SFB 823

The speed of transition revisited

Eric Naevdal, Martin Wagner

Nr. 10/2017

Discussion Paper

The Speed of Transition Revisited

Eric Nævdal
Frisch Centre
Oslo, Norway

Martin Wagner*
Faculty of Statistics
Technical University Dortmund
Dortmund, Germany
&
Institute for Advanced Studies
Vienna, Austria
&
Bank of Slovenia
Ljubljana, Slovenia

Abstract

The speed of transition literature appears to have overlooked the fact that due to the dynamic nature of the economy the post–transition economic performance influences optimal behavior already during transition. We illustrate the implications of this neglect using the well–known model of Aghion and Blanchard (1994, Section 6.4). The correct solution differs in several respects from the "approximate" solution presented by Aghion and Blanchard. First, unemployment is increasing up to a certain endogenous point in time, when, second, the remaining state sector is closed down. This point in time can be defined as the end of transition. The correct solution is based on transforming the problem to a type of a dynamic optimization problem often encountered in resource economics: a scrap value problem with free terminal time.

JEL Classification: C61, E61, P20

Keywords: Transition, Optimal Unemployment Rate, Dynamic Optimization

*Corresponding author: Faculty of Statistics, Technical University Dortmund, Vogelpothsweg 87, D-44227 Dortmund, Germany; Tel.: ++49+231 755 3174, Email: mwagner@statistik.tu-dortmund.de

1 Introduction

The transition from a state controlled economy to a market economy has received considerable attention over the last 25 years. Many formerly centrally planned economies have gone a long way or have even completed their transition to market based economies. However, some other centrally planned countries as well as a considerable number of semi-developed countries are just starting or will in all likelihood embark on this process in the next few years. In addition many countries, such as large oil producers, are still burdened with large inefficient state sectors and/or state owned companies operating under soft budget constraints that will at some point have to have to address efficiency enhancing reforms to avoid detrimental economic development. Also, in a different context some authors, e.g. Brynjolfsson and McAfee (2011), discuss that even well-developed market economies are facing potentially dramatic transition processes due to the technological revolution ongoing under labels such as “digitalization”, “industry 4.0” or “internet of things”. It is indeed possible that these developments will necessitate huge reallocation of workforce across sectors. This process is in many ways formally similar to the type of transition problem referred to here. Massive technological change may lead to part of the economy finding itself with too many workers with a resultant need for large-scale economic reorganization.

One important aspect, turning back to transition from a centrally planning to a market based economy, of such a process is the macroeconomic management of the transition through the transfer of resources, such as capital and labor, from the state controlled sector to the emerging private sector. In this paper we argue that an important aspect of this macroeconomic management problem has been overlooked. This is the issue of when and how the transition period should end. At some point in time the transfer of resources is completed and we will show that this leads to the introduction of a constraint on the optimal policy that has considerable impact on the transition process already before the end of the transition period.

This point is illustrated using the model Aghion and Blanchard (1994), which is an early, important contribution with a solid microeconomic fundament to the literature on economic transition from centrally planned to market economies. In their Section 6.4 they present a dynamic optimization model to determine the optimal speed of transition and the resultant optimal unemployment rate (see also the description in Roland, 2000).[1] When solving the

[1]For detailed discussion concerning labor market dynamics in transition see Boeri (2000). Our contribution here is of a methodological nature and thus several aspects of labor market dynamics that are found to be

dynamic optimization problem, Aghion and Blanchard do not, in fact, derive the exact solution but only an "approximate" solution, which neglects the behavior of the economy after the state sector has been closed down, see in particular their footnote 33 on p. 305. Due to the dynamic nature of the economy the post–transition economic performance influences the optimal behavior during transition and thus influences the optimal path also whilst the state sector still employs people. The behavior of the economy at the state sector closure is neglected also in other speed of transition models. Brixiova and Yousef (2000) assume a constant closure rate of the state sector, which may also lead to welfare losses and different dynamic behavior compared to optimal closure. Burda (1993) also finds a constant optimal unemployment rate, where again the effect of state sector closure is not analyzed in detail. Castanheira and Roland (2000) avoid the problem by assuming that there is no unemployment and that capital can be be moved freely from the old to the new sector.

