

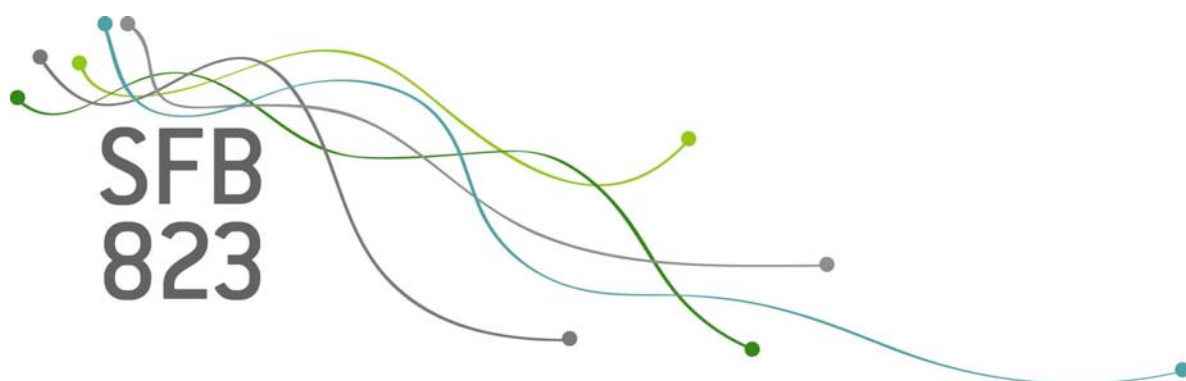
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Pseudolikelihood estimation of the stochastic frontier model

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PSEUDOLIKELIHOOD ESTIMATION OF THE STOCHASTIC FRONTIER MODEL

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ABSTRACT. Stochastic frontier analysis is a popular tool to assess firm performance. Almost universally it has been applied using maximum likelihood estimation. An alternative approach, pseudolikelihood estimation, which decouples estimation of the error component structure and the production frontier, has been adopted in several advanced settings. To date, no formal comparison has yet to be conducted comparing these methods in a standard, parametric cross sectional framework. We seek to produce a comparison of these two competing methods using Monte Carlo simulations. Our results indicate that pseudolikelihood estimation enjoys almost identical performance to maximum likelihood estimation across a range of scenarios, and out performs maximum likelihood estimation in settings where the distribution of inefficiency is incorrectly specified.

1. INTRODUCTION

The study of firm performance has a long history in economics. Accounting for the presence of inefficiency was a vexing econometric issue until a composed error approach was proposed by Aigner, Lovell & Schmidt (1977) and Meeusen & van den Broeck (1977). This approach, stochastic frontier analysis (SFA) treats the error term in a standard regression model as stemming from two sources, noise/measurement error and firm level inefficiency. These two separate components can be identified given that inefficiency operates in one direction on the firm; for example, in a production context, it lowers output. SFA is almost universally

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implemented using maximum likelihood (ML). However, an alternative method of moments (MoM) approach was also suggested (e.g. in Aigner et al. 1977, Olson, Schmidt & Waldman 1980) which decouples estimation of the frontier and the unknown parameters of the noise and inefficiency distribution. To date, several simulation based papers exist which have compared the relative performance of ML and MoM estimation of the stochastic frontier model (Olson et al. 1980, Coelli 1995, Behr & Tente 2008).

In this paper we compare an alternative stochastic frontier estimator, based on pseudolikelihood (PL) estimation, to both ML and MoM. Similar to the MoM, PL estimation proceeds by decoupling estimation of the frontier and the parameters of the error component. The PL approach was first suggested by Fan, Li & Weersink (1996) who needed to decouple estimation of the frontier and the error components because they placed no structure on the frontier, requiring nonparametric estimation. What is interesting about the PL econometric framework proposed in Fan et al. (1996) is that while they used it in a nonparametric context, it is equally applicable in a parametric setting, which seems to have gone unnoticed in the applied stochastic frontier literature. Although Fan et al. (1996, pg. 466) acknowledge the applicability of the PL estimator in the parametric context, noting “. . . if $g(x_i)$ is linear then $\hat{E}[y_i|x_i]$. . . can be replaced by the least squares prediction of y_i given x_i .”, it has not been used in applied settings nor has its performance been properly adjudicated against ML. Another important reason to study the PL estimator is that it was recently proposed in a three step procedure to recover both persistent and time varying inefficiency in a panel data stochastic frontier model (Kumbhakar, Lien & Hardaker 2014, Kumbhakar, Wang & Horncastle 2015), which would otherwise require optimization of a complicated maximum likelihood function.

