Essays in Macroeconomics: The Role of Housing for Monetary and Fiscal Policy

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This dissertation is dedicated to my mother Mürüvvet and my father Halim.

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Chapter 1

Introduction

The analysis of monetary and fiscal policy belongs to the central issues in macroeconomics. This thesis provides new insights concerning current monetary and fiscal policy issues by accounting for the role of housing. Recent studies focusing on the interplay of housing and the macroeconomy have outlined the importance of housing along several dimensions, like for business cycles or asset pricing.[1] This thesis analyzes the implications of housing for the conduct of monetary and fiscal policy. In the models used in this thesis, the importance of housing stems from its usage as collateral in the presence of financial market imperfections. When debt repayment cannot be enforced, lenders will request collateral from borrowers. This collateral is typically the house of a borrower. In such a borrower-lender framework underlying each chapter of this thesis, we study the following policy issues.

First, we analyze in chapter 2 the preferential tax treatment of housing, like the deductibility of mortgage interest payments from income that is observed in many industrialized countries. Housing subsidies are shown to be optimal once one accounts for the role of housing as collateral. Second, we quantify in chapter 3 that is coauthored with Andreas Schabert the macroeconomic effects of the Federal Reserve's (Fed) purchases of mortgage-

[1] Research on the interplay of housing and the macroeconomy has gained much attention in recent years. Two recent economics handbooks, The Handbook of Regional and Urban Economics, Volume 5 (2015), and The Handbook of Macroeconomics, Volume 2 (forthcoming), devote a chapter to this topic.

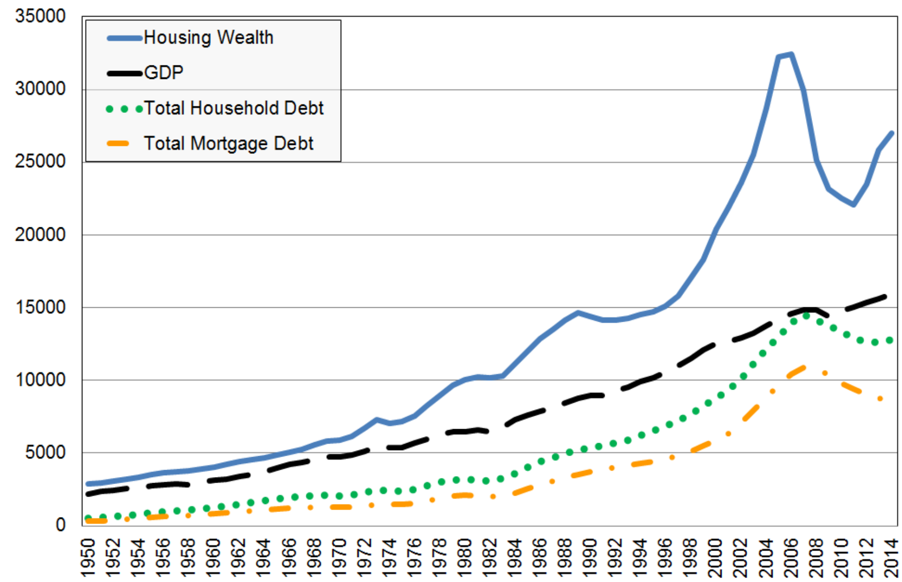

Figure 1.1: Household Wealth, Household Debt, and GDP (in $billions of 2009). Following Iacoviello (2010), we compute housing wealth as the sum of the market value of owner-occupied and rented homes. Data are from the Federal Reserve's Flow of Funds Accounts. All data are deflated.

backed securities, which is one of the unconventional policy measures the Fed used for the first time in its history during the financial crisis. We find that the purchases fostered economic activity. Third, chapter 4 provides a theoretical framework with occasionally binding collateral constraints in which government spending is more effective in recessions compared to expansions consistent with what is found by recent empirical research. Moreover, based on this framework we quantify the differences between government spending multipliers in recessions and expansions and find that these are considerably large.

Figure 1 underlines the importance of housing as component of wealth and in serving as collateral for private loans. We subdivide total household wealth into housing wealth (solid line) and non-housing wealth. We find that in the US housing wealth accounted for about 40% of total household wealth on average between 1950 and 2014 with a maximum of 48% in 2005. Moreover, the ratio of housing wealth to annual GDP (dashed line) was 1.4 on average and 2.3 when it reached its maximum in 2005. Further, total household debt (dotted line) made up 56% of annual GDP on average, with a maximum of almost 100% in 2007, while total mortgage debt (dashed-dotted line) amounted to 38% of GDP on average with a maximum of 73% again in 2007. The ratios of total household debt to GDP and total mortgage debt to GDP as well as the ratio of total mortgage debt to total household debt increased over time. Houses served as collateral for about 70% of the total debt of US households on average between 1950 and 2014. While in 1950 this number was 60%, in 2009 it reached its maximum of 76%. Housing as collateral for private debt has thus gained in importance.

Figure 2 illustrates the relationship between housing wealth and consumption. It shows the annual percentage changes of housing wealth (solid line), aggregate consumption (dashed line), and non-durable consumption (dotted line) for 1960-2014. As the figure suggests, consumption is correlated with housing wealth. The unconditional contemporaneous correlation between aggregate consumption and housing wealth is 0.53 and between non-durable consumption and housing wealth is 0.3.

As will be discussed below, one possible explanation for this correlation of consumption and housing wealth is the collateral effect. As Figure 1 has made clear, private debt is widely collateralized by housing. Hence, if housing wealth changes, this means that the value of collateral and thereby the borrowing capacity of households changes, which in turn affects their consumption possibilities. This collateral effect can be of large importance for the transmission of monetary and fiscal policy, especially when a policy affects housing wealth, which may lead to an amplification of the initial effect of the policy measure. Since typically borrowers have a high marginal propensity to consume out of wealth this amplification may be large.

The reaction of consumption plays a crucial role for the transmission of monetary and fiscal policy. While in neoclassical dynamic stochastic general equilibrium (DSGE) models based on optimizing representative agents and flexible prices, in which consumption is a function of permanent income,

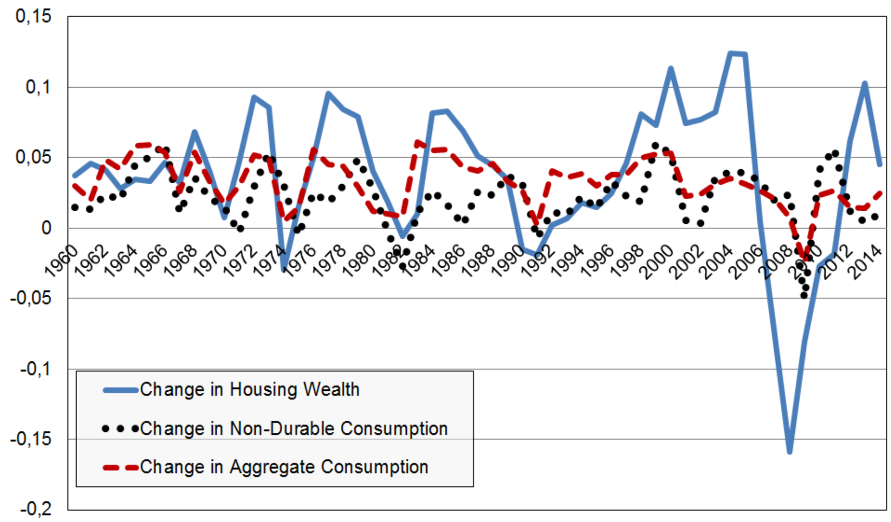

Figure 1.2: Housing Wealth and Consumption. Annual Percentage Changes.

monetary and fiscal policy are largely ineffective, Keynesian models, in which consumption depends on current disposable income, imply large effects of those. Due to these contradictory predictions of the traditional theories, the "New Neoclassical Synthesis" (NNS) integrated Keynesian price stickiness into neoclassical DSGE models with optimizing representative agents. Goodfriend and King (1997) and Linnemann and Schabert (2003) are early examples for the analysis of monetary and fiscal policy, respectively, within these types of frameworks, also called New Keynesian DSGE models.

This approach relaxed the assumption of flexible prices in DSGE models, while another strand in the literature relaxed the assumption of one representative agent to better describe the behavior of aggregate consumption and hence to better understand the effectiveness of monetary and fiscal policy. This strand developed theoretical models based on the study of aggregate consumption behavior following the Euler equation approach of Hall (1978), who derived an Euler equation from the optimizing behavior of a representative rational agent. His theoretical conclusion that the (marginal utility of) consumption follows a random walk with trend and cannot be predicted by lagged variables other than consumption was rejected by the data. To better fit the data, Campbell and Mankiw (1989) considered a model in which half of the households are optimizing and the other half are rule-of-thumb households, who consume all their disposable income in each period. They found that their model can better explain the behavior of aggregate consumption. In a model with such a household sector, Mankiw (2000) analyzed fiscal policy in his famous “savers-spenders theory”. Based on this framework, several papers analyzed fiscal as well monetary policy integrating rule-of-thumb households in NNS models with sticky prices.[2] One can conclude that the inclusion of rule-of-thumb households into the NNS framework has substantial implications for monetary as well as fiscal policy.

Finally, based on the work of Zeldes (1989) and Kiyotaki and Moore (1997), Iacoviello (2004) derived an aggregate Euler equation from a model in which a fraction of households are collateral constrained and their debt is collateralized by housing. Compared to the assumption that rule-of-thumb households have no access to credit markets at all and hence neither save nor borrow, the collateral constraint can be interpreted as an endogenous outcome in the absence of debt enforcement. Iacoviello (2004) found that the value of collateral, i.e. housing, has a non-negligible effect on aggregate consumption through the borrowing capacity of households, as suggested by Figure 2. A general equilibrium version of this model with sticky prices, presented in Iacoviello (2005), confirmed this finding.[3] This new generation of NNS models with household heterogeneity and financial market imperfections was as well recently used for the analysis of monetary and fiscal policy.[4] The inclusion of collateral constrained borrowers implies an important effect – the collateral or housing wealth effect – that was ignored in the

[2]See e.g. Gali et al. (2004), Coenen and Straub (2005), Gali et al. (2007), Cogan et al. (2010), Cwik and Wieland (2011), and Coenen et al. (2013).

[3]Iacoviello (2010) illustrates the relationship between housing wealth and consumption in a theoretical framework and reviews existing empirical literature. He concludes that “a considerable portion of the effect of housing wealth on consumption could reflect the influence of changes in housing wealth on borrowing against such wealth.” (p. 11). For further studies on this issue see e.g. Case et al. (2005), Campbell and Cocco (2007), Carroll et al. (2011), and Mian and Sufi (2011).

[4]See e.g. Monacelli (2008, 2009), Calza et al. (2013), Khan and Reza (2013), Schabert (2014), and Andres et al. (2014, 2015).

previous models.

In each chapter of this thesis, we consider models, which incorporate household sectors consisting of lenders and borrowers who face a collateral constraint. Accounting for the role of housing as collateral, we provide studies concerning current monetary and fiscal policy issues ranging from subsidization of housing, over the Fed's purchases of mortgage-backed securities to the different levels of effectiveness of government spending in recessions and expansions.

The second chapter of this thesis deals with one important issue concerning housing, namely its preferential tax treatment, which can be observed in many industrialized countries. In the US, for instance, total housing subsidies added up to 220 billion dollars in 2011, corresponding to 1.5% of GDP. We find that these subsidies can be justified by means of an optimal taxation approach. We study optimal taxation of housing in the presence of collateral constraints and find that it is optimal to subsidize housing of collateral constrained households to disburden them. This result can be understood as a rationale for the preferential tax treatment of housing. Since the subsidy is financed to the largest extent by a housing tax on patient savers it can be interpreted as redistribution from wealth-rich lenders to wealth-poor borrowers.

As reference case, we also consider a representative agent version of the model, which provides intuitive results that are in line with the principle of optimal taxation that goods with lower elasticities should be taxed at a higher rate, which turns out to be housing in the baseline calibration. Hence, the housing tax rate is positive in the representative agent version, while it is negative for constrained households in the full version. The main result of optimal housing subsidies for constrained households is robust to several parameter variations. This chapter provides a rationale for housing subsidies based on capital market imperfections. In contrast, previous studies have focused on externalities accompanied with homeownership, like its impact on the education of children (Green and White, 1997), which should be internalized through these subsidies. In that sense, it adds a new dimension on this issue and indicates a new path for future research.

The third chapter analyzes an unconventional policy measure – the purchases of mortgage-backed securities (MBS) – that has been used by the Fed for the first time in its history. To counteract the effects of the financial crisis on the economy, the Fed implemented unconventional policy measures, like large-scale asset purchases, also called quantitative easing (QE), since it hit the zero lower bound on the policy rate in the aftermath of the financial crisis.

We analyze the macroeconomic effects of these purchases, which added up to more than $2 trillion in both MBS purchase programs conducted in the first and third round of quantitative easing programs, henceforth referred to as QE1 and QE3. We find that they had considerable expansionary effects on output, consumption and prices providing a rationale for this new type of policy measure.

The analysis of MBS purchases is conducted within a theoretical framework to better understand the mechanisms through which they affect macroeconomic variables. Moreover, since it is the first time that such a policy is conducted, data on it are scarce. To this end, we develop a New Keynesian DSGE model with banks conducting costly financial intermediation. Intermediation costs increase in the amount of loans, which are granted to impatient households and collateralized by housing and hence can be interpreted as MBS and decrease in the amount of reserves, which are supplied by the central bank only in exchange for eligible assets. Besides government bonds, with the announcement of the purchase program during QE1 also MBS became eligible. Hence, by purchasing MBS, the central bank can influence MBS yields, which in turn has effects on the behavior of households and banks.

We calibrate the model to US data and to empirical evidence on yield effects of the observed MBS purchases. Based on this calibration, we compute the effects of the observed MBS purchases in QE1 and QE3. We find expansionary effects of the purchases on macroeconomic aggregates, like GDP, consumption, employment, and inflation. MBS purchases imply a positive wealth effect with regard to impatient borrowers since they reduce MBS yields and increase house prices as well as inflation. This enables impatient households to borrow more and hence to increase consumption and housing. The effectiveness of the program in stimulating consumption, employment and GDP lies in the fact that it affects those households that have a high marginal propensity to consume out of wealth.

In particular, our results indicate that MBS purchases during QE1 led to maximum quarterly increases in GDP, consumption, and employment of 1.12%, 1.13%, and 1.67%, respectively, and to cumulative increases of 2.24%, 2.36%, and 3.36%, respectively. For QE3, we find that the program led to maximum quarterly increases in GDP of 0.86%, in consumption of 0.85%, and in employment of 1.28%. The cumulative increases were 1.62%, 1.69%, and 2.43%, respectively. The smaller effects of QE3 are due to the smaller size of the program. We further conclude that central bank asset purchases have counteracted deflationary effects from the crisis shock.

The analysis in chapter 4 is motivated by growing empirical evidence indicating that fiscal policy is more effective in recessions compared to expansions. It provides a theoretical model with occasionally binding collateral constraints that illustrates a mechanism that makes government spending less effective in expansions. Based on this model, we then quantify the differences in the effectiveness of government spending in recessions and expansions.

We again consider a household sector consisting of patient lenders and collateral constrained impatient borrowers. The collateral constraint plays a crucial role since it is not always binding but occasionally, which leads to a non-linearity. In particular, the collateral constraint becomes slack in expansions, while it binds in recessions. We compute government spending multipliers for expansions and recessions and find that a fiscal stimulus is more effective in a recession than in an expansion. The intuition for this result is that in expansions the collateral constraint that impatient households face is slack. A slack collateral constraint means that the household is at its unconstrained optimum and hence largely insensitive to changes in disposable income. On the contrary, in recessions, impatient households find themselves at their borrowing limit, which means that they consume less than they would if they could borrow more. Thus, in recessions, their decisions are substantially affected by their disposable income.

Calibrating the model to US data, we find an impact multiplier of 1.6 in an average recession and of 1 in an average expansion. Moreover, the cumulative multiplier for 1 year (2 years) is about 1.5 (1.2) in recessions, whereas the corresponding multiplier is about 0.9 (0.7) in expansions. We provide sensitivity analyses for a large range for key parameters and find that the differences in recession and expansion multipliers remain considerably large.

To summarize, the studies in this thesis emphasize the importance of housing for monetary and fiscal policy, especially in serving as collateral for private loans. They show how accounting for this role of housing provides novel insights concerning current monetary and fiscal policy issues ranging from housing subsidies, over the Fed's MBS purchases to the effectiveness of government spending in recessions and expansions.

Chapter 2

Housing, Collateral Constraints, and Fiscal Policy

2.1 Introduction

Housing is subject to a preferential tax treatment in many industrialized countries. In the US, total housing subsidies added up to 220 billion dollars in 2011, corresponding to 1.5% of GDP (US Budget, 2011). Also in European countries the values of total housing subsidies in percent of GDP were in that range, e.g. 0.9% in Germany, 1.1% in France and 1.4% in Spain in 2000 (ECB, 2003).

The two most important housing subsidies are the deductibility of mortgage interest payments from income and the tax exemption of imputed rents on owner-occupied housing. In the US, the former amounted to 105 billion dollars while the latter added up to 38 billion dollars in 2011 (US Budget, 2011). These two subsidies accounted for 65% of total housing subsidies.

However, the view of economists on this preferential tax treatment of housing is controversial. On the one hand, it is criticized by researchers like Poterba (1992) and Gervais (2002), among others, who argue that this treatment leads to a welfare loss since it distorts investment decisions of individuals towards housing. These studies are in line with Rosen who writes in the Handbook of Public Economics that "paternalism and political considerations seem to be the sources of this policy" (1985, p. 380).

On the other hand, there are proponents of this treatment who argue that homeownership is accompanied with externalities which are internalized through these subsidies. For instance, Green and White (1997) stress the positive impact of homeownership on the education of children and DiPasquale and Glaeser (1999) state that homeowners are "better citizens" in the sense that they are more involved in local organizations.

In contrast to these papers, this chapter gives a rationale for housing subsidies based on market imperfections.[1] We assume that private loans are not enforceable and, therefore, have to be collateralized by housing. Looking at the data makes the importance of housing as a component of wealth and the relevance of its usage as collateral clear. First, housing makes up a large part of total household wealth as well as total national wealth. In the US, housing wealth accounts for almost half of total household wealth and is larger than annual GDP with an average ratio of housing wealth to GDP of about 1.5 from 1952-2008 (Iacoviello, 2009). Secondly, in 2010 residential mortgage debt amounted to 77% of GDP in the US and to 47% in Germany, to 41% in France and to 64% in Spain (Hypostat, 2010). To the best of our knowledge, this work is the first one that studies optimal taxation of housing in the presence of collateral constraints.

The structure of the model is as follows. We consider a household sector that relates to Kiyotaki and Moore's (1997) model with two types of agents who differ in their discount factors, patient and impatient ones. Due to this difference in patience we get lenders, the patient agents, and borrowers, the impatient ones, in equilibrium. While for the former the collateral constraint is irrelevant in equilibrium, it is of importance for the latter. As in Iacoviello (2005), housing plays a dual role for households. First, it delivers utility to the agents together with consumption and leisure and secondly, private loans are collateralized by housing. The government, that is assumed to have

[1] This chapter is based on Polattimur (2013b). A shortened version was published as Polattimur (2013a).

access to a commitment technology, has exogenous expenditures that have to be financed by two taxes, a housing property tax that can differ for the two types of agents and a labor income tax. The different housing tax rates for the two types can be understood as follows. The patient households for whom the collateral constraint is irrelevant will always own a larger house than the impatient ones and, therefore, are taxed at another rate than the impatient and hence wealth-poor agents.

The main result of this chapter is that it provides a rationale for housing subsidies. In the presence of collateral constraints, optimal fiscal policy should subsidize housing of the impatient households, for whom the collateral constraint is relevant, to disburden them. This subsidy has to be financed to the largest extent by a housing tax on the patient households and to a smaller part by a labor income tax. Hence, this can be interpreted as redistribution from wealth-rich patient households with a higher housing stock to wealth-poor impatient households with a lower housing stock.

The main result of housing subsidies for impatient households is robust to several parameter variations and can be attributed for the most part to the collateral constraint. To illustrate this point, we analyze the effects of the discount rate difference of the types of agents on housing subsidies in comparison to the effects of the collateral constraint, with the result that the former plays a minor role.

We also consider a representative agent version of the model as reference case. Thereby we can understand how the inclusion of a durable good, housing, per se affects optimal fiscal policy compared to standard models. Furthermore, this allows us to compare the results of the representative agent version to existing literature. The representative agent version delivers intuitive results that are in line with the principle of optimal taxation that goods with lower elasticities should be taxed at a higher rate. For the benchmark calibration, this turns out to be housing and, hence, the housing tax rate is positive in the representative agent version, while it is negative for collateral constrained agents in the full version.

This study further relates to the work of Eerola and Määttänen (2009) that considers optimal taxation of housing in a dynamic representative agent model with fairly general preferences and an extended tax system compared to the model of this chapter. However, the results of the representative agent version of our model are compatible with their results. Another closely related paper is Monacelli (2008) that considers a model with two types of agents with different patience rates and collateral constraints similar to the one of this chapter. While Monacelli analyzes optimal *monetary* policy in that framework, he points out that also the analysis of optimal fiscal policy in such a model would be of interest, which is done in this chapter.

The rest of the chapter is organized as follows. In Section 2.2, the model with two types of agents, firms and the government is described, the Ramsey problem is set up and the equilibrium conditions for the steady state are derived. In Section 2.3, the results for the full as well as the representative agent version are presented and a sensitivity analysis is given. Section 2.4 concludes.

2.2 The Model

In this section, we present the model with a household sector consisting of two types of agents, a production sector consisting of two types of firms and the government. Concerning the household sector, we follow Kiyotaki and Moore (1997), who pioneered the models with two types of agents, patient and impatient ones, resulting in an equilibrium with lenders and borrowers. We assume that private debt contracts are not enforceable and have to be collateralized by housing as in Iacoviello (2005). Therefore, a household can only borrow up to a fraction m of his expected end of period housing wealth. Additionally to its usage as collateral, housing delivers utility together with consumption and leisure.

Like in Favilukis et al. (2012), we consider a two-sector production side, such that both housing demand and supply are modeled explicitly. There are two types of firms, one of which produces non-durable consumption goods and the other durable housing.

The government levies a flat-rate tax on labor income and a housing property tax that can differ for the two types of agents and issues one-period bonds to finance an exogenous stream of government expenditures. It has no access to lump-sum taxes. The reason why housing tax rates can differ is that a patient household will own a larger house than an impatient one. Hence, rather than taxing degrees of patience differently, we can understand this as taxing the ones with a larger house at a higher rate than the ones with a smaller house. Due to the usage of housing as collateral that is only relevant for the borrowers, who will be the impatient agents in equilibrium, we will see that the housing tax rates will differ markedly.

2.2.1 Households

There is a continuum of households consisting of two types, patient and impatient ones. They differ in their discount factors $1 > \beta > \beta' > 0$ with β being the discount factor of patient and β' of impatient households. Henceforth, variables of patient (impatient) households are denoted without (with) a prime, while aggregate variables are denoted with a superscript T (e.g. c_t^T, for total consumption). The population share of patient households is s. Borrowing between the two types of households is modeled as follows. A household can borrow an amount $\frac{b_t}{1+r_{t-1}} < 0$ in period $t-1$ and has to pay back b_t in period t, where r_{t-1} is the real interest rate on loans between $t-1$ and t. Since we assume that private debt contracts are not enforceable, there is a limit on private debt given by a fraction m of the expected end of period housing wealth

$$b_{t+1}^{(\prime)} \geq -m p_{h,t+1} h_t^{(\prime)}, \tag{2.1}$$

where m denotes the exogenous pledgeable fraction of housing, $p_{h,t+1}$ the end of period real price of housing, and $h_t^{(\prime)}$ the household's housing stock. As we will see below, this constraint will become relevant for impatient households, while it will be irrelevant for patient ones.

Both types of households derive utility from consumption $c_t^{(\prime)}$ and housing $h_t^{(\prime)}$ and disutility from labor $n_t^{(\prime)}$ and maximize the infinite sum of expected utility. Their objective is given by

$$\sum_{t=0}^{\infty} \beta^{(\prime)t} u(c_t^{(\prime)}, h_t^{(\prime)}, n_t^{(\prime)}). \tag{2.2}$$

We consider the following CRRA-specification of the utility function

$$u(c_t, h_t, n_t) = \frac{c_t^{1-\mu^c}}{1-\mu^c} + \frac{h_t^{1-\mu^h}}{1-\mu^h} - \frac{n_t^{1+\mu^n}}{1+\mu^n}, \tag{2.3}$$

where $\mu^{c(h)}$ denotes the inverse of the intertemporal elasticity of substitution in consumption (housing) and μ^n the inverse of the Frisch elasticity of labor supply.

Patient Households

The representative patient household generates income from working $w_t n_t$, with w_t being the real wage rate and the return of bond holdings b_t^g. Labor income is taxed at the rate τ_t^n. Every period the household can adjust its stock of housing according to $h_t - (1-\delta_h)h_{t-1}$ at the price of housing $p_{h,t}$, with δ_h being the depreciation rate of housing. The value of the housing stock owned by the household in period t, $p_{h,t}h_t$, is taxed at the rate τ_t^h. Thus, we consider a housing property tax, that is proportional to the value of the current housing stock and is paid every period. The budget constraint of the patient households is given by

$$\begin{aligned} & c_t + p_{h,t}\left[\left(1+\tau_t^h\right)h_t - (1-\delta_h)h_{t-1}\right] + \frac{b_{t+1}^g}{R_t^g} + \frac{b_{t+1}}{R_t} \\ = \quad & \left(1-\tau_t^n\right) w_t n_t + b_t^g + b_t, \end{aligned} \tag{2.4}$$

where c_t denotes consumption, $\frac{b_{t+1}^g}{R_t^g}$ investment in new government bonds with the relating gross interest rate $R_t^g = 1 + r_t^g$ and b_t privately issued debt with the gross interest rate $R_t = 1 + r_t$. The patient household will hold positive amounts of $b_t^g > 0$ and $b_t > 0$ and hence be the lender in equilibrium. That's why the collateral constraint (2.1) will be irrelevant for patient households: $b_{t+1} > 0 > -mp_{h,t+1}h_t$.

Impatient Households

The budget constraint of the representative impatient household analogously reads

$$
\begin{aligned}
& c_t' + p_{h,t}\left[\left(1+\tau_t'^h\right)h_t' - (1-\delta_h)h_{t-1}'\right] + \frac{b_{t+1}'^g}{R_t^g} + \frac{b_{t+1}'}{R_t} \qquad (2.5)\\
= \quad & (1-\tau_t^n)\, w_t n_t' + b_t'^g + b_t'.
\end{aligned}
$$

Since we rule out short sales in government bonds, the impatient households will set $b_{t+1}'^g = b_t'^g = 0$. Furthermore, this type will be the private borrower in equilibirium, i.e. $b_{t+1}' = -\frac{s}{1-s}b_{t+1} < 0$, following from the market clearing condition for private debt $(1-s)\,b_{t+1}' + sb_{t+1} = 0$. Hence, the collateral constraint (2.1) will become relevant here. Therefore, there is a a limit on the obligations of impatient households which is given by $b_{t+1}' \geq -mp_{h,t+1}h_t'$.

2.2.2 Government

The government levies a flat-rate tax on labor income τ_t^n and a housing property tax $\tau_t^{(\prime)h}$ and issues one-period bonds ($b_t^{(\prime)g} \geq 0\ \forall t \geq 0$) to finance an exogenous stream of government expenditures (g_t):

$$
g_t - \frac{b_{t+1}^g}{R_t^g} + b_t^g = s\tau_t^h p_{h,t}h_t + (1-s)\,\tau_t'^h p_{h,t}h_t' + \tau_t^n w_t n_t^T, \qquad (2.6)
$$

where $n_t^T = sn_t + (1-s)\,n_t'$ denotes total labor supply. As mentioned before, the different housing tax rates τ_t^h and $\tau_t'^h$ can be understood as taxing the wealthier agents which will be the patient households in equilibrium at a rate that differs from (and will be higher than) the one for the wealth-poor impatient households which will own smaller houses in equilibrium.

2.2.3 Firms

The production side of the economy is characterized by two sectors, one of which produces consumption goods y_c and the other housing y_h. In both sectors, there is a continuum of firms, which are assumed to produce with the same technology for simplicity. The representative firm of each sector produces its output with labor according to

$$\begin{aligned} y_{c,t} &= n_{c,t}^{T} \\ y_{h,t} &= n_{h,t}^{T}, \end{aligned}$$

where total labor input in both sectors sum up to total labor demand $n_t^{T,D} = n_{c,t}^T + n_{h,t}^T$, which meets total labor supply given by $n_t^{T,S} = sn_t + (1-s)\, n_t'$ in equilibrium leading to $sn_t + (1-s)\, n_t' = n_{c,t}^T + n_{h,t}^T = n_t^T$. Labor is assumed to be totally mobile between the two sectors leading to a wage rate that is the same for both sectors.

2.2.4 Competitive Equilibrium

We now describe the competitive equilibrium of the private sector and then set up the Ramsey problem.

Patient Households

A patient household chooses the values of c_t, h_t, n_t, b_{t+1}^g and b_{t+1} to maximize (2.2) subject to the budget constraint (2.4) leading to the first order conditions

$$\begin{aligned} h_t^{-\mu^h} &= \left(1+\tau_t^h\right) p_{h,t} c_t^{-\mu^c} && (2.7) \\ &\quad -\beta c_{t+1}^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} \\ n_t^{\mu^n} &= \left(1-\tau_t^n\right) w_t c_t^{-\mu^c} && (2.8) \\ c_t^{-\mu^c} &= \beta R_t^g c_{t+1}^{-\mu^c} && (2.9) \\ c_t^{-\mu^c} &= \beta R_t c_{t+1}^{-\mu^c}. && (2.10) \end{aligned}$$

Equation (2.7) describes housing demand. In the optimum, the marginal utility of current housing $h_t^{-\mu^h}$ equals the marginal utility of foregone consumption $c_t^{-\mu^c}$ at the gross price of housing $\left(1+\tau_t^h\right) p_{h,t}$ less the discounted marginal utility of next period's consumption $\beta c_{t+1}^{-\mu^c}$ achieved from selling the house after depreciation $(1-\delta_h)$ at the price $p_{h,t+1}$. Equation (2.8), that is fairly standard, describes labor supply of a patient household and equates the marginal rate of substitution between consumption and leisure $\frac{n_t^{\mu^n}}{c_t^{-\mu^c}}$ to the net real wage rate $(1-\tau_t^n)\,w_t$. Equations (2.9) and (2.10) are Euler equations with respect to public and private lending.

Impatient Households

An impatient household chooses the values of c'_t, h'_t, n'_t and b'_{t+1} to maximize (2.2) subject to the budget constraint (2.5) *and* the collateral constraint (2.1) leading to the first order conditions

$$\begin{aligned} h_t'^{-\mu^h} &= \left(1+\tau_t^h\right) p_{h,t} c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} \qquad (2.11)\\ &\quad +\omega_t m p_{h,t+1} \\ n_t'^{\mu^n} &= \left(1-\tau_t^n\right) w_t c_t'^{-\mu^c} \qquad (2.12)\\ \omega_t &= \frac{c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} R_t}{R_t} \qquad (2.13)\end{aligned}$$

and the complementary slackness conditions

$$\omega_t \left(b'_{t+1} + m p_{h,t+1} h'_t\right) = 0,\ \ b'_{t+1} + m p_{h,t+1} h'_t \geq 0,\ \ \omega_t \geq 0.$$

Equation (2.11) describes housing demand of an impatient household. The term $\omega_t m p_{h,t+1}$ stems from the collateral constraint, with ω_t being the multiplier on this constraint. Equation (2.12) is the labor supply function of an impatient household. Equation (2.13) is the modified Euler equation resulting from the fact that the impatient household is borrowing constrained. In the steady state, the collateral constraint will be binding as we can see from (2.10) which becomes $\frac{1}{R} = \beta$ and (2.13) leading to $\omega = c'^{-\mu^c}\left(1/R - \beta'\right) = c'^{-\mu^c}\left(\beta - \beta'\right) > 0$. Finally, from the complementary slackness conditions we get $b' + m p_h h' = 0 \Leftrightarrow b' = -m p_h h'$.

Furthermore, the transversality conditions $\lim_{t\to\infty} \beta^t u_t^c \frac{-b_{t+1}^g}{R_t^g} = 0$ and $\lim_{t\to\infty} \beta^t u_t^c \frac{b'_{t+1}}{R_t} = 0$ must hold, of which the latter is redundant due to the collateral constraint that is more restrictive.

