Markus CSLOVJECSEK, Brugg; Samuel INNIGER, Brugg

Sound Learning in Math Classrooms: How Children Teach us to Teach

Sound and motion highly comply with the way young children learn and with their unremitting inquiry of function and coherence. Many important steps in children's ways of exploring the world are anchored in the manner they deal with sound and motion. Even later on, when they grow up, musical activity is nearly always experience-related and connected with cognitive processes. Math textbooks use visual art tables and specific other learning materials to involve pupils in mathematical situations. However, as a rule they do not encourage activities with sound or motion. In this paper, we discuss why and how musical (sound), kinaesthetic (motion), and tactile (touch) impulses are of value to learning and show how we further intend to develop this approach in a transdisciplinary EU Comenius Project.

1. Is Math a Sounding Discipline?

Since classical antiquity, music and mathematics have been described as a wonderful pair. However, the valuable relation between the two disciplines has been associated neither with primary school mathematics nor with the topics of music instruction at school. Topics of an interdisciplinary discussion have rather been questions of harmonics, acoustics, the digitalization of sound, the pitch of instruments, or the mathematical calculability of composition and interpretation. Obviously, domains dealing with these topics do not have much in common with the curriculum of a primary school.

Our interest is in low-level connections between mathematics and music (Lakoff, & Núnez 2000). It originates in observations made in musically active classrooms. Based on these first-hand observations, our interest was not directed to the so-called *transfer effects*[6] or to collect, use or produce *Songs for Teaching*[7]. Surely, counting or reciting formulas by singing songs may be fun for children and in some situations, notably as mnemonics, such activities may be helpful. However, the relations we had observed

[6] Superficial interpretations of findings in brain research cause short circuits such as "music makes you smart" and prepares the field of simply using music as a learning machine (Jäncke 2008).

[7] Songs for Teaching are educational songs, chants, and rap songs to teach content across the curriculum to students of all ages. There are numerous materials developed and sold by professional providers over the last few years. E.g. http://www.songsforteaching.com

In F. Caluori, H. Linneweber-Lammerskitten & C. Streit (Hrsg.), *Beiträge zum Mathematikunterricht 2015*. Münster: WTM-Verlag

in the classrooms went much beyond this utilitarian use. They include at least shared conceptual structures and strategies, "functionally important in each domain" (Bamberger (2000).

2. Classrooms with Enhanced Music Education - Enable Transformative Practice Zones

The field of our first observations was a Swiss schoolproject in the late nineteen eighties. The long-term development of the project '*Musik macht Schule*'[8] goes along with Bresler's (2003, 2004) suggestion of Transformative Practice Zones (TPZ) as "spaces as well as ways of interacting and thinking, where the participants are touched and often transformed in the process." In the starting phase of the project, primary school teachers decided to teach five hours of music a week with their classes. Despite the correspondingly reduced hours in mathematics and languages, the curriculum remained unchanged for all subjects. No additional homework was allowed. After two years, the competences of the experimental group in the time-shortened subjects did not deviate from those of the control group (Weber, Spychiger & Patry 1993). This is tosay that in this setting, the reduction of 20% of math lessons did not have a negative impact on the measured output in maths learning. In a second phase, other astonishing effects where observed (Spychiger 1995, 2001; Cslovjecsek 1997; Cslovjecsek, Spychiger 1998): it became obvious that pupils and teachers started to link music with other subjects. The daily work with music soon urged out of its topical borders and the musical perspective offered interesting practices and tasks in the teaching and learning of languages, mathematics, and general studies. Students and teachers realized that many topics and methods in school were basically full of sound and potential for musical activities.

In the same time, mathematics education in Switzerland has developed in the direction of exploratory learning (Dewey 1934), the recognition of different learning paths (Hengartner 1999), the "concentration on fundamental ideas" and the "turning away from instruction in tiny steps in favour of a conceptual entireness of the learning situation" (Selter&Spiegel 1997; Wittmann 1998). The path from perception to mental conception is understood as process that also depends on the learner. It consciously comes with individual and social interpretation and requires openness, time, and space for creativity. To facilitate the postulated learning processes, specific materials have been introduced into the math classroom. According to

[8] Swiss project on „Schools with Enhanced Music Education"

Krauthausen (1998), criteria for learning materials for mathematics are as follows:

> […] adequate representation of the structure of the mathematical facts which are to be taught; multi-purposed possibilities of usage; possibility of continuation of the started learning-process; simple handling and simple structure; easy possibility of transfer to graphical representations; easy practicability of mental operations; potential of discovering individual and differentiated strategies for finding solution and the social interexchange in the process; continuous availability for all students and demo version for the class; low price, stability, and environmentally harmless material.

