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Number Lines as an Instrument for Solving Problem on Relative Values

1. Introduction

In teaching relative values, teachers in Japan usually use number lines because it is believed that they promote understanding of content related to relative values. That is to say, when learning relative values, it is necessary to relate to two actual quantities to the corresponding ratio. The advantage of using number lines is that it visually demonstrates these relationships. However, it is not known if number lines are used as an instrument for thinking by elementary school students, in promoting their understandings of relative values.

The purpose of this study was to elucidate the conditions for understanding number lines by fifth graders. The problems of relative values were classified into the following three types, and data were analyzed for each type. The first type: Compared value and base quantity are already known, and relative value is to be answered. The second type: Base quantity and relative value are already known, and compared quantity is to be answered. The third type: Compared quantity and relative value are already known, and base quantity is to be answered.

2. Study I

Procedure

Participants were 55 fifth graders from two classes of a elementary school. Two question sheets (A and B) were prepared, both of which contain three problems on relative value. The question sheet A was distributed to one of the classes (Group A, $n = 22$), and the question sheet B was distributed to the other class (Group B, $n = 23$). The participants were asked to solve all problems. The three problems on question sheets A and B were exactly the same except that Group A were asked to draw a number line when solving problems, whereas Group B had already printed number lines applicable to the problems on their sheet. The problems presented to participants are as follows.

Problem 1: There is a 30 cm stick. You cut 18 m for work. The length you cut is how many times the length of the original stick? (The first type)

Problem 2: A class consists of a total of 40 students. The number of boys in the class was 0.6 times the total number of the class. How many boys are there in this class? (The second type)

Problem 3: A boy gave 15 marbles to his sister. The number of marbles that the boy gave to his sister was 0.6 times the marbles that he originally had. How many marbles did the boy originally have? (The third type)

Results

The purpose of this study was to examine students' understanding of the three third problem types. Therefore, responses to the problems were evaluated based on the adequacy of the formulae written by students. If the correct formula was used, but a wrong answer was written on the answer sheet due to a miscalculation, it was regarded as a correct answer.

The percentages of question answered correctly in Group A, which was asked to draw a number line was 9 %, 67 %, and 45 % for problems 1, 2, and 3, respectively. On the other hand, question answered correctly in Group B which provided a number line were 87 %, 78 %, and 78 %, respectively.

In Group A, very few students were able to draw appropriate number lines for the presented problems. Many of those students who could not draw appropriate number lines were not able to fill in relative value (ratio) on the number lines for any problem. Although clear conclusions cannot be drawn due to the limited number of participants, it appears that students who were able to draw number lines for any one of the question had higher percentages of correctly answered problems than those who were not able to draw the lines. In addition, the reason for the lower score for problem 1 in Group A is that many students had the misconception that “the bigger value is the dividend and the smaller value is the divisor in division,” and they solved the problem based on this misunderstanding, even though the “dividend” was smaller than the “divisor.”

Discussion

The above results indicated the following two points. Firstly, the comparison between Group A and B indicated that Group B had higher percentages of correct answers for three problems than Group A. Furthermore, in Group A, students who drew appropriate number lines tended to have higher percentages of correctly answered questions for the three problems than those who were not able to draw the lines. These findings indicated that the presentation of a number line is an important instrument in the thought process of problem solving.

Secondly, a few students were able to draw appropriate number lines for Group A, which indicated that the automatic use of number lines is difficult for fifth graders. This suggests that many students do not fully understand

the number line, and that there is a need to consider effective instructional strategies to teach the number line itself.

3. Study II

Procedure

Pre-test, question sheet A used in Study I was given to a fifth grader C. Then, she was taught number lines based on the prepared teaching objectives. After that a post-test, using the same content as the pre-test, was given to C to examine the effect of the instructions. Two fourth graders, K and H, also participated in the instruction session as their participation was necessary to compare each person's height to show and confirm that a ratio between shadow and height is the same for everyone. This paper only reports the results of C.