Firms

In both sectors, the representative firm maximizes profits according to

$$\max_{n_{c,t}^T} \Pi_{c,t} = \max_{n_{c,t}^T} \left(n_{c,t}^T - w_t n_{c,t}^T \right)$$

in the final consumption goods sector and

$$\max_{n_{h,t}^T} \Pi_{h,t} = \max_{n_{h,t}^T} \left(p_{h,t} n_{h,t}^T - w_t n_{h,t}^T \right)$$

in the housing sector leading to the first order conditions

$$w_t = 1 \text{ and } p_{h,t} = 1.$$

Aggregate Resource Constraint

Finally, due to identical production technologies and perfect mobility of labor between the two sectors, the aggregate resource constraint is given by (see Appendix 2.5.1)

$$c_t^T + g_t + p_{h,t} h_t^T = y_{c,t} + p_{h,t} y_{h,t} + (1 - \delta_h) p_{h,t} h_{t-1}^T. \tag{2.14}$$

2.2.5 The Ramsey Problem

We assume that the government has access to a commitment technology and is able to bind itself to its policy. The government chooses the values of h_t, c_t, n_t, h'_t, c'_t, n'_t and the tax rates τ_t^h, $\tau_t'^h$ and τ_t^n in order to maximize social welfare subject to the private sector equilibrium conditions, the resource and the implementability constraint summarized in Appendix 2.5.2, while financing an exogenous stream of government expenditures $\{g_t\}_{t=0}^{\infty}$. Following Monacelli (2008), in this economy with two types of agents, social welfare is measured by the weighted sum of utility of the two types

$$\sum_{t=0}^{\infty} \beta^t s u(c_t, h_t, n_t) + \beta'^t (1 - s) u(c'_t, h'_t, n'_t)$$

and the aggregate discount rate is defined as $\widetilde{\beta} = \beta^s \beta'^{(1-s)}$ to be used as discount rate for the constraints. For the mathematical formulation of the Ramsey problem see Appendix 2.5.2, where also the first-order conditions of the Ramsey problem and the steady state are derived.

2.3 Results

This section presents and discusses optimal taxation results of the model. First, as a natural starting point of the analysis, results for the representative agent version, which can be derived analytically, will be given. The relation of these results to existing literature on optimal taxation will be discussed. Then, numerical results for the full version of the model will be given and compared with the results of the representative agent version in order to point out the role of the collateral constraint. Finally, we will compare the role of the difference in discount rates against the role of the collateral constraint and present sensitivity analyses.

2.3.1 Representative Agent Version

By setting the discount rate of the impatient agents equal to the one of the patient agents, $\beta' = \beta$, the model collapses to a representative agent version. For this version, we can derive analytical solutions for the steady state tax rates which are the labor income tax τ^n and the housing property tax τ^h.

The optimal steady state tax rate on labor income is given by (see Appendix 2.5.3)

$$\tau^n = \frac{\phi\left(\mu^n + \mu^c\right)}{1 + \phi\left(1 + \mu^n\right)}$$

and is positive for $\phi > 0$. It only depends on the multiplier on the implementability constraint, $\phi \geq 0$, and the parameters μ^c and μ^n.

The optimal steady state tax rate on housing is given by (see Appendix 2.5.3)

$$\tau^h = \frac{\phi}{\underbrace{1 - \phi\left(\mu^h - 1\right)}_{>0}} \underbrace{\left(\mu^h - \mu^c\right)}_{(i)} \underbrace{\left(1 - \beta\left(1 - \delta_h\right)\right)}_{(ii)}. \tag{2.15}$$

This equation reflects two features of housing: (i) can be attributed to the fact that housing delivers utility like consumption and (ii) to the durability of housing. For $\phi > 0$, the sign of τ^h, thus whether housing should be taxed or subsidized, only depends on the term (i) in (2.15). The term (ii) in (2.15) can be neglected for the sign of τ^h, since $1 - \underbrace{\beta(1-\delta_h)}_{\in(0,1)}$ is positive. Further, the analysis has to be restricted to values of $\mu^h < \frac{1}{\phi} + 1$ implying $\phi\left(\mu^h - 1\right) < 1$, which ensures that the second derivatives are negative resulting in maxima (see Appendix 2.5.3). Thus, the first term in (2.15) is as well irrelevant for the sign of τ^h.

Hence, whether housing should be taxed or subsidized only depends on the inverses of the intertemporal elasticities of substitution μ^c and μ^h. From principles of optimal taxation, we know that goods with lower elasticities should be taxed at a higher rate. Since we do not consider a consumption tax at all, which means a zero tax on consumption, whether housing should be taxed or subsidized depends on whether its intertemporal elasticity of substitution is lower or higher than the one of consumption. There are three cases:

1. For $\mu^c = \mu^h$, housing and consumption should be treated identically due to identical intertemporal elasticities of substitution, leading to an optimal tax rate on housing of zero.

2. If the elasticity of housing is smaller than the one of consumption, i.e. $\frac{1}{\mu^c} > \frac{1}{\mu^h} \Leftrightarrow \mu^c < \mu^h$, the optimal housing tax rate is positive.

3. For $\mu^c > \mu^h$, the optimal housing tax rate is negative since the elasticity of consumption is smaller than the one of housing, $\frac{1}{\mu^c} < \frac{1}{\mu^h}$.

These results are compatible with the ones of Eerola and Määttänen (2009) who consider a more general representative agent framework with capital and optimal taxation of capital in addition to housing.

While the term (ii) in (2.15) is irrelevant for the sign of τ^h, it has a large effect on the size of it. For the baseline calibration (see Table 2.1), for instance, it reduces the housing tax by more than 97%. However, the higher δ_h is, i.e. the lower the durability of housing is, the smaller is the impact of (ii) on the size of τ^h. Notice, that (ii) disappears for the case $\delta_h = 1$, where the durability of housing is assumed away and housing fully depreciates within one period.

2.3.2 Results of the Full Version

Since analytical results are not available for the full version, we consider numerical results for the steady state, where the collateral constraint is binding, as we have seen before in Section 2.2.4. For comparison, we also give numerical results for the representative agent version in the baseline calibration.

Calibration

In this section, the baseline calibration of the model is described. Following Iacoviello (2005), one time period is set to one quarter and the discount factor of patient households to $\beta = 0.99$ leading to a steady state gross real interest rate of $R = 1.01$, which is equivalent to an annual real interest rate of 4%. The discount factor of impatient households is set to $\beta' = 0.95$ by Iacoviello (2005) as a compromise of the estimates given in the literature, which is adopted here. However, in section 2.3.3, we will consider a variation in β' between 0.95 and 0.97 to see how this affects the results. In order to get a wage share of patient households equal to $\frac{swn}{swn+(1-s)wn'} = 0.64$ as in Iacoviello (2005), we set $s = 0.62$, while we will also show in the sensitivity analyses 2.3.4 how a variation in population shares alters the results. Furthermore, we set the pledgeable fraction of housing to $m = 0.55$ resulting from an estimation of Iacoviello (2005). Hence, an impatient agent can only borrow up to 55% of the value of his house. We will also consider in section 2.3.3, how a variation in m between 0 and 1, which covers all relevant values for m, affects the results. The depreciation rate of housing is set following Davis and Heathcote (2005), who estimate an annual rate of 1.41%. According to this, we set $\delta_h = 0.0035$ for a quarter.

In the calibration of the utility parameters μ^c and μ^n we follow King and Rebelo (1999), who say that the basic RBC model with log utility in consumption implies a labor supply elasticity of 4. Hence, we set $\mu^c = 1$ and $\mu^n = 1/4$, while we will also conduct robustness checks for both of these parameters in section 2.3.4.

Since the aim of the chapter is to evaluate optimal taxation of housing, the utility parameter of housing μ^h is calibrated in order to match an empirical fact on housing. According to Iacoviello (2009), where some stylized facts about housing, that should be matched when calibrating models of housing, are listed, total housing stock is on average 1.5 times as large as annual GDP in the US between 1952 and 2008. Therefore, we set the parameter μ^h in order to match this value. Since in the model one time period is one quarter and hence y in the notation of the model denotes quarterly GDP, we have to multiply this value by four and to match the ratio of the value of the total housing stock to quarterly GDP of $\frac{h^T}{y} = 6$ (since $p_{h,t} = 1$). This is achieved by setting $\mu^h = 1.75$ leading to an elasticity of $\frac{1}{\mu^h} = 4/7$. Nevertheless, we will also give sensitivity results concerning the parameter μ^h in section 2.3.4.

For the calibration of governmental variables g and b^g we use data from the Worldbank.[2] In 2010, US general government final consumption expenditures amounted to 17% of annual GDP. Since both, government expenditures and GDP are flow variables, the ratio is the same for a time period of one quarter, $\frac{g}{y} = 0.17$. Moreover, US total central government debt made up 76.8% of annual GDP in 2010. Since government debt is a stock variable, this value again has to be multiplied by four. Hence, the ratio we have to match in terms of quarterly GDP is given by $\frac{b^g}{y} = 3$. These values of the governmental variables are achieved by setting $g = 0.172$ and $b^g = 3.1$. The baseline parameter calibration is summarized in Table 2.1.

Given this parameter calibration we compute the steady state numerically, which delivers the optimal values of consumption, housing, and labor for both types of agents as well as the optimal tax rates τ^h, τ'^h and τ^n.

[2] Data from Worldbank web page: "Central Government Debt, Total" and "General Government Final Consumption Expenditure" on http://data.worldbank.org/.

Table 2.1: Baseline Parameter Calibration.

Description	Source/Target	Parameter	Value
Discount factor patient households	Iaco. 2005	β	0.99
Discount factor impatient households	Iaco. 2005	β'	0.95
Pledgeable fraction of housing	Iaco. 2005	m	0.55
Depreciation rate of housing	D&H 2005	δ_h	0.0035
Share of the patient households	wage share = 0.64	s	0.62
Inverse of Frisch elasticity	K&R 1999	μ^n	1/4
Inverse of IES in consumption	K&R 1999	μ^c	1
Inverse of IES in housing	$h^T/y = 6$	μ^h	1.75
Government expenditures	$g/y = 0.17$	g	0.172
Government debt	$b^g/y = 3$	b^g	3.1

Numerical Results

The results of the full and the representative agent version for the baseline calibration are summarized in Table 2.2. Notice, that the optimal tax rate on housing in the representative agent version is close to zero but still positive ($\tau^h = 0.2\%$), while for the full version, we get two housing tax rates that differ both markedly from zero. The optimal housing tax rate for patient households is $\tau^h = 1.65\%$ and the one for impatient households $\tau'^h = -2.72\%$. Thus, for the baseline calibration, it is optimal to subsidize housing of impatient and hence constrained households and to tax patient ones in the full version, while in the representative agent version housing is taxed at a positive rate close to zero. Hence, the subsidy for impatient households results from the heterogeneity in patience rates and the collateral constraint, that are absent in the representative agent version.

To see how this subsidy optimally is financed, we consider the government budget (2.6) in the steady state

$$g + (1-\beta)\, b^g = \tau^n n^T + s\tau^h h + (1-s)\, \tau'^h h'. \tag{2.16}$$

Expenditures are given by $g + (1-\beta)\, b^g = 0.203$ and revenues by $\tau^n n^T + s\tau^h h + (1-s)\, \tau'^h h' = 0.1887 + 0.0668 - 0.0526 = 0.203$. We see, that the labor income tax finances government expenditures, while the housing subsidy for impatient households is financed for the most part by a housing tax on the patient households. Therefore, the housing tax rate on the patient households is much larger than the tax rate on housing in the representative agent version. This point becomes clearer, when we consider the case $g = b^g = 0$ (last column of Table 2.2). For this case, the left hand side of the government budget (2.16) is zero, $g + (1-\beta)\, b^g = 0$, and we observe a large decline in the labor income tax rate. On the right hand side of (2.16), we have revenues from taxing labor income equal to $\tau^n n^T = 0.029$, revenues from taxing housing of patient households given by $s\tau^h h = 0.069$ and housing subsidies for impatient households equal $(1-s)\,\tau'^h h' = -0.098$. We see that the largest part, more than 70%, of housing subsidies, is financed by taxing housing of patient households. Hence, this can be interpreted as a redistribution from wealth-rich, i.e. patient households with a higher housing stock ($h = 6.5$), to wealth-poor households with a lower housing stock ($h' = 5.1$).

To link these results to the empirical findings described in the introduction, we compute the ratio of total housing subsidies to GDP given by $\frac{-(1-s)\tau'^h h'}{swn+(1-s)wn'}$. For the baseline calibration, we get a ratio of 5.24%. Hence, according to the model the granted subsidies in the US that added up to 1.5% of GDP in 2011 seem to be lower than what would be optimal. On the other hand, the model is likely to overestimate housing subsidies since it does not incorporate physical capital. Housing is the only component of wealth in the model, while in the US it accounts for half of total household wealth (see e.g. Iacoviello, 2009).

2.3.3 Discounting vs. Collateral Constraint

The result of subsidizing impatient agents' housing is due to two features of the model, as we have seen in the previous section, the different discount rates of the two types and the collateral constraint, while the former is necessary for the latter. Without different discount rates, the model collapses to the representative agent version where private borrowing and hence the collateral constraint are irrelevant.

The aim of this section is to analyze how these two features affect housing

Table 2.2: Numerical Results - Comparison.

Version	Repr. Agent	Full Version	
Calibration	Baseline	Baseline	$g = b^g = 0$
c	0.8161	0.7999	0.9485
h	9.6310	6.5323	7.4249
n	1.0218	1.0630	1.0954
c'	–	0.8316	1.0136
h'	–	5.0929	6.6759
n'	–	0.9100	0.8398
τ^n	0.1795	0.1878	0.0296
τ^h	0.0020	0.0165	0.0149
τ'^h	–	−0.0272	−0.0388

subsidies. Therefore, we first define the two effects related to these two features. Housing subsidies stemming from the collateral constraint that are granted by the Ramsey planner in order to soften the constraint and hence originate from the market friction are attributed to the *collateral effect*, whereas housing subsidies that purely stem from the difference in discounting and hence are based on preferences are attributed to the *discount rate effect.* To identify how housing subsidies are affected by these two effects, we conduct the following experiment. First, we consider a variation in the pledgeable fraction of housing, m, reaching from 0 to 1 and illustrate in Figure 2.1 how this affects the housing tax rates τ^h and τ'^h, private debt given by $(1-s)\,mh'$, the difference in housing stocks of the two agents, $h-h'$, the tightness of the collateral constraint measured by $\omega = c'^{-\mu^c}\left(\beta - \beta'\right)$ (see (2.13)) and redistribution measured by the ratio of revenues from taxing housing of the patient agents to the subsidies that impatient agents receive: $Red = -\frac{sh\tau^h}{(1-s)h'\tau'^h}$. The plots are given for the benchmark calibration but with equal population shares, $s = 0.5$, for convenience in aggregation. Then, we do the same for a variation in the borrowers' discount rate between $\beta' = 0.95$ and $\beta' 0.97$.

First, consider the lower limit, $m = 0$, where private borrowing is zero and hence the collateral effect is shut down (see Iacoviello (2005) for a similar experiment). Since the link between borrowing and housing of the impatient

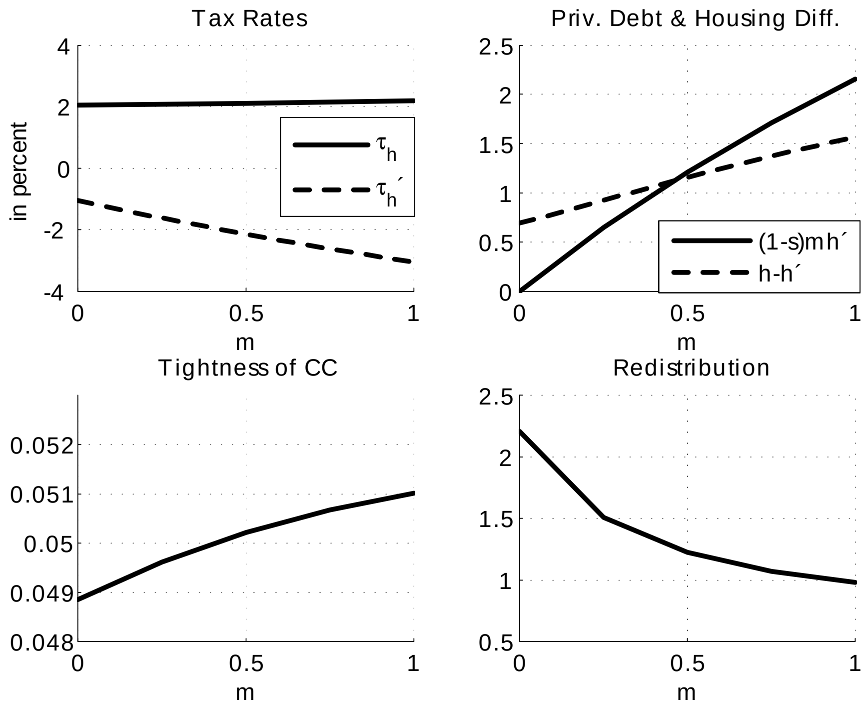

Figure 2.1: Effects of varying the pledgeable fraction of housing m for the baseline calibration with $s = 0.5$.

household is cut off, in this case, the resulting level of subsidies is only due the discount rate effect. Then, the variation in m between the lower and the upper limit, $m = 1$, where housing is fully pledgeable, illustrates the role of the collateral effect compared to the discount rate effect for a given $\beta' = 0.95$. Figure 2.1 shows that a higher pledgeable fraction of housing leads to a larger amount of private debt (panel 2) and hence to a tighter collateral constraint (due to lower consumption) (panel 3) resulting in a higher level of housing subsidies for the constrained households (panel 1, dashed line), whereas the tax rate on the patient agents does not change much (panel 1, solid line). This is due to the fact that the collateral constraint and hence the parameter m is not directly relevant for the patient agents. Thus, the level of redistribution (panel 4), as it is measured here, decreases in m since

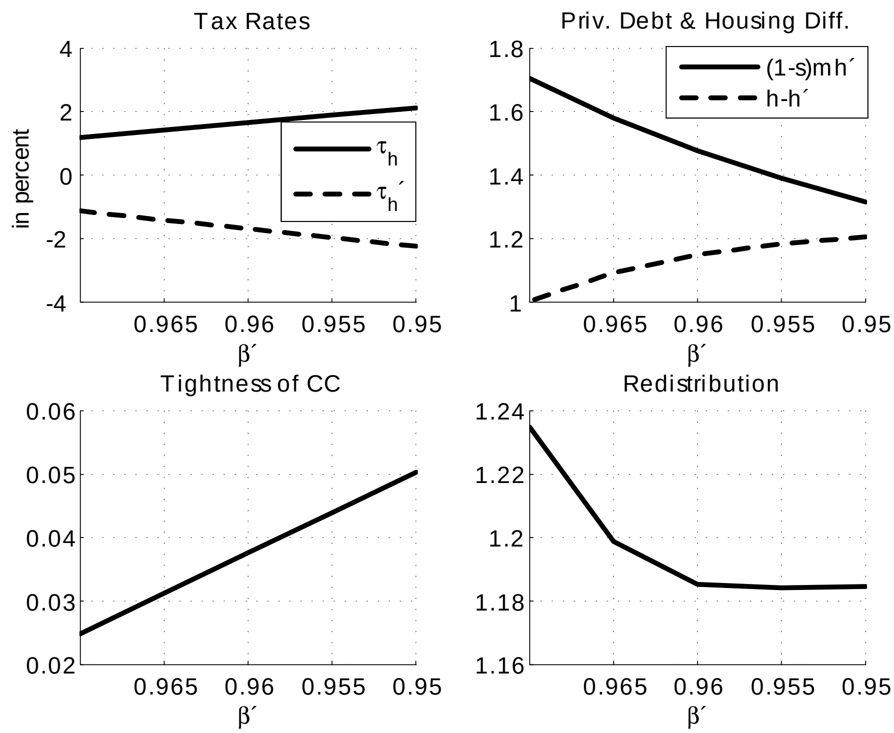

Figure 2.2: Effects of varying the impatient agents' discount rate β' for the baseline calibration with $s = 0.5$.

housing subsidies to impatient agents rise faster than housing tax revenues from patient ones do.

For $m = 0$, where the collateral channel is shut down, the resulting subsidy is $\tau'^h = -1.04\%$, whereas for the baseline case of $m = 0.55$ it more than doubles to $\tau'^h = -2.24\%$. This makes clear that housing subsidies not only result from a difference in preference parameters but are also due to the market friction, the collateral constraint. Regarding the rates just mentioned and taking into account that the discount rate channel dampens the effect of the collateral channel, which is discussed below, more than half of the resulting subsidies can be attributed to the collateral constraint in the baseline calibration.

Figure 2.2 plots the results for a variation in β'. Notice that β' decreases, i.e. the difference in discount rates increases from left to right on the abscissa. The higher this difference is, the larger is the housing subsidy for impatient agents τ'^h (panel 1, dashed line) and the housing tax for patient agents τ^h (panel 1, solid line). In contrast to the variation in m, the variation in β' affects both rates equally. As for a higher m, the level of redistribution (panel 4) decreases in the difference in discount rates for the same reason. In contrast, unlike a higher m leading to higher borrowing, a larger discount rate difference lowers borrowing since it reduces housing of the impatient agents. Hence, we can conclude that the discount rate effect dampens the collateral effect since it reduces private borrowing. Here, the collateral constraint becomes tighter due to the larger difference in discount rates ($\omega = c'^{-\mu^c} \left(\beta - \beta'\right)$).

2.3.4 Sensitivity Analyses

In the previous section, we have seen that the main result of optimality of housing subsidies to impatient agents is robust to variations in the parameters m and β'. In this section, we will check whether it is also robust to changes in the parameters μ^c, μ^h and s. Two interesting questions come here in mind. The first question is, what happens if the intertemporal elasticities are changed such that $\mu^h < \mu^c$. Since we have seen that this changed the sign of the housing tax in the representative agent version, it is interesting to see how this change will affect optimal taxation in the full version. Another question we will explore is how changing the share of lenders s affects the results. We will consider the case where both types have equal shares $s = 0.5$. Table 2.3 summarizes the results.

First of all, we can conclude from Table 2.3, that for every parameter variation we consider, it remains optimal to subsidize housing of impatient households and to tax housing of patient ones.

In the third column where we lower μ^h, housing demand rises and both types have higher housing stocks ($\frac{h^T}{y} \approx 8.2$) compared to the baseline calibration in column 2 of Table 2.3. Although τ^h is lower, tax revenues from taxing housing of patient agents are higher due to their higher housing stock

Table 2.3: Numerical Results - Robustness.

	Baseline Calibration, except			
	$-$	$\mu^h = 1.5$	$\mu^c = 2$	$s = 0.5$
c	0.7999	0.7933	0.8713	0.7950
h	6.5323	9.2023	6.8915	5.9908
n	1.0630	1.0965	1.1581	1.0595
c'	0.8316	0.8396	0.8945	0.8198
h'	5.0929	6.8126	5.6412	4.7861
n'	0.9100	0.8739	0.9383	0.9370
τ^n	0.1878	0.1882	0.2124	0.1935
τ^h	0.0165	0.0150	0.0124	0.0212
τ'^h	-0.0272	-0.0281	-0.0366	-0.0224

$h = 9.2$. Therefore, subsidies for impatient households can increase slightly.

In column 4, we set $\mu^c = 2 > \mu^h = 1.75$. We see that, in contrast to the representative agent version, there is no important change in the tax rates. Moreover, τ'^h becomes larger while τ^h decreases, since households attach a higher value to housing compared to consumption. As a result, both types work more to own a larger house, while the labor income tax increases.

In column 5 the share of lenders in the economy is lower than in the baseline calibration. This means that there are less wealth-rich households in the economy who finance the housing subsidy. Therefore, the tax rates τ^n and τ^h are higher while the subsidy τ'^h is lower. As a result, both types of households have lower consumption and housing levels.

Summing up, in every variation we considered, m, β', μ^h and s, the main result of the chapter holds. It is optimal to disburden the impatient and hence constrained households by subsidizing their housing.

2.4 Conclusion

Housing subsidies that can be observed in many industrialized countries have been subject to macroeconomic research since many years. Nevertheless, there is no definite conclusion one can draw from this research. While the opponents point at inefficiencies resulting from housing subsidies due to distortions in investment decisions of agents, the proponents argue that subsidies internalize externalities accompanied with homeownership.

This chapter, in which we have studied optimal taxation of housing in a borrower-lender framework resulting from different discount rates with housing being used as collateral for private loans, provides results in favor of housing subsidies. The main result of this chapter is that optimal fiscal policy should disburden impatient borrowers by subsidizing their housing in the presence of collateral constraints. We find that this subsidy has to be financed to the largest extent by a housing tax on the patient and unconstrained households and to a smaller part by a labor income tax. Hence, a redistribution from patient/unconstrained households to impatient/constrained ones takes place.

In this framework housing subsidies result from two features of the model, the different discount rates of the two types of agents and the collateral constraint. We have seen that for the baseline calibration more than half of the subsidy can be attributed to the collateral constraint. Hence, housing subsidies not only result from the difference in preference parameters but rather are due to the market friction in our model. Moreover, the sensitivity analyses showed that the main result of housing subsidies for constrained households is robust to several parameter variations.

Furthermore, we considered a representative agent version of the model, which delivers intuitive results in line with the principles of optimal taxation that goods with lower elasticities should be taxed at a higher rate. In our benchmark calibration, this turns out to be housing. Hence, the housing tax rate is positive in the representative agent version, while it is negative for constrained households in the full version.

This chapter provides a rationale for housing subsidies other than externalities that have been focused on in previous studies and indicates a new path for further research. An extension of the model could be the addition of inter-generational heterogeneity in an overlapping generations model as in Gervais (2002). The life cycle behavior of agents could also have substantial implications and should also be accounted for when trying to measure the effects of housing subsidies on social welfare. This is left for future research.

2.5 Appendix

2.5.1 Aggregate Resource Constraint

Consolidation of the budget constraints (2.4), (2.5) and (2.6) delivers

$$
\begin{aligned}
& sc_t + (1-s)\, c'_t + sp_{h,t}\left[\left(1+\tau_t^h\right) h_t - (1-\delta_h)h_{t-1}\right] \\
& + (1-s)\, p_{h,t}\left[\left(1+\tau_t'^h\right) h'_t - (1-\delta_h)h'_{t-1}\right] + g_t \\
= \; & s\,(1-\tau_t^n)\, w_t n_t + (1-s)\,(1-\tau_t^n)\, w_t n'_t \\
& + s\tau_t^h p_{h,t} h_t + (1-s)\, \tau_t'^h p_{h,t} h'_t,
\end{aligned}
$$

since the terms b_t, b'_t and b_t^g cancel out. With $x_t^T = sx_t + (1-s)\, x'_t$ for aggregate variables this becomes to

$$
\begin{aligned}
& c_t^T + p_{h,t}\left[\left(1+\tau_t^h\right) h_t^T - (1-\delta_h)h_{t-1}^T\right] + g_t \\
= \; & (1-\tau_t^n)\, w_t n_t^T + \tau_t^h p_{h,t} h_t^T,
\end{aligned}
$$

which can further be simplified to

$$
\begin{aligned}
& c_t^T + p_{h,t} h_t^T + g_t \\
= \; & w_t n_t^T + p_{h,t}(1-\delta_h)h_{t-1}^T.
\end{aligned}
$$

Inserting the production functions, we get (2.14).

2.5.2 Solution of the Full Version

Summary of Private Sector Equilibrium Conditions

Summarizing the private sector equilibrium conditions delivers

$$
\begin{aligned}
h_t^{-\mu^h} &= \left(1+\tau_t^h\right) p_{h,t} c_t^{-\mu^c} - \beta c_{t+1}^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} \\
n_t^{\mu^n} &= \left(1-\tau_t^n\right) w_t c_t^{-\mu^c} \\
c_t^{-\mu^c} &= \beta R_t c_{t+1}^{-\mu^c} \\
c_t^{-\mu^c} &= \beta R_t^g c_{t+1}^{-\mu^c} \\
h_t'^{-\mu^h} &= \left(1+\tau_t'^h\right) p_{h,t} c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} \left(1-\delta_h\right) p_{h,t+1} + \omega_t m p_{h,t+1} \\
n_t'^{\mu^n} &= \left(1-\tau_t^n\right) w_t c_t'^{-\mu^c} \\
\omega_t &= \frac{c_t'^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c} R_t}{R_t} \\
c_t' &+ \left(1+\tau_t'^h\right) p_{h,t} h_t' = \left(1-\tau_t^n\right) w_t n_t' + (1-\delta_h) p_{h,t} h_{t-1}' - \frac{b_{t+1}'}{R_t} + b_t' \\
0 &= \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} \left(R_i^g\right)^{-1}\right) \left[g_t - s\tau_t^h p_{h,t} h_t - (1-s)\,\tau_t'^h p_{h,t} h_t' - \tau_t^n w_t n_t^T\right] + b_0^g \\
y_{c,t} &= n_{c,t}^T, \quad y_{h,t} = n_{h,t}^T, \quad w_t = 1, \quad p_{h,t} = 1 \\
h_t^T &= s h_t + (1-s)\, h_t', \quad n_t^T = s n_t + (1-s)\, n_t', \;\; c_t^T = s c_t + (1-s)\, c_t' \\
n_t^T &= n_{c,t}^T + n_{h,t}^T, \quad b_{t+1}' \geq -m p_{h,t+1} h_t', \\
c_t^T &+ g_t + p_{h,t} h_t^T = y_{c,t} + p_{h,t} y_{h,t} + (1-\delta_h) p_{h,t} h_{t-1}^T.
\end{aligned}
$$

Eliminating prices and using $\frac{\beta^t c_t^{-\mu^c}}{c_0^{-\mu^c}} = \prod_{i=0}^{t-1} \left(R_i^g\right)^{-1}$ the conditions above can be reduced to

$$
\begin{aligned}
h_t^{-\mu^h} &= \left(1+\tau_t^h\right) c_t^{-\mu^c} - \beta\left(1-\delta_h\right) c_{t+1}^{-\mu^c} \\
n_t^{\mu^n} c_t^{\mu^c} &= \left(1-\tau_t^n\right) \\
R_t^g &= R_t = \frac{c_t^{-\mu^c}}{\beta c_{t+1}^{-\mu^c}} \\
h_t'^{-\mu^h} &= \left(1+\tau_t'^h\right) c_t'^{-\mu^c} - \beta'\left(1-\delta_h\right) c_{t+1}'^{-\mu^c} + m\left[\frac{c_t'^{-\mu^c}}{c_t^{-\mu^c}} \beta c_{t+1}^{-\mu^c} - \beta' c_{t+1}'^{-\mu^c}\right] \\
n_t'^{\mu^n} c_t'^{\mu^c} &= \left(1-\tau_t^n\right) \\
c_t' &+ \left(1+\tau_t'^h\right) h_t' = \left(1-\tau_t^n\right) n_t' + (1-\delta_h) h_{t-1}' + \frac{m h_t'}{c_t^{-\mu^c}} \beta c_{t+1}^{-\mu^c} - m h_{t-1}' \\
0 &= \sum_{t=0}^{\infty} \left(\frac{\beta^t c_t^{-\mu^c}}{c_0^{-\mu^c}}\right) \left[g_t - s\tau_t^h h_t - (1-s)\,\tau_t'^h h_t' - \tau_t^n n_t^T\right] + b_0^g \\
s c_t &+ (1-s)\, c_t' + g_t + s h_t + (1-s)\, h_t' \\
&= s n_t + (1-s)\, n_t' + \left(1-\delta_h\right) \left(s h_{t-1} + (1-s)\, h_{t-1}'\right)
\end{aligned}
$$

given $b_0^g > 0$ and $b_0 > 0$.

The Ramsey Problem

The Ramsey problem reads

$$J=\sum_{t=0}^{\infty}\left\{\begin{array}{c}\beta^{t}su(c_{t},h_{t},n_{t})+\beta'^{t}\left(1-s\right)u(c'_{t},h'_{t},n'_{t})\\ +\widetilde{\beta}^{t}\lambda_{t,1}\left[h_{t}^{-\mu^{h}}-\left(1+\tau_{t}^{h}\right)c_{t}^{-\mu^{c}}+\beta\left(1-\delta_{h}\right)c_{t+1}^{-\mu^{c}}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,2}\left[n_{t}^{\mu^{n}}c_{t}^{\mu^{c}}-1+\tau_{t}^{n}\right]+\widetilde{\beta}^{t}\lambda_{t,3}\left[n_{t}'^{\mu^{n}}c_{t}'^{\mu^{c}}-1+\tau_{t}^{n}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,4}\left[\begin{array}{c}h_{t}'^{-\mu^{h}}-\left(1+\tau_{t}'^{h}\right)c_{t}'^{-\mu^{c}}+\beta'\left(1-\delta_{h}\right)c_{t+1}'^{-\mu^{c}}\\ -m\left(c_{t}'^{-\mu^{c}}c_{t}^{\mu^{c}}\beta c_{t+1}^{-\mu^{c}}-\beta'c_{t+1}'^{-\mu^{c}}\right)\end{array}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,5}\left[\begin{array}{c}-c'_{t}-\left(1+\tau_{t}'^{h}\right)h'_{t}+\left(1-\tau_{t}^{n}\right)n'_{t}+(1-\delta_{h})h'_{t-1}\\ +\frac{mh'_{t}}{c_{t}^{-\mu^{c}}}\beta c_{t+1}^{-\mu^{c}}-mh'_{t-1}\end{array}\right]\\ +\widetilde{\beta}^{t}\lambda_{t,6}\left[\begin{array}{c}-sc_{t}-\left(1-s\right)c'_{t}-g_{t}-sh_{t}-\left(1-s\right)h'_{t}\\ +sn_{t}+\left(1-s\right)n'_{t}+\left(1-\delta_{h}\right)\left(sh_{t-1}+\left(1-s\right)h'_{t-1}\right)\end{array}\right]\\ +\beta^{t}\lambda_{7}\frac{c_{t}^{-\mu^{c}}}{c_{0}^{-\mu^{c}}}\left[\begin{array}{c}g_{t}-s\tau_{t}^{h}h_{t}-\left(1-s\right)\tau_{t}'^{h}h'_{t}\\ -\tau_{t}^{n}\left(sn_{t}+\left(1-s\right)n'_{t}\right)\end{array}\right]+\beta^{t}\lambda_{7}b_{0}^{g}\end{array}\right\}$$

where $\lambda_{t,i}$ denotes the Lagrange multiplier on constraint i in period t, while the multiplier λ_7 on the intertemporal government budget constraint, which is derived below, has no time index since it is an intertemporal constraint. The first order conditions of the Ramsey problem are as well derived below, where also the steady state of the problem is given.

