[27] As emphasized by Buiter and Sibert (2005), low sovereign bond yields might also have been the result of the European Central Bank's willingness to accept Greek government bonds as risk-free collateral for loans.

came to an end after bad fiscal shocks led to solvency problems in the aftermath of the financial crisis of 2008/09. While Greece is not an emerging economy, its recent history still suggests that the mechanism described in this chapter is an empirically plausible one.

2.4.3 Welfare Analysis

Given the results of the previous section, the welfare effects of increasing the degree of monetary conservatism are not obvious. While a higher value for α has the benefit of lowering the mean and variance of inflation, it also leads to a higher average debt burden and more frequent default events that are associated with temporary periods of costly autarky. In addition, the increased volatility of public spending will tend to have an adverse impact on household welfare as well.

To quantify the welfare implications of central bank independence, I calculate the welfare measure ω. It is the percentage increase in public consumption that households in an economy without central bank independence need to be given in each period to achieve the same welfare as in the respective economy with monetary conservatism of degree α, where household welfare is given by

$$\mathcal{U} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(g_t, \pi_t) \right],$$

with

$$U(g_t, \pi_t) = u(g_t) - \psi(\pi_t).$$

Table 2.1 shows that the computed values for ω are positive and increasing in α, with values ranging from almost zero ($\alpha = \alpha_\theta$) to 0.1% ($\alpha = 10\alpha_\theta$).[28] The benefits of monetary conservatism (lower and more stable inflation) thus outweigh the associated costs (more average debt, less average supply of the public good, more volatile fiscal policy), leading to a small net welfare gain. The welfare analysis implies that the optimal degree of monetary conservatism involves a central bank that does not respond to the state of the economy and implements a constant inflation rate ($\alpha \to \infty$). This result is consistent with the findings of Nuño and Thomas (2015) who also show that the gains of eliminating the time-inconsistency problem related to inflation dominate the costs of having a less flexible monetary policy. However, this chapter shows that the superiority of such an unresponsive monetary policy regime holds even when the fiscal authority is not benevolent and

[28] The unconditional expectation of discounted life-time utility $\mathcal{U}$ is calculated by computing the sum of discounted simulated utilities for 2000 periods and taking the average value over 2500 samples, where the first 500 observations are discarded for each sample to reduce the role of initial conditions.

subject to political economy considerations.

2.5 Conclusion

This chapter has studied the effectiveness and desirability of monetary conservatism in a quantitative model that accounts for three frictions that are important for many emerging economies: (i) incomplete financial markets, (ii) default risk, and (iii) political distortions. In the model, fiscal policy is set by a fiscal authority that cannot commit to future policy and exhibits a deficit bias. Monetary policy is chosen by a central bank that also lacks commitment and might care more about inflation than the fiscal authority and society. The chapter has shown that the delegation of monetary policy to an inflation-averse central bank successfully reduces average inflation but is associated with a higher average debt burden, more frequent debt crises and more volatile fiscal policy. A welfare analysis has shown that the costs associated with these adverse effects are outweighed by the benefits of lower and more stable inflation.

2.A Appendix

2.A.1 First-Order Conditions for the Policy Problems

I will first cover the decision problem of the central bank and then derive the first-order condition associated with the fiscal policy problem.[29] Before doing so, I introduce the notation $s \equiv (b, \tau)$ and $\hat{s} \equiv (\pi, s)$.

GEE for the Central Bank

For an interior solution, the first-order condition for the central bank's problem is

$$\frac{\partial \hat{\mathcal{M}}(\hat{s})}{\partial \pi} = 0,$$

or

$$u_g(g)\frac{\partial \hat{\mathcal{G}}(\hat{s})}{\partial \pi} - \alpha \psi_\pi(\pi) + \beta \mathbb{E}_{\tau'|\tau}\left[\frac{\partial \mathcal{M}(s')}{\partial b'}\right]\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi} = 0. \tag{2.29}$$

For the central bank, the value $\mathcal{M}(s)$ satisfies

$$\mathcal{M}(s) = u(\mathcal{G}(s)) - \alpha \psi(\Pi(s)) + \beta \mathbb{E}_{\tau'|\tau}\left[\mathcal{M}(\mathcal{B}(s), \tau')\right].$$

Differentiating $\mathcal{M}(s)$ with respect to b yields

$$\frac{\partial \mathcal{M}(s)}{\partial b} = u_g(g)\frac{\partial \mathcal{G}(s)}{\partial b} - \alpha \psi_\pi(\pi)\frac{\partial \Pi(s)}{\partial b} + \beta \mathbb{E}_{\tau'|\tau}\left[\frac{\partial \mathcal{M}(s')}{\partial b'}\right]\frac{\partial \mathcal{B}(s)}{\partial b}. \tag{2.30}$$

By using the first-order condition (2.29) to replace $\beta \mathbb{E}_{\tau'|\tau}[\partial \mathcal{M}(s')/\partial b']$ in condition (2.30), one obtains

$$\frac{\partial \mathcal{M}(s)}{\partial b} = u_g(g)\frac{\partial \mathcal{G}(s)}{\partial b} - \alpha \psi_\pi(\pi)\frac{\partial \Pi(s)}{\partial b} - \left(u_g(g)\frac{\partial \hat{\mathcal{G}}(\hat{s})}{\partial \pi} - \alpha \psi_\pi(\pi)\right)\frac{\frac{\partial \mathcal{B}(s)}{\partial b}}{\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi}}.$$

By using the conditions

$$\frac{\partial \hat{\mathcal{G}}(\hat{s})}{\partial \pi} = \lambda \pi^{-2} b + \Delta_q \frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi}, \tag{2.31}$$

$$\frac{\partial \mathcal{G}(s)}{\partial b} = \lambda \pi^{-2} b \frac{\partial \Pi(s)}{\partial b} - \Delta_\lambda + \Delta_q \frac{\partial \mathcal{B}(s)}{\partial b}, \tag{2.32}$$

[29] The derivations are similar as in Cuadra and Sapriza (2008) and Niemann et al. (2013b) who derive GEEs for related models with political frictions or monetary-fiscal policy interactions (see Section 2.1 for details).

which are derived by differentiating the government budget constraint with respect to π and b, this condition can further be rewritten as

$$\begin{aligned}\frac{\partial \mathcal{M}(s)}{\partial b} &= u_g(g)\left(\lambda\pi^{-2}b\frac{\partial \Pi(s)}{\partial b} - \Delta_\lambda + \Delta_q\frac{\partial \mathcal{B}(s)}{\partial b}\right) - \alpha\psi_\pi(\pi)\frac{\partial \Pi(s)}{\partial b} \\ &\quad - \left(u_g(g)\left(\lambda\pi^{-2}b + \Delta_q\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi}\right) - \alpha\psi_\pi(\pi)\right)\frac{\frac{\partial \mathcal{B}(s)}{\partial b}}{\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi}},\end{aligned}$$

or

$$\frac{\partial \mathcal{M}(s)}{\partial b} = -u_g(g)\Delta_\lambda + \Delta_\alpha\left(\frac{\partial \Pi(s)}{\partial b} - \frac{\frac{\partial \mathcal{B}(s)}{\partial b}}{\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi}}\right). \tag{2.33}$$

As in Section 2.3, I use the definitions

$$\begin{aligned}\Delta_\alpha &\equiv u_g(g)\lambda\pi^{-2}b - \alpha\psi_\pi(\pi), \\ \Delta_\lambda &\equiv \lambda\pi^{-1} + 1 - \lambda, \\ \Delta_q &\equiv \lambda q_N(b',\tau) + (1-\lambda)q_R(b',\tau) + \left(\lambda\frac{\partial q_N(b',\tau)}{\partial b'} + (1-\lambda)\frac{\partial q_R(b',\tau)}{\partial b'}\right)b'.\end{aligned}$$

By eliminating $\partial\hat{\mathcal{G}}(\hat{s})/\partial\pi$ in (2.29) via (2.31), one obtains

$$\Delta_\alpha + u_g(g)\Delta_q\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi} + \beta\mathbb{E}_{\tau'|\tau}\left[\frac{\partial \mathcal{M}(s')}{\partial b'}\right]\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi} = 0. \tag{2.34}$$

After updating (2.33) one period ahead and using it to eliminate $\partial\mathcal{M}(s')/\partial b'$ in (2.34), one arrives at

$$0 = u_g(g)\Delta_q\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi} + \Delta_\alpha - \beta\mathbb{E}_{\tau'|\tau}\left[u_g(g')\Delta'_\lambda - \Delta'_\alpha\left(\frac{\partial \Pi(s')}{\partial b'} - \frac{\frac{\partial \mathcal{B}(s')}{\partial b'}}{\frac{\partial \hat{\mathcal{B}}(\hat{s}')}{\partial \pi'}}\right)\right]\frac{\partial \hat{\mathcal{B}}(\hat{s})}{\partial \pi},$$

which is the generalized Euler equation for the central bank presented in Section 2.3.

GEE for the Fiscal Authority

The first-order condition for the fiscal policy problem is given by

$$0 = \theta u_g(g)\Delta_q + \beta\mathbb{E}_{\tau'|\tau}\left[\mu\frac{\partial \mathcal{F}(s')}{\partial b'} + (1-\mu)\frac{\partial \mathcal{F}^*(s')}{\partial b'}\right]. \tag{2.35}$$

The value $\mathcal{F}(s)$ satisfies

$$\mathcal{F}(s) = \theta u(\mathcal{G}(s)) - \psi(\Pi(s)) + \beta \mathbb{E}_{\tau'|\tau} \begin{bmatrix} \mu \mathcal{F}(\mathcal{B}(s), \tau') \\ +(1-\mu) \mathcal{F}^*(\mathcal{B}(s), \tau') \end{bmatrix}.$$

Differentiating $\mathcal{F}(s)$ with respect to b yields

$$\frac{\partial \mathcal{F}(s)}{\partial b} = \theta u_g(g) \frac{\partial \mathcal{G}(s)}{\partial b} - \psi_\pi(\pi) \frac{\partial \Pi(s)}{\partial b} + \beta \mathbb{E}_{\tau'|\tau} \begin{bmatrix} \mu \frac{\partial \mathcal{F}(s')}{\partial b'} \\ +(1-\mu) \frac{\partial \mathcal{F}^*(s')}{\partial b'} \end{bmatrix} \frac{\partial \mathcal{B}(s)}{\partial b}. \quad (2.36)$$

Using the first-order condition (2.35), (2.36) can be written as

$$\frac{\partial \mathcal{F}(s)}{\partial b} = \theta u_g(g) \frac{\partial \mathcal{G}(s)}{\partial b} - \psi_\pi(\pi) \frac{\partial \Pi(s)}{\partial b} - \theta u_g(g) \Delta_q \frac{\partial \mathcal{B}(s)}{\partial b}.$$

When combined with (2.32), this expression can be written as

$$\begin{aligned} \frac{\partial \mathcal{F}(s)}{\partial b} &= \theta u_g(g) \left(\lambda \pi^{-2} b \frac{\partial \Pi(s)}{\partial b} - \Delta_\lambda + \Delta_q \frac{\partial \mathcal{B}(s)}{\partial b} \right) \\ &\quad - \psi_\pi(\pi) \frac{\partial \Pi(s)}{\partial b} - \theta u_g(g) \Delta_q \frac{\partial \mathcal{B}(s)}{\partial b}, \end{aligned}$$

which reduces to

$$\frac{\partial \mathcal{F}(s)}{\partial b} = \Delta_\theta \frac{\partial \Pi(s)}{\partial b} - \theta u_g(g) \Delta_\lambda, \quad (2.37)$$

when using the definition $\Delta_\theta \equiv \theta u_g(g) \lambda \pi^{-2} b - \psi_\pi(\pi)$ from Section 2.3. For the party currently not in office, the value $\mathcal{F}^*(s)$ satisfies

$$\mathcal{F}^*(s) = u(\mathcal{G}(s)) - \psi(\Pi(s)) + \beta \mathbb{E}_{\tau'|\tau} \begin{bmatrix} \mu \mathcal{F}^*(\mathcal{B}(s), \tau') \\ +(1-\mu) \mathcal{F}(\mathcal{B}(s), \tau') \end{bmatrix}.$$

Differentiating $\mathcal{F}^*(s)$ with respect to b yields

$$\frac{\partial \mathcal{F}^*(s)}{\partial b} = u_g(g) \frac{\partial \mathcal{G}(s)}{\partial b} - \psi_\pi(\pi) \frac{\partial \Pi(s)}{\partial b} + \beta \mathbb{E}_{\tau'|\tau} \begin{bmatrix} \mu \frac{\partial \mathcal{F}^*(s')}{\partial b'} \\ +(1-\mu) \frac{\partial \mathcal{F}(s')}{\partial b'} \end{bmatrix} \frac{\partial \mathcal{B}(s)}{\partial b}. \quad (2.38)$$

By rewriting the first-order condition (2.35), one obtains the expression

$$\beta \mathbb{E}_{\tau'|\tau} \left[\frac{\partial \mathcal{F}^*(s')}{\partial b'} \right] = \frac{1}{1-\mu} \left[-\theta u_g(g) \Delta_q - \beta \mu \mathbb{E}_{\tau'|\tau} \left[\frac{\partial \mathcal{F}(s')}{\partial b'} \right] \right]. \quad (2.39)$$

Inserting (2.39) into (2.38) yields

$$\begin{aligned}\frac{\partial \mathcal{F}^*(s)}{\partial b} &= u_g(g)\frac{\partial \mathcal{G}(s)}{\partial b} - \psi_\pi(\pi)\frac{\partial \Pi(s)}{\partial b} \\ &+ \left[\begin{array}{c} \frac{\mu}{1-\mu}\left[-\theta u_g(g)\Delta_q - \beta\mu\mathbb{E}_{\tau'|\tau}\left[\frac{\partial \mathcal{F}(s')}{\partial b'}\right]\right] \\ +(1-\mu)\beta\mathbb{E}_{\tau'|\tau}\left[\frac{\partial \mathcal{F}(s')}{\partial b'}\right]\end{array}\right]\frac{\partial \mathcal{B}(s)}{\partial b} \\ &= u_g(g)\frac{\partial \mathcal{G}(s)}{\partial b} - \psi_\pi(\pi)\frac{\partial \Pi(s)}{\partial b} - \mu\frac{\theta u_g(g)}{1-\mu}\Delta_q\frac{\partial \mathcal{B}(s)}{\partial b} \\ &+\beta\frac{1-2\mu}{1-\mu}\mathbb{E}_{\tau'|\tau}\left[\frac{\partial \mathcal{F}(s')}{\partial b'}\right]\frac{\partial \mathcal{B}(s)}{\partial b}.\end{aligned}$$

With (2.37), this expression can be written as

$$\begin{aligned}\frac{\partial \mathcal{F}^*(s)}{\partial b} &= u_g(g)\frac{\partial \mathcal{G}(s)}{\partial b} - \psi_\pi(\pi)\frac{\partial \Pi(s)}{\partial b} - \mu\frac{\theta u_g(g)}{1-\mu}\Delta_q\frac{\partial \mathcal{B}(s)}{\partial b} \\ &+\beta\frac{1-2\mu}{1-\mu}\mathbb{E}_{\tau'|\tau}\left[\Delta'_\theta\frac{\partial \Pi(s')}{\partial b'} - \theta u_g(g')\Delta'_\lambda\right]\frac{\partial \mathcal{B}(s)}{\partial b}.\end{aligned} \tag{2.40}$$