Teaching Objectives

I Students were asked to fully complete activities to draw a line that was p times a given arbitrary length. Next, they were asked to calculate the values using actual measured values by three type's formulas.

II The structure of the number line indicating the relationship between an actual measured value and a relative value was carefully taught to students.

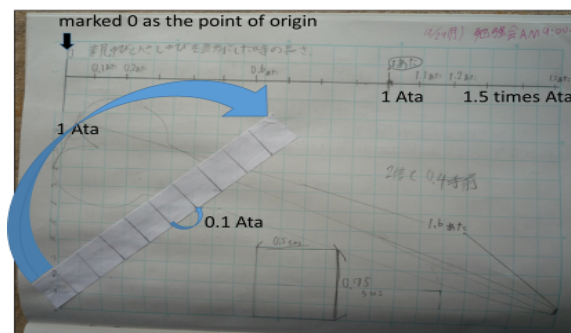
III The second type, obtaining a compared quantity, was introduced first to students.

IV The first and the third types were introduced by expression transformations using the unknown as \square in the formula of the second type.

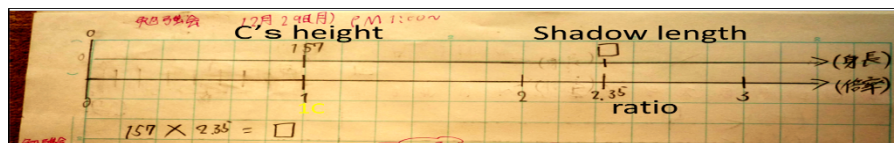
Five Teaching Steps and Teaching Process

Step 1. Teaching Number Lines The students were instructed on how to draw a number line and 0.1 figure, and how to measure a length p times.

Step 2. Making a My Ruler by Themselves The students were asked to make a ruler of themselves by using a paper tape, the height of each student was regarded as 1C (in C's case; the unitary name is the name of each student). The students were instructed that 1C was divided into 10 or 100 equal parts represented as 0.1 or 0.01, respectively.



Step 3. Application to the Second type Formula C also expressed the measured length of her shadow in Step 2 above as a formula, and calculated as $157\text{cm} \times 2.35 = 368.95\text{cm}$ (the shadow was 2.35 times of her length).



Step 4.

Application to the First Type for Obtaining a Ratio The students were asked to calculate the number of times K's height (i.e., 142 cm) would be based on H's height (i.e., 140 cm). First, the ratio was set as \square , and the students were asked to express it on a number line. C wrote and transformed her formulae in the following order: $140 \times \square = 142$, $\square = 142 \div 140$, and $\square = 1.014$.

Step 5. Application to the Third Type for Obtaining Base Quantity Participant C was able to correctly calculate and answer, 140 cm, using the following formula, $\square \times 1.014 = 142$ and $142 \div 1.12 = \square$.

Results

The results of the pre-test indicated that C made errors in writing formulae using \square for the three problems and using number lines inappropriately. Moreover, she wrote incorrect answers for all the problems. This condition was identical to the students in Study I.

The results of the post-test indicated that C was able to write the number line and the formula correctly for Problem 1. For Problem 2, the number line, the formula, and the answer were correct; however, she was not able to write the formula using \square correctly. For problem 3, C was not able to draw a number line; however, she wrote the following formula, $\square = 15 \div 0.6$, $\square = 25$, when instructed about the number line. In this case, C did not have confidence in her answer, because she perceived her answer as too large for the result of division. After the instructor taught C to consider the relationship between the quotient and the dividend when the divisor was less than 1 and over 1, she was able to fully understand the answer.

Discussion

Initially, C was not able to solve word problems at all; however, she came to be able to draw a number line and solve them by writing a formula using \square . This is because she acquired knowledge experientially that her height became the base quantity measured by the My Ruler, and that she projected the My Ruler into the word problems, which made it easier for her to select and confirm the base quantity.