Intertemporal Government Budget Constraint

The intertemporal government budget constraint is derived as follows. We write the government budget (2.6) for $t+1$ and solve for

$$b_{t+1}^{g}=s\tau_{t+1}^{h}p_{h,t+1}h_{t+1}+\left(1-s\right)\tau_{t+1}'^{h}p_{h,t+1}h'_{t+1}+\tau_{t+1}^{n}w_{t+1}n_{t+1}^{T}-g_{t+1}+\frac{b_{t+2}^{g}}{R_{t+1}^{g}}$$

and insert this in the one for t

$$
\begin{aligned}
g_t \quad - \quad & \frac{1}{R_t^g}\left[\begin{array}{c} s\tau_{t+1}^h p_{h,t+1}h_{t+1} + (1-s)\,\tau_{t+1}'^h p_{h,t+1}h_{t+1}' \\ +\tau_{t+1}^n w_{t+1}n_{t+1}^T - g_{t+1} + \frac{b_{t+2}^g}{R_{t+1}^g} \end{array}\right] + b_t^g \\
= \quad & s\tau_t^h p_{h,t}h_t + (1-s)\,\tau_t'^h p_{h,t}h_t' + \tau_t^n w_t n_t^T .
\end{aligned}
$$

This can be rewritten as

$$
\begin{aligned}
g_t \quad + \quad & \frac{g_{t+1}}{R_t^g} - \frac{b_{t+2}^g}{R_t^g R_{t+1}^g} + b_t^g = s\tau_t^h p_{h,t}h_t + (1-s)\,\tau_t'^h p_{h,t}h_t' \\
& + \frac{s\tau_{t+1}^h p_{h,t+1}h_{t+1} + (1-s)\,\tau_{t+1}'^h p_{h,t+1}h_{t+1}'}{R_t^g} + \tau_t^n w_t n_t^T + \frac{\tau_{t+1}^n w_{t+1}n_{t+1}^T}{R_t^g}.
\end{aligned}
$$

Iterating on this we get with the transversality condition on government debt the intertemporal government budget constraint

$$
\begin{aligned}
& \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right) g_t + b_0^g \\
= \quad & \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right) s\tau_t^h p_{h,t}h_t + (1-s)\,\tau_t'^h p_{h,t}h_t' + \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right)\tau_t^n w_t n_t^T \\
\Leftrightarrow \quad & \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right)\left[g_t - s\tau_t^h p_{h,t}h_t - (1-s)\,\tau_t'^h p_{h,t}h_t' - \tau_t^n w_t n_t^T\right] + b_0^g = 0.
\end{aligned}
$$

Inserting (2.44) delivers the form used in the formulation of the Ramsey problem.

First Order Conditions and Steady State

The first order conditions of the Ramsey problem can be summarized by:

$$
\begin{aligned}
\lambda_{t,1} c_t^{-\mu^c} + \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} s h_t &= 0 \\
\lambda_{t,2} + \lambda_{t,3} - \lambda_{t,5} n_t' - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} n_t^T &= 0 \\
\lambda_{t,4} c_t'^{-\mu^c} + \lambda_{t,5} h_t' + \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} (1-s) h_t' &= 0 \\
\widehat{\beta}^t \frac{s c_t}{\mu^c} + \lambda_{t,1} \left(1 + \tau_t^h\right) + \lambda_{t,2} n_t^{\mu^n} c_t^{2\mu^c} - \lambda_{t,4} m c_t'^{-\mu^c} \beta c_{t+1}^{-\mu^c} c_t^{2\mu^c} & \\
+\lambda_{t,5} m h_t' \beta c_{t+1}^{-\mu^c} c_t^{2\mu^c} - \lambda_{t,6} \frac{s c_t^{\mu^c+1}}{\mu^c} - \widehat{\beta}^t \frac{\lambda_7}{c_0^{-\mu^c}} \begin{bmatrix} g_t - s\tau_t^h h_t \\ -(1-s)\tau_t'^h h_t' - \tau_t^n n_t^T \end{bmatrix} & \\
-\widehat{\beta} \lambda_{t-1,1} (1-\delta_h) + \widehat{\beta} \lambda_{t-1,4} m c_{t-1}'^{-\mu^c} c_{t-1}^{\mu^c} - \widehat{\beta} \lambda_{t-1,5} m h_{t-1}' c_{t-1}^{\mu^c} &= 0 \\
\widehat{\beta}^t h_t^{-\mu^h} - \lambda_{t,1} \frac{\mu^h}{s} h_t^{-\mu^h-1} - \lambda_{t,6} - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} \tau_t^h + \lambda_{t+1,6} \widetilde{\beta} (1-\delta_h) &= 0 \\
-\widehat{\beta}^t n_t^{\mu^n} + \lambda_{t,2} \frac{\mu^n}{s} c_t^{\mu^c} n_t^{\mu^n-1} + \lambda_{t,6} - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} \tau_t^n &= 0 \\
\overline{\beta}^t \frac{(1-s) c_t'}{\mu^c} + \lambda_{t,3} n_t'^{\mu^n} c_t'^{2\mu^c} + \lambda_{t,4} \left[\left(1 + \tau_t'^h\right) + m c_t^{\mu^c} \beta c_{t+1}^{-\mu^c}\right] & \\
-\frac{\lambda_{t,5}}{\mu^c c_t'^{-\mu^c-1}} - \frac{\lambda_{t,6} (1-s)}{\mu^c c_t'^{-\mu^c-1}} - \overline{\beta} \lambda_{t-1,4} [1 - \delta_h + m] &= 0 \\
\overline{\beta}^t h_t'^{-\mu^h} - \lambda_{t,4} \frac{\mu^h}{(1-s)} h_t'^{-\mu^h-1} - \frac{\lambda_{t,5}}{(1-s)} \begin{bmatrix} \left(1 + \tau_t'^h\right) \\ -m c_t^{\mu^c} \beta c_{t+1}^{-\mu^c} \end{bmatrix} - \lambda_{t,6} & \\
-\widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} \tau_t'^h + \widetilde{\beta} \lambda_{t+1,5} \frac{(1 - \delta_h - m)}{(1-s)} + \widetilde{\beta} \lambda_{t+1,6} (1-\delta_h) &= 0 \\
-\overline{\beta}^t n_t'^{\mu^n} + \lambda_{t,3} \frac{\mu^n}{(1-s)} c_t'^{\mu^c} n_t'^{\mu^n-1} + \lambda_{t,5} \frac{(1-\tau_t^n)}{(1-s)} & \\
+\lambda_{t,6} - \widehat{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} \tau_t^n &= 0
\end{aligned}
$$

with $\widehat{\beta}^t = \frac{\beta^t}{\widetilde{\beta}^t} = \left(\frac{\beta}{\beta^s \beta'^{(1-s)}}\right)^t = \left[\left(\frac{\beta}{\beta'}\right)^{1-s}\right]^t$ and $\overline{\beta}^t = \frac{\beta'^t}{\widetilde{\beta}^t} = \left(\frac{\beta'}{\beta^s \beta'^{(1-s)}}\right)^t = \left[\left(\frac{\beta'}{\beta}\right)^s\right]^t$.

Assuming that we are initially in the steady state ($c_0 = c$ for $t = 0$), where variables without subscript denote steady state values henceforth, these conditions read in the steady state

$$
\begin{aligned}
\lambda_1 c^{-\mu^c} + \lambda_7 sh &= 0 \quad (2.17)\\
\lambda_2 + \lambda_3 - \lambda_5 n' - \lambda_7 n^T &= 0 \quad (2.18)\\
\lambda_4 c'^{-\mu^c} + \lambda_5 h' + \lambda_7 (1-s) h' &= 0 \quad (2.19)\\
\frac{sc}{\mu^c} + \lambda_1 \left[1 + \tau^h - \widehat{\beta}(1-\delta_h)\right] + \lambda_2 (1-\tau^n) c^{\mu^c} & \\
+\lambda_4 m c'^{-\mu^c} c^{\mu^c} \left(\widehat{\beta} - \beta\right) + \lambda_5 m h' c^{\mu^c} \left(\beta - \widehat{\beta}\right) - \lambda_6 \frac{s c^{\mu^c+1}}{\mu^c} & \quad (2.20)\\
-\lambda_7 c^{\mu^c} \left[g - s\tau^h h - (1-s)\tau'^h h' - \tau^n n^T\right] &= 0 \quad (2.21)\\
h^{-\mu^h} \left(1 - \lambda_1 \frac{\mu^h}{sh}\right) + \lambda_6 \left[\widetilde{\beta}(1-\delta_h) - 1\right] - \lambda_7 \tau^h &= 0 \quad (2.22)\\
-n^{\mu^n} + \lambda_2 \frac{\mu^n (1-\tau^n)}{sn} + \lambda_6 - \lambda_7 \tau^n &= 0 \quad (2.23)\\
\frac{(1-s) c'}{\mu^c} + \lambda_3 (1-\tau^n) c'^{\mu^c} + \lambda_4 \begin{bmatrix} 1 + \tau'^h + m\beta \\ -\overline{\beta}(1-\delta_h + m) \end{bmatrix} & \\
-\frac{\lambda_5}{\mu^c c'^{-\mu^c-1}} - \frac{\lambda_6 (1-s)}{\mu^c c'^{-\mu^c-1}} &= 0 \quad (2.24)\\
\lambda_5 \left[\frac{\widetilde{\beta}(1-\delta_h - m) - 1 - \tau'^h + m\beta}{1-s}\right] & \\
+h'^{-\mu^h} \left(1 - \lambda_4 \frac{\mu^h}{(1-s) h'}\right) + \lambda_6 \left[\widetilde{\beta}(1-\delta_h) - 1\right] - \lambda_7 \tau'^h &= 0 \quad (2.25)\\
-n'^{\mu^n} + \lambda_3 \frac{\mu^n (1-\tau^n)}{(1-s) n'} + \lambda_5 \frac{(1-\tau^n)}{(1-s)} + \lambda_6 - \lambda_7 \tau^n &= 0. \quad (2.26)
\end{aligned}
$$

The private sector equilibrium conditions that determine the steady state together with the first order conditions of the Ramsey problem (2.17)-(2.26) are given by

$$
\begin{aligned}
h^{-\mu^h} &= c^{-\mu^c}\left[\left(1+\tau^h\right)-\beta\left(1-\delta_h\right)\right] && (2.27)\\
n^{\mu^n}c^{\mu^c} &= \left(1-\tau^n\right) && (2.28)\\
R^g &= R=\frac{1}{\beta}\\
h'^{-\mu^h} &= c'^{-\mu^c}\left[\begin{array}{c}\left(1+\tau'^h\right)-\beta'\left(1-\delta_h\right)\\ +m\left(\beta-\beta'\right)\end{array}\right] && (2.29)\\
n'^{\mu^n}c'^{\mu^c} &= \left(1-\tau^n\right) && (2.30)\\
c' &= n'\left(1-\tau^n\right)+h'\left[m\left(\beta-1\right)-\delta_h-\tau'^h\right] && (2.31)\\
g+\left(1-\beta\right)b^g &= s\tau^h h+\left(1-s\right)\tau'^h h'+\tau^n\left(sn+\left(1-s\right)n'\right) && (2.32)\\
sc+\left(1-s\right)c'+g &= sn+\left(1-s\right)n'-\delta_h sh-\delta_h\left(1-s\right)h'. && (2.33)
\end{aligned}
$$

2.5.3 Representative Agent Version

Solution

The first order conditions of the representative household are given by (with $u_t^x=\frac{\partial u}{\partial x_t}$)

$$
\begin{aligned}
u_t^h+\beta u_{t+1}^c\left(1-\delta_h\right)p_{h,t+1} &= u_t^c\left(1+\tau_t^h\right)p_{h,t} && (2.34)\\
u_t^n &= -u_t^c w_t\left(1-\tau_t^n\right) && (2.35)\\
u_t^c &= u_{t+1}^c\beta R_t^g && (2.36)
\end{aligned}
$$

and the transversality condition on bonds holds $\lim_{t\to\infty}\beta^t u_t^c\frac{b_{t+1}^g}{R_t^g}=0$. Inserting (2.36) in (2.34) delivers the relationship between the marginal utilities of housing and consumption

$$
\frac{u_t^h}{u_t^c}=\left(1+\tau_t^h\right)p_{h,t}-\frac{\left(1-\delta_h\right)}{R_t^g}p_{h,t+1}. \quad (2.37)
$$

The first order conditions of the firms lead to the real wage rate $w_t=1$and the price of housing $p_{h,t}=1$ and the aggregate resource constraint reads

$$c_t + g_t + h_t - (1-\delta_h)h_{t-1} = n_t \tag{2.38}$$

The Ramsey problem is to maximize social welfare subject to the aggregate resource constraint (2.38) and the implementability constraint (2.46), which is derived below, and can be written as

$$\begin{aligned} J &= \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t, h_t, n_t) + \phi \left[u_t^c c_t + u_t^h h_t + u_t^n n_t \right] \\ +\rho_t \left[-c_t - g_t - h_t + n_t + (1-\delta_h)h_{t-1} \right] \end{array} \right\} \\ &\quad + \phi \left[(1-\delta_h) p_{h,0} h_{-1} + b_0^g \right]. \end{aligned}$$

Insertion of the marginal utilities leads to

$$\begin{aligned} J &= \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t, h_t, n_t) + \phi \left\{ c_t^{1-\mu^c} + h_t^{1-\mu^h} - n_t^{1+\mu^n} \right\} \\ +\rho_t \left[-c_t - g_t - h_t + n_t + (1-\delta_h)h_{t-1} \right] \end{array} \right\} \\ &\quad + \phi \left[(1-\delta_h) p_{h,0} h_{-1} + b_0^g \right]. \end{aligned}$$

The first order conditions of the Ramsey problem are given by

$$\frac{\partial J}{\partial c_t} = 0 \quad \Rightarrow \quad \rho_t = c_t^{-\mu^c} \left[1 + \phi \left(1 - \mu^c \right) \right] \tag{2.39}$$

$$\frac{\partial J}{\partial n_t} = 0 \quad \Rightarrow \quad \rho_t = n_t^{\mu^n} \left[1 + \phi \left(1 + \mu^n \right) \right]. \tag{2.40}$$

Equating (2.39) and (2.40) we get the optimal labor income tax

$$\begin{aligned} (1-\tau_t^n) &\overset{(2.35)}{=} \frac{n_t^{\mu^n}}{c_t^{-\mu^c}} = \frac{\left[1 + \phi \left(1 - \mu^c \right) \right]}{\left[1 + \phi \left(1 + \mu^n \right) \right]} \qquad (2.41) \\ \Rightarrow \tau^n &= 1 - \frac{\left[1 + \phi \left(1 - \mu^c \right) \right]}{\left[1 + \phi \left(1 + \mu^n \right) \right]} = \frac{\phi \left(\mu^n + \mu^c \right)}{1 + \phi \left(1 + \mu^n \right)} > 0 \text{ for } \phi > 0. \end{aligned}$$

The first order condition with respect to housing is given by

$$\begin{aligned}
\frac{\partial J}{\partial h_t} = 0 \quad & \Rightarrow \quad h_t^{-\mu^h} + \phi\left(1-\mu^h\right) h_t^{-\mu^h} - \rho_t + \beta\rho_{t+1}\left(1-\delta_h\right) = 0 \\
& \Leftrightarrow \quad \rho_t = h_t^{-\mu^h} + \phi\left(1-\mu^h\right) h_t^{-\mu^h} + \rho_{t+1}\beta\left(1-\delta_h\right) \\
& \Rightarrow \quad {}^{(2.39)} c_t^{-\mu^c}\left[1+\phi\left(1-\mu^c\right)\right] = h_t^{-\mu^h} + \phi\left(1-\mu^h\right) h_t^{-\mu^h} \\
& \qquad + c_{t+1}^{-\mu^c}\left[1+\phi\left(1-\mu^c\right)\right]\beta\left(1-\delta_h\right) \\
& \Rightarrow \quad \left[1+\phi\left(1-\mu^c\right)\right] = \underbrace{\frac{h_t^{-\mu^h} + c_{t+1}^{-\mu^c}\beta\left(1-\delta_h\right)}{c_t^{-\mu^c}}}_{\left(1+\tau_t^h\right)} \\
& \qquad + \phi\left(1-\mu^h\right)\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} + \underbrace{\frac{c_{t+1}^{-\mu^c}\beta}{c_t^{-\mu^c}}}_{1/R_t^g}\phi\left(1-\mu^c\right)\left(1-\delta_h\right).
\end{aligned}$$

Hence, the optimal tax rate on housing can be written as

$$\begin{aligned}
\tau_t^h &= \phi\left(1-\mu^c\right) - \phi\left(1-\mu^h\right)\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} - \frac{\phi\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g} \\
&= \phi\left[1-\mu^c - \left(1-\mu^h\right)\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} - \frac{\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&= {}^{(2.43)}\phi\left[1-\mu^c - \left(1-\mu^h\right)\left[\left(1+\tau_t^h\right) - \frac{\left(1-\delta_h\right)}{R_t^g}\right] - \frac{\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&= \phi\left[1-\mu^c - \left(1-\mu^h\right)\left(1+\tau_t^h\right) + \frac{\left(1-\mu^h\right)\left(1-\delta_h\right)}{R_t^g} - \frac{\left(1-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
&= \phi\left[1-\mu^c - 1 + \mu^h - \left(1-\mu^h\right)\tau_t^h + \frac{\left(1-\mu^h-1+\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
\Rightarrow \quad & \tau_t^h\left[1+\phi\left(1-\mu^h\right)\right] = \phi\left[\mu^h - \mu^c - \frac{\left(\mu^h-\mu^c\right)\left(1-\delta_h\right)}{R_t^g}\right] \\
\tau_t^h &= \frac{\phi}{1+\phi\left(1-\mu^h\right)}\left(\mu^h-\mu^c\right)\left(1-\frac{\left(1-\delta_h\right)}{R_t^g}\right),
\end{aligned}$$

with $\frac{h_t^{-\mu^h}}{c_t^{-\mu^c}} = \left(1+\tau_t^h\right) - \frac{(1-\delta_h)}{R_t^g}$ following from (2.43). In the steady state with $R_t^g = \beta^{-1}$ the optimal housing tax rate is given by

$$\tau^h = \frac{\phi\left(\mu^h - \mu^c\right)}{1-\phi\left(\mu^h - 1\right)}\left(1-\beta\left(1-\delta_h\right)\right). \tag{2.42}$$

Second Derivatives

The second derivatives with respect to consumption, labor, and housing are given by

$$\begin{aligned}
\frac{\partial^2 J}{\partial c_t^2} &= -\underbrace{\mu^c c_t^{-1-\mu^c}}_{>0}\left[1-\phi\left(\mu^c-1\right)\right] \\
\frac{\partial^2 J}{\partial n_t^2} &= -\underbrace{\mu^n n_t^{-1+\mu^n}}_{>0}\left[1+\phi\left(\mu^n+1\right)\right] \\
\frac{\partial^2 J}{\partial h_t^2} &= -\underbrace{\mu^h h_t^{-1-\mu^h}}_{>0}\left[1-\phi\left(\mu^h-1\right)\right].
\end{aligned}$$

The second derivative with respect to labor is always negative, the ones for consumption and housing for $\mu^c < \frac{1}{\phi}+1$ and $\mu^h < \frac{1}{\phi}+1$, which is the case in our calibrations since ϕ is typically small. This ensures that we get maxima. Further, this implies $\phi\left(\mu^h-1\right) < 1$.

Derivation of the Implementability Constraint

The implementability constraint is derived as follows. We write (2.37) in the form

$$R_t^g = \frac{(1-\delta_h)p_{h,t+1}}{\left(1+\tau_t^h\right)p_{h,t} - \frac{u_t^h}{u_t^c}} \tag{2.43}$$

and rewrite condition (2.36) $u_t^c = u_{t-1}^c\left(\beta R_{t-1}^g\right)^{-1}$ and insert $u_{t-1}^c = u_{t-2}^c\left(\beta R_{t-2}^g\right)^{-1} \Rightarrow u_t^c = u_{t-2}^c\beta^{-2}\left(R_{t-1}^g\right)^{-1}\left(R_{t-2}^g\right)^{-1}$. Iterating on this we get

$$\beta^t u_t^c = u_0^c\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}. \tag{2.44}$$

Thus, we can rewrite the transversality condition as (with $u_0^c > 0$)

$$\lim_{t\to\infty} \prod_{i=0}^{t-1} \left(R_i^g\right)^{-1} \frac{b_{t+1}^g}{R_t^g} = 0.$$

Now, we solve the household budget constraint for period $t+1$ for b_{t+1}^g

$$\begin{aligned} b_{t+1}^g &= c_{t+1} + \left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1} + \frac{b_{t+2}^g}{R_{t+1}^g} \\ &\quad - \left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1} - (1-\delta_h) p_{h,t+1} h_t \end{aligned}$$

and insert this in the one for period t to get

$$\begin{aligned} & c_t + \left(1+\tau_t^h\right) p_{h,t} h_t \\ & + \frac{1}{R_t^g} \left[\begin{array}{c} c_{t+1} + \left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1} + \frac{b_{t+2}^g}{R_{t+1}^g} \\ - \left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1} - (1-\delta_h) p_{h,t+1} h_t \end{array} \right] - b_t^g \\ = \;& \left(1-\tau_t^n\right) w_t n_t + (1-\delta_h) p_{h,t} h_{t-1}. \end{aligned}$$

This can be rewritten as

$$\begin{aligned} & c_t + \frac{c_{t+1}}{R_t^g} + \left(1+\tau_t^h\right) p_{h,t} h_t - \frac{(1-\delta_h) p_{h,t+1} h_t}{R_t^g} \\ & + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} + \frac{b_{t+2}^g}{R_t^g R_{t+1}^g} \\ = \;& \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g. \end{aligned}$$

We now collect the terms with h_t, factor out h_t and insert (2.43)

$$\begin{aligned} & h_t \left[\left(1+\tau_t^h\right) p_{h,t} - \frac{(1-\delta_h) p_{h,t+1}}{R_t^g} \right] \\ = \;& {}^{(2.43)} h_t \left[\left(1+\tau_t^h\right) p_{h,t} - \frac{(1-\delta_h) p_{h,t+1}}{(1-\delta_h)\, p_{h,t+1}} \left(\left(1+\tau_t^h\right) p_{h,t} - \frac{u_t^h}{u_t^c} \right) \right] \\ = \;& h_t \left[\left(1+\tau_t^h\right) p_{h,t} - \left(\left(1+\tau_t^h\right) p_{h,t} - \frac{u_t^h}{u_t^c} \right) \right] = h_t \frac{u_t^h}{u_t^c}. \end{aligned} \tag{2.45}$$

Thus, we can rewrite the budget constraint again to get

$$\begin{aligned}
& c_t + \frac{c_{t+1}}{R_t^g} + h_t \frac{u_t^h}{u_t^c} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} + \frac{b_{t+2}^g}{R_t^g R_{t+1}^g} \\
= \quad & \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g .
\end{aligned}$$

Inserting the budget constraint of $t+2$ then delivers

$$\begin{aligned}
& c_t + \frac{c_{t+1}}{R_t^g} + h_t \frac{u_t^h}{u_t^c} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} \\
& + \frac{1}{R_t^g R_{t+1}^g} \left[\begin{array}{c} c_{t+2} + \left(1+\tau_{t+2}^h\right) p_{h,t+2} h_{t+2} + \frac{b_{t+3}^g}{R_{t+2}^g} \\ - \left(1-\tau_{t+2}^n\right) w_{t+2} n_{t+2} - (1-\delta_h) p_{h,t+2} h_{t+1} \end{array} \right] \\
= \quad & \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g .
\end{aligned}$$

We can rewrite this as

$$\begin{aligned}
& c_t + \frac{c_{t+1}}{R_t^g} + \frac{c_{t+2}}{R_t^g R_{t+1}^g} + h_t \frac{u_t^h}{u_t^c} + \frac{\left(1+\tau_{t+1}^h\right) p_{h,t+1} h_{t+1}}{R_t^g} \\
- \quad & \frac{(1-\delta_h) p_{h,t+2} h_{t+1}}{R_t^g R_{t+1}^g} + \frac{\left(1+\tau_{t+2}^h\right) p_{h,t+2} h_{t+2}}{R_t^g R_{t+1}^g} + \frac{b_{t+3}^g}{R_t^g R_{t+1}^g R_{t+2}^g} \\
= \quad & \left(1-\tau_t^n\right) w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} \\
+ \quad & \frac{\left(1-\tau_{t+2}^n\right) w_{t+2} n_{t+2}}{R_t^g R_{t+1}^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g .
\end{aligned}$$

Repeating the steps above in (2.45) we get

$$
\begin{aligned}
& c_t + \frac{c_{t+1}}{R_t^g} + \frac{c_{t+2}}{R_t^g R_{t+1}^g} + h_t \frac{u_t^h}{u_t^c} + \frac{h_{t+1}}{R_t^g} \frac{u_{t+1}^h}{u_{t+1}^c} \\
& + \frac{\left(1+\tau_{t+2}^h\right) p_{h,t+2} h_{t+2}}{R_t^g R_{t+1}^g} + \frac{b_{t+3}^g}{R_t^g R_{t+1}^g R_{t+2}^g} \\
= \; & (1-\tau_t^n)\, w_t n_t + \frac{\left(1-\tau_{t+1}^n\right) w_{t+1} n_{t+1}}{R_t^g} \\
& + \frac{\left(1-\tau_{t+2}^n\right) w_{t+2} n_{t+2}}{R_t^g R_{t+1}^g} + (1-\delta_h) p_{h,t} h_{t-1} + b_t^g .
\end{aligned}
$$

Iterating on this and using the transversality conditions, we get the intertemporal budget constraint with the initial endowments of h_{-1} and b_0^g

$$
\begin{aligned}
& \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} (R_i^g)^{-1} \right) c_t + \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} (R_i^g)^{-1} \right) h_t \frac{u_t^h}{u_t^c} \\
= \; & \sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} (R_i^g)^{-1} \right) (1-\tau_t^n)\, w_t n_t + (1-\delta_h) p_{h,0} h_{-1} + b_0^g ,
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
\sum_{t=0}^{\infty} \left(\prod_{i=0}^{t-1} (R_i^g)^{-1} \right) \left[c_t + h_t \frac{u_t^h}{u_t^c} - (1-\tau_t^n)\, w_t n_t \right] & \\
+ (1-\delta_h) p_{h,0} h_{-1} + b_0^g &= 0.
\end{aligned}
$$

By eliminating prices with (2.44) and (2.35) we get the implementability constraint

$$
\begin{aligned}
& \sum_{t=0}^{\infty} \beta^t \frac{u_t^c}{u_0^c} \left[c_t + h_t \frac{u_t^h}{u_t^c} + \frac{u_t^n}{u_t^c} n_t \right] + (1-\delta_h) p_{h,0} h_{-1} + b_0^g = 0 \\
\Leftrightarrow & \sum_{t=0}^{\infty} \beta^t \left[u_t^c c_t + u_t^h h_t + u_t^n n_t \right] + (1-\delta_h) p_{h,0} h_{-1} u_0^c + b_0^g u_0^c = 0 \qquad (2.46)
\end{aligned}
$$

Chapter 3

Macroeconomic Effects of the Federal Reserve's MBS Purchases

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3.1 Introduction

After hitting the zero lower bound on interest rates in the aftermath of the financial crisis, the US Federal Reserve (Fed) made use of several unconventional policy measures. Large-scale asset purchase (LSAP) programs, also known as quantitative easing (QE), were one of these measures. Within these programs the Fed not only purchased treasuries (as done by the Bank of Japan before, and the Bank of England and the European Central Bank after the Fed), but also mortgage-backed securities (MBS), which even made up the larger part of the purchased assets in the first round of quantitative easing (QE1).

In November 2008, when QE1 started, the Fed announced that it would purchase MBS worth up to $500 billion issued by the government-sponsored enterprises Fannie Mae and Freddie Mac. Fed chairman Bernanke argued in 2008, that "housing and housing finance played a central role in precipitating the current crisis" and concluded that "steps that stabilize the housing market will help stabilize the economy as well" (Bernanke, 2008). On March 18, 2009 the Fed decided to expand the program and to purchase "up to an additional $750 billion of agency mortgage-backed securities" in order to "provide greater support to mortgage lending and housing markets" (FOMC, 2009), such that the announced total purchases added up to $1250 billion in QE1. As Figure 3.1 illustrates, actual purchases during QE1 did not fully reach the announced amount but summed up to $1120 billion. Finally, in 2012, the Fed decided to purchase agency MBS (as part of the QE3 program) at the amount of $40 billion each month. In his press conference on September 13, 2012, Bernanke explained how the program was intended to work: "The program of MBS purchases should increase the downward pressure on long-term interest rates more generally, but also on mortgage rates specifically, which should provide further support to the housing sector by encouraging home purchases and refinancing" (Bernanke, 2012). As shown in Figure 3.1, total MBS purchases during QE3 added up to $900 billion. Compared to QE1, the QE3 program was not only smaller in total size but also, the purchases in each quarter were smaller due to the longer duration of the program.

The aim of this chapter is to provide an analysis that quantifies the effects of MBS purchases in the Fed's QE programs on macroeconomic variables. While there is some empirical research on how these purchases influenced MBS yields and mortgage rates (e.g. Hancock and Passmore 2011, 2012, 2015), we aim at measuring their effects on real activity and aggregate goods prices in a dynamic general equilibrium model with sticky prices and financial frictions. In particular, we try to provide an assessment on how macroeconomic aggregates, i.e. GDP, consumption and employment, reacted to MBS purchases in a macroeconomic framework that is calibrated to match Hancock and Passmore's (2011, 2015) estimates for the effects on MBS yields.

The analysis is conducted in a framework that consists of two types of households, patient and impatient ones, as in Kiyotaki and Moore (1997). While patient households hold deposits at financial intermediaries, the latter

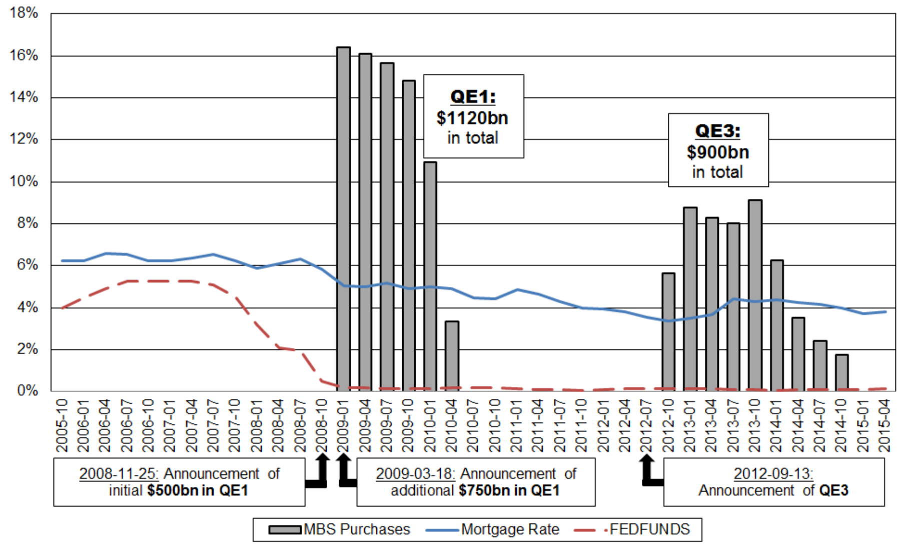

Figure 3.1: MBS Purchases in % of GDP, Annual Federal Funds Rate, and Annual 30-Year Fixed Mortgage Rate, 2005Q4-2015Q2. Source: FRED.

grant loans to impatient households, which are collateralized by their housing, as in Iacoviello (2005). Banks face costs increasing in the amount of loans they supply and decreasing in the amount of reserves, similar to Curdia and Woodford (2011). To allow for non-neutrality of asset purchases, we follow Schabert (2015) and assume that the central bank rations supply of reserves and supplies money only in exchange for eligible assets.[1] The central bank can then influence the yield of an eligible asset (like treasury bills or MBS) when it purchases it with high powered money at an above-market price. Regarding impatient borrowers MBS purchases imply a positive wealth effect since they reduce MBS yields and increase house prices as well as inflation. This alleviates the borrowing constraint of impatient households, who increase their consumption and housing. The effectiveness of the program in stimulating consumption, employment and GDP lies in

[1] Applying a much more stylized model (in particular, without financial market frictions), Schabert (2015) shows that a central bank can increase welfare by money rationing and purchasing eligible assets in a state contingent way.

the fact that it affects those households that have a high marginal propensity to consume out of wealth.

