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Notes on the students' solutions of Mathematical Olympiad problems

Introduction. The quality of mathematics education in schools of Latvia can be evaluated by several criteria: on national level – by the results of centralized examination, by diagnostic tests, by students' achievements in educational Olympiads, and in international comparison – by analysis of results of students' assessment programs such as TIMSS and PISA. These statistics identify the major problems in mathematics education.

The level of mathematical knowledge of Latvian students. Last year's (2014) results of centralized examination in mathematics for high school students demonstrate that a considerable large part of them had moderate, superficial mathematical knowledge – the mean score of the results was 43.34%. Especially students' performance in problem solving was poor – the mean score in this part of examination was only 15.10%. Latvian students demonstrate quite an average level of proficiency in mathematics in international comparison as well. In PISA 2012 study, students achieved a mean score 491 which is below the OECD average score 494. The relevant criterion in this assessment was the number of top performers in mathematics. Only 3.7% of participants demonstrated good problem solution skills, which is below the average OECD score 4.4% (OECD, 2014). Searching for ways to fill the gaps in mathematics education and to improve students' mathematical knowledge, the National Centre for Education produces various diagnostic tests and conducts the analysis of results. Some of the most recent results in primary school indicate that students have little experience in solving non-standard problems, which may be due to the lack of such problems in textbooks, as well as to the difficulties that teachers face in applying modern didactic methods (Krastina, Vituma, 2014).

To improve the quality of mathematics education, there were amendments made to mathematics curriculum in elementary education in 2013, emphasizing the students' mastering of mathematical problem solving skills, and the development of thinking. In this scope the teachers' guide books could be supplemented by non-standard mathematics problems and didactic advice to introduce students to the methodology of problem solving.

Considering the fact that the set of Mathematical Olympiad problems differs rather significantly from the tasks included in textbooks, one can find in students' Olympiad works most diverse problem-solving approaches, which to a certain extent characterize their knowledge, way of thinking,

In F. Caluori, H. Linneweber-Lammerskitten & C. Streit (Hrsg.), *Beiträge zum Mathematikunterricht 2015*. Münster: WTM-Verlag

originality of ideas, and views on the solution of the problem. The analysis of these works could help school teachers develop a methodology of mathematics classes.

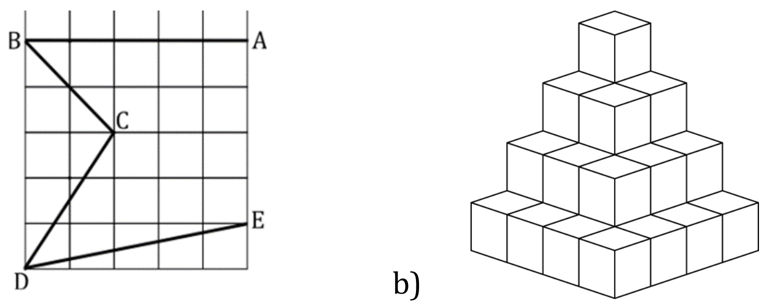
Geometry problems in Open Mathematical Olympiad. Open Mathematical Olympiad (OMO), organized for every student from the 1st till the 12th grade, can test their competences in problem solution. Because of the large number of participants (approximately 3000 in the last few years), students' Olympiad works implicitly reflect the teaching-learning methods used in the schools of Latvia. Some of the works reveal the gaps in the students' mathematical knowledge.

The OMO problem set contains many challenging problems including the tasks of combinatorial and Euclidean geometry. Here the notes on solutions of 3 geometry problems will be presented. Two problems for 6th and 10th grade are of combinatorial type. The third problem for 8th grade can be solved in combinatorial way or by using results of Euclidean geometry.

Problem 1. (OMO39, 6th grade.) Dissect the square into two equal polygons a) hexagons; b) heptagons.

Problem 2. (OMO41, 8th grade.) Line segments AB, BC, CD and DE are constructed in the vertices of orthogonal lattice (see figure 1.a)). Which of the angles is bigger: ∠ABC or ∠ CDE?

a)



b)

Figure 1. a) Problem 2; **b)** Problem 3.

Problem 3. (OMO38, 10th grade.) A tower is constructed from equal cubes of size 1 x 1 x 1 (see figure 1.b). Is it possible to construct such a tower a) from duo-blocks of size 1 x 1 x 2; b) from L – shaped blocks containing 3 cubes?

Students' solutions. Both parts of the first problem can be solved in different ways: case a): joining the opposite sides of square by broken line; case b): joining the opposite vertices. There were 16% of students that drew the answers straight away. 36% of participants solved only case a), 3% solved only case b). A quarter of students did not find any correct dissection. An investigation of drafts made by students showed that they were interested in this problem, sometimes drawing more than 20 figures. Some of the students analyzed the configurations of polyominoes, whereas the sense of the given problem is based on the symmetry principle. There were some creative solutions too: one of the participants dissected every unit square of the given figure into two triangles to determine the correct dissection of the square into hexagons. Another student constructed two equal hexagons inside the square and then filled the empty places to get the answer.