Focusing as mentioned only on the Aghion and Blanchard model we derive in this paper the correct solution to the dynamic optimization problem and show that by including end–of–transition effects the resultant optimal path has several interesting features. It turns out that a proper analysis of the model gives rise to richer dynamics than might be expected when resorting to what Aghion and Blanchard label "turnpike" approximation. In particular we show that (correct) optimal paths have the following properties: Up to a certain point in time, say τ, the government assumes an active role on the labor market by shrinking the inefficient state sector. This is done at an increasing rate, hence the optimal unemployment rate is not constant. At time τ the government closes down the (remaining) inefficient state sector and does not intervene in the labor market any further. Hence, at time τ the unemployment rate jumps and from there on gradually moves towards zero. Thus, in this model the end of transition occurs at time τ, where the remaining state sector is closed down in a discontinuous fashion. It holds that τ is endogenous and has to be chosen optimally by the government. Furthermore, it holds that the correct optimal unemployment rate is larger than the rate proposed by Aghion and Blanchard over the transition period. Thus, the path obtained by Aghion and Blanchard leads to welfare losses. The reason for the lower unemployment rate over a longer period leading to welfare losses is that the lower unemployment rate leads to

relevant are, as in the model of Aghion and Blanchard (1994), neglected. Also, of course, this type of model has to be interpreted in a stylized fashion with respect to the role of the state in an economy. All aspects of state activity present also in market based economies are not in the focus of the paper and simply neglected to focus on the core issue. For the same reason we also abstract from reform uncertainty and potential reform reversal, issues discussed in Dewatripont and Roland (1995) or Fernandez and Rodrik (1991).

slower job growth in the more efficient private sector and thus unnecessarily delays output growth compared to the optimal faster transition path.

The results presented here may perhaps be best understood by noticing that the problem is formally similar to the problem of extracting an exhaustible resource. The stock of individuals employed in the state sector is the resource that can be "mined". Discounting and the fact that the resource at some point is exhausted gives rise to extraction paths that are non-constant. The difference to the resource problem lies in that the process of mining a resource yields profits that represent instantaneous benefits, whereas in the present model, mining (i.e. unemployment) is costly. This explains why models of exhaustible resources predict that resources should be mined at a decreasing rate whereas the present model prescribes that unemployment should increase over the interval $[0, \tau)$.

The paper is organized as follows: In Section 2 we set up and analyze the Aghion and Blanchard (1994) model in detail and Section 3 briefly concludes.

2 The Model and the Correct Solution

We restrict the description of the model on the dynamic optimization problem presented in Section 6.4 of Aghion and Blanchard (1994) and discuss only those parts of the analysis presented in their paper in detail that are immediately relevant here.

Denote with $E(t)$ the number of people employed in the state sector (with constant marginal productivity x), with $N(t)$ the number of people employed in the emerging private sector (with constant marginal productivity y) and with $U(t)$ the number of unemployed people at time t. Population is normalized to one, i.e. $E(t)+N(t)+U(t) = 1$. At the outset of transition, employment in the state sector drops from 1 to $E(0) < 1$. Aghion and Blanchard (1994) develop an efficiency-wage based explanation for costly labor adjustment between the old state and the new private sector. In particular, they derive the following relationship for the speed of job creation in the new private sector (developed in their equation (9) on page 298):[2]

$$\dot{N} = f(U) = a\left[\frac{U}{U+ca}\right]\left[y - rc - \left(\frac{b}{1-U}\right)\right] \tag{1}$$

with a, b, c and r positive constants. a is a parameter indicating how much the speed in private job creation is affected by profits per worker in the private sector, b is unemployment benefits,

[2] To avoid overloaded notation we sometimes skip the time index t.

r is the discount rate and c is a constant indicating how much the equilibrium wage responds to improvements in employment prospects for the unemployed. The cost of job creation in the private sector is given by $\frac{1}{2ar}(f(U))^2$. The state sector declines over time and the government chooses the speed of closure and hence of unemployment.