The model is calibrated applying US data from 1990Q1 to 2008Q3, such that effects from the financial crisis are not taken into account. During this period, the Fed supplied money essentially according to a "treasury only" regime. With the announcement of QE1 also MBS became eligible. Since the federal funds rate was effectively zero during the period of the purchase programs (see Figure 3.1), the analyses are conducted at the zero lower bound. We specify the banking cost function in a parsimonious way with two parameters, a level parameter and an elasticity of banking costs with regard to loans/reserves. The level parameter is calibrated to match average MBS yields between 1990Q1 and 2008Q3 and the elasticity parameter to match empirical evidence on yield effects of MBS purchases reported by Hancock and Passmore (2011, 2015). Based on this calibration we aim at quantifying the (untargeted) effects of the purchases in QE1 and later in QE3 on output, consumption and employment as well as inflation. Specifically, our results indicate that during QE1 GDP and consumption increased by 0.63%, and total hours worked by 0.95% in the announcement period. In the subsequent period, where purchases started and the program was extended, the reactions became larger and reached their maximum. In this period, the increase in GDP was 1.12%, in total consumption 1.13%, in total hours worked 1.67%, and in inflation 0.45%. The cumulative effects, which are computed for the period of a binding zero lower bound, which covers 9 quarters for our QE1 simulation, indicate that in total MBS purchases in QE1 increased GDP by 2.24%, total consumption by 2.36%, and total hours worked by 3.36%. In comparison, the effects during QE3 were largest in the announcement period, in which GDP increased by 0.86%, total consumption by 0.85%, total hours by 1.28%, and inflation by 0.45%. As cumulative effects, we get an increase in GDP by 1.62%, in total consumption by 1.69%, and in total hours worked by 2.43%. These smaller values for QE3 are due to the fact that the program in QE3 was about 30% smaller in total size and the purchases had a different schedule.

Besides the aforementioned empirical literature, there is, to the best of our knowledge, only one paper by Dai, Dufourt, and Zhang (2013) that analyzes MBS purchases in a theoretical framework. They find that MBS purchases are useful in stabilizing the housing market but less effective in stabilizing GDP and employment, depending on the degree of segmentation of credit branches (corporate vs. mortgage loans).[2] While we try to assess the effects of MBS purchases in isolation, there is a growing literature that studies the effects of LSAP programs in general with purchases of treasuries, corporate bonds and/or MBS, like e.g. Chen et al. (2012), Curdia and Woodford (2011), Del Negro et al. (2011), Gertler and Karadi (2011, 2013), and Hörmann and Schabert (2015).

The rest of the chapter is organized as follows. In Section 3.2, the model with a household sector consisting two types of agents, a banking sector, a firm sector with monopolistic competition and the public sector consisting of the treasury and the central bank is described. In Section 3.3, the model is calibrated, MBS purchases during QE1 and QE3 are described in detail and their effects are analyzed. Section 3.4 concludes.

3.2 The Model

In this section, we present the model. We follow Kiyotaki and Moore (1997) and consider two types of agents, patient and impatient ones. The former will save and the latter borrow in equilibrium. Intermediation between these two types is conducted by financial intermediaries which collect deposits from savers and grant loans to borrowing households. We assume that debt contracts are not enforceable and are collateralized by housing (see Iacoviello, 2005). Mortgage loans are assumed to be tradable and will be equivalent to mortgage-backed securities (MBS). The treasury issues one-period bonds, which are held by financial intermediaries and the central bank. Following Schabert (2015), we assume that the central bank supplies

[2] They conclude that this is due to the small share of residential investment in GDP and the absence of spillovers of MBS purchases on other credit markets. However, the connection of housing to the real economy that is more important seems to be its usage as collateral for private loans, which is why a reduction of MBS yields and mortgage rates should have macroeconomic effects, even if other yields are not affected by MBS purchases.

money only against eligible assets, here, treasuries and MBS. The central bank sets the policy rate and can further control the amount of money supplied against eligible assets, e.g. it can increase the supply of reserves by purchasing MBS. For the simulation of the financial crisis we will use a banking cost shock as in Christoffel and Schabert (2015) and to make the zero lower bound binding a shock to the discount factor of patient households, as in Christiano et al. (2011). Henceforth, upper-case letters denote nominal variables, while lower-case letters denote real variables.

3.2.1 Households

There is a continuum of households of mass 1 consisting of two types, patient ones, which are indexed with p and have a share of $0 < s < 1$ of total population, and impatient ones (indexed with i and share $(1-s)$). They only differ with regard to their subjective discount factors: $1 > \beta^p > \beta^i > 0$. Both types of households derive utility from consumption $c_{*,t}$, housing $h_{*,t}$ and disutility from labor $n_{*,t}$ $(* = i, p)$ and maximize the infinite sum of expected utility. Their objective is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{*,t}, h_{*,t}, n_{*,t}) \tag{3.1}$$

where $\beta = z_t^\beta \beta^p$ is the discount factor for patient and $\beta = \beta^i$ for impatient households with $1 > z_t^\beta \beta^p > \beta^i > 0$. We consider a shock z_t^β on the discount factor of patient agents with $\log z_t^\beta = \rho_\beta \log z_{t-1}^\beta + \varepsilon_t^\beta$, where $\varepsilon_t^\beta \sim n.i.d.\left(0, \sigma_\beta^2\right)$ and $0 < \rho_\beta < 1$ that is capable of making the zero lower bound binding, as e.g. in Christiano et al. (2011). We use the following CRRA-specification of the utility function

$$u(c_{*,t}, h_{*,t}, n_{*,t}) = \frac{c_{*,t}^{1-\mu^c}}{1-\mu^c} + \gamma^h \frac{h_{*,t}^{1-\mu^h}}{1-\mu^h} - \gamma^n \frac{n_{*,t}^{1+\mu^n}}{1+\mu^n}. \tag{3.2}$$

We first describe the problem of a representative patient household and then the one of a representative impatient household.

Patient Households A patient household p enters a period t with deposits $D_{p,t-1}$ held at financial intermediaries and real housing $h_{p,t-1}$. Neglecting borrowing from financial intermediaries (which would never occur in equilibrium), its budget constraint is given by

$$P_t c_{p,t} + P_t p_{h,t}(h_{p,t} - h_{p,t-1}) + D_{p,t}/R_t^D = D_{p,t-1} + P_t w_t n_{p,t} + P_t \tau_{p,t} + P_t \delta_{p,t}, \tag{3.3}$$

where the left hand side contains expenditures for consumption, $P_t c_{p,t}$, with P_t denoting the aggregate price level, and housing, $P_t p_{h,t} h_{p,t}$, with the real house price $p_{h,t}$, and new holdings of deposits $D_{p,t}$ at the price $1/R_t^D$, while the right hand side shows deposits from the preceding period as well as labor income, $P_t w_t n_{p,t}$, transfers from the public sector, $P_t \tau_{p,t}$, and profits of firms and retailers $P_t \delta_{p,t}$ due to the assumption that patient households are the owners of firms and retailers. A patient household chooses the values of $c_{p,t}$, $h_{p,t}$, $n_{p,t}$ and $d_{p,t} = D_{p,t}/P_t$ to maximize (3.1) subject to the budget constraint (3.3) and a non-negativity constraint $D_{p,t} \geq 0$, leading to the first order conditions

$$\gamma^h h_{p,t}^{-\mu^h} = p_{h,t} c_{p,t}^{-\mu^c} - z_t^\beta \beta^p E_t c_{p,t+1}^{-\mu^c} p_{h,t+1}, \tag{3.4}$$

$$\gamma^n n_{p,t}^{\mu^n} = w_t c_{p,t}^{-\mu^c}, \tag{3.5}$$

$$\frac{1}{R_t^D} = z_t^\beta \beta^p E_t \frac{c_{p,t+1}^{-\mu^c}}{c_{p,t}^{-\mu^c} \pi_{t+1}}. \tag{3.6}$$

Equation (3.4) describes housing demand of a patient household. In the optimum, marginal utility of current housing $\gamma^h h_{p,t}^{-\mu^h}$ equals marginal utility of foregone consumption $c_{p,t}^{-\mu^c}$ at the price of housing $p_{h,t}$ less the discounted marginal utility of next period's expected consumption $z_t^\beta \beta^p E_t c_{p,t+1}^{-\mu^c}$ achieved from selling the house at the expected price $E_t p_{h,t+1}$. Equation (3.5) describes labor supply of a patient household and (3.6) is the Euler equation for deposits. Further, the transversality conditions hold.

Impatient Households Since an impatient household values current consumption more than a patient one, it will become borrower in equilibrium.

We assume that its debt is non-enforceable and is collateralized by housing. A household i can borrow from intermediaries in nominal terms an amount $B_{i,t}^M / R_t^L < 0$ in period t and pays back $B_{i,t}^M$ in period $t+1$, where R_t^L is the gross nominal interest rate on these loans. We follow Iacoviello (2005) and assume that borrowing is limited by a (fraction) of the expected value of housing at the beginning of the subsequent period when the loan matures

$$B_{i,t}^M \geq -\phi E_t P_{t+1} p_{h,t+1} h_{i,t}, \tag{3.7}$$

where ϕ denotes the (exogenous) pledgeable fraction of housing. An impatient household i enters a period t with a mortgage loan $B_{i,t-1}^M < 0$ and real housing $h_{i,t-1}$. It has expenditures for consumption, $P_t c_{i,t}$, and housing, $P_t p_{h,t} h_{i,t}$, gets transfers, $P_t \tau_{i,t}$, earns labor income $P_t w_t n_{i,t}$, and borrows by issuing mortgage loans $B_{i,t}^M / R_t^L$. Neglecting deposits held at financial intermediaries (which would never occur in equilibrium), the budget constraint of an impatient household i reads

$$P_t c_{i,t} + P_t p_{h,t} \left[h_{i,t} - h_{i,t-1} \right] + B_{i,t}^M / R_t^L = B_{i,t-1}^M + P_t w_t n_{i,t} + P_t \tau_{i,t}. \tag{3.8}$$

An impatient household i chooses the values of $c_{i,t}$, $h_{i,t}$, $n_{i,t}$, and $b_{i,t}^M = B_{i,t}^M / P_t$ to maximize (3.1) subject to the collateral constraint (3.7) and the budget constraint (3.8) leading to the first order conditions

$$\gamma^h h_{i,t}^{-\mu^h} = c_{i,t}^{-\mu^c} p_{h,t} - \beta^i E_t c_{i,t+1}^{-\mu^c} p_{h,t+1} - \omega_t \phi E_t \pi_{t+1} p_{h,t+1}, \tag{3.9}$$

$$\gamma^n n_{i,t}^{\mu^n} = w_t c_{i,t}^{-\mu^c}, \tag{3.10}$$

$$\frac{c_{i,t}^{-\mu^c}}{R_t^L} = \beta^i E_t \frac{c_{i,t+1}^{-\mu^c}}{\pi_{t+1}} + \omega_t, \tag{3.11}$$

with ω_t being the multiplier on the collateral constraint (3.7) and the complementary slackness conditions $\omega_t (b_{i,t}^M + \phi E_t \pi_{t+1} p_{h,t+1} h_{i,t}) = 0$, $b_{i,t}^M + \phi E_t \pi_{t+1} p_{h,t+1} h_{i,t} \geq 0$, and $\omega_t \geq 0$. Equation (3.9) describes housing demand of an impatient household. Here, the additional term $\omega_t \phi E_t \pi_{t+1} p_{h,t+1}$, which is subtracted on the right hand side and hence increases housing demand of a borrower, stems from the fact that housing has an additional value as collateral for loans for impatient agents that is absent for patient savers. Equation (3.10) describes the labor supply decision of an impatient household and (3.11) is a modified Euler equation for mortgage debt taking into account the collateral constraint (3.7). In equilibrium, the condition for a binding borrowing constraint in the steady state, is given by $\omega = c_i^{-\mu^c}\left(\frac{1}{R^L} - \frac{\beta^i}{\pi}\right)$. Hence, ω is larger zero, i.e. the borrowing constraint is binding, if $\frac{1}{R^L} > \frac{\beta^i}{\pi}$, which we can be ensured by setting β^i to a sufficiently low value. Furthermore, the transversality conditions must hold.

3.2.2 Financial Intermediaries

There is a continuum of identical perfectly competitive financial intermediaries (banks) of mass 1 indexed with b. Banks face costs of managing loans, for which we consider an ad-hoc cost function Ξ_t, following Curdia and Woodford (2011). Bank b's budget constraint is given by

$$\begin{aligned} & P_t\pi^B_{b,t} + D_{b,t-1} + B_{b,t}/R^G_t + B^M_{b,t}/R^L_t + M^H_{b,t} + I_{b,t}\left(R^m_t - 1\right) \qquad (3.12) \\ + \quad & P_t\Xi_t\left(B^M_{b,t}, Q_{b,t}\right) = \frac{D_{b,t}}{R^D_t} + B_{b,t-1} + B^M_{b,t-1} + M^H_{b,t-1}, \end{aligned}$$

where $\pi^B_{b,t}$ denotes its profits. The term $I_{b,t}\left(R^m_t - 1\right)$ in (3.12) denotes costs associated with the acquisition of new central bank money since the central bank discounts eligible assets at the rate R^m_t. A bank b collects deposits from (patient) households $D_{b,t} = sD_{p,t}$, pays an interest R^D_t on them and holds money $M^H_{b,t}$. Further, it holds government bonds $B_{b,t}$ which deliver an interest of R^G_t and supplies mortgage loans, $B^M_{b,t} = (s-1)B^M_{i,t}$ at the interest rate R^L_t. We assume that mortgage loans can in principle be traded frictionlessly and at any amount between different financial intermediaries. Thus, in this model, they are equivalent to mortgage backed securities (MBS). The term $\Xi_t(B^M_{b,t}, Q_{b,t})$ denotes costs of supplying loans as an increasing function of the volume of loans supplied $\partial\Xi_t(B^M_{b,t}, Q_{b,t})/\partial B^M_{b,t} > 0$. As in Curdia and Woodford (2011), we assume that these costs are further decreasing in reserves, i.e. $\partial\Xi_t(B^M_{b,t}, Q_{b,t})/\partial Q_{b,t} < 0$ where $Q_{b,t} = M^H_{b,t-1} + I_{b,t}$, inducing a positive bank demand for high powered money.

Both types of bank assets, i.e. treasuries and MBS, are assumed to be eligible and can, therefore, be used to get new reserves from the central bank. In accordance with the Fed's pre-crisis money supply policy, we assume that treasuries are fully eligible. We further assume that the central bank decides in each period how much MBS are purchased. i.e. how much reserves are supplied against MBS, for which it sets the instrument $\left(z_t^i - 1\right)$. We model MBS purchases of the central bank as a random event, i.e. these purchases are assumed to be unexpected and transitory. The central bank further sets the price of money in terms of eligible assets R_t^m, which we therefore call the repo rate or the policy rate. Notably, the federal funds rate, which actually served as the policy rate of the US Federal Reserve, closely relates to the rate on treasury repurchase agreements before the financial crisis, see e.g. Bech et al. (2012). New money injections $I_{b,t}$ that a bank receives from the central bank are then limited by the money supply constraint (see Schabert (2015) for a corresponding money supply constraint without MBS)

$$I_{b,t} \leq \frac{B_{b,t-1}}{R_t^m} + \left(z_t^i - 1\right) \frac{B_{b,t-1}^M}{R_t^m}, \tag{3.13}$$

where z_t^i measures the (exogenously determined) MBS purchases by the central bank, which will be quantified to match the Fed's QE1 and QE3 programs. Below, we will describe in detail how the data generating process for z_t^i is chosen to match the size and the time pattern of the Fed's MBS purchases. Note that since the Fed only purchased Agency MBS in the programs, the term $\left(z_t^i - 1\right)$ will be measured by Agency MBS purchased by the Fed as share of total Agency MBS outstanding. In the steady state, we set $z^i = 1$ such that only government bonds are eligible.

A bank b maximizes the present value of future profits $\max E_0 \sum_{k=0}^{\infty} \vartheta_{t,t+k} \pi_{b,t+k}^B$ subject to its budget constraint (3.12) and the money supply constraint (3.13), where $\vartheta_{t,t+k}$ denotes the stochastic discount factor of banks. With $\vartheta_{t,t+1} = \vartheta_{t,t+k+1}/\vartheta_{t,t+k}$ and η_t denoting the multiplier on the money supply constraint of banks (3.13), the first order conditions with respect to deposits, government bonds, MBS, money holdings and injections are given by

$$\frac{1}{R_t^D} = E_t \frac{\vartheta_{t,t+1}}{\pi_{t+1}}, \tag{3.14}$$

$$\frac{1}{R_t^G} = \frac{1}{R_t^D}\left(1 + E_t \frac{\eta_{t+1}}{R_{t+1}^m}\right), \tag{3.15}$$

$$\frac{1}{R_t^L} = \frac{1}{R_t^D}\left(1 + E_t \eta_{t+1} \frac{\left(z_{t+1}^i - 1\right)}{R_{t+1}^m}\right) - \frac{\partial \Xi_t}{\partial b_{b,t}^M}, \tag{3.16}$$

$$1 = \frac{1}{R_t^D} - E_t \vartheta_{t,t+1} \frac{\partial \Xi_{t+1}}{\partial m_{b,t}^H}, \tag{3.17}$$

$$R_t^m = 1 - \frac{\partial \Xi_t}{\partial i_{b,t}} - \eta_t, \tag{3.18}$$

and the complementary slackness conditions

$$\begin{aligned} \eta_t \left(-i_{b,t+k} + \frac{b_{b,t+k-1}}{\pi_{t+k} R_{t+k}^m} + \left(z_{t+k}^i - 1\right) \frac{b_{b,t+k-1}^M}{\pi_{t+k} R_{t+k}^m}\right) &= 0, \\ -i_{b,t+k} + \frac{b_{b,t+k-1}}{\pi_{t+k} R_{t+k}^m} + \left(z_{t+k}^i - 1\right) \frac{b_{b,t+k-1}^M}{\pi_{t+k} R_{t+k}^m} &\geq 0, \ \eta_t \geq 0. \end{aligned}$$

Note, that the stochastic discount factor of banks will in equilibrium equal the one of patient households (see (3.6) and (3.14)). We further get the following relationships between the interest rates R_t^D, R_t^G and R_t^L. When government bonds are not eligible or the money supply constraint (3.13) is not binding, the interest rates on deposits and bonds are identical: $R_t^D = R_t^G$. Otherwise, we get $R_t^G < R_t^D$ due to the eligibility of bonds. Moreover, since $1 \geq \left(z_{t+1}^i - 1\right)$, we have $R_t^G < R_t^L$ due to the fact that mortgage loans increase banking costs. When MBS are not eligible ($z^i = 1$), the MBS yield exceeds the deposit rate, $R_t^L > R_t^D$, due to the costs associated with mortgage loans. Then, when MBS purchases are announced and become eligible, the expected eligibility ($E_t z_{t+1}^i > 1$) will reduce R_t^L, as can be seen from (3.16), provided that the money supply constraint is expected to be binding ($E_t \eta_{t+1} > 0$). Finally, equation (3.17) equates the costs of holding deposits to the costs of holding money and equation (3.18) delivers a condition, under which the money supply constraint is binding, $\eta_t = 1 -$

$R_t^m - \frac{\partial \Xi_t}{\partial i_{b,t}} > 0$ if $1 - R_t^m > \frac{\partial \Xi_t}{\partial i_{b,t}}$ or $\left| \frac{\partial \Xi_t}{\partial i_{b,t}} \right| > R_t^m - 1$, i.e. if the marginal (negative) effect of injections on banking costs is sufficiently large.

3.2.3 Firms

A continuum of perfectly competitive identical firms indexed with j produce the intermediate good according to $IO_{j,t} = \left(n_{j,t}^T \right)^\alpha$, where $\alpha \in (0,1)$. The firm hires labor $n_{j,t}^T$ at a common rate w_t to produce its output $IO_{j,t}$, which it sells to the retailers at the price $P_{J,t}$. Hence a firm j solves $\max P_{J,t} \left(n_{j,t}^T \right)^\alpha - P_t w_t n_{j,t}^T$ leading to the first order condition

$$P_{J,t} \alpha \left(n_{j,t}^T \right)^{\alpha-1} = P_t w_t \tag{3.19}$$

and to profits of $(1-\alpha) P_{J,t} \left(n_{j,t}^T \right)^\alpha$ that are distributed to the patient households which are assumed to own these firms.

Price stickiness is introduced in a standard way. A continuum of monopolistically competitive retailers indexed with k buy intermediate goods at the price $P_{J,t}$, re-package them according to $IO_t = \int_0^1 IO_{j,t} dj$, differentiate them into $y_{k,t} = IO_{k,t}$ and sell the distinct goods $y_{k,t}$ at the price $P_{k,t}$ to perfectly competitive bundlers. They bundle them to the final good $y_t = \left(\int_0^1 y_{k,t}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$, where $\epsilon > 1$, which is sold at the price P_t. Hence a retailer k faces the demand function $y_{k,t} = (P_{k,t}/P_t)^{-\epsilon} y_t$ and sets its own price $P_{k,t}$ according to this and taking $P_{J,t}$ as given. Following Calvo (1983), we assume that each period only a fraction $1-\theta$ of retailers is allowed to change his price. The other fraction $\theta \in [0,1)$ adjusts the price according to full indexation to the steady state inflation rate: $P_{k,t} = \pi P_{k,t-1}$. Defining $\widetilde{Z}_t = P_{k,t}^*/P_t$ with the optimal price of retailers $P_{k,t}^*$, the optimal real price can be written recursively as $\widetilde{Z}_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, with $Z_{1,t} = c_{p,t}^{-\mu^c} y_t mc_t + \theta z_t^\beta \beta^p E_t \left(\frac{\pi_{t+1}}{\pi} \right)^\epsilon Z_{1,t+1}$ and $Z_{2,t} = c_{p,t}^{-\mu^c} y_t + \theta z_t^\beta \beta^p E_t \left(\frac{\pi_{t+1}}{\pi} \right)^{\epsilon-1} Z_{2,t+1}$ (for details see Appendix 3.5.1).

Due to perfectly competitive bundlers, the aggregate price level P_t for final goods is given by $P_t^{1-\epsilon} = \int_0^1 P_{k,t}^{1-\epsilon} dk$ (zero profit condition) and can be written as $P_t^{1-\epsilon} = (1-\theta)\left(P_{k,t}^*\right)^{1-\epsilon} + \theta\pi^{1-\epsilon}P_{t-1}^{1-\epsilon} \Leftrightarrow 1 = (1-\theta)\widetilde{Z}_t^{1-\epsilon} + \theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon-1}$ (see Appendix 3.5.1). Moreover, aggregate output is given by $y_t = \frac{\left(n_t^T\right)^{\alpha}}{v_t}$ where $v_t = \int\limits_0^1 \left(P_{k,t}/P_t\right)^{-\epsilon} dk$ is a measure of price dispersion, which can be written recursively as $v_t = (1-\theta)\widetilde{Z}_t^{-\epsilon} + \theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon} v_{t-1}$ (see Appendix 3.5.1). Profits of intermediate goods producing firms and retailers that are distributed to a patient households p are collected in the term $P_t\delta_{p,t}$.

3.2.4 Public Sector

The treasury issues one-period bonds which are held by financial intermediaries $B_{b,t}$ and the central bank $B_{C,t}$. Hence, total demand for government bonds in period t is given by $B_t + B_{C,t}$ with $B_t = \int_0^1 B_{b,t}db$. As in Schabert (2015), we assume that bonds are supplied following a simple rule $B_{T,t} = \Gamma B_{T,t-1}$, where $\Gamma > \beta^p$ is the growth rate of total government bonds. The treasury receives seigniorage revenues from the central bank $P_t\tau_t^m$ and pays lump-sum transfers $P_t\tau_t = sP_t\tau_{p,t} + (1-s)P_t\tau_{i,t}$ to households to balance its budget

$$B_{T,t-1} + P_t\tau_t = B_{T,t}/R_t^G + P_t\tau_t^m,$$

where we assume that transfers are identical for both types, $\tau_{p,t} = \tau_{i,t}$.

The central bank supplies money outright $M_{t-1}^H = \int_0^1 M_{b,t-1}^H db$ and through repurchase agreements $M_t^R = \int_0^1 M_{b,t}^R db$ and holds treasuries, $B_{C,t}$,and MBS, $B_{C,t}^M$,that yield interest earnings. New money injections in each period are given by $I_t = M_t^H - M_{t-1}^H + M_t^R$, where $I_t = \int_0^1 I_{b,t-1}db$. The budget constraint of the central bank reads

$$\begin{aligned} & B_{C,t}/R_t^G + B_{C,t}^M/R_t^L + P_t\tau_t^m \\ = \quad & B_{C,t-1} + B_{C,t-1}^M + R_t^m\left(M_t^H - M_{t-1}^H\right) + \left(R_t^m - 1\right)M_t^R. \end{aligned}$$

Assuming that the central bank transfers all its interest earnings given by $P_t\tau_t^m = \left(1 - 1/R_t^G\right)B_{C,t} + \left(1 - 1/R_t^L\right)B_{C,t}^M + \left(R_t^m - 1\right)\left(M_t^H - M_{t-1}^H\right) + \left(R_t^m - 1\right)M_t^R$ to the treasury, we get the following relationship between the evolution of assets held and money supplied by the central bank $B_{C,t} - B_{C,t-1} + B^M_{C,t} - B^M_{C,t-1} = M^H_t - M^H_{t-1}$. Assuming that initial assets and liabilities satisfy $B_{C,-1} + B^M_{C,-1} = M^H_{-1}$, the balance sheet of the central bank reads

$$B_{C,t} + B^M_{C,t} = M^H_t.$$

Finally, with the market clearing condition for government bonds as well as $B^M_{C,t} = \left(z^i_t - 1\right) B^M_{b,t-1}$, the balance sheet can be written as $B_{T,t} - B_t + \left(z^i_t - 1\right) B^M_{b,t-1} = M^H_t$. The policy rate is set by the central bank following a feedback rule respecting the zero lower bound given by

$$R^m_t = \max\left\{1, \left(R^m_{t-1}\right)^{\rho_R} \left(R^m\right)^{1-\rho_R} \left(\pi_t/\pi\right)^{\rho_\pi(1-\rho_R)} \left(y_t/y\right)^{\rho_y(1-\rho_R)}\right\},$$

where variables without time index denote their steady state values, $R^m > 1$ is an average policy rate chosen by the central bank, $1 > \rho_R \geq 0$, $\rho_\pi \geq 0$ and $\rho_y \geq 0$. Given that we do not model real growth and that there is thus no trend in real money demand (that would have to be accommodated by an increasing outright money supply), the central bank sets the ration of money supplied under repos to money supplied outright equal to one ($M^R_t = M^H_t$), which ensures non-negative injections in equilibrium.

3.2.5 Equilibrium

The labor market clears according to $n^T_t = sn_{p,t} + (1-s)n_{i,t}$ and consolidation of budget constraints delivers the aggregate resource constraint

$$y_t = \Xi_t + sc_{p,t} + (1-s)\, c_{i,t}$$

implying market clearing in the goods market. Finally, the market clearing condition for the housing market with fixed supply H holds $sh_{p,t} + (1-s)\, h_{i,t} = H$. The set of equilibrium conditions is summarized and the equilibrium is defined in Appendix 3.5.2.

3.3 MBS Purchases in QE1 and QE3

In this section, we evaluate the Fed's MBS purchases applying the model developed in the previous section. First, we study the purchases during the QE1 program, announced on November 25, 2008, where credit markets were disrupted and MBS yields had risen markedly (see e.g. Hancock and Passmore, 2011). Moreover, the federal funds rate reached its zero lower bound at that time. To account for these circumstances before QE1, we simulate a crisis using a banking cost shock and a discount rate shock. The former captures the rise in MBS yields and the latter makes the zero lower bound binding. Second, we examine the MBS purchases in the QE3 program in 2012, where the functioning of mortgage markets had improved substantially (see e.g. Hancock and Passmore, 2015). Therefore, here we only use the discount rate shock to make the zero lower bound binding. To derive a solution of the model including the possibility that the zero lower bound is binding, we use the OccBin toolkit by Guerrieri and Iacoviello (2015b) for Dynare (2011), which facilitates the analysis of an occasionally binding constraint. Before these analyses are conducted, we describe how the model is calibrated.

3.3.1 Calibration

The model is calibrated using standard parameter values whenever it is possible, to facilitate comparisons. We further calibrate the remaining parameters that describe the processes governing the MBS purchases using detailed information of the Fed's purchase programs and the parameters of the banking cost function to replicate unconditional moments of MBS yields as well as their responses to the policy intervention using evidence provided by Hancock and Passmore (2011, 2015).

One time period is assumed to be a quarter. To calculate the long run values of the rates we consider, we use quarterly US data from the FRED database for the time period 1990Q1 to 2008Q3, excluding data of the recent financial crisis. The reason for beginning in 1990Q1 is that after the mid-1980s the housing finance system was restructured and the mortgage market got better integrated into the capital market (see e.g. Iacoviello, 2004). As we interpret the rate R^L as the MBS yield, for which we have no data, we calculate its mean as follows. Hancock and Passmore (2011) argue that the mortgage rate is a mark-up over the MBS yield, and report a mean MBS yield for the time period July 2000 to March 2004, which is 5.95%. In this time period the mean 30-year fixed mortgage rate is 6.57%, i.e. it is by a factor 1.1 larger than the MBS yield. Assuming that this mark-up is a time-invariant value for the entire sample and using the mean mortgage rate for 1990Q1 to 2008Q3 (7.34%), the resulting MBS yield is 6.64% annually and 1.62% quarterly for the time period 1990Q1 to 2008Q3. In the US, the mean net inflation rate and the means of the empirical counterparts of the monetary policy rate, the treasury bill rate, the deposit rate and the MBS yield between 1990Q1 and 2008Q3 are given by $\pi - 1 = 0.46\%$, $R^m - 1 = 1.06\%$, $R^G - 1 = 1.09\%$, $R^D - 1 = 1.10\%$, and $R^L - 1 = 1.62\%$.[3]

Given the values for the long run inflation and deposit rates, the discount factor of patient households follows from equation (3.6), implying $\beta^p = \frac{\pi}{R^D} = 0.9937$. We set the fraction of constrained consumers to 0.4, i.e. $s = 0.6$, which is in the range of what Kaplan et al. (2014) estimate for the share of hand-to-mouth consumers in the US. Following Guerrieri and Iacoviello (2015a), we set the utility parameters to $\mu^c = \mu^h = \mu^n = 1$, implying a Frisch labor supply elasticity of 1 and log utility in consumption and housing. The parameters γ^n and γ^h are calibrated such that total hours worked in the steady state is $n^T = 0.33$ and housing wealth to quarterly GDP is $p_h H/y = 6$ as total housing wealth to annual GDP is about 1.5 in the US (see Iacoviello, 2009) implying $\gamma^n = 5.9$ and $\gamma^h = 0.055$.

The total housing stock is normalized to 1 and the parameters concerning the production sector are set to $\alpha = 2/3$, $\theta = 0.75$, $\epsilon = 21$ following Iacoviello (2005). We further need to assign values for the discount factor of impatient households β^i and the pledgeable fraction of housing ϕ. For this, we follow also Iacoviello (2005), who summarizes some estimates of the discount factors and concludes that $\beta^i = 0.95$ is an appropriate value for impatient households' discount rate, which is adopted here. We further apply

[3]See Appendix 3.5.4 for further details on the data and their sources.

Table 3.1: Baseline Parameter Calibration.

Description	Source/Target	Parameter	Value
Discount factor patient households	π/R^D	β^p	0.9937
Discount factor impatient households	Iaco. (2005)	β^i	0.95
Pledgeable fraction of housing	Iaco. (2005)	ϕ	0.55
Share of the patient households	KVW (2014)	s	0.6
Inverse of Frisch elasticity	G&I (2015a)	μ^n	1
Inverse of IES in consumption	G&I (2015a)	μ^c	1
Inverse of IES in housing	G&I (2015a)	μ^h	1
Weight of housing in utility	$p_h/y = 6$	γ^h	0.055
Weight of labor in utility	$n^T = 0.33$	γ^n	5.9
Production function	Iaco. (2005)	α	2/3
Price rigidity	Iaco. (2005)	θ	0.75
Elasticity of substitution	Iaco. (2005)	ϵ	21
Fixed housing supply	normalized	H	1
Banking cost function	$R^L - 1 = 1.62\%$	κ	0.001
Banking cost function	16 bp drop in R^L	ι	0.5

his estimate for the pledgeable fraction of housing for impatient households and set $\phi = 0.55$.