Updating (2.37) and (2.40) one period ahead and inserting the resulting expressions into the first-order condition (2.35) leads to

$$\begin{aligned}0 &= \theta u_g(g)\Delta_q + \beta\mu\mathbb{E}_{\tau'|\tau}\left[\Delta'_\theta\frac{\partial \Pi(s')}{\partial b'} - \theta u_g(g')\Delta'_\lambda\right] \\ &+\beta(1-\mu)\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c} u_g(g')\frac{\partial \mathcal{G}(s')}{\partial b'} - \psi_\pi(\pi')\frac{\partial \Pi(s')}{\partial b'} - \mu\frac{\theta u_g(g')}{1-\mu}\Delta'_q\frac{\partial \mathcal{B}(s')}{\partial b'} \\ +\beta\frac{1-2\mu}{1-\mu}\mathbb{E}_{\tau''|\tau'}\left[\Delta''_\theta\frac{\partial \Pi(s'')}{\partial b''} - \theta u_g(g'')\Delta''_\lambda\right]\frac{\partial \mathcal{B}(s')}{\partial b'}\end{array}\right].\end{aligned}$$

After rearranging this condition a little bit, one finally arrives at the generalized Euler equation for the fiscal authority:

$$\begin{aligned}0 &= \theta u_g(g)\Delta_q \\ &-\mu\beta\mathbb{E}_{\tau'|\tau}\left[\theta u_g(g')\Delta'_\lambda - \Delta'_\theta\frac{\partial \Pi(s')}{\partial b'}\right] \\ &+(1-\mu)\beta\mathbb{E}_{\tau'|\tau}\left[u_g(g')\frac{\partial \mathcal{G}(s')}{\partial b'} - \psi_\pi(\pi')\frac{\partial \Pi(s')}{\partial b'}\right] \\ &-\mu\beta\mathbb{E}_{\tau'|\tau}\left[\theta u_g(g')\Delta'_q\frac{\partial \mathcal{B}(s')}{\partial b'}\right] \\ &+(2\mu-1)\beta\mathbb{E}_{\tau'|\tau}\left[\beta\mathbb{E}_{\tau''|\tau'}\left[\theta u_g(g'')\Delta''_\lambda - \Delta''_\theta\frac{\partial \Pi(s'')}{\partial b''}\right]\frac{\partial \mathcal{B}(s')}{\partial b'}\right].\end{aligned}$$

2.A.2 Numerical Solution

The numerical solution algorithm extends the algorithm proposed by Hatchondo et al. (2010) for a standard sovereign default model as in Arellano (2008) to a setting with two optimizing authorities. The algorithm computes the policy and value functions $\mathcal{X}^r(b,\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M},\mathcal{B},\mathcal{G},\Pi\}$, and $\mathcal{X}^d(\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M},\mathcal{G},\Pi\}$. As in Hatchondo et al. (2010), I approximate these functions on discrete grids for debt and tax revenues, using cubic spline interpolation to allow for off-grid values of b and τ. To reduce the computational burden, I exploit the additive separability of the period objective function and reformulate the within-period interaction between the fiscal authority and the central bank without changing equilibrium outcomes.

Reformulated Model

Conditional on repayment, the problem of the central bank can also be written as

$$\mathcal{M}^r(b,\tau) = \max_{\pi \geq \pi_{min}} \left\{-\alpha\psi(\pi) + \tilde{\mathcal{M}}^r(a,\tau)\right\} \;\; s.t. \;\; a = \left(\lambda\pi^{-1} + 1 - \lambda\right) b.$$

The variable a combines the inflation rate π and the debt position b to a single intra-period state variable.[30] In equilibrium, the value functions for the repayment case then satisfy

$$\begin{aligned} \mathcal{M}^r(b,\tau) &= -\alpha\psi(\Pi^r(b,\tau)) + \tilde{\mathcal{M}}^r(\mathcal{A}^r(b,\tau),\tau), \\ \mathcal{F}^r(b,\tau) &= -\psi(\Pi^r(b,\tau)) + \tilde{\mathcal{F}}^r(\mathcal{A}^r(b,\tau),\tau), \\ \mathcal{F}^{*r}(b,\tau) &= -\psi(\Pi^r(b,\tau)) + \tilde{\mathcal{F}}^{*r}(\mathcal{A}^r(b,\tau),\tau), \end{aligned}$$

with

$$\mathcal{A}^r(b,\tau) = \left(\lambda\Pi^r(b,\tau)^{-1} + 1 - \lambda\right) b.$$

The fiscal policy problem now is given by

$$\tilde{\mathcal{F}}^r(a,\tau) = \max_{b'} \left\{ \begin{array}{c} \theta u(\tau - a + [\lambda q_N(b',\tau) + (1-\lambda) q_R(b',\tau)]\, b') \\ +\beta \mathbb{E}_{\tau'|\tau} \left[\begin{array}{c} \mu \mathcal{F}(b',\tau') \\ +(1-\mu)\mathcal{F}^*(b',\tau') \end{array} \right] \end{array} \right\}.$$

[30]For the case of i.i.d. tax revenues, a could also include τ. For persistent tax revenues however, the current value τ will be needed to form expectations about τ'.

The intra-period continuation values for the central bank and the political party not in office satisfy

$$\tilde{\mathcal{M}}^r(a,\tau) = \left\{u(\tilde{\mathcal{G}}^r(a,\tau)) + \beta\mathbb{E}_{\tau'|\tau}\left[\mathcal{M}(\tilde{\mathcal{B}}^r(a,\tau),\tau')\right]\right\},$$

and

$$\tilde{\mathcal{F}}^{*r}(a,\tau) = \left\{u(\tilde{\mathcal{G}}^r(a,\tau)) + \beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c}\mu\mathcal{F}^*(\tilde{\mathcal{B}}^r(a,\tau),\tau') \\ +(1-\mu)\mathcal{F}(\tilde{\mathcal{B}}^r(a,\tau),\tau')\end{array}\right]\right\}.$$

In the default case, the central bank solves

$$\mathcal{M}^d(\tau) = \max_{\pi\geq\pi_{min}}\left\{-\alpha\psi(\pi) + \tilde{\mathcal{M}}^d(\tau)\right\}.$$

It is obvious that the central bank is not able to affect the behavior of the fiscal authority in the default case since the inflation rate does not have an impact on the government budget constraint. Regardless of the state τ, the inflation policy then satisfies $\alpha\psi_\pi(\Pi^d(\tau)) = 0$. Government spending is given as $\mathcal{G}^d(\tau) = \tau - \phi(\tau)$. The value functions satisfy

$$\mathcal{M}^d(\tau) = \left\{\begin{array}{c} u(\mathcal{G}^d(\tau)) - \alpha\psi(\Pi^d(\tau)) \\ +\beta\mathbb{E}_{\tau'|\tau}\left[\delta\mathcal{M}(0,\tau') + (1-\delta)\mathcal{M}^d(\tau')\right]\end{array}\right\},$$

$$\mathcal{F}^d(\tau) = \left\{\begin{array}{c} \theta u(\mathcal{G}^d(\tau)) - \psi(\Pi^d(\tau)) \\ +\delta\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c}\mu\mathcal{F}(0,\tau') \\ (1-\mu)\mathcal{F}^*(0,\tau')\end{array}\right] \\ +(1-\delta)\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c}\mu\mathcal{F}^d(\tau') \\ (1-\mu)\mathcal{F}^{*d}(\tau')\end{array}\right]\end{array}\right\},$$

and

$$\mathcal{F}^{*d}(\tau) = \left\{\begin{array}{c} u(\mathcal{G}^d(\tau)) - \psi(\Pi^d(\tau)) \\ +\delta\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c}\mu\mathcal{F}^*(0,\tau') \\ (1-\mu)\mathcal{F}(0,\tau')\end{array}\right] \\ +(1-\delta)\beta\mathbb{E}_{\tau'|\tau}\left[\begin{array}{c}\mu\mathcal{F}^{*d}(\tau') \\ (1-\mu)\mathcal{F}^d(\tau')\end{array}\right]\end{array}\right\}.$$

Solution Algorithm

The numerical solution algorithm consists of the following steps:

1. Construct discrete grids for debt $[\underline{b},\overline{b}]$, tax revenues $[\underline{\tau},\overline{\tau}]$ and the intra-period state variable $[\underline{a},\overline{a}]$.

2. Choose initial values for the policy and value functions $\mathcal{X}^r_{start}(b,\tau)$ and $\mathcal{X}^d_{start}(\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M},\mathcal{G},\Pi\}$, at all grid points $(b,\tau) \in [\underline{b},\overline{b}] \times [\underline{\tau},\overline{\tau}]$.

3. Set $\mathcal{X}^j_{next} = \mathcal{X}^j_{start}$, $j \in \{r,d\}$ and fix an error tolerance ε.

   (a) For each grid point combination $(a,\tau) \in [\underline{a},\overline{a}] \times [\underline{\tau},\overline{\tau}]$, compute the policies $\tilde{\mathcal{B}}^r(a,\tau)$ and $\tilde{\mathcal{G}}^r(a,\tau)$ that solve the fiscal policy problem, and calculate the associated values $\tilde{\mathcal{X}}^r(a,\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M}\}$.

   (b) For each grid point combination $(b,\tau) \in [\underline{b},\overline{b}] \times [\underline{\tau},\overline{\tau}]$, compute the inflation rate $\Pi^r_{new}(b,\tau)$ that solves the monetary policy problem, and calculate the associated fiscal policies and values $\mathcal{X}^r_{new}(b,\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M},\mathcal{B},\mathcal{G}\}$.

   (c) For each revenue value $\tau \in [\underline{\tau},\overline{\tau}]$, compute the policies $\mathcal{G}^d_{new}(\tau)$ and $\Pi^d_{new}(\tau)$ that satisfy $\alpha\psi_\pi(\Pi^d_{new}(\tau)) = 0$ and $\mathcal{G}^d_{new}(\tau) = \tau - \phi(\tau)$, as well as the associated values $\mathcal{X}^d_{new}(\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M}\}$.

   (d) If $|\mathcal{X}^r_{new}(b,\tau) - \mathcal{X}^r_{next}(b,\tau)| < \varepsilon$ and $\left|\mathcal{X}^d_{new}(\tau) - \mathcal{X}^d_{next}(\tau)\right| < \varepsilon$, $\mathcal{X} \in \{\mathcal{F}, \mathcal{F}^*, \mathcal{M}, \mathcal{G}, \Pi\}$, for all grid point combinations, go to step 4, else set $\mathcal{X}^j_{next} = \mathcal{X}^j_{new}$, $j \in \{r,d\}$ and repeat step 3.

4. Take $\mathcal{X}^j_{new}(\cdot)$, $j \in \{r,d\}$, as approximations of the respective equilibrium objects in the infinite-horizon economy.

I use discrete grids with equidistant grid points. Since the grid for the intra-period state variable a is directly related to the debt grid, I set $[\underline{a},\overline{a}] = [\underline{b},\overline{b}]$. The asymmetric default costs lead to a kink at $\tau = \tilde{\tau}$ in $\mathcal{X}^d(\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M},\mathcal{G},\Pi\}$. To address this discontinuity in an approriate way, I partition the τ-grid for the default case into two parts as in Hatchondo et al. (2010). I choose an error tolerance of $\varepsilon = 10^{-5}$.

Following Hatchondo et al. (2010), I solve for the infinite-horizon limit of a finite-horizon model version. I thus first compute the value and policy functions for the final period problem where no borrowing decision is made and use the resulting objects as initial values $\mathcal{X}^j_{start}$, $j \in \{r,d\}$, for step 2. Note that in the final period, the central bank can effectively choose government spending g via the government budget constraint: $g = \tau - \left(\lambda\pi^{-1} + 1 - \lambda\right)b$.

For step 3a, the debt policy $\tilde{\mathcal{B}}^r(a,\tau)$ is computed via a global non-linear optimizer. First, the algorithm performs a grid search over a pre-defined grid for b'. Then, the solution to this grid search is used as an initial guess for the Nelder-Mead algorithm. The optimization step delivers values for $\tilde{\mathcal{M}}^r(\cdot)$, $\tilde{\mathcal{F}}^r(\cdot)$ and $\tilde{\mathcal{F}}^{*r}(\cdot)$ that are associated with the optimal debt and government spending response functions $\tilde{\mathcal{G}}^r(\cdot)$ and $\tilde{\mathcal{B}}^r(\cdot)$.

Given the response functions and continuation values obtained in step 3a, step 3b computes the inflation policy $\Pi^r_{new}(b,\tau)$ that solves the central bank problem for each grid point combination $(b,\tau) \in [\underline{b},\overline{b}] \times [\underline{\tau},\overline{\tau}]$. Using the calculated inflation policy $\Pi^r_{new}(b,\tau)$, the equilibrium fiscal policies and continuation values are computed by evaluating the functions $\tilde{\mathcal{X}}^r(a,\tau)$, $\mathcal{X} \in \{\mathcal{F},\mathcal{F}^*,\mathcal{M},\mathcal{B},\mathcal{G}\}$, at $a = \left(\lambda \Pi^r_{new}(b,\tau)^{-1} + 1 - \lambda\right) b$.

To accurately compute expected values for the optimization in step 3a, it is important to account for the discontinuity generated by the discrete default decision. To illustrate this, take a look at the continuation value for the central bank in the repayment case:

$$\mathbb{E}_{\tau'|\tau}\left[\mathcal{M}_{next}(b',\tau')\right] = \int_0^{\hat{\tau}(b')} \mathcal{M}^d_{next}(\tau') f_\tau(\tau'|\tau) d\tau' + \int_{\hat{\tau}(b')}^{\infty} \mathcal{M}^r_{next}(b',\tau') f_\tau(\tau'|\tau) d\tau',$$

where $f_\tau(\cdot)$ is the conditional probability density function for future tax revenues τ'. One can characterize the default decision of the fiscal authority via a threshold $\hat{\tau}(b)$ for tax revenues that satisfies $\mathcal{F}^r(b,\hat{\tau}(b)) - \mathcal{F}^d(\hat{\tau}(b)) = 0$. Given the debt position b, $\hat{\tau}(b)$ is the lowest τ-value for which the fiscal authority prefers to repay its debt. For $\tau < \hat{\tau}(b')$, the fiscal authority finds it optimal to default. I compute the default threshold $\hat{\tau}(b)$ via bisection method. Following Hatchondo et al. (2010), I use Gauss-Legendre quadrature nodes and weights to approximate the integrals above.

Chapter 3

Markovian Households

3.1 Introduction

During the last three decades, a large empirical and theoretical literature has evolved that stresses the importance of analyzing household decision making not just at the aggregate household level but also at the intra-household level.[1] Within this literature, several model environments have been developed that allow to study how the interaction between different household members shapes the allocation of household resources. Broadly, these models can be divided into two categories: cooperative and non-cooperative models. In both categories, the household decision process has mostly been studied in static environments. While there a few recent papers that consider cooperative households in a dynamic context, dynamic non-cooperative models of the household are still the exception.[2]

The contribution of this chapter is to develop a framework that allows to understand how cooperation and lack thereof affects the decision making of a two-person household (a couple) in a dynamic environment with endogenous labor supply (and uncertainty). More specifically, the model studies the consumption-savings problem of a household whose individual members cannot commit to future actions and might not cooperate. First, I characterize the basic properties of household decision making with and without cooperation. Then, I use a calibrated model version with incomplete financial markets and idiosyncratic income risk to explore the implications of lack of cooperation for precautionary savings, intra-household risk sharing and welfare.

[1] See Chiappori and Donni (2011), Chiappori and Mazzocco (2014) and Chiappori and Meghir (2015) for recent surveys of this literature.

[2] Examples of dynamic models that use a cooperative framework are Mazzocco (2007, 2008), Ligon (2011), Ortigueira and Siassi (2013). Examples of non-cooperative household models that study dynamic decision problems are Browning (2000), Konrad and Lommerud (2000) and Hertzberg (2012). This chapter is particularly related to the latter study. Details can be found at the end of this section.

