an interval I, yielding $\{\mathcal{N}(s, \sigma^2) \mid \sigma \in I\}$ as a set of distributions effective for Angel in that situation.

But we cannot do with just arbitrary subsets of the set of all subprobabilities on state space S. We want also to characterize possible outcomes, i.e., sets of distributions over the state space for composite games. This means that we will want to average over intermediate states, which in turn requires measurability of the functions involved. Hence we require measurable sets of subprobabilities as possible outcomes. We also impose a condition on measurability on the interplay between distributions on states and reals for measuring the probabilities of sets of states. This leads to the definition of a stochastic effectivity function.

Modeling all this requires some preparations by fixing the range of a stochastic effectivity function through a suitable functor. Put for a measurable space S

$$\mathcal{V}(S) := \{V \subseteq \mathcal{B}(\$(S)) \mid V \text{ is upper closed}\} \tag{10}$$

thus if $V \in \mathcal{V}(S)$, then $A \in V$ and $A \subseteq B$ together imply $B \in V$. A measurable map $f : S \to T$ induces a map $\mathcal{V}(f) : \mathcal{V}(S) \to \mathcal{V}(T)$ upon setting

$$\mathcal{V}(f)(V) := \{W \in \mathcal{B}(\$(T)) \mid \$(f)^{-1}[W] \in V\} \tag{11}$$

for $V \in \mathcal{V}(S)$, then clearly $\mathcal{V}(f)(V) \in \mathcal{V}(T)$.

Note that $\mathcal{V}(S)$ has not been equipped with a σ-algebra, so the usual notion of measurability between measurable spaces cannot be applied. In particular, $\mathcal{V}$ is not an endofunctor on the category of measurable spaces. We will not discuss functorial aspects of $\mathcal{V}$ here in detail, referring the reader to [8] instead.

It would be most convenient if we could work in a monad — after all, the semantics pertaining to composition of games is modelled appropriately using a composition operator, as demonstrated, e.g., in [33]. Markov transition systems are based on the Kleisli morphisms for the Giry monad, and the functor assigning each set upper closed subsets of the power set form a monad as well [9, Example 2.4.10]. So one might want to capitalize on the composition of these monads. Alas, it is well known that the composition of two monads is not necessarily a monad, and this particular composition has defied so far all attempts at establishing the properties of a monad. Consequently, this approach does not work, and one has to resort to ad-hoc methods simulating the properties of a monad (or of a Kleisli tripel) [41]. This is what we will do in the sequel.

Preparing for this, we require some properties pertaining to measurability, when dealing with the composition of distributions when discussing composite games. This will be provided in the following way. Let $H \in \mathcal{B}(\$(S) \otimes [0, 1])$ be a measurable subset of $\$(S) \times [0, 1]$ indicating a quantitative assessment of subprobabilities (a typical example could be

$$\{\langle \mu, q \rangle \mid \mu \in \boldsymbol{\beta}(A, > q), q \in \mathbb{Q} \cap [0, 1]\}$$

for some $A \in \mathcal{B}(S)$; the set $\boldsymbol{\beta}(A, > q)$ is defined in (5)). Fix some real q and consider the set

$$H_q := \{\mu \mid \langle \mu, q \rangle \in H\}$$

of all measures evaluated through q. We ask for all states s such that this set is effective for s. They should come from a measurable subset of S. It turns out that this is not enough, we