We consider the following cost function of banks

$$\Xi_t = z_t^{\Xi}\kappa\left(\frac{b_{b,t}^M}{m_{t-1}^H/\pi_t + i_t}\right)^{\iota},$$

where z_t^{Ξ} denotes a banking cost shock following $\log z_t^{\Xi} = \rho_{\Xi}\log z_{t-1}^{\Xi} + \varepsilon_t^{\Xi}$ with $\varepsilon_t^{\Xi} \sim n.i.d.\left(0, \sigma_{\Xi}^2\right)$ and $0 < \rho_{\Xi} < 1$, as in Christoffel and Schabert (2015), which we will use for the simulation of the financial crisis. The parameters of the cost function, κ and ι, are set as follows. We calibrate the level parameter κ to match average MBS yields between 1990Q1 and 2008Q3, i.e. such that in the steady state $R^L - 1 = 1.62\%$ is matched. The calibration of the elasticity parameter ι is based on the following empirical results. According to Hancock and Passmore (2011), annual MBS yields fell between the announcement of QE1 in November 2008 and the start of the actual purchases in January 2009 by about 0.69 percentage points. This 69 basis points decline in the annual rate corresponds to a 16 basis points

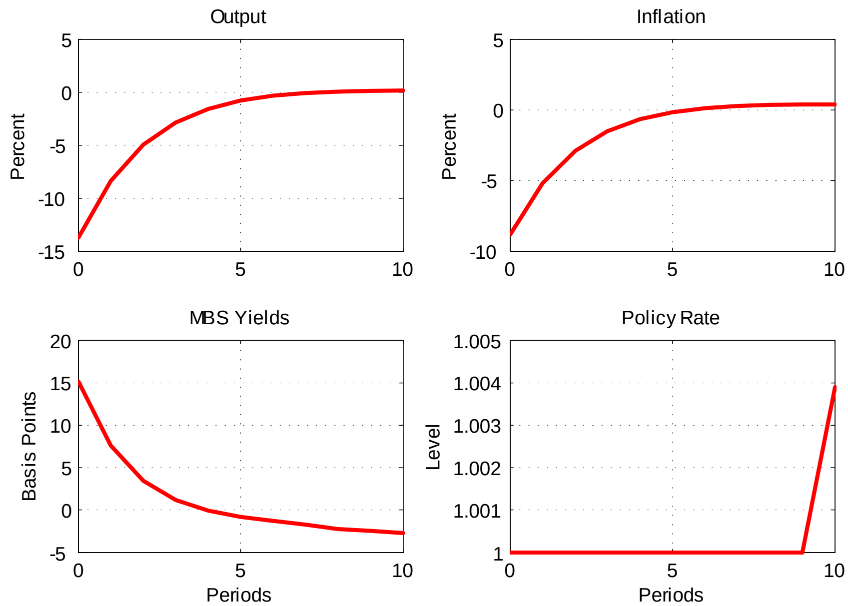

Figure 3.2: Responses to Crisis Shocks (without Policy Interventions) Preceding QE1.

decline per quarter, which we apply as one calibration target. Moreover, Hancock and Passmore (2015) find for MBS purchases during QE3 that weekly purchases of \$10 billion in June 2013 reduced annual MBS yields by 0.5 basis points weekly. On a quarterly basis, this would mean that a purchase of \$120 billion in 2013Q3 reduced annual MBS yields by 6 basis points corresponding to a decline of 1.5 basis points in quarterly yields, which is our second target. We apply a value for ι such that our model matches both of these empirically observed yields effects during QE1 and QE3. The resulting parameter values for the banking cost function are $\kappa = 0.001$ and $\iota = 0.5$. Our baseline calibration is summarized in Table 3.1.

3.3.2 Financial Crisis and Zero Lower Bound

The circumstances, under which MBS purchases took place, can be summarized as follows. QE1 was preceded by the financial crisis that led to large drops in output, inflation and the policy rate, which hit its zero lower bound and remained there for a long time period. Moreover, it led to a rise in MBS yields. More precisely, according to Hancock and Passmore (2014), during the crisis period, annual MBS yields were on average 63 basis points higher than what their normal pricing regression predicts. This corresponds to 15 basis points in quarterly yields. In contrast, the purchases in QE3 were conducted in an environment where the economy had recovered mainly but where the zero lower bound was still binding. To simulate scenarios that account for these circumstances, we use the discount factor shock and the banking cost shock.[4]

Our simulation of the financial crisis preceding QE1 is shown in Figure 3.2. In this simulation, we aim at replicating the 15 basis points increase in MBS yields together with a large drop in output and inflation that leads to a binding zero lower bound for a time period that at least spans the purchase period. We achieve this with a banking cost shock with a magnitude of $\varepsilon_0^{\Xi} = 0.35$ and an autocorrelation of $\rho_{\Xi} = 0.95$ together with a discount rate shock with a magnitude of $\varepsilon_0^{\beta} = 0.004$ and an autocorrelation of $\rho_{\beta} = 0.99$. The zero lower bound binds for 10 quarters spanning the purchases in QE1. For QE3, we only use the discount rate shock (with $\varepsilon_0^{\beta} = 0.006$) to make the zero lower bound binding for 11 quarters since the purchases during QE3 span 10 quarters. Note, that the timing is such that the shocks for the simulation of the economic environments before the purchase programs occur in period 0 and the announcements of the purchases will take place in period 1.

3.3.3 Simulating the Fed's MBS Purchases

QE1 We define 2008Q4 when the MBS purchases first were announced, as the first quarter of QE1. Table 3.2 shows the MBS purchases of the Fed in billions of dollars as well as relative to total agency MBS outstanding.

As can be seen in Table 3.2, total purchases at the end of the QE1 program exceeded $500 billion, which was the stated volume in the first

[4] Plots of impulse response functions of selected variables to these shocks are given and described in Appendix 3.5.5.

Table 3.2: FED MBS Purchases and Shock Processes in QE1.

Quarter	08Q4	09Q1	09Q2	09Q3	09Q4	10Q1	10Q2	Total
Quarter of QE1	1	2	3	4	5	6	7	7
MBS Purchases in:								
\$ billions	0	236	231	225	216	160	49	1117
% of total AMBS	0	4.7	4.5	4.3	4.0	2.9	0.9	21
$z_t^{i,QE1} - 1$	0	0.047	0.045	0.042	0.037	0.03	0.022	

announcement. This is due to the fact that the Fed expanded the program after the first announcement while it was running. On March 18, 2009, the Fed announced that it would expand the program and purchase "up to an additional $750 billion of agency mortgage-backed securities" (FOMC, 2009). Hence, to approximate the observed MBS purchases, given by the black solid line in Figure 3.3, by the shock z_t^i, we have to split it into two parts, the initially announced (labeled with A) and the expansion (labeled with B). To approximate these paths, we have to take a closer look into the announcements. Concerning the volume of the initially announced program the Fed's press release notifies that "purchases of up to $500 billion in MBS will be conducted" and concerning the duration of the program that it is "expected to take place over several quarters" (Fed, 2008). In accordance with this, we approximate the initially announced part of the program by an AR(1) process given by $\log\left(z_t^{i,A}\right) = \rho_i^A \log\left(z_{t-1}^{i,A}\right) + \varepsilon_{t-1}^{i,A}$. We set $\varepsilon_1^{i,A} = 0.047$ such that the initial purchase equals the observed one and $\rho_i^A = 0.54$ such that this part of the program in isolation leads to significant purchases over six quarters adding up to 10% of total US Agency MBS outstanding, which is what $500 billion made up in 2008Q4. This process A is depicted by the dashed-dotted green line in Figure 3.3.

For the second part of the program that was announced towards the end of 2009Q1 (labeled with B), we assume that actual purchases started in the subsequent quarter, namely 2009Q2. Hence, we also model this shock as an announced one. This second shock is used to approximate the actual purchases from 2009Q2 to 2010Q2 given the AR(1) process of the first shock. For this, we have to assume an AR(2) process given by

Figure 3.3: Observed Purchases during QE1 in % of Total US Agency MBS Outstanding and their Approximation through $z_t^{i,QE1}$.

$\log\left(z_t^{i,B}\right) = \rho_{1,i}^B \log\left(z_{t-1}^{i,B}\right) + \rho_{2,i}^B \log\left(z_{t-2}^{i,B}\right) + \varepsilon_{t-1}^{i,B}$, with $\varepsilon_1^{i,B} = 0.0195$, $\rho_{1,i}^B = 1.45$, and $\rho_{2,i}^B = -0.6$. The process B is shown by the dotted blue line in Figure 3.3. The process for $z_t^{i,QE1}$ as the aggregate QE1 shock process is the sum of the two processes given by $z_t^{i,QE1} = z_t^{i,A} + z_t^{i,B}$. It is depicted by the dashed red line in Figure 3.3 and approximates the observed purchases (black solid line) quite well.

We model the shocks as announced shocks since the purchase programs of the Fed, during QE1 as well as QE3, exhibited a lag between the announcement of the program and its implementation of one period. In accordance with this practice, a shock $\varepsilon_t^i > 0$ will not affect z_t^i but z_{t+1}^i, i.e. MBS purchases and injections will only be affected with a one-period delay. Nevertheless, knowing that the shock has occurred in t, agents can react and hence, in particular, R_t^L will respond to the shock in period t even without any change in MBS purchases and injections.

Table 3.3: FED MBS Purchases and Shock Processes in QE3.

Quarter	12Q4	13Q1	13Q2	13Q3	13Q4	14Q1	14Q2	14Q3	14Q4	Total
Q. of QE3	2	3	4	5	6	7	8	9	10	10
Purch. in:										
\$ billions	92	144	137	134	155	106	61	42	31	902
% AMBS	1.6	2.5	2.4	2.3	2.6	1.8	1.0	0.7	0.5	15
$z_t^{i,QE3} - 1$	0.016	0.024	0.026	0.025	0.022	0.017	0.013	0.008	0.003	

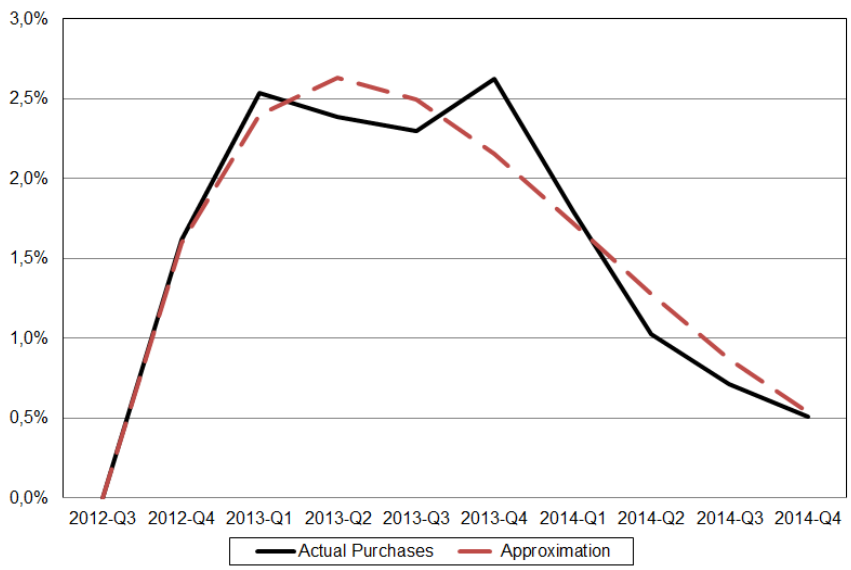

Figure 3.4: Observed Purchases during QE3 in % of Total US Agency MBS Outstanding and their Approximation through $z_t^{i,QE3}$.

QE3 The announcement of MBS purchases as part of QE3 was made on September 13, 2012. Hence, we take 2012Q3 as the announcement period of the MBS purchase program during QE3. In Table 3.3 again absolute and relative MBS purchases by the Fed are shown. We see that from quarter 2 to 6 of the intervention period of QE3 the purchases were smaller than in the corresponding quarters in QE1. Moreover, the total purchases were smaller in QE3 in absolute ($\frac{902}{1117} \approx 0.8$) as well as relative terms ($\frac{0.15}{0.21} \approx 0.71$). The difference in relative terms is larger since the amount of total Agency MBS outstanding was larger during the QE3 period. We approximate the observed MBS purchase program during QE3 by an AR(2) process for z_t^i given by $\log\left(z_t^{i,QE3}\right) = \rho_{i,1}\log\left(z_{t-1}^{i,QE3}\right) + \rho_{i,2}\log\left(z_{t-1}^{i,QE3}\right) + \varepsilon_{t-1}^i$, with $\varepsilon_1^i = 0.016$, $\rho_{i,1} = 1.5$ and $\rho_{i,1} = -0.605$. Observed purchases in QE3 and their approximation are shown in Figure 3.4. Again our shock process approximates observed purchases quite well.

3.3.4 Effects of MBS Purchases at the Zero Lower Bound

In this section, we present quantitative results regarding the macroeconomic effects of the approximated Fed's MBS purchases. The analysis is conducted separately for the purchases in QE1 and QE3, where we treat the policy interventions as realization of two distinct specifications of data generating processes for the particular policy instrument z_t^i.

QE1 Figure 3.5 shows the reaction of selected variables to the MBS purchase shock $z_t^{i,QE1}$. The figure is constructed as follows. First, we simulate the financial crisis with the banking cost shock and the shock to the discount factor of patient households, as described in Section 3.3.2. The zero lower bound is binding for 10 quarters as of the crisis period, i.e. it spans the whole purchase period. Then we let the economy return to its steady state. In a second simulation, we use the same shocks as before and add the MBS purchase shock $z_t^{i,QE1}$ in the period subsequent to the crisis shocks, i.e. period 1. The lines in Figure 3.5 show the difference between the reactions of the selected variables for the simulation with MBS purchase compared to the simulation without it. Hence, it shows the isolated effects stemming from the MBS purchase shock.

Our results indicate that in response to the MBS purchase shock that mimics the observed program of the Fed during QE1, GDP and total consumption increased by 0.63%, and total hours even by 0.95% in the announcement period, where MBS yields dropped by 16 basis points in accordance with Hancock and Passmore (2011). Note that MBS yields already fell in the announcement period without purchases due to the expected eligibility of MBS. In the subsequent period, where 4.7% of total Agency MBS outstanding were purchased, the reactions became even larger and reached their maximum. In this period, the increase in GDP was 1.12%, in total consumption 1.13%, in total hours worked 1.67%, and inflation 0.45%.

In the second period, MBS yields fell by 34 basis points since additional MBS were purchased. This led to an increase in house prices (+0.66%) and hence borrowing (MBS: +5.72%), which enabled constrained impatient households, who have a higher propensity to consume out of wealth, to increase consumption (+2.19%) and housing (+5.36%). Moreover, borrowers benefitted from debt deflation since MBS purchases led to an unexpected rise in inflation. On the other hand, since the deposit rate decreased with banking costs and MBS yields, patient households saved less, consumed more and also worked more (+2.44%) due to fewer interest earnings.

We further compute cumulative effects for the horizon of 9 quarters, for which the zero lower bound is binding. We find that over this horizon GDP increased by 2.24%, total consumption by 2.36%, and total hours worked by 3.36%. To sum up, MBS purchases of the central bank had expansionary effects on consumption, employment, and output since they benefitted impatient households with a high marginal propensity to consume out of wealth.

Finally, Figure 3.5 shows another important effect of MBS purchases, the increase in inflation with a maximum increase in the gross inflation rate of 0.45% in the second quarter of the intervention. Hence, central bank asset purchases have dampened the deflationary effects from the crisis shock.

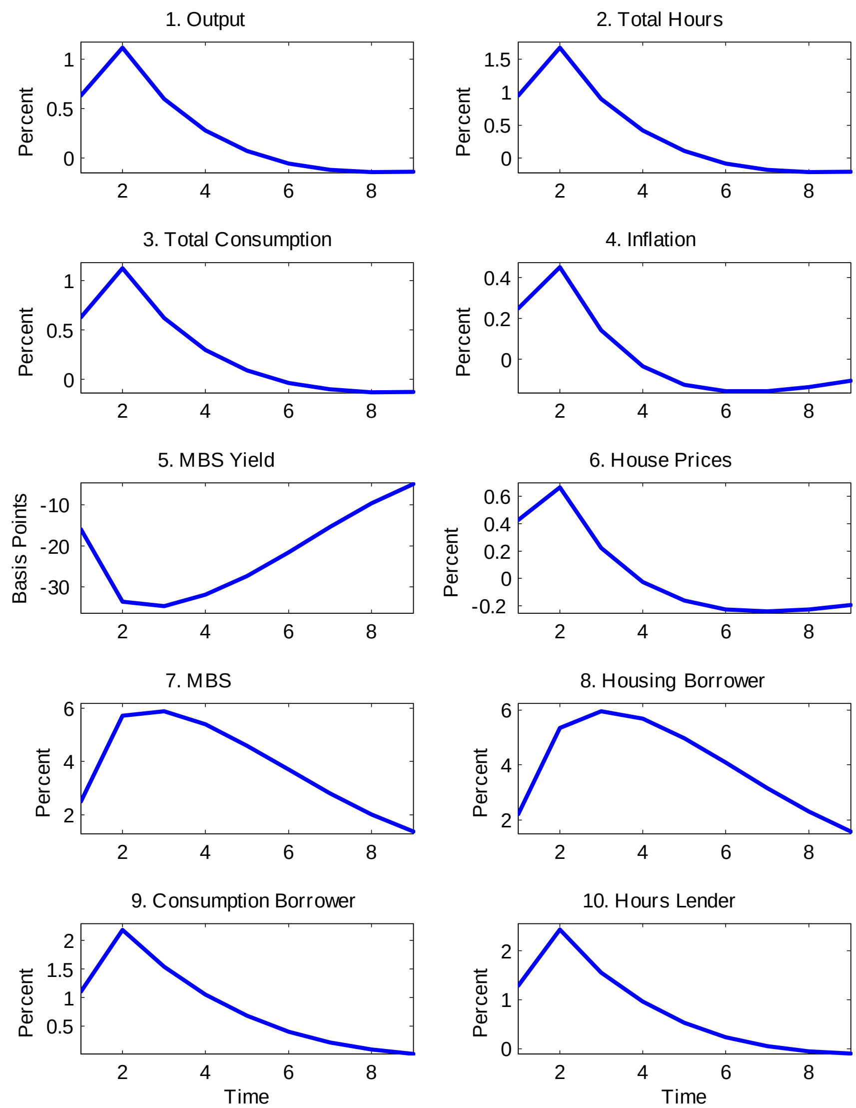

Figure 3.5: Effects of MBS Purchases in QE1.

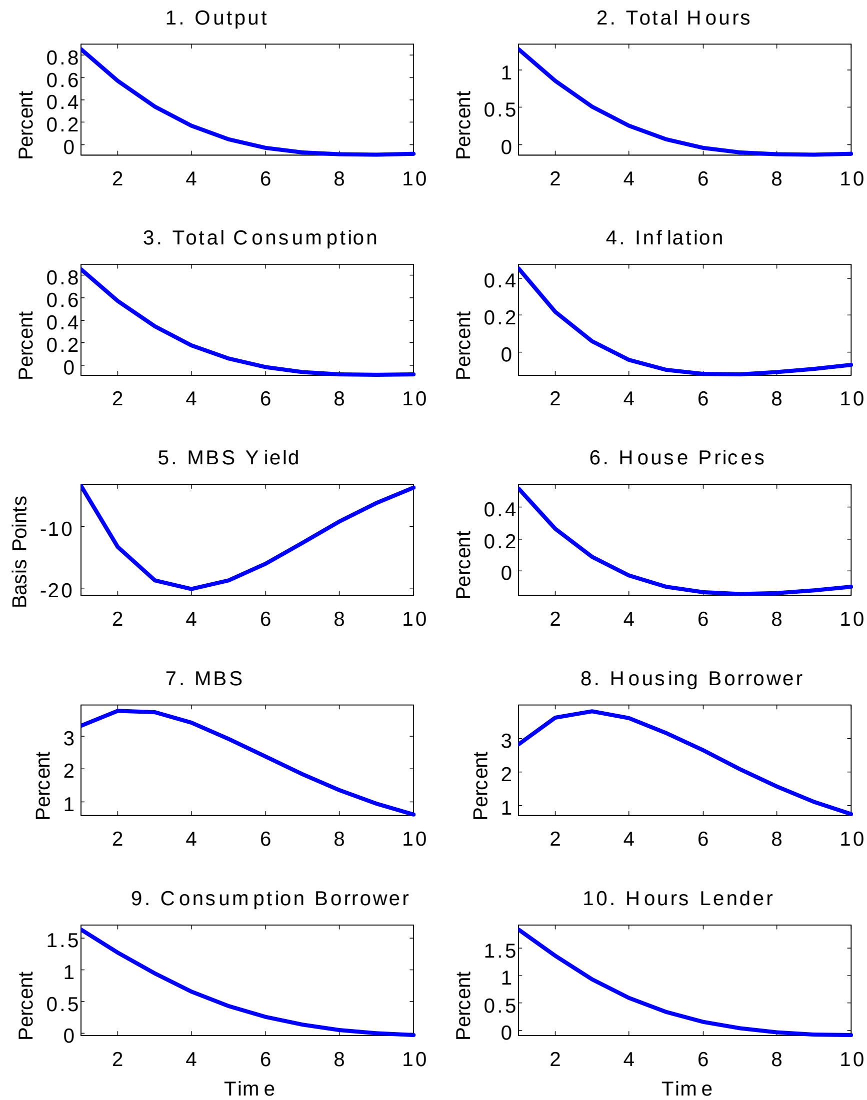

Figure 3.6: Effects of MBS Purchases in QE3.

QE3 Figure 3.6 shows the reaction of selected variables to the MBS purchase shock $z_t^{i,QE3}$. This figure is constructed in the same way as Figure 3.5 with the exception that here the banking cost shock was not used (see Section 3.3.2). Moreover, the zero lower bound is binding here for 10 quarters as of the announcement of QE3 since the purchase program spanned 10 quarters.

In contrast to the effects in QE1, the responses of macroeconomic variables to the MBS purchase program in QE3 are largest in the announcement period, although the response of MBS yields is hump-shaped. In QE3, the maximum increase of GDP was 0.86%, of total consumption 0.85%, of total hours worked 1.28%, and of inflation 0.45%. As the cumulative effects for the horizon of 10 quarters indicate, MBS purchases during QE3 increased GDP by 1.62%, total consumption by 1.69%, and total hours worked by 2.43% in total.

Comparison of QE1 and QE3 In each period of QE3, the drop in MBS yields was smaller than in QE1 and in total smoother. The reason is that the purchases in periods 2 to 6 of the intervention were smaller in QE3 compared to QE1 and that total MBS purchases in QE3 relative to total Agency MBS outstanding amounted to only about 70% of the purchases during QE1 (15% in QE3 vs. 21% in QE1). Moreover, the reaction of aggregate variables was largest in the announcement period in QE3, while in QE1 they reached their maximum in the second period of the intervention. This is due to the fact that the Fed expanded the program in QE1 while it was running. The second announcement in 2009Q1 had further expansionary effects. With QE1, the Fed reacted with a program containing a few MBS purchases of large size to rapidly counteract the rise in MBS yields and help the housing market to stabilize. In contrast, the QE3 program in 2012 was conducted when the economic situation was far better than in 2008. The program spanned a longer time period (10 quarters) and showed roughly constant purchases. The path of the purchases in QE3 was announced more clearly with announced purchases of $40 billion each month. Hence, not only the size of the purchase program but also its timing and its announcement are important for its effectiveness. To sum up, MBS purchase programs have successfully stimulated aggregate demand, output, and prices.

3.3.5 Sensitivity Analysis

In this section, we provide a sensitivity analysis. We vary the curvature parameter of housing in the utility function and set it to $\mu^h = 2$. We only compare the results for QE1 since this was the program with larger volume and larger effects. We have to adapt the parameters γ^h, κ and ι to induce a steady state ratio of housing wealth to GDP of $p_h/y = 6$, a steady state net MBS yield of $R^L - 1 = 1.62\%$, and a drop of 16 basis points in MBS yields in the announcement period. Therefore, we set these parameters to $\gamma^h = 0.059$, $\kappa = 0.00185$, and $\iota = 0.58$.

For this parametrization, the effects of MBS purchases on aggregate variables in QE1 are slightly larger. The maximum increases in output, total consumption, and hours worked, which are reached in period 2, are 1.2%, 1.23%, and 1.8%, respectively, and the cumulative increases are 2.31%, 2.6%, and 3.47%, respectively. What are the reasons for these larger effects? On the one hand, the higher value for μ^h leads to a smaller reaction of housing demand (borrowers: +3.49% vs. +5.36%). Consequently, borrowing (MBS) increases less than in the benchmark calibration (+3.84% vs. +5.72%). On the other hand, the higher value for μ^h strengthens the effects on consumption (borrowers: +2.65% vs. +2.19%) and hours worked (lenders: +2.82% vs. +2.44%). In sum, the larger reaction of consumption and hours worked outweighs the smaller increase in borrowing, which leads to larger expansionary effects on aggregate variables.

3.4 Conclusion

After the financial crisis, the Fed implemented for the first time in its history unconventional policy measures, like purchases of mortgage-backed securities. Hence, the discussion about the effectiveness of these types of interventions is vivid. This chapter provides an analysis that quantifies the effects of MBS purchases of the Fed in the QE programs on macroeconomic variables. Our results suggest that MBS purchases by the Fed had non-negligible effects on macroeconomic variables. Specifically, our results indicate that MBS purchases during QE1 increased GDP and total consumption by 0.63%, and total hours worked by 0.95% in the announcement period. In the second quarter, where purchases started and the program was expanded, the program had its maximum effects with increases in GDP, consumption, hours worked, and inflation of 1.12%, 1.13%, 1.67%, and 0.45%, respectively. Moreover, we find that in total the QE1 MBS purchase program increased GDP by 2.24%, total consumption by 2.36%, and total hours worked by 3.36%.

For QE3, we find that the program led to a maximum quarterly increase in GDP of 0.86%, in consumption of 0.85%, in hours worked of 1.28%, and in inflation of 0.45% in the announcement period. Moreover, the cumulative effects in QE3 summed up to an increase of 1.62% in GDP, of 1.69% in total consumption, and of 2.43% in total hours worked.

Reserves that were supplied to banks in exchange for MBS reduced banking costs and hence MBS yields which in turn increased house prices. This positive wealth effect for borrowers with a high marginal propensity to consume out of wealth let them increase consumption. On the other hand, lenders increased labor supply due to lower interest earnings. In total, consumption, hours worked and GDP increased. Moreover, the MBS purchases led to an increase in inflation suggesting that they have been useful to counteract deflationary effects of the crisis.

As our analyses showed, not only the size but also the construction of the purchase program is important for its effectiveness. Therefore, questions concerning the timing and the design of tapering could be addressed with this type of model, which is left for future research.

3.5 Appendix

3.5.1 Details of the Firms' Problem

Retailers' Price

The price setting problem of a retailer reads

$$\frac{\partial}{\partial P_{k,t}} E_t \left\{ \sum_{s=0}^{\infty} \theta^s q_{t,t+s} \left(\pi^s P_{k,t} - P_{J,t+s}\right) \left(\frac{P_{k,t}}{P_{t+s}}\right)^{-\epsilon} y_{t+s} \right\} = 0$$

with the stochastic discount factor $q_{t,t+s} = \left(z_t^{\beta} \beta^p\right)^s E_t \frac{c_{p,t+s}^{-\mu^c}}{c_{p,t}^{-\mu^c}} \frac{P_t}{P_{t+s}}$ leading to the first order condition for the optimal price $P_{k,t}^*$

$$P_{k,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum\limits_{s=0}^{\infty} \theta^s q_{t,t+s} \left(\pi^s\right)^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s} P_{J,t+s}}{E_t \sum\limits_{s=0}^{\infty} \theta^s q_{t,t+s} \left(\pi^s\right)^{1-\epsilon} P_{t+s}^{\epsilon} y_{t+s}}. \tag{3.20}$$

Inserting $q_{t,t+s} = \left(z_t^{\beta} \beta^p\right)^s E_t \frac{c_{p,t+s}^{-\mu^c}}{c_{p,t}^{-\mu^c}} \frac{P_t}{P_{t+s}}$ and marginal costs of retailers $mc_t = \frac{P_{J,t}}{P_t}$, (3.20) can be written as

$$P_{k,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum\limits_{s=0}^{\infty} \left(\theta z_t^{\beta} \beta^p\right)^s c_{p,t+s}^{-\mu^c} \left(\pi^s\right)^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s} mc_{t+s}}{E_t \sum\limits_{s=0}^{\infty} \left(\theta z_t^{\beta} \beta^p\right)^s c_{p,t+s}^{-\mu^c} \left(\pi^s\right)^{1-\epsilon} P_{t+s}^{\epsilon-1} y_{t+s}}. \tag{3.21}$$

We define $\widetilde{Z}_t = P_{k,t}^*/P_t$ such that we can rewrite the denominator and the numerator of (3.21) recursively as $\widetilde{Z}_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, with $Z_{1,t} = c_{p,t}^{-\mu^c} y_t mc_t + \theta z_t^{\beta} \beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon} Z_{1,t+1}$ and $Z_{2,t} = c_{p,t}^{-\mu^c} y_t + \theta z_t^{\beta} \beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon-1} Z_{2,t+1}$.