In the model, both household members consume a (private) consumption good and supply labor, subject to a single joint household budget constraint. The household has access to a risk-less one-period bond that allows the transfer of resources across periods. The individual household members are infinitely-lived and exhibit spousal altruism, i.e. they care about the utility that their spouse derives from its own consumption and leisure. The household members may differ with respect to the wage rate that they earn on the labor market, their utility function and their degree of spousal altruism. The interaction between the household members is modeled as a Markov-perfect game (see Maskin and Tirole, 2001) with the household's asset position as the single endogenous state variable. In each period, the equilibrium choices of the household members are hence characterized by policy functions that only depend on the joint asset position. Since the Markov-perfect equilibrium concept is a refinement of the subgame-perfect equilibrium concept, it guarantees that the decisions of the household members are time consistent.[3] While the household members take the allocation of future consumption and working time as given, they fully understand that these actions will depend on future household asset holdings which they can affect via their current consumption and labor supply decisions.

I consider two cases for the household problem. In the first case, the household members agree on a joint household objective function that reflects their relative degree of altruism. Based on this objective, the two individuals cooperate and jointly decide about the intra-household allocation of consumption and working time, as well as savings. In the second case, the household members do not cooperate. Instead, the allocation of household resources is determined in a non-cooperative game played by the two individuals. More specifically, the household members choose their consumption and labor supply to maximize their own objective function, taking as given the decisions of the spouse.

For both cases, the equilibrium can be characterized by a set of intuitive optimality conditions. While the optimal labor supply conditions are standard and do not differ for the cooperative and non-cooperative household, the allocation of consumption within the household and over time is chosen differently. In the cooperative case, these decisions are associated with a sharing rule and a standard Euler equation. For a given level of savings, the sharing rule states that the ratio of the household members' marginal utilities of consumption is equal to the inverse of the ratio of their respective weights in the joint household objective. As in the collective model of the household proposed by Chiappori (1988, 1992), this sharing rule guarantees the efficient distribution of income within the household. The non-cooperative solution to the household problem does not involve an

[3]By requiring household members to only condition their decisions on the minimal payoff-relevant state, equilibria that involve reputational considerations based on trigger strategies with complex history-dependence are ruled out by construction.

explicit sharing rule but an additional Euler equation instead. There are therefore two Euler equations in this case, one for each of the two household members. Relative to the Euler equation associated with the cooperative household, these two Euler equations feature an additional term which can be interpreted as a wedge that distorts the consumption-savings trade-off. More specifically, this wedge reflects the household members' inability to control the decisions of their spouse as well as their lack of commitment and disagreement about the valuation of spousal consumption and labor supply.

Under the assumption that household members are homogeneous, i.e. they have the same preferences with respect to their own consumption and leisure, face the same wage rate and exhibit the same degree of altruism, I show analytically that the cooperative and the non-cooperative solutions to the household problem coincide when the household members are perfectly altruistic, i.e. when they place the same weight on their own utility and spousal utility. When the household members are imperfectly altruistic and place a lower weight on spousal utility than on their own, the non-cooperative household exhibits an undersaving (or overborrowing) bias relative to the cooperative case, which is captured by the wedge in the Euler equation. For a household member, saving increases the amount of resources available to the household in the next period. However, these additional resources will encourage the spouse to consume more and work less in the next period, leaving less resources for the saving household member to consume. When imperfectly altruistic, the household member thus has an incentive to save less and consume more in the current period. This savings distortion that is due to imperfect spousal altruism is very similar to the intertemporal distortion found in models with quasi-geometric discounting (see e.g. Laibson, 1997). In fact, as in previous work by Hertzberg (2012), I show that there is a direct relationship between the behavior of the non-cooperative household and that of a representative quasi-geometric household. More specifically, I show a direct link between the degree of spousal altruism and the short-run discount factor of the quasi-geometric household (see e.g. Laibson, 1997).[4] Using numerical examples, I show that the savings distortion remains when household members are heterogeneous and investigate how outcomes vary with the exact type of heterogeneity.

Given that lack of cooperation leads to undersaving when imperfectly altruistic household members do not cooperate, it is interesting to ask how costly the savings distortion is in terms of welfare. To answer this question in a quantitatively meaningful way, I study a version of the household problem with incomplete financial markets in which the two household members are both subject to idiosyncratic labor productivity shocks. When

[4]The relation between the results of this chapter and the ones obtained by Hertzberg (2012) will be discussed at the end of this section.

financial markets only provide partial insurance against idiosyncratic labor income risk, a role for precautionary savings emerges. In addition, when the household members' idiosyncratic income risks are not perfectly positively correlated, spousal labor supply adjustments are a useful instrument to share risks within the household and reduce the impact of adverse shocks on household consumption (see e.g. Ortigueira and Siassi, 2013). The main finding is that the savings distortion can substantially reduce precautionary savings of non-cooperative households relative to cooperative ones when the individual members are imperfectly altruistic. As a result, non-cooperative households rely more on labor supply adjustments to smooth consumption in response to bad shocks, making intra-household risk-sharing more important for these types of households relative to cooperative ones. However, non-cooperative households not only experience more volatile labor supply but also more volatile consumption compared to cooperative households since they have lower buffer stock savings. A welfare exercise reveals that the welfare costs of lack of cooperation are sizable for even modest deviations from perfect spousal altruism.

This chapter is related to the large and growing literature on multi-person households (see e.g. Chiappori and Donni, 2011). As mentioned earlier, despite its size, there are relatively few papers that study such households in a dynamic context (see Chiappori and Mazzocco, 2014, for details). Two exceptions that use an intertemporal version of the collective household model to study consumption-savings problems with endogenous labor supply as in this chapter are Mazzocco (2008), who estimates the preferences of the household members, and Ortigueira and Siassi (2013), who study the importance of intra-household risk sharing in a general equilibrium incomplete markets model.

Examples of non-cooperative two-person households in a dynamic context are Browning (2000), Konrad and Lommerud (2000) and Doepke and Tertilt (2014).[5] Browning (2000) studies a non-cooperative savings problem in a two-period model with two household members where the husband might not live as long as the wife. Konrad and Lommerud (2000) use an intertemporal model to study investment in human capital. In their two-stage model, the within-household allocation is determined non-cooperatively at the first stage and via Nash-bargaining at the second stage, with the non-cooperative outcome as the threat point. Doepke and Tertilt (2014) build a non-cooperative household model with human capital investment to study the impact of transfers to females on growth.

Another paper that develops a non-cooperative dynamic model is Hertzberg (2012), which is closely related to this chapter. The author also studies the consumption-savings

[5] Examples of studies that look at non-cooperative decision making in a static context are Lundberg and Pollak (1993), Konrad and Lommerud (1995), Chen and Woolley (2001), Lechene and Preston (2011) and Del Boca and Flinn (2012).

problem of a non-cooperative household with two (potentially) imperfectly altruistic individual members that lack commitment and relates it to the cooperative case with perfect commitment. However, there are several differences between our studies. First, his model environment is a deterministic one with a finite time horizon and exogenous labor income, whereas the one in this chapter features an infinite time horizon, endogenous labor supply and (for an application) uncertainty. Second, to analyze his model, he studies the subgame-perfect equilibrium of the non-cooperative game between the household members, while I study the interaction as a stationary Markov-perfect game. More specifically, he derives closed form solutions for the model that show how imperfect altruism leads to an undersaving bias. To derive these solutions, Hertzberg (2012) relies on specific functional forms for the utility functions of the household members. By contrast, I analyze the equilibrium of the non-cooperative game via time-invariant first-order conditions. In particular, I derive generalized Euler equations for the household members similar to Krusell et al. (2002), which demonstrate the existence of a savings wedge that distorts intertemporal decisions making. These Euler equations admit a very intuitive interpretation and allow me to illustrate the determinants of the savings distortion. In addition, I do not have to assume specific functional forms to show that non-cooperative households save less than cooperative ones when the household members are imperfectly altruistic. Methodologically, our two studies can thus be seen as complementary. Third, as in his paper, a direct link between the behavior of the non-cooperative household and a quasi-geometric representative household is established. Again, I do not need to assume specific functional forms for the utility functions to obtain this finding. A further difference between our studies is that I perform a welfare analysis of the costs of lack of cooperation in the absence of commitment by using a calibrated model with uncertainty, whereas he uses numerical examples to illustrate the welfare properties of his deterministic life-cycle model. These welfare comparisons can be viewed as complementary since Hertzberg (2012) highlights a life-cycle savings motive for the household, whereas I consider a precautionary savings motive.

Due to the link between non-cooperative two-person households and quasi-geometric representative households, this chapter is also related to the literature on quasi-geometric discounting. In particular, it relates to Harris and Laibson (2001), Krusell et al. (2002) and Chatterjee and Eyigungor (2014) who study the decision problem of an infinitely-lived representative household with quasi-geometric preferences.

The remainder of this chapter is organized as follows. In Section 3.2, I present the model and discuss the implications of lack of cooperation for household decision making. Section 3.3 introduces labor income risk into the model of Section 3.2 and investigates how non-cooperative interaction between the household members affects precautionary savings, intra-household risk sharing and welfare. Section 3.4 concludes.

3.2 Model

The model considers the dynamic decision problem of a two-person household. Time is discrete, starts in $t = 0$ and goes on forever. A household consists of two infinitely-lived individuals, one male (M) and one female (F), and will be interpreted as a married couple. Throughout the paper, I will not model household dissolution and assume that the individuals form a couple at period $t = 0$ and never break up. The objective of household member $i \in \{F, M\}$ is given by

$$\sum_{t=0}^{\infty} \beta^t U_i(c_{it}, c_{-it}, n_{it}, n_{-it}),$$

where

$$U_i(c_{it}, c_{-it}, n_{it}, n_{-it}) = u_i(c_{it}, n_{it}) + \theta_i u_{-i}(c_{-it}, n_{-it}).$$

The period objective function U_i is the sum of two parts.[6] The first part, u_i, is the utility that household member i derives from its own consumption c_i and labor supply n_i. This function satisfies $u_{c,i}, -u_{n,i} > 0$ and $u_{cc,i}, u_{nn,i} < 0$, where $u_{x,i}$ ($u_{xx,i}$) denotes the first (second) derivative of u_i with respect to the argument $x \in \{c, n\}$. The second part, $\theta_i u_{-i}$, reflects household member i's altruism towards household member $-i \equiv \{F, M\} \setminus i$, where the parameter $\theta_i > 0$ measures the degree of altruism.[7] For $\theta_i = 1$, i is perfectly altruistic and places the same weight on its "private" utility u_i and the utility u_{-i} that its spouse derives from consuming c_{-i} and working n_{-i}. For $\theta_i < 1$ ($\theta_i > 1$), individual i places a lower (higher) weight on the utility that its spouse $-i$ derives from its consumption and leisure than on its own private utility u_i. The two household members might differ from each other with respect to their wage rate $w_i > 0$, utility function u_i and altruism θ_i.[8] The wage rates w_i are exogenous and constant, i.e. the model is a partial equilibrium one. The

[6] Mazzocco (2008) uses the same preferences in the context of an intertemporal collective household model.

[7] In the literature, individual preferences that involve this type of altruism are sometimes also referred to as "caring preferences" (see e.g. Chiappori, 1992, or Chen and Woolley, 2001).

[8] I abstract from heterogeneity in discount factors since this case has already received a lot of attention in the literature (see e.g. Hertzberg, 2013, Jackson and Yariv, 2014, Schaner, 2015, and references therein) and is well understood. In particular, it is known that heterogeneous time preferences lead to a time-inconsistency problem even when a group of individuals decides collectively, i.e. under cooperation. In this chapter, I want to highlight a time-inconsistency problem that arises due to other factors and explore how it depends on household cooperation.

household does not face any uncertainty about the future.[9]

The household faces the joint period budget constraint

$$w_F n_{Ft} + w_M n_{Mt} + a_t (1 + r_t) = c_{Ft} + c_{Mt} + a_{t+1}. \tag{3.1}$$

It has access to a one-period bond a_{t+1} that yields a net return of r_{t+1} in the subsequent period. A negative asset position means that the household is in debt. As noted by Hertzberg (2012), for 2002, the General Social Survey documents that the majority of married couples in the United States (53.35%) share their financial resources (see Smith et al., 2011). The assumption of a joint household budget thus is an empirically plausible one. In addition, it is also an assumption that is common in the literature (see e.g. Mazzocco, 2008, and Ortigueira and Siassi, 2013).

I assume that the interest rate $r_{t+1} = r(a_{t+1})$ can vary with the savings position: $\partial r(a_{t+1})/\partial a_{t+1} \leq 0$. In particular, $r(a_{t+1})$ may decrease with a_{t+1} for negative asset values values, i.e. the interest rate increases with the size of the credit. The relationship between the interest rate and the asset position is exogenously imposed and might reflect e.g. the lender's marginal costs of monitoring and enforcing a loan. When making their decisions, the household members internalize that the interest rate changes with savings a_{t+1}. The debt-elastic interest rate schedule is introduced for two reasons. First, since I analyze a partial equilibrium model with exogenous prices, an interior and unique solution for the steady state asset level is not generally assured.[10] The debt-elasticity of the interest rate $r(a_{t+1})$ provides a mechanism that induces an interior and unique steady state even for given prices.[11] Second, for analytical and computational tractability, it is not feasible to introduce a standard debt limit $a_{t+1} \geq \underline{a}$ into the model. More specifically, I will need differentiability of the equilibrium objects for the analysis which does not necessarily hold when a debt limit is imposed. As will be shown in greater detail below, the debt-elastic interest rate will have very similar implications for the household savings decision as an ad hoc debt limit.

The interaction between the household members is modeled as a dynamic game. I assume that the household members act in a time consistent way and are not able to commit to actions beyond the current period. I follow Krusell et al. (2002) and restrict attention to stationary Markov-perfect equilibria. The strategies of the household members are thus assumed to be Markov, i.e. they are only conditioned on the minimal payoff-relevant

[9] Section 3.3 will introduce uncertainty about the household members' future labor productivities.

[10] This will even be true when $r_{t+1} = \bar{r}$ and $\beta = 1/(1+\bar{r})$, where $\bar{r}$ is a positive constant.

[11] Schmitt-Grohé and Uribe (2003) also consider a debt-elastic interest rate to induce stationarity in a model of a small open economy with a constant world interest rate.

state (see Maskin and Tirole, 2001), which is the joint asset position a_t in this case. The Markov-perfect equilibrium (MPE) is a refinement of the subgame-perfect (Nash) equilibrium and therefore ensures that the equilibrium strategies are time consistent. It also rules out the possibility of reputational considerations that are based on the use of trigger strategies. As already mentioned above, I follow Krusell et al. (2002) and restrict attention to stationary equilibria with differentiable policy and value functions. This restriction is made to facilitate the analysis by allowing the derivation of first-order conditions that intuitively highlight the main forces of the model. Since the game may in principle still feature many equilibria, I follow Krusell et al. (2002) and additionally restrict attention to the equilibrium that is the infinite-horizon limit of a finite-horizon version of the household problem.[12] Uniqueness of this particular equilibrium follows from a standard backward induction argument. I will first look at a household whose members optimize under full cooperation, and then consider the case of non-cooperative household members.

3.2.1 Household Problem under Cooperation

Under cooperation, the household members agree on an objective function for the household and jointly decide about current consumption and labor supply of both household members as well as savings without commitment. The literature on multi-person households typically refers to this case as the "collective model of the household" (see e.g. Chiappori, 1988, 1992). In the cooperative case, the household objective is given as

$$\sum_{t=0}^{\infty} \beta^t U\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right),$$

with period objective function

$$U\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right) = U_F\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right) + \mu U_M\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right),$$

where $\mu > 0$ is a constant relative Pareto weight. As in Browning et al. (2006), the period objective can be rewritten as

$$U\left(c_{Ft}, c_{Mt}, n_{Ft}, n_{Mt}\right) = \left(1 + \mu\theta_M\right) u_F(c_{Ft}, n_{Ft}) + \left(\mu + \theta_F\right) u_M(c_{Mt}, n_{Mt}).$$

When the household members agree on how to evaluate intra-household allocations, the household decision problem is equivalent to that of a utilitarian planner who assigns constant weights $1 + \mu\theta_M$ and $\mu + \theta_F$ to the utility functions u_F and u_M and chooses consump-

[12] Details about the (static) household problem in the final period are given in Appendix 3.A.3.

tion and labor supply of the household members without commitment. In the remainder, I will assume that $\mu = 1$, i.e. the weights placed on u_F and u_M only reflect the relative altruism of the household members. If e.g. $\theta_M > \theta_F$ holds, the household objective assigns a higher weight to u_F compared to u_M.