Theoretically, a key advantage of PL – and analogously MoM – is that when either of the distributions of the error component are misspecified, consistent estimators of the shape of the production frontier should still be produced.¹ As Kumbhakar & Lovell (2003, pg. 93) note, referencing MoM, that two-stage methods “. . . use distributional assumptions only in the second step, and so the first-step estimators are robust to distributional assumptions on v_i and u_i ”. Under distributional misspecification, ML estimation may potentially produce biased and/or inconsistent estimators. Another practical advantage of PL in comparison to ML is that it potentially lessens the (numerical) maximization complexity due to the fact

¹The intercept will still be biased as it depends on the unknown, nonzero mean of the error component.

that it reduces the number of variables over which to perform the optimization. Hence, again analogously to MoM, PL could be a promising alternative to ML.

A further contribution of the paper is that it shed more light on the relative comparison of ML and MoM. To date, papers comparing ML and MoM (Olson et al. 1980, Coelli 1995, Behr & Tente 2008) have focused on estimation of the parameters of the model (slope coefficients and variance parameters of the composed error distribution). All three studies show that both estimators have their strengths and weaknesses. Yet, the three studies come to somewhat diverging conclusions. While Olson et al. (1980, pg.80) reason that “[f]or all sample sizes below 400 and for λ less than 3.16, [MoM] is preferred. But, even for higher sample sizes and variance ratios, the additional efficiency of the [ML] may not be worth the extra trouble required to compute it.”, Coelli (1995, pg. 264) concludes that “[o]verall, these results suggest the ML estimator should be preferred to the [MoM] estimator . . .”. Lastly, based on their simulation results, Behr & Tente (2008, pg. 16) suggest that “. . . method of moment estimation should be considered an alternative to maximum likelihood estimation . . .”. However, it should be noted that all three studies draw their conclusions on a generic data generation procedure deliberately for cases with no covariates, arguing that covariates should not matter for relative performance.² An additional focus of our simulations is to learn if the presence of covariates has any effect on the earlier claims of Olson et al. (1980), Coelli (1995) and Behr & Tente (2008).

Naturally, no Monte Carlo investigation is above reproach and sampling and specification issues naturally preclude making definitive outcomes. However, ignoring covariates in the comparison between the one-step ML approach and the two-stage procedures, MoM and PL, eliminates the key advantage that the latter two methods may possess over ML. Given that earlier papers comparing ML and MoM did not extensively model the frontier, we believe that our simulation results here are instructive. Our main results are twofold: First, when the distribution of the inefficiency term is correctly specified, all three methods have relative similar performance when estimating returns to scale, inefficiency levels and firm output and second, when the distribution of inefficiency is misspecified, PL appears to be the dominant method for the majority of sample size/signal-to-noise ratio scenarios considered. In combination, this may suggest use of the PL estimator in settings where there is uncertainty as to the correct distribution of inefficiency.

²To be precise, the study of Olson et al. (1980) presents one experiment based on real data with four covariates. However, this experiment is limited from several perspectives (number of replications, maintained assumptions, etc.).

2. ESTIMATION OF THE STOCHASTIC FRONTIER MODEL

The stochastic production frontier of Aigner et al. (1977) and Meeusen & van den Broeck (1977), across n firms is

$$(1) \quad y_i = m(\mathbf{x}_i; \boldsymbol{\beta}) - u_i + v_i = m(\mathbf{x}_i; \boldsymbol{\beta}) + \varepsilon_i, \quad i = 1, \dots, n,$$

where u_i captures inefficiency (shortfall from maximal output), v_i captures outside influences beyond the control of the producer and the production frontier is $m(\mathbf{x}_i; \boldsymbol{\beta})$. We assume that u_i and v_i are independent of one another as well as the covariates \mathbf{x}_i . Due to the fact that inefficiency can only affect output in one direction, we have that $E(u_i) = \mu > 0$. In contrast, v_i can contribute positively or negatively to output and we assume $E(v_i) = 0$; thus, $E(\varepsilon_i) \neq 0$. The most basic formulation of the stochastic frontier model is to assume that $v_i \sim N(0, \sigma_v^2)$ and $u_i \sim N_+(0, \sigma_u^2)$. In this case, ML estimation proceeds by optimizing $\mathcal{L} = \prod_{i=1}^n f(\varepsilon_i)$, where $\varepsilon_i = y_i - m(\mathbf{x}_i; \boldsymbol{\beta})$ yielding

$$(2) \quad \ln \mathcal{L}(\beta, \lambda, \sigma) = -n \ln \sigma + \sum_{i=1}^n \ln \Phi(-\varepsilon_i \lambda / \sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^n \varepsilon_i^2,$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal random variable and we have set $\sigma = \sigma_u + \sigma_v$ and $\lambda = \frac{\sigma_u}{\sigma_v}$.