Aggregate Price Level

The aggregate price level P_t for final goods is given by $P_t^{1-\epsilon} = \int\limits_0^1 P_{k,t}^{1-\epsilon} dk$ due to perfectly competitive bundlers (zero profit condition). Since all retail prices were set according to (3.20) in the past, we can write this as

$$\begin{aligned} P_t^{1-\epsilon} &= \int_0^1 P_{k,t}^{1-\epsilon} dk = (1-\theta) \sum_{s=0}^{\infty} \theta^s \left(\pi^s P_{k,t-s}^*\right)^{1-\epsilon} \\ &= (1-\theta)\left(P_{k,t}^*\right)^{1-\epsilon} + \theta \pi^{1-\epsilon} P_{t-1}^{1-\epsilon}. \end{aligned}$$

Hence the aggregate price level is given by

$$\begin{aligned} P_t^{1-\epsilon} &= (1-\theta)\left(P_{k,t}^*\right)^{1-\epsilon} + \theta \pi^{1-\epsilon} P_{t-1}^{1-\epsilon} \\ &\Rightarrow \ 1 = (1-\theta)\widetilde{Z}_t^{1-\epsilon} + \theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon-1}. \end{aligned} \tag{3.22}$$

Aggregate Output

Aggregate output of intermediate goods producers who behave identically is given by $IO_t = \left(n_t^T\right)^{\alpha}$. Moreover, $IO_t = \int_0^1 IO_{k,t} dk = \int_0^1 y_{k,t} dk$ holds due to market clearing. Equalizing these equations for intermediate output and inserting $y_{k,t} = \left(P_{k,t}/P_t\right)^{-\epsilon} y_t$, we get $\left(n_t^T\right)^{\alpha} = \int_0^1 \left(P_{k,t}/P_t\right)^{-\epsilon} y_t dk \Leftrightarrow y_t = \frac{\left(n_t^T\right)^{\alpha}}{v_t}$ where $v_t = \int_0^1 \left(P_{k,t}/P_t\right)^{-\epsilon} dk$ is a measure of price dispersion. With the assumption that there is no initial price dispersion, we can write v_t as

$$v_t = (1-\theta)\sum_{l=0}^{\infty} \theta^l \left(\pi^l P_{k,t-l}^* / P_t\right)^{-\epsilon},$$

where $P_{k,t}^*$ is the optimal price set by the price adjusting firms at time t. This term can further be simplified with v_t

$$v_t = (1-\theta)\sum_{l=0}^{\infty} \theta^l \left(\pi^l P_{k,t-l}^* / P_t\right)^{-\epsilon} = (1-\theta)\sum_{l=0}^{\infty} \theta^l \left(\pi^l\right)^{-\epsilon} \widetilde{Z}_{t-l}^{-\epsilon} \prod_{s=1}^{l} \pi_{t+1-s}^{\epsilon}$$

with $\widetilde{Z}_t = P^*_{k,t}/P_t$ and $\sum_{l=1}^{\infty}\prod_{s=1}^{l}\pi^{\epsilon}_{t+1-s} = \sum_{l=1}^{\infty}(P_{t-l}/P_t)^{-\epsilon}$. Taking differences, we can write v_t in a more compact recursive way:

$$v_t - \theta\pi^{-\epsilon}\pi^{\epsilon}_t v_{t-1} = (1-\theta)\,\widetilde{Z}_t^{-\epsilon} \Leftrightarrow v_t = (1-\theta)\,\widetilde{Z}_t^{-\epsilon} + \theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon} v_{t-1}.$$

3.5.2 Equilibrium

Definition 1 *A rational expectations equilibrium is a set of sequences* $\{c_{p,t}$, $h_{p,t}$, $n_{p,t}$, $c_{i,t}$, $h_{i,t}$, $n_{i,t}$, $p_{h,t}$, w_t, π_t, ω_t, $b^M_{i,t}$, R^m_t, R^G_t, R^D_t, R^L_t, η_t, Ξ_t, b^M_t, m^H_t, $\vartheta_{t,t+1}$, i_t, b_t, n^T_t, mc_t, $\widetilde{Z}_t$, $Z_{1,t}$, $Z_{2,t}$, y_t, v_t, $b_{T,t}$, m^R_t, $\tau_t\}_{t=0}^{\infty}$ *satisfying the optimality conditions of patient households*

$$\begin{aligned}
\gamma^h h_{p,t}^{-\mu^h} &= p_{h,t}c_{p,t}^{-\mu^c} - z^{\beta}_t\beta^p E_t c_{p,t+1}^{-\mu^c} p_{h,t+1}, && (3.23)\\
\gamma^n n_{p,t}^{\mu^n} &= w_t c_{p,t}^{-\mu^c}, && (3.24)\\
\frac{1}{R^D_t} &= z^{\beta}_t\beta^p E_t \frac{c_{p,t+1}^{-\mu^c}}{c_{p,t}^{-\mu^c}\pi_{t+1}}, && (3.25)
\end{aligned}$$

impatient households

$$\begin{aligned}
\gamma^h h_{i,t}^{-\mu^h} &= c_{i,t}^{-\mu^c}p_{h,t} - \beta^i E_t c_{i,t+1}^{-\mu^c}p_{h,t+1} - \omega_t\phi E_t\pi_{t+1}p_{h,t+1}, && (3.26)\\
\gamma^n n_{i,t}^{\mu^n} &= w_t c_{i,t}^{-\mu^c}, && (3.27)\\
\frac{c_{i,t}^{-\mu^c}}{R^L_t} &= \beta^i E_t\frac{c_{i,t+1}^{-\mu^c}}{\pi_{t+1}} + \omega_t, && (3.28)\\
c_{i,t} &+ p_{h,t}\left[h_{i,t} - h_{i,t-1}\right] + \frac{b^M_{i,t}}{R^L_t} = \frac{b^M_{i,t-1}}{\pi_t} + w_t n_{i,t} + \tau_t, && (3.29)\\
b^M_{i,t} &= -\phi E_t\pi_{t+1}p_{h,t+1}h_{i,t},\ \ \textit{if } \omega_t > 0, && (3.30)\\
\textit{or } b^M_{i,t} &> -\phi E_t\pi_{t+1}p_{h,t+1}h_{i,t},\ \ \textit{if } \omega_t = 0, && (3.31)
\end{aligned}$$

banks

$$\begin{aligned}
\frac{1}{R_t^D} &= E_t \frac{\vartheta_{t,t+1}}{\pi_{t+1}}, && (3.32)\\
\frac{1}{R_t^G} &= \frac{1}{R_t^D}\left(1 + E_t \frac{\eta_{t+1}}{R_{t+1}^m}\right), && (3.33)\\
\frac{1}{R_t^L} &= \frac{1}{R_t^D}\left(1 + E_t \eta_{t+1} \frac{\left(z_{t+1}^i - 1\right)}{R_{t+1}^m}\right) - \frac{\partial \Xi_t}{\partial b_t^M}, && (3.34)\\
1 &= \frac{1}{R_t^D} - E_t \vartheta_{t,t+1} \frac{\partial \Xi_{t+1}}{\partial m_t^H}, && (3.35)\\
R_t^m &= 1 - \frac{\partial \Xi_t}{\partial i_t} - \eta_t, && (3.36)\\
\Xi_t &= z_t^{\Xi} \kappa \left(\frac{b_t^M}{m_{t-1}^H/\pi_t + i_t}\right)^{\iota} && (3.37)\\
b_t^M &= (s-1) b_{i,t}^M, && (3.38)\\
i_t &= \frac{b_{t-1}}{\pi_t R_t^m} + \left(z_t^i - 1\right) \frac{b_{t-1}^M}{\pi_t R_t^m},\ \ \textit{if}\ \eta_t > 0, && (3.39)\\
\textit{or}\ i_t &< \frac{b_{t-1}}{\pi_t R_t^m} + \left(z_t^i - 1\right) \frac{b_{t-1}^M}{\pi_t R_t^m},\ \ \textit{if}\ \eta_t = 0, && (3.40)
\end{aligned}$$

firms

$$\begin{aligned}
w_t &= \alpha \left(n_t^T\right)^{\alpha - 1} mc_t, && (3.41)\\
n_t^T &= s n_{p,t} + (1-s)\, n_{i,t} && (3.42)\\
\widetilde{Z}_t &= \frac{\epsilon}{\epsilon - 1} Z_{1,t}/Z_{2,t}, && (3.43)\\
Z_{1,t} &= c_{p,t}^{-\mu^c} y_t mc_t + \theta z_t^{\beta} \beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon} Z_{1,t+1}, && (3.44)\\
Z_{2,t} &= c_{p,t}^{-\mu^c} y_t + \theta z_t^{\beta} \beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon - 1} Z_{2,t+1}, && (3.45)\\
y_t &= \frac{\left(n_t^T\right)^{\alpha}}{v_t}, && (3.46)\\
v_t &= (1-\theta)\, \widetilde{Z}_t^{-\epsilon} + \theta v_{t-1} \left(\frac{\pi_t}{\pi}\right)^{\epsilon}, && (3.47)\\
1 &= (1-\theta)\, \widetilde{Z}_t^{1-\epsilon} + \theta \left(\frac{\pi_t}{\pi}\right)^{\epsilon - 1}, && (3.48)
\end{aligned}$$

the conditions for the treasury

$$
\begin{aligned}
b_{T,t} &= \Gamma \frac{b_{T,t-1}}{\pi_t}, && (3.49)\\
\frac{b_{T,t-1}}{\pi_t} + \tau_t &= \frac{b_{T,t}}{R_t^G} + \left(1 - \frac{1}{R_t^G}\right)(b_{T,t} - b_t) && (3.50)\\
&+ \left(1 - \frac{1}{R_t^L}\right)\left(z_t^i - 1\right) b_{t-1}^M + (R_t^m - 1)\left(m_t^H - \frac{m_{t-1}^H}{\pi_t} + m_t^R\right),
\end{aligned}
$$

the central bank

$$
\begin{aligned}
i_t &= m_t^H - \frac{m_{t-1}^H}{\pi_t} + m_t^R && (3.51)\\
m_t^H &= m_t^R && (3.52)\\
m_t^H &= b_{T,t} - b_t + \left(z_t^i - 1\right) b_{t-1}^M && (3.53)\\
R_t^m &= \left(R_{t-1}^m\right)^{\rho_R} (R^m)^{1-\rho_R} (\pi_t/\pi)^{\rho_\pi(1-\rho_R)} (y_t/y)^{\rho_y(1-\rho_R)} && (3.54)
\end{aligned}
$$

the market clearing conditions

$$
\begin{aligned}
y_t &= \Xi_t + s c_{p,t} + (1-s)\, c_{i,t} && (3.55)\\
H &= s h_{p,t} + (1-s)\, h_{i,t} && (3.56)
\end{aligned}
$$

and transversality conditions, given the fixed housing supply H, initial values $b_{-1} > 0$, $b_{T,-1} > 0$, $m_{-1}^H > 0$, $\pi_{-1} > 0$, $v_{-1} = 1$, and the exogenous processes for $\{\, z_t^\beta,\ z_t^i,\ z_t^\Xi\}_{t=0}^\infty$ and i.i.d. innovations with mean zero $\{\varepsilon_t^\beta,\ \varepsilon_t^i,\ \varepsilon_t^\Xi\}_{t=0}^\infty$ and $\frac{\partial \Xi_t}{\partial b_t^M} = \iota \frac{\Xi_t}{b_t^M}$, $\frac{\partial \Xi_t}{\partial m_{t-1}^H} = -\iota \frac{\Xi_t}{\pi_t\left(m_{t-1}^H/\pi_t + i_t\right)}$, and $\frac{\partial \Xi_t}{\partial i_t} = -\iota \frac{\Xi_t}{m_{t-1}^H/\pi_t + i_t}$.

3.5.3 Steady State

The deterministic steady state follows from the equilibrium defined above. The steady state value of each variable is given by the variable where the time index is dropped. We can reduce the equilibrium conditions for the steady state as follows. First, consider der firms' optimality conditions $\widetilde{Z}_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, where $Z_1 = \frac{c_p^{-\mu^c} y\, mc}{1-\theta\beta^p}$ and $Z_2 = \frac{c_p^{-\mu^c} y}{1-\theta\beta^p}$ in the steady state.

Hence, we get $\widetilde{Z} = \frac{\epsilon}{\epsilon-1} mc$ and with the steady state mark-up of $\frac{\epsilon}{\epsilon-1} \Rightarrow mc = \frac{\epsilon-1}{\epsilon} \Rightarrow \widetilde{Z} = 1$. From this, we get $v = \frac{(1-\theta)\widetilde{Z}^{-\epsilon}}{1-\theta} = 1$ and hence $y = \left(n^T\right)^{\alpha}$. Moreover, the policy rate rule delivers $R^m = R^m > 1$, the bond supply rule $\pi = \Gamma$ and we get $R^D = \frac{\pi}{\beta^p}$ from the patient households, and $\vartheta = \beta^p$ from the banks' first order conditions. Finally, we have $m^R = m^H$. The values of $\{c_p$, h_p, n_p, c_i, h_i, n_i, p_h, w, ω, b_i^M, R^G, R^L, η, Ξ, b^M, m^H, i, b, n^T, y, b_T, $\tau\}$ that satisfy the conditions (3.57)-(3.78) mark the steady state of the model:

$$\gamma^h h_p^{-\mu^h} = p_h c_p^{-\mu^c} \left(1 - \beta^p\right) \tag{3.57}$$

$$\gamma^n n_p^{\mu^n} = w c_p^{-\mu^c} \tag{3.58}$$

$$\gamma^h h_i^{-\mu^h} = c_i^{-\mu^c} p_h \left(1 - \beta^i\right) - \omega\phi\pi p_h \tag{3.59}$$

$$\gamma^n n_i^{\mu^n} = w c_i^{-\mu^c} \tag{3.60}$$

$$\omega = c_i^{-\mu^c} \left(\frac{1}{R^L} - \frac{\beta^i}{\pi}\right) > 0 \tag{3.61}$$

$$b_i^M = -\phi\pi p_h h_i \tag{3.62}$$

$$c_i = b_i^M \left(\frac{1}{\pi} - \frac{1}{R^L}\right) + w n_i + \tau \tag{3.63}$$

$$\frac{1}{R^G} = \frac{1}{R^D}\left(1 + \frac{\eta}{R^m}\right) \tag{3.64}$$

$$\frac{1}{R^L} = \frac{1}{R^D} - \frac{\partial \Xi}{\partial b^M} \tag{3.65}$$

$$1 = \frac{1}{R^D} - \beta^p \frac{\partial \Xi}{\partial m^H} \tag{3.66}$$

$$R^m = 1 - \frac{\partial \Xi}{\partial i} - \eta \tag{3.67}$$

$$b^M = (s-1) b_i^M \tag{3.68}$$

$$i = \frac{b}{\pi R^m} \tag{3.69}$$

$$\frac{\epsilon}{\epsilon - 1} w = \alpha \left(n^T\right)^{\alpha - 1} \tag{3.70}$$

$$n^T = s n_p + (1-s)\, n_i \tag{3.71}$$

$$y = \left(n^T\right)^{\alpha} \tag{3.72}$$

$$
\begin{aligned}
\tau &= b_T\left(\frac{1}{R^G}-\frac{1}{\pi}\right)+\left(1-\frac{1}{R^G}\right)(b_T-b) && (3.73)\\
&\quad +(R^m-1)\,m^H\left(2-\frac{1}{\pi}\right)\\
i &= m^H\left(2-\frac{1}{\pi}\right) && (3.74)\\
m^H &= b_T-b && (3.75)\\
y &= \Xi+sc_p+(1-s)\,c_i && (3.76)\\
H &= sh_p+(1-s)\,h_i && (3.77)\\
\Xi &= \kappa\left(\frac{b^M}{m^H/\pi+i}\right)^{\iota} && (3.78)
\end{aligned}
$$

with $\frac{\partial\Xi}{\partial b^M}=\iota\frac{\Xi}{b^M}$, $\frac{\partial\Xi}{\partial m^H}=-\iota\frac{\Xi}{\pi(m^H/\pi+i)}$, and $\frac{\partial\Xi}{\partial i}=-\iota\frac{\Xi}{m^H/\pi+i}$, given parameter values for β^i and $R^D>R^G$ ensuring a binding borrowing and money supply constraint, respectively, in the steady state.

3.5.4 Data

In this section, we briefly describe the data used in this study. Our main source is the FRED database (https://research.stlouisfed.org/fred2/).

To calibrate the long run value of R^m, we use the time series on the effective federal funds rate (FEDFUNDS), of R^D the three-month certificate of deposit (CD3M), of R^G the one-year treasury constant maturity rate (DGS1), and of π the GDP implicit price deflator (GDPDEF). Moreover, as described in the text, for the approximation of R^L, we use the 30-year fixed mortgage rate (MORTGAGE30US).

To calibrate the shock processes, we use the time series on mortgage-backed securities held by the Federal Reserve (MBST) from FRED and the time series on US Agency MBS outstanding from the website of the Securities Industry and Financial Markets Association (SIFMA) (www.sifma.org). Finally, we use the GDP time series from FRED to get MBS held by the Fed in percent on GDP as it is shown in Figure 3.1.

3.5.5 Impulse Response Functions

The impulse responses displayed in the figures below show deviations of each variable x_t from its steady state value x, i.e. $x_t^{obs} = 100 \log x_t/x$, in response to shocks of one standard deviation. As an exception variables with a star (*) show deviations in basis points, variables with two stars (**) deviations in levels, and variables with three stars (***) levels. We consider shocks that are sufficiently small to ensure a binding borrowing constraint for impatient households (3.7) and a binding money supply constraint (3.13). The standard deviations for the shocks are set to $\sigma_\beta = 0.006$ and $\sigma_\Xi = 0.1$ and the parameters of autocorrelation of the shock processes to $\rho_\beta = 0.7$ and $\rho_\Xi = 0.95$. The IRFs refer to the benchmark calibration given in Table 3.1.

The IRFs of the shock to the discount factor, $\varepsilon_t^\beta > 0$, are shown in Figure 3.7. This shock leads to a drop in the deposit rate as well as in consumption and output since the patient households become more patient and delay consumption. Thus, employment and inflation decrease as well. The drop in output and inflation lead to a large drop in the policy rate. That's why this shock is effective in making the zero lower bound binding. This shock has contractionary effects on consumption, employment, and output.

As Figure 3.8 shows, the banking cost shock, $\varepsilon_t^\Xi > 0$, lets banks increase money holdings and reduce loan supply in order to offset the increase in banking costs. Hence, MBS yields rise and house prices fall. This tightens the collateral constraint of borrowers, ω_t increases, who reduce their consumption and housing. In total, the shock has contractionary effects on output and consumption as well as on wages and inflation.

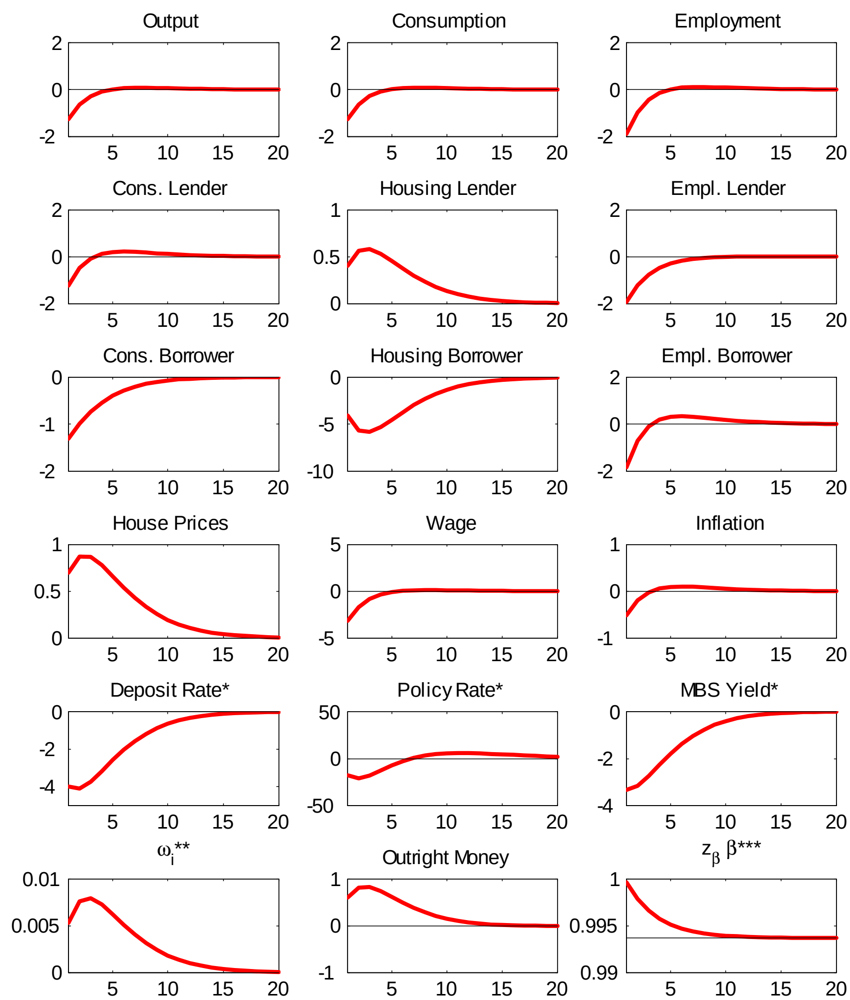

Figure 3.7: Impulse response functions to a shock to the discount factor of patient households. Percentage deviations from steady state, except for: * deviation in basis points, ** deviation in levels, and *** in levels.

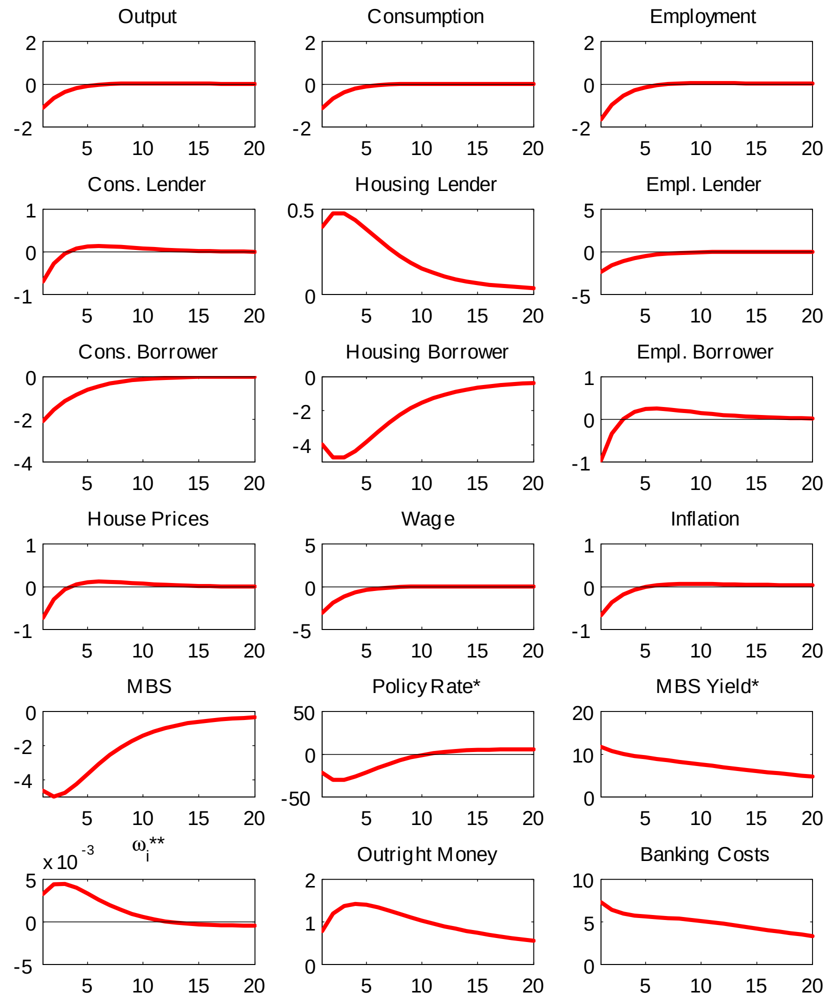

Figure 3.8: Impulse response functions to a shock to the discount factor of patient households. Percentage deviations from steady state, except for: * deviation in basis points, ** deviation in levels.

Chapter 4

Countercyclical Fiscal Multipliers

4.1 Introduction

One central issue of fiscal policy analysis is how an increase in government spending affects output. More specifically, the question is how large the government spending multiplier with respect to output is, i.e. how large the increase in output in dollar terms in response to a one dollar increase in government spending is. Recent studies indicate that the size of government spending multipliers depends on the state of the economy. Hence, the new question researchers put forward is *when* rather than whether government spending multipliers are large. Specifically, recent empirical papers suggest that the size of multipliers differs over the business cycle. For instance, Auerbach and Gorodnichenko (2012) deliver empirical evidence for countercyclical multipliers in the US, using a regime-switching vector autoregression model. Moreover, using annual data for nineteen OECD countries, Tagkalakis (2008) shows that the effectiveness of fiscal policy is higher in recessions due to binding borrowing constraints.[1]

[1] For further empirical evidence on countercyclical fiscal multipliers see e.g. Baum and Koester (2011), Bachmann and Sims (2012), Batini et al. (2012), Mittnik and Semmler (2012), Baum et al. (2012), Auerbach and Gorodnichenko (2013), Candelon and Lieb (2013), Fazzari et al. (2015), and Caggiano et al. (2015).

While empirical evidence on the cyclicality of multipliers is growing, there are only a few papers with theoretical models that can generate and explain countercyclical government spending multipliers. The aim of this chapter is to provide a theoretical model based on occasionally binding collateral constraints that illustrates a mechanism leading to countercyclical multipliers and to quantify the differences in multipliers in recessions and expansions on the grounds of this model. Our approach is in the lines of Tagkalakis (2008), who suggests borrowing constraints for explaining countercyclical multipliers. We implement this idea in a New Keynesian DSGE model with two types of households, patient Ricardian and impatient non-Ricardian collateral constrained households, whose borrowing limits are tied to the value of their houses, as in Iacoviello (2005). This collateral constraint builds the key element of our analysis since it is occasionally binding and hence leads to a non-linearity in our model. We show that in a situation in which the collateral constraint binds, which is the case in our recession scenario, a fiscal stimulus is more effective than in a situation where it does not bind, which is the case in our expansion scenario. The intuition for this result is that in the expansion scenario impatient households are less sensitive to changes in disposable income since their collateral constraint is not binding. A slack collateral constraint means that the household is at its unconstrained optimum and hence largely insensitive to changes in disposable income. On the contrary, in the recession scenario, impatient households find themselves at their borrowing limit and thus at their constrained optimum, which means that they consume less than they would if they could borrow more. Therefore, in the recession, their decisions are substantially affected by their current disposable income.

We observe the following effects on disposable income. First, a fiscal shock raises wages as a consequence of price rigidity, which leads to a larger increase in consumption and housing of impatient households in recessions. Second, in recessions the higher housing stock of impatient households together with an increase or, at least, a smaller drop in house prices boosts the borrowing limit of impatient households, which amplifies and prolongs the initial stimulus. This is what we label as *collateral effect*. In contrast, in expansions, the borrowing limit even decreases due to a large house price drop. Third, impatient households benefit from an unexpected increase in inflation in response to the government spending shock, which deflates their debt. On the contrary, depending on the financing of government spending, higher taxes eventually lower disposable income. These changes in disposable income affect substantially the decisions of impatient households in recessions, while they are largely insensitive to them in expansions.

The evaluation of our model concerning the analysis of government spending multipliers in expansions and recessions is conducted as follows. First, we simulate expansion and recession scenarios that are representative for the US. For the time period after World War II, the average expansion led to an increase in the cyclical component of real GDP by $2-2.5\%$, which we match when simulating a representative expansion scenario. For the simulation of a representative recession scenario, we use the same shock series as for the expansion scenario, but with negative sign. We find that for the expansion scenario the collateral constraint becomes slack, while it binds in the recession scenario. Then, we compute multipliers for the same government spending shock, first, when it occurs in the expansion scenario and, second, when it occurs in the recession scenario and compare them. The quantitative results of our model calibrated to US data can be summarized as follows. First, we find a large difference in the impact multiplier, which is considerably larger than 1 in recessions, while it is about 1 in expansions (1.6 vs. 1.04). Second, cumulative multipliers for the horizon of 1 year and 2 years are 1.47 and 1.18, respectively, in recessions, whereas the corresponding multipliers are 0.89 and 0.74, respectively, in expansions. Again, the differences are quite large with 64% for the horizon of 1 year and still 60% for 2 years. We provide sensitivity analyses for a large range for key parameters and find that the differences in recession and expansion multipliers remain considerably large.

Besides the aforementioned empirical literature, this chapter relates to theoretical work on state-dependent government spending multipliers. One state of the economy that leads to large government spending multipliers and that has been focused on recently is an economic environment in which the nominal interest rate is stuck at the zero lower bound (see e.g. Eggertsson (2010), Christiano et al. (2011), Woodford (2011), and Fernández-Villaverde et al. (2015)). Multipliers are large in this environment since the interest rate cannot counteract the effects of a fiscal stimulus. However, this state occurs very infrequent and did not matter until the recent financial crisis in the US. Moreover, Canzoneri et al. (2015) analyze countercyclical government spending multipliers in the presence of financial frictions. In a model with countercyclical bank intermediation costs, they find countercyclical multipliers of about 2 for recessions and about 1 for expansions, which is roughly consistent with what we find. One drawback of their approach is that they have to rely on countercyclical banking costs at the outset.

Further, this chapter extends the approach used in several papers, like e.g. Gali et al. (2007), which incorporate rule-of-thumb households into standard DSGE models. Depending on the fraction of these households, these models are capable of generating large multipliers, but not state-dependent ones. Another important drawback of their approach is that it relies on an ad-hoc constraint, which implies that a fraction of households do not borrow or save at all. In contrast, in our approach, both types of households are optimizing and have access to credit markets. The collateral constraint is the outcome of a costly enforcement problem between borrowers and savers and hence endogenous. Moreover, including collateral constrained rather than rule-of-thumb households in DSGE models seems empirically more relevant. Kaplan et al. (2014) estimate that a sizeable fraction of the population in major industrialized countries consists of hand-to-mouth consumers. They distinguish between two types of them, the "poor hand-to-mouth" with neither liquid nor illiquid wealth, which can be understood as the rule-of-thumb households in e.g. Gali et al. (2007) and the "wealthy hand-to-mouth" who own no liquid wealth but a large amount of illiquid wealth, like houses. The impatient collateral constrained households in our model own houses but do not hold any other assets and hence can be understood as "wealthy hand to mouth". Kaplan et al. (2014) figure out that the population share of the "wealthy" is twice as large as the share of the "poor hand-to-mouth" in the US and argue that former should be incorporated in models analyzing fiscal policy to have an undistorted view of the effects of fiscal stimuli on aggregate consumption. Finally, there are some studies that analyze government spending in frameworks with collateral constrained households (e.g. Khan and Reza (2013), Andres et al., 2015), but these do not consider the possibility of the collateral constraint to become slack.

The rest of this chapter is organized as follows. In section 4.2, the model framework is described. In section 4.3, we calibrate the model to US data, describe the simulation of representative expansion and recession scenarios, and discuss the results. Moreover, we provide some robustness checks at the end of this section. Section 4.4 concludes.

4.2 The Model

In this section, we present the model. The household sector consists of two types of agents, patient and impatient ones. While the former will save, the latter will be borrowers in equilibrium. As in Iacoviello (2005), their borrowing is collateralized by housing due to costly enforcement of loans. In the production sector, we introduce monopolistic competition of retailers and Calvo (1983) price setting leading to price stickiness. The treasury finances its expenditures by collecting lump sum taxes and issuing one-period bonds. Further, we consider a fiscal rule that inhibits a feedback of public debt to taxes. Finally, the central bank sets its policy rate according to a Taylor-type feedback rule.

4.2.1 Households

There is a continuum of households consisting of two types, patient (indexed with p) and impatient ones (indexed with i) with discount factors $1 > \beta^p > \beta^i > 0$. Both types of households derive utility from consumption $c_{*,t}$, housing $h_{*,t}$ and disutility from labor $n_{*,t}$ ($* = i, p$) and maximize the infinite sum of expected utility. Their objective is given by

$$E_0 \sum_{t=0}^{\infty} \beta^{*,t} u(c_{*,t}, h_{*,t}, n_{*,t}). \tag{4.1}$$

We consider the following CRRA-specification of the utility function

$$u(c_{*,t}, h_{*,t}, n_{*,t}) = z_t^c \frac{c_{*,t}^{1-\mu^c}}{1-\mu^c} + z_t^h \gamma^h \frac{h_{*,t}^{1-\mu^h}}{1-\mu^h} - \gamma^n \frac{n_{*,t}^{1+\mu^n}}{1+\mu^n}, \tag{4.2}$$

where $z_t^{c(h)}$ is a (housing) demand shock with $\log\left(z_t^{c(h)}\right) = \rho_{c(h)} \log\left(z_{t-1}^{c(h)}\right) + \varepsilon_t^{c(h)}$, where $\varepsilon_t^{c(h)} \sim n.i.d.\left(0, \sigma_{c(h)}^2\right)$ and $0 < \rho_{c(h)} < 1$. Here, $\mu^{c(h)}$ denotes the inverse of the intertemporal elasticity of substitution in consumption (housing) and μ^n the inverse of the Frisch elasticity of labor supply, γ^h and γ^n are weights on housing and labor in the utility function. We assume that $\mu^{c,h,n} > 0$ and $\gamma^{h,n} > 0$.

Let the population share of the patient households be $0 < s < 1$. We first describe the problem of a representative patient household and then the one of a representative impatient household.

Patient Households In equilibrium patient households are the savers in our model. They can invest their savings in physical capital $i_{p,t}$, hold government bonds $b_{p,t}^G$, and lend to impatient borrowers $b_{p,t}$. The rental rate for physical capital is denoted by r_t^k, the gross bond yield by R_t^G, and the gross loan rate by R_t. The budget constraint of a patient household p in period t in real terms is given by

$$\begin{aligned} & c_{p,t} + i_{p,t} + (1+\kappa_h)\, p_{h,t} h_{p,t} + b_{p,t}^G + b_{p,t} + \tau_{p,t} \\ = \quad & p_{h,t} h_{p,t-1} + \frac{R_{t-1}^G}{\pi_t} b_{p,t-1}^G + \frac{R_{t-1}}{\pi_t} b_{p,t-1} + w_t n_{p,t} + r_t^k k_{p,t} + \delta_{p,t} \end{aligned} \tag{4.3}$$

where the left hand side contains expenditures for consumption $c_{p,t}$, investment in physical capital $i_{p,t}$, purchase of a new house $(1+\kappa_h)\, p_{h,t} h_{p,t}$, and lump sum taxes, $\tau_{p,t}$. The real price of housing is denoted with $p_{h,t}$. Following Bajari et al. (2013), we assume that housing is associated with transaction costs that are proportional to the value of the newly purchased house.[2] Moreover, we assume that housing entails linear maintenance costs as in Cocco (2004). For simplicity, both of these costs are pooled in the term $\kappa_h p_{h,t} h_{p,t}$.

On the right hand side of (4.3), we have the revenues from selling the old house at the current price $p_{h,t} h_{p,t-1}$, revenues from bond holdings $\frac{R^G_{t-1}}{\pi_t} b^G_{p,t-1}$, repayment of previous period loans $\frac{R_{t-1}}{\pi_t} b_{p,t-1}$, with π_t being the inflation rate, labor income $w_t n_{p,t}$, capital income $r^k_t k_{p,t}$, and profits of firms and retailers $\delta_{p,t}$ since patient households are the owners of firms and retailers.

Physical capital is due to investment adjustment costs and accumulates according to

$$k_{p,t+1} = i_{p,t} - \kappa_i \frac{\left(i_{p,t} - i_{p,t-1}\right)^2}{i_p} + \left(1 - \delta_k\right) k_{p,t}, \tag{4.4}$$

where i_p is the steady state value of investment in physical capital of a patient household, $\delta_k > 0$ its depreciation rate, and $\kappa_i > 0$ a parameter reflecting the size of these costs.