Given the focus on MPE, the household problem is formulated recursively.[13] In each period, the household solves

$$\max_{c_F,c_M,n_F,n_M} \sum\nolimits_{i\in\{F,M\}} (1+\theta_{-i})\, u_i(c_i,n_i) + \beta\mathcal{V}(a'(a,c_F,c_M,n_F,n_M)),$$

where savings a' are given by the period budget constraint

$$a' = w_F n_F + w_M n_M + a\,(1+r(a)) - c_F - c_M.$$

The continuation value $\mathcal{V}$ satisfies

$$\mathcal{V}(a) = \sum\nolimits_{i\in\{F,M\}} (1+\theta_{-i})\, u_i(\mathcal{C}_i(a),\mathcal{N}_i(a)) + \beta\mathcal{V}(\mathcal{A}(a)),$$

where $\mathcal{C}_F\,(a)$, $\mathcal{C}_M\,(a)$, $\mathcal{N}_F\,(a)$, $\mathcal{N}_M\,(a)$ and $\mathcal{A}\,(a)$ are the policy functions that determine consumption, labor supply and savings in the subsequent period.[14] These functions only depend on the beginning-of-period asset position. Since the household members cannot commit to future actions, they take these policies as given. However, they recognize that they can affect the future allocation via the asset position a'. In a stationary equilibrium, $c_i = \mathcal{C}_i\,(a)$ and $n_i = \mathcal{N}_i\,(a)$, $i \in \{F,M\}$, as well as $a' = \mathcal{A}\,(a)$ hold. The policy functions that govern future decisions thus coincide with the policy functions that determine the optimal decisions in the current period. The MPE for the household problem under cooperation is formally defined as follows[15]:

Definition 3. *A stationary Markov-perfect equilibrium for the household problem under cooperation is given by a set of functions* $\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M,\mathcal{V}\}$ *such that for all a,*

(i) $\{\mathcal{X}\,(a)\}_{\mathcal{X}\in\{\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M\}} = \arg\max_{c_F,c_M,n_F,n_M} \left\{ \begin{array}{l} \sum_{i\in\{F,M\}} (1+\theta_{-i})\, u_i(c_i,n_i) \\ +\beta\mathcal{V}(a'(a,c_F,c_M,n_F,n_M)) \end{array} \right\}$,

(ii) $\mathcal{V}(a) = \sum_{i\in\{F,M\}} (1+\theta_{-i})\, u_i(\mathcal{C}_i(a),\mathcal{N}_i(a)) + \beta\mathcal{V}(\mathcal{A}(a))$,

(iii) $a'(a,c_F,c_M,n_F,n_M) = w_F n_F + w_M n_M + a\,(1+r(a)) - c_F - c_M$,

[13] The time indices are hence dropped and a prime is used to denote variables of the next period.

[14] Note that the continuation value $\mathcal{V}$ of the household can only be written as a single value for the whole household since the individual members share the same discount factor.

[15] The definition of the equilibrium is formulated by using notation similar to that used in Niemann et al. (2013b) for a public policy problem.

(iv) $\mathcal{A}(a) = a'(a, \mathcal{C}_F(a), \mathcal{C}_M(a), \mathcal{N}_F(a), \mathcal{N}_M(a))$.

The equilibrium for the household decision problem under full cooperation between the two household members can be characterized by the conditions stated in the following proposition:

Proposition 1. *The Markov-perfect equilibrium for the household decision problem under cooperation satisfies the conditions*

$$\begin{aligned}
\frac{u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a))}{u_{c,M}(\mathcal{C}_M(a), \mathcal{N}_M(a))} &= \frac{1+\theta_F}{1+\theta_M}, && (3.2)\\
-u_{n,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) &= u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) w_i, i \in \{F, M\}, && (3.3)\\
u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) &= \beta u_{c,F}(\mathcal{C}_F(\mathcal{A}(a)), \mathcal{N}_F(\mathcal{A}(a))) && (3.4)\\
&\quad \times \left(1 + r(\mathcal{A}(a)) + \mathcal{A}(a)\frac{\partial r(a')}{\partial a'}\right), \\
\mathcal{A}(a) + \mathcal{C}_F(a) + \mathcal{C}_M(a) &= w_F \mathcal{N}_F(a) + w_M \mathcal{N}_M(a) + a(1 + r(a)), && (3.5)
\end{aligned}$$

for all a.

Proof. See Appendix 3.A.1.

Condition (3.2) states that the cooperative solution to the household problem allocates consumption and working time of the household members such that the ratio of the household members' marginal utility of consumption $u_{c,F}/u_{c,M}$ equals the inverse of the ratio of the respective utility weights $(1+\theta_F)/(1+\theta_M)$. In the literature, this condition is often referred to as a "sharing rule" (see e.g. Chiappori, 1992). If $\theta_i > \theta_{-i}$, the optimal intra-household allocation implies that the marginal utility of consumption is higher for i relative to $-i$. Condition (3.3) is a standard labor supply condition which requires that the marginal rate of substitution between consumption and labor supply of household member i equals the respective wage rate w_i. Condition (3.4) is the Euler equation for the household that governs how resources are allocated intertemporally. In principle, the set of equilibrium conditions also involves an Euler equation for household member M. However, this condition is redundant here since the sharing rule (3.2) implies that the growth rate of the marginal utility of consumption is equalized across household members. Condition (3.5) is simply the household budget constraint evaluated at the equilibrium policies. Given consumption and labor supply of the household members, this budget constraint determines equilibrium savings $a' = \mathcal{A}(a)$. Note that the exact values for the degree of altruism of the household members do not matter for the household allocation if $\theta_F = \theta_M$.

The equilibrium conditions (3.2)-(3.5) are standard in the literature that uses the collective household model in a dynamic context (see e.g. Mazzocco, 2008, or Ortigueira and Siassi, 2013). One exception is the term $r(a') + a' \times (\partial r(a')/\partial a')$ that appears in the Euler equation (3.4), measuring the (assumed) marginal effect of a' on borrowing costs.[16] However, the debt-elastic interest rate has a very similar effect on the saving behavior of the household as an ad hoc borrowing constraint $a' \geq \underline{a}$ that prevents the household from reducing its asset position below the level $\underline{a}$. Define the "risk-free rate" $\bar{r} = r(0)$. Just like the debt-elastic interest rate, a binding borrowing constraint $a' = \underline{a}$ would also imply that $u_{c,i} \geq \beta u'_{c,i}(1+\bar{r})$ holds (see Ortigueira and Siassi, 2013). In both cases, the household will not borrow beyond a certain level, either because of being outright rationed or due to an interest rate schedule that increases with debt.

When the household members agree on a joint household objective function and collectively make their decisions based on it, the implemented outcome does not depend on whether the household can commit to future actions or not, i.e. the optimal cooperative household allocation under commitment is time consistent (see Appendix 3.A.2). Since the optimal household allocation under commitment is Pareto optimal with respective to the joint household objective function, the cooperative solution without commitment is Pareto optimal as well, making the cooperative household solution a useful benchmark for the non-cooperative case.

3.2.2 Household Problem without Cooperation

Now assume that the individual household members do not cooperate. The interaction between the household members is modeled as a non-cooperative simultaneous-move game. In each period, the household members F and M simultaneously choose consumption c_i and labor supply n_i, taking as given the decisions of their spouse c_{-i} and n_{-i}. In addition, they also take as given the policy functions that determine the allocation of household resources in the next period $\mathcal{X}(a')$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{N}_F, \mathcal{C}_M, \mathcal{N}_M\}$, given savings a'.

From the perspective of household member i, the joint household budget constraint is

$$a' = w_i n_i + w_{-i} n_{-i} + a(1 + r(a)) - c_i - c_{-i}. \tag{3.6}$$

Given the consumption and labor supply decisions of both household members, savings a' are determined residually to satisfy the budget constraint. The household members however internalize the impact of their own decisions on household savings. The decision

[16] To economize on notation, I will occasionally exploit stationarity and write a' instead of $\mathcal{A}(a)$.

problem of household member $i \in \{F,M\}$ is given by

$$\max_{c_i,n_i} u_i(c_i,n_i) + \theta_i u_{-i}(c_{-i},n_{-i}) + \beta \mathcal{V}_i(a'(a,c_i,c_{-i},n_i,n_{-i})), \tag{3.7}$$

where spousal consumption c_{-i} and labor supply n_{-i} are taken as given, and savings a' are given by the budget constraint (3.6).

The continuation value $\mathcal{V}_i$ is defined recursively as

$$\mathcal{V}_i(a) = u_i(\mathcal{C}_i(a),\mathcal{N}_i(a)) + \theta_i u_{-i}(\mathcal{C}_{-i}(a),\mathcal{N}_{-i}(a)) + \beta \mathcal{V}_i(\mathcal{A}(a)). \tag{3.8}$$

The definition of the MPE for the non-cooperative household problem is as follows:

Definition 4. *A stationary Markov-perfect equilibrium for the household problem without cooperation is given by a set of functions $\{\mathcal{A},\mathcal{C}_F,\mathcal{C}_M,\mathcal{N}_F,\mathcal{N}_M,\mathcal{V}_F,\mathcal{V}_M\}$ such that for all a,*

(i) $\{\mathcal{X}(a)\}_{\mathcal{X}\in\{\mathcal{C}_i,\mathcal{N}_i\}} = \arg\max_{c_i,n_i} \left\{ \begin{array}{c} u_i(c_i,n_i) + \theta_i u_{-i}(\mathcal{C}_{-i}(a),\mathcal{N}_{-i}(a)) \\ +\beta \mathcal{V}_i(a'(a,c_i,\mathcal{C}_{-i}(a),n_i,\mathcal{N}_{-i}(a))) \end{array} \right\}, i \in \{F,M\},$

(ii) $\mathcal{V}_i(a) = u_i(\mathcal{C}_i(a),\mathcal{N}_i(a)) + \theta_i u_{-i}(\mathcal{C}_{-i}(a),\mathcal{N}_{-i}(a)) + \beta \mathcal{V}_i(\mathcal{A}(a)),\ i \in \{F,M\},$

(iii) $a'(a,c_F,c_M,n_F,n_M) = w_F n_F + w_M n_M + a(1+r(a)) - c_F - c_M,$

(iv) $\mathcal{A}(a) = a'(a,\mathcal{C}_F(a),\mathcal{C}_M(a),\mathcal{N}_F(a),\mathcal{N}_M(a)).$

The key difference relative to Definition 3 is that the two household members now independently make their consumption and working decisions based on their own objective, with the joint household budget constraint providing the link between the individual household members' actions. Condition (i) requires that the strategies chosen by the household members in equilibrium are indeed optimal responses to each other, i.e. the policy functions form a Nash equilibrium. The equilibrium conditions for the non-cooperative household problem are given by the following proposition:

Proposition 2. *The Markov-perfect equilibrium for the household decision problem without cooperation satisfies the conditions (3.3),(3.5), and*

$$\begin{aligned} u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a)) &= \beta u_{c,i}(\mathcal{C}_i(\mathcal{A}(a)),\mathcal{N}_i(\mathcal{A}(a))) \\ &\times \left(1 + r(\mathcal{A}(a)) + \mathcal{A}(a)\frac{\partial r(a')}{\partial a'} + \Gamma_i(\mathcal{A}(a))\right),\ i \in \{F,M\}, \end{aligned} \tag{3.9}$$

with

$$\Gamma_i(a) = \left(\theta_i \frac{u_{c,-i}\left(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)\right)}{u_{c,i}\left(\mathcal{C}_i(a), \mathcal{N}_i(a)\right)} - 1\right)\left[\frac{\partial \mathcal{C}_{-i}(a)}{\partial a} - w_{-i}\frac{\partial \mathcal{N}_{-i}(a)}{\partial a}\right], \tag{3.10}$$

for all a.

Proof. See Appendix 3.A.1.

The optimal labor supply conditions are the same for the non-cooperative and the cooperative household problem. The important difference is that the ratio $u_{c,F}/u_{c,M}$ now is not pinned down by a sharing rule (see condition (3.2)) but by two generalized Euler equations instead. In contrast to the cooperative case, two intertemporal optimality conditions thus matter for the equilibrium outcome. Following Krusell et al. (2002), the Euler equations are called generalized due to the presence of the function Γ_i which involves derivatives of the policy functions $\mathcal{C}_{-i}$ and $\mathcal{N}_{-i}$ with respect to the asset position.

As shown in Appendix 3.A.2, the non-cooperative solution to the household problem under commitment features the same equilibrium conditions as in the case without commitment given by Proposition 2, except that Γ_i does not appear in the Euler equations for the commitment case. The function Γ_i can thus be interpreted as a wedge that distorts the optimal consumption-savings decision of household member i relative to the commitment case. In contrast to the cooperative household problem, lack of commitment therefore matters when the household members do not cooperate.

The wedge admits an intuitive interpretation. To see this, it is helpful to multiply Γ_i with $u_{c,i}$, which yields

$$\left(\theta_i u_{c,-i}\left(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)\right) - u_{c,i}\left(\mathcal{C}_i(a), \mathcal{N}_i(a)\right)\right) \times \left[\frac{\partial \mathcal{C}_{-i}(a)}{\partial a} - w_{-i}\frac{\partial \mathcal{N}_{-i}(a)}{\partial a}\right].$$

The expression in squared brackets measures the change in household resources that a marginal increase in wealth a induces by changing the consumption and working decisions of household member $-i$. The expression in round brackets measures the marginal valuation of this change in resources from the perspective of household member i. Suppose that $\partial \mathcal{C}_{-i}(a)/\partial a > 0$ and $\partial \mathcal{N}_{-i}(a)/\partial a = 0$, i.e. there is no wealth effect on labor supply and a marginal increase in assets a only leads to an increase in consumption of household member $-i$, leaving its labor supply decision unchanged. The first term in round brackets, $\theta_i u_{c,-i}$, is the marginal increase in utility that individual i derives from the increase in spousal consumption due to its altruism. The second term, $-u_{c,i}$, is the marginal decrease in utility that household member i experiences because a higher value of c_{-i} reduces the amount of resources that i can spend on its own consumption. The term in round brackets thus measures the net marginal change in utility of household member i associated with a change in available resources given by the term in squared brackets. If the two terms in round brackets sum up to zero, the wedge Γ_i disappears from the Euler equation. If the marginal valuation does not equal zero, the wedge Γ_i will deviate from zero as well and distort the consumption-savings trade-off relative to the commitment case.

When the household members are heterogeneous, it is difficult to make a clear statement about how the wedges Γ_F and Γ_M affect the behavior of the household in detail. Therefore, the next section will consider homogeneous household members, i.e. household members which share the same attributes $x_i = x_{-i} = x$ for $x \in \{\theta, u, w\}$, which implies that $\mathcal{X}_i(a) = \mathcal{X}_{-i}(a) = \mathcal{X}(a)$ for $\mathcal{X} \in \{\mathcal{C}, \mathcal{N}\}$ in a symmetric equilibrium. Under this assumption, one can isolate the role of lack of cooperation for household decision making. The case of heterogeneous household members will further be examined in Section 3.2.5.