The government is only concerned with efficiency and chooses employment in the state sector[3] to maximize the present discounted value of output. This optimization problem is given by:

$$\max_{E(t)} \int_0^\infty \left[E(t)x + N(t)y - \frac{1}{2ar} f\left(U(t)\right)^2 \right] e^{-rt} dt \tag{2}$$

subject to:

$$\dot{N}(t) = f(U(t)) \tag{3}$$

$$N(0) = 0 \tag{4}$$

$$E(t) + N(t) + U(t) = 1 \tag{5}$$

and non-negativity of $E(t)$, $N(t)$ and $U(t)$.

Based on the relation that $E(t) + N(t) + U(t) = 1$, one immediately observes that the problem can equivalently be formulated by using $U(t)$ as the control and by eliminating $E(t)$, which leaves us with only $U(t)$ and $N(t)$ in both the objective function and the constraints.[4] This formulation of the problem is given by:

$$\max_{U \in [0,1]} \int_0^\infty \left[(1 - N(t) - U(t))x + N(t)y - \frac{1}{2ar} f\left(U(t)\right)^2 \right] e^{-rt} dt \tag{6}$$

subject to:

$$\dot{N}(t) = f(U(t)) \tag{7}$$

$$N(t) \in [0, 1] \tag{8}$$

$$N(t) + U(t) \leq 1 \tag{9}$$

[3] See below that this is equivalent to choosing unemployment.

[4] We perform this substitution to have U, postulated to be constant along optimal paths by Aghion and Blanchard (1994), as the control variable.

Note first that an optimal path may have one of the following properties: There exists a $\tau < \infty$ such that $\tau = \inf_{t \geq 0}(N(t) + U(t) = 1)$ *or* condition (9) is not binding for any finite t. These two cases will be discussed separately below. Before doing so, an important property of the model is derived in Proposition 1.

Proposition 1 *Along any path it holds that $N(t) < 1$ for all $t < \infty$.*

Proof: For values of $N(t)$ sufficiently close to 1, the largest possible value of $\dot{N}(t)$ is given by setting $U(t) = 1 - N(t)$. The ordinary differential equation $\dot{N}(t) = f(1 - N(t))$ has a stable steady state at $N = 1$, hence $N(t)$ approaches 1 only asymptotically. $\square$

An additional problem with the model is that the Hamiltonian may have two local maxima with respect to U. However, this problem can easily be dispensed with: If $\widehat{U}$ is the larger of these two maxima, then it is easy to show that there is some value $\widetilde{U} < \widehat{U}$ such that $f(\widetilde{U}) = f(\widehat{U})$. If this is the case, then $\widetilde{U}$ leads to the same rate of job creation at a lesser cost, so $\widehat{U}$ cannot be optimal. Hence, we can disregard the possibility of two local maxima of the Hamiltonian in the sequel.

Let us now turn to study the possible optimal paths in detail. We start with the case that the constraint (9) becomes binding for the first time at some $\tau < \infty$. Given that state sector employment is monotonically non-increasing, it follows that for $t \geq \tau$ the control problem has a trivial optimal solution. Denote with $N(t, N_\tau)$ the solution to the differential equation $\dot{N}(t) = f(1 - N(t))$ solved over (τ, ∞) with initial condition $N(\tau) = N_\tau$. Note that it trivially holds that $\frac{\partial N(\tau, N_\tau)}{\partial N_\tau} = 1$ and also note that up to now both τ and N_τ are unspecified.

The objective function of the optimization problem from τ onwards is given by:

$$V(\tau, N_\tau) = \int_\tau^\infty \left[N(t, N_\tau) y - \frac{1}{2ar} f(1 - N(t, N_\tau))^2 \right] e^{-rt} dt \tag{10}$$

Note the following relationships for the partial derivatives of the objective function (10):

$$\frac{\partial V(\tau, N_\tau)}{\partial \tau} = -\left[N(t, N_\tau) y - \frac{1}{2ar} f(1 - N(t, N_\tau))^2 \right] e^{-r\tau} \tag{11}$$

$$\begin{aligned} \frac{\partial V(\tau, N_\tau)}{\partial N_\tau} &= \int_\tau^\infty \left[y + \frac{1}{ar} f(1 - N(t, N_\tau)) f'(1 - N(t, N_\tau)) \right] e^{-rt} dt \\ &= \frac{y}{r} e^{-r\tau} + \int_\tau^\infty \left[\frac{1}{ar} f(1 - N(t, N_\tau)) f'(1 - N(t, N_\tau)) \right] e^{-rt} dt \end{aligned} \tag{12}$$