A patient household chooses the values of $c_{p,t}$, $h_{p,t}$, $n_{p,t}$, $i_{p,t}$, $b^G_{p,t}$ and $b_{p,t}$ to maximize (4.1) subject to the budget constraint (4.3) and the capital accumulation equation (4.4), leading to the first order conditions

$$\begin{aligned}
u^h_{p,t} &= u^c_{p,t} p_{h,t} \left(1 + \kappa_h\right) - \beta^p E_t u^c_{p,t+1} p_{h,t+1} && (4.5)\\
-u^n_{p,t} &= w_t u^c_{p,t} && (4.6)\\
\xi_t &= \beta^p \xi_{t+1} \left(1 - \delta_k\right) - \beta^p u^c_{p,t+1} r^k_{t+1} && (4.7)\\
u^c_{p,t} &= \xi_t \left(\frac{2\kappa_i}{i_p} \left(i_{p,t} - i_{p,t-1}\right) - 1\right) - \beta^p \xi_{t+1} \frac{2\kappa_i \left(i_{p,t+1} - i_{p,t}\right)}{i_p} && (4.8)\\
u^c_{p,t} &= \beta^p E_t \frac{R^G_t}{\pi_{t+1}} u^c_{p,t+1} = \beta^p E_t \frac{R_t}{\pi_{t+1}} u^c_{p,t+1}, && (4.9)
\end{aligned}$$

with $u^j_{*,t} = \frac{\partial u}{\partial j_{*,t}}$ denoting the marginal utility with respect to the argument ($j = c, h, n$) and ξ_t being the multiplier on (4.4).

[2] For further examples for nonconvex housing adjustment costs see e.g. Flavin and Nakagawa (2008) and Iacoviello and Pavan (2013).

Equation (4.5) describes housing demand of a patient household. Marginal costs in terms of forgone consumption today $u_{p,t}^{c} p_{h,t} (1+\kappa_h)$ equal marginal utility the house delivers today $u_{p,t}^{h}$ plus the discounted benefit achieved from selling the house tomorrow in terms of consumption tomorrow $\beta^p E_t u_{p,t+1}^{c} p_{h,t+1}$. Equation (4.6) describes labor supply of a patient household and equates the marginal rate of substitution between consumption and leisure $\frac{u_{i,t}^{n}}{u_{i,t}^{c}}$ to the real wage rate w_t. Equation (4.7) and (4.8) describe optimal investment in physical capital and the equations in (4.9) are Euler equations for bonds and loans.

Impatient Households Since impatient households value current consumption more than patient ones, they will become borrowers in equilibrium and do not hold any assets. When a household i borrows in real terms an amount $b_{i,t-1} > 0$ in period $t-1$, it has to pay back $\frac{R_{t-1}}{\pi_t} b_{i,t-1}$ in period t. Following Guerrieri and Iacoviello (2015a), an impatient household i can only borrow up to a limit given by

$$b_{i,t} \leq \gamma^b \frac{b_{i,t-1}}{\pi_t} + \left(1-\gamma^b\right) \frac{\phi p_{h,t+1} h_{i,t} \pi_{t+1}}{R_t}, \tag{4.10}$$

where $0 < \gamma^b < 1$ denotes inertia in the borrowing limit and ϕ the (exogenous) pledgeable fraction of housing. This specification of the collateral constraint is more flexible and captures the sluggish response of mortgage debt to house prices (see Guerrieri and Iacoviello, 2015a).

An impatient household i has expenditures for consumption $c_{i,t}$, and new housing $(1+\kappa_h) p_{h,t} h_{i,t}$, pays back previous period loans $\frac{R_{t-1}}{\pi_t} b_{i,t-1}$, and lumps sum taxes $\tau_{i,t}$. On the income side, it has revenues from selling the old house $p_{h,t} h_{i,t-1}$, labor income $w_t n_{i,t}$, and a new loan $b_{i,t}$. Hence, the budget constraint of an impatient household i reads

$$c_{i,t} + (1+\kappa_h) p_{h,t} h_{i,t} + \frac{R_{t-1}}{\pi_t} b_{i,t-1} + \tau_{i,t} = p_{h,t} h_{i,t-1} + b_{i,t} + w_t n_{i,t}. \tag{4.11}$$

An impatient household i chooses the values of $c_{i,t}$, $h_{i,t}$, $n_{i,t}$, and $b_{i,t}$ to maximize (4.1) subject to the collateral constraint (4.10) and the budget constraint (4.11), leading to the first order conditions

$$\begin{aligned}
u^h_{i,t} &= u^c_{i,t} p_{h,t} \left(1+\kappa_h\right) - \beta^i E_t u^c_{i,t+1} p_{h,t+1} \qquad (4.12)\\
&\quad -\omega_t \left(1-\gamma^b\right) \frac{\phi p_{h,t+1} \pi_{t+1}}{R_t} \\
-u^n_{i,t} &= w_t u^c_{i,t} \qquad (4.13)\\
u^c_{i,t} &= \beta^i E_t \frac{R_t}{\pi_{t+1}} u^c_{i,t+1} + \omega_t - \gamma^b \frac{\beta^i \omega_{t+1}}{\pi_{t+1}} \qquad (4.14)
\end{aligned}$$

with ω_t being the multiplier on the collateral constraint and the complementary slackness conditions

$$\begin{aligned}
\omega_t \left(-b_{i,t} + \gamma^b \frac{b_{i,t-1}}{\pi_t} + \left(1-\gamma^b\right) \frac{\phi p_{h,t+1} h_{i,t} \pi_{t+1}}{R_t} \right) &= 0, \\
-b_{i,t} + \gamma^b \frac{b_{i,t-1}}{\pi_t} + \left(1-\gamma^b\right) \frac{\phi p_{h,t+1} h_{i,t} \pi_{t+1}}{R_t} \geq 0, \ \omega_t &\geq 0.
\end{aligned}$$

Equation (4.12) describes housing demand of an impatient household, again equating marginal costs with marginal utility but with the additional term $\omega_t \left(1-\gamma^b\right) \frac{\phi p_{h,t+1} \pi_{t+1}}{R_t}$ compared to a patient household. This term stems from the collateral constraint and reflects the additional benefit of housing to borrowers resulting from its usage as collateral for loans, which we label as the "collateral value" of housing. Equation (4.13) describes labor supply of an impatient household and (4.14) is a modified Euler equation for borrowing accounting for the collateral constraint. Furthermore, the transversality conditions must hold.

Finally, inserting (4.9) in (4.14) delivers the following steady state condition under which the collateral constraint is binding

$$\omega = u^c_i \frac{1 - \frac{\beta^i}{\beta^p}}{1 - \gamma^b \frac{\beta^i}{\pi}} > 0.$$

Since $u^c_i > 0$ and the nominator of the quotient is positive because of $\beta^p > \beta^i$ and the denominator as well due to $\pi > 1 > \beta^i$ and $0 < \gamma^b < 1$, the collateral constraint is binding in the steady state.

4.2.2 Firms

A continuum of perfectly competitive identical firms indexed with j produce the intermediate good according to $IO_{j,t} = z_t^p n_{j,t}^{\alpha} k_{j,t}^{1-\alpha}$, where $\alpha \in (0,1)$ and z_t^p is a productivity shock with $\log(z_t^p) = \rho_p \log\left(z_{t-1}^p\right) + \varepsilon_t^p$ and $\varepsilon_t^p \sim n.i.d.\left(0, \sigma_p^2\right)$ and $0 < \rho_p < 1$. The firm j hires labor $n_{j,t}$ at the real wage rate w_t and physical capital $k_{j,t}$ at the rate r_t^k to produce its output $IO_{j,t}$, which it sells to the retailers at the price $P_{J,t}$. Hence a firm j solves $\max P_{J,t} z_t^p n_{j,t}^{\alpha} k_{j,t}^{1-\alpha} - P_t w_t n_{j,t} - P_t r_t^k k_{j,t}$ leading to the first order conditions

$$P_{J,t} z_t^p \alpha n_{j,t}^{\alpha-1} k_{j,t}^{1-\alpha} = P_t w_t \tag{4.15}$$

$$P_{J,t} z_t^p n_{j,t}^{\alpha} (1-\alpha) k_{j,t}^{-\alpha} = P_t r_t^k \tag{4.16}$$

and to zero profits.

A continuum of monopolistically competitive retailers indexed with k buy intermediate goods at the price $P_{J,t}$, repackage them according to $IO_t = \int_0^1 IO_{j,t} dj$, differentiate them into $y_{k,t} = IO_{k,t}$ and sell the distinct goods $y_{k,t}$ at the price $P_{k,t}$ to perfectly competitive bundlers. They bundle them to the final good $y_t = \left(\int_0^1 y_{k,t}^{\frac{\epsilon-1}{\epsilon}} dk\right)^{\frac{\epsilon}{\epsilon-1}}$, where $\epsilon > 1$, which is sold at the price P_t. Hence a retailer k sets its price $P_{k,t}$ facing the demand function $y_{k,t} = \left(P_{k,t}/P_t\right)^{-\epsilon} y_t$ and taking $P_{J,t}$ as given. Following Calvo (1983), we assume that each period only a fraction $1-\theta$ of retailers is allowed to change its price. The other fraction $\theta \in [0,1)$ indexes the price to the steady state inflation rate according to $P_{k,t} = \pi P_{k,t-1}$. Defining $\widetilde{Z}_t = P_{k,t}^*/P_t$ with the optimal price of retailers $P_{k,t}^*$, the optimal real price can be written recursively as $\widetilde{Z}_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, with $Z_{1,t} = c_{p,t}^{-\mu^c} y_t mc_t + \theta\beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon} Z_{1,t+1}$ and $Z_{2,t} = c_{p,t}^{-\mu^c} y_t + \theta\beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon-1} Z_{2,t+1}$ where $mc_t = \frac{P_{J,t}}{P_t}$ denotes real marginal costs of retailers (for details see Appendix 4.5.1).

Due to perfectly competitive bundlers the aggregate price level P_t for final goods is given by $P_t^{1-\epsilon} = \int_0^1 P_{k,t}^{1-\epsilon} dk$ (zero profit condition) and can be written as $P_t^{1-\epsilon} = (1-\theta)\left(P_{k,t}^*\right)^{1-\epsilon} + \theta\pi^{1-\epsilon} P_{t-1}^{1-\epsilon} \Leftrightarrow 1 = (1-\theta)\widetilde{Z}_t^{1-\epsilon} +$

$\theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon-1}$ (see Appendix 4.5.1). Moreover, aggregate output is given by $y_t = \frac{z_t^p n_t^\alpha k_t^{1-\alpha}}{v_t}$ where $v_t = \int\limits_0^1 \left(P_{k,t}/P_t\right)^{-\epsilon} dk$ is a measure of price dispersion, which can be written recursively as $v_t = (1-\theta)\,\widetilde{Z}_t^{-\epsilon} + \theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon} v_{t-1}$ (see Appendix 4.5.1). Profits of retailers are distributed to patient households in a lump-sum way.

4.2.3 Public Sector

The treasury has (exogenous) expenditures which it finances by collecting lump sum taxes and issuing one-period bonds, which are held by patient households: $b_t^G = s b_{p,t}^G$. We assume that taxes are identical for both types $\tau_{p,t} = \tau_{i,t} = \tau_t$. The government budget constraint hence reads

$$g_t + \frac{R_{t-1}^G b_{t-1}^G}{\pi_t} = b_t^G + \tau_t.$$

We consider the following fiscal rule

$$\frac{\tau_t - \tau}{y} = \rho_\tau \frac{b_{t-1}^G - b^G}{y},$$

where $0 < \rho_\tau < 1$ is the feedback parameter for the reaction of taxes to debt and where variables without subscript denote steady state values of these variables. Whenever previous period's debt differs from its steady state level, taxes will react. If, for instance, an increase in government spending leads to an increase in debt, then the fiscal rule implies that taxes are raised to stabilize debt. Hence, the larger ρ_τ, the more of an increase in government spending is tax financed and the smaller ρ_τ, the more of it is debt financed. Government spending evolves according to

$$\log\frac{g_t}{g} = \rho_g \log\frac{g_{t-1}}{g} + \varepsilon_t^G,$$

where g denotes the steady state value of government spending and $\varepsilon_t^G \sim n.i.d.\left(0, \sigma_G^2\right)$ a government spending shock with $0 < \rho_g < 1$ being the parameter for the persistence of this shock.

The policy rate R_t is set by the central bank following a feedback rule given by

$$R_t = (R_{t-1})^{\rho_R} (R)^{1-\rho_R} (\pi_t/\pi)^{\rho_\pi(1-\rho_R)} (y_t/y)^{\rho_y(1-\rho_R)},$$

where $R > 1$ is an average policy rate chosen by the central bank, $\rho_R \geq 0$, $\rho_\pi \geq 0$ and $\rho_y \geq 0$.

4.2.4 Equilibrium

Factor markets clear according to $n_t = sn_{p,t} + (1-s)\, n_{i,t}$ and $k_t = sk_{p,t}$ and the credit market according to $sb_{p,t} = (1-s)\, b_{i,t}$. The consolidation of budget constraints delivers the aggregate resource constraint $y_t = sc_{p,t} + (1-s)c_{i,t} + g_t + si_{p,t} + \Psi_t$ with Ψ_t denoting total housing transaction and maintenance costs $\Psi_t = s\kappa_h p_{h,t} h_{p,t} + (1-s)\kappa_h p_{h,t} h_{i,t} = \kappa_h p_{h,t} H$, where $H = sh_{p,t} + (1-s)h_{i,t}$ and H is the fix level of housing supply. The set of equilibrium conditions is summarized and the equilibrium is defined in Appendix 4.5.2.

4.3 Simulation

In this section, we analyze whether the model developed in the previous section is capable of generating countercyclical government spending multipliers. To this end, we first simulate expansion and recession scenarios that are representative for the US. Given these scenarios, we can analyze how the same government spending shock affects GDP and other variables depending on whether it occurs in an expansion or in a recession. Since in an average expansion, in which the cyclical component of GDP rises by $2-2.5\%$, the collateral constraint becomes slack, this will potentially lead to (large) differences in expansion and recession multipliers. Throughout our analysis, we use a toolkit for solving models with occasionally binding constraints, OccBin, developed by Guerrieri and Iacoviello (2015b). Before we come to the analysis, we first calibrate the model to the US economy.

Table 4.1: Baseline Parameter Calibration.

Description	Source/Target	Parameter	Value
Discount factor patient households	G&I 2015a	β^p	0.995
Discount factor impatient households	G&I 2015a	β^i	0.99
Pledgeable fraction of housing	G&I 2015a	ϕ	0.9
Share of the patient households	G&I 2015a	s	0.6
Inverse of Frisch elasticity	G&I 2015a	μ^n	1
Inverse of IES in consumption	G&I 2015a	μ^c	1
Inverse of IES in housing	G&I 2015a	μ^h	1
Weight of housing in utility	$\frac{\Psi}{GDP} = 2\%$	γ^h	0.05
Weight of labor in utility	$n = 0.33$	γ^n	9.5
Inertia borrowing constraint	G&I 2015a	γ^b	0.5
Investment adjustment costs	G&I 2015a	κ_i	5
Capital depreciation rate	G&I 2015a	δ_k	0.025
Housing trans. & mainten. costs	see text	κ_h	0.06
Production function	G&I 2015a	α	2/3
Price rigidity	G&I 2015a	θ	0.9
Elasticity of substitution	G&I 2015a	ϵ	6

4.3.1 Calibration

Table 4.1 summarizes the baseline parameter calibration. One time period is assumed to be a quarter and the total housing stock is normalized to $H = 1$. In general, our calibration follows Guerrieri and Iacoviello (2015a). First, we adopt the following parameters from their calibration. We set the discount factor of patient households to $\beta^p = 0.995$, which together with a gross quarterly steady state inflation rate of $\pi = 1.005$ implies an annual real interest rate of 2% or $R = 1.01$. The parameters of the utility function are set to $\mu^n = 1$, implying a Frisch labor supply elasticity of 1, $\mu^c = 1$, and $\mu^h = 1$ implying log utility in consumption and housing. We set the pledgeable fraction of housing to $\phi = 0.9$, the capital depreciation rate to $\delta_k = 0.025$ and $\varepsilon = 6$ leading to a steady state mark-up of 20%. In line with their calibration, we further set the labor share in production to $\alpha = 2/3$.

Further parameters are set to values in the lines of the estimation results of Guerrieri and Iacoviello (2015a). The impatient households' discount factor is set to $\beta^i = 0.99$, the parameter for borrowing inertia to $\gamma^b = 0.5$, the investment adjustment cost parameter to $\kappa_i = 5$ and the Calvo parameter to $\theta = 0.9$. Finally, we set the autocorrelation coefficients of the shock processes that are used for the simulation of expansions and recessions to slightly smaller values than Guerrieri and Iacoviello (2015a) to ensure that the reversion to the steady state does not take too long. Therefore, we set $\rho_h = 0.85$, $\rho_c = 0.7$ and $\rho_p = 0.7$.

The remaining parameters are calibrated as follows. We set $\gamma^n = 9.5$ such that total hours worked in the steady state is $n = 0.33$ and $\gamma^h = 0.05$, which leads to $\frac{\Psi}{GDP} = 2\%$ in line with what we find in the National Income and Product Account (NIPA).[3] The share of impatient households is set to $1 - s = 0.4$, which is in the range of what Kaplan et al. (2014) estimate for the share of hand-to-mouth consumers in the US. We set κ_h as follows. Smith et al. (1988) estimate that housing transaction costs make up 8-10% of the value of the house and Harding et al. (2007) find for a sample from the American Housing Survey average annual maintenance costs of 1.4% of the house's value. Since maintenance costs always incur, but transaction costs only when a transaction takes place, we set the parameter to a conservative value of $\kappa_h = 0.06$.

Further, we set the parameters of the policy rate rule to values that are in the lines of what is typically used in the literature: $\rho_R = 0.8$, $\rho_\pi = 1.5$, and $\rho_y = 0.08$. Following Canzoneri et al. (2015), we set the feedback parameter of the fiscal rule to $\rho_\tau = 0.02$. We set the steady state ratio of government debt to annual GDP to 60% and of government spending to GDP to 20% in line with US data. Finally, we estimate the persistence parameter of the government spending shock for the time period that Guerrieri and Iacoviello (2015a) consider in their estimation, which is 1985Q1 to 2011Q4. The resulting point estimate from our regression of the HP-filtered (with a smoothing parameter of 1600) government spending series on its own lag is $\rho_g = 0.76$. This very close to 0.75, which is what Brückner and Pappa (2012) find as the average value of this parameter for several OECD countries. The resulting 95% confidence band from our regression for ρ_g is

[3] NIPA Tables 5.4.6.42 and 43 report values for real home improvements and brokers' commissions, respectively. On average, these expenditures made up about 2% of GDP for the time period of 2007-2014.

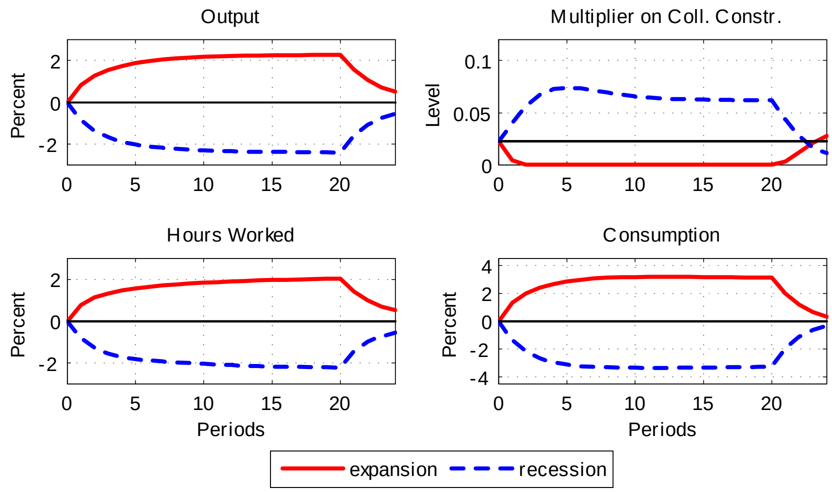

Figure 4.1: Simulation of Expansion and Recession Scenarios.

[0.63; 0.88], for which we will report multipliers in the robustness checks.

4.3.2 Simulating Expansions and Recessions

The analysis of countercyclical government spending multipliers requires expansion and recession scenarios that are representative for the US. For this, we estimate the average increase (fall) in GDP during expansions (recessions) using an HP-filtered (with a smoothing parameter of 1600) GDP series for the time period after World War II. We find that on average the cyclical component of GDP rose by $2-2.5\%$ during expansions, whereas the average drop during recessions was slightly larger. To avoid asymmetry in the starting point of our analysis, we calibrate the shock series to match the average expansion and use the same series of shocks with opposite sign for the recession scenario.

Since, on the one hand, demand shocks and, on the other hand, supply shocks may lead to an expansion or a recession, we use both types of shocks for the simulation of our representative scenarios. For the expansion scenario, we consider positive innovations to the consumption and housing demand shocks as well as to the TFP shock between period 1 and 20 such that GDP increases by 2.25%. For the recession scenario, we use the same shock series, but with negative sign. Figure 4.1 illustrates our simulation of expansions (solid line) and recessions (dashed line). It shows the reactions of output, the multiplier on the collateral constraint, ω_t, hours worked, and total consumption to the shock series. Except for the multiplier on the collateral constraint, that shows the level of ω_t, the figure shows percentage deviations of the variables from their steady state values.

In both scenarios, we see a comovement of aggregate variables. In the expansion scenario, output, total hours worked, and total consumption increase, while all three variables fall in the recession scenario. However, the drops in the recession are slightly larger than the increases in the expansion, although the shock series does not differ except for the sign. The reason for this lies in the reaction of the multiplier ω_t. While the multiplier rises strongly in the recession, its drop in the expansion is bounded by 0, which is the point, where the collateral constraint becomes slack. As we can see, in the expansion scenario the multiplier is stuck at 0, i.e. the collateral constraint is slack, from period 2 to 21. The fact that the collateral constraint becomes slack in the expansion scenario leads to the slight asymmetry in the reactions shown in Figure 4.1, but as we will see below it will influence multipliers considerably.[4]

4.3.3 Government Spending in Expansions and Recessions

Having simulated representative expansion and recession scenarios, we now turn to the analysis of the effectiveness of government spending depending on in which of these two scenarios it occurs. For this, we impose a positive government spending shock in the magnitude of 1% of its steady state value on the one hand in the expansion and on the other hand in the recession scenario. We then isolate the effects of the government spending shock as follows. First, we simulate an expansion without government spending (as in the previous section) and denote the resulting path of any variable x_t

[4] The asymmetric effects of occasionally binding collateral constraints on macroeconomic variables is studied in detail in Guerrieri and Iacoviello (2015a).

by $\Upsilon_E^{x_t,\neg g}$. Then, we simulate an expansion with the government spending shock and denote the resulting path of any variable x_t by $\Upsilon_E^{x_t,g_t}$. Finally, we subtract the impulse response functions (IRFs) of the former simulation from those of the latter. The resulting IRFs, given by $\Upsilon_E^{x_t} = \Upsilon_E^{x_t,g_t} - \Upsilon_E^{x_t,\neg g}$, which are displayed by the solid lines in Figure 4.2, show the isolated effects of the government spending shock in the expansion scenario. We follow the same procedure to obtain the effects of the same government spending shock in the recession scenario denoted by $\Upsilon_R^{x_t} = \Upsilon_R^{x_t,g_t} - \Upsilon_R^{x_t,\neg g}$ and shown by the dashed lines in Figure 4.2.

First of all, in both scenarios, the government spending shock leads to an increase in income, hours worked and wages. However, there are qualitative differences. While in the expansion the government spending shock increases income by 0.2%, hours worked by 0.3%, and wages by 0.4%, in the recession the increases are larger: 0.32%, 0.48%, and 0.88%, respectively. Moreover, in both scenarios, a patient household's consumption falls, whereas consumption and housing of an impatient household rise. While the drop in a patient household's consumption is almost the same in recessions and expansions, the initial rise in an impatient household's consumption and housing stock is about 3 times larger in recessions. Further, in the recession house prices slightly increase initially and then fall, while their fall in the expansion is more pronounced. This is reflected in the reaction of borrowing. The increase or, at least, smaller drop in house prices together with the larger increase in the housing stock of an impatient household lead to an increase in borrowing in the recession, whereas borrowing falls in the expansion. Total consumption, which is comprised of the consumption of the two types, increases in the recession persistently, while in the expansion it only rises slightly on impact and then falls. Finally, inflation goes up in both scenarios and after a small initial drop, the real interest rate increases as well.

How can we explain these differences? The intuition for this result is that in the expansion scenario impatient households are less sensitive to changes in disposable income since their collateral constraint is not binding. A slack collateral constraint means that the household is at its un-

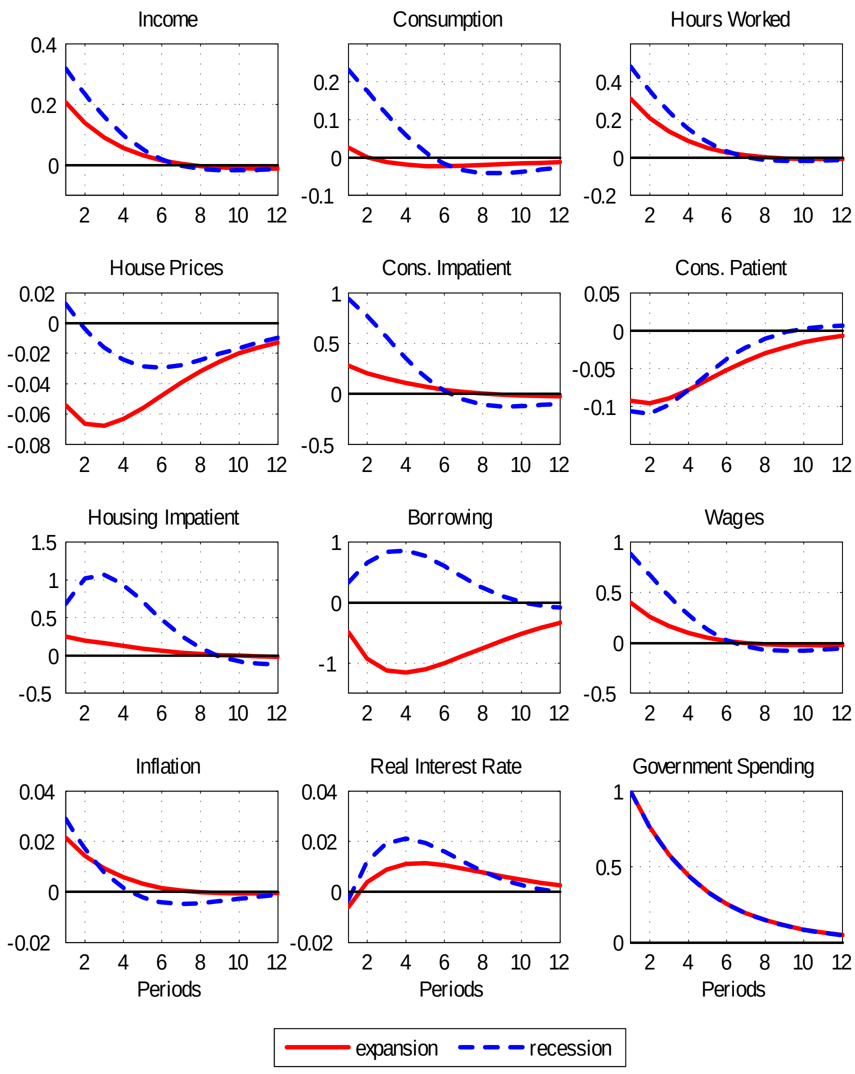

Figure 4.2: Effects of the same Government Spending Shock in the Expansion (solid line) and in the Recession Scenario (dashed line).

constrained optimum and hence largely insensitive to changes in disposable income. Although the household could borrow more, if it wanted, it does not so since this would be suboptimal. For the sake of argument, consider the first order condition (4.14) of impatient households with no inertia in the borrowing limit, $\gamma^b = 0$. For a slack collateral constraint, we get $\omega_t = 0$ and $u^c_{i,t} = \beta^i E_t \frac{R_t}{\pi_{t+1}} u^c_{i,t+1}$, such that the marginal utility of current consumption equals the expected discounted marginal utility of future consumption multiplied by the real interest rate. On the contrary, in the recession scenario, impatient households find themselves at their borrowing limit and thus at their constrained optimum. This means that they consume less than they would if they could borrow more. Therefore, in the recession, their decisions are substantially affected by their current disposable income. For a binding collateral constraint, we have $\omega_t > 0$ implying $u^c_{i,t} > \beta^i E_t \frac{R_t}{\pi_{t+1}} u^c_{i,t+1}$, i.e. the marginal utility of current consumption is larger than the expected discounted marginal utility of future consumption multiplied by the real interest rate. Therefore, in this state, an impatient household will increase current consumption towards its unconstrained optimum, whenever it is possible.

We observe the following effects on disposable income. First, a fiscal shock raises wages as a consequence of price rigidity. Since aggregate demand rises and prices are sticky, output and hence labor demand rise leading to an increase in wages. This leads to a larger increase in consumption and housing of impatient households in recessions because households are sensitive to disposable income. Second, in recessions the higher housing stock of impatient households together with an increase or, at least, a smaller drop in house prices boosts the borrowing limit of impatient households, which amplifies and prolongs the initial stimulus. This is what we label as *collateral effect*. In contrast, in expansions, the borrowing limit even decreases due to a large house price drop. Third, impatient households benefit from an unexpected increase in inflation in response to the government spending shock, which deflates their debt. These changes in disposable income affect strongly the decisions of impatient households in recessions, while they are largely insensitive to them in expansions.

More precisely, the mechanism behind the collateral effect works as follows. Due to the negative wealth effect of expected future tax increases implied by the government spending shock, Ricardian patient households want to reduce consumption and housing. Housing supply, however, is fixed at $H = 1$ and held by the two types of households. Hence, housing of impatient households must increase in the same amount as housing of patient households falls. How this reallocation takes place depends on the state of the economy. Consider now the first order condition describing impatient households' housing demand (4.12)

$$u_{i,t}^{h} = u_{i,t}^{c} p_{h,t} \left(1 + \kappa_{h}\right) - \beta^{i} E_{t} u_{i,t+1}^{c} p_{h,t+1} - \omega_{t} \left(1 - \gamma^{b}\right) \frac{\phi p_{h,t+1} \pi_{t+1}}{R_{t}}.$$

In the expansion, where ω_t is zero, the last term in this equation, which reflects the value of housing stemming from the fact that it serves as collateral, disappears. Impatient households are at their unconstrained optimum and are not willing to change their housing stock unless a sufficiently large drop in house prices occurs. Therefore, in the expansion, we observe a large house price drop in combination with a small reallocation of housing. In contrast, in the recession, impatient households, which are at their constrained optimum, are willing to increase their housing at the outset. The increase in disposable income enables them to do so, which is why house prices initially rise. Moreover, due to the additional collateral value of housing for $\omega_t > 0$, housing demand of impatient households is large, which is why the reallocation is larger in the recession. In sum, these reactions of house prices and housing stocks boost the borrowing limit of impatient households in recessions, which in turn increases their consumption and housing and amplifies the initial stimulus.

Comparing Multipliers To figure out the quantitative differences in the effectiveness of government spending in expansions and recessions, we compute government spending multipliers ($GSMs$) for three horizons. Let GSM_j with $j \in \{E, R\}$ denote the corresponding multiplier in the expansion (E) or the recession (R) scenario. The government spending multiplier for

Table 4.2: Government Spending Multipliers.

Multiplier	Expansion GSM_E	Recession GSM_R
GSM^I	1.04	1.60
GSM^{1Y}	0.89	1.47
GSM^{2Y}	0.74	1.18

the horizon T is then given by

$$GSM_j^T = \frac{y}{g} \frac{\sum_{t=1}^{T} \Upsilon_j^{y_t}}{\sum_{t=1}^{T} \widehat{g}_t},$$

where $\Upsilon_j^{y_t}$ denotes the isolated effect of government spending on GDP in quarter t (see above), $\widehat{g}_t$ denotes the deviation of government spending from its steady state value in the same quarter, and $\frac{y}{g}$ is the steady state ratio of GDP to government spending. For $T = 1$, we get the impact multiplier, GSM^I, which shows the effect of government spending in the first quarter. For the longer horizons, we get cumulative government spending multipliers. In particular, we consider the 1-year cumulative multiplier GSM_j^{1Y}, which results from $T = 4$, and the 2-year cumulative multiplier GSM_j^{2Y} resulting from $T = 8$.

Table 4.2 summarizes the resulting multipliers for our baseline calibration. The impact recession multiplier is considerably larger 1, $GSM_R^I = 1.6$, while the impact expansion multiplier is about 1, $GSM_E^I = 1.04$. The impact recession multiplier is thus 54% larger than the impact expansion multiplier. This difference becomes even larger in the subsequent periods, which is reflected in the difference of cumulative multipliers. The 1-year and 2-year cumulative recession multipliers are still larger 1, $GSM_R^{1Y} = 1.47$ and $GSM_R^{2Y} = 1.18$, whereas the cumulative expansion multipliers are noticeably smaller one, $GSM_E^{1Y} = 0.89$ and $GSM_E^{2Y} = 0.74$. The difference between recession and expansion multipliers is 64% for the horizon of 1 year and still 60% for 2 years.