3.2.3 Homogeneous Household Members

When the household members share the same attributes, the equilibrium for the cooperative and the non-cooperative household problem can be characterized by the following lemma:

Lemma 1. *If household members are homogeneous, i.e. $x_i = x_{-i} = x$ for $x \in \{\theta, u, w\}$, the Markov-perfect equilibrium for the household decision problem satisfies the conditions*

$$u_c(\mathcal{C}(a), \mathcal{N}(a))w = -u_n(\mathcal{C}(a), \mathcal{N}(a)), \tag{3.11}$$

$$u_c(\mathcal{C}(a), \mathcal{N}(a)) = \beta u_c(\mathcal{C}(\mathcal{A}(a)), \mathcal{N}(\mathcal{A}(a))) \times \left(1 + r(\mathcal{A}(a)) + \mathcal{A}(a)\frac{\partial r(a')}{\partial a'} + \Gamma(\mathcal{A}(a))\right), \tag{3.12}$$

with

$$\Gamma(a) = \begin{cases} (\theta - 1)\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right], & \textit{without cooperation,} \\ 0, & \textit{with cooperation,} \end{cases} \tag{3.13}$$

as well as

$$\mathcal{A}(a) + 2\mathcal{C}(a) = 2w\mathcal{N}(a) + a(1 + r(a)), \tag{3.14}$$

for all a.

Proof. Without differences in altruism, wage rates and utility functions $\mathcal{X}_F = \mathcal{X}_M = \mathcal{X}$ holds for $\mathcal{X} \in \{\mathcal{C}, \mathcal{N}\}$, which implies that the first-order conditions for F and M coincide. As a result, only one of the two optimal labor supply conditions and (for the non-cooperative household solution) one Euler equation is needed to pin down the equilibrium outcome. For the cooperative household solution, the sharing rule (3.2) becomes redundant as it is always satisfied for homogeneous household members.

As summarized by the following corollary, perfect altruism and the absence of within-household heterogeneity imply that the equilibrium outcome does not depend on the household members' ability to cooperate.

Corollary 1. *If household members are homogeneous and perfectly altruistic ($\theta = 1$), the outcome of the MPE under cooperation coincides with the outcome of the MPE without cooperation.*

Proof. If $\theta = 1$, $\Gamma(a) = 0$ holds regardless of whether household members cooperate or not. The equilibrium conditions (3.11)-(3.14) then are identical for the household decision problems with and without cooperation.

In contrast to the cooperative solution to the household problem, the exact value of the degree of altruism θ matters for the non-cooperative solution if $\theta \neq 1$. In particular, θ governs the magnitude (and the sign) of the savings distortion given by the wedge Γ_i. To understand the savings distortion, suppose that $\partial \mathcal{C}(a)/\partial a > 0$ as well as $\partial \mathcal{N}(a)/\partial a \leq 0$ hold.[17] With $u_c > 0$, it then follows from (3.13) that $\Gamma(a) \lesseqqgtr 0$, if $\theta \lesseqqgtr 1$. If individual household members cannot commit to future actions and do not cooperate, imperfect altruism ($\theta < 1$) hence leads a two-person household to save less (or borrow more) relative to the case of perfect altruism ($\theta = 1$) and thus relative to the cooperative case (see Corollary 1). The presence of imperfect altruism effectively lowers the marginal return from saving for a household member. While an additional unit of resources transfered into the next period yields the marginal return $1 + r(a') + a' \times (\partial r(a')/\partial a')$, it will also encourage the spouse to consume more ($\partial \mathcal{C}(a')/\partial a' > 0$) and work less ($\partial \mathcal{N}(a')/\partial a' \leq 0$), which tends to reduce the resources available for the saving household member to consume. If the household member is perfectly altruistic ($\theta = 1$), this change in future spousal consumption and labor supply is valued like a change in its own consumption and working time. As a result $\Gamma = 0$ holds and the Euler equation has the same shape as in the cooperative case. If the household member is imperfectly altruistic ($\theta < 1$), the increase in future spousal consumption and labor supply is valued less than the decline in utility due to the decrease in available resources, leading to a negative wedge $\Gamma < 0$. In the current period by contrast, the household member can directly allocate financial resources towards its

[17] For standard utility functions that are used in the literature, these properties usually hold.

own consumption. As a result, it has an incentive to reduce savings and allocate more wealth into the present, where it can directly consume the resources itself.[18]

The intuition behind the savings distortion of the non-cooperative household is similar to that in political economy models of public debt (see Persson and Svensson, 1989, and Alesina and Tabellini, 1990), where turnover risk and disagreement between political parties result in a deficit bias. In these models, the incumbent party has an incentive to front-load public spending since it might not be in charge of allocating resources in future periods and disagrees with the way these resources are spent by its successor. Aguiar and Amador (2011) and Chatterjee and Eyigungor (2014) demonstrate that there is in fact a direct link between the behavior of a government in such a political economy model and the behavior of a benevolent policy maker who exhibits quasi-geometric discounting (see Laibson, 1997, or Krusell et al., 2002). Similarly, as first demonstrated by Hertzberg (2012) for a finite-horizon model without labor supply, one can show a direct link between a two-person household which consists of non-cooperative, imperfectly altruistic individuals that discount geometrically and a representative household which discounts in a quasi-geometric fashion.

3.2.4 Imperfect Spousal Altruism and Quasi-Geometric Discounting

Consider the decision problem of a quasi-geometric household who consumes, works and saves (see e.g. Laibson, 1997, or Krusell et al., 2002).[19] In a given period $t \geq 0$, this household values the stream of current and future consumption and working time $\{c_{t+s}, n_{t+s}\}_{s=0}^{\infty}$ according to

$$u(c_t, n_t) + \delta \sum_{s=1}^{\infty} \beta^s u(c_{t+s}, n_{t+s}),$$

where u is the same period utility function used before, β a standard (long-run) discount factor and $\delta > 0$ the household's short-run discount factor. For $\delta = 1$, the household has standard time-consistent preferences. In this case, the household therefore does not have an incentive to deviate from a plan made in the past about future actions. If the short-run discount factor deviates from one, the household exhibits preference reversals over

[18] Another way to think about the undersaving (or overborrowing) bias is pointed out by Hertzberg (2012, p. 13) who proves the existence of such a bias for a finite-horizon setting with exogenous income (see Proposition 1 in his paper). When the household members share their financial wealth, savings are a public good and imperfect altruism leads the members to contribute less to the provision of this good than with perfect spousal altruism.

[19] In the literature, quasi-geometric discounting sometimes is also referred to as quasi-hyperbolic discounting. However, Krusell et al. (2002) point out that, mathematically speaking, quasi-geometric is the more appropriate term to describe this type of discounting.

time, making such commitments to future actions time inconsistent. More specifically, for $\delta < 1$, the household has an incentive to delay costly actions into future periods. In the context of the consumption-savings problem in this chapter, the household would like to delay (costly) saving today and commit to save more in the future. However, in the next period $t+1$, the household will again be tempted to delay saving, making a commitment to save more in the future time inconsistent. For $\delta > 1$, the household's temptation goes into the opposite direction, leading to an "oversaving bias". As pointed out by Laibson (1997), the decision problem of such a quasi-geometric household can be modeled as a dynamic game between multiple successive selves of the household.

To analyze this game, I follow Krusell et al. (2002) and study the stationary MPE that is the infinite-horizon limit of the MPE of the finite-horizon household problem.[20] In recursive notation, the decision problem is given as

$$\max_{c,n} u(c,n) + \delta\beta\mathcal{V}(a'(a,c,n)),$$

where savings a' are given via the period budget constraint

$$a' = wn + a(1+r(a)) - c,$$

and the continuation value $\mathcal{V}$ satisfies

$$\mathcal{V}(a) = u(\mathcal{C}(a),\mathcal{N}(a)) + \beta\mathcal{V}(\mathcal{A}(a)).$$

The MPE for the decision problem of the quasi-geometric household is formally defined as follows:

Definition 5. *A stationary Markov-perfect equilibrium for the decision problem of the quasi-geometric household is given by a set of functions* $\{\mathcal{A},\mathcal{C},\mathcal{N},\mathcal{V}\}$ *such that for all a,*

(i) $\{\mathcal{X}(a)\}_{\mathcal{X}\in\{\mathcal{C},\mathcal{N}\}} = \arg\max_{c,n}\{u(c,n) + \delta\beta\mathcal{V}(a'(a,c,n))\}$,

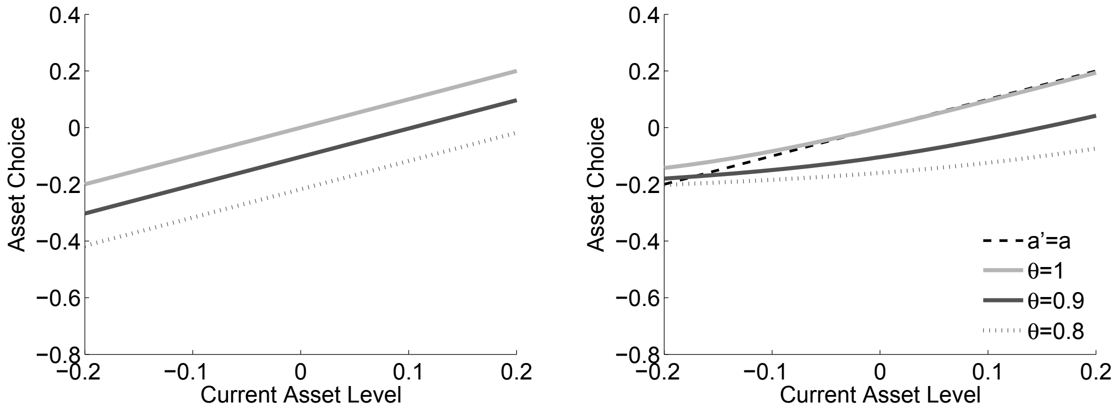
(ii) $\mathcal{V}(a) = u(\mathcal{C}(a),\mathcal{N}(a)) + \beta\mathcal{V}(\mathcal{A}(a))$,

(iii) $a'(a,c,n) = wn + a(1+r(a)) - c$,

(iv) $\mathcal{A}(a) = a'(a,\mathcal{C}(a),\mathcal{N}(a))$.

[20] In addition, I again assume differentiability of the policy and value functions. Bernheim et al. (2015) focus on subgame-perfect equilibria in general and study under which conditions quasi-geometric households can overcome their commitment problems by using different (non-Markov) strategies.

Figure 3.1: Savings policy function



(a) Constant interest rate ($\partial r/\partial a = 0$) (b) Debt-elastic interest rate ($\partial r/\partial a \leq 0$)

The next proposition summarizes the link between the quasi-geometric representative household and the non-cooperative two-person household:

Proposition 3. *If $\delta = \theta$, the Markov-perfect equilibrium for the decision problem of the quasi-geometric household can be characterized by the same conditions given by Lemma 1 for the non-cooperative household, except that $\mathcal{A}(a) + \mathcal{C}(a) = w\mathcal{N}(a) + a(1 + r(a))$ replaces condition (3.14).*

Proof. See Appendix 3.A.1.

The direct link between the short-run discount factor δ and the degree of altruism θ demonstrates that imperfect spousal altruism leads a non-cooperative two-person household to effectively discount in a quasi-geometric fashion even though its members exhibit standard geometric discounting. The only difference between the equilibrium conditions for the quasi-geometric household and those for the non-cooperative household is that consumption and labor supply are total household quantities in the former case and individual quantities in the latter case. The nature of the trade-offs is however the same. Hertzberg (2012) has previously established the direct link between the short-run discount factor and spousal altruism for a finite-horizon model without endogenous labor supply. In contrast to Hertzberg (2012), the derivation of this result in this chapter does not rely on a specific functional form for the utility function, a finite time-horizon and exogenous household income. However, Hertzberg (2012) can show that the non-cooperative household behaves in the aggregate like the representative quasi-geometric household, whereas I only show that the trade-off faced by the two types of households is very close.

An important implication of Proposition 3 is that the behavior of the non-cooperative two-person household exhibits the basic properties highlighted for (infinitely-lived) quasi-geometric households in the literature (see e.g. Krusell et al., 2002, Chatterjee and Eyigungor, 2014). In particular, when household members are imperfectly altruistic ($\theta < 1$) and the interest rate is constant by assumption, i.e. $r(a)$ is always debt-inelastic ($\partial r(a)/\partial a = 0$), there is no interior steady state a^* with $\mathcal{A}(a^*) = a^*$ unless the (constant) interest rate is given by $r = 1/\beta - 1 - \Gamma(a^*)$. If however, $\beta(1+r) = 1$ and $\theta < 1$ hold, the wedge $\Gamma(a)$ is negative and the non-cooperative household keeps on accumulating debt (see Figure 3.1a), leading consumption and/or leisure to sharply decline in the long run.[21] As shown by Figure 3.1b, the debt-elastic interest rate introduces a mechanism that discourages the persistent accumulation of debt and induces an interior steady state a^*.

3.2.5 The Role of Within-Household Heterogeneity

In Section 3.2.3, I have demonstrated that in the absence of within-household heterogeneity, lack of cooperation distorts household decision making when individual household members are imperfectly altruistic. I will now examine how heterogeneity between household members affects the behavior of the non-cooperative household.

Model Specification

To study the impact of within-household heterogeneity on non-cooperative household decision making, I use numerical examples. More specifically, I choose a baseline calibration for the model under the assumption that household members are homogeneous and then perform comparative statics with respect to a specific attribute x_i, with $x \in \{\theta, w\}$, leaving the other attributes unchanged. In particular, I will consider mean-preserving spreads for one specific attribute $x \in \{\theta, w\}$, keeping the average value constant across household members: $x_F = (1-\varepsilon)x$ and $x_M = (1+\varepsilon)x$, $\varepsilon \in [0, 1)$. I do not consider preference heterogeneity for the numerical exercises, i.e. $u_i = u_{-i} = u$.

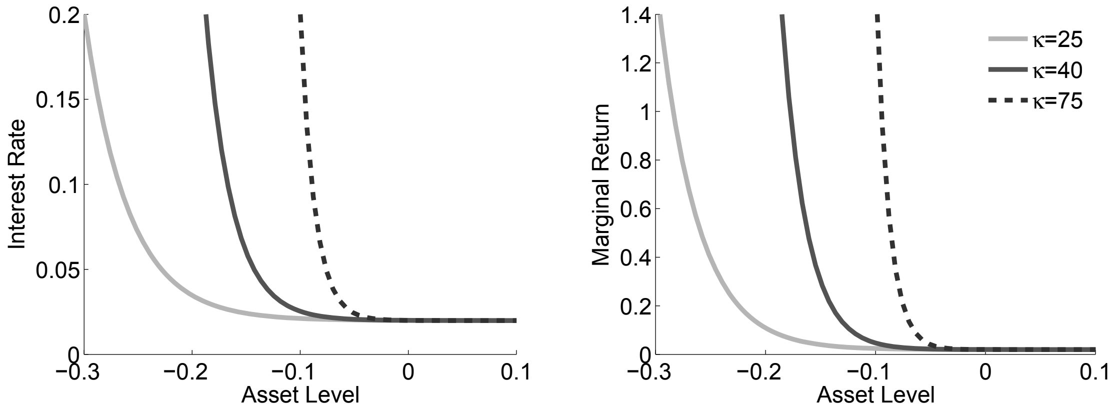
For the utility function, I use the specification

$$u(c, n) = \alpha \ln c + (1-\alpha) \ln(1-n).$$

A consumption share of $\alpha = 0.3$ is chosen to match a steady-state working time of one third for the cooperative household. The average wage rate w is normalized to one, i.e. $w_F = w_M = 1$ in the baseline scenario without heterogeneity.

[21] For the case $\theta > 1$, the opposite effect would lead to persistent accumulation of savings.