Now the optimization problem corresponding to the case considered can be rewritten as a scrap value problem with free terminal time, i.e. τ is to be chosen optimally as well:

$$\max_{U\in[0,1],\tau\in[0,\infty)}\left[\int_0^\tau\left[(1-N-U)x+Ny-\frac{1}{2ar}f\left(U\right)^2\right]e^{-rt}dt+V\left(\tau,N\left(\tau\right)\right)\right] \tag{13}$$

subject to (7), (8) and (9).

Problems of this type are studied in Seierstad and Sydsæter (1987, Theorem 3 and Note 2, p. 182–184), where necessary conditions for optimality are presented. The Hamiltonian corresponding to this problem is given by $H=(1-N-U)x+Ny-\frac{1}{2ar}f(U)^2+\mu f(U)$, where we ignore, for brevity, the other constraints (8) and (9) and the associated multipliers. It is straightforward but cumbersome to present the solution including these additional terms in the Hamiltonian. It can be shown that these constraints will not be binding, except possibly at $t=0$ and $t=\infty$.[5]

Necessary conditions for optimality are given by:

$$-x-\frac{1}{ar}f\left(U\right)f'\left(U\right)+\mu f'\left(U\right)=0 \tag{14}$$

$$\dot{\mu}=r\mu+x-y \tag{15}$$

Furthermore, the following transversality condition has to hold:

$$\mu\left(\tau\right)e^{-r\tau}=\frac{\partial V\left(\tau,N_\tau\right)}{\partial N_\tau} \tag{16}$$

The optimal terminal time τ is found from:

$$He^{-r\tau}+\frac{\partial V\left(\tau,N_\tau\right)}{\partial\tau}=0 \tag{17}$$

Equation (15) gives the following solution for $\mu(t)$:

$$\mu(t)=\frac{y-x}{r}+Ke^{rt} \tag{18}$$

Here K is a constant whose value has to be determined from the transversality condition (16).

Remark 1 *The solution proposed by Aghion and Blanchard (1994) is derived from the above differential equation* (18) *by setting $K=0$. This implies a constant value of the costate variable $\mu(t)\equiv\frac{y-x}{r}$ and thus a constant unemployment rate. Inserting $\mu=\frac{y-x}{r}$ in equation* (14)

[5] In fact, it can be shown that the only possible case where any other constraint than $U(t)+N(t)\leq 1$ is binding for $t<\infty$ is the case where $U\left(0\right)=1$, in which case $\tau=0$.

leads to the solution proposed by Aghion and Blanchard (1994), see their equation (26) on p. 309.

As noted in Aghion and Blanchard (1994) and also mentioned in the introduction, this cannot be the correct solution for all values of t, since due to private sector job creation (which happens at a constant rate for constant unemployment) at some point the unemployment rate has to decline. We show below, however, that even before the end of transition, the optimal unemployment rate is not constant.

Let us next determine K, or to be more precise, let us determine whether it is equal to zero or not for optimal paths. This can be achieved by inserting (18) and (12) into the transversality condition (16). After some rearrangements this yields:

$$Ke^{r\tau} = \frac{x}{r} + \int_{\tau}^{\infty} \left[\frac{1}{ar} f\left(1 - N\left(t, N_{\tau}\right)\right) f'\left(1 - N\left(t, N_{\tau}\right)\right) \right] e^{-r(t-\tau)} dt \tag{19}$$

In order to sign K, we need to sign the last term in the square brackets in this equation. The following proposition is helpful.