Alternatively, MoM proceeds by estimating the model in (1) using ordinary least squares (OLS). From there, MoM uses the second and third moment conditions for ε to estimate σ_v^2 and σ_u^2 . With the estimate of σ_u^2 , the intercept of $m(\mathbf{x}_i; \boldsymbol{\beta})$ can be shifted up to account for the non-zero mean of the composed error. See Kumbhakar & Lovell (2003) or Greene (2008) for a more detailed account of the exact moment conditions.

PL estimation of the stochastic frontier model proceeds – analogously to MoM – by estimating the model in (1) using OLS. Then, the variance parameters are estimated by maximizing³

$$(3) \quad \ln \mathcal{L}(\lambda) = -n \ln \hat{\sigma} + \sum_{i=1}^n \ln \Phi \left[\frac{-\hat{\varepsilon}_i \lambda}{\hat{\sigma}} \right] - \frac{1}{2\hat{\sigma}^2} \sum_{i=1}^n \hat{\varepsilon}_i^2,$$

³Note that this optimization is over the single unknown parameter λ as, from Fan et al. (1996), σ can be concentrated out with the normal-half normal distributional assumptions.

where $\hat{\varepsilon}_i = \hat{\varepsilon}_{i,OLS} - \frac{\sqrt{2\lambda\hat{\sigma}}}{\sqrt{\pi(1+\lambda^2)}}$ and $\hat{\varepsilon}_{i,OLS}$ are the residuals from OLS estimation of (1) and

$$\hat{\sigma} = \sqrt{\frac{\frac{1}{n} \sum_{j=1}^n \hat{\varepsilon}_{j,OLS}^2}{1 - \frac{2\lambda^2}{\pi(1+\lambda)}}}.$$

Subsequently, a consistent estimator for the intercept of the production frontier is given by:

$$\hat{\beta}_0 = \hat{\beta}_{0,OLS} + \hat{E}(u_j) = \hat{\beta}_{0,OLS} + \sqrt{\frac{2}{\pi}} \hat{\sigma}_u.$$

After shifting the OLS frontier upwards by the expected value of the inefficiency term, all of the estimators are unbiased and consistent (see Aigner et al. 1977, Kumbhakar & Lovell 2003, Greene 2008). For any of the ML, MoM, or PL estimators, an estimator for expected firm level inefficiency can be obtained through the conditional expectation of u given ε following the approach of Jondrow, Lovell, Materov & Schmidt (1982). Measurement of technical efficiency (Battese & Coelli 1988) follows from

$$(4) \quad \widehat{TE}_i = \hat{E}(e^{-u_i} | \hat{\varepsilon}_j) = \frac{\Phi(\hat{\mu}_{*j}/\hat{\sigma}_* - \hat{\sigma}_*)}{\Phi(\hat{\mu}_{*j}/\hat{\sigma}_*)} \cdot e^{(\frac{1}{2}\hat{\sigma}_*^2 - \hat{\mu}_{*j})},$$

where $\mu_* = -\varepsilon\sigma_u^2/\sigma^2$ and $\sigma_*^2 = \sigma_u^2\sigma_v^2/\sigma^2$.

One caveat that we mention here is that the assumption of a constant variance for firm level inefficiency, u_i , is crucial for both MoM and PL to produce consistent first stage estimators. In the setting where the variance of u_i varies across a set of covariates, the ignorance of the error structure which MoM and PL possess, can produce inconsistent estimators of the production frontier parameters (Parmeter & Kumbhakar 2014). The potential development of PL or MoM approaches which can handle this level of heterogeneity may prove elusive, but would certainly be welcome.

3. MONTE CARLO SIMULATION

3.1. Data generating process and performance criteria. To assess the performance of the PL approach to ML and MoM, we turn to Monte Carlo experiments. Rather than generate data from a generic production function, we instead base our simulations around a real world dataset. Specifically, we use the Philippines rice dataset which has become a benchmark example in applied efficiency analysis, serving as the dominant heuristic illustration in Coelli, Rao, O' Donnell & Battese (2005) and also appearing recently in Rho &

Schmidt (2015). The data are composed of 43 farmers observed annually for eight years. Even though the data constitutes a panel, we will ignore this aspect for our purposes. The output variable is tonnes of freshly threshed rice with the main input variables being area of planted rice (hectares), total labor used (man-days of family and hired-labor) and fertilizer used (kilograms). There is also a fourth input, other inputs, which is measured relative to farm 17 in the data via the Laspeyres index for 1991.⁴