Figure 3.2: Interest rate schedule



(a) $r(a)$ (b) $r(a) + a \times (\partial r / \partial a)$

Following Schmitt-Grohé and Uribe (2003), the interest rate schedule is specified as

$$r(a) = \bar{r} + \psi \left(\exp\left(-\kappa a\right) - 1\right).$$

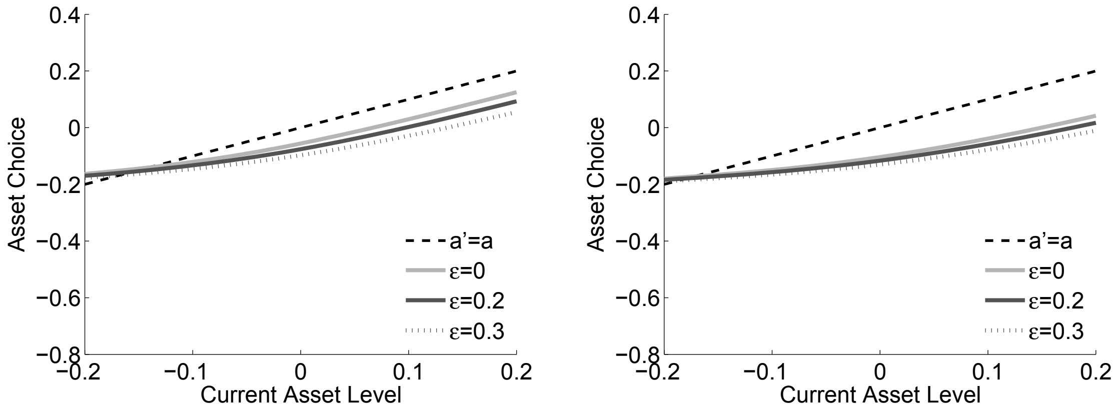
The parameters ψ and κ are set to 0.0001 and 25, whereas $\bar{r}$ is set to 0.02. The interest rate schedule $r(a)$ is shown in Figure 3.2 for different κ-values. If the household saves, the interest rate is virtually invariant to the asset position and increasing for negative asset values. The chosen parametrization for $r(a)$ ensures that the interest rate smoothly moves away from $\bar{r}$ for negative asset values. This property keeps the policy functions differentiable and makes the use of standard computational methods feasible.[22] The parameter κ governs the steepness of the interest rate schedule. For $\psi = 0$, the interest rate is always constant ($r(a) = \bar{r}$) and thus independent of the asset position. The discount factor β is set to $1/(1+\bar{r})$. The interest rate schedule $r(a)$ thus implies a steady-state asset value of zero for the cooperative household solution.

Degree of Altruism

For the non-cooperative case, Figure 3.3 depicts the savings policy function for different θ- and ε-values. The overall picture is that an increase in the spread ε tends to increase the household's incentive to borrow, leading to a lower (negative) steady-state asset value. The intra-household allocation of consumption and labor supply always reflects relative spousal altruism. As in the cooperative case, the ratio of the marginal utility of consump-

[22] Appendix 3.A.3 provides details on the numerical solution algorithm used in this chapter.

Figure 3.3: Savings policy function and differences in the degree of altruism θ_i



(a) $\theta = 0.95$

(b) $\theta = 0.9$

tion $u_{c,F}$ and $u_{c,M}$ is also given by the ratio $(1+\theta_F)/(1+\theta_M)$ in the non-cooperative case.[23] In contrast to the cooperative case, this outcome is not the result of an explicit sharing rule but determined by two Euler equations instead. Since savings differ for a given combination of θ_F and θ_M, consumption and labor supply are not the same in the cooperative and non-cooperative case unless $\theta_F = \theta_M = 1$.

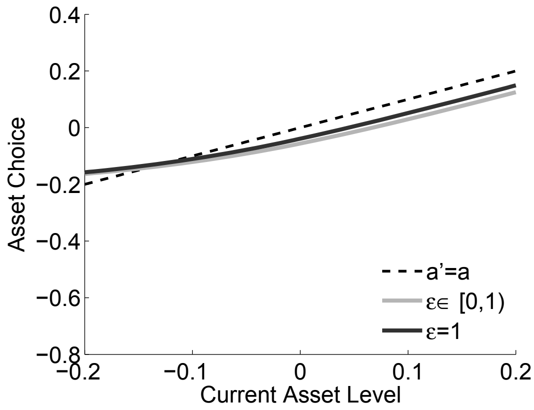
Wage Rate

It is well documented that female workers tend to receive a lower wage rate in the labor market than male workers. This phenomenon is also referred to as the ”gender pay gap” in the literature (see e.g. Blau and Kahn, 2000). Does this pay gap matter in the context of the household problem studied in this chapter? It turns out that it does not if $\varepsilon \in [0,1)$. Again, I consider mean-preserving spreads ε for the household members, such that $w_F = 1-\varepsilon$ and $w_M = 1+\varepsilon$ since the average wage rate has been normalized to one. While different spreads ε imply different expressions for Γ_i, the effect of these differences cancels out across household members, leaving the savings policy of the household and the ratio $u_{c,F}/u_{c,M}$ unaffected. Labor supply of both household members is however adjusted to the changed intra-household wage distribution.

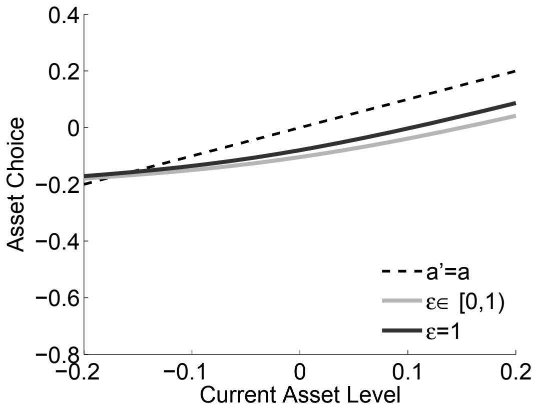
For $\varepsilon = 1$, household savings are affected relative to $\varepsilon \in [0,1)$. In this case, the labor supply condition does not hold with equality since the female household member finds it optimal not to participate in the labor market, i.e. $n_F = 0$. This non-participation leads to an asymmetry in the Euler equations for F and M that does not cancel out since the

[23]The properties highlighted in this paragraph are also shown by Hertzberg (2012) for his model.

Figure 3.4: Savings policy function and differences in the wage rate w_i



(a) $\theta = 0.95$



(b) $\theta = 0.9$

derivative $\partial \mathcal{N}_F(a)/\partial a$ is not present in Γ_M. As a result, the overborrowing bias is relaxed for M and savings are higher compared to $\varepsilon \neq 1$ (see Figure 3.4).

3.3 Household Problem with Labor Income Risk

In the deterministic model environment studied so far, the household savings motive was driven by the household's impatience, as given by the discount factor β, relative to $1/(1+\bar{r})$. To allow for a realistic evaluation of the quantitative and welfare implications of lack of cooperation, this section studies a calibrated version of the model presented in Section 3.2 with idiosyncratic labor income risk and incomplete financial markets. Financial markets only provide partial insurance against income fluctuations since the household can only trade a non-state contingent one-period bond a'. In addition, the debt-elastic interest rate schedule $r(a')$ makes it costly to borrow against future labor income in response to adverse productivity shocks. As a result, there now is a role for precautionary savings such that household savings are not only governed by the household's impatience. In contrast to most incomplete markets models (see Heathcote et al., 2009), the household does not face income risk at the "aggregate" household level.[24] Instead, the two household members are both subject to idiosyncratic, i.e. member-specific, labor productivity shocks that are not perfectly positively correlated. In this case, as highlighted e.g. by Attanasio et al. (2005), adjustment of spousal labor supply offers an additional insurance channel that allows the

[24] A notable exception within the incomplete markets literature are Ortigueira and Siassi (2013) who study the importance of intra-household risk sharing in a general equilibrium incomplete markets model with endogenous labor supply.

household to smooth consumption in response to idiosyncratic shocks.

With labor income risk, the household budget constraint is given by

$$a' = e_F w_F n_F + e_M w_M n_M + a\,(1 + r(a)) - c_F - c_M, \tag{3.15}$$

where e_i is the random labor productivity of household member $i \in \{F, M\}$. When $e_i w_i$ is defined as the effective wage rate of household member i, changes in productivity e_i can be interpreted as shocks to the wage rate. The household members are assumed to have perfect information about the productivity value of their spouse e_{-i}. Labor productivities e_i follow first-order Markov processes with discrete support $e_i \in \{e_1, ..., e_I\}$. I assume that the productivities might be correlated across household members. It will however be important that the shocks are not perfectly positively correlated. In such a case, the productivity shocks would have the same impact as a single "aggregate shock" at the household level and intra-household risk sharing via spousal labor supply would not be possible.

When the household members cooperate, the decision problem under uncertainty is given by

$$\max_{c_F, c_M, n_F, n_M} \sum\nolimits_{i \in \{F,M\}} (1 + \theta_{-i})\, u_i(c_i, n_i) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}(a'(a, e, c_F, c_M, n_F, n_M), e') \right],$$

where savings are given by the period budget constraint (3.15), $e = (e_F, e_M)$ summarizes the exogeneous household productivity state and $\mathbb{E}_{e'|e}[\cdot]$ denotes the conditional expectation operator. The value and policy functions now depend on asset holdings a as well as on the labor productivities of both household members e. The continuation value $\mathcal{V}$ satisfies

$$\mathcal{V}(a, e) = \sum\nolimits_{i \in \{F,M\}} (1 + \theta_{-i})\, u_i(\mathcal{C}_i(a, e), \mathcal{N}_i(a, e)) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}(\mathcal{A}(a, e), e') \right].$$

Without cooperation, household member $i \in \{F, M\}$ takes spousal consumption c_{-i} and labor supply n_{-i} as given and solves

$$\max_{c_i, n_i} u_i(c_i, n_i) + \theta_i u_{-i}(c_{-i}, n_{-i}) + \beta \mathbb{E}_{e'|e} \left[\mathcal{V}_i(a'(a, e, c_i, c_{-i}, n_i, n_{-i}), e') \right],$$

where savings are given by

$$a' = e_i w_i n_i + e_{-i} w_{-i} n_{-i} + a\,(1 + r(a)) - c_i - c_{-i},$$

and $\mathcal{V}_i$ satisfies

$$\mathcal{V}_i(a,e) = u_i(\mathcal{C}_i(a,e),\mathcal{N}_i(a,e)) + \theta_i u_{-i}(\mathcal{C}_{-i}(a,e),\mathcal{N}_{-i}(a,e)) + \beta \mathbb{E}_{e'|e}\left[\mathcal{V}_i(\mathcal{A}(a,e),e')\right].$$

The definitions of the MPE for the household problems under uncertainty are straightforward extensions of Definition 3 and Definition 4, and are therefore omitted here. With productivity shocks, the conditions that describe the MPE outcome for the cooperative and the non-cooperative household problem are also very similar to the ones in the model of Section 3.2. The only changes are that: (i) the wage rate w_i is replaced by the effective wage rate $e_i w_i$, (ii) the minimal payoff-relevant state now includes the individual productivity values summarized by e in addition to the asset position a, and (iii) the household members have to form expectations with respect to future productivities e', conditional on current productivities e.

In the remainder of this section, I will assume that the household members share the same attributes x_i, with $x \in \{\theta, u, w\}$, to isolate the impact of income risk on household behavior. In addition, the productivities e_F and e_M will follow the same Markov process. Importantly, the realizations of the idiosyncratic productivity shocks will not be perfectly positively correlated across household members. As a result, transitory differences between the household members will occur, generating the possibility of intra-household risk sharing.

3.3.1 Model Calibration

The model with labor income risk is specified as follows. In contrast to Section 3.2.5, I will now follow Ortigueira and Siassi (2013) and use the additively-separable utility function

$$u(c,n) = \alpha \frac{c^{1-\gamma}-1}{1-\gamma} + (1-\alpha)\frac{(1-n)^{1-\eta}-1}{1-\eta}, \alpha \in (0,1), \gamma, \eta > 0,$$

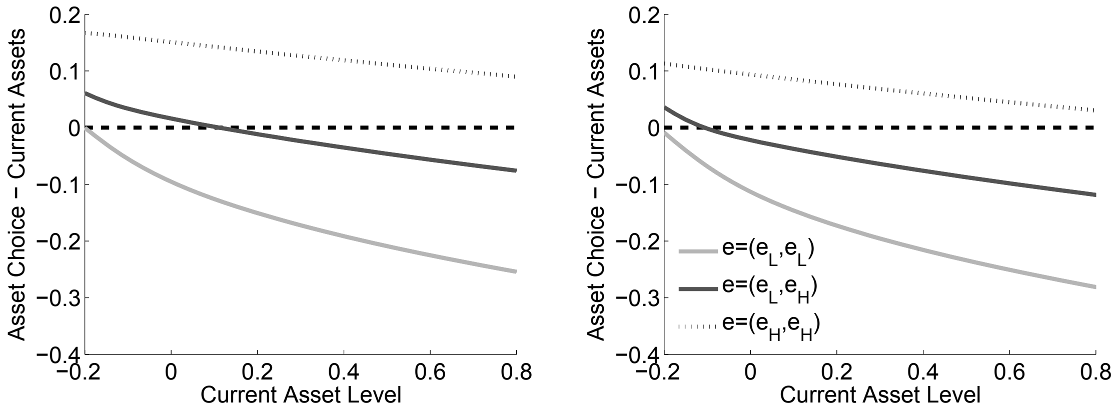
which is widely used in the incomplete markets literature.[25]

Following Ortigueira and Siassi (2013), I set the parameter γ to a standard value of 2 and η to a value of 3. The consumption share α is set to 0.4 to match an average working time of 36% (see Floden and Lindé, 2001), which implies a Frisch elasticity of roughly 60%.[26] For the interest rate schedule $r(a)$, I keep the parameter values from

[25]Of course, the utility function used in Section 3.2.5 is a special case of the one used here for $\gamma \to 1$ and $\eta \to 1$.

[26]For the chosen utility function, the Frisch elasticity of labor supply is $\frac{1}{\eta}\frac{1-n^*}{n^*}$, where n^* is the average working time of the (symmetric) household members (see Ortigueira and Siassi, 2013).

Figure 3.5: Savings policy function with labor income risk ($\theta = 0.9$)



(a) With cooperation

(b) Without cooperation

Section 3.2.5. A model period corresponds to one year. Labor productivity follows the log-normal AR(1)-process

$$e_{it} = e_{it-1}^{\rho} \exp(\sigma \varepsilon_{it}), \varepsilon_{it} \overset{i.i.d.}{\sim} \mathcal{N}(0,1).$$

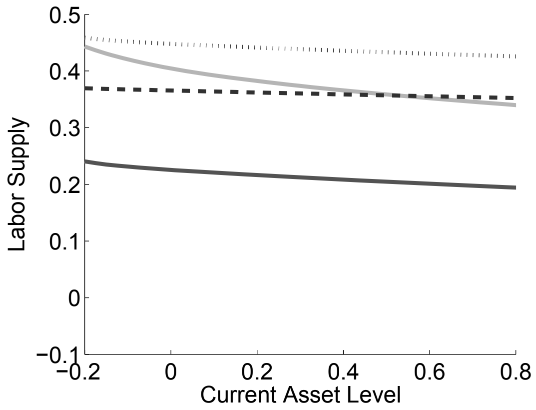
The productivity parameters ρ and σ are specified as in Floden and Lindé (2001). Using the Panel Study of Income Dynamics data set, they estimate the values $(\rho, \sigma) = (0.9136, 0.2064)$ based on wage data for the United States. I approximate the productivity process as a two-state Markov chain via the Rouwenhorst method as described in Kopecky and Suen (2010), assuming that labor productivity shocks are uncorrelated across household members. Labor productivity e_i can thus take on one of two values: $e_i \in \{e_L, e_H\}$, with $e_L < e_H$. Following Domeij and Floden (2006), the discount factor β is set to 0.95. This value satisfies the condition $\beta(1+\bar{r}) < 1$, which is standard in the incomplete markets literature to prevent households from accumulating assets without bound.[27]

3.3.2 Precautionary Savings and Imperfect Altruism

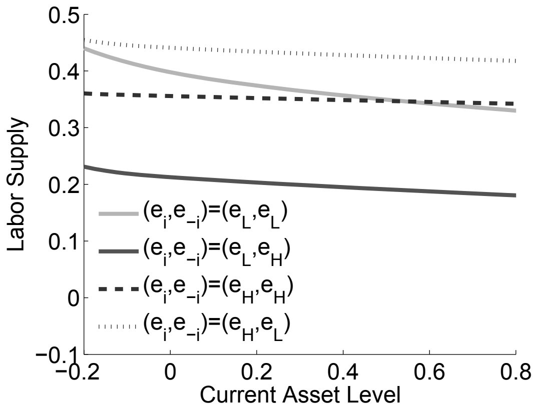
First, I assess how imperfect altruism affects precautionary savings. Figure 3.5 depicts the savings policy function for a cooperative household (left panel) and a non-cooperative one

[27]When labor supply is exogenous and $\beta(1+\bar{r}) = 1$ holds, the precautionary savings motive leads to the accumulation of infinitely high asset holdings in the long run (see e.g. Ljungqvist and Sargent, 2004). By contrast, Marcet et al. (2007) show that when labor supply is endogenous, household savings will be (possibly large but) finite even for $\beta(1+\bar{r}) = 1$.