Music, sound, and motion highly comply with these criteria, but - perhaps due to the lack of tradition of a corresponding approach - they hardly have been considered working material or tools for teaching and learning. If at all, they appeared in educational material rather coincidentally and as a means of decoration.

3. Creative Potential of an Integrated Approach in Music Education

There is no justification for the limitation of educational material to verbal, visual, haptic, and mathematically-abstract approaches; acoustic (sound and music), kinaesthetic (motion), and tactile (touch) approaches are also of high value, especially with respect to acting and experiencing of primary school age children. The integration of these types of experiences in mathematical instruction has as good as no tradition – but it seems to have a great potential in several aspects:

Children's Thinking Paths: Sound and movement give access to important active learning paths of children in the classroom; insight of teachers in unexpected thinking paths of their pupils is promoted (Gardner 1991). As shown in *Snappings, Clappings and the Representation of Numbers* (Cslovjecsek, Linneweber-Lammerskitten 2011), children's thinking paths do not follow disciplinary traditions and borders.

Music Becomes a 'Language': When understanding sound and music simultaneously as a tool for teaching and learning, children assist in further developing sound and movement as an additional medium of teaching. When integrated in the curriculum, musical thinking and acting becomes a timeline in class for pupils and their teachers. It turns into a mode of operation and a starting point for further projects. By means of this double implementation of music in education, its potential takes on lasting effects: a group of the aforementioned third graders was working for a longer period of time on finding out the total number of possibilities combining 222-patterns with given body percussion movements. In the music class, they started composing instrumental music based on 222-patterns and became

interested in listening and discussing compositions of *Philip Glass*[9]. More and more they understood themselves as experts in 'Clapping Music'.

New 'Moving Rooms' in the Conjunction of Math and Music: While analyzing such classroom activities in today's mathematics and music curricula, we realized that both are approached in a highly desirable way. When integrating musical activities in a math classroom, it is important to know about the disciplinary goals addressed. Many learning opportunities are simultaneously mathematical and musical (Bamberger 2000) and very likely linked to language learning and visual competencies as well. Furthermore, there are goals in understanding learning and design processes as well as dynamics in individuals and groups. In the mentioned example, the pupils' intervention changed the focus away from the representation of natural numbers by the positional notation system (i.e. decimal system) to a game with musical patterns and then back to the relevance of patterns and orders for the representation of numbers.

Learning in Cooperation: Human beings flourish, learn, and connect more when they are cooperating as opposed to competing or working in an isolated fashion (Hertz-Lazarowitz & Miller, 1992; Sharan, 1994; Slavin, 1996). In order to manage cooperative learning processes, teachers need to find approaches with less teacher-centred settings, open to students' needs and promoting initiative, providing opportunities and space for team processes and unexpected product results.

Promoting Innovation and Flexibility: The ability to recognize and compare forms and complete patterns based on their qualitative properties is an important prerequisite for creating arithmetical and geometrical theories. In fact, mathematics can be understood as the science of pattern (Devlin, 1994). The confrontation of the mathematical content and the processes with musical resources offers a great creative potential:

- Acoustic patterns are the basis of different types of music: rhythmic accompanying patterns (drums, keyboard, guitar, bass) in rock, pop and folk music, sound patterns (sampling technique) in techno music, and melodic patterns in minimal music. All types of (musical) expression have a form. This fact plays a central role both in the reception (perception), the production (composition), and the reproduction (interpretation) of music. Examples of this include the prelude, interlude, and the cadence in the arrangement of a song or in the setting of poetry to mu-

[9] Philip Morris Glass (*1937) is considered one of the most influential composers of the late 20th century. He described himself as a composer of 'music with repetitive structures'.

sic, the sequence of refrains and verses in a strophic song, the rhyme scheme in a text, the construction of a melody, and the rhythmic structure of the movement processes in sports.

