Ingrida VEILANDE, Riga

Tasks on orthogonal configurations in extracurricular activities

Introduction. Why mathematical Circles? Mathematical Circles or Clubs (MC) are organized in the schools, at the universities, or in the Centers of Educational Research. There are different reasons of establishing them. For example, the project MALU at the University of Munster involve gifted children to raise the level of their mathematical competencies and to research their problem solving abilities (Rott, 2013). Moscow Center of Continued Mathematical Education[1] defines the goal to hold and to expand the traditions of mathematical education. The mathematics educators and the teachers from different countries propose that MC are necessary to improve the students mathematical knowledge (Thompson, 2009) or to prepare the students for Mathematical Olympiads (Koichu, Andzans, 2009), or to improve the problem solving skills of students and to collect the information about the challenging problems and to provoke the creativity of the bachelor students of pedagogy (Prescot, Pressick-Kilborn, 2015).

Preparation to the Open Mathematical Olympiad. Mathematical Circles organized in Latvia have additional important goal – to prepare students for Mathematical Olympiads. Two Mathematical Olympiads – State Olympiad and Open Mathematical Olympiad (OMO) - are organized by the Extramural School of Mathematics at the University of Latvia. At the State Olympiad can take a part only the best students. At the OMO can participate any student from the 1st till the 12th grade. This is very popular event - in recent years, the number of participants has been reaching 3000. Nevertheless the average score of gained results on the Olympiad is low, for example, the average score of the high school students' usually is less the third or the forth part of the maximal score 50 points. Considering the data offered by PISA about the assessment of the 15 – years old students (PISA, 2013), the principals of schools in Latvia reported that approximately a third part of students can attend to the MC, about a half of students can visit after-school lessons in mathematics, more than 90% of students can participate at the Mathematical Olympiads. Comparison of these data with the results on OMO shows implicitly that the students of these age group do not use by the school offered possibilities actively to prepare well for the Olympiad. The problem set on OMO is quite different from the problems of com-

[1] Homepage of Moscow Center of Continued Mathematical Education (in Russian). Retrieved from http://www.mccme.ru/head/celi.htm

In Institut für Mathematik und Informatik Heidelberg (Hrsg.), *Beiträge zum Mathematikunterricht 2016* (S. x–y). Münster: WTM-Verlag

pulsory mathematics. Therefore the question originate - what auxiliary materials could be recommended for teachers and leaders of the MC and of other extracurricular mathematical activities, who prepare the students for Olympiads?

Problems on changeable configuration. To solve the problems of OMO is not necessary to use very deep mathematical knowledge. In every problem set are included the problems of recreational type too along with the problems of number theory, algebra, and geometry. The problems of combinatorial geometry are offered every year. The students of younger grades and older students as well mostly use the method of trial and error to solve the problems of such type, but they do not understand exactly what does it means “the proof problem”.

The problems about the orthogonal configurations with changeable structure are very useful to master the main problem solving principles - to read and to understand givens, to make the purposeful experiments, to draw the pictures, to classify, to master different heuristic strategies, to hypothesize, to explain, to discuss, and to prove.

The orthogonal *changeable configuration* can be defined in the following way:

The objects are given in the actions' field. The start position of objects is described generally or it is given by definite configuration. The configuration changes step by step according to the rules. Orthogonality of configuration is determined by the actions' field and/or the specific configuration of objects.

Usually the *actions field* is a rectangle consisting from the unit squares, orthogonal lattice, squared plane, and marked straight line. The *objects* can be points, numbers, unit squares, figures, bricks, colors etc.

Dependent case:	**Independent case:**
Create an algorithm to reach the end position	Detect whether the process is finite or continuous
Find the shortest algorithm	What could be the end position?
Prove that the process converges to the fixed end position	What are specific characteristics of the process?
Prove that the desired result is not possible	Change the start position to get some specific end position

Table 1. Categories of problems on changeable configurations

The problems on changeable configurations can be categorized according to the content and the imperative of the problem. The *dependent case* is given when the solver has to use given conditions to change the configuration. The *independent case* has to be researched when the discrete configuration changes itself with respect of the givens (see Table 1). Each of these cases can be classified by the demand of the problem. In some cases these problems cover the questions of algorithm theory.