Figure 3.6: Policy function for labor supply of household member i ($\theta = 0.9$)



(a) With cooperation



(b) Without cooperation

(right panel), assuming that the degree of altruism is $\theta = 0.9$ in both cases.[28] Since the household members are symmetric, the productivity states $e = (e_L, e_H)$ and $e = (e_H, e_L)$ lead to the same savings policy function. As probably expected given the results of Section 3.2, the non-cooperative household saves less in all possible states.

Table 3.1 lists the average assets of the non-cooperative household relative to its annual household labor income for different degrees of altruism. These values are calculated based on 75 million simulated model periods. When the household members cooperate or do not cooperate but are perfectly altruistic ($\theta = 1$), the average asset position is 36.52% of annual total household income. For the non-cooperative household, savings are already notably lower (29.26%) when the members only exhibit small deviations from perfect altruism ($\theta = 0.98$). The undersaving bias induced by imperfect spousal altruism and lack of cooperation thus tends to dominate the precautionary saving motive. When the degree of altruism is further reduced to $\theta = 0.90$, the household holds substantially less assets relative to the cooperative case (4.14%).

3.3.3 Intra-Household Risk Sharing

When household member i is hit by a low productivity shock, its spouse can work more to increase household earnings and thereby reduce the impact of i's bad labor market outcome on household consumption. This risk-sharing aspect of spousal labor supply can be seen in Figure 3.6 which displays the labor supply policy function for a household mem-

[28] When the household members share the same permanent attributes, the exact value of θ is irrelevant for the behavior of the cooperative household.

ber i. The interesting two cases are the states $e = (e_L, e_H)$ and $e = (e_H, e_L)$. Compared to the good state $e = (e_H, e_H)$, a bad shock to the labor productivity of household member $-i$ leads to an increase in i's working time, partially recovering $-i$'s income loss. If household member i is in a low productivity state, a bad shock to $-i$'s productivity leads to a much weaker response of i's labor supply. As shown by Figure 3.5, in this case, the household relies more on its savings (or borrowing) to smooth consumption. Since lack of cooperation increases the household's willingness to borrow (or dissave) in response to adverse shocks, its members need to work less for a given asset position and productivity state.

However, one should not infer from Figures 3.6 that the non-cooperative household smooths consumption in a more effective way compared to a cooperative one. To avoid large adjustments in consumption and leisure, the cooperative household accumulates large asset holdings that are used as a buffer stock in bad times. By contrast, the non-cooperative household tends to have lower asset holdings which then require larger changes in consumption and labor supply in response to bad shocks, increasing the volatility of consumption and labor supply.

3.3.4 The Welfare Cost of Lack of Cooperation

Given the impact of lack of cooperation on the behavior of a household, it is interesting to ask how much non-cooperative decision making matters in terms of welfare. To address this question, I calculate the welfare-equivalent permanent change in consumption ζ that the members of a non-cooperative household need to experience to achieve the same expected life-time utility as in the cooperative case. Formally, I solve for the ζ-value that satisfies

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U\left(c_{Ft}^C, c_{Mt}^C, n_{Ft}^C, n_{Mt}^C\right)\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U\left((1+\zeta)\,c_{Ft}^N, (1+\zeta)\,c_{Mt}^N, n_{Ft}^N, n_{Mt}^N\right)\right],$$

with

$$U\left(c_F, c_M, n_F, n_M\right) = (1+\theta_M)\,u_F(c_F, n_F) + (1+\theta_F)\,u_M(c_M, n_M),$$

where the sequences of consumption and labor supply for the members of a household with ($j = C$) and without cooperation ($j = N$) are denoted as $\{c_{Ft}^j, c_{Mt}^j, n_{Ft}^j, n_{Mt}^j\}_{t=0}^{\infty}$. Remember that $\theta_F = \theta_M = \theta$ holds. I calculate the unconditional expectation of life-time utility for the two household types by computing the sum of realized discounted utilities for 2500 periods and taking the average value over 50,000 samples. For each sample, the first 1000 observations are not used to reduce the impact of initial conditions.

	$\theta = 1$	$\theta = 0.98$	$\theta = 0.96$	$\theta = 0.94$	$\theta = 0.92$	$\theta = 0.90$
Average assets	36.52	29.26	22.35	15.84	9.75	4.14
Welfare measure	0	0.32	0.64	0.95	1.25	1.54

Table 3.1: Average asset holdings of the non-cooperative household (in % of total household income) and welfare measure ζ (in %) for different degrees of spousal altruism θ

The results are displayed in Table 3.1. The welfare cost of lack of cooperation ζ decreases monotonically with the degree of spousal altruism θ for $\theta < 1$. For small deviations from perfect altruism ($\theta = 0.98$), the calculated ζ-value is already notable (0.32%). For values lower than $\theta = 0.94$, ζ exceeds one percent, going up to 1.54% for $\theta = 0.9$. Lack of cooperation therefore entails sizable welfare losses for the household members for even modest deviations from perfect spousal altruism.

3.4 Conclusion

This chapter has studied the consumption-savings problem of a two-person household whose individual members cannot commit to future actions and might not cooperate. The interaction between the individual household members was modeled as a Markov-perfect game. Intuitive first-order conditions were derived that illustrate how lack of cooperation leads to a savings distortion relative to the case of full cooperation. More specifically, a non-cooperative household tends to save less (borrow more) than a cooperative one when its members exhibit imperfect spousal altruism. A calibrated model version with incomplete markets and idiosyncratic labor income risk was used to quantify the implications of lack of cooperation for precautionary savings and welfare. Even modest deviations from perfect altruism were shown to induce a decline in precautionary savings for the non-cooperative household, leading to substantial welfare losses relative to the cooperative case.

3.A Appendix

3.A.1 Proofs

This section contains the proofs of the propositions in the main text.

Proof of Proposition 1.

Under full cooperation of the household members, the first-order conditions for consumption c_i and labor supply n_i are

$$\begin{aligned}
(1+\theta_{-i})\, u_{c,i}(c_i,n_i)+\beta\frac{\partial\mathcal{V}(a')}{\partial a'}\frac{\partial a'}{\partial c_i} &= 0,\\
(1+\theta_{-i})\, u_{n,i}(c_i,n_i)+\beta\frac{\partial\mathcal{V}(a')}{\partial a'}\frac{\partial a'}{\partial n_i} &= 0,
\end{aligned}$$

with $i\in\{F,M\}$. Differentiating the budget constraint with respect to consumption c_i and labor supply n_i yields

$$\frac{\partial a'}{\partial c_i} = -1, \tag{3.16}$$

$$\frac{\partial a'}{\partial n_i} = w_i. \tag{3.17}$$

Using these derivatives, the first-order conditions can be written as

$$(1+\theta_{-i})\, u_{c,i}(c_i,n_i) = \beta\frac{\partial\mathcal{V}(a')}{\partial a'}, \tag{3.18}$$

$$-(1+\theta_{-i})\, u_{n,i}(c_i,n_i) = \beta\frac{\partial\mathcal{V}(a')}{\partial a'}w_i, \tag{3.19}$$

or combined as

$$-u_{n,i}(c_i,n_i)=u_{c,i}(c_i,n_i)w_i. \tag{3.20}$$

By combining the first-order conditions for c_F and c_M, one can derive the sharing rule

$$u_{c,F}(c_F,n_F)=\frac{1+\theta_F}{1+\theta_M}u_{c,M}(c_M,n_M). \tag{3.21}$$

Using

$$\mathcal{V}(a)=\textstyle\sum_{i\in\{F,M\}}(1+\theta_{-i})\, u_i(\mathcal{C}_i(a),\mathcal{N}_i(a))+\beta\mathcal{V}(\mathcal{A}(a)),$$

and

$$\mathcal{A}(a)=w_F\mathcal{N}_F(a)+w_M\mathcal{N}_M(a)+a(1+r(a))-\mathcal{C}_F(a)-\mathcal{C}_M(a),$$

the derivatives of $\mathcal{V}(a)$ and $\mathcal{A}(a)$ are given by

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i\in\{F,M\}} \left[\begin{array}{c}(1+\theta_{-i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{C}_i(a)}{\partial a} \\ +(1+\theta_{-i})\,u_{n,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{N}_i(a)}{\partial a}\end{array}\right] \\ &\quad +\beta\frac{\partial \mathcal{V}(\mathcal{A}(a))}{\partial a'}\frac{\partial \mathcal{A}(a)}{\partial a},\end{aligned} \tag{3.22}$$

and

$$\frac{\partial \mathcal{A}(a)}{\partial a} = \sum\nolimits_{i\in\{F,M\}} \left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right] + 1 + r(a) + a\frac{\partial r(a)}{\partial a}. \tag{3.23}$$

After combining (3.22) with (3.18) for $i=F$ and (3.23),

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i\in\{F,M\}} \left[\begin{array}{c}(1+\theta_{-i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{C}_i(a)}{\partial a} \\ +(1+\theta_{-i})\,u_{n,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\frac{\partial \mathcal{N}_i(a)}{\partial a}\end{array}\right] \\ &\quad +(1+\theta_M)\,u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))\left[\begin{array}{c}\sum_{i\in\{F,M\}}\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right] \\ +1+r(a)+a\frac{\partial r(a)}{\partial a}\end{array}\right],\end{aligned}$$

and using (3.20) to replace $u_{n,i}(c_i,n_i)$ with $-u_{c,i}(c_i,n_i)w_i$ for $i\in\{F,M\}$, one obtains

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i\in\{F,M\}} \left[(1+\theta_{-i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[\frac{\partial \mathcal{C}_i(a)}{\partial a} - w_i\frac{\partial \mathcal{N}_i(a)}{\partial a}\right]\right] \\ &\quad +(1+\theta_M)\,u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))\left[\begin{array}{c}\sum_{i\in\{F,M\}}\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right] \\ +1+r(a)+a\frac{\partial r(a)}{\partial a}\end{array}\right],\end{aligned}$$

This expression can further be rewritten by using (3.21) to rewrite

$$(1+\theta_M)\,u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))\sum\nolimits_{i\in\{F,M\}}\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right],$$

as

$$\sum\nolimits_{i\in\{F,M\}}\left[(1+\theta_{-i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right]\right],$$

which yields

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= \sum\nolimits_{i\in\{F,M\}} \left[(1+\theta_{-i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[\frac{\partial \mathcal{C}_i(a)}{\partial a} - w_i\frac{\partial \mathcal{N}_i(a)}{\partial a}\right]\right] \\ &\quad +\sum\nolimits_{i\in\{F,M\}} \left[(1+\theta_{-i})\,u_{c,i}(\mathcal{C}_i(a),\mathcal{N}_i(a))\left[w_i\frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a}\right]\right] \\ &\quad +(1+\theta_M)\,u_{c,F}(\mathcal{C}_F(a),\mathcal{N}_F(a))\left(1+r(a)+a\frac{\partial r(a)}{\partial a}\right),\end{aligned}$$

or
$$\frac{\partial \mathcal{V}(a)}{\partial a} = (1+\theta_M)\, u_{c,F}(\mathcal{C}_F(a), \mathcal{N}_F(a)) \left(1 + r(a) + a\frac{\partial r(a)}{\partial a}\right).$$

Updating this expression one period ahead and plugging it into (3.18) for $i = F$ yields the Euler equation for the cooperative household

$$u_{c,F}(c_F, n_F) = \beta u_{c,F}(\mathcal{C}_F(a'), \mathcal{N}_F(a')) \left(1 + r\left(a'\right) + a'\frac{\partial r(a')}{\partial a'}\right).$$

Proof of Proposition 2.

Household member $i \in \{F, M\}$ solves the decision problem (3.7), taking as given current spousal decisions c_{-i} and n_{-i} as well as next period's intra-household allocation given by $\mathcal{X}_i(a')$ and $\mathcal{X}_{-i}(a')$, with $\mathcal{X} \in \{\mathcal{C}, \mathcal{N}\}$. The optimal consumption and labor supply decisions of the household member satisfy

$$\begin{aligned} u_{c,i}(c_i, n_i) + \beta \frac{\partial \mathcal{V}_i(a')}{\partial a'} \frac{\partial a'}{\partial c_i} &= 0, \\ u_{n,i}(c_i, n_i) + \beta \frac{\partial \mathcal{V}_i(a')}{\partial a'} \frac{\partial a'}{\partial n_i} &= 0. \end{aligned}$$

Using the derivatives (3.16) and (3.17), the first-order conditions can be written as

$$u_{c,i}(c_i, n_i) = \beta \frac{\partial \mathcal{V}_i(a')}{\partial a'}, \tag{3.24}$$

$$-u_{n,i}(c_i, n_i) = \beta \frac{\partial \mathcal{V}_i(a')}{\partial a'} w_i, \tag{3.25}$$

or combined as

$$-u_{n,i}(c_i, n_i) = u_{c,i}(c_i, n_i) w_i. \tag{3.26}$$

Using (3.8) and

$$\mathcal{A}(a) = w_i \mathcal{N}_i(a) + w_{-i}\mathcal{N}_{-i}(a) + a(1 + r(a)) - \mathcal{C}_i(a) - \mathcal{C}_{-i}(a),$$

the derivatives of $\mathcal{V}_i(a)$ and $\mathcal{A}(a)$ with respect to a can be calculated:

$$\begin{aligned} \frac{\partial \mathcal{V}_i(a)}{\partial a} = {} & u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{C}_i(a)}{\partial a} + u_{n,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{N}_i(a)}{\partial a} \\ & + \theta_i u_{c,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \frac{\partial \mathcal{C}_{-i}(a)}{\partial a} + \theta_i u_{n,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} \\ & + \beta \frac{\partial \mathcal{V}_i(\mathcal{A}(a))}{\partial a'} \frac{\partial \mathcal{A}(a)}{\partial a}, \end{aligned} \tag{3.27}$$

and

$$\frac{\partial \mathcal{A}(a)}{\partial a} = w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} + w_{-i} \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_{-i}(a)}{\partial a} + 1 + r(a) + a \frac{\partial r(a)}{\partial a}. \quad (3.28)$$

Combined, the conditions (3.24), (3.27) and (3.28) yield

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{C}_i(a)}{\partial a} + u_{n,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{N}_i(a)}{\partial a} \\
&\quad + \theta_i u_{c,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \frac{\partial \mathcal{C}_{-i}(a)}{\partial a} + \theta_i u_{n,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} \\
&\quad + u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{A}(a)}{\partial a} \\
&= u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{C}_i(a)}{\partial a} + u_{n,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \frac{\partial \mathcal{N}_i(a)}{\partial a} \\
&\quad + \theta_i u_{c,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \frac{\partial \mathcal{C}_{-i}(a)}{\partial a} + \theta_i u_{n,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} \\
&\quad + u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \begin{pmatrix} w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} + w_{-i} \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_{-i}(a)}{\partial a} \\ +1 + r(a) + a \frac{\partial r(a)}{\partial a} \end{pmatrix},
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left[\frac{\partial \mathcal{C}_i(a)}{\partial a} - w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} \right] \\
&\quad + \theta_i u_{c,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) \left[\frac{\partial \mathcal{C}_{-i}(a)}{\partial a} - w_{-i} \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} \right] \\
&\quad + u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left[w_i \frac{\partial \mathcal{N}_i(a)}{\partial a} - \frac{\partial \mathcal{C}_i(a)}{\partial a} \right] \\
&\quad + u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left[w_{-i} \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} - \frac{\partial \mathcal{C}_{-i}(a)}{\partial a} \right] \\
&\quad + u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left(1 + r(a) + a \frac{\partial r(a)}{\partial a} \right),
\end{aligned}$$

by using the labor supply condition (3.26) to replace $u_{n,i}(c_i, n_i)$ with $-u_{c,i}(c_i, n_i) w_i$ and $u_{n,-i}(c_{-i}, n_{-i})$ with $-u_{c,-i}(c_{-i}, n_{-i}) w_{-i}$. After collecting terms, this expression can be reduced to

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(a)}{\partial a} &= u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \left(1 + r(a) + a \frac{\partial r(a)}{\partial a} \right) \\
&\quad + \left[\theta_i u_{c,-i}(\mathcal{C}_{-i}(a), \mathcal{N}_{-i}(a)) - u_{c,i}(\mathcal{C}_i(a), \mathcal{N}_i(a)) \right] \\
&\quad \times \left[\frac{\partial \mathcal{C}_{-i}(a)}{\partial a} - w_{-i} \frac{\partial \mathcal{N}_{-i}(a)}{\partial a} \right].
\end{aligned} \quad (3.29)$$

After updating (3.29) one period ahead and combining it with (3.24), one obtains the Euler equation for household member i:

$$\begin{aligned}u_{c,i}(c_i,n_i) &= \beta u_{c,i}(\mathcal{C}_i(a'),\mathcal{N}_i(a'))\left(1+r\left(a'\right)+a'\frac{\partial r(a')}{\partial a'}\right) \\ &\quad +\beta\left[\theta_i u_{c,-i}(\mathcal{C}_{-i}(a'),\mathcal{N}_{-i}(a'))-u_{c,i}(\mathcal{C}_i(a'),\mathcal{N}_i(a'))\right] \\ &\quad \times\left[\frac{\partial \mathcal{C}_{-i}(a')}{\partial a'}-w_{-i}\frac{\partial \mathcal{N}_{-i}(a')}{\partial a'}\right].\end{aligned}$$

Proof of Proposition 3.

The first-order conditions for the decision problem of the quasi-geometric household are given by

$$u_c(c,n) = \delta\beta\frac{\partial \mathcal{V}(a')}{\partial a'}, \tag{3.30}$$

$$-u_n(c,n) = \delta\beta\frac{\partial \mathcal{V}(a')}{\partial a'}w, \tag{3.31}$$

where I used that $\partial a'/\partial c=-1$ and $\partial a'/\partial n=w$. By combining these two conditions, one obtains the standard labor supply condition

$$-u_n(c,n)=u_c(c,n)w. \tag{3.32}$$

Differentiating $\mathcal{V}(a)$ with respect to a yields

$$\frac{\partial \mathcal{V}(a)}{\partial a}=u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{C}(a)}{\partial a}+u_n(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{N}(a)}{\partial a}+\beta\frac{\partial \mathcal{V}(\mathcal{A}(a))}{\partial a'}\frac{\partial \mathcal{A}(a)}{\partial a},$$

and differentiating $\mathcal{A}(a)$ with respect to a gives

$$\frac{\partial \mathcal{A}(a)}{\partial a}=w\frac{\partial \mathcal{N}(a)}{\partial a}-\frac{\partial \mathcal{C}(a)}{\partial a}+1+r(a)+a\frac{\partial r(a)}{\partial a}.$$

Together with condition (3.30), the two expressions above can be combined to

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{C}(a)}{\partial a}+u_n(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{N}(a)}{\partial a} \\ &\quad +\frac{1}{\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{A}(a)}{\partial a} \\ &= u_c(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{C}(a)}{\partial a}+u_n(\mathcal{C}(a),\mathcal{N}(a))\frac{\partial \mathcal{N}(a)}{\partial a} \\ &\quad +\frac{1}{\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\left(w\frac{\partial \mathcal{N}(a)}{\partial a}-\frac{\partial \mathcal{C}(a)}{\partial a}+1+r(a)+a\frac{\partial r(a)}{\partial a}\right),\end{aligned}$$

which can be written as

$$\begin{aligned}\frac{\partial \mathcal{V}(a)}{\partial a} &= u_c(\mathcal{C}(a),\mathcal{N}(a))\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right] \\ &\quad + \frac{1}{\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\left[w\frac{\partial \mathcal{N}(a)}{\partial a} - \frac{\partial \mathcal{C}(a)}{\partial a}\right] \\ &\quad + \frac{1}{\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\left(1 + r(a) + a\frac{\partial r(a)}{\partial a}\right) \\ &= u_c(\mathcal{C}(a),\mathcal{N}(a))\left(1 - \frac{1}{\delta}\right)\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right] \\ &\quad + \frac{1}{\delta}u_c(\mathcal{C}(a),\mathcal{N}(a))\left(1 + r(a) + a\frac{\partial r(a)}{\partial a}\right),\end{aligned}$$

by using condition (3.32). Updating this condition one period ahead and combining it with condition (3.30) yields the Euler equation for the quasi-geometric household:

$$u_c(c,n) = \beta u_c(\mathcal{C}(a'),\mathcal{N}(a'))\left(1 + r\left(a'\right) + a'\frac{\partial r(a')}{\partial a'} + \Gamma(a')\right),$$

with

$$\Gamma(a) = (\delta - 1)\left[\frac{\partial \mathcal{C}(a)}{\partial a} - w\frac{\partial \mathcal{N}(a)}{\partial a}\right].$$

3.A.2 Household Problem under Commitment

This section derives the optimality conditions for the cooperative and the non-cooperative household problem under commitment.

Household Problem under Cooperation

Under commitment, the cooperative household chooses current and future consumption, labor supply and savings $\{c_{Ft}, n_{Ft}, c_{Mt}, n_{Mt}, a_{t+1}\}_{t=0}^{\infty}$ to maximize the joint objective function $\sum_{t=0}^{\infty}\beta^t\sum_{i\in\{F,M\}}(1+\theta_{-i})\,u_i(c_{it},n_{it})$ subject to a sequence of budget constraints (3.1) for all periods $t \geq 0$. The first-order conditions for this problem are

$$\begin{aligned}(1+\theta_{-i})\,u_{c,i}(c_{it},n_{it}) &= \lambda_t, i\in\{F,M\}, \\ -(1+\theta_{-i})\,u_{n,i}(c_{it},n_{it}) &= \lambda_t w_i, i\in\{F,M\}, \\ \lambda_t &= \beta\lambda_{t+1}\left(1 + r(a_{t+1}) + a_{t+1}\frac{\partial r(a_{t+1})}{\partial a_{t+1}}\right),\end{aligned}$$

with λ_t denoting the Lagrange multiplier associated with the household budget constraint of period t. After using $\lambda_{t+s} = (1+\theta_{-i})\,u_{c,i}(c_{it+s},n_{it+s})$ to eliminate the multipliers λ_t

and λ_{t+1}, these conditions reduce to

$$\begin{aligned}
\frac{u_{c,F}(c_{Ft},n_{Ft})}{u_{c,M}(c_{Mt},n_{Mt})} &= \frac{1+\theta_F}{1+\theta_M}, \\
-u_{n,i}(c_{it},n_{it}) &= u_{c,i}(c_{it},n_{it})w_i, i \in \{F,M\}, \\
u_{c,F}(c_{Ft},n_{Ft}) &= \beta u_{c,F}(c_{Ft+1},n_{Ft+1})\left(1+r(a_{t+1})+a_{t+1}\frac{\partial r(a_{t+1})}{\partial a_{t+1}}\right),
\end{aligned}$$

which are the same conditions as listed by Proposition 1 for the case without commitment when written using sequential notation.[29] Under cooperation, the household hence does not have an incentive to deviate from the optimal plan under commitment when re-optimizing from period to period, i.e. the optimal plan under commitment is time consistent.

The cooperative solution to the household problem under commitment is only time consistent since the household members have standard time-consistent preferences and there is (by assumption) no disagreement about how to evaluate consumption and labor supply within and across periods. It is therefore crucial that the household members share the same discount factor because heterogeneous time preferences would introduce disagreement into the household problem (see e.g. Jackson and Yariv, 2014). In addition, for the time-consistency result, it is also important that the interest rate charged in a given period does not depend on the actions of the household in the subsequent period. If, for instance, the household could default on a loan ($a < 0$) and the lender would charge an actuarially fair interest rate that reflects the risk of default as a function of the size of the credit as in Chatterjee et al. (2007), a time-inconsistency problem would arise. Only a household who can commit to a plan for all future actions would internalize the adverse effect that a default has on the interest rate in the previous period, leading to a different outcome relative to the case without commitment.

Household Problem without Cooperation

The non-cooperative interaction between the household members under commitment is modeled as an open-loop Nash game. Time-consistent (or subgame-perfect) outcomes are thus not generally achieved. At period $t = 0$, the household members now separately choose their own current and future consumption, labor supply as well as savings $\{c_{it},n_{it},a_{t+1}\}_{t=0}^{\infty}$ to maximize the own objective function $\sum_{t=0}^{\infty}\beta^t U_i(c_{it},c_{-it},n_{it},n_{-it})$ subject to a sequence of budget constraints (3.1) for all periods $t \geq 0$, taking all current and future decisions of the spouse $\{c_{-it},n_{-it}\}_{t=0}^{\infty}$ as parametrically given.

[29]The household budget constraint is the same as well.

The first-order conditions for household member $i \in \{F,M\}$ are

$$\begin{aligned} u_{c,i}(c_{it},n_{it}) &= \lambda_{it}, \\ -u_{n,i}(c_{it},n_{it}) &= \lambda_{it} w_i, \\ \lambda_{it} &= \beta \lambda_{it+1} \left(1 + r(a_{t+1}) + a_{t+1} \frac{\partial r(a_{t+1})}{\partial a_{t+1}}\right), \end{aligned}$$

where λ_{it} denotes i's Lagrange multiplier associated with the period t budget constraint. After eliminating the multipliers λ_{it} and λ_{it+1} via $\lambda_{it+s} = u_{c,i}(c_{it+s}, n_{it+s})$, these conditions reduce to

$$\begin{aligned} -u_{n,i}(c_{it},n_{it}) &= u_{c,i}(c_{it},n_{it}) w_i, i \in \{F,M\}, \\ u_{c,i}(c_{it},n_{it}) &= \beta u_{c,i}(c_{it+1},n_{it+1}) \left(1 + r(a_{t+1}) + a_{t+1} \frac{\partial r(a_{t+1})}{\partial a_{t+1}}\right), i \in \{F,M\}. \end{aligned}$$

Relative to the conditions listed by Proposition 2, the only but important difference is that the term Γ_i does not appear in the Euler equations above, i.e. $\Gamma_i(a_{t+1}) = 0$ for the commitment case. Note that the degree of altruism θ_i does not enter the equilibrium conditions above and is thus irrelevant for the equilibrium outcome.

3.A.3 Numerical Solution

The model from Section 3.2 is solved via a time-iteration algorithm (see Judd, 1998).[30] It involves the following six steps:

1. Construct a discrete grid for household asset holdings $[\underline{a}, \overline{a}]$.
2. Choose initial values for the policy functions $\mathcal{X}_{start}(a)$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{C}_M, \mathcal{N}_F, \mathcal{N}_M\}$, at all grid points $a \in [\underline{a}, \overline{a}]$.
3. Set $\mathcal{X}_{next}(a) = \mathcal{X}_{start}(a)$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{C}_M, \mathcal{N}_F, \mathcal{N}_M\}$, for all $a \in [\underline{a}, \overline{a}]$ and choose an error tolerance ε.
4. For each grid point $a \in [\underline{a}, \overline{a}]$, compute the policies $\mathcal{X}_{new}(a)$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{C}_M, \mathcal{N}_F, \mathcal{N}_M\}$, that satisfy the equilibrium conditions, given next period's policy functions $\mathcal{X}_{next}(a)$.
5. If $|\mathcal{X}_{new}(a) - \mathcal{X}_{next}(a)| < \varepsilon$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{C}_M, \mathcal{N}_F, \mathcal{N}_M\}$, for all $a \in [\underline{a}, \overline{a}]$, go to step 6, else set $\mathcal{X}_{next}(a) = \mathcal{X}_{new}(a)$ and repeat step 4.

[30] It is straightforward to modify the algorithm to solve the model with labor income risk discussed in Section 3.3.

6. Take $\mathcal{X}_{new}(a)$, $\mathcal{X} \in \{\mathcal{A}, \mathcal{C}_F, \mathcal{C}_M, \mathcal{N}_F, \mathcal{N}_M\}$, as approximations of the respective equilibrium objects $\mathcal{X}(a)$ in the infinite-horizon model.

The policies in step 4 are computed via a non-linear equation solver. Given the policy functions $\mathcal{X}_{next}$ that determine the allocation in the subsequent period, the equation solver finds values for current savings, consumption and labor supply that satisfy the equilibrium conditions at all grid points. In the cooperative case, these conditions are the ones listed by Proposition 1. In the non-cooperative case, Proposition 2 summarizes the relevant equilibrium conditions. To approximate the policy functions, I use Chebyshev polynomials. The use of Chebyshev polynomials preserves the differentiability of the policy functions which is crucial here since the equilibrium conditions in the non-cooperative case involve derivatives of future policy functions with respect to savings.

Since I want to solve for the infinite-horizon limit of the finite-horizon model version, the initial values used in step 2 need to be the policies that solve the final-period problem of the household. In the final period, no savings decision takes place such that the initial values for the savings policy $\mathcal{A}_{start}(a)$ are set to zero. For the final-period problem of the cooperative household, the equilibrium conditions are given by the ones listed by Proposition 1 except that the Euler equation (3.4) now is dropped from the set of equilibrium conditions, leaving four conditions for four variables. In the non-cooperative case, the equilibrium cannot be determined when the two Euler equations are not present anymore. Therefore, I assume a Stackelberg timing for the non-cooperative household problem in the final period which allows to compute policies for the final period problem. The assumption of Stackelberg leadership is only made for the final period. For the other periods, the household members act simultaneously.[31] Alternatively, similar to Hertzberg (2012), one can also assume that savings in the final period are split evenly between the two household members who then both optimize independently subject to the private budget constraint

$$0 = w_i n_i + (a/2)(1 + r(a)) - c_i.$$

Under this assumption, the algorithm again converges to the same policy functions. The same holds true when the cooperative solution is used for the last period instead of the non-cooperative Stackelberg solution.

[31] Hertzberg (2012) considers an alternative version of his model with Stackelberg timing in every period.

Part III

Concluding Remarks

Concluding Remarks

This thesis has presented three essays that study how governments and households make decisions in the absence of commitment, contributing to the literature on time-inconsistency problems in macroeconomics. Chapters 1 and 2 have investigated the interaction between monetary and fiscal policy when a government is not able to commit to future policies, including debt repayment. Chapter 3 was concerned with the role of cooperation for the trade-offs faced by two-person households whose individual members lack commitment.

While the three chapters highlight the role of lack of commitment for different applications and hence make individual contributions, they have several things in common. They all use the Markov-perfect equilibrium concept to study decision making without commitment. They all make use of computational methods to evaluate the quantitative implications of the respective models. They all highlight that macroeconomists need to be careful when making assumptions about agents and the way they interact since these assumptions can strongly affect the implications that lack of commitment has for the predictions of a model.

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