- Musical patterns are audible and can be written down: they can be heard in tone colors, in time structure, in volume ratios, in pitches or harmonic sequences. Often, several different patterns on different levels can be distinguished simultaneously. However, the ear is directed towards singular aspects.
- Movement sequences and patterns are perceptible: they are palpable and visible. Here, the dimensions of the expression are space, time, and energy. They can be transformed into sound as well as be written down. When working with body percussion, sound and movement merge in such a way that movement patterns simultaneously produce acoustic patterns and vice versa.
- Audible material is open to be adapted based on individual prerequisites: The material for musical work with shapes and movement ranges from the simplest pattern (walking, clapping, etc.) to very complex forms. Based on the teaching situation, it is the teacher's task to invent meaningful variations and to create prerequisites for dealing with simpler or more complex forms. Surprising and interesting ideas often originate from the children themselves.

Learning Beyond Disciplines: Looking at teaching and learning situations based on musical activities, the focus soon goes beyond the disciplinary learning and takes challenging aspects of learning at school into account:

- Heterogeneity: Not all children find it equally easy to create serial sequences. It is advisable to work patiently with short sequences, repetitions, and variations until the level can be raised.
- Responsibility: The basis for music making is to be with one self and with the others simultaneously. In order to make one's own contribution at the correct time, it is essential to empathize with the other contributors.
- Creativity: Finding own solutions within an agreed framework awakens creative abilities. Creativity is further developed by the recognition and discussion of alternative solutions found by others.
- Self-assurance: Self-assurance of a child can be enhanced when it has to produce a pattern, present a solution, or act out a pattern and stay with it whilst other children perform a different one.

- Self-efficacy: Children learn that their ideas can be important contributions for the learning process. In this way, they understand that their own thinking and acting can be meaningful for learning.

Stimulating Teachers' Curiosity: As described before, integrating musical activity in other school subjects is an innovative and creative approach and a challenge for teaching and teachers, even when the proposed activities do not presuppose highly developed musical abilities. In a small field study in Switzerland, we proposed an activity about listening to the sound of falling coins with three different levels of difficulty to be chosen by the teacher depending on the capabilities of the class. Answering a questionnaire, the involved teachers told us that they were very surprised and happy about such easy ways to include listening abilities into math's lessons. They had been impressed about the children's attention and the ideas coming up after working with. Some teachers asked the children to invent and describe their own instructions for similar tasks. Often, in spite of not being taught anymore, the input lasted for a longer period in the agenda of the class. It seems that even teachers with high interest in both subjects are not putting into practice the simplest possibilities to incorporate aspects of music in their math lessons (Cslovjecsek 2003).

Discovering New Landscapes: While discussing shared learning situations with colleagues from other disciplinary backgrounds or other expertise, we learn a lot; as musicians, as mathematicians, and as teachers. Based on the collaboration and linked knowledge in our example (Cslovjecsek, Linneweber-Lammerskitten 2011), we embarked on the challenging idea to express numbers by means of music:

The sign-value notation of numbers opens possibilities for numerous variations of representing the same number – some of these will be aesthetically and /or mathematically interesting, others boring. […] What would be a good musical representation that is easily understood and captures the central idea of a positional system of a given basis n?

4. How to Proceed

In the EU-funded Comenius Project "EMP-Maths: Sounding Ways Into Mathematics", we discuss interrelations, similarities, and differences of mathematics and music learning. We are aware of the fact that integrated teaching and learning within a school setting is a very challenging goal. Crossing the borders of disciplines is a complex problem. Moreover, this complex problem should be solved in a transdisciplinary way, which means: action-centered, participative, and interdisciplinary (Klein 2002).

Based on realistic classroom activities (action-oriented approach), we work together with classroom teachers and specialists in didactics and education (participatory approach) from the fields of mathematics and music (interdisciplinary approach).

In a first step, a practice-oriented description of the theoretical background was elaborated. The state-of-the-art papers from the involved countries (Hilton et al. 2015) show that mathematics and music are mostly thought of as separate disciplines. Nevertheless, in every country, we observe evidence for projects and suggestions for interdisciplinary teaching and learning. We also see, however, that teachers feel inadequately trained to teach the two subjects in an interdisciplinary way. Therefore, in order to stimulate the teachers' creativity and the search for new "moving rooms" in the conjunction of mathematics and music as described above, we work on an interdisciplinary guidance document, the Teacher Handbook. The elaboration process entails meetings and creative workshops in different European cities. We intensively discuss the content of the Handbook, experiment new approaches, and test proven practice activities.

After the completion of the necessary theoretical resource base, the project is now entering the testing and dissemination phase. In Continuing Professional Development courses, we present, perform, discuss, and further develop the material with teachers from all across Europe. The project works intensively with online platforms, which will feature a collection of good practice examples and possibilities of user-to-user interactions. Materials will be published continuously on www.emportfolio.eu

Literatur

Bamberger, J. (2000). Music, Maths and Science: Towards an Integrated Curriculum. Journal for Learning Through Music, 32–35.