Proposition 2 *Along an optimal path, $f'\left(U\left(t\right)\right) > 0$ for all t.*

Proof: First, note that for any choice of τ and N_{τ} there is a segment $[\tau + d, \infty)$ such that $f'\left(1 - N\left(t, N_{\tau}\right)\right) = f'\left(U\left(t\right)\right) > 0$ for all $t \in [\tau + d, \infty)$. This is a straightforward implication of $U\left(t\right)$ becoming small as $N\left(t\right)$ goes to 1. In particular, this implies that paths where $f'\left(U\left(t\right)\right) > 0$ for all t are always feasible if $U\left(t\right)$ is chosen to be small enough. Second, note that for every $\widehat{U}$ such that $f'(\widehat{U}) < 0$, there is a value $\widetilde{U} < \widehat{U}$ such that $f(\widetilde{U}) = f(\widehat{U})$ and $f'(\widetilde{U}) > 0$. Since $\widetilde{U}$ and $\widehat{U}$ give the same rate of job creation, but higher values of U are more costly, it follows that for the optimal choice of U it will always hold that $f'\left(U\left(t\right)\right) > 0$. Taken together these two facts imply that it is always possible to choose paths such that $f'\left(U\left(t\right)\right) > 0$ for all t and it is not optimal to choose any other paths. Thus, the proposition follows. $\square$

Proposition 2 implies that the second term on the right hand side of (19), $f'\left(1 - N\left(t, N_{\tau}\right)\right)$, is positive and hence, the right hand side is positive. Consequently, it follows that K is positive. This implies that $\mu(t)$ is not constant over time and thus the optimal unemployment rate is also not constant over time. In fact it follows that the optimal unemployment rate

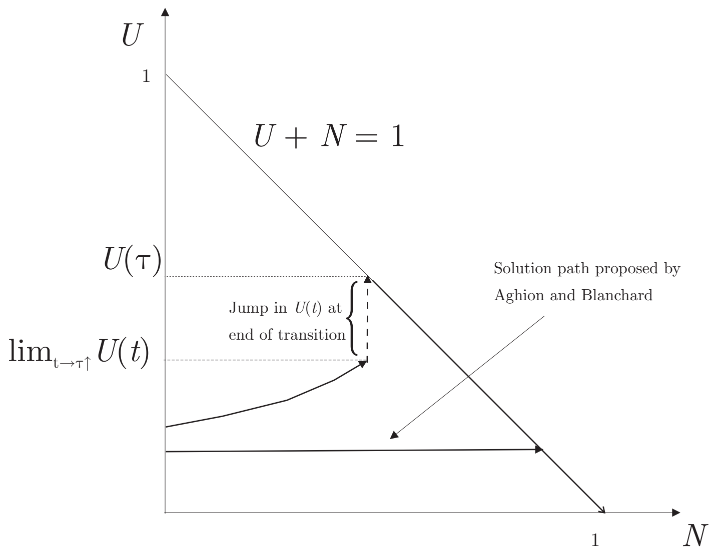

Figure 1: The transition process in (N, U)-space. The optimal unemployment rate increases continuously until time τ, where the remaining state sector is closed and the unemployment rate jumps to $1 - N(\tau)$ to gradually decline to zero afterwards. The figure also shows the lower constant unemployment rate corresponding to the solution of Aghion and Blanchard.

is increasing over time until τ, which is to be determined from equation (17). The fact that $K > 0$ implies that $\mu(t)$ is larger than $\frac{y-x}{r}$ for all $t < \tau$. This implies that $U(t)$ corresponding to the optimal solution is larger than derived in Aghion and Blanchard. Consequently it follows that the transition period is shorter than suggested by Aghion and Blanchard.

There is another interesting feature: The optimal unemployment rate is discontinuous at time τ and hence the optimal path for the unemployment rate is as illustrated in Figure 1, with the result derived analytically below.

Proposition 3 *For an optimal path of the unemployment rate it holds that* $\lim\limits_{t\to\tau^-} U(t) < 1 - N(t)$*. This implies that* $U(t)$ *is discontinuous at* τ*.*

Proof: The proof is by contradiction, therefore assume that $\lim\limits_{t\to\tau^-} U(t) = 1 - N(t)$. Then equation (17) implies that $\mu(\tau) f(1 - N(\tau)) = 0$. This in turn implies, since $N(\tau) < 1$ (which follows from Proposition 1), that $\mu(\tau) = 0$. Then equation (18) implies that $K < 0$, since $y > x$ by assumption. However, $K < 0$ is in contradiction with (19). This shows the proposition. □