To allow for various sample sizes we first estimate the translog production function based on the real data and set these estimates as true parameter values. We then take smooth samples from the four main inputs following the approach of Silverman (1986). We vary λ from $\{0.562, 1.000, 1.778\}$ so that noise to signal is equal to, greater than and less than 1. We set $\sigma = 1$ across all scenarios given the invariance of the methods as pointed out by Olson et al. (1980) (i.e. doubling σ should lead to a quadrupling of the mean square errors). For the noise term, we assume a normal distribution. As the assumption about the inefficiency distribution is of special interest, we analyze two cases. First we focus on the performance of the estimators when the distributional assumption is correctly specified. Specifically, inefficiency is generated from a half normal distribution and we assume that it stems from a half normal distribution. Alternatively, we assess the three methods when inefficiency is generated from an exponential distribution but we still assume that it stems from a half normal distribution. Each case considers 12 (3 values for λ by 4 sample sizes) scenarios for a total of 24 scenarios. Each scenario is replicated $R = 10,000$ times.

Due to the fact that productivity and efficiency analysis is generally applied to estimate returns to scale (RTS), predict expected firm output or measure individual technical efficiency, we evaluate the performance of the methods based on these measures. For each measure, our performance criterion is the median (across the 10,000 simulations) of mean square error (MMSE) between the estimated and the true value:

$$MMSE = \text{median}_{r=1, \dots, R} \frac{1}{n} \sum_{i=1}^n (\hat{M}_{i,r} - M_{i,r})^2.$$

where $M_{i,r}$ is the true value of our measure for the i th firm in the r th simulation (M being RTS, expected output or technical efficiency) and $\hat{M}_{i,r}$ the estimated value. The Monte Carlo experiments are conducted in R (version 3.2.3) and all code is available upon request.

⁴See Coelli et al. (2005, Appendix 2) for a more detailed description of the data.

3.2. Correctly Specified Distribution. Table 1 shows the results for the setting where the distribution of u_i is correctly specified. The results show in general that each method is superior in a particular scenario. PL and MoM perform relatively better than ML when the sample size is small and/or when λ is low. With increasing λ and increasing sample size the relative performance of ML improves. However, consistent with Olson et al. (1980), even when ML performs better the gains are not that substantial. For the comparison between PL and MoM the results suggest that in the perfectly specified case, MoM has the comparative advantages when the sample size is small and λ is small.

Regarding the estimation of RTS, the results show that PL and MoM (note that these estimates are the first step OLS estimates) estimate the RTS more accurate when the sample size is relatively small (i.e. $n=100$). For all other scenarios, the RTS estimates of all three methods are nearly the same. For the estimation of expected output, across all sample sizes, the MoM estimator is the most accurate for predicting expected output when λ is small. When λ is 1.778, MoM is the worst method across all sample sizes. PL is here the best method when the sample size is small and is as good as ML when the sample size is large (i.e. $n=800$). With respect to the estimation of individual efficiency, MoM is always the best method. However, there is only one scenario ($n=100$ and $\lambda=0.562$) where any noticeable difference between MoM and PL arises. For larger values of λ or n the results for ML are also indistinguishable.

3.3. Misspecified Distribution. As noted earlier, we expect that PL and MoM to be less affected by misspecified distributional assumptions pertaining to inefficiency than ML. To assess this presumption, we conducted the same scenarios as above but used an incorrect distributional assumption for the inefficiency term to estimate the model. Tables 2 shows the results for the same 12 scenarios discussed above but by generating firm level inefficiency from an exponential distribution.

Several key differences with the correctly specified setting emerge. Regarding the estimation of RTS, ML is the superior method except for scenarios with small sample sizes (i.e. $n=100$). These results are somewhat surprising as one might expect that the two step estimators would estimate RTS more accurately (due to the fact that the estimated shape of the production function is unbiased). We have two explanations which might explain this result. First, the performance criterion RTS does not exactly measure the estimation of the single β , but has additionally some weighting of the β . Second, it could be the case that even with a misspecified distribution that the convoluted error term's distribution is close to the

TABLE 1. Estimation Results of PL, ML and MoM for the correctly specified distribution case, 10,000 Simulations