Bresler, L. (2003). Out of the trenches: The joys (and risks) of cross-disciplinary collaborations. Council of Research in Music Education, 152, pp. 17-39.

Bresler, L. (2004). Crossing the Borders of Music and Arts Education: The Challenge, the Danger and the Opportunities. In F. Niermann, & C. Wimmer, (Eds.). Musik lernen – ein Leben lang (pp. 274-282). Wien: UE.

Cslovjecsek, M. (1997). Zur Frage der Zeichensysteme in der Praxis des Unterrichtens . In: Gymnasium Helveticum. Aarau: Sauerländer, Vol. 51. Jg., Nr. 3; pp. 134-142.

Cslovjecsek, M., Linneweber-Lammerskitten, H. (2011). Snappings, Clappings and the Representation of Numbers. In: The New Jersey Mathematics Teacher, 69 (1), 10-12.

Cslovjecsek, M., Spychiger, M. (1998). Mus ik oder Mus ik nicht? - Musik als Unterrichtsprinzip: Eine Unterrichtshilfe für Lehrpersonen der Volksschule. Hölstein: Verlag SWCH. Retrieved from http://www.creafon.com/content/mus_ik.php

Cslovjecsek, M. (2001/2004). Mathe macht Musik - Impulse zum musikalischen Unterricht mit dem Zahlenbuch (Vol 1-3), Zug: Klett und Balmer. Retrieved from http://www.mamu.ch/en

Cslovjecsek, M. (2003). Mathe macht Musik – ein alter Zugang zur Welt der Zahlen neu entdeckt. In Sache Wort Zahl – Zeitschrift für Grundschulpädagogik Heft 51, (Jg. 31 pp. 48-50). Köln: Aulis-Verlag.

Devlin, K. (1994). Mathematics: The Science of Patterns: The Search for Order in Life, Mind and the Universe. New York : Scientific American Library.

Dewey, J. (1934). Art as experience. New York: Capricorn Books.

Gardner, H. (1983). Frames of mind: The theory of multiple intelligences. New York: Basic Books.

Gardner, H. (1991). The unschooled mind: How children think and how schools should teach. New York: Basic Books.

Hengartner, E. (1999). Mit Kindern lernen. Zug: Klett und Balmer.

Hertz-Lazarowitz, R., & Miller, N. (Eds.). (1992). Interaction in cooperative groups: The theoretical anatomy of group learning. Cambridge: Cambridge University Press.

Hilton, C. et al. (2015). A review of Literature. European Music Portfolio: Sounding Ways Into Mathematics. Online Publication. www.maths.emportfolio.eu

Jäncke, L. (2008). Macht Musik schlau? Bern: Huber Verlag.

Krauthausen, G. (1998). Lernen – Lehren – Lehren lernen. Zur mathematikdidaktischen Lehrerbildung am Beispiel der Primarstufe. Leipzig: Klett Grundschulverlag.

Lakoff, G. & Núnez R. E. (2000). Where mathematics comes from – How the embodied mind brings mathematics into being. New York: Basic Books.

Selter, Ch. & Spiegel, H. (1997). Wie Kinder rechnen. Düsseldorf: Klett

Sharan, S. (Ed.) (1994). Handbook on cooperative learning methods. Westport: Greenwood Press.

Slavin, R. E. (1996). Research for the future, research on cooperative learning and achievement: What we know, what we need to know. Contemporary Educational Psychology, 21, 43-69.

Spychiger, M. (1995). Mehr Musikunterricht an den öffentlichen Schulen? Entwicklung eines zeichentheoretisch orientierten Begründungsansatzes als Alternative zur aussermusikalischen Argumentation. Hamburg: Verlag Dr. Kovac.

Spychiger, M. (2001). Understanding musical activity and musical learning as sign processes: Toward a semiotic approach to music education. The Journal of Aesthetic Education, 35(1), 53-67.

Weber, E. & Spychiger, M. & Patry, J.-L. (1993). Musik macht Schule. Biographie und Ergebnisse eines Schulversuchs mit erweitertem Musikunterricht. Essen: Die Blaue Eule.

Wittmann E.C. (1998). Design und Erforschung von Lernumgebungen als Kern der Mathematikdidaktik. Beiträge zur Lehrerbildung, 16(3), 329-342.