To complete our analysis it remains to be shown that the second case, where condition (9) does not become binding for any finite t, cannot lead to optimal paths. Note first that, also in this case, $K \neq 0$, because $K = 0$ implies a constant unemployment rate (compare Remark 1). This follows from inserting (18) in (14), both of which now have to hold for all $t \geq 0$ for optimal paths. Since a constant unemployment rate implies a constant job creation rate, eventually the unemployment rate has to decrease because of constant population size. Thus, $K \neq 0$. This implies that $\mu(t)$ diverges to either plus or minus infinity, depending upon the sign of K. However, such a path of $\mu(t)$ cannot fulfill the necessary condition (14) for all $t \geq 0$, since both $f(U)$ and $f'(U)$ are bounded. This shows that indeed such paths cannot be optimal.

3 Conclusions

Using the model for the optimal speed of transition of Aghion and Blanchard (1994, Section 6.4) we discuss in this note some properties of optimal solutions of such speed of transition models. The analysis has to take into account the behavior of the economy after the closure of the state sector, which has repercussions on the optimal behavior during transition. Such problems can be formulated as scrap value problems with free terminal time, which typically arise in exhaustible resource extraction problems.

The correct optimal unemployment paths derived in this paper differ in two respects from the "approximate" solution presented in Aghion and Blanchard (1994). First, the optimal unemployment rate is increasing over time until, second, the state sector is shut down entirely at a certain point in time. This leads to a discontinuity in the unemployment rate at this point in time. The point in time where the government closes the inefficient remaining state sector entirely defines (in this stylized model) the end of transition. Afterwards the government does not assume an active role in the labor market. Note again, as discussed in the introduction, that the non-constancy of the optimal unemployment rate is a similarity to the solutions typically found for exhaustible resource extraction problems.

The analysis put forward in this paper for the Aghion and Blanchard (1994) model appears by analogy useful also for other transition models. On a more general scale we speculate, based on the results in this paper, that it might be fruitful for the transition literature to borrow further insights from resource economics.

Acknowledgements

This work originated whilst Eric Nævdal was a post doctoral fellow at Princeton University and affiliated with the Norwegian University of Life Sciences and Martin Wagner was a visiting scholar at Princeton University. The hospitality of Princeton University is gratefully acknowledged. Martin Wagner gratefully acknowledges financial support from Deutsche Forschungsgemeinschaft via the Collaborative Research Center 823: *Statistical Modelling of Nonlinear Dynamic Processes* and from the Jubiläumsfonds of the Oesterreichische Nationalbank via several grants. The usual disclaimer applies.

References

Aghion, P., Blanchard, O.J., 1994. On the speed of transition in central europe. NBER Macroeconomics Annual, 283–319.

Boeri, T., 2000. *Structural Change, Welfare Systems, and Labour Reallocation: Lessons from the Transition of Formerly Planned Economies.* Oxford: Oxford University Press.

Brixiova, Z., Yousef, T., 2000. Labor market adjustment in transition economies with on-the-job search. *Economics Letters* **67**, 223–229.

Brynjolfsson, E., McAfee, A., 2011. *Race Against The Machine: How The Digital Revolution is Accelerating Innovation, Driving Productivity, and Irreversibly Transforming Employment and the Economy.* Lexington, MA: Digital Frontier Press.

Burda, M.C., 1993. Unemployment, labor market institutions and structural change. *Economic Policy* **8**, 102–137.

Castanheira, M., Roland, G., 2000. The optimal speed of transition: A general equilibrium analysis. *International Economic Review* **41**, 219–239.

Dewatripont, M., Roland, G., 1995. The design of reform packages under uncertainty. *American Economic Review* **85**, 1207–1223.

Fernandez, R., Rodrik, D., 1991. Resistance to reform: Status-quo bias in the presence of individual-specific uncertainty. *American Economic Review* **81**, 1146–1155.

Roland, G., 2000. *Transition and Economics: Politics, Markets and Firms*. Cambridge, MA: MIT Press.

Seierstad, A., Sydsæter, K., 1987. *Optimal Control Theory with Economic Applications*. Amsterdam: North-Holland.