4.3.4 Robustness Checks

In this section, we present sensitivity analyses concerning parameters that have been shown to be important for the effectiveness of fiscal policy. First, we vary the persistence of the government spending shock, ρ_g, for which our regression provides a 95% confidence interval of $[0.63; 0.88]$. Therefore, we consider how varying ρ_g between 0.6 and 0.9 affects multipliers. Second, we consider how a variation of ρ_τ, i.e. tax vs. debt financing of government spending, affects multipliers. Third, we provide multipliers for a range of the share of impatient households reaching from 25% to 40%, which is the range estimated by Kaplan et al. (2014) for the share of hand-to-mouth consumers in the US. Fourth, we vary the pledgeable fraction of housing, ϕ, between 0.7 and the maximum of 1. Finally, we consider a variation of the Calvo parameter, θ, within a range that spans 0.75, which is typically used in the literature and 0.92, which is the exact estimate of Guerrieri and Iacoviello (2015a).

The results of our robustness checks are plotted in Figure 4.3. In each row of Figure 4.3, we plot the multipliers resulting from the variation of one particular variable, holding all other variables as in the baseline calibration. Again, solid lines depict expansion multipliers and dashed lines recession multipliers. The left panels show impact multipliers, the middle panels cumulative multipliers for 1 year and the right panels for 2 years.

Persistence of Government Spending The first row of Figure 4.3 shows multipliers resulting from a variation of ρ_g. Consistent with what Gali et al. (2007) report, we find that multipliers are smaller, the larger the persistence of the government spending shock is. However, over the whole range we consider, the differences between recession and expansion multipliers are large. Since the initial shock to government spending is unexpected and the process is known once a shock has appeared, ρ_g determines how much of the total increase in g is unexpected and how much of it is anticipated. The larger ρ_g is, the larger is the proportion of the total increase in g that is anticipated. Therefore, the negative wealth effect is larger for higher persistence. This is why the multipliers fall in the persistence of gov-

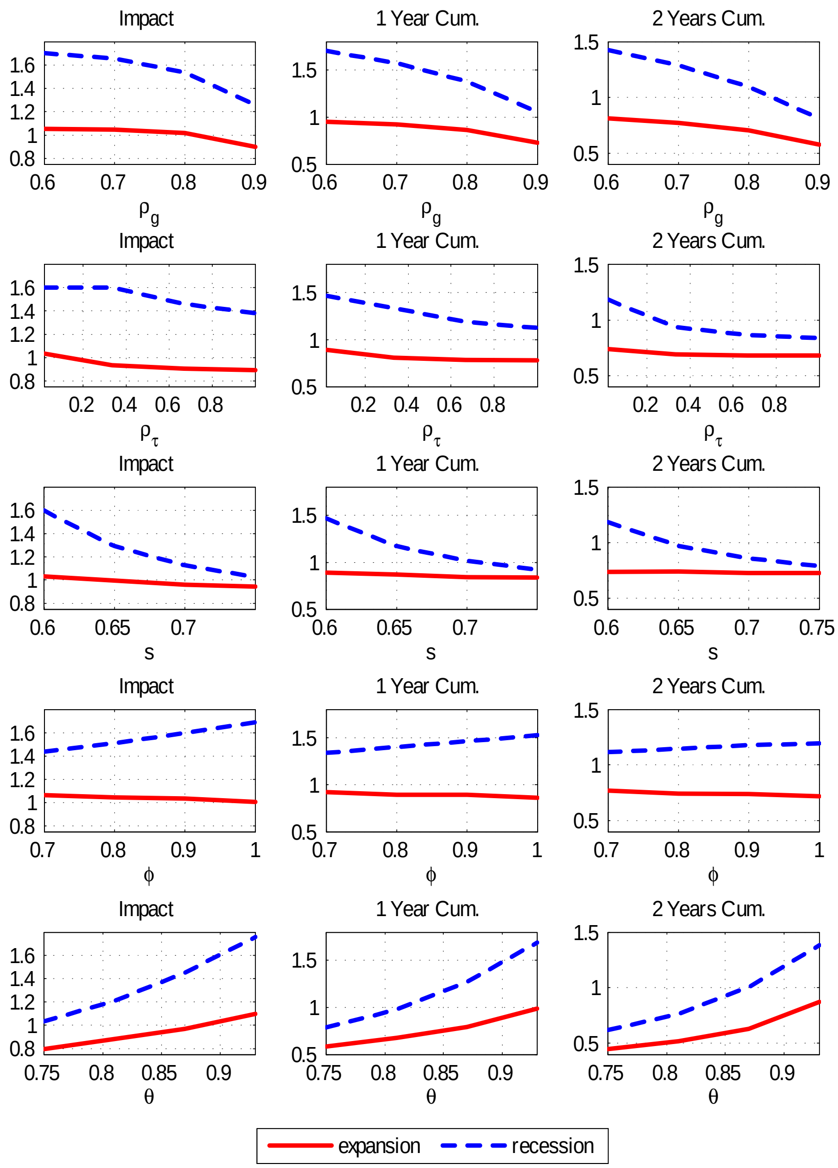

Figure 4.3: Robustness Checks: Each row shows impact, 1-year, and 2-year cumulative multipliers in recessions and expansions for a parameter variation.

ernment spending. In contrast, for a low persistence, e.g. $\rho_g = 0.6$, the negative wealth effect is small and outweighed by the positive effects on disposable income and the collateral effect described before.

To make this point more clear, consider Figure 4.4, which plots multipliers for $\rho_g = 0,$ i.e. for a government spending shock process with no persistence at all. Since ρ_g also determines the total size of the government spending shock, we impose a series of unexpected shocks with $\varepsilon_t^G = 0.01 \cdot (0.76)^{t-1}$ for $t = 1, ..., 12$, which mimics the shock process in the baseline calibration and hence makes the two scenarios better comparable. The impact recession and expansion multipliers are slightly smaller for the shock process without persistence, while cumulative multipliers are considerably larger, especially in recessions. The cumulative recession multipliers without anticipation are $GSM_R^{1Y} = 1.93$ and $GSM_R^{2Y} = 1.85$, whereas in the baseline calibration with $\rho_g = 0.76$ they are quite smaller: $GSM_R^{1Y} = 1.47$ and $GSM_R^{2Y} = 1.18$.

As mentioned before, when there is no anticipation, the negative wealth effect is small and the positive effect on impatient households comes to bear, which leads to the hump-shaped pattern in many of the IRFs in Figure 4.4. In particular, house prices show a hump-shaped reaction and increase persistently for this shock process due to the hump-shaped reaction of wages. Thus, the collateral effect and the amplification are more pronounced since the increase in the borrowing limit is larger. The conclusion of this exercise is that the less of an increase in government spending is anticipated, the more effective the fiscal stimulus is.

Tax vs. Debt Financing The second row of Figure 4.3 shows multipliers resulting from a variation of the debt feedback parameter in the fiscal rule, ρ_τ. Multipliers are smaller, the larger ρ_τ is, i.e. the more of an increase in government spending is tax financed. Nevertheless, as can be seen in Figure 4.3, the difference between recession and expansion multipliers for the whole range of ρ_τ is quite large. Multipliers fall in ρ_τ because higher taxes reduce ceteris paribus current disposable income of impatient households, which dampens their reactions that lead to large multipliers. Hence, the timing of taxation is not irrelevant in our model due to the binding

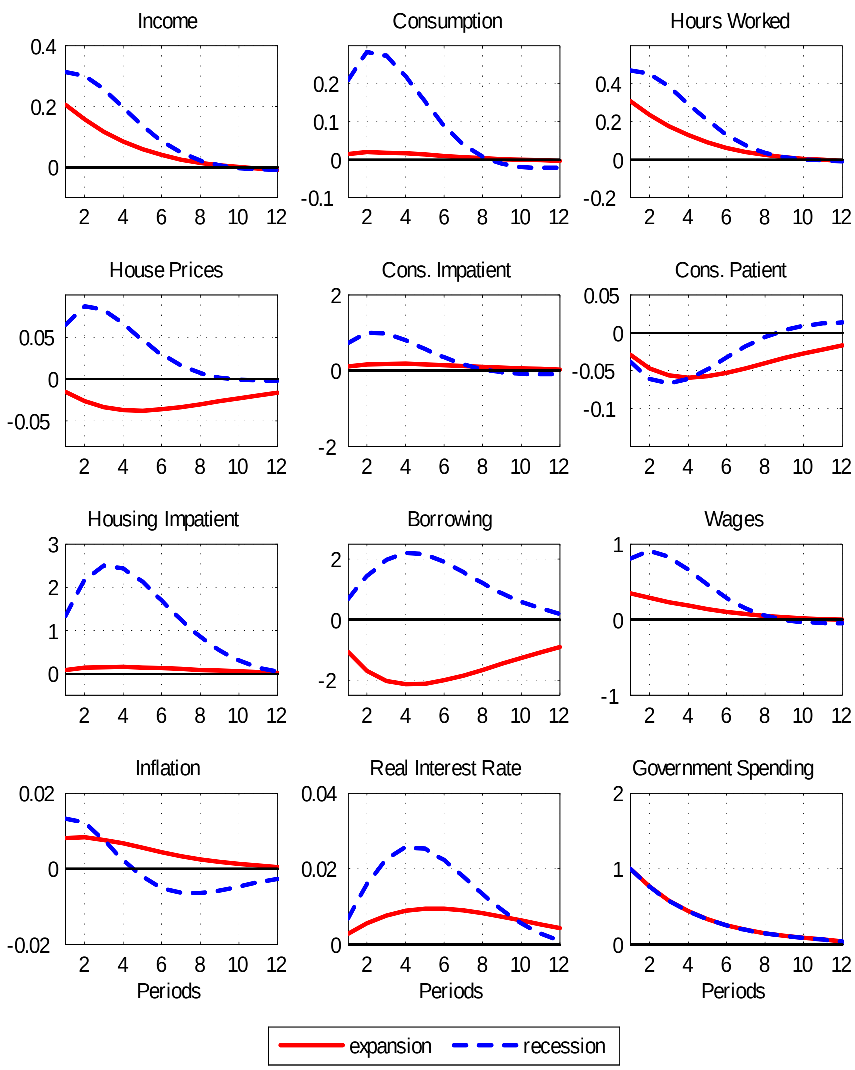

Figure 4.4: Robustness Checks: No persistence in government spending: $\rho_g = 0$. For a better comparability, we reproduce the government spending shock of the baseline calibration with unexpected shocks in each period.

collateral constraint, which implies a higher marginal utility of current consumption. Canzoneri et al. (2015) find as well that multipliers are larger when government spending is debt financed based on a similar mechanism.

Share of Patient Households The third row of Figure 4.3 shows multipliers resulting from a variation of the share of patient households, s, reaching from 0.6 to 0.75 since Kaplan et al. (2014) estimate a range of $25 - 40\%$ for the share of hand-to-mouth consumers in the US. We observe that multipliers fall in s, but even for the largest value of $s = 0.75$, we observe a difference in expansion and recession multipliers in Figure 4.3. The larger the share of patient Ricardian households, the more pronounced is the negative wealth effect in the aggregate. On the other hand, the smaller the share of impatient households, the fewer households are sensitive to changes in disposable income and hence the weaker is the positive effect on multipliers in the aggregate. Therefore, multipliers are larger, the larger the share of impatient households is. Consistent with our findings, Gali et al. (2007) find that multipliers are higher, the larger the share of rule-of-thumb households in their model is.

Pledgeable Fraction of Housing The fourth row of Figure 4.3 shows the multipliers resulting from a variation of ϕ. We see that for higher ϕ recession multipliers increase and the difference in recession and expansion multipliers becomes larger. Over the whole range for ϕ, we have considerable differences between expansion and recession multipliers. A higher pledgeable fraction of housing implies a higher marginal collateral value of a house and hence higher borrowing. This amplifies the positive effects on impatient households and multipliers are larger for larger ϕ. Andres et al. (2015) find as well larger multipliers for a higher level of private debt.

Price Stickiness The last row of Figure 4.3 shows the multipliers resulting from a variation of the Calvo parameter θ. Here, we see that the higher the level of price stickiness is, the larger multipliers are. Again, we see considerable differences in the multipliers for the whole range of θ we consider. Multipliers increase in θ because the initial wage increase that triggers the positive effect on impatient households is larger, the smaller the fraction of firms that can adjust prices is. The larger wage increase leads to a larger increase in impatient households' consumption, housing, and borrowing. Hence the higher θ, the stronger is this positive effect on impatient households, which outweighs the negative wealth effect on patient ones. Gali et al. (2007) find similar results for a variation of the Calvo parameter.

In summary, for all variations for the key parameters considered, the differences between expansion and recession multipliers remain large and the mechanisms leading to these differences are robust.

4.4 Conclusion

Recent empirical literature suggests that government spending multipliers are countercyclical, i.e. large in recessions and small in expansions. While empirical literature on this topic is growing, theoretical work is rare. In this chapter, we try to contribute to fill this gap. We provide a theoretical model based on occasionally binding collateral constraints that illustrates a mechanism leading to countercyclical government spending multipliers. Based on this framework, we quantify the differences between multipliers in recessions and expansions. In our framework, collateral constraints bind in recessions and become slack in expansions. Our model, calibrated to the US, generates impact recession multipliers of 1.6 and impact expansion multipliers of 1.04. Moreover, the one year (two years) cumulative multiplier is 1.47 (1.18) in recessions and 0.89 (0.74) in expansions. Hence, we observe large differences in recession and expansion multipliers on impact as well as for longer horizons.

The intuition for this result is that in the expansion scenario impatient households are largely insensitive to changes in disposable income since their collateral constraint is not binding and they are at their unconstraint optimum. On the contrary, in the recession scenario, impatient households find themselves at their borrowing limit and thus at their constrained optimum. Therefore, their decisions are substantially affected by changes in their disposable income. The increase in wages, the implied higher borrowing limit, and the unexpected increase in inflation let impatient households increase their consumption in recessions, while their reaction is much smaller in expansions.

We provide several robustness checks and find that for a large range for key variables the differences in recession and expansion multipliers remain considerably large. Finally, we find that a fiscal stimulus is more effective, the less of it is anticipated.

4.5 Appendix

4.5.1 Details of the Firms' Problem

Retailers' Price

The price setting problem of a retailer reads

$$\frac{\partial}{\partial P_{k,t}} E_t \left\{ \sum_{s=0}^{\infty} \theta^s q_{t,t+s} \left(\pi^s P_{k,t} - P_{J,t+s}\right) \left(\frac{P_{k,t}}{P_{t+s}}\right)^{-\epsilon} y_{t+s} \right\} = 0$$

with the stochastic discount factor $q_{t,t+s} = (\beta^p)^s E_t \frac{c_{p,t+s}^{-\mu^c}}{c_{p,t}^{-\mu^c}} \frac{P_t}{P_{t+s}}$ leading to the first order condition for the optimal price $P_{k,t}^*$

$$P_{k,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum\limits_{s=0}^{\infty} \theta^s q_{t,t+s} \left(\pi^s\right)^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s} P_{J,t+s}}{E_t \sum\limits_{s=0}^{\infty} \theta^s q_{t,t+s} \left(\pi^s\right)^{1-\epsilon} P_{t+s}^{\epsilon} y_{t+s}}. \tag{4.17}$$

Inserting $q_{t,t+s} = (\beta^p)^s E_t \frac{c_{p,t+s}^{-\mu^c}}{c_{p,t}^{-\mu^c}} \frac{P_t}{P_{t+s}}$ and marginal costs of retailers given by $mc_t = \frac{P_{J,t}}{P_t}$, (4.17) can be written as

$$P_{k,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum\limits_{s=0}^{\infty} (\theta\beta^p)^s c_{p,t+s}^{-\mu^c} \left(\pi^s\right)^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s} mc_{t+s}}{E_t \sum\limits_{s=0}^{\infty} (\theta\beta^p)^s c_{p,t+s}^{-\mu^c} \left(\pi^s\right)^{1-\epsilon} P_{t+s}^{\epsilon-1} y_{t+s}}. \tag{4.18}$$

We define $\widetilde{Z}_t = P_{k,t}^*/P_t$ such that we can rewrite the denominator and the numerator of (4.18) recursively as $\widetilde{Z}_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, with $Z_{1,t} = c_{p,t}^{-\mu^c} y_t mc_t + \theta\beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon} Z_{1,t+1}$ and $Z_{2,t} = c_{p,t}^{-\mu^c} y_t + \theta\beta^p E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon-1} Z_{2,t+1}$.

Aggregate Price Level

The aggregate price level P_t for final goods is given by $P_t^{1-\epsilon} = \int_0^1 P_{k,t}^{1-\epsilon} dk$ due to perfectly competitive bundlers (zero profit condition). Since all retail prices were set according to (4.17) in the past, we can write this as

$$\begin{aligned}
P_t^{1-\epsilon} &= \int_0^1 P_{k,t}^{1-\epsilon} dk = (1-\theta) \sum_{s=0}^{\infty} \theta^s \left(\pi^s P_{k,t-s}^*\right)^{1-\epsilon} \\
&= (1-\theta)\left(P_{k,t}^*\right)^{1-\epsilon} + \theta \pi^{1-\epsilon} P_{t-1}^{1-\epsilon}.
\end{aligned}$$

Hence, the aggregate price level is given by

$$P_t^{1-\epsilon} = (1-\theta)\left(P_{k,t}^*\right)^{1-\epsilon} + \theta \pi^{1-\epsilon} P_{t-1}^{1-\epsilon} \Rightarrow 1 = (1-\theta)\widetilde{Z}_t^{1-\epsilon} + \theta \left(\frac{\pi_t}{\pi}\right)^{\epsilon-1}.$$

Price Dispersion

With the assumption that there is no initial price dispersion, we can write v_t with $P_{k,t}^*$ being the optimal price set by the price adjusting firms at time t as

$$\begin{aligned}
v_t &= (1-\theta)\left(P_{k,t}^*/P_t\right)^{-\epsilon} + (1-\theta)\theta\left(\pi P_{k,t-1}^*/P_t\right)^{-\epsilon} \\
&\quad + (1-\theta)\theta^2\left(\pi^2 P_{k,t-2}^*/P_t\right)^{-\epsilon} + ... \\
&= (1-\theta)\sum_{l=0}^{\infty}\theta^l \left(\pi^l P_{k,t-l}^*/P_t\right)^{-\epsilon} \\
&= (1-\theta)\sum_{l=0}^{\infty}\theta^l \left(\pi^l P_{k,t-l}^*/P_{t-l}\right)^{-\epsilon}\left(P_{t-l}/P_t\right)^{-\epsilon} \\
&= (1-\theta)\sum_{l=0}^{\infty}\theta^l \left(\pi^l\right)^{-\epsilon}\widetilde{Z}_{t-l}^{-\epsilon}\prod_{s=1}^{l}\pi_{t+1-s}^{\epsilon}
\end{aligned}$$

with $\widetilde{Z}_t = P_{k,t}^*/P_t$ and $\sum\limits_{l=1}^{\infty}\prod\limits_{s=1}^{l}\pi_{t+1-s}^{\epsilon} = \pi_t^{\epsilon} + \pi_t^{\epsilon}\pi_{t-1}^{\epsilon} + \pi_t^{\epsilon}\pi_{t-1}^{\epsilon}\pi_{t-1}^{\epsilon} + ... = \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} + \underbrace{\left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon}\left(\frac{P_{t-2}}{P_{t-1}}\right)^{-\epsilon}}_{(P_{t-2}/P_t)^{-\epsilon}} + \underbrace{\left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon}\left(\frac{P_{t-2}}{P_{t-1}}\right)^{-\epsilon}\left(\frac{P_{t-3}}{P_{t-2}}\right)^{-\epsilon}}_{(P_{t-3}/P_t)^{-\epsilon}} + ... = \sum\limits_{l=1}^{\infty}\left(\frac{P_{t-l}}{P_t}\right)^{-\epsilon}$. Taking differences, we can write v_t in a more compact recursive way:

$$
\begin{aligned}
v_t - \theta\pi^{-\epsilon}\pi_t^{\epsilon}v_{t-1} &= (1-\theta)\sum_{l=0}^{\infty}\theta^l\left(\pi^l\right)^{-\epsilon}\widetilde{Z}_{t-l}^{-\epsilon}\prod_{s=1}^{l}\pi_{t+1-s}^{\epsilon} \\
&- \theta\pi^{-\epsilon}\pi_t^{\epsilon}(1-\theta)\sum_{l=0}^{\infty}\theta^l\left(\pi^l\right)^{-\epsilon}\widetilde{Z}_{t-1-l}^{-\epsilon}\prod_{s=1}^{l}\pi_{t-s}^{\epsilon} \\
= (1-\theta)\widetilde{Z}_t^{-\epsilon} &+ (1-\theta)\theta\pi^{-\epsilon}\widetilde{Z}_{t-1}^{-\epsilon}\pi_t^{\epsilon} + (1-\theta)\theta^2\pi^{-2\epsilon}\widetilde{Z}_{t-2}^{-\epsilon}\pi_t^{\epsilon}\pi_{t-1}^{\epsilon} + ... \\
&- (1-\theta)\theta\pi^{-\epsilon}\widetilde{Z}_{t-1}^{-\epsilon}\pi_t^{\epsilon} - (1-\theta)\theta^2\pi^{-2\epsilon}\widetilde{Z}_{t-2}^{-\epsilon}\pi_t^{\epsilon}\pi_{t-1}^{\epsilon} - ... \\
\Leftrightarrow v_t &= (1-\theta)\widetilde{Z}_t^{-\epsilon} + \theta\left(\frac{\pi_t}{\pi}\right)^{\epsilon}v_{t-1}.
\end{aligned}
$$

4.5.2 Equilibrium

Definition 2 *A rational expectations equilibrium is a set of sequences* $\{c_{p,t}$, $h_{p,t}$, $n_{p,t}$, $k_{p,t}$, $i_{p,t}$, $c_{i,t}$, $h_{i,t}$, $n_{i,t}$, $p_{h,t}$, w_t, π_t, ω_t, $b_{i,t}$, R_t, n_t, mc_t, $\widetilde{Z}_t$, $Z_{1,t}$, $Z_{2,t}$, y_t, v_t, g_t, b_t^G, τ_t, r_t^k, k_t, $\xi_t\}_{t=0}^{\infty}$ *satisfying the optimality conditions of patient households*

$$u_{p,t}^h = u_{p,t}^c p_{h,t}(1+\kappa_h) - \beta^p E_t u_{p,t+1}^c p_{h,t+1}, \tag{4.19}$$

$$-u_{p,t}^n = w_t u_{p,t}^c, \tag{4.20}$$

$$k_{p,t+1} = i_{p,t} - \kappa_i\frac{(i_{p,t}-i_{p,t-1})^2}{i_p} + (1-\delta_k)k_{p,t}, \tag{4.21}$$

$$\xi_t = \beta^p\xi_{t+1}(1-\delta_k) - \beta^p u_{p,t+1}^c r_{t+1}^k, \tag{4.22}$$

$$u_{p,t}^c = \xi_t\left(\frac{2\kappa_i}{i_p}(i_{p,t}-i_{p,t-1}) - 1\right) - \beta^p\xi_{t+1}\frac{2\kappa_i}{i_p}(i_{p,t+1}-i_{p,t}), \tag{4.23}$$

$$u_{p,t}^c = \beta^p E_t\frac{R_t}{\pi_{t+1}}u_{p,t+1}^c, \tag{4.24}$$

impatient households

$$\begin{aligned}
u_{i,t}^{h} &= u_{i,t}^{c} p_{h,t}\left(1+\kappa_{h}\right)-\beta^{i} E_{t} u_{i,t+1}^{c} p_{h,t+1} \\
&\quad -\omega_{t}\left(1-\gamma^{b}\right) \frac{\phi p_{h,t+1} \pi_{t+1}}{R_{t}} \qquad (4.25)\\
-u_{i,t}^{n} &= w_{t} u_{i,t}^{c} \qquad (4.26)\\
u_{i,t}^{c} &= \beta^{i} E_{t} \frac{R_{t}}{\pi_{t+1}} u_{i,t+1}^{c}+\omega_{t}-\gamma^{b} \frac{\beta^{i} \omega_{t+1}}{\pi_{t+1}} \qquad (4.27)\\
c_{i,t} &+ \left(1+\kappa_{h}\right) p_{h,t} h_{i,t}+\frac{R_{t-1}}{\pi_{t}} b_{i,t-1}+\tau_{t} \qquad (4.28)\\
&= p_{h,t} h_{i,t-1}+b_{i,t}+w_{t} n_{i,t} \\
b_{i,t} &= \gamma^{b} \frac{b_{i,t-1}}{\pi_{t}}+\left(1-\gamma^{b}\right) \frac{\phi p_{h,t+1} h_{i,t} \pi_{t+1}}{R_{t}} \text{ if } \omega_{t}>0, \qquad (4.29)\\
\text{or } b_{i,t} &< \gamma^{b} \frac{b_{i,t-1}}{\pi_{t}}+\left(1-\gamma^{b}\right) \frac{\phi p_{h,t+1} h_{i,t} \pi_{t+1}}{R_{t}} \text{ if } \omega_{t}=0, \qquad (4.30)
\end{aligned}$$

firms

$$\begin{aligned}
y_{t} &= \frac{z_{t}^{p} n_{t}^{\alpha} k_{t}^{1-\alpha}}{v_{t}} \qquad (4.31)\\
n_{t} &= s n_{p,t}+(1-s) n_{i,t}, \qquad (4.32)\\
k_{t} &= s k_{p,t}, \qquad (4.33)\\
w_{t} &= m c_{t} z_{t}^{p} \alpha n_{t}^{\alpha-1} k_{t}^{1-\alpha}, \qquad (4.34)\\
r_{t}^{k} &= m c_{t} z_{t}^{p} n_{t}^{\alpha}(1-\alpha) k_{t}^{-\alpha}, \qquad (4.35)\\
\widetilde{Z}_{t} &= \frac{\epsilon}{\epsilon-1} Z_{1,t} / Z_{2,t}, \qquad (4.36)\\
Z_{1,t} &= u_{p,t}^{c} y_{t} m c_{t}+\theta \beta^{p} E_{t}\left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon} Z_{1,t+1}, \qquad (4.37)\\
Z_{2,t} &= u_{p,t}^{c} y_{t}+\theta \beta^{p} E_{t}\left(\frac{\pi_{t+1}}{\pi}\right)^{\epsilon-1} Z_{2,t+1}, \qquad (4.38)\\
v_{t} &= (1-\theta) \widetilde{Z}_{t}^{-\epsilon}+\theta v_{t-1}\left(\frac{\pi_{t}}{\pi}\right)^{\epsilon}, \qquad (4.39)\\
1 &= (1-\theta) \widetilde{Z}_{t}^{1-\epsilon}+\theta\left(\frac{\pi_{t}}{\pi}\right)^{\epsilon-1}, \qquad (4.40)
\end{aligned}$$

the public sector conditions

$$g_t + \frac{R_{t-1}b^G_{t-1}}{\pi_t} = b^G_t + \tau_t, \tag{4.41}$$

$$\frac{\tau_t - \tau}{y} = \rho_\tau \frac{b^G_{t-1} - b^G}{y}, \tag{4.42}$$

$$\log \frac{g_t}{g} = \rho_g \log \frac{g_{t-1}}{g} + \varepsilon^G_t, \tag{4.43}$$

$$R_t = (R_{t-1})^{\rho_R} (R)^{1-\rho_R} (\pi_t/\pi)^{\rho_\pi(1-\rho_R)} (y_t/y)^{\rho_y(1-\rho_R)}, \tag{4.44}$$

the market clearing conditions

$$y_t = sc_{p,t} + (1-s)c_{i,t} + g_t + si_{p,t} + \kappa_h p_{h,t} H \tag{4.45}$$

$$H = sh_{p,t} + (1-s)h_{i,t} \tag{4.46}$$

and transversality conditions, with $u^j_{*,t} = \frac{\partial u}{\partial j_{*,t}}$ *denoting marginal utility with respect to* $j \in (c,h,n)$ *of type* $* \in (p,i)$*, given the fixed housing supply* $H > 0$, $\Psi_t = s\kappa_h p_{h,t} h_{p,t} + (1-s)\kappa_h p_{h,t} h_{i,t} = \kappa_h p_{h,t} H$*, initial values* $b^G_{-1} > 0$, $k_{-1} > 0$, $\pi_{-1} > 0$, $v_{-1} = 1$*, and the exogenous processes for* $\{z^h_t,\ z^c_t,\ z^p_t\}^\infty_{t=0}$ *and i.i.d. innovations with mean zero* $\{\varepsilon^h_t,\ \varepsilon^c_t,\ \varepsilon^p_t,\ \varepsilon^G_t\}^\infty_{t=0}$*. Herein, variables without subscript denote their steady state values.*

4.5.3 Steady State

The deterministic steady state follows from the equilibrium defined above. The steady state value of each variable is denoted without time index. We can reduce the equilibrium conditions for the steady state as follows. First, consider the firms' optimality condition $\widetilde{Z}_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, where $Z_1 = \frac{c_p^{-\mu^c} y\, mc}{1-\theta\beta^p}$ and $Z_2 = \frac{c_p^{-\mu^c} y}{1-\theta\beta^p}$ in the steady state. Hence, we get $\widetilde{Z} = \frac{\epsilon}{\epsilon-1} mc$ and with the steady state mark-up of $\frac{\epsilon}{\epsilon-1} \Rightarrow mc = \frac{\epsilon-1}{\epsilon} \Rightarrow \widetilde{Z} = 1$. From this, we get $v = \frac{(1-\theta)\widetilde{Z}^{-\epsilon}}{1-\theta} = 1$. Moreover, for a given steady state inflation rate π and we get $R = \frac{\pi}{\beta^p}$ and $r^k = \frac{1}{\beta^p} - 1 + \delta_k$. The values of $\{c_p,\ h_p,\ n_p,\ c_i,\ h_i,\ n_i,\ p_h,\ w,\ \omega,\ b_i,\ y,\ n,\ k,\ \tau\}$ that satisfy the conditions (4.47)-(4.60) mark the steady state of the model:

$$
\begin{aligned}
u_p^h &= u_p^c p_h \left(1 + \kappa_h - \beta^p\right) && (4.47)\\
-u_p^n &= w u_p^c && (4.48)\\
u_i^h &= u_i^c p_h \left(1 + \kappa_h - \beta^i\right) - \omega \left(1 - \gamma^b\right) \phi p_h \beta^p && (4.49)\\
-u_i^n &= w u_i^c && (4.50)\\
\omega &= u_i^c \frac{1 - \frac{R}{\pi}\beta^i}{1 - \frac{\gamma^b}{\pi}\beta^i} && (4.51)\\
b_i &= \frac{1 - \gamma^b}{1 - \frac{\gamma^b}{\pi}} \phi p_h h_i \beta^p && (4.52)\\
c_i + \kappa_h p_h h_i + \tau &= b_i \left(1 - \frac{R}{\pi}\right) + w n_i && (4.53)\\
n &= s n_p + (1 - s)\, n_i && (4.54)\\
\frac{\epsilon}{\epsilon - 1} w &= \alpha n^{\alpha - 1} k^{1-\alpha} && (4.55)\\
\frac{\epsilon}{\epsilon - 1} r^k &= n^{\alpha} (1 - \alpha)\, k^{-\alpha} && (4.56)\\
y &= n^{\alpha} k^{1-\alpha} && (4.57)\\
g &= b^G \left(1 - \frac{R^G}{\pi}\right) + \tau && (4.58)\\
y &= s c_p + (1 - s) c_i + g + s i_p + \kappa_h p_h H && (4.59)\\
H &= s h_p + (1 - s)\, h_i && (4.60)
\end{aligned}
$$

with $b^G = 4 \cdot 0.6y$, $g = 0.2y$ and $u_*^j = \frac{\partial u}{\partial j_*}$, given parameter values for β^p, β^i and π ensuring a binding collateral constraint in the steady state.

Chapter 5

Concluding Remarks

This thesis has presented three essays that emphasize the role of housing for the conduct of monetary and fiscal policy. In the presence of financial market imperfections, private debt has to be collateralized, which is typically done by pledging houses. In each chapter of this thesis, we have used models, which incorporate household sectors consisting of lenders and borrowers who face a collateral constraint that ties their borrowing limit to the value of their houses.

In chapter 2, we have shown that housing subsidies, which can be observed in several industrialized countries, are optimal in the presence of collateral constrained households. Chapter 3 has analyzed the macroeconomic effects of the Fed's MBS purchases and has shown that these had considerable effects on economic activity and prices providing a rationale for this new type of monetary policy measure. Chapter 4 has provided a theoretical framework in which collateral constraints are occasionally binding, which leads to government spending multipliers that are large in recessions and small in expansions consistent with recent empirical evidence.

In our frameworks, housing wealth affects consumption of collateral constrained borrowers and thereby aggregate consumption. Hence, whenever a policy affects housing wealth of borrowers, this will amplify the effects of this policy measure. Since borrowers have a high marginal propensity to consume out of wealth, this amplification may become large. We have seen that this housing wealth/collateral channel played a significant role for the effects in chapters 3 and 4.

To summarize, we have shown that accounting for the role of housing in borrower-lender frameworks with financial market imperfections, has important implications for monetary and fiscal policy. The increased interest of researchers on the interplay of housing and the macroeconomy in recent years leads to various new frameworks incorporating housing, which help to better understand the role that housing plays for the macroeconomy. The analyses within this theses suggest, that analyzing the implications of housing for other policy questions within these new frameworks may deliver interesting insights.

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