Examples of problems. The problems chosen for the MC have to fulfil some of the following criteria – here must be an introductory problem, problems have to be challenging, the levels of difficulty must increase, various heuristic methods have to be master at the problem solution.

Introductory problem. What do you see in this picture? What connections are between the squares? (See Figure 1.)

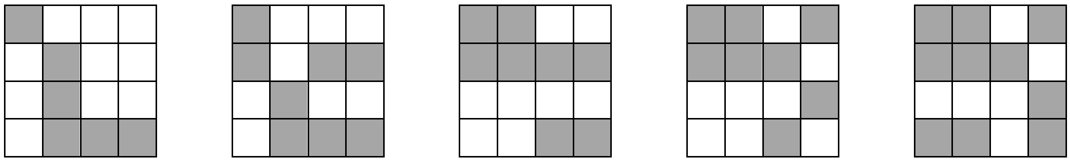

Figure 1. Colored squares for introductory problem

This problem was offered for 5th – 9th grade students from some Mathematical circles in Riga and the Extracurricular School of Mathematics in Valmiera district. They mostly responded about the number of colored unit squares, explored the properties of symmetry, and tried to detect different polyominoes. No one of them detected the change of the configuration. They can't immediately find how from the first square one can get the next squares. The observation of the part that does not changes gives the correct conclusion about the recoloring of the row or of the column. Such problem is good to introduce students with additional inquiry based tasks, for example, "Try to recolor the region of different form in the square"; "Detect all the groups of permutations"; "Investigate whether the properties of the coloring depend on the size of the square or on the type of the recolored region".

Second problem is useful for research what happens if the problem's conditions are changed?

Problem 2. The square 4 x 4 contains all the pluses except the corner square's neighbour containing a minus. It is allowed to reverse all signs in one whole row or column. Can we get only all pluses in the square?

The solution of the problem can be find by simple observation of the square 2 x 2 containing the minus sign. Adding the condition that it is allowed to change the signs in diagonals too, the solution becomes more general by the implementation of the method of invariants.

To raise students' interest in problem solving it is necessary to pose the problems with exciting content. Here is an example where the research on simpler special case can give the relevant information. The following problem can be solved using modular arithmetic by modulus 4 and the creation and solution of the system of appropriate equations.

Problem 3. Robber wants to crack the code on the safe to open it. The safe will open when all arrows will point in the same direction (see Figure 2). All arrows in the row (column) turn by 90 degrees in the same direction according to the wish of the user. How the robber can open the safe?

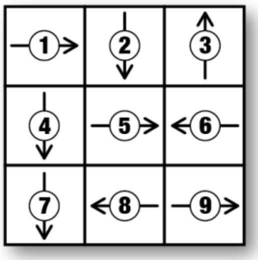

Figure 2. Problem3. The arrows on the safe

Conclusions. The solution of the problems of changeable configurations teach the students to work regularly and systematically, to change the viewpoint, to imagine, to try extraordinary ideas. The participation at Mathematical circles fortify students' mathematical proficiency, develop students' mathematical intelligence, foster students' creativity, and help them to prepare for Mathematics Olympiads.

References

Koichu, B., Andzans, A. (2009) *Mathematical creativity and giftedness in out-of-school activities*. In Leikin, R., Berman, A., Koichu, B. (Ed). Creativity in mathematics and the education of gifted students. Sense Publishers Pp 285-308

OECD (2013), *PISA 2012 Results: What Makes Schools Successful? Resources, Policies and Practices (Volume IV),* PISA , OECD Publishing. Pp. 112 – 116 Retrieved from *http://dx.doi.org/10.1787/9789264201156-en*

Prescot, A., Pressick-Kilborn, K. (2015) It's great to be doing maths! Engaging primary students in a lunchtime club. *Australian Primary Mathematics Classroom,* vol. 20 (3), pp. 34 – 39. Retrieved from https://opus.lib.uts.edu.au/handle/10453/35462

Rott, B. (2013) Mathematisches Problemlosen. Ergebnisse Einer Empirischen Studie. WTM, Munster, pp. 123 – 128

Thompson, J. H. (2009) Enhancing Problem Solving Through Math Clubs. *Action Research Projects.* Paper 24. University of Nebraska – Lincoln. Retrieved from http://digitalcommons.unl.edu/mathmidactionresearch/24