Second problem OMO41 can be solved by the investigation of different configurations of the rectangle of size 2 x 3 unit squares. Such an original approach can be named „a proof without words" (see figure 2).

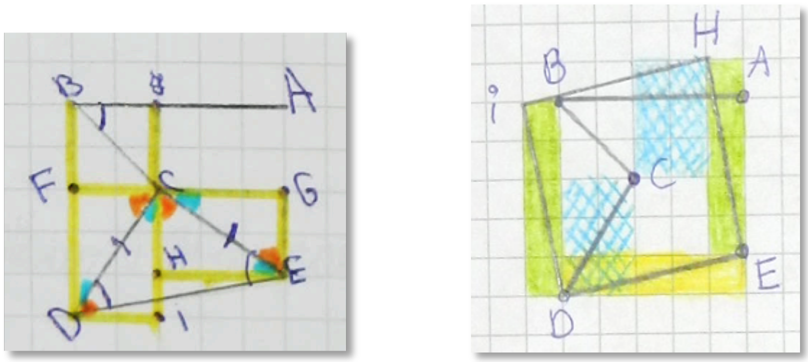


Figure 2. Examples of solution of the problem OMO41.

Using this method, the problem was solved by 4.6% of participants. All of the students who tried to prove the equivalence of given angles using the results of Euclidean geometry did not solve the problem. Some students noted the possibility to turn the angle ∠ CDE, but did not complete the construction to find the similar triangles and to use the Pythagorean theorem.

The 10th grade problem consists of two different parts. 27% of students demonstrated good spatial imagination. They interpreted the construction of L-shaped blocks by coloring all separate layers, or used the enumeration of unit cubes of the tower, or drew 3-dimensional constructions. Case a) need to be proved. Only 7% of students did the correct proof of impossibility of the given construction. One of the shortest ways of solution is the use of the method of invariants by implementation of coloring of the unit cubes in chessboard fashion. Most students tried to research examples of the construction, but none made the full enumeration of all variants. Figure 3 shows a student's experiment to complete the construction by duo-blocks. The question marks suggest to remind the well-known problem about the covering of chessboard by dominoes if two squares are cut out from the same diagonal of the board (Soifer, 2010).

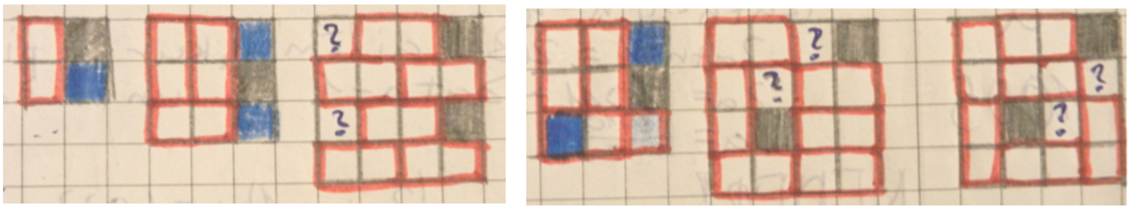


Figure 3. Schematic constructions of the tower by duo-blocks.

Combinatorial geometry problems are problems that almost all participants of Olympiads try to solve. The evaluation of students' works characterizes the differences in their performance and shows the deficiencies that have to be eliminated. A significant part of students are not flexible at the stage of problem analysis: they chose only one way of solution and did not change their viewpoint. Students do not know the main solution methods of geometry tasks. Students have difficulties with mathematically correct argumentation, explanation, and justification. Regardless of the fact that the students did not solve the problems completely, they had interesting, lively ideas that have to be developed.

Conclusion. Many challenging Olympiad problems can be introduced in mathematics classes in an appropriate way. Geometry tasks develop students' imagination and promote students' skills of mathematical argumentation. So the first problem could be to supplement the topic on symmetry, where the initial question, for example, could be: in how many ways can the rectangle be dissected by one or more straight lines? The second problem can be recommended to the topic on the similar triangles, application of Pythagorean theorem, or rotation of figures. The last problem is useful in combinatorial calculations and in mastering heuristic problem solution methods and general reasoning methods.

Literature

Krastina, E., Vituma, M. (2014) *Diagnostic work in mathematics for 6th grade in school year 2013/2014: analysis of results and recommendations.* (in Latvian) URL (26.02.2015):http://visc.gov.lv/vispizglitiba/eksameni/dokumenti/metmat/2013_2014_ddarbs_matem_6kl_met_mat.pdf

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