λ	Sample Size	Returns to Scale (MMSE)		Production Value (MMSE)			Efficiency (MMSE)		
		PL/MoM	ML	PL	ML	MoM	PL	ML	MoM
0.562	$n = 100$	0.093	0.098	0.235	0.268	0.208	0.066	0.083	0.054
0.562	$n = 200$	0.041	0.041	0.165	0.173	0.141	0.044	0.050	0.041
0.562	$n = 400$	0.019	0.019	0.120	0.119	0.102	0.028	0.028	0.026
0.562	$n = 800$	0.009	0.009	0.067	0.068	0.065	0.016	0.017	0.016
1.000	$n = 100$	0.075	0.081	0.171	0.252	0.166	0.017	0.042	0.016
1.000	$n = 200$	0.033	0.033	0.083	0.096	0.081	0.009	0.012	0.008
1.000	$n = 400$	0.015	0.015	0.041	0.043	0.041	0.004	0.005	0.004
1.000	$n = 800$	0.008	0.008	0.021	0.021	0.021	0.002	0.002	0.002
1.778	$n = 100$	0.055	0.061	0.101	0.122	0.105	0.009	0.016	0.010
1.778	$n = 200$	0.025	0.024	0.048	0.047	0.050	0.004	0.004	0.004
1.778	$n = 400$	0.012	0.011	0.023	0.022	0.024	0.002	0.002	0.002
1.778	$n = 800$	0.006	0.005	0.011	0.011	0.012	0.001	0.001	0.001

MMSE: median of the mean square error between the estimated and the true value over all replications.

TABLE 2. Estimation Results of PL, ML and MoM for the misspecified distribution case, 10,000 Simulations

λ	Sample Size	Return to Scale		Production Value			Efficiency		
		PL/MoM	ML	PL	ML	MoM	PL	ML	MoM
0.562	$n = 100$	0.110	0.121	0.271	0.345	0.260	0.051	0.081	0.051
0.562	$n = 200$	0.048	0.048	0.177	0.210	0.179	0.040	0.050	0.043
0.562	$n = 400$	0.023	0.022	0.120	0.134	0.131	0.035	0.039	0.037
0.562	$n = 800$	0.011	0.011	0.099	0.105	0.112	0.033	0.035	0.037
1.000	$n = 100$	0.108	0.121	0.204	0.311	0.233	0.021	0.042	0.026
1.000	$n = 200$	0.049	0.045	0.143	0.176	0.189	0.022	0.029	0.031
1.000	$n = 400$	0.023	0.020	0.124	0.138	0.195	0.023	0.027	0.036
1.000	$n = 800$	0.011	0.010	0.118	0.123	0.207	0.024	0.026	0.040
1.778	$n = 100$	0.107	0.109	0.173	0.235	0.236	0.015	0.035	0.031
1.778	$n = 200$	0.048	0.038	0.124	0.140	0.248	0.015	0.019	0.051
1.778	$n = 400$	0.023	0.016	0.103	0.103	0.282	0.014	0.016	0.056
1.778	$n = 800$	0.011	0.007	0.094	0.090	0.305	0.014	0.014	0.057

MMSE: median of the mean square error between the estimated and the true value over all replications.

correctly specified setting, i.e. the exponential distribution is not sufficiently different to the half normal one. However, for both other performance criteria, the estimation of expected output and individual efficiency, PL is almost always the dominant method. In addition,

in contrast to the correctly specified distribution case, the performance difference are partly substantial.

Naturally, no Monte Carlo investigation is above reproach and it could be that ML outperforms PL when λ is higher than 1.778 or for an entire different set of parameter values or production frontier structures. However, the range of λ from 0.562 to 1.778 is of practical relevance, and the translog functional form is a common modeling approach. Hence, our results suggest that the PL estimator has some practical value for applied researchers.

4. CONCLUSIONS

In this paper we investigated the PL estimator's ability to estimate parameters from the stochastic frontier model. The PL approach decouples estimation of the production frontier and the parameters of the error components. A commonly held notion is that under distributional misspecification this decoupling can provide a consistent estimator of the production structure. Using a Monte Carlo investigation based on a publicly available dataset we compared the performance of ML and PL under correct specification of the distribution of the inefficiency term as well as the more practical setting (precisely because it is unknown in reality) where this distribution is misspecified. For measures of returns to scale, expected output and individual technical efficiency, PL is seen as holding its ground, or outperforming ML, across nearly all the scenarios we considered.

Given the optimistic performance of PL a potential future avenue for research would include development of the correct standard errors for the ML components in the second stage. As Coelli (1995, pg. 251) notes “[t]he unpopularity of the [MoM] estimator may also be due to access to estimated standard errors”. It is well known that two step estimators typically require a correct to the standard errors. Currently, no such correction for the standard errors from the second stage of the PL approach exists. Further, a limitation of both MoM and PL in current practice is the inability to include z -variables which can influence the parameters of the distribution of inefficiency. An extension along these lines would